1 Required Exercise 1

1. Augmented grammar:

$$
\begin{aligned}
&S' \to S \\
&S \to SA \,|\, SB \,|\, a \\
&A \to S+ \\
&B \to S-
\end{aligned}
$$

Canonical LR(0) collection:

(a) Initially, $I_0 = \text{CLOSURE}(\{[S' \to \cdot S]\}) = \{[S' \to \cdot S], [S \to \cdot SA], [S \to \cdot SB], [S \to \cdot a]\}$.

- $\text{GOTO}(I_0, S) = \text{CLOSURE}(\{[S' \to S\cdot], [S \to S \cdot A], [S \to S \cdot B]\}) = \{[S' \to S\cdot], [S \to S \cdot A], [S \to S \cdot B], [A \to \cdot S+], [B \to \cdot S-], [S \to \cdot SA], [S \to \cdot SB], [S \to \cdot a]\} = I_1$.
- $\text{GOTO}(I_0, a) = \text{CLOSURE}(\{[S \to a\cdot]\}) = \{[S \to a\cdot]\} = I_2$
- All other transfers lead to errors.

(b) Consider I_1.

- $\text{GOTO}(I_1, \$) = \text{Accept}$.
- $\text{GOTO}(I_1, S) = \text{CLOSURE}(\{[A \to S \cdot +], [B \to S \cdot -], [S \to S \cdot A], [S \to S \cdot B]\}) = \{[A \to S \cdot +], [B \to S \cdot -], [S \to S \cdot A], [S \to S \cdot B], [A \to \cdot S+], [B \to \cdot S-], [S \to \cdot SA], [S \to \cdot SB], [S \to \cdot a]\} = I_3$.
- $\text{GOTO}(I_1, A) = \text{CLOSURE}(\{[S \to SA\cdot]\}) = \{[S \to SA\cdot]\} = I_4$.
- $\text{GOTO}(I_1, B) = \text{CLOSURE}(\{[S \to SB\cdot]\}) = \{[S \to SB\cdot]\} = I_5$.
- $\text{GOTO}(I_1, a) = \text{CLOSURE}(\{[S \to a\cdot]\}) = \{[S \to a\cdot]\} = I_2$.
- All other transfers lead to errors.

(c) Consider I_2: All transfers lead to errors.

(d) Consider I_3.

- $\text{GOTO}(I_3, S) = \text{CLOSURE}(\{[A \to S \cdot +], [B \to S \cdot -], [S \to S \cdot A], [S \to S \cdot B]\}) = \{[A \to S \cdot +], [B \to S \cdot -], [S \to S \cdot A], [S \to S \cdot B], [A \to \cdot S+], [B \to \cdot S-], [S \to \cdot SA], [S \to \cdot SB], [S \to \cdot a]\} = I_3$.
- $\text{GOTO}(I_3, A) = \text{CLOSURE}(\{[S \to SA\cdot]\}) = \{[S \to SA\cdot]\} = I_4$.
- $\text{GOTO}(I_3, B) = \text{CLOSURE}(\{[S \to SB\cdot]\}) = \{[S \to SB\cdot]\} = I_5$.
- $\text{GOTO}(I_3, a) = \text{CLOSURE}(\{[S \to a\cdot]\}) = \{[S \to a\cdot]\} = I_2$.
- $\text{GOTO}(I_3, +) = \text{CLOSURE}(\{[A \to S + \cdot]\}) = \{[A \to S + \cdot]\} = I_6$.
- $\text{GOTO}(I_3, -) = \text{CLOSURE}(\{[B \to S - \cdot]\}) = \{[B \to S - \cdot]\} = I_7$.
- All other transfers lead to errors.

(e) Consider I_4: All transfers lead to errors.

(f) Consider I_5: All transfers lead to errors.

(g) Consider I_6: All transfers lead to errors.

(h) Consider I_7: All transfers lead to errors.

