

# Lecture 4

## Linked List

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# Our Roadmap

- ◆ Linked List Definition
- ◆ Linked List Operators
- ◆ Illustration Example

# Representing a Sequence of Data

- ◆ An ordered collection of items (position matters)
  - ◆ Array, lists, stacks, and queues
- ◆ What did you study before? Array!
- ◆ Advantages of using an array
  - ◆ Easy and efficient access to any item in the sequence
  - ◆ `item[i]`: return the *i*-th element in array `item`
  - ◆ Every item can be accessed in constant time
  - ◆ This feature of arrays is known as “random access”
  - ◆ Very compact (in terms of memory)
- ◆ Disadvantages of using an array ?

# Disadvantages of an Array

- ◆ Have to specify an initial array size
- ◆ Resize an array is possible, but not so easy
- ◆ Difficult to insert/delete elements at arbitrary positions
  - ◆ Delete **10** in array A, time complexity?

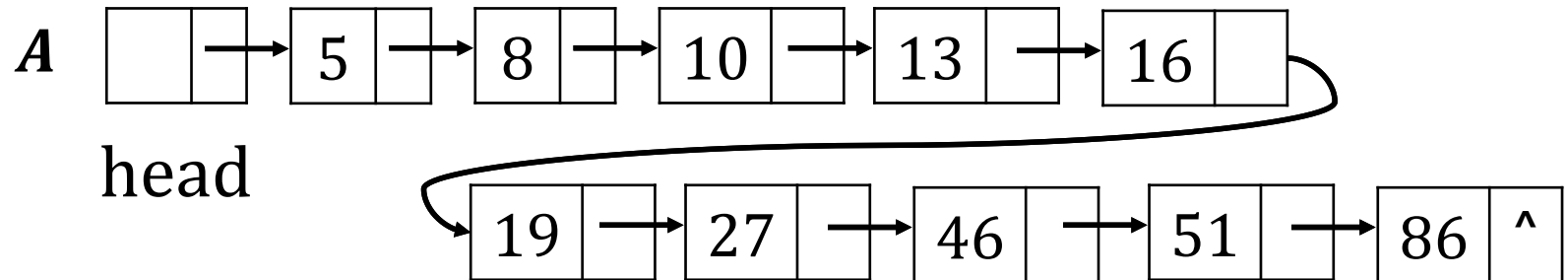
|          |   |   |    |    |    |    |    |    |    |    |
|----------|---|---|----|----|----|----|----|----|----|----|
| <b>A</b> | 5 | 8 | 10 | 13 | 16 | 19 | 27 | 46 | 51 | 86 |
|----------|---|---|----|----|----|----|----|----|----|----|

|          |   |   |  |    |    |    |    |    |    |    |
|----------|---|---|--|----|----|----|----|----|----|----|
| <b>A</b> | 5 | 8 |  | 13 | 16 | 19 | 27 | 46 | 51 | 86 |
|----------|---|---|--|----|----|----|----|----|----|----|

|          |   |   |    |    |    |    |    |    |    |  |
|----------|---|---|----|----|----|----|----|----|----|--|
| <b>A</b> | 5 | 8 | 13 | 16 | 19 | 27 | 46 | 51 | 86 |  |
|----------|---|---|----|----|----|----|----|----|----|--|

# A Linked List

- ◆ Alternative Representation of a sequence. Example:



- ◆ A linked list stores a sequence of elements in separate nodes
- ◆ Each node contains: a single item, a “link” to the node containing the next item: 

|    |   |
|----|---|
| 13 | → |
|----|---|
- ◆ The last node in the linked list has a link value of “NULL”: 

|    |   |
|----|---|
| 86 | ^ |
|----|---|
- ◆ The linked list as a whole is represented by a variable that hold a reference to the first node (e.g., *A*)

# Array vs. Linked List in Memory

- ◆ In an array, the elements occupy consecutive memory locations:

|       |       |       |       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 5     | 8     | 10    | 13    | 16    | 19    | 27    | 46    | 51    | 86    |
| 0x100 | 0x104 | 0x108 | 0x112 | 0x116 | 0x120 | 0x124 | 0x128 | 0x132 | 0x136 |

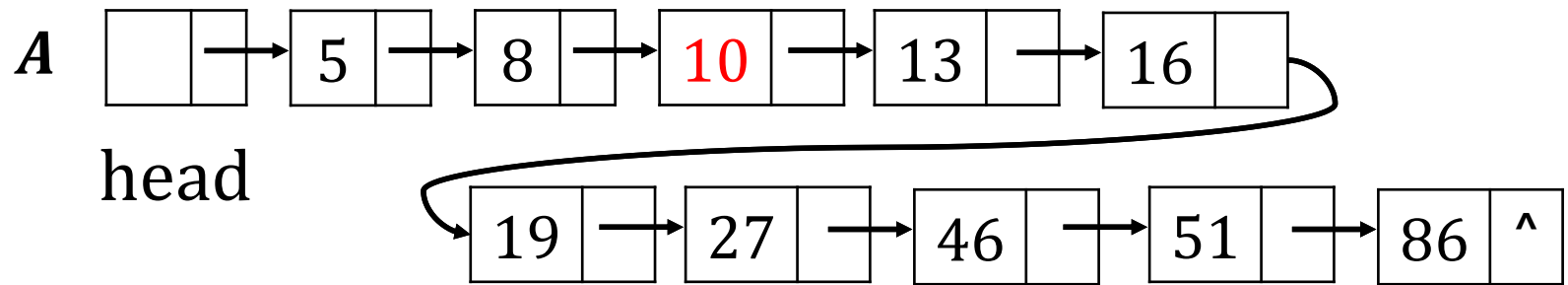
- ◆ In linked list, each node is a distinct object. The nodes do NOT have to be next to each other in memory. That's why we need the links to get from one node to the next.

|       |       |
|-------|-------|
| 0x100 | 8     |
|       | 0x480 |
| ...   | ...   |
| 0x240 | 13    |
|       | 0x640 |
| ...   | ...   |
|       | 5     |
|       | 0x100 |
| ...   | ...   |
| 0x480 | 10    |
|       | 0x240 |
| ...   | ...   |
| 0x640 | 16    |
|       | 0x800 |
| ..... | ...   |

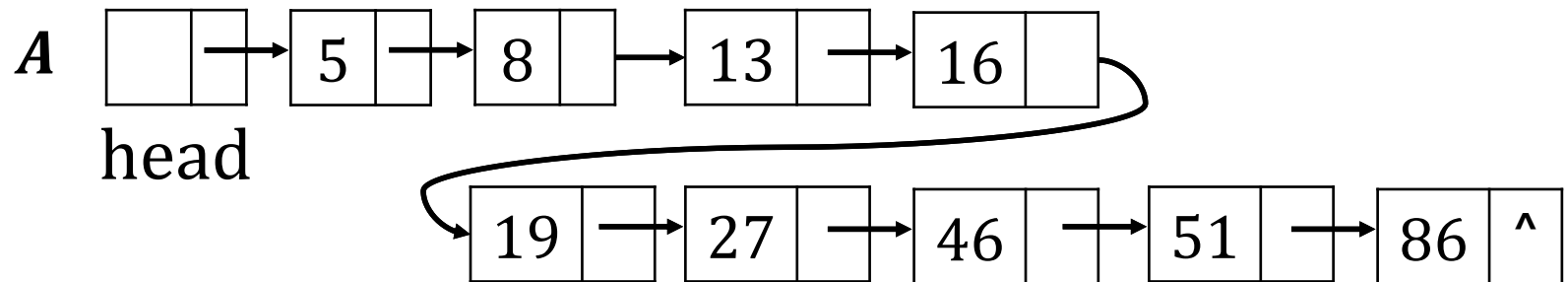


# Features of Linked List

- ◆ It can grow without limit (not fixed length)
- ◆ Easy to insert/delete an element
- ◆ Delete 10 in Linked List A, before:

