

Chapter 2: Regular Expressions & Lexical Analysis

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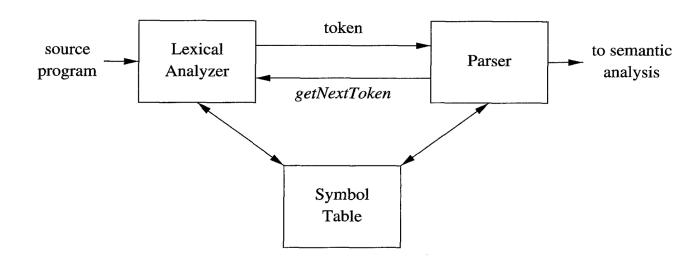
The chapter numbering in lecture notes does not follow that in the textbook.

Outline

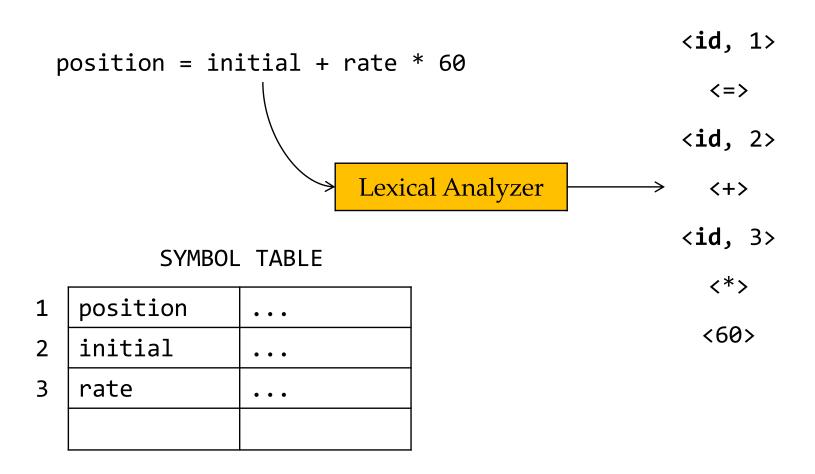
- The Role of Lexers: Recognizing Tokens
- Regular Expressions (for specifying tokens)
- Finite Automata (for recognizing patterns)

The Role of Lexical Analyzer

- Read the input characters of the source program, group them into lexemes, and produces a sequence of tokens
- Add lexemes into the symbol table when necessary



The Role of Lexical Analyzer



Tokens, Patterns, and Lexemes

- A *lexeme* is a string of characters that is a lowest-level syntactic unit in programming languages
- A *token* is a syntactic category representing a class of lexemes. Formally, it is a pair <token name, attribute value>
 - Token name: an abstract symbol representing the kind of the token
 - Attribute value (optional) points to the symbol table
- Each token has a particular *pattern*: a description of the form that the lexemes of the token may take

Examples

TOKEN	INFORMAL DESCRIPTION	SAMPLE LEXEMES
if	characters i, f	if
else	characters e, 1, s, e	else
${f comparison}$	<pre>< or > or <= or >= or !=</pre>	<=, !=
\mathbf{id}	letter followed by letters and digits	pi, score, D2
${f number}$	any numeric constant 3.14159, 0, 6.02e2	
literal	anything but ", surrounded by "'s	"core dumped"

Consider the C statement: printf("Total = %d\n", score);

Lexeme	printf	score	"Total = %d\n"	(• • •
Token	id	id	literal	left_parenthesis	• • •

Attributes for Tokens

- When more than one lexeme match a pattern, the lexical analyzer must provide additional information, named *attribute values*, to the subsequent compiler phases
 - Token names influence parsing decisions
 - Attribute values influence semantic analysis, code generation etc.
- For example, an **id** token is often associated with: (1) its lexeme, (2) type, and (3) the location at which it is first found. Token attributes are stored in the symbol table.

Lexical Errors

• When none of the patterns for tokens match any prefix of the remaining input

• Example: int 3a = a * 3;

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Specification of Tokens

- Regular expression (正则表达式, regexp for short) is an important notation for specifying lexeme patterns
- Content of this part
 - Strings and Languages (串和语言)
 - Operations on Languages (语言上的运算)
 - Regular Expressions
 - Regular Definitions (正则定义)
 - Extensions of Regular Expressions

Strings and Languages

- Alphabet (字母表): any finite set of symbols
 - Examples of symbols: letters, digits, and punctuations
 - Examples of alphabets: {1, 0}, ASCII, Unicode
- A **string** (串) over an alphabet is a <u>finite</u> sequence of symbols drawn from the alphabet
 - The length of a string s, denoted |s|, is the number of symbols in s (i.e., cardinality)
 - Empty string (空串): the string of length 0, ϵ

Terms (using banana for illustration)

- Prefix (前缀) of string s: any string obtained by removing 0 or more symbols from the end of s (ban, banana, ϵ)
- Proper prefix (真前缀): a prefix that is not ϵ and not s itself (ban)
- Suffix (后缀): any string obtained by removing 0 or more symbols from the beginning of s (nana, banana, ϵ).
- Proper suffix (真后缀): a suffix that is not ϵ and not equal to s itself (nana)

Terms Cont.

