ASTR 333: Homework 2

1. The best theory to explain the formation of the Solar System is the Nebular Hypothesis, in which a large cloud (nebula) of cold gas (sprinkled with dust) collapsed under its own gravity. The cloud would have spun faster and faster as it contracted, due to conservation of angular momentum. Because of this, the cloud flattened into a rotating disk with a protostellar bulge at the centre. The Sun (dominating the mass of the Solar System) formed at the centre, and the planets coalesced in the disk, accounting for the general dynamical properties of their orbits. In the following questions, you'll explore the current distribution of angular momentum in our Solar System. Use the "Clearing Time Spreadsheet" as a starting point for your study of our Solar System.
    1. Calculate the orbital angular momentum (L) of each of the planets in our Solar System, assuming the planets have circular orbits. What is the total orbital angular momentum of the Solar System planets? Use units of the orbital angular momentum of Earth. Use the formula: $L_{orbit} = m_{planet}\, a_{planet}\, v_{planet}$.
    2. Estimate the angular momenta of the Sun (27 days), Earth (24 hours) and Jupiter (10 hours) due to their rotations. For a solid body, angular momentum from its rotation is: $0.4\, m_{planet}\, R_{planet}\, v_{equator}$.
    3. Estimate the angular momentum of the Moon as it orbits the Earth.
    4. In the Solar System, where is the bulk of the angular momentum?
    5. In the Earth-Moon system, where is the bulk of the angular momentum?

Answer:

1. $L_{Mercury} = 3.285 \times 10^{23} kg \ * \ 5.834 \times 10^{7} km \ * \ 47.89\ km/s$

    $= 9.17797004 \times 10^{32}\ kg * km^2/s$

    $L_{Venus} = 4.867 \times 10^{24}\ kg \ * \ 1.077 \times 10^{8}\ km \ * \ 35.03\ km/s$

    $= 1.83618818 \times 10^{34}\ kg * km^2/s$

    $L_{Earth} = 5.972 \times 10^{24}\ kg \ * \ 1.496 \times 10^{8}\ km * 29.79\ km/s$

    $= 2.66147196 \times 10^{34}\ kg * km^2/s$

    $L_{Mars} = 6.39 \times 10^{23}\ kg \ * \ 2.227 * 10^{8} km * 24.13\ km/s$

    $= 3.43382689 \times 10^{33}\ kg * km^2/s$

    $L_{Jupiter} = 1.898 \times 10^{27}\ kg \ * 7.780 * 10^{8} km * 13.06\ km/s$

    $= 1.92849706 \times 10^{37}\ kg * km^2/s$

$L_{Saturn} = 5.683 \times 10^{26}\ kg\ *\ 1.427 * 10^{9} km * 9.64\ km/s$

$= 7.81769392\times10^{36}\ kg * km^{2}/s$

$L_{Uranus} = 8.681 \times 10^{25} kg\ *\ 2.869 * 10^{9} km * 6.81\ km/s$

$= 1.69608423\times10^{36}\ kg * km^{2}/s$

$L_{Neptune} = 1.024 \times 10^{26} kg\ *\ 4.497 * 10^{9} km * 5.43\ km/s$

$= 2.5004759\times10^{36}\ kg * km^{2}/s$

In Earth Units:

$L_{Mercury} = 0.034\ Earth\ Units$

$L_{Venus} = 0.690\ Earth\ Units$

$L_{Earth} = 1\ Earth\ Units$

$L_{Mars} = 0.129\ Earth\ Units$

$L_{Jupiter} = 724.6\ Earth\ Units$

$L_{Saturn} = 293.7\ Earth\ Units$

$L_{Uranus} = 63.7\ Earth\ Units$

$L_{Neptune} = 93.95\ Earth\ Units$

Total angular momentum = $(9.17797004\times10^{32})$+ $(1.83618818\times10^{34})$+ $(2.66147196\times10^{34})$+ $(3.43382689\times10^{33})$+$(1.92849706\times10^{37})$+ $(7.81769392\times10^{36})$+$(1.69608423\times10^{36})$+$(2.5004759\times10^{36})$ = $3.13485529\times10^{37} kg\ *\ km^{2}/s$= 884.103 Earth Units