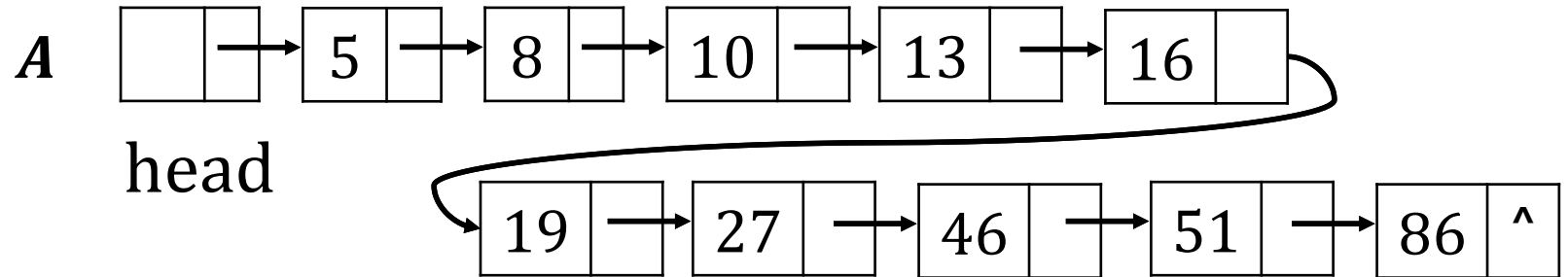


- ◆ After:

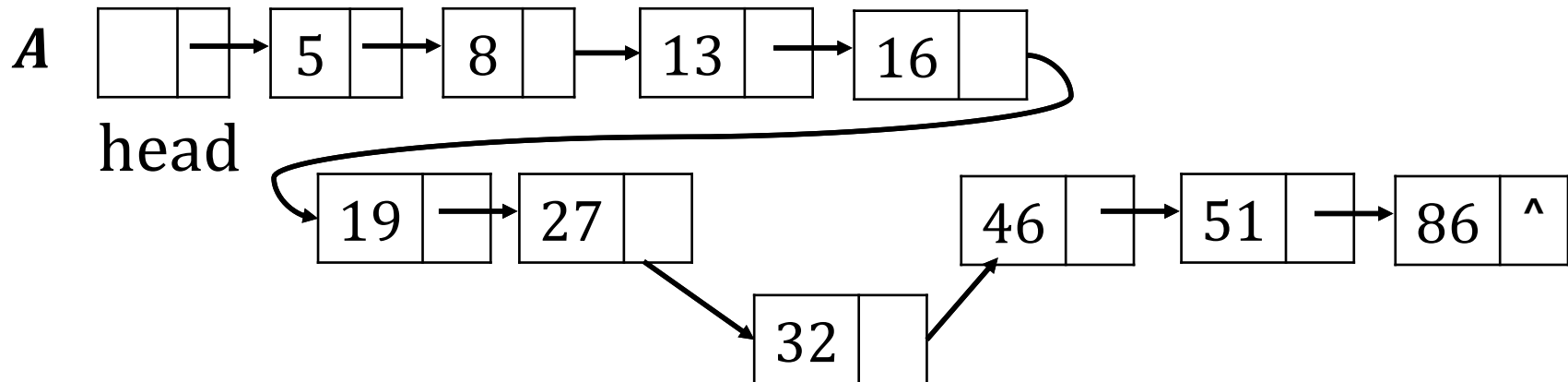


# Features of Linked List

- ◆ Insert 32 in Linked List A, before:



- ◆ After:



- ◆ Time Complexity?

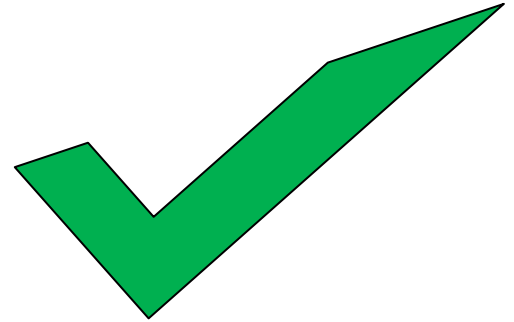


# Features of Linked List

- ◆ Disadvantages of Linked List
  - ◆ They do not provide random access
  - ◆ Need to “walk down” the list to access an item
  - ◆ The links take up additional memory
  - ◆ Not compact (in terms of Memory)
- ◆ Linked List vs. Array
  - ◆ Space complexity
  - ◆ Time Complexity: Insert, Delete, Find

# Our Roadmap

- ◆ Linked List Definition
- ◆ Linked List Operators
- ◆ Illustration Example

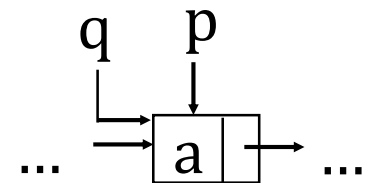
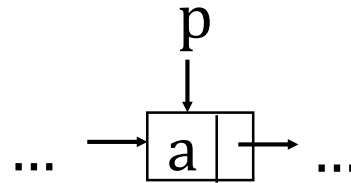


# Basic Operators of Linked List

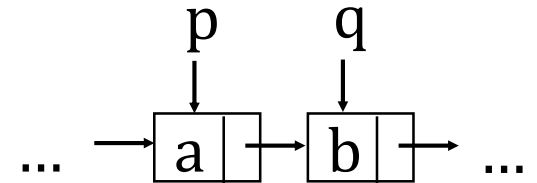
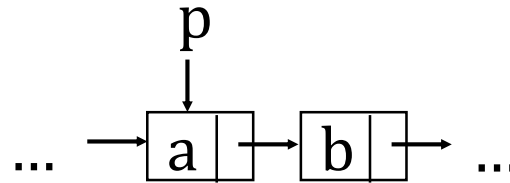
Before

After

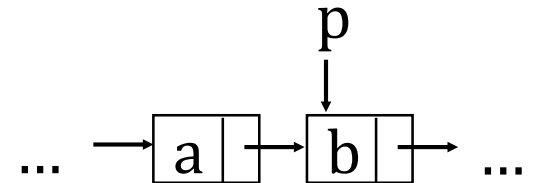
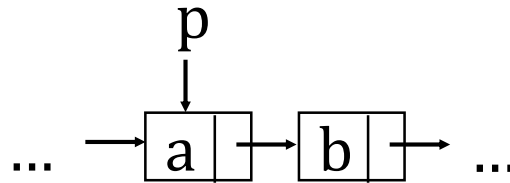
①  $q \leftarrow p$