The final canonical LR(0) collection is $\{I_0, I_1, I_2, I_3, I_4, I_5, I_6, I_7\}$. Also, we have FIRST and FOLLOW sets:

$$\text{FIRST}(S') = \text{FIRST}(S) = \text{FIRST}(A) = \text{FIRST}(B) = \{a\}$$

$$\text{FOLLOW}(S') = \{\$\}$$

$$\text{FOLLOW}(S) = \text{FOLLOW}(A) = \text{FOLLOW}(B) = \{\$, a, +, -\}$$

Then we calculate the ACTION.

(a) Consider I_0.

- Since $[S \to \cdot a] \in I_0$ and $\text{GOTO}(I_0, a) = I_2$, ACTION$[0, a]$ is "shift 2".

- Since $\text{GOTO}(I_0, S) = I_1$, $\text{GOTO}(0, S) = 1$.

(b) Consider I_1.

- Since $[S \to \cdot a] \in I_1$ and $\text{GOTO}(I_1, a) = I_2$, ACTION$[1, a]$ is "shift 2".
- Since $[S' \to S\cdot] \in I_1$, ACTION$[1, \$]$ is "accept".
- Since $\text{GOTO}(I_1, S) = I_3$, $\text{GOTO}(1, S) = 3$.
- Since $\text{GOTO}(I_1, A) = I_4$, $\text{GOTO}(1, A) = 4$.
- Since $\text{GOTO}(I_1, B) = I_5$, $\text{GOTO}(1, B) = 5$.

(c) Consider I_2.

- Since $[S \to a\cdot] \in I_2$ and $\text{FOLLOW}(S) = \{\$, a, +, -\}$, ACTION$[2, \$]$, ACTION$[2, a]$, ACTION$[2, +]$ and ACTION$[2, -]$ are "reduce $S \to a$".

(d) Consider I_3.

- Since $[S \to \cdot a] \in I_3$ and $\text{GOTO}(I_3, a) = I_2$, ACTION$[3, a]$ is "shift 2".
- Since $[A \to S \cdot +] \in I_3$ and $\text{GOTO}(I_3, +) = I_6$, ACTION$[3, +]$ is "shift 6".
- Since $[B \to S \cdot -] \in I_3$ and $\text{GOTO}(I_3, -) = I_7$, ACTION$[3, -]$ is "shift 7".
- Since $\text{GOTO}(I_3, S) = I_3$, $\text{GOTO}(3, S) = 3$.
- Since $\text{GOTO}(I_3, A) = I_4$, $\text{GOTO}(3, A) = 4$.
- Since $\text{GOTO}(I_3, B) = I_5$, $\text{GOTO}(3, B) = 5$.

(e) Consider I_4.

- Since $[S \to SA\cdot] \in I_4$ and $\text{FOLLOW}(S) = \{\$, a, +, -\}$, ACTION$[4, \$]$, ACTION$[4, a]$, ACTION$[4, +]$ and ACTION$[4, -]$ are "reduce $S \to SA$".

(f) Consider I_5.

- Since $[S \to SB\cdot] \in I_5$ and $\text{FOLLOW}(S) = \{\$, a, +, -\}$, ACTION$[5, \$]$, ACTION$[5, a]$, ACTION$[5, +]$ and ACTION$[5, -]$ are "reduce $S \to SB$".

(g) Consider I_6.

- Since $[A \to S + \cdot] \in I_6$ and $\text{FOLLOW}(A) = \{\$, a, +, -\}$, ACTION$[6, \$]$, ACTION$[6, a]$, ACTION$[6, +]$ and ACTION$[6, -]$ are "reduce $A \to S+$".

(h) Consider I_7.

- Since $[B \to S - \cdot] \in I_7$ and $\text{FOLLOW}(B) = \{\$, a, +, -\}$, ACTION$[7, \$]$, ACTION$[7, a]$, ACTION$[7, +]$ and ACTION$[7, -]$ are "reduce $B \to S-$".