- Substring (子串) of s: any string obtained by removing any prefix and any suffix from s (banana, nan, ϵ)
- Proper substring (真子串): a substring that is not ϵ and not equal to s itself (nan)
- Subsequence (子序列): any string formed by removing 0 or more not necessarily consecutive symbols from *s* (bnn)



How many substrings & subsequences does banana have?

(Two substrings are different if they have different start/end index)

String Operations (串的运算)

- **Concatenation** (连接): the concatenation of two strings *x* and *y*, denoted *xy*, is the string formed by appending *y* to *x*
 - x = dog, y = house, xy = doghouse
- Exponentiation (幂/指数运算): $s^0 = \epsilon$, $s^1 = s$, $s^i = s^{i-1}s$
 - x = dog, $x^0 = \epsilon$, $x^1 = dog$, $x^3 = dogdogdog$

Language (语言)

- A **language** is any **countable set**¹ of strings over some fixed alphabet
 - The set containing only the empty string, that is $\{\epsilon\}$, is a language, denoted \emptyset
 - The set of all grammatically correct English sentences
 - The set of all syntactically well-formed C programs

¹ In mathematics, a countable set is a set with the same cardinality (number of elements) as some subset of the set of natural numbers. A countable set is either a finite set or a countably infinite set.

Operations on Languages (语言的运算)

• 并,连接,Kleene闭包,正闭包



Stephen C. Kleene

OPERATION 4	DEFINITION AND NOTATION
$Union ext{ of } L ext{ and } M$	$L \cup M = \{s \mid s \text{ is in } L \text{ or } s \text{ is in } M\}$
$Concatenation ext{ of } L ext{ and } M$	$LM = \{ st \mid s \text{ is in } L \text{ and } t \text{ is in } M \}$
$Kleene\ closure\ { m of}\ L$	$L^* = \cup_{i=0}^{\infty} L^i$
$Positive\ closure\ { m of}\ L$	$L^+ = \cup_{i=1}^{\infty} L^i$

The exponentiation of L can be defined using concatenation. L^n means concatenating L n times.

https://en.wikipedia.org/wiki/Stephen_Cole_Kleene

Examples

• $L = \{A, B, ..., Z, a, b, ..., z\}$ 52 English letters

•
$$D = \{0, 1, ..., 9\}$$
 10 digits

LUD	{A, B,, Z, a, b,, z, 0, 1,,9}	
LD	the set of 520 strings of length two, each consisting of one letter followed by one digit	
L^4	the set of all 4-letter strings	
L*	the set of all strings of letters, including ϵ	
$L(L \cup D)^*$?	
D ⁺	?	

Note: L, D might seem to be the alphabets of letters and digits. We define them to be languages: all strings happen to be of length one.

Regular Expressions - For Describing Languages/Patterns

Rules that define regexps over an alphabet Σ :

- **BASIS**: two rules form the basis:
 - ϵ is a regexp, $L(\epsilon) = {\epsilon}$
 - If a is a symbol in Σ , then a is a regexp, and $L(a) = \{a\}$
- **INDUCTION:** Suppose **r** and **s** are regexps denoting the languages L(**r**) and L(**s**)
 - (r) | (s) is a regexp denoting the language $L(r) \cup L(s)$
 - (r)(s) is a regexp denoting the language L(r)L(s)
 - (r)* is a regexp denoting (L(r))*
 - (r) is a regexp denoting L(r), that is, additional parentheses do not change the language an expression denotes.

Regular Expressions Cont.

- Following the rules, regexps often contain unnecessary pairs of parentheses. We may drop some if we adopt the conventions:
 - Precedence (优先级): closure * > concatenation > union |
 - Associativity (结合性): All three operators are left associative, meaning that operations are grouped from the left.
 - o For example, a | b | c would be interpreted as (a | b) | c
- Example: (a) \mid ((b)*(c)) can be simplified as a \mid b*c

Regular Expressions Examples

- Let $\Sigma = \{a, b\}$
 - a | b denotes the language {a, b}
 - (a|b)(a|b) denotes {aa, ab, ba, bb}
 - \blacksquare a* denotes { ϵ , a, aa, aaa, ...}
 - (a|b)* denotes the set of all strings consisting of 0 or more a's or b's: { ϵ , a, b, aa, ab, ba, bb, aaa, ...}
 - a la*b denotes the string a and all strings consisting of 0 or more a's and ending in b: {a, b, ab, aab, aaab, ...}

Regular Language (正则语言)

- A **regular language** is a language that can be defined by a regexp
- If two regexps r and s denote the same language, they are *equivalent*, written as r = s

(a|b)(a|b)

=

aa|ab|ba|bb

?

Algebraic Laws

• Each law below asserts that expressions of two different forms are equivalent

LAW	DESCRIPTION	
r s=s r	is commutative	
r (s t) = (r s) t	is associative	
r(st) = (rs)t	Concatenation is associative	
r(s t) = rs rt; (s t)r = sr tr	Concatenation distributes over	
$\epsilon r = r\epsilon = r$	ϵ is the identity for concatenation	
$r^* = (r \epsilon)^*$	ϵ is guaranteed in a closure	
$r^{**} = r^*$	* is idempotent	

| can be viewed as + in arithmetics, concatenation can be viewed as \times , * can be viewed as the power operator.