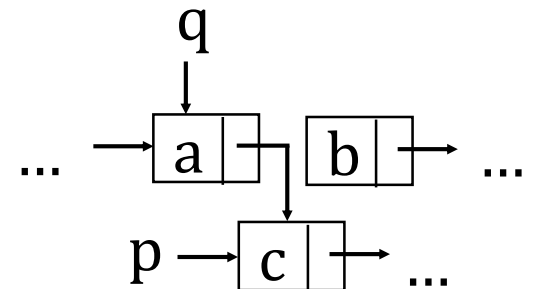
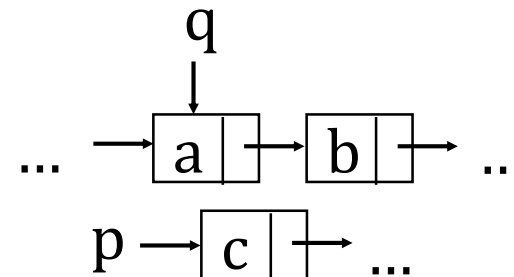
②  $q \leftarrow \text{next of } p$



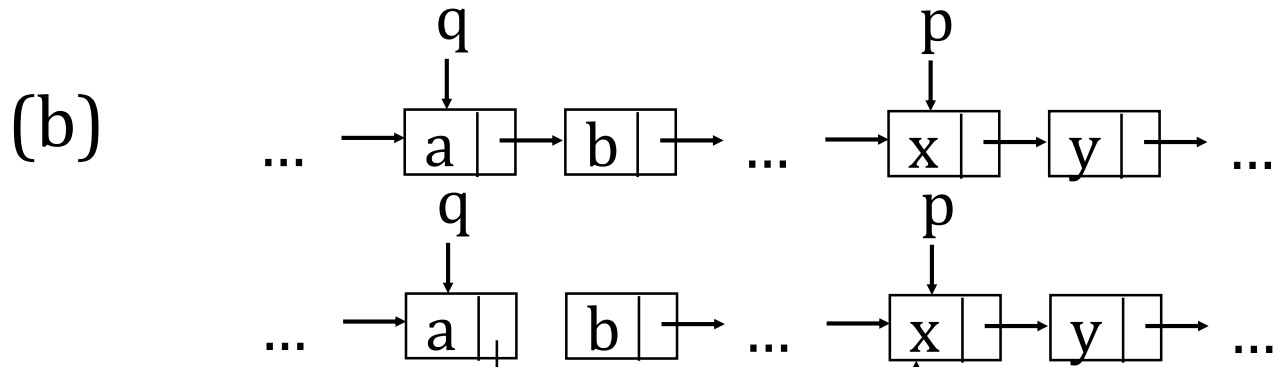
③  $p \leftarrow \text{next of } p$



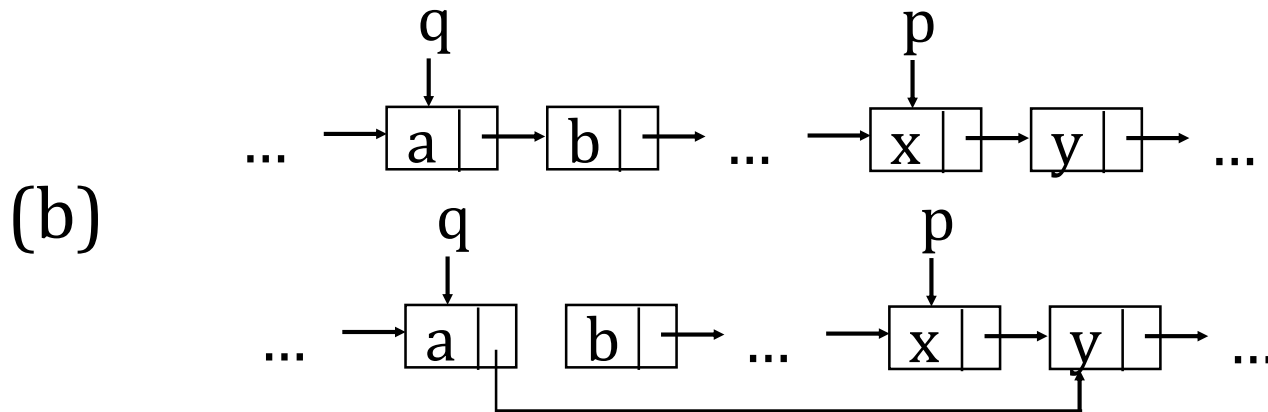
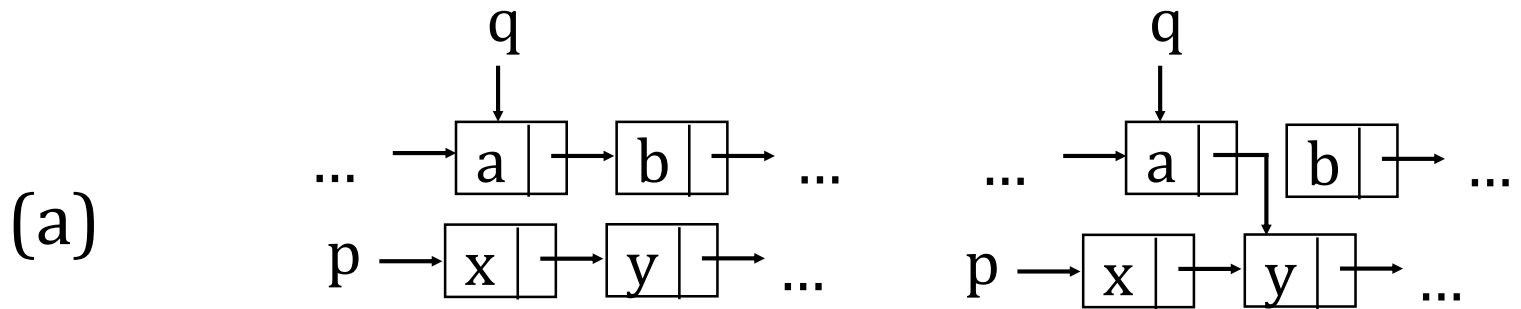
④ next of  $q \leftarrow p$   
(a)



# Basic Operators of Linked List

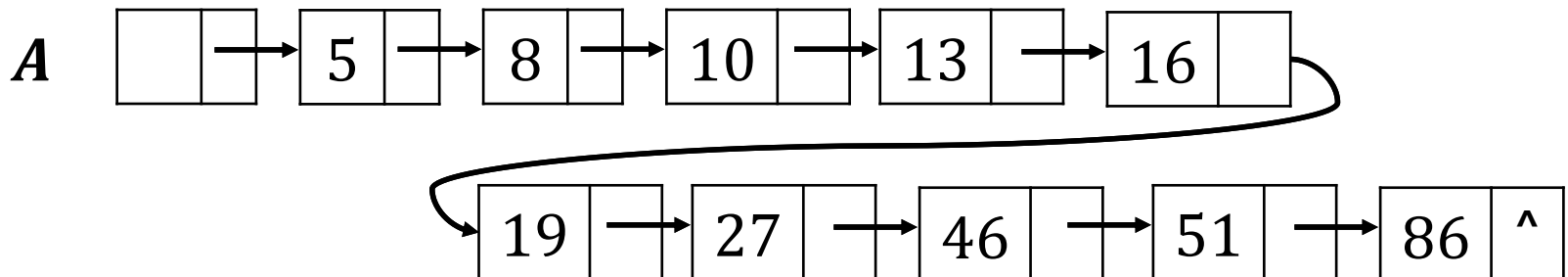


⑤ next of q  $\leftarrow$  next of p



# Traverse a Linked List

- ◆ Many tasks require us to traverse or “walk down” a linked list
- ◆ Recursion Pseudocode
- ◆ **Algorithm:** traverse(A):
  1. if (A=NULL)
  2.       return
  3. else
  4.     **print A.value**
  5.     traverse(A.next)