Thus, the parsing table is

STATE	ACTION				GOTO		
	a	$+$	$-$	\$	S	A	B
0	s2				1		
1	s2			acc	3	4	5
2	r3	r3	r3	r3			
3	s2	s6	s7		3	4	5
4	r1	r1	r1	r1			
5	r2	r2	r2	r2			
6	r4	r4	r4	r4			
7	r5	r5	r5	r5			

Here the production numbers are

$$S \to SA \tag{1}$$
$$S \to SB \tag{2}$$
$$S \to a \tag{3}$$
$$A \to S+ \tag{4}$$
$$B \to S- \tag{5}$$

2. Yes. There is no conflict.

3. Yes.

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	\$	$aaaa+++\$$	shift to 2
(2)	0 2	$\$a$	$aaa+++\$$	reduce by $S \to a$
(3)	0 1	$\$S$	$aaa+++\$$	shift to 2
(4)	0 1 2	$\$Sa$	$aa+++\$$	reduce by $S \to a$
(5)	0 1 3	$\$SS$	$aa+++\$$	shift to 2
(6)	0 1 3 2	$\$SSa$	$a+++\$$	reduce by $S \to a$
(7)	0 1 3 3	$\$SSS$	$a+++\$$	shift to 2
(8)	0 1 3 3 2	$\$SSSa$	$+++\$$	reduce by $S \to a$
(9)	0 1 3 3 3	$\$SSSS$	$+++\$$	shift to 6
(10)	0 1 3 3 3 6	$\$SSSS+$	$++\$$	reduce by $A \to S+$
(11)	0 1 3 3 4	$\$SSSA$	$++\$$	reduce by $S \to SA$
(12)	0 1 3 3	$\$SSS$	$++\$$	shift to 6
(13)	0 1 3 3 6	$\$SSS+$	$+\$$	reduce by $A \to S+$
(14)	0 1 3 4	$\$SSA$	$+\$$	reduce by $S \to SA$
(15)	0 1 3	$\$SS$	$+\$$	shift to 6
(16)	0 1 3 6	$\$SS+$	\$	reduce by $A \to S+$
(17)	0 1 4	$\$SA$	\$	reduce by $S \to SA$
(18)	0 1	$\$S$	\$	accept

2 Required Exercise 2

1. The augmented grammar and FIRST and FOLLOW sets are calculated in 1.

   Canonical LR(1) collection:

   (a) Initially, $I_0 = \text{CLOSURE}(\{[S' \to \cdot S, \$]\}) = \{[S' \to \cdot S, \$], [S \to \cdot SA, \$], [S \to \cdot SB, \$], [S \to \cdot a, \$], [S \to \cdot SA, a], [S \to \cdot SB, a], [S \to \cdot a, a]\}$.

   - $\text{GOTO}(I_0, S) = \text{CLOSURE}(\{[S' \to S\cdot, \$], [S \to S \cdot A, \$], [S \to S \cdot B, \$], [S \to S \cdot A, a], [S \to S \cdot B, a]\}) = \{[S' \to S\cdot, \$], [S \to S \cdot A, \$], [S \to S \cdot B, \$], [S \to S \cdot A, a], [S \to S \cdot B, a], [A \to \cdot S+, \$], [B \to \cdot S-, \$], [A \to \cdot S+, a], [B \to \cdot S-, a], [S \to \cdot SA, +], [S \to \cdot SB, +], [S \to \cdot a, +], [S \to \cdot SA, -], [S \to \cdot SB, -], [S \to \cdot a, -], [S \to \cdot SA, a], [S \to \cdot SB, a], [S \to \cdot a, a]\} = I_1$.
   - $\text{GOTO}(I_0, a) = \text{CLOSURE}(\{[S \to a\cdot, \$], [S \to a\cdot, a]\}) = \{[S \to a\cdot, \$], [S \to a\cdot, a]\} = I_2$
   - All other transfers lead to errors.

   (b) Consider I_1.