Regular Definitions (正则定义)

• For notational convenience, we can give names to certain regexps and use those names in subsequent expressions

If Σ is an alphabet of basic symbols, then a *regular definition* is a sequence of definitions of the form:

$$d_1 \rightarrow r_1$$

$$d_2 \rightarrow r_2$$

$$\cdots$$

$$d_n \rightarrow r_n$$

where:

- Each d_i is a new symbol not in Σ and not the same as the other d's
- Each r_i is a regexp over the alphabet $\Sigma \cup \{d_1, d_2, ..., d_{i-1}\}$

Each new symbol denotes a regular language. The second rule means that you may reuse previously-defined symbols.

Examples

Regular definition for C identifiers

Regexp for C identifiers

```
(A|B|...|Z|a|b|...|z|_)((A|B|...|Z|a|b|...|z|_)|(0|1|...|9))*
```

Extensions of Regular Expressions

- **Basic operators:** union |, concatenation, and Kleene closure * (proposed by Kleene in 1950s)
- A few **notational extensions**:
 - One or more instances: the unary, postfix operator *

$$\circ r^+ = rr^*, r^* = r^+ \mid \epsilon$$

Zero or one instance: the unary postfix operator?

$$\circ r? = r \mid \epsilon$$

Character classes: shorthand for a logical sequence

$$\circ [a_1 a_2 ... a_n] = a_1 | a_2 | ... | a_n$$

$$\circ [a-e] = a | b | c | d | e$$

• The extensions are only for notational convenience, they do not change the descriptive power of regexps

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 - NFA & DFA
 - NFA → DFA
 - Regexp → NFA
 - Combining NFAs

Finite Automata (有穷自动机)

- Finite automata are the simplest machines to recognize patterns
- They take a string as input and output "yes" (pattern is matched) or "no" (pattern is unmatched).
 - Nondeterministic finite automata (NFA, 非确定有穷自动机): A symbol can label several edges out of the same state (allowing multiple target states), and the empty string ϵ is a possible label.
 - Deterministic finite automata (DFA, 确定有穷自动机): For each state and for each symbol in the input alphabet, there is exactly one edge with that symbol leaving that state.
- NFA and DFA recognize the same languages, regular languages, which regexps can describe.

Nondeterministic Finite Automata

- An **NFA** is a 5-tuple, consisting of:
 - 1. A finite set of states *S*
 - 2. A set of input symbols Σ , the *input alphabet*. We assume that the empty string ϵ is never a member of Σ
 - 3. A *transition function* that gives, for each state, and for each symbol in $\Sigma \cup \{\epsilon\}$ a set of *next states*
 - 4. A *start state* (or initial state) s_0 from S
 - 5. A set of *accepting states* (or *final states*) *F*, a subset of *S*

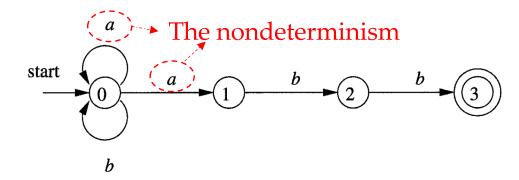
NFA Example

•
$$S = \{0, 1, 2, 3\}$$

The NFA can be represented as a Transition Graph:

•
$$\Sigma = \{a, b\}$$

• Start state: 0



- Accepting states: {3}
- Transition function

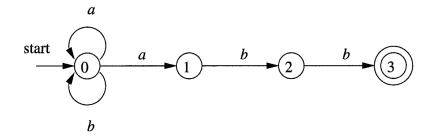
■
$$(0, a) \rightarrow \{0, 1\}$$
 $(0, b) \rightarrow \{0\}$

$$(0, b) \rightarrow \{0\}$$

■
$$(1, b) \rightarrow \{2\}$$
 $(2, b) \rightarrow \{3\}$

$$(2, b) \rightarrow \{3\}$$

Transition Table

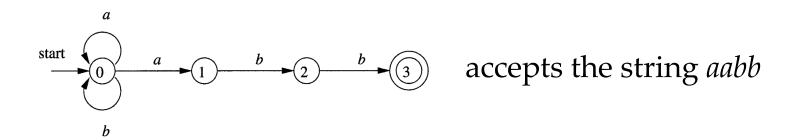


- Another representation of an NFA
 - Rows correspond to states
 - Columns correspond to the input symbols or ϵ
 - The table entry for a <u>state-input pair</u> lists the set of next states
 - Ø: the transition function has no information about the state-input pair (the move is not allowed)

STATE	$\langle \overline{a} \rangle$	b	ϵ
(0)	$\{0,1\}$	{0}	Ø
1	Ø	$\{2\}$	Ø
2	Ø	$\{3\}$	Ø
3	Ø	Ø	Ø