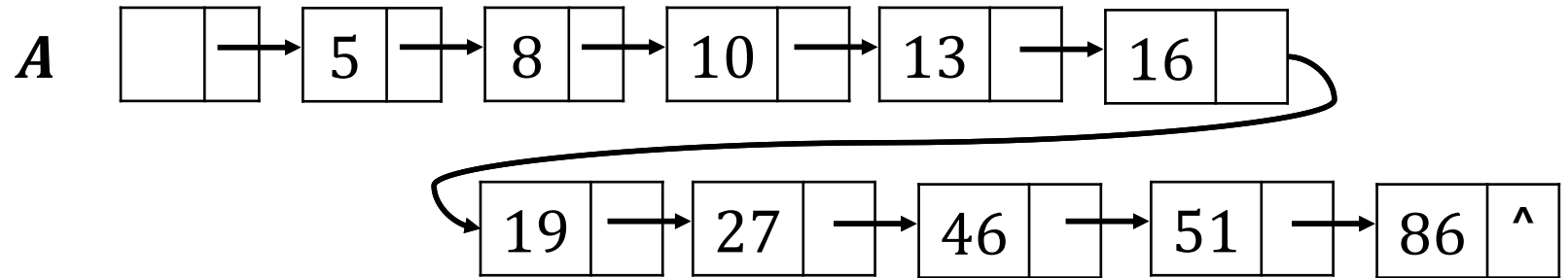


# Traverse a Linked List

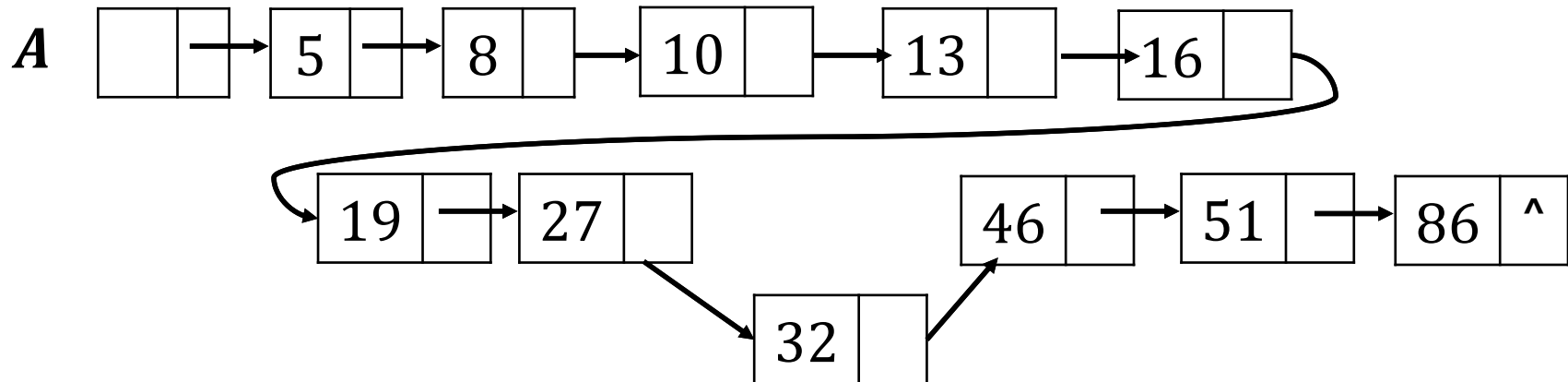
- ◆ It can also be done using iteration (for loops, while loops, etc.)
- ◆ Iteration Pseudocode
- ◆ **Algorithm:** traverseIteration(A):
  1. node trav  $\leftarrow$  A
  2. While (trav  $\neq$  NULL)
  3.       **print** trav.value
  4.       trav  $\leftarrow$  trav.next
- ◆ We use iteration in the following operators, but you can try to use recursion to implement these operators.

# Inserting an Item at Position i

- ◆ Insert 32 in Linked List A at position 8, before:



- ◆ After:



- ◆ How to do that?

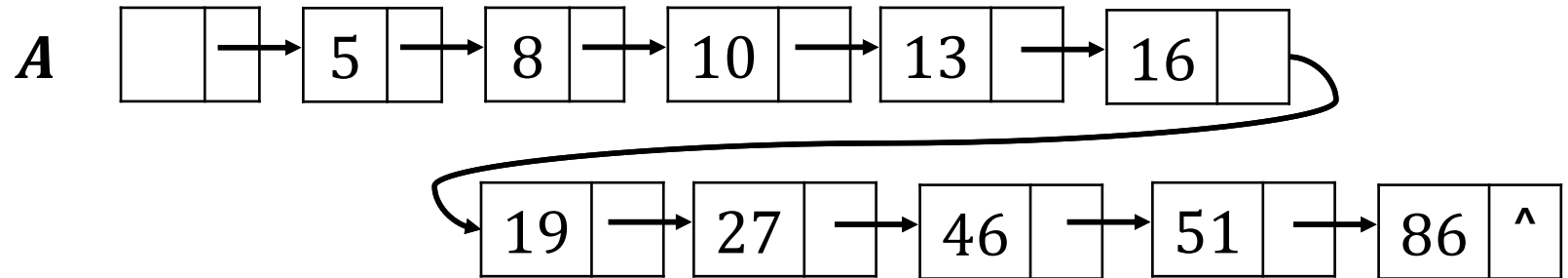
# Inserting an Item at Position $i$

- ◆ **Problem:** insert node  $q$  in Linked List  $A$  at Position  $i$
- ◆ **Algorithm:** insertNode( $A$ , node  $q$ ,  $i$ ):
  1.  $a \leftarrow 0$ , node  $p \leftarrow A$ ,
  2. **while** ( $i-1 > a$ )
  3.         $p \leftarrow p.next$
  4.         $a \leftarrow a + 1$
  5.  $tmp \leftarrow p.next$
  6.  $p.next \leftarrow q$
  7.  $q.next \leftarrow tmp$
  8. **return**  $A$
- ◆ Time Complexity:  **$O(n)$**
- ◆ Space Complexity:  **$O(1)$**

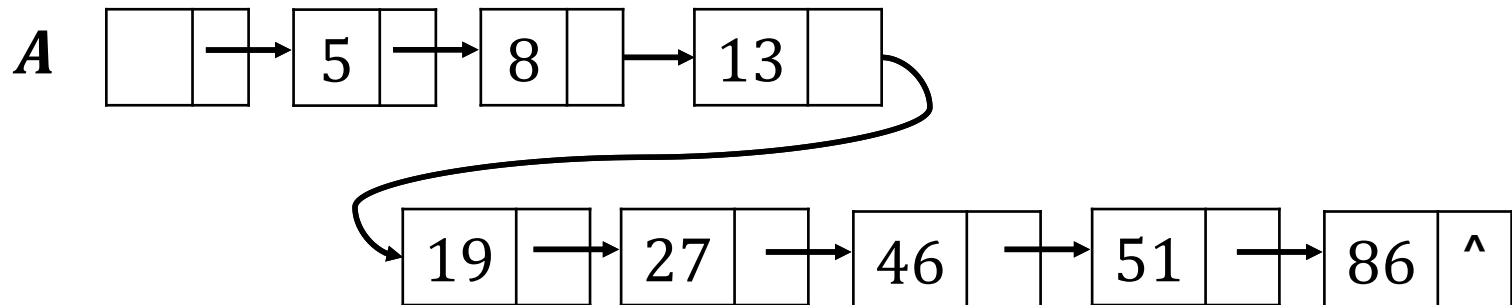


# Deleting an Item at Position i

- ◆ Delete position 5 in Linked List A, before:



- ◆ After:



- ◆ How to do that?

