Homework 1

Textbook reading

We are starting with part of Chapter 2 of Durrett. After a few days of learning counting techniques, we will go back to Chapter 1 and work more systematically through the book.

Please read Section 2.1 of the textbook, stopping when you get to Example 2.6. We will come back to Example 2.6 when we work on Chapter 1.

Example solutions

16 horses race in the Kentucky Derby. How many possible results are there for win, place, and show (first, second, and third)? (3,360)

There are 16 possibilities for the horse in first place. Whatever horse is in first place, there are 15 possibilities remaining for second place. Given the horses in first and second place, there are 14 possibilities remaining for third place. By the multiplication rule, there are (16)(15)(14) = 3360 total possible results.

Durrett's notation for this type of counting is $P_{16,3}$, the number of permutations three objects from a set of size 16.

In a class of 19 students, 7 will get A's. In how many ways could we pick these students? (50,388)

We choose 7 students out of the 19 students in the class. The order that we pick the students in does not matter. There are (19)(18)(17)(16)(15)(14)(13) / 7! = 50388 ways to do this.

Durrett's notation for this kind of counting is $C_{19,7}$, the number of combinations of 7 things taken from a set of 19.

Homework questions

For each of the following questions, explain how to count the described things.

- Give your method for counting, following the model of my example solutions above.
- If any of the numbers you use in your counting match Durrett's special notations $P_{n,k}$ or $C_{n,k}$, give those notations, as in my example solutions.
- I did the counting for you in each exercise. Your job is to explain the reasoning to get the given numbers.

How many possible batting orders are there for nine baseball players? (362,880)

A batting order is a permutation of all nine players (nine objects from a set of nine). There are $P_{9,9}$ = 9! batting orders. We have 9! = 362,880.

A tourist wants to visit six of America's ten largest cities. In how many ways can she do this if the order of her visits is (a) important or (b) not important. (part (a) 151,200; part (b) 210)

(a) Picking six cities in order is a permutation of six objects (cities) taken from a set of ten. There are $P_{10,6}$ = 151,200 such permutations.

(b) Picking six cities without an order is the same as choosing six objects (the cities to visit) out of a set of ten. There are $C_{10,6}$ = 210 ways to do that.

How many license plates are possible if the first three places are occupied by letters and the last three by numbers? (17,576,000) How many are possible if the three letters are all different and the three numbers are all different? (11,232,000)

We use the multiplication rule. There are six slots to fill, three with letters and three with digits.

In the first situation, we choose successively the three letters (26 choices at each stage) and then the three digits (10 choices at each stage). This gives us (26)(26)(26)(10)(10)(10) = 17,576,000 possibilities.

We could reason out the second situation using the multiplication rule in six stages. To practice with the concepts from Section 2.1, though we think of two stages. The first stage is picking three distinct letters in order. That is the same as picking a permutation of three objects from a set of 26. There are $P_{26,3}$ ways to do that. The second stage is picking three distinct digits in order. That is the same as picking a permutation of three objects from a set of 10. There are $P_{10,3}$ ways to do that. Combining these choices with the multiplication rule, we find a total of $P_{26,3}$ $P_{10,3}$ = (15600)(720) = 11,232,000 possibilities.

The Duke basketball team has 10 women who can play guard and 12 women who can play the other three positions. At the start of the game, the coach gives the referee a starting lineup that lists who will play left guard, right guard, left forward, center, and right forward. In how many ways can this be done? (118,800)

We use the multiplication rule. First we count ways the coach can pick the two guards (left and right—order matters). This choice is a permutation of two objects from a set of 10, and so there are $P_{10,2}$ possibilities. Then we count ways the coach can assign the other three positions. This choice is a permutation of three objects from a set of 12, and so there are $P_{12,3}$ possibilities. Combining these successive choices with the multiplication rule, we find $P_{10,2}$ $P_{12,3}$ = 118,800 total possibilities.

A basketball team has 5 players more than 6 feet tall and 6 who are less than 6 feet. How many ways can they have their picture taken, if the 5 taller players stand in a row behind the 6 shorter players, who are sitting on a row of chairs?

We use the multiplication rule. We first put the taller players in order and then put the shorter players in order. There are $P_{5,5} = 5!$ ways to put the taller players in order and $P_{6,6} = 6!$ ways to put the shorter players in order. Combining these gives $(5!)(6!) = 86{,}400$ ways to arrange the players for the picture.