   - $\text{GOTO}(I_1, \$) = \text{Accept}$.
   - $\text{GOTO}(I_1, S) = \text{CLOSURE}(\{[A \to S \cdot +, \$], [B \to S \cdot -, \$], [A \to S \cdot +, a], [B \to S\cdot -, a], [S \to S\cdot A, +], [S \to S\cdot B, +], [S \to S\cdot A, -], [S \to S\cdot B, -], [S \to S\cdot A, a], [S \to S \cdot B, a]\}) = \{[A \to S \cdot +, \$], [B \to S \cdot -, \$], [A \to S \cdot +, a], [B \to S \cdot -, a], [S \to S\cdot A, +], [S \to S\cdot B, +], [S \to S\cdot A, -], [S \to S\cdot B, -], [S \to S\cdot A, a], [S \to S\cdot B, a], [A \to \cdot S+, +], [A \to \cdot S+, -], [A \to \cdot S+, a], [B \to \cdot S-, +], [B \to \cdot S-, -], [B \to \cdot S-, a], [S \to \cdot SA, +], [S \to \cdot SB, +], [S \to \cdot a, +], [S \to \cdot SA, -], [S \to \cdot SB, -], [S \to \cdot a, -], [S \to \cdot SA, a], [S \to \cdot SB, a], [S \to \cdot a, a]\} = I_3$.
   - $\text{GOTO}(I_1, A) = \text{CLOSURE}(\{[S \to SA\cdot, \$], [S \to SA\cdot, a]\}) = \{[S \to SA\cdot, \$], [S \to SA\cdot, a]\} = I_4$.
   - $\text{GOTO}(I_1, B) = \text{CLOSURE}(\{[S \to SB\cdot, \$], [S \to SB\cdot, a]\}) = \{[S \to SB\cdot, \$], [S \to SB\cdot, a]\} = I_5$.
   - $\text{GOTO}(I_1, a) = \text{CLOSURE}(\{[S \to a\cdot, +], [S \to a\cdot, -], [S \to a\cdot, a]\}) = \{[S \to a\cdot, +], [S \to a\cdot, -], [S \to a\cdot, a]\} = I_6$.
   - All other transfers lead to errors.

   (c) Consider I_2: All transfers lead to errors.

   (d) Consider I_3.

- GOTO(I_3, S) = CLOSURE$(\{[A \to S \cdot +, +], [A \to S \cdot +, -], [A \to S \cdot +, a], [B \to S \cdot -, +], [B \to S \cdot -, -], [B \to S \cdot -, a], [S \to S \cdot A, +], [S \to S \cdot B, +], [S \to S \cdot A, -], [S \to S \cdot B, -], [S \to S \cdot A, a], [S \to S \cdot B, a]\}) = \{[A \to S \cdot +, +], [A \to S \cdot +, -], [A \to S \cdot +, a], [B \to S \cdot -, +], [B \to S \cdot -, -], [B \to S \cdot -, a], [S \to S \cdot A, +], [S \to S \cdot B, +], [S \to S \cdot A, -], [S \to S \cdot B, -], [S \to S \cdot A, a], [S \to S \cdot B, a], [A \to \cdot S +, +], [A \to \cdot S +, -], [A \to \cdot S +, a], [B \to \cdot S -, +], [B \to \cdot S -, -], [B \to \cdot S -, a], [S \to \cdot SA, +], [S \to \cdot SB, +], [S \to \cdot a, +], [S \to \cdot SA, -], [S \to \cdot SB, -], [S \to \cdot a, -], [S \to \cdot SA, a], [S \to \cdot SB, a], [S \to \cdot a, a]\} = I_7$.
- GOTO(I_3, A) = CLOSURE$(\{[S \to SA \cdot, +], [S \to SA \cdot, -], [S \to SA \cdot, a]\}) = \{[S \to SA \cdot, +], [S \to SA \cdot, -], [S \to SA \cdot, a]\} = I_8$.
- GOTO(I_3, B) = CLOSURE$(\{[S \to SB \cdot, +], [S \to SB \cdot, +], [S \to SB \cdot, a]\}) = \{[S \to SB \cdot, +], [S \to SB \cdot, +], [S \to SB \cdot, a]\} = I_9$.
- GOTO(I_3, a) = CLOSURE$(\{[S \to a \cdot, +], [S \to a \cdot, -], [S \to a \cdot, a]\}) = \{[S \to a \cdot, +], [S \to a \cdot, -], [S \to a \cdot, a]\} = I_6$.
- GOTO$(I_3, +)$ = CLOSURE$(\{[A \to S + \cdot, \$], [A \to S + \cdot, a]\}) = \{[A \to S + \cdot, \$], [A \to S + \cdot, a]\} = I_{10}$.
- GOTO$(I_3, -)$ = CLOSURE$(\{[B \to S - \cdot, \$], [B \to S - \cdot, a]\}) = \{[B \to S - \cdot, \$], [B \to S - \cdot, a]\} = I_{11}$.
- All other transfers lead to errors.

