

→ Calculando la tasa de convergencia

método en diferencias en grad!

$$\begin{cases} y' = f(t, y) \\ y(a) = y_0 \\ t \in [a, b] \end{cases}$$

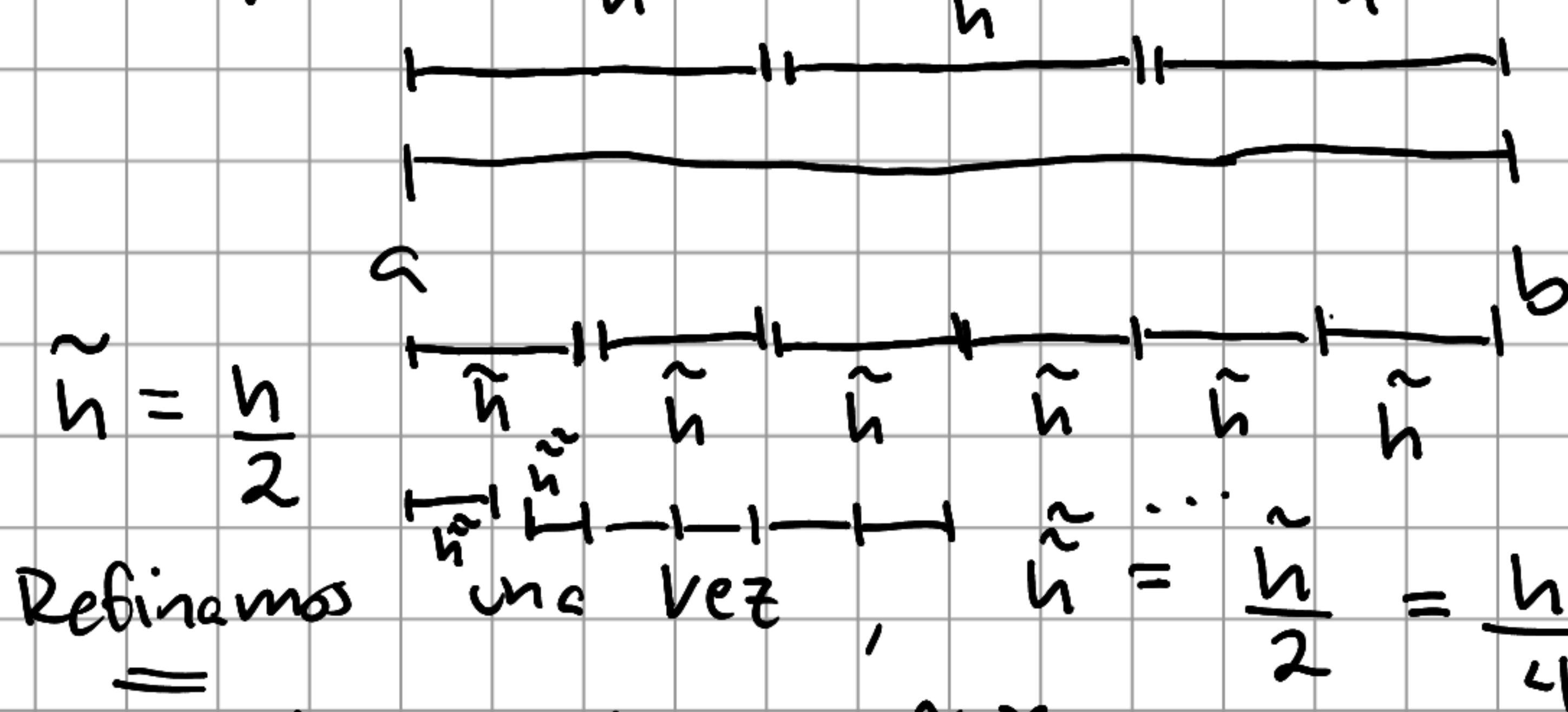
$$\hat{y}_0 = y_0, h = \frac{b-a}{N}$$

$$\hat{y}_{i+1} = \hat{y}_i + h \cdot \phi(t_i, \hat{y}_i), i = 0 \dots N-1$$

i) Error de convergencia: (suposición)

$$e_i = |y(t_i) - \hat{y}_i| \leq C h^r, r \in \mathbb{Z}^+$$

ii) Suponemos $e_i = C h^r$



iii) $h, \tilde{h} \quad \tilde{e}_i = C \tilde{h}^r = C \left(\frac{h}{2}\right)^r$

$$\frac{\tilde{e}_i}{e_i} = \frac{C \tilde{h}^r}{C h^r} = \left(\frac{h}{2}\right)^r \cdot \frac{1}{h^r} = \left(\frac{1}{2}\right)^r = \frac{1}{2^r}$$

$$\Rightarrow \log\left(\frac{\tilde{e}_i}{e_i}\right) = \log\left(\frac{1}{2^r}\right) = r \log\left(\frac{1}{2}\right)$$

$$\Rightarrow r = \log\left(\frac{\tilde{e}_i}{e_i}\right) / \log\left(\frac{1}{2}\right)$$

Para sistemas:

Método en diferencias en gra:

$$*) \quad \underline{y}' = \underline{f}(t, \underline{y}) \quad t \in (a, b)$$

$$\underline{y}(a) = \underline{y}_0$$

$$**) \quad \begin{aligned} \hat{\underline{y}}_0 &= \underline{y}_0 & h &= \frac{b-a}{N} \\ \hat{\underline{y}}_{i+1} &= \hat{\underline{y}}_i + h \underline{\phi}(t_i, \hat{\underline{y}}_i), \quad i=0, \dots, n-1 \end{aligned}$$

i) Error de convergencia

$$e_i = \|\underline{y}(t_i) - \hat{\underline{y}}_i\|_p \leq C h^r$$

donde $\underline{a} \in \mathbb{R}^n$, $\|\underline{a}\|_p = \sum_{j=1}^n |a_j|^p \quad 1 \leq p < \infty$

$$\|\underline{a}\|_\infty = \max_j |a_j|$$