# Deleting an Item at Position $i$

- ◆ **Problem:** delete node in Linked List A at Position  $i$
- ◆ **Algorithm:** deleteNode(A,  $i$ ):
  1.  $a \leftarrow 0$ , node  $p \leftarrow A$ ,
  2. **while** ( $i-1 > a$ )
  3.            $p \leftarrow p.next$
  4.            $a \leftarrow a + 1$
  5.  $p.next \leftarrow p.next.next$
  6. **return** A
- ◆ Time Complexity:  **$O(n)$**
- ◆ Space Complexity:  **$O(1)$**

# Finding an Item at Position i

◆ **Problem:** Find value x in Linked List A

◆ **Algorithm:** findValue(A, x):

```
1. a ← 0, node p ← A,  
2. while (p!=NULL)  
4.     if (x = p.value)  
5.         return p  
6.     p ← p.next  
7. return -1
```

◆ Time Complexity:  **$O(n)$**

◆ Space Complexity:  **$O(1)$**

# Updating an Item at Position i

- ◆ **Problem:** Update nodes with value x to y in Linked List A
- ◆ **Algorithm:** updateNodes(A, x):
  1.  $a \leftarrow \emptyset$ , node  $p \leftarrow A$ ,
  2. **while** ( $p \neq \text{NULL}$ )
  4.        **if** ( $x = p.\text{value}$ )
  5.                 $p.\text{value} \leftarrow y$
  6.         $p \leftarrow p.\text{next}$
  7. **return** A
- ◆ Time Complexity:  **$O(n)$**
- ◆ Space Complexity:  **$O(1)$**

# Our Roadmap

- ◆ Linked List Definition
- ◆ Linked List Operators
- ◆ Illustration Example



# Operators on polynomials

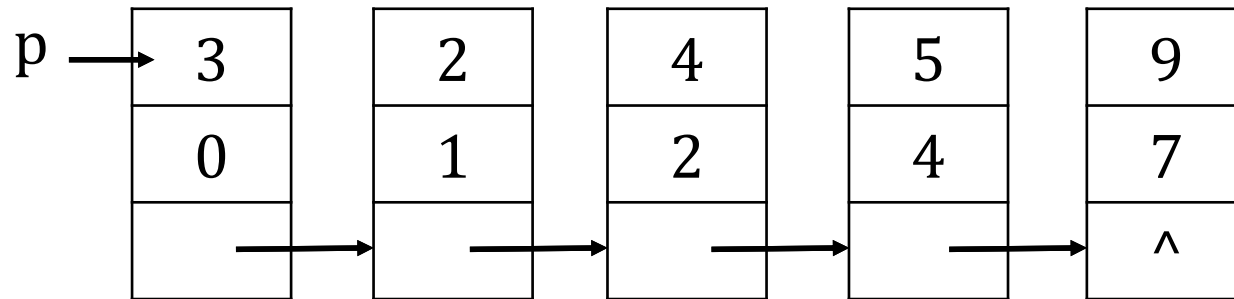
- ◆ **Polynomials:**  $p(x) = p_0 + p_1x + p_2x^2 + \dots + p_nx^n$
- ◆ a set of ordered pairs of  $\langle p_i, i \rangle$  where  $p_i$  is the coefficient and  $i$  is the exponent.
- ◆ We use linked list store the  $\langle p_i, i \rangle$  pairs of  $p(x)$
- ◆ Without loss of generality, we skip all nodes w/  $p_i = 0$
- ◆ Node representation:

```
node polyItem{  
    float coef    // record  $p_i$   
    int  expo    // record exponent  
    node next    // reference to next polyItem  
}
```

- ◆ **Question:** how about use array?

# Finding degree of a Polynomials

◆ **Polynomials:**  $p(x) = 3 + 2x + 4x^2 + 5x^4 + 9x^7$



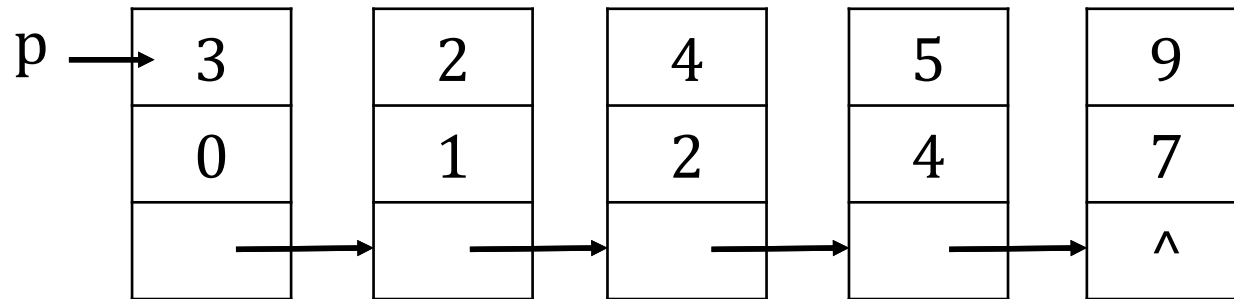
◆ Degree of  $p(x)$ : 7

◆ **Algorithm:** findDegree(p):

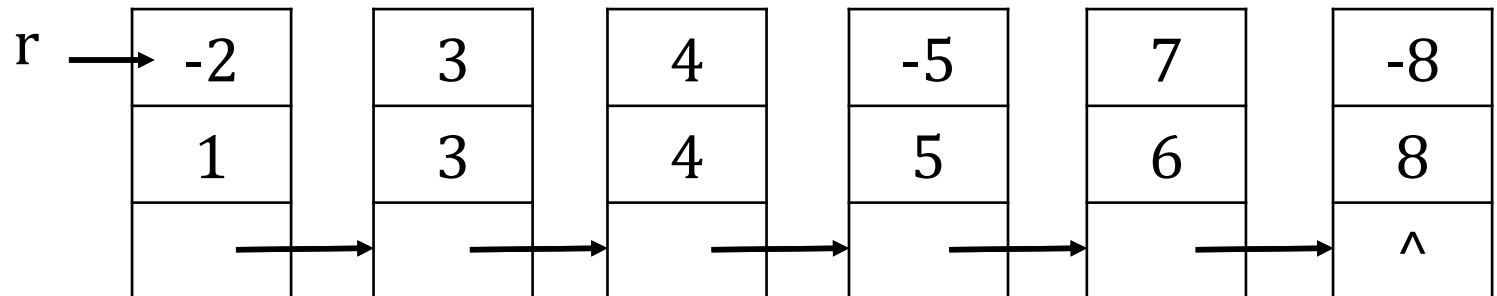
1. node tmp  $\leftarrow$  p
2. **While** (tmp.next  $\neq$  NULL)
3.        tmp  $\leftarrow$  tmp.next
4. **return** tmp.expo