(e) Consider I_4: All transfers lead to errors.

(f) Consider I_5: All transfers lead to errors.

(g) Consider I_6: All transfers lead to errors.

(h) Consider I_7.

- GOTO(I_7, S) = CLOSURE$(\{[A \to S \cdot +, +], [A \to S \cdot +, -], [A \to S \cdot +, a], [B \to S \cdot -, +], [B \to S \cdot -, -], [B \to S \cdot -, a], [S \to S \cdot A, +], [S \to S \cdot B, +], [S \to S \cdot A, -], [S \to S \cdot B, -], [S \to S \cdot A, a], [S \to S \cdot B, a]\}) = \{[A \to S \cdot +, +], [A \to S \cdot +, -], [A \to S \cdot +, a], [B \to S \cdot -, +], [B \to S \cdot -, -], [B \to S \cdot -, a], [S \to S \cdot A, +], [S \to S \cdot B, +], [S \to S \cdot A, -], [S \to S \cdot B, -], [S \to S \cdot A, a], [S \to S \cdot B, a], [A \to \cdot S +, +], [A \to \cdot S +, -], [A \to \cdot S +, a], [B \to \cdot S -, +], [B \to \cdot S -, -], [B \to \cdot S -, a], [S \to \cdot SA, +], [S \to \cdot SB, +], [S \to \cdot a, +], [S \to \cdot SA, -], [S \to \cdot SB, -], [S \to \cdot a, -], [S \to \cdot SA, a], [S \to \cdot SB, a], [S \to \cdot a, a]\} = I_7$.
- GOTO(I_7, A) = CLOSURE$(\{[S \to SA \cdot, +], [S \to SA \cdot, -], [S \to SA \cdot, a]\}) = \{[S \to SA \cdot, +], [S \to SA \cdot, -], [S \to SA \cdot, a]\} = I_8$.
- GOTO(I_7, B) = CLOSURE$(\{[S \to SB \cdot, +], [S \to SB \cdot, +], [S \to SB \cdot, a]\}) = \{[S \to SB \cdot, +], [S \to SB \cdot, +], [S \to SB \cdot, a]\} = I_9$.
- GOTO(I_7, a) = CLOSURE$(\{[S \to a \cdot, +], [S \to a \cdot, -], [S \to a \cdot, a]\}) = \{[S \to a \cdot, +], [S \to a \cdot, -], [S \to a \cdot, a]\} = I_6$.
- GOTO$(I_7, +)$ = CLOSURE$(\{[A \to S + \cdot, +], [A \to S + \cdot, -], [A \to S + \cdot, a]\}) = \{[A \to S + \cdot, +], [A \to S + \cdot, -], [A \to S + \cdot, a]\} = I_{12}$.
- GOTO$(I_3, -)$ = CLOSURE$(\{[B \to S - \cdot, +], [B \to S - \cdot, -], [B \to S - \cdot, a]\}) = \{[B \to S - \cdot, +], [B \to S - \cdot, -], [B \to S - \cdot, a]\} = I_{13}$.
- All other transfers lead to errors.