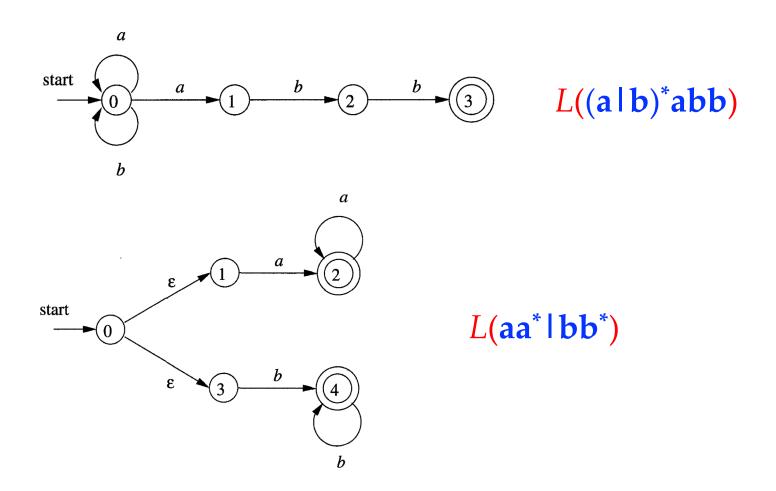
Acceptance of Input Strings

- An NFA accepts an input string x if and only if
 - There is a path in the transition graph from the start state to one accepting state, such that the symbols along the path form x (ϵ labels are ignored).



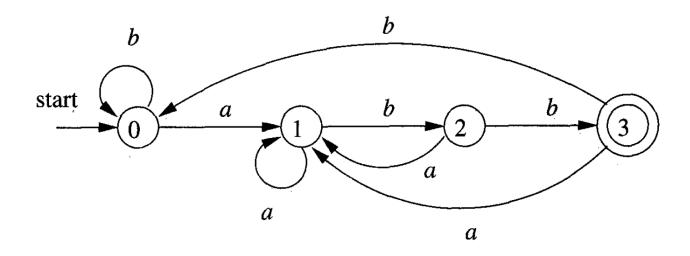
- The language defined or accepted by an NFA
 - The set of strings labelling some path from the start state to an accepting state

NFA and Regular Languages



Deterministic Finite Automata (DFA)

- A **DFA** is a special NFA where:
 - There are no moves on input ϵ
 - For each state *s* and input symbol *a*, there is exactly one edge out of *s* labeled *a* (i.e., exactly one target state)



Simulating a DFA

- Input:
 - String *x* terminated by an end-of-file character **eof**.
 - DFA D with start state s_0 , accepting states F, and transition function move
- **Output:** Answer "yes" if *D* accepts *x*; "no" otherwise

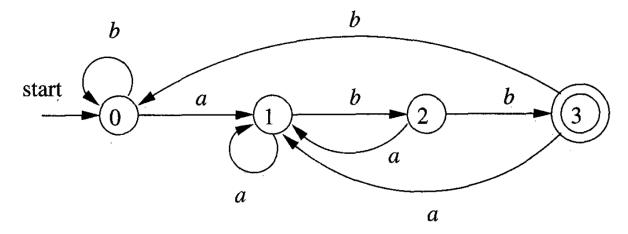
```
s = s<sub>0</sub>;
c = nextChar();
while ( c != eof ) {
    s = move(s, c);
    c = nextChar();
}
if ( s is in F ) return "yes";
else return "no";
```

We can see from the algorithm:

• DFA can efficiently accept/reject strings (i.e., recognize patterns)

DFA Example

• Given the input string *ababb*, the DFA below enters the sequence of states 0, 1, 2, 1, 2, 3 and returns "yes"





What's the language defined by this DFA?

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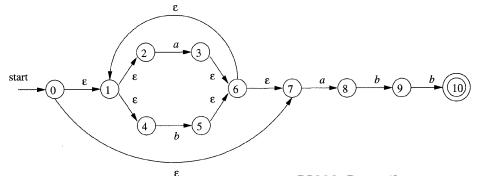
From Regular Expressions to Automata

- Regexps concisely & precisely describe the patterns of tokens
- DFA can efficiently recognize patterns (comparatively, the simulation of NFA is less straightforward*)
- When implementing lexical analyzers, regexps are often converted to DFA:
 - Regexp \rightarrow NFA \rightarrow DFA
 - Algorithms: Thompson's construction + subset construction

^{*} There may be multiple transitions at a state when seeing a symbol

Conversion of an NFA to a DFA

- The subset construction algorithm (子集构造法)
 - Insight: Each state of the constructed DFA corresponds to a set of NFA states
 - Why? Because after reading the input $a_1a_2...a_n$, the DFA reaches one state while the NFA may reach multiple states
 - Basic idea: The algorithm simulates "in parallel" all possible moves an NFA can make on a given input string to map a set of NFA states to a DFA state.

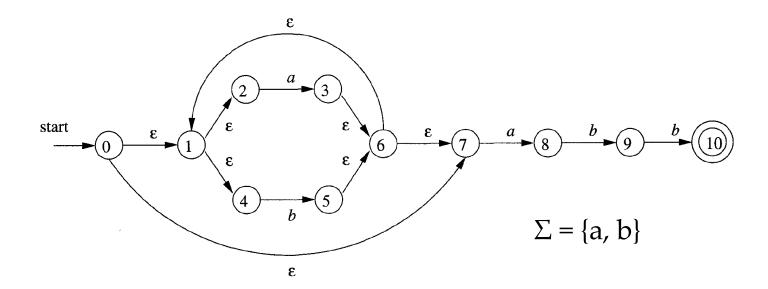


After reading "a", the NFA may reach any of these states:

3, 6, 1, 7, 2, 4, 8

Example for Algorithm Illustration

- The NFA below accepts the string babb
 - There exists a path from the start state 0 to the accepting state 10, on which the labels on the edges form the string *babb*



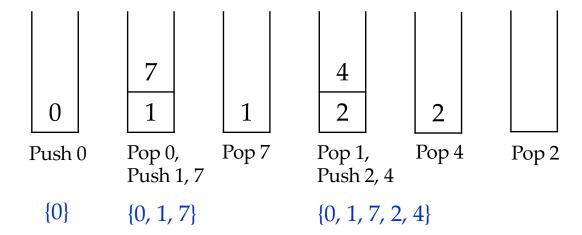
Subset Construction Technique

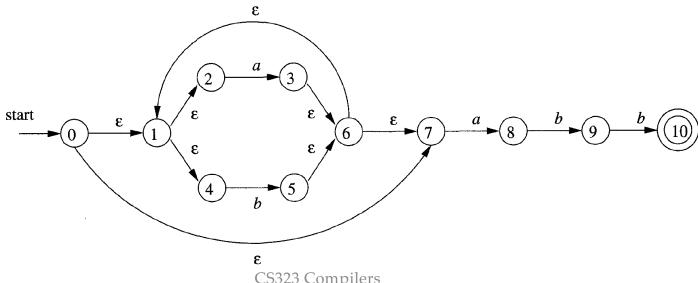
- Operations used in the algorithm:
 - ϵ -closure(s): Set of NFA states reachable from NFA state s on ϵ -transitions alone
 - ϵ -closure(T): Set of NFA states reachable from some NFA state s in set T on ϵ -transitions alone
 - o That is, $\bigcup_{s \text{ in } T} \epsilon closure(s)$
 - *move*(*T*, *a*): Set of NFA states to which there is a transition on input symbol *a* from some state *s* in *T* (i.e., the target states of those states in *T* when seeing *a*)

Subset Construction Technique

- Computing ϵ -closure(T)
 - It is a graph traversal process (only consider ϵ edges)
 - Computing ϵ -closure(s) is the same (when T has only one state)

• ϵ -closure(0) = ?

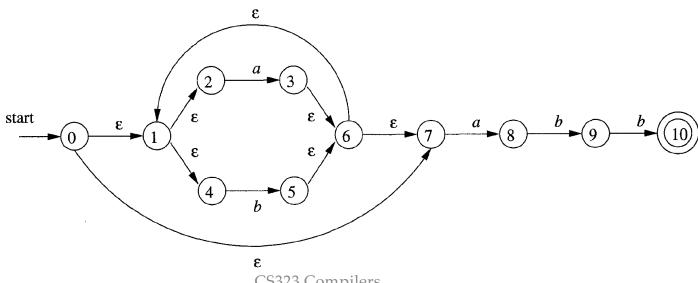




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Exercise

• ϵ -closure({3, 8}) = ?



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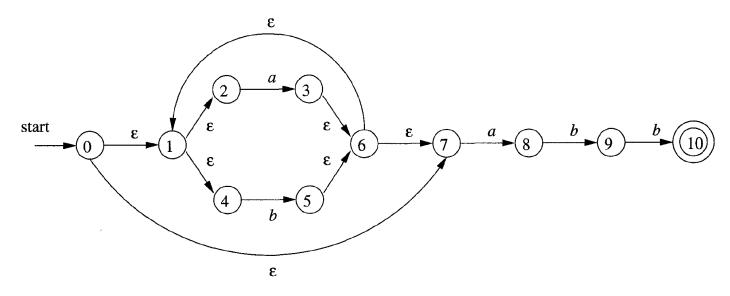
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Subset Construction Technique Cont.

- The construction of the DFA *D*'s states, *Dstates*, and the transition function *Dtran* is also a search process
 - Initially, the only state in *Dstates* is ϵ -closure(s_0) and it is unmarked
 - o Unmarked state means that its next states have not been explored

```
while ( there is an unmarked state T in Dstates ) { mark T; for ( each input symbol a ) { // find the next states of T U = \epsilon\text{-}closure(move(T, a)); if ( U is not in Dstates ) add U as an unmarked state to Dstates; Dtran[T, a] = U; }
```

- Initially, Dstates only has one unmarked state:
 - ϵ -closure(0) = {0, 1, 2, 4, 7} -- A
- Dtran is empty

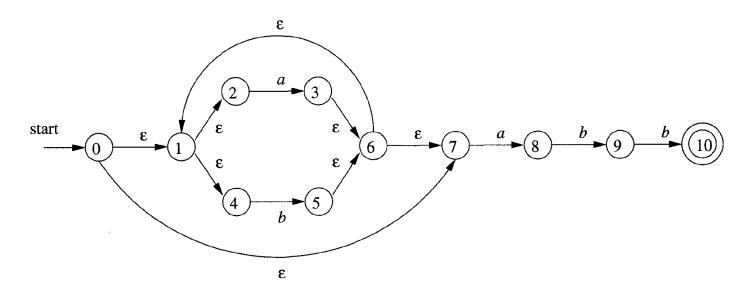


{0, 1, 2, 4, 7} -- A

 ϵ -closure(move[A, a])

