

→ Métodos Numéricos para sistemas de EDOs.

1) Un problema de valor inicial de primer orden para un sistema de tamaño n se define como!

$\underbrace{\hspace{10em}}_{\text{"n-sistema"}}$

$$y_1' = \frac{dy_1}{dt} = f_1(t, y_1, \dots, y_n)$$

$$y_2' = \frac{dy_2}{dt} = f_2(t, y_1, \dots, y_n) \quad t \in (a, b)$$

\vdots

$$\frac{dy_n}{dt} = f_n(t, y_1, \dots, y_n)$$

$y_i : \mathbb{R} \rightarrow \mathbb{R} \quad i=1, \dots, n$ con condición inicial

$$y_1(a) = y_{1,0}$$

$$y_2(a) = y_{2,0}$$

\vdots

$$y_n(a) = y_{n,0}$$

2) Notación vectorial:

$$\underline{y} : \mathbb{R} \rightarrow \mathbb{R}^n, \quad \underline{y} = [y_1, y_2, \dots, y_n]^T$$

$$[\underline{y}]_i = y_i$$

(*) P.VI:

$$\begin{aligned} \underline{y}' &= \underline{f}(t, \underline{y}) \quad t \in (a, b) \\ \underline{y}(a) &= \underline{y}_0 \end{aligned}$$

3) Teorema de Taylor vectorial para una variable independiente

$$h = \frac{b-a}{N}, \quad N : \# \text{ subintervalos}$$

t_{i+1} , alrededor $\underline{t_i}$

$$\underline{y}(t_{i+1}) = \underline{y}(t_i) + h \underline{y}'(t_i) + \frac{h^2}{2!} \underline{y}''(t_i) + \dots$$

$$+ \frac{h^m}{m!} \underline{y}^{(m)}(t_i) + \frac{h}{(m+1)!} \underline{y}^{(m+1)}(\xi_i)$$

$$\underline{y}^{(k)} = \frac{d}{dt} [\underline{y}]$$

$$[\xi_i]_j \in (t_i, t_{i+1})$$

Suposición: y_j sea derivable $(m+1)$ veces. $j = 1, \dots, n$

3) Ej. método de Euler, método de Taylor de orden $m=1$

$$\begin{aligned} \underline{y}(t_{i+1}) &= \underline{y}(t_i) + h \underline{y}'(t_i) + \frac{h^2}{2} \underline{y}^{(2)}(\xi_i) \\ &= \underline{y}(t_i) + h \underline{f}(t_i, \underline{y}(t_i)) + \frac{h^2}{2} \underline{y}^{(2)}(\xi_i) \end{aligned}$$

• Truncamos el residuo

$$\begin{cases} \hat{\underline{y}}_0 = \underline{y}(a) \\ \hat{\underline{y}}_{i+1} = \hat{\underline{y}}_i + h \underline{f}(t_i, \hat{\underline{y}}_i) \end{cases} \quad i=1, \dots, N$$

$\underline{f}(t_i, \hat{\underline{y}}_{1,i}, \dots, \hat{\underline{y}}_{n,i})$

$$\begin{bmatrix} \hat{\underline{y}}_{1,i+1} \\ \vdots \\ \hat{\underline{y}}_{n,i+1} \end{bmatrix} = \begin{bmatrix} \hat{\underline{y}}_{1,i} \\ \vdots \\ \hat{\underline{y}}_{n,i} \end{bmatrix} + h \begin{bmatrix} \underline{f}_1(t_i, \hat{\underline{y}}_i) \\ \vdots \\ \underline{f}_n(t_i, \hat{\underline{y}}_i) \end{bmatrix}$$

donde $\hat{\underline{y}}_{k,i} :=$ es la aproximación de la solución
 $\underline{y}_k(t_i)$

4) método de Taylor de orden $m=2$.

$$\underline{y}(t_{i+1}) = \underline{y}(t_i) + h \underline{y}'(t_i) + \frac{h^2}{2} \underline{y}^{(2)}(t_i) + \frac{h^3}{3!} \underline{y}^{(3)}(\xi_i)$$

Truncamos el residuo

$$[\xi_i]; \in (t_i, t_{i+1})$$

$$\hat{\underline{y}}_0 = \underline{y}_0$$

$$\hat{\underline{y}}_{i+1} = \hat{\underline{y}}_i + h [\underline{f}(t_i, \hat{\underline{y}}_i) + \underline{f}'(t_i, \hat{\underline{y}}_i)]$$

5) Supongamos $n=2$

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} f_1(t, y_1, y_2) \\ f_2(t, y_1, y_2) \end{bmatrix}$$

$$\underline{f}' := \frac{d}{dt} [\underline{f}] = \begin{bmatrix} \frac{d}{dt} [f_1(t, y_1, y_2)] \\ \frac{d}{dt} [f_2(t, y_1, y_2)] \end{bmatrix}$$

$k=1,2$

$$= \frac{d}{dt} [f_k(t, y_1, y_2)] = \frac{\partial f_k}{\partial t} \cdot \frac{dt}{dt} + \frac{\partial f_k}{\partial y_1} \cdot \frac{dy_1}{dt} + \frac{\partial f_k}{\partial y_2} \cdot \frac{dy_2}{dt}$$

$$= \frac{\partial f_k}{\partial t} \cdot 1 + \frac{\partial f_k}{\partial y_1} \cdot y_1' + \frac{\partial f_k}{\partial y_2} \cdot y_2'$$

$$\begin{array}{c} \nearrow \quad \longleftarrow \quad \longleftarrow \\ \frac{\partial \underline{f}}{\partial t} \quad \quad \quad \underline{M} \cdot \underline{y}' \end{array}$$

$M \in \text{matriz } n \times n.$

5) Para n en grad, n es un entero.

$$\frac{d}{dt} f(t, \bar{y}) = \frac{\partial f}{\partial t} + \underbrace{\frac{\partial f}{\partial \underline{y}}}_{m} \underline{y}'$$

$$\frac{\partial f}{\partial \underline{y}} := \text{Jacobiana}$$

$$\left[\frac{\partial f}{\partial \underline{y}} \right]_{\underline{k}, \underline{l}} = \frac{\partial f_k}{\partial y_l}$$

$$\begin{aligned} \Rightarrow \left[\frac{\partial f}{\partial \underline{y}} \underline{y}' \right]_{\underline{l}} &= \sum_{k=1}^n \left[\frac{\partial f}{\partial \underline{y}} \right]_{\underline{l}, k} \cdot [\underline{y}']_k && \begin{array}{l} k : \text{ renglon} \\ l : \text{ columna} \end{array} \\ &= \sum_{k=1}^n \frac{\partial f_l}{\partial y_k} \cdot \frac{dy_k}{dt} && \left[\begin{array}{c} \text{---} \\ \left[\begin{array}{c} \vdots \\ \vdots \end{array} \right] \end{array} \right] \end{aligned}$$

6) $n=2$.

$$\left[\frac{\partial f}{\partial \underline{y}} \underline{y}' \right]_1 = \sum_{k=1}^2 \frac{\partial f_1}{\partial y_k} \cdot y'_k = \frac{\partial f_1}{\partial y_1} y'_1 + \frac{\partial f_1}{\partial y_2} y'_2$$

$$\left[\frac{\partial f}{\partial \underline{y}} \underline{y}' \right]_s = \sum_{k=1}^2 \frac{\partial f_s}{\partial y_k} \cdot y'_k \quad s=1, 2$$