(i) Consider I_8: All transfers lead to errors.

(j) Consider I_9: All transfers lead to errors.

(k) Consider I_{10}: All transfers lead to errors.

(l) Consider I_{11}: All transfers lead to errors.

(m) Consider I_{12}: All transfers lead to errors.

(n) Consider I_{13}: All transfers lead to errors.

The final canonical LR(0) collection is $\{I_0, I_1, \cdots, I_{13}\}$.

Then we calculate the ACTION.

(a) Consider I_0.

- Since $[S \to \cdot a, \$] \in I_0$ and $\text{GOTO}(I_0, a) = I_2$, ACTION$[0, a]$ is "shift 2".
- Since $\text{GOTO}(I_0, S) = I_1$, $\text{GOTO}(0, S) = 1$.

(b) Consider I_1.

- Since $[S \to \cdot a, +/-/a] \in I_1$ and $\text{GOTO}(I_1, a) = I_6$, ACTION$[1, a]$ is "shift 6".
- Since $[S' \to S\cdot, \$] \in I_1$, ACTION$[1, \$]$ is "accept".
- Since $\text{GOTO}(I_1, S) = I_3$, $\text{GOTO}(1, S) = 3$.
- Since $\text{GOTO}(I_1, A) = I_4$, $\text{GOTO}(1, A) = 4$.
- Since $\text{GOTO}(I_1, B) = I_5$, $\text{GOTO}(1, B) = 5$.

(c) Consider I_2.

- Since $[S \to a\cdot, \$/a] \in I_2$, ACTION$[2, \$]$, ACTION$[2, a]$ are "reduce $S \to a$".

(d) Consider I_3.

- Since $[S \to \cdot a, +/-/a] \in I_3$ and $\text{GOTO}(I_3, a) = I_6$, ACTION$[3, a]$ is "shift 6".
- Since $[A \to S \cdot +, \$/a] \in I_3$ and $\text{GOTO}(I_3, +) = I_{10}$, ACTION$[3, +]$ is "shift 10".
- Since $[B \to S \cdot -, \$/a] \in I_3$ and $\text{GOTO}(I_3, -) = I_{11}$, ACTION$[3, -]$ is "shift 11".
- Since $\text{GOTO}(I_3, S) = I_7$, $\text{GOTO}(3, S) = 7$.
- Since $\text{GOTO}(I_3, A) = I_8$, $\text{GOTO}(3, A) = 8$.
- Since $\text{GOTO}(I_3, B) = I_9$, $\text{GOTO}(3, B) = 9$.

(e) Consider I_4.

- Since $[S \to SA\cdot, \$/a] \in I_4$, ACTION$[4, \$]$, ACTION$[4, a]$ are "reduce $S \to SA$".

(f) Consider I_5.

- Since $[S \to SB\cdot, \$/a] \in I_5$, ACTION$[5, \$]$, ACTION$[5, a]$ are "reduce $S \to SB$".

(g) Consider I_6.

- Since $[S \to a\cdot, +/-/a] \in I_6$, ACTION$[6, +]$, ACTION$[6, -]$ and ACTION$[6, a]$ are "reduce $S \to a$".

(h) Consider I_7.

- Since $[S \to \cdot a, +/-/a] \in I_7$ and $\text{GOTO}(I_7, a) = I_6$, ACTION$[7, a]$ is "shift 6".
- Since $[A \to S \cdot +, +/-/a] \in I_7$ and $\text{GOTO}(I_7, +) = I_{12}$, ACTION$[7, +]$ is "shift 12".
- Since $[B \to S \cdot -, +/-/a] \in I_7$ and $\text{GOTO}(I_7, -) = I_{13}$, ACTION$[7, -]$ is "shift 13".
- Since $\text{GOTO}(I_3, S) = I_7$, $\text{GOTO}(3, S) = 7$.
- Since $\text{GOTO}(I_3, A) = I_8$, $\text{GOTO}(3, A) = 8$.
- Since $\text{GOTO}(I_3, B) = I_9$, $\text{GOTO}(3, B) = 9$.