- $= \epsilon$ -closure({3, 8})
- $= \{1, 2, 3, 4, 6, 7, 8\}$

- We get an unseen state {1, 2, 3, 4, 6, 7, 8} -- B
- Update Dstates: {A, B}
- Update Dtran: $\{[A, a] \rightarrow B\}$

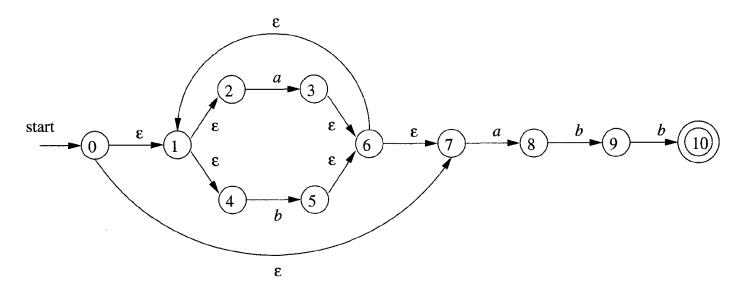


{0, 1, 2, 4, 7} -- A

 ϵ -closure(move[A, b])

- $= \epsilon$ -closure({5})
- $= \{1, 2, 4, 5, 6, 7\}$

- We get an unseen state {1, 2, 4, 5, 6, 7} -- C
- Update Dstates: {A, B, C}
- Update \overline{Dtran} : {[A, a] \rightarrow B, [A, b] \rightarrow C}

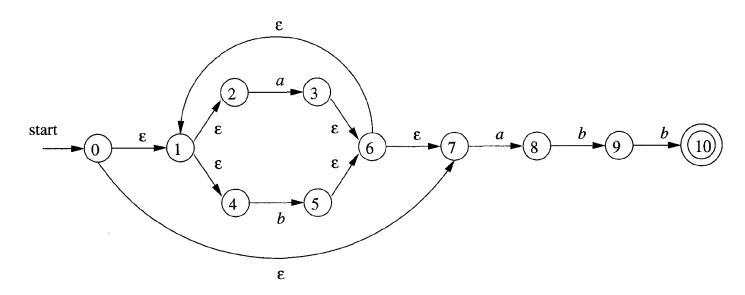


{1, 2, 3, 4, 6, 7, 8} -- B

 ϵ -closure(move[B, a])

- $= \epsilon$ -closure({3, 8})
- $= \{1, 2, 3, 4, 6, 7, 8\}$

- The state {1, 2, 3, 4, 6, 7, 8} already exists (B)
- No need to update Dstates: {A, B, C}
- Update Dtran: $\{[A, a] \rightarrow B, [A, b] \rightarrow C, [B, a] \rightarrow B\}$



- Eventually, we will get the following DFA:
 - Start state: A; Accepting states: {E}

NFA STATE	DFA STATE	a	b
$\{0, 1, 2, 4, 7\}$	\overline{A}	B	C
$\{1, 2, 3, 4, 6, 7, 8\}$	B	B	D
$\{1, 2, 4, 5, 6, 7\}$	C	$\mid B \mid$	C
$\{1, 2, 4, 5, 6, 7, 9\}$	D	B	E
$\{1, 2, 4, 5, 6, 7, 10\}$	E	B	C

This DFA can be further minimized: A and C have the same moves on all symbols and can be merged.

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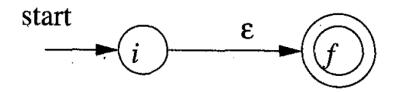
Regular Expression to NFA

Thompson's construction algorithm (Thompson构造法)

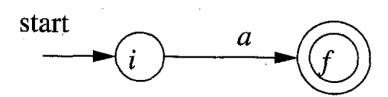
- The algorithm works recursively by splitting a regular expression into subexpressions, from which the NFA will be constructed using the following rules:
 - Two basis rules (基本规则): handle subexpressions with no operators
 - Three inductive rules (归纳规则): construct larger NFAs from the smaller NFAs for subexpressions

Two basis rules:

1. The empty expression ϵ is converted to

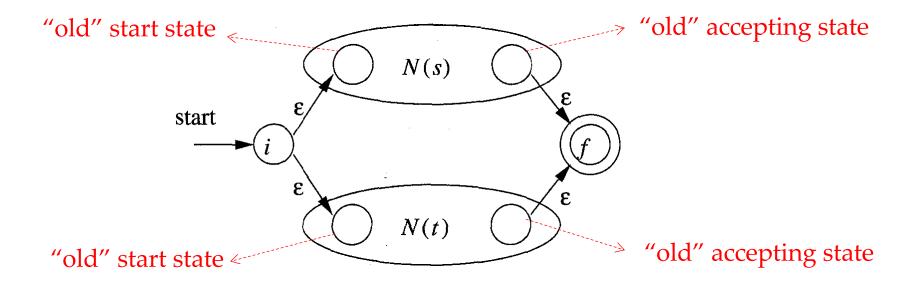


2. Any subexpression *a* (a single symbol in input alphabet) is converted to



The inductive rules: the union case

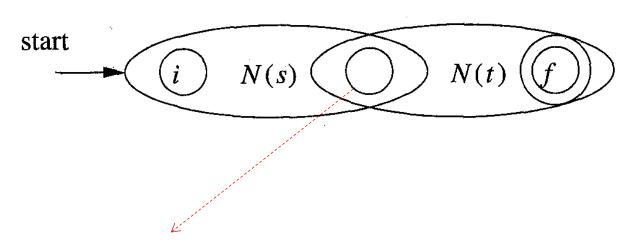
• $s \mid t : N(s)$ and N(t) are NFAs for subexpressions s and t



