

Homework 10

Due date: Apr 16, 2020, 9:30am

Instructions

Exercise 1 & 2 require writing a proof. A proof is a sequence of valid arguments that can be traced by the reader. Think of your proof as a story: Have a story-line in your proof instead of having various disconnected parts. Write in full sentences explaining what you are showing. Don't place any formulas in your proof without any explanation.

1. (15 points)

Let S be a set defined as follows:

Base case: 4 is an element of S

Recursive case: If x is in S , then x^2 is in S .

Use structural induction to prove that all element in S are even, where an even number is defined as a number that can be written in the form $2 \cdot k$ for an integer k .

2. (15 points)

We define a perfect ternary tree as follows:

Base case: A single node is a perfect ternary tree.

Recursive case: If T_1 , T_2 , and T_3 are perfect ternary trees with roots r_1 , r_2 , and r_3 respectively, and T_1 , T_2 , and T_3 have distinct nodes and T_1 , T_2 , and T_3 have the same height, then the structure with root r and with an edge from r to root r_1 , an edge from r to r_2 , and an edge from r to r_3 is a perfect ternary tree.

Use structural induction to prove that a perfect ternary tree has $(3^{h+1} - 1) / 2$ nodes where h is the height of the tree.

3. (12 points)

Draw a binary search tree with the nodes `cat`, `cow`, `dog`, `duck`, `goat`, `goose`, `hen`, `horse`, `mouse`, `pig`, `rooster`, `sheep`, `turkey` and that meets the requirement below.

Node values are compared lexicographically:

a) The tree is complete.

b) The tree has the greatest possible height.

c) The tree is height-balanced and has the greatest possible height among the height-balanced trees with the given nodes.

4. (8 points)

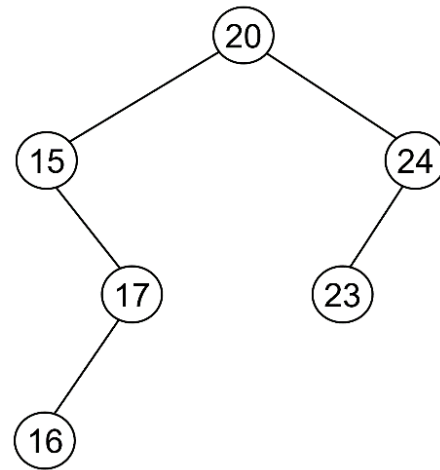
Determine the balance factor of the specified nodes and decide whether the tree is an AVL tree. The numbers represent the comparable node values.

a) Balance factor of 20:

Balance factor of 15:

Balance factor of 17:

Is the tree an AVL tree?

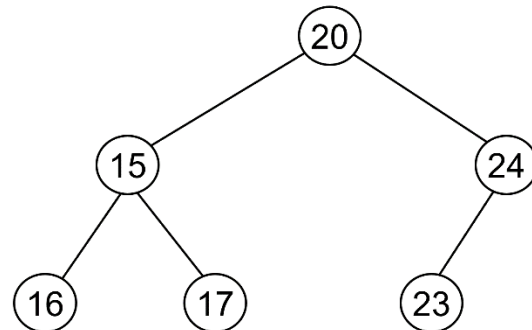


b) Balance factor of 20:

Balance factor of 15:

Balance factor of 16:

Is the tree an AVL tree?



Submission

Submit a single PDF file (preferred) or a single MS Word document with your solutions. No other file formats are accepted. If you prefer to write (or draw) your solution by hand and you do not have a scanner, take pictures of your hand-written solutions and embed the pictures in a Word document.