(i) Consider I_8.

- Since $[S \to SA\cdot, +/-/a] \in I_8$, ACTION$[8, +]$, ACTION$[8, -]$ and ACTION$[8, a]$ are "reduce $S \to SA$".

(j) Consider I_9.

- Since $[S \to SB\cdot, +/-/a] \in I_9$, ACTION$[9, +]$, ACTION$[9, -]$ and ACTION$[9, a]$ are "reduce $S \to SB$".

(k) Consider I_{10}.

- Since $[A \to S + \cdot, \$/a] \in I_{10}$, ACTION$[10, \$]$, ACTION$[10, a]$ are "reduce $A \to S+$".

(l) Consider I_{11}.

- Since $[B \to S - \cdot, \$/a] \in I_{11}$, ACTION$[11, \$]$, ACTION$[11, a]$ are "reduce $B \to S-$".

(m) Consider I_{12}.

- Since $[A \to S + \cdot, +/-/a] \in I_{12}$, ACTION$[12, +]$, ACTION$[12, -]$ and ACTION$[12, a]$ are "reduce $A \to S+$".

(n) Consider I_{13}.

- Since $[B \to S - \cdot, +/-/a] \in I_{13}$, ACTION$[13, +]$, ACTION$[13, -]$ and ACTION$[13, a]$ are "reduce $B \to S-$".

Thus, the parsing table is

STATE	ACTION				GOTO		
	a	$+$	$-$	\$	S	A	B
0	s2				1		
1	s6			acc	3	4	5
2	r3			r3			
3	s6	s10	s11		7	8	9
4	r1			r1			
5	r2			r2			
6	r3	r3	r3				
7	s6	s12	s13		7	8	9
8	r1	r1	r1				
9	r2	r2	r2				
10	r4			r4			
11	r5			r5			
12	r4	r4	r4				
13	r5	r5	r5				

2. Yes. There is no conflict.

3. Yes.

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	\$	$aaaa---\$$	shift to 2
(2)	0 2	$\$a$	$aaa---\$$	reduce by $S \to a$
(3)	0 1	$\$S$	$aaa---\$$	shift to 6
(4)	0 1 6	$\$Sa$	$aa---\$$	reduce by $S \to a$
(5)	0 1 3	$\$SS$	$aa---\$$	shift to 6
(6)	0 1 3 6	$\$SSa$	$a---\$$	reduce by $S \to a$
(7)	0 1 3 7	$\$SSS$	$a---\$$	shift to 6
(8)	0 1 3 7 6	$\$SSSa$	$---\$$	reduce by $S \to a$
(9)	0 1 3 7 7	$\$SSSS$	$---\$$	shift to 13
(10)	0 1 3 7 7 13	$\$SSSS-$	$--\$$	reduce by $B \to S-$
(11)	0 1 3 7 9	$\$SSSB$	$--\$$	reduce by $S \to SB$
(12)	0 1 3 7	$\$SSS$	$--\$$	shift to 13
(13)	0 1 3 7 13	$\$SSS-$	$-\$$	reduce by $B \to S-$
(14)	0 1 3 9	$\$SSB$	$-\$$	reduce by $S \to SB$
(15)	0 1 3	$\$SS$	$-\$$	shift to 11
(16)	0 1 3 11	$\$SS-$	\$	reduce by $B \to S-$
(17)	0 1 5	$\$SB$	\$	reduce by $S \to SB$
(18)	0 1	$\$S$	\$	accept

3 Required Exercise 3

1. Using the result from 2 as the base, we try to find out the core of each state:

   (a) I_0: $\{[S' \to \cdot S], [S \to \cdot SA], [S \to \cdot SB]\}$

   (b) I_1: $\{[S' \to S\cdot], [S \to S \cdot A], [S \to S \cdot B], [A \to \cdot S+], [B \to \cdot S-], [S \to \cdot SA], [S \to \cdot SB], [S \to \cdot a]\}$

