Discrete Math. Assignment 06 12210731 陈信

Q.1.

(a): For minimum, we ~~need~~ don't need it to be connected. So

it's has no edge ~~a chain~~. So minimum number is ~~1~~ 0.

For maximum,

we connect every two vertices. but without repetition.

so number is $(n-1)+\cdots+1=\frac{n(n-1)}{2}$

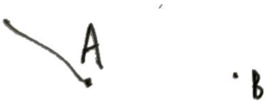
(b). For minimum, $\text{degree}_{min}(A)=0$, since we ~~must~~ don't need edge to connect

For maximum, we have every vertex connect to others

so $\text{degree}_{max}(A)=n-1$

(c). since G is a simple edge, so there's no two different edges connect same two vertices.

Assume A, B in the graph has two least number of degree



1° if no edge between A and B

$\deg(\min_A)=\frac{n-1}{2}$ $\deg(\min_B)=\frac{n-1}{2}$ $\frac{n-1}{2}+\frac{n-1}{2}=n-1>n-2$

so there must exists a point A and B both have edge on.

2° if there is a edge between then A can reach B.

For every ~~edge~~ vertices has ~~two~~ greater edge. there must be a path between

so. G must be connected.

Q.2:

1° since K_n is complete map $\overline{K_n}$ is isolated vertices. contained by with edges

2° since $K_{m,n}$ contains all edges between two maps

~~with~~ which contains m, n vertices respectively

so $\overline{K_{m,n}} = K_m \cup K_n$ contains all K_m edges and K_n edges

$(m+n)$ vertices

3° $\overline{C_n}$ consists all n $v_1 \cdots v_n$

and every edges. $\{v_1, v_3\}, \{v_1, v_4\} \cdots \{v_1, v_{n-1}\}$.

$\{v_2, v_4\} \cdots \{v_2, v_n\}$.

$\{v_3, v_5\} \cdots \{v_3, v_n\}$. $\{v_3, v_n\}$.

$\{v_{n-2}, v_n\}$.

$(n-3)+(n-3)+(n-2)+\cdots+1$

$= n-3+\frac{(n-2)(n-3)}{2}$

$= n-3+\frac{n^2-5n+6}{2}$

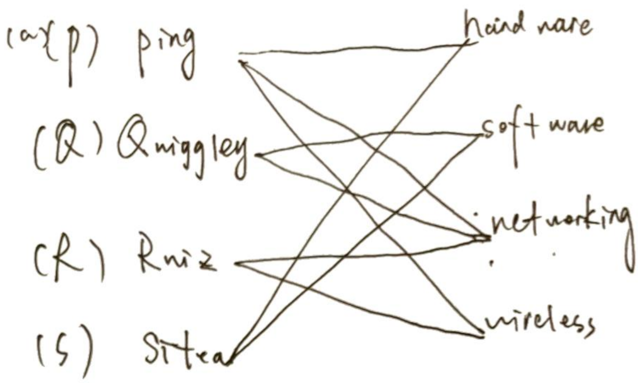
$= \frac{n^2}{2}-\frac{3}{2}n$. edges

4° $\overline{Q_n}$ is a graph contained by n vertices

Two vertices are adjacent if and only if the bit string represented by binary differs more than one bit.

Q3.

(a)



(b) For every subsets A of V_1 all satisfy $|N(A)| \geq |A|$

1° if $|A| = 1$ satisfy

2° if $|A| = 2$ if we chose $\{P, Q\}$ ✓ $\{Q, R\}$ ✓

if $\{P, R\}$ ✓ $\{Q, S\}$ ✓

if $\{P, S\}$ ✓ $\{R, S\}$ ✓

3° if $|A| = 3$ if $\{P, Q, R\}$ ✓ if $\{P, Q, S\}$ ✓ $\{P, R, S\}$ ✓

if $\{Q, R, S\}$ ✓

4° if $|A| = 4$ ✓

so according to Hall's marriage theorem, there must be a complete match

Ping — hardware Rwiz — wireless.

Quiggley — networking Sttea — software

Q4. since G and $\overline{G}$ is isomorphic and $G \cup \overline{G} = K_n$

K_n has n vertices and $\frac{n(n-1)}{2}$ edges so G and $\overline{G}$ must both have $\frac{n(n-1)}{4}$ edges

and $\frac{n(n-1)}{4}$ must be an Integer

so $n \mid 4$ or $(n-1) \mid 4$ so $n \equiv 1$ or $0 \pmod 4$.