By construction, the NFAs have only one start state and one accepting state

The inductive rules: the concatenation case

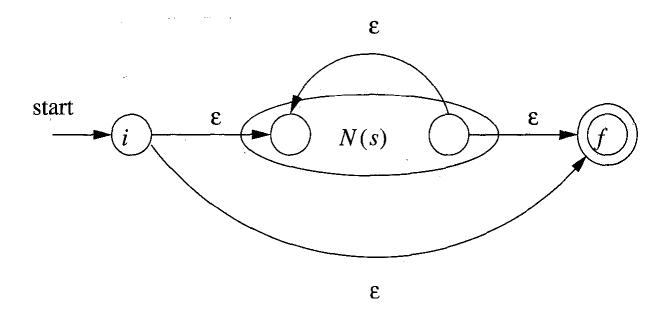
• st: N(s) and N(t) are NFAs for subexpressions s and t



Merging the accepting state of N(s) and the start state of N(t)

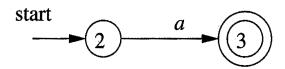
The inductive rules: the Kleene star case

• $s^*: N(s)$ is the NFA for subexpression s

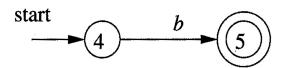


Use Thompson's algorithm to construct an NFA for the regexp $r = (\mathbf{a} \mid \mathbf{b})^* \mathbf{a}$

1. NFA for the first a (apply basis rule #1)

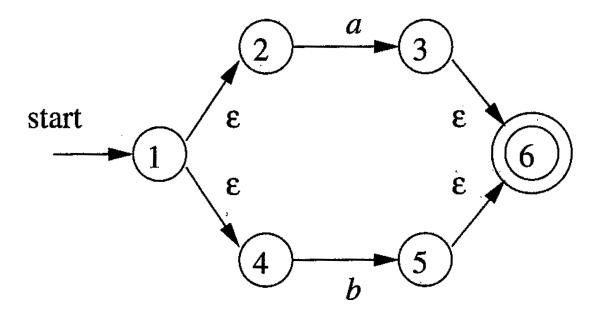


2. NFA for the first **b** (apply basis rule #1)



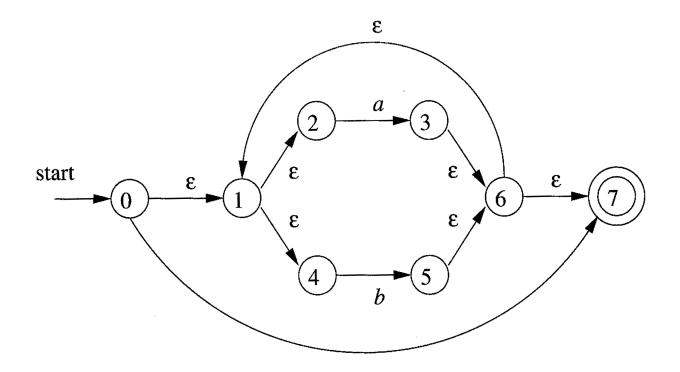
Example
$$r = (a|b)^*a$$

3. NFA for (a|b) (apply inductive rule #1)



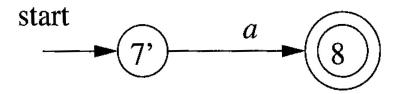
Example $r = (a|b)^*a$

4. NFA for (a|b)* (apply inductive rule #3)



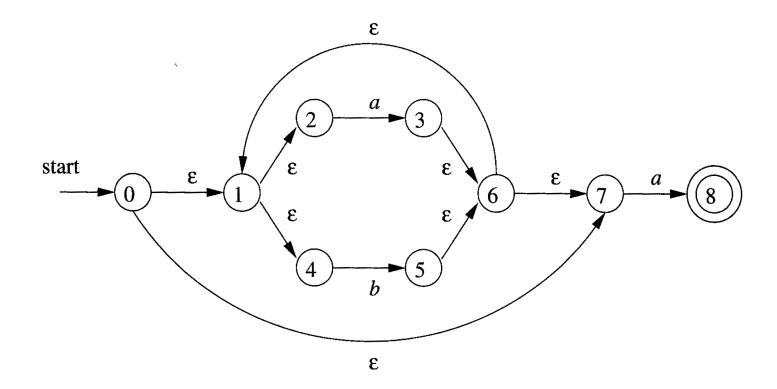
Example
$$r = (a|b)^*a$$

5. NFA for the second **a** (apply basic rule #1)



Example $r = (a|b)^*a$

6. NFA for (a|b)*a (apply inductive rule #2)

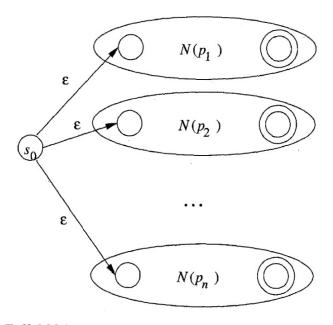


Outline

- The Role of Lexers: Recognizing Tokens
- Regular Expressions (for specifying tokens)
- Finite Automata (for recognizing patterns)
 - NFA & DFA
 - NFA → DFA
 - Regexp → NFA
 - Combining NFAs