   (c) I_2: $\{[S \to a\cdot]\}$

   (d) I_3: $\{[A \to S \cdot +], [B \to S \cdot -], [S \to S \cdot A], [S \to S \cdot B], [A \to \cdot S+], [B \to \cdot S-], [S \to \cdot SA], [S \to \cdot SB], [S \to \cdot a]\}$

   (e) I_4: $\{[S \to SA\cdot]\}$

   (f) I_5: $\{[S \to SB\cdot]\}$

   (g) I_6: $\{[S \to a\cdot]\}$

(h) I_7: $\{[A \to S \cdot +], [B \to S \cdot -], [S \to S \cdot A], [S \to S \cdot B], [A \to \cdot S+], [B \to \cdot S-], [S \to \cdot SA], [S \to \cdot SB], [S \to \cdot a]\}$

(i) I_8: $\{[S \to SA\cdot]\}$

(j) I_9: $\{[S \to SB\cdot]\}$

(k) I_{10}: $\{[A \to S + \cdot]\}$

(l) I_{11}: $\{[B \to S - \cdot]\}$

(m) I_{12}: $\{[A \to S + \cdot]\}$

(n) I_{13}: $\{[B \to S - \cdot]\}$

It could be seen that, I_2 and I_6 have the same core, I_3 and I_7 have the same core, I_4 and I_8 have the same core, I_5 and I_9 have the same core, I_{10} and I_{12} have the same core and I_{11} and I_{13} have the same core.

If we try to merge them, we could get a new parsing table:

STATE	ACTION				GOTO		
	a	$+$	$-$	\$	S	A	B
0	s2				1		
1	s2			acc	3	4	5
2	r3	r3	r3	r3			
3	s2	s6	s7		3	4	5
4	r1	r1	r1	r1			
5	r2	r2	r2	r2			
6	r4	r4	r4	r4			
7	r5	r5	r5	r5			

The correspondence is

NEW STATE	OLD STATES
0	0
1	1
2	2, 6
3	3, 7
4	4, 8
5	5, 9
6	10, 12
7	11, 13

2. Yes. There is no conflict.

3. Yes.

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	\$	$aaaa+-+\$$	shift to 2
(2)	0 2	$\$a$	$aaa+-+\$$	reduce by $S \to a$
(3)	0 1	$\$S$	$aaa+-+\$$	shift to 2
(4)	0 1 2	$\$Sa$	$aa+-+\$$	reduce by $S \to a$
(5)	0 1 3	$\$SS$	$aa+-+\$$	shift to 2
(6)	0 1 3 2	$\$SSa$	$a+-+\$$	reduce by $S \to a$
(7)	0 1 3 3	$\$SSS$	$a+-+\$$	shift to 2
(8)	0 1 3 3 2	$\$SSSa$	$+-+\$$	reduce by $S \to a$
(9)	0 1 3 3 3	$\$SSSS$	$+-+\$$	shift to 6
(10)	0 1 3 3 3 6	$\$SSSS+$	$-+\$$	reduce by $A \to S+$
(11)	0 1 3 3 4	$\$SSSA$	$-+\$$	reduce by $S \to SA$
(12)	0 1 3 3	$\$SSS$	$-+\$$	shift to 7
(13)	0 1 3 3 7	$\$SSS-$	$+\$$	reduce by $B \to S-$
(14)	0 1 3 5	$\$SSB$	$+\$$	reduce by $S \to SB$
(15)	0 1 3	$\$SS$	$+\$$	shift to 6
(16)	0 1 3 6	$\$SS+$	\$	reduce by $A \to S+$
(17)	0 1 4	$\$SA$	\$	reduce by $S \to SA$
(18)	0 1	$\$S$	\$	accept