Q5

(a). if: when $u=v$ $dis(u,v)=0 \geq 0$ so proved

only if: if $dist(u,v) \geq 0$ and $dist(u,v)=0$.

any $dis(u,v) \geq 0$ positive edges so $u=v$

(b) $dist(u,v) = dist(u,w_1) + dist(w_1,w_2) + \dots + dist(w_n,v)$

~~since it's~~ so we can definitely find a way back

$dist(v,u) = dist(v,w_n) + \dots + dist(w_1,u)$ so $dist(u,v) = dist(v,u)$

(c) Let's prove it by contradiction

if $dist(u,v) > dist(u,w) + dist(w,v)$

then $dist(u,v)$ should be replaced by RHS.

so it's not the minimum length of path from u to v.

so $dist(u,v) \leq dist(u,w) + dist(w,v)$.

Q.6. (a) prove it by contradiction if we have circles of length 1

then $x \to x$ not true

if we have length 2 then $x \to y$ $y \to x$ so it doesn't exist

so R-B tournament cannot have cycles of length 1 or 2

(b) antisymmetric: always

reflexive : never

irreflexive : always.

transitive : ~~never~~ sometimes

(c). only if: since antisymmetric irreflexive is always true.

if it's transitive so $A \to B$, $B \to C$ implies $A \to C$ for every pair. $A \to C$

$C \to A$ can't both exist so no cycles of 3 ~~every elements $x \leq y$ is for sure so it's strict total ordering~~

if: since ~~it's transitive~~ no circles of 3

then $x \to y$, $y \to z$ must ~~contain~~ not contain $z \to x$ since there must be connection between x and z so $x \to z$

so it's transitive

and we can have every $x \leq y$ clearly so it's comparable.

Q7.

since the edge is not contained in any simple circuit,

As the graph is connected simple graph. so it connects two seperate sub-connected graph, it's like a bridge edge. So according to the concept of cut edge. With the removal of the edge, there will be disconnected sub-graph (more connected component)

So it's a cut edge

Q.8 since G is regular every vertix has same degree every vertex has Assume n degree

Consider two endpoint A and B

edge between A and B is vertex in $L(G)$.

and it has $2(n-1) = 2n-2$ neighbours which means

every vertex in $L(G)$ $(2n-2)$ ~~vertex~~ degree $2 \mid 2n-2$ so

According to theorem, every vertex has even degree, It has an Euler circuit

Q.9. Let's prove it by induction.

Q_1: it definitely has. a Hamilton circuit

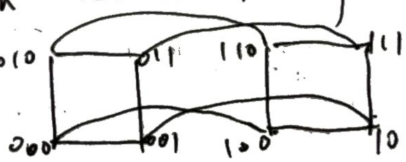
Let's assume. Q_n has one Hamilton circuit

So we have to prove Q_{n+1} also has one Hamilton circuit

we have to create Q_{n+1} by Q_n by the following steps:

we add "0" before every vertex of Q_n.

and we add "1" before for another Q_n and connect every two vertex only different in the first digit like:



by doing these, we have Q_{n+1}

we go along Q_n's Hamilton circuit and go to another Q_n through the connected bridge and pass the same circuit in another direction,

So we must have a Hamilton circuit.

Q10. According to Euler's formula.

$r+v-2=e$. (planar simple graph)

$\sum d(\text{regions}) = 2e \geq 3r$. (degree of every region is at least 3)

$2r+2v-4 \geq 3r$. $r \leq 2v-4$

$e \leq 3v-6$.

Graph with m edges n vertices. $\overline{G}$ has $\frac{n(n-1)}{2}-m$ edges.

$\frac{n(n-1)}{2}-m \leq 3n-6$ $\quad \frac{n(n-1)}{2} \leq 3n+m-6$ and $m \leq 3n-6$

$\frac{n(n-1)}{2} \leq 6n-12$.

$\frac{n^2}{2}-\frac{n}{2}-6n+12 \leq 0$ $\quad n^2-13n+24 \leq 0$

$n \leq 10$.

so G with at least 11 vertices

G or $\overline{G}$ is nonplanar.

Q11. since no simple circuits of lenght 4 or less

so every regions has degree(r) ≥ 5

$\sum$ degree $= 2e \geq 5r$

$r+v-2=e$. $\leftarrow$ so $r \leq \frac{2}{5}e$.

~~$e \leq 3v-6$~~

$r = e+2-v \leq \frac{2}{5}e$ $\quad \frac{3}{5}e \leq v-2$ $\quad e \leq \frac{5}{3}v-\frac{10}{3}$ $(v \geq 4)$

Q12. Since there are 17 students.

Let's assume. one student. discuss question, so. he at least discuss the same question with other 6 students, let's assume it's question 1.