Combining NFAs

- Why? In the lexical analyzer, we need a single automaton to recognize lexemes matching any pattern
- How? Introduce a new start state with ϵ -transitions to each of the start states of the NFAs for pattern p_i

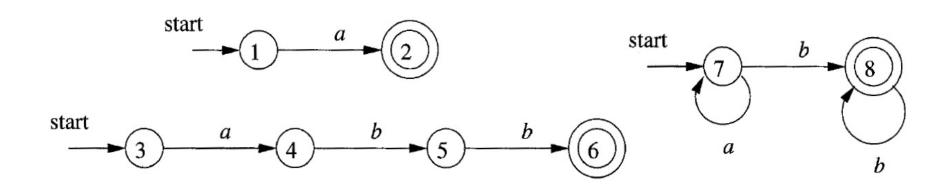


- The language that can be accepted by the big NFA is the union of the languages that can be accepted by the small NFAs
- Different accepting states correspond to different patterns

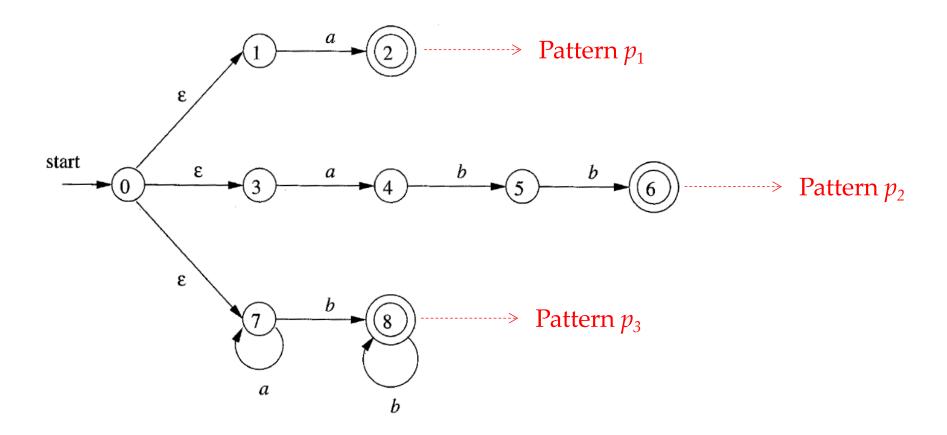
DFAs for Lexical Analyzers

- Convert the NFA for all the patterns into an equivalent DFA, using the subset construction algorithm
- An accepting state of the DFA corresponds to a subset of the NFA states, in which at least one is an accepting NFA state
 - If there are more than one accepting NFA state, this means that conflicts arise (the prefix of the input string matches multiple patterns)
 - Upon conflicts, find the first pattern whose accepting state is in the set and make that pattern the output of the DFA state

- Suppose we have three patterns: p_1 , p_2 , and p_3
 - **a** {action A_1 for pattern p_1 }
 - **abb** {action A_2 for pattern p_2 }
 - $\mathbf{a}^*\mathbf{b}^+$ {action A_3 for pattern p_3 }
- We first construct an NFA for each pattern

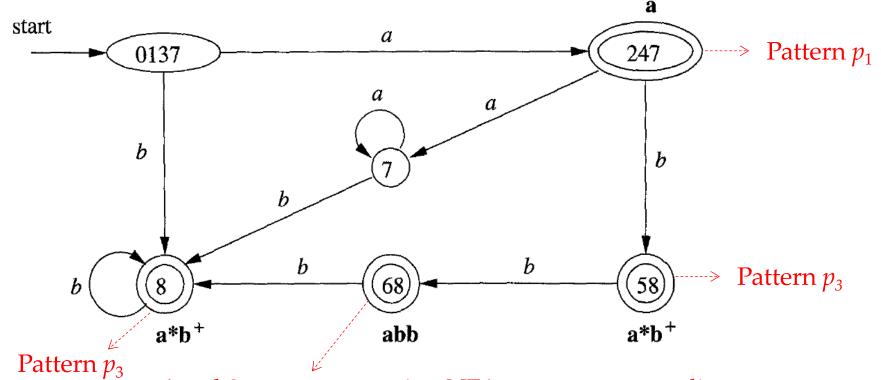


• Combining the three NFAs



start 0 ε 3 a 4 b 5 b 6

Converting the big NFA to a DFA



6 and 8 are two accepting NFA states corresponding to two patterns. We choose Pattern p2, which is specified before p3

Reading Tasks

- Chapter 3 of the dragon book
 - 3.1 The role of the lexical analyzer
 - 3.3 Specification of tokens
 - 3.4 Recognition of tokens (lab content)
 - 3.5 The lexical-analyzer generator Lex (lab content)
 - 3.6 Finite automata
 - 3.7 From regular expressions to automata
 - 3.8 Design of a lexical analyzer generator
 - \circ 3.8.1 3.8.3, the remaining can be skipped