# Adding two polynomials

◆  $p(x) = 3 + 2x + 4x^2 + 5x^4 + 9x^7$



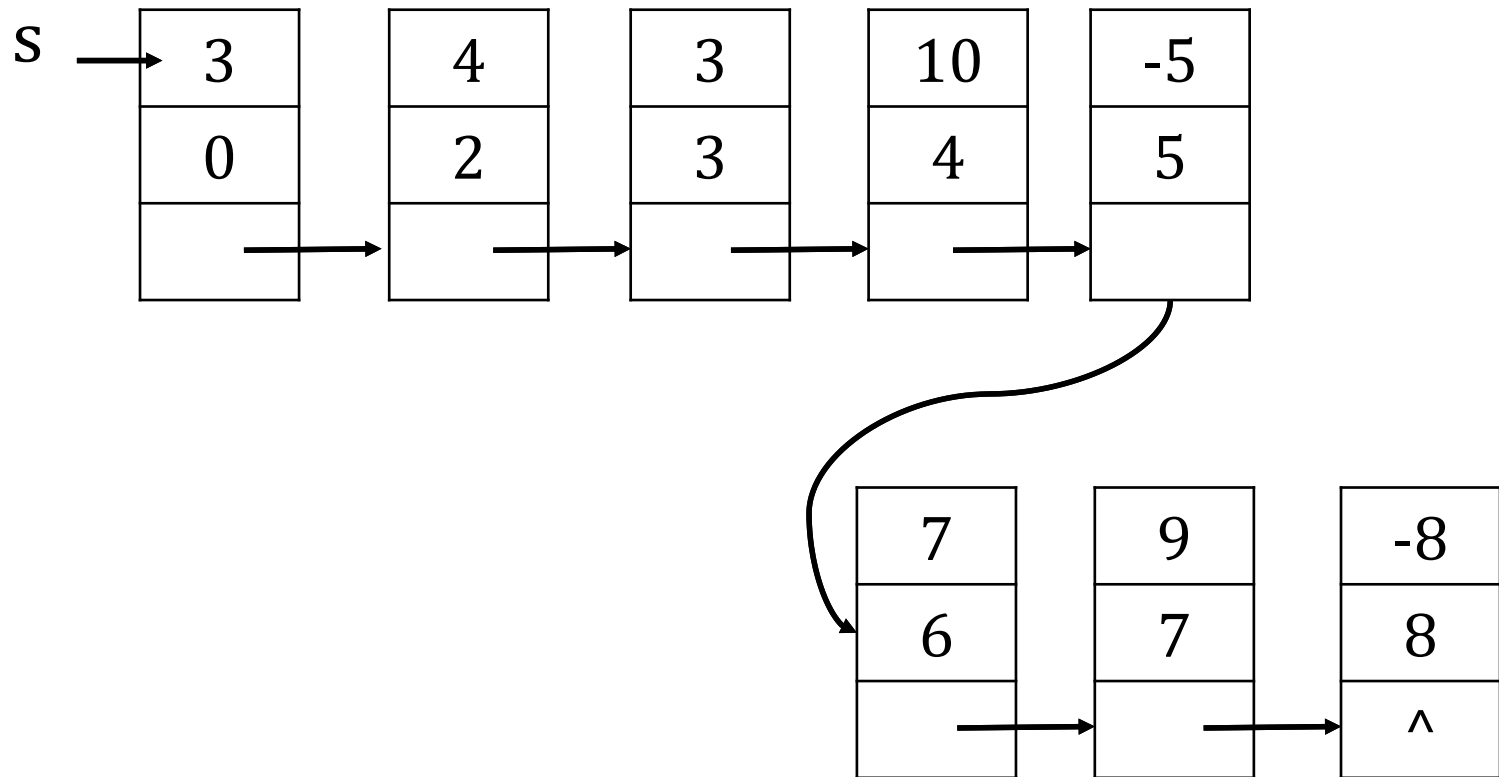
◆  $r(x) = -2x + 3x^3 + 5x^4 - 5x^5 + 7x^6 - 8x^8$





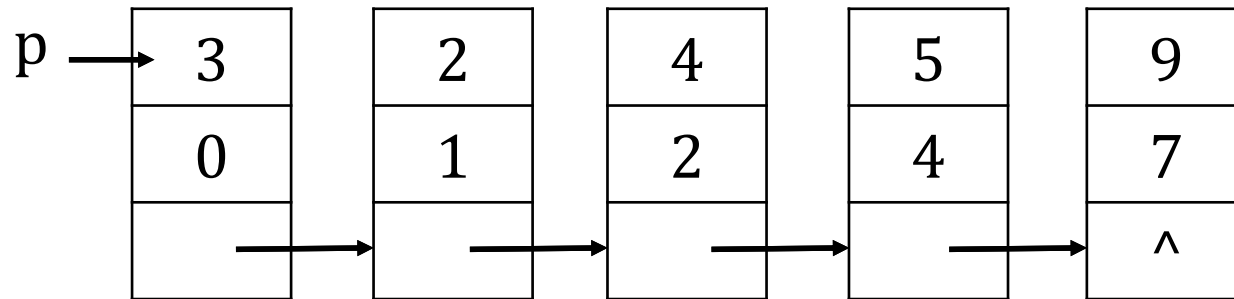
# Adding two polynomials

◆  $s(x) = p(x) + r(x)$   
 $= 3 + 4x^2 + 3x^3 + 10x^4 - 5x^5 + 7x^6 + 9x^7 - 8x^8$