1° if any two of the 6 students discuss. question 1.

so. ~~at least~~ proof done

2° let's see one of the students. who will only discuss 2 or 3.

so he will at least discuss the same question with other 3

let's assum it's question 2; so there will be a 4-size cycle. students

if it's question, then there will be 3-size cycle so. proof done

Q.13

(a) T_1 . T_2 . T_3  T_4  T_5 

$V(1)=1$ $V(2)=1$ $V(3)=3$. $V(4)=$

$V(n)= V(n-1)+V(n-2)+1$. 2 v 6

$\underbrace{V(n)+1}_{P(n)} = (V(n-1)+1)+(V(n-2)+1)$

$P(n)$ $P(n)=2f_k$.

$x^2 = x+1+1 >$ $x^2-x-2=0$ $(x-2)(x+1)=0$

$V(n) = \alpha 2^n + \beta\cdot(-1)^n$. $V(1)= 2\alpha-\beta=1$ $6\alpha = 2$ $\alpha=\frac{1}{3}$ $V(n)=2f(n)-1$

$V(2) = 4\alpha+\beta=1$ $\beta=-\frac{1}{3}$

$V(n)=\frac{1}{3}2^n-\frac{1}{3}(-1)^n = \frac{1}{3}(2^n-(-1)^n)$

$L(1)=1$ $L(2)=1$ $L(3)=2$ $L_4=3$ $L_5=5$ $L(n)=f(n)$

$\cancel{L(n)=L(n-1)+L(n-2)}$

$\cancel{x^2=x+1 \quad x^2-x-1=0 \quad x=\frac{1\pm\sqrt5}{2} \quad \alpha\left(\frac{1-\sqrt5}{2}\right)^n+\beta\left(\frac{1+\sqrt5}{2}\right)^n} = L(n)$

$\cancel{L(1)=\alpha\frac{1-\sqrt5}{2}+\beta\frac{1+\sqrt5}{2}=1}$

$\cancel{\therefore L(n)=\frac{\sqrt5}{5}\left(\frac{1+\sqrt5}{2}\right)^n-\frac{\sqrt5}{5}\left(\frac{1-\sqrt5}{2}\right)^n}$. $\cancel{\frac{1}{2}(\alpha+\beta)+\frac{\sqrt5}{2}(\beta-\alpha)=1}$. $\alpha+\beta=0$ $\beta-\alpha=\frac{2\sqrt5}{5}$

$\cancel{L(2)=\frac{6-2\sqrt5}{4}\alpha+\frac{6+2\sqrt5}{4}\beta=1}$ $\beta=\frac{\sqrt5}{5}$ $\alpha=-\frac{\sqrt5}{5}$

$\cancel{\frac{3}{2}(\alpha+\beta)+\frac{\sqrt5}{2}(\beta-\alpha)=1}$

$I(n)=V(n)-L(n)$.

$= \cancel{\frac{1}{3}(2^n-(-1)^n)-\frac{\sqrt5}{5}\left(\frac{1+\sqrt5}{2}\right)^n+\frac{\sqrt5}{5}\left(\frac{1-\sqrt5}{2}\right)^n} = 2f(n)-1-f(n)=f(n)-1$

(b) $T(n) = \begin{cases} 0, & n=1,2. \\ n-2, & n\geq 2 \end{cases}$

since when $n\geq 3$. root's left sub tree provide the maximum depth, which is exactly $T(n-1)$ and so $T(n)=T(n-1)+1$.

Q.14. $32 * 2 \uparrow 53-84/*-$.

$\underbrace{32*}_{6}$ 36 | 2. 2. | 4. | 32

so ans = 32.

Q.15

(a) we start from a.

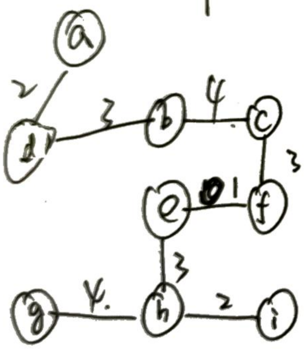
~~a~~	0
~~b~~	INF → 5
~~c~~	INF → 9
~~d~~	INF → 2
~~e~~	INF → 9
~~f~~	INF → 11 → 10
~~g~~	INF → 8
h	INF → 10
i	INF

~~so after~~

every time we choose the min path from the list ~~then~~ queue-out a vertex every time and add it to the list if the path is shorter. so a→f is

a→d→e→f.

(b) we start from vertex a.



a	0	✓
b	5→3	✓
c	4	✓
d	2	✓
e	7→5→1	✓
f	6→3	✓
g	6→4	✓
h	8→10→3	✓
i	4→2	✓