2. $L_{Sun}\ due\ to\ rotation\ =\ 0.4 * 1.989 \times 10^{30}\ kg * 1.997\ km/s$

$= 1.5888132\times10^{30}\ kg * km/s$

$L_{Earth}\ due\ to\ rotation\ =\ 0.4 * 5.972 \times 10^{24}\ kg * 0.44704\ km/s$

$= 1.06788915\times10^{24}\ kg * km/s$

$L_{Jupiter}\ due\ to\ rotation\ =\ 0.4 * 1.898 \times 10^{27}\ kg * 11.9444\ km/s$

$= 9.068222\times10^{27} kg * km/s$

3. $L_{Moon} = 5.972 \times 10^{24}\ kg * 3.844 * 10^{5} km\ *\ 1.022\ km/s$

$= 2.34614081 \times 10^{30} kg * km^2/s$

4. In the Solar System, Jupiter contributes the bulk of its angular momentum.

5. Most of the angular momentum of the Earth-Moon system comes from the orbital motion of the Moon around the Earth.

2. Calculate the rate of shrinkage of Jupiter that is required to account for the release of internal energy. The potential energy of a spherical planet with the internal structure like Jupiter or Saturn is -0.75GM^2/R. Consider a similar model for Saturn. Why doesn't the model make sense for Saturn?

Answer:

Internal Power of Jupiter = $4 \times 10^{17} W = 4 \times 10^{17} J/s$

$$\frac{dP}{dt} = \frac{dP}{dR} \times \frac{dR}{dt}$$

$$\frac{d}{dR}\left(\frac{-0.75GM^2}{R}\right) = \frac{0.75GM^2}{R^2} = 0.75 * 6.67408 \times 10^{-11} * (1.898 \times 10^{27})^2 \, kg/(6.99 \times 10^{7})^2 m$$

$$= 3.69054696 \times 10^{28}$$

$$4 \times 10^{17} J/s = 3.69054696 \times 10^{28} * \frac{dR}{dt}$$

$$\frac{dR}{dt} = \frac{4 \times 10^{17}}{3.69054696 \times 10^{28}} = 1.32031561 \times 10^{-11} m/s$$

This same model doesn’t make sense for Saturn because it has half as large of an internal energy source as Jupiter and since Saturn has less of primordial heat, another source (separation of helium and hydrogen) is at work generating its power.

3. Where did the water on Earth come from? Outline the evidence for the different ideas and which is the most favoured today.

Answer: There are many possible scenarios for the origin of water on Earth that are currently proposed:

1. There was an early delivery of water to Earth during its accretion, ie. water was available from the beginning.
    a. First hypothesis: water and other volatiles degassed from Earth’s interior during its formation. This is currently the most accepted hypothesis.

i. Evidence: Gas emissions from volcanoes are mainly composed of CO_2, H_2O, sulfates, nitrogen and rare gases, which corresponds with the composition of the atmosphere, oceans and sediments.

b. Second Hypothesis: Last few planetary embryos (formed in the outer asteroid belt) during accretion carried the bulk of water currently present on Earth.

   i. Evidence: Abundance of deuterium in oceans (more than mantle but less than comets) indicates a source from beyond the orbit of Mars (via extraterrestrial carriers).

2. There was a continuous delivery (again by extraterrestrial carriers) of water throughout the eons.
   a. Evidence: Frank et al. (1986) showed that Earth is annually hit by a large amount of small cometary bodies. Calculations indicate that $2.2 - 8.5 \times 10^{21} kg$of water has reached Earth, which is 3 times the mass of water in present day oceans

4. How does the oceanic crust differ from the continental crust? Where are the oldest rocks on Earth? How do we know this?

