7) Método del Trapecio.

$$\hat{\underline{y}}_{i+1}$$

(P)
$$\begin{cases} \hat{\underline{y}}_0 = y_0 \\ \hat{\underline{y}}_{i+1} = \hat{\underline{y}}_i + \frac{h}{2}\left[\underline{f}(t_i, \hat{\underline{y}}_i) + \underline{f}(t_{i+1}, \hat{\underline{y}}_{i+1}) \right] \end{cases}$$

$i = 0, \dots, N$

Método numérico de la forma siguiente:

$$\begin{cases} \hat{\underline{y}}_0 = \underline{y}_0 \\ \hat{\underline{y}}_{i+1} = \underline{G}_i(\underline{f}, h, \hat{\underline{y}}_0, \hat{\underline{y}}_1, \dots, \hat{\underline{y}}_i, \hat{\underline{y}}_{i+1}) \end{cases}$$

El error de truncamiento $\underline{\tau}_{i+1}(h)$ se define como:

$$\underline{\tau}_{i+1}(h) := \frac{1}{h}\left[\underline{y}(t_{i+1}) - \underline{G}_i(\underline{f}, h, \underline{y}_0, \underline{y}(t_1), \underline{y}(t_2), \dots \underline{y}(t_i), y(t_{i+1})) \right]$$

$$= O(h^r)$$

Decimos que una función g es de orden $O(h^r)$ si y solo si $\| \underline{g} \| \leq C h^r$

El error de truncamiento es de orden r

8) El error de truncamiento del método del trapecio es de orden $r = 2$.

$$(*1)\quad y(t_{i+1}) - G_i(f, h, y(t_i), y(t_{i+1}))$$

$$= y(t_{i+1}) - \Big\{ y(t_i) + \frac{h}{2}\Big[\underbrace{f(t_i, y(t_i))}_{y'(t_i)} + \underbrace{f(t_{i+1}, y(t_{i+1}))}_{y'(t_{i+1})}\Big]$$

$$= \cancel{y(t_i)} + h\,\cancel{y'(t_i)} + \frac{1}{2}h^2\,\cancel{y''(t_i)} + O(h^3)$$
$$- \Big\{ \cancel{y(t_i)} + \frac{h}{2}\Big[\cancel{y'(t_i)} + \big(\cancel{y'(t_i)} + h\,\cancel{y''(t_i)} + O(h^2)\big)\Big] \Big\}$$

$$= O(h^3) - \frac{h}{2}O(h^2) = O(h^3) + O(h^3)$$
$$= O(h^3)$$

$$\tau_{i+1} := \frac{1}{h}\Big[y(t_{i+1}) - G_i(f, h, y(t_i), y(t_{i+1}))\Big]$$

$$= \frac{1}{h}\big[O(h^3)\big] = O(h^2)$$