→ Métodos Numéricos para sistemas de EDOs.

1) Un problema de valor inicial de primer orden para un $\underbrace{\text{sistema de tamaño } n}_{\text{" n - sistema "}}$ se define como:

$$y_1' = \frac{dy_1}{dt} = f_1(t, y_1, \dots, y_n)$$

$$y_2' = \frac{dy_2}{dt} = f_2(t, y_1, \dots, y_n) \qquad t \in (a, b)$$

$$\vdots$$

$$\frac{dy_n}{dt} = f_n(t, y_1, \dots, y_n)$$

$y_i : \mathbb{R} \to \mathbb{R}$ $\quad i = 1, \dots, N$ $\quad$ con condición inicial

$$y_1(a) = y_{1,0}$$
$$y_2(c) = y_{2,0}$$
$$\vdots$$
$$y_n(a) = y_{n,0}$$

2) Notación vectorial:

$$\underline{y} : \mathbb{R} \to \mathbb{R}^n , \quad \underline{y} = [y_1, y_2, \ldots, y_n]^t$$

$$[\underline{y}]_i = y_i$$

$$\left\{ \begin{array}{ll} \underline{(*)}\ \text{P.V.I:} \quad \underline{y}' = \underline{f}(t, \underline{y}) & t \in (a,b) \\ \underline{y}(a) = \underline{y}_0 & \end{array} \right.$$

3) Teorema de Taylor vectorial para una variable independiente

$h = \frac{b-a}{N}$, N: # subintervalos

t_{i+1}, alrededor t_i

$$\underline{y}(t_{i+1}) = \underline{y}(t_i) + h\underline{y}'(t_i) + \frac{h^2}{2!}\underline{y}''(t_i) + \ldots$$
$$+ \frac{h^m}{m!}\underline{y}^{(m)}(t_i) + \frac{h^{m+1}}{(m+1)!}\underline{y}^{(m+1)}(\underline{\xi}_i)$$

$$\underline{y}^{(k)} = \frac{d}{dt}[\underline{y}]$$

$[\underline{\xi}_i]_j \in (t_i, t_{i+1})$

$j = 1, \ldots, n$

Suposición: y_j sea derivable $(m+1)$ veces. $j = 1, \ldots, n$

3) Ej. método de Euler, Método de Taylor de orden $m=1$

$$\underline{y}(t_{i+1}) = \underline{y}(t_i) + h\,\underline{y}'(t_i) + \frac{h^2}{2}\,\underline{y}^{(2)}(\xi_i)$$

$$= \underline{y}(t_i) + h\,\underline{f}(t_i, \underline{y}(t_i)) + \frac{h^2}{2}\,\underline{y}^{(2)}(\xi_i)$$

• Truncamos el residuo

$$\begin{cases} \hat{\underline{y}}_0 = \underline{y}(a) \\ \hat{\underline{y}}_{i+1} = \hat{\underline{y}}_i + h\,\underline{f}(t_i, \hat{\underline{y}}_i) \quad i=1,\dots,N \end{cases}$$

$$f_1(t_i, \hat{y}_{1,i}, \dots, \hat{y}_{n,i})$$

$$\begin{bmatrix} \hat{y}_{1,i+1} \\ \vdots \\ \hat{y}_{n,i+1} \end{bmatrix} = \begin{bmatrix} \hat{y}_{1,i} \\ \vdots \\ \hat{y}_{n,i} \end{bmatrix} + h \begin{bmatrix} f_1(t_i, \hat{\underline{y}}_i) \\ \vdots \\ f_n(t_i, \hat{\underline{y}}_i) \end{bmatrix}$$

donde $\hat{y}_{k,i}$:= es la aproximación de la solución $y_k(t_i)$

4) Método de Taylor de orden $m=2$.

$$\underline{y}(t_{i+1}) = \underline{y}(t_i) + h\,\underline{y}'(t_i) + \frac{h^2}{2}\,\underline{y}^{(2)}(t_i) + \frac{h^3}{3!}\,\underline{y}^{(3)}(\underline{\xi}_i)$$

Truncamos el residuo $\qquad [\underline{\xi}_i]_j \in (t_i, t_{i+1})$

$$\begin{cases} \hat{\underline{y}}_0 = \underline{y}_0 \\ \hat{\underline{y}}_{i+1} = \hat{\underline{y}}_i + h\left[\underline{f}(t_i, \hat{\underline{y}}_i) + \frac{h}{2}\underline{f}'(t_i, \hat{\underline{y}}_i)\right] \end{cases}$$

5) Supongamos $n=2$
$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} f_1(t, y_1, y_2) \\ f_2(t, y_1, y_2) \end{bmatrix}$$

$$\underline{f}' := \frac{d}{dt}[\underline{f}] = \begin{bmatrix} \frac{d}{dt}[f_1(t, y_1, y_2)] \\ \frac{d}{dt}[f_2(t, y_1, y_2)] \end{bmatrix}$$

$k = 1, 2$

$$\frac{d}{dt}[f_k(t, y_1, y_2)] = \frac{\partial f_k}{\partial t}\cdot\frac{dt}{dt} + \frac{\partial f_k}{\partial y_1}\cdot\frac{dy_1}{dt} + \frac{\partial f_k}{\partial y_2}\cdot\frac{dy_2}{dt}$$

$$= \underbrace{\frac{\partial f_k}{\partial t}\cdot 1}_{\frac{\partial \underline{f}}{\partial t}} + \underbrace{\frac{\partial f_k}{\partial y_1}\cdot y_1' + \frac{\partial f_k}{\partial y_2}\cdot y_2'}_{M\cdot\underline{y}'}$$

$M \in$ matriz $n \times n$.

5) Para n en gral, n es un entero.

$$\frac{d}{dt}\underline{f}(t,\bar{y}) = \frac{\partial \underline{f}}{\partial t} + \underbrace{\frac{\partial \underline{f}}{\partial \underline{y}}}_{m}\underline{y}'$$

$\frac{\partial \underline{f}}{\partial \underline{y}}$:= Jacobiano

$$\left[\frac{\partial \underline{f}}{\partial \underline{y}}\right]_{k,\ell} = \frac{\partial f_k}{\partial y_\ell}$$

k : renglon
ℓ : columna

$$\left[\frac{\partial \underline{f}}{\partial \underline{y}}\underline{y}'\right]_{\ell} = \sum_{k=1}^{n}\left[\frac{\partial \underline{f}}{\partial \underline{y}}\right]_{\ell,k}\cdot[\underline{y}']_k$$

$$= \sum_{k=1}^{n}\frac{\partial f_\ell}{\partial y_k}\cdot\frac{dy_k}{dt}.$$

m

6) ** $n=2$.

$$\left[\frac{\partial \underline{f}}{\partial \underline{y}}\underline{y}'\right]_1 = \sum_{k=1}^{2}\frac{\partial f_1}{\partial y_k}\cdot y_k' = \frac{\partial f_1}{\partial y_1}y_1' + \frac{\partial f_1}{\partial y_2}\cdot y_2'$$

$$\left[\frac{\partial \underline{f}}{\partial \underline{y}}y'\right]_s = \sum_{k=1}^{2}\frac{\partial f_s}{\partial y_k}\cdot y_k' \qquad s=1,2$$