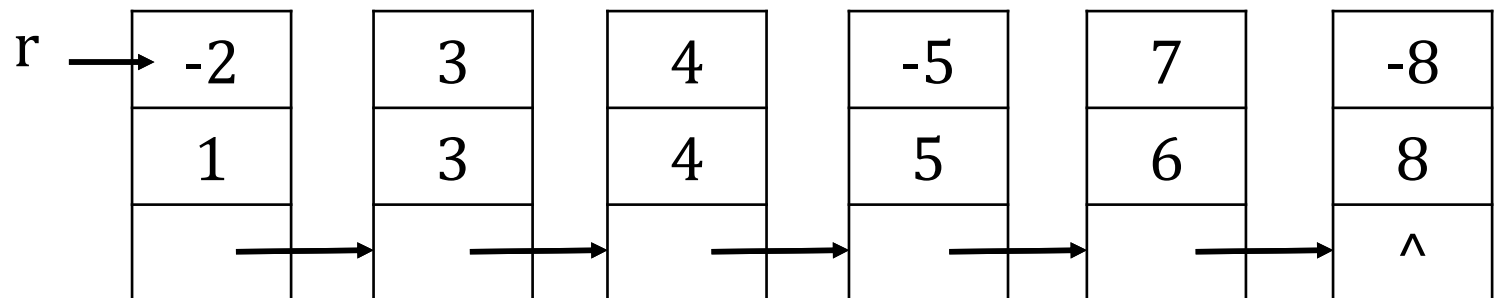


# Subtracting two polynomials

◆  $p(x) = 3 + 2x + 4x^2 + 5x^4 + 9x^7$

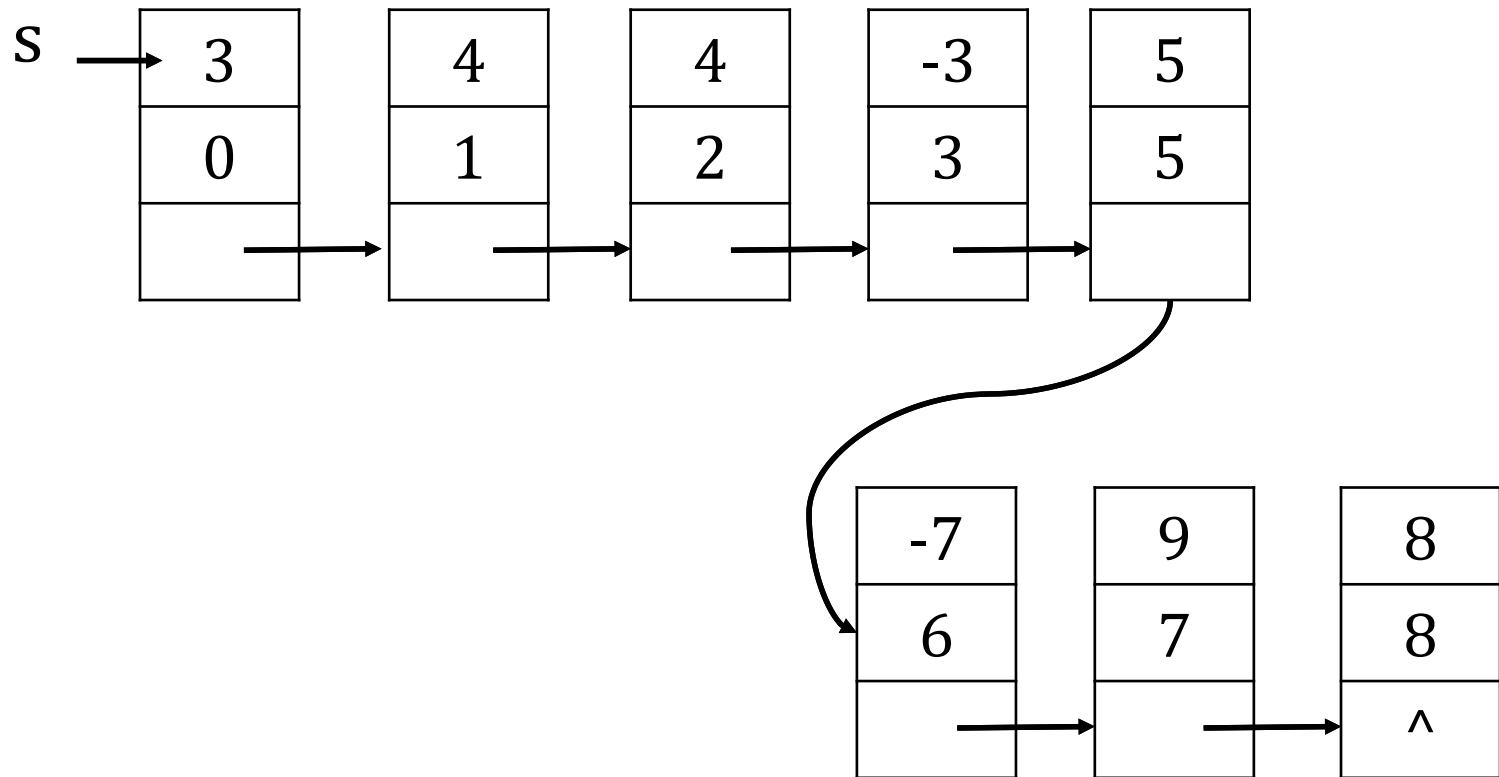


◆  $r(x) = -2x + 3x^3 + 5x^4 - 5x^5 + 7x^6 - 8x^8$



# Subtracting two polynomials

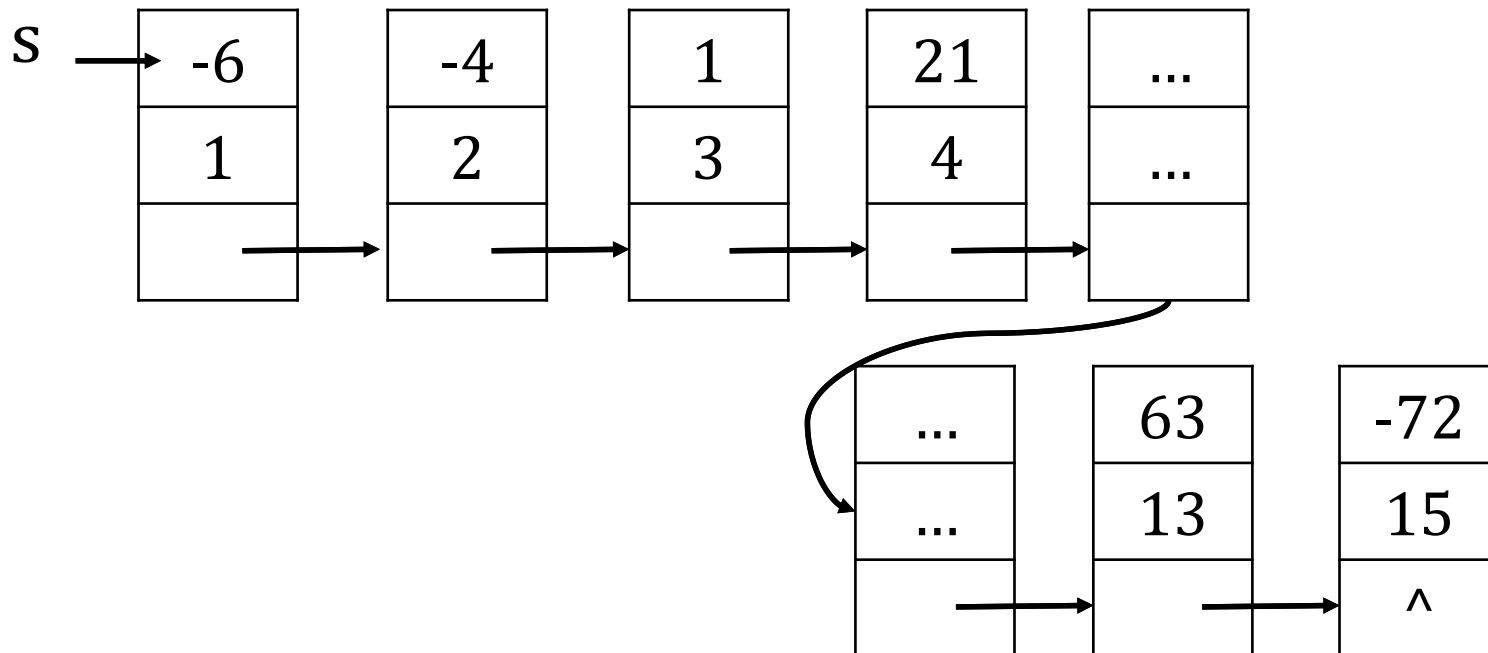
◆  $s(x) = p(x) - r(x)$   
 $= 3 + 4x + 4x^2 - 3x^3 + 5x^5 - 7x^6 + 9x^7 + 8x^8$



# Multiplying two polynomials

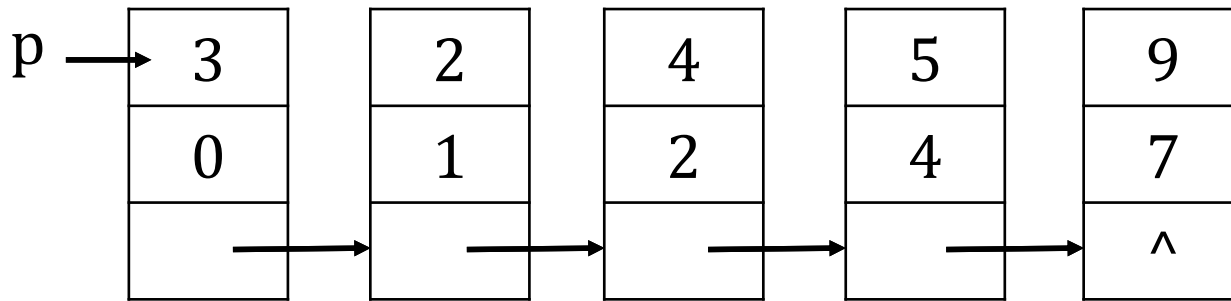
- ◇  $p(x) = 3 + 2x + 4x^2 + 5x^4 + 9x^7$
- ◇  $r(x) = -2x + 3x^3 + 5x^4 - 5x^5 + 7x^6 - 8x^8$
- ◇  $s(x) = p(x) * r(x)$   

$$= -6x - 4x^2 + x^3 + 21x^4 - 3x^5 + 31x^6 + 9x^7 + 11x^8 - 41x^9 + 30x^{10} + 45x^{11} - 85x^{12} + 63x^{13} - 72x^{15}$$