   Answer: The oceanic crust and the continental crust differ in many ways:
   1. The average density of the oceanic crust is greater than that of the continental crust. This leads to the continental crust floating higher on the mantle than the oceanic crust.
   2. The continental crust is thicker than the oceanic crust.
   3. The oceanic crust can be recycled into the mantle (at subduction zones when it runs into the continental crust). Meanwhile, the continental crust is rarely recycled (but it can be covered).

   The oldest rocks on Earth are located in Canada (4.0 Ga). By measuring the variation of rare earth elements neodymium and samarium, geologists were able to determine the age of these rocks.

5. Calculate the tidal heating power for Io, Europa, Ganymede and Callisto and compare them to the insolation. Take Im k_2 = 0.01.

   Answer: $E_{Tidal} = \; - Im(k_2)\frac{21}{2}e^2\frac{1}{G}(\frac{2\pi R}{P_{orbit}})^5$

$E_{Tidal} = \; - 0.01\frac{21}{2}e^{2}\frac{1}{G}(\frac{2\pi R}{P_{orbit}})^{5}$

Where $G \; = \; 6.67408 \times 10^{-11}$

For Io,

$R_{Io} = \; 1821600\,m, \; e_{Io} = 0.0041, \; P_{Io} = \; 1.769137786\,d$

Therefore, $E_{Io} =- \; 2.99724153\times10^{38}\,W$

Io has a reflectivity of 60%, this means that 60% of the incoming insolation is reflected.

For Europa,

$R_{Europa} = \; 1560800\,m, \; e_{Europa} = 0.009, \; P_{Europa} = \; 3.551181\,d$

Therefore, $E_{Europa} =- \; 2.04670142\times10^{37}\,W$

Europa has a reflectivity of 70%, i.e. 70% of incoming insolation is reflected. Therefore, Europa has the lowest level of insolation.

For Ganymede,

$R_{Ganymede} = 2634100\,m, \; e_{Ganymede} = 0.0013, \; P_{Ganymede} = 7.15455296\; d,$

Therefore, $E_{Ganymede} =- \; 1.76129749\times10^{35}\,W$

Ganymede has reflectivity of 40%, i.e. 40% of incoming insolation is reflected.

For Callisto,

$R_{Castillo} = \; 2410300\,m, \; e_{Castillo} = 0.0074, \; P_{Castillo} = 16.6890184\; d$

Therefore, $E_{Castilo} =- \; 5.30105376\times10^{34}\,W$

Callisto has reflectivity of 20%, i.e 20% of incoming insolation is reflected. Therefore, Callisto has the highest level of insolation.

6. Calculate the terminal velocity of a person falling on Titan, Venus and Mars. On Earth it takes the value of 50 m/s.

Answer:

$V_{terminal} = \sqrt{(2mg)/(\rho A C_d)}$

$Terminal\,Velocity\,on\,Earth \; = \; 50\,m/s \; = \; \sqrt{(2mg_{earth})/(\rho_{earth} A C_d)}$

The mass of the person, area and drag coeff. remains the same on all planets. So,

$\sqrt{(2m/(AC_d)} = 50/(\sqrt{g_{earth}/\rho_{earth}})$

$V_{Titan} = (50/(\sqrt{g_{earth}/\rho_{earth}})) * \sqrt{g_{Titan}/\rho_{Titan}} = (50/\sqrt{9.807/1.2041}) * \sqrt{1.352/5.29804} = 8.85\ m/s$

$V_{Venus} = (50/(\sqrt{g_{earth}/\rho_{earth}})) * \sqrt{g_{Venus}/\rho_{Venus}} = (50/\sqrt{9.807/1.2041}) * \sqrt{8.87/67} = 6.37\ m/s$

$V_{Mars} = (50/(\sqrt{g_{earth}/\rho_{earth}})) * \sqrt{g_{Mars}/\rho_{Mars}} = (50/\sqrt{9.807/1.2041}) * \sqrt{3.271/0.02} = 224.06\ m/s$