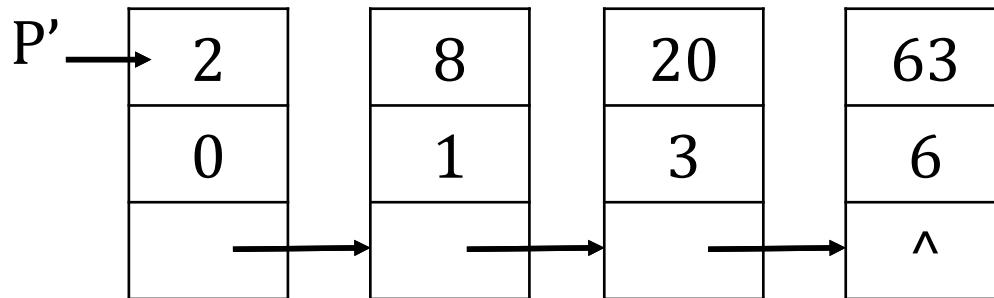


# Differentiating of a polynomial

◆  $p(x) = 3 + 2x + 4x^2 + 5x^4 + 9x^7$

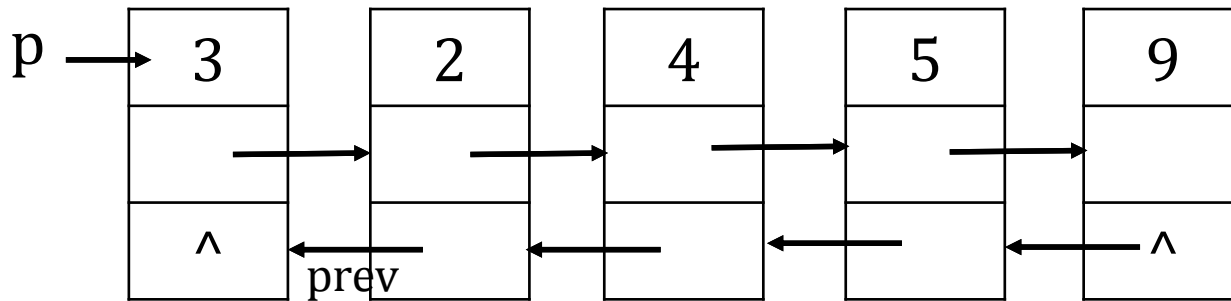


◆  $p'(x) = 2 + 8x + 20x^3 + 63x^6$



# Other variants of Lined List

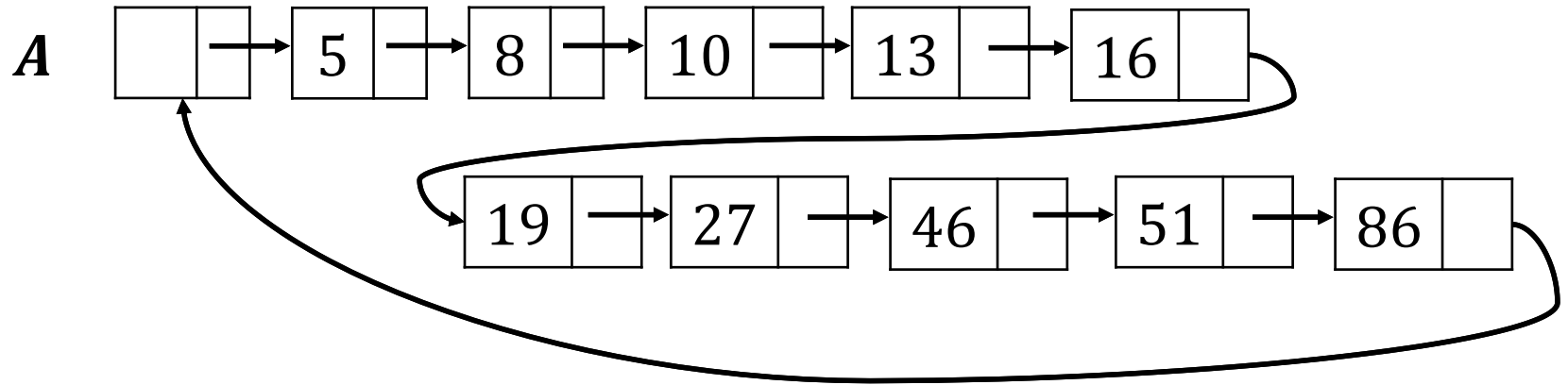
## ◆ Double linked list



- ◆ add a prev reference to each node: refers to the previous node
- ◆ allow us to “back up” from a given node

# Other variants of Lined List

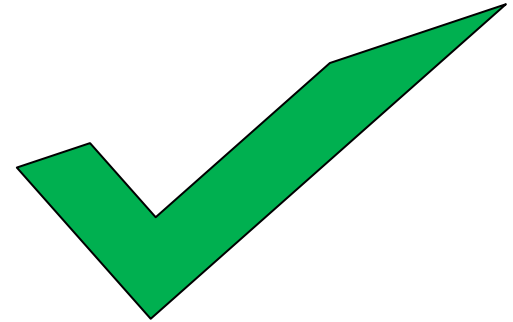
## ◆ Circular linked list



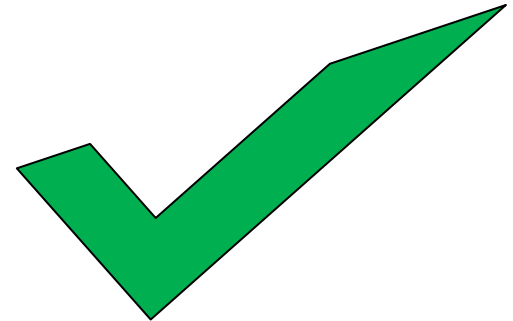
- ◆ Is it a empty list?  $\text{head.next} = \text{head}$  ?
- ◆ Is it the end of list?  $\text{tmp.next} = \text{head}$ ?

# Our Roadmap

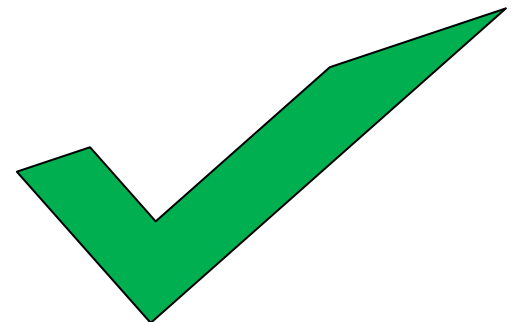
- ◆ Linked List Definition



- ◆ Linked List Operators



- ◆ Illustration Example





Thank You!