Trabalho 4 (Transformada)

Descrição do trabalho:

http://webserver2.tecgraf.puc-rio.br/~mgattass/visao/trb/T4.html (http://webserver2.tecgraf.puc-rio.br/~mgattass/visao/trb/T4.html)

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In [1]:

```
from google.colab import drive
drive.mount('/content/drive', force_remount=True)
```

```
Mounted at /content/drive
```

In [2]:

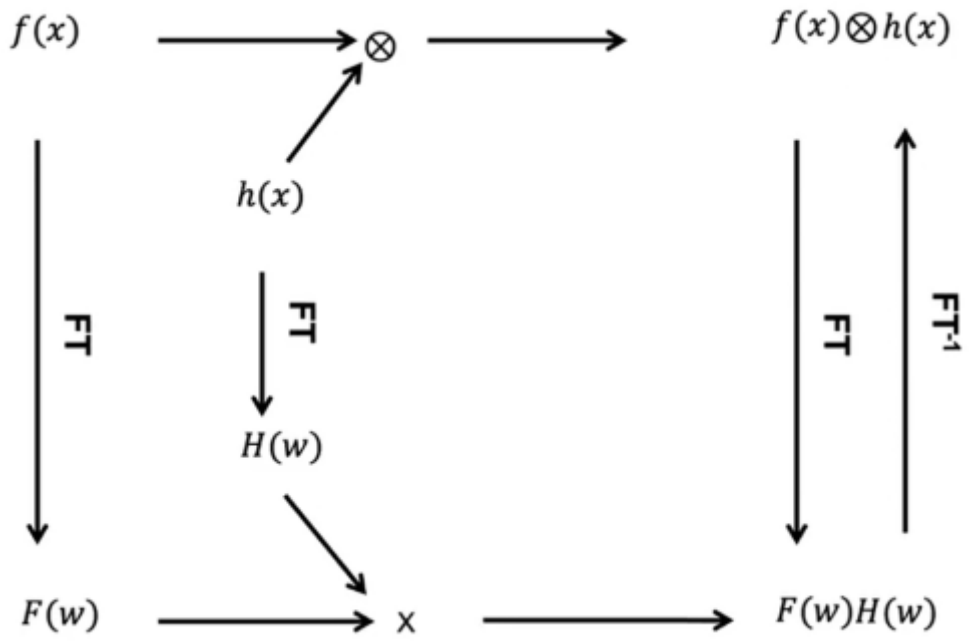
```
cd "drive/MyDrive/Doutorado/Disciplinas/[2022.2] [PUC-Rio] Visão Computacional - Professor: Marcelo Gattass/Trabalhos/Trabalho 4 - Transformada/"
```

```
/content/drive/MyDrive/Doutorado/Disciplinas/[2022.2] [PUC-Rio] Visão Computacional - Professor: Marcelo Gattass/Trabalhos/Trabalho 4 - Transformada
```

In [3]:

```
!pwd
```

```
/content/drive/MyDrive/Doutorado/Disciplinas/[2022.2] [PUC-Rio] Visão Computacional - Professor: Marcelo Gattass/Trabalhos/Trabalho 4 - Transformada
```



Imports

In [4]:

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import LogNorm
from scipy import fftpack
from scipy import signal
import math
```

1. Implemente uma transformada de Fourier.

Baseado em: "Understanding the FFT Algorithm"

https://jakevdp.github.io/blog/2013/08/28/understanding-the-fft/ (https://jakevdp.github.io/blog/2013/08/28/understanding-the-fft/)

DFT

In [5]:

```
def DFT(x):
    """Compute the discrete Fourier Transform of the 1D array x"""
    x = np.asarray(x, dtype=float)
    N = x.shape[0]
    n = np.arange(N)
    k = n.reshape((N, 1))
    M = np.exp(-2j * np.pi * k * n / N)
    return np.dot(M, x)
```

In [6]:

```
type(1j) # Número complexo em Python
```

Out[6]:

```
complex
```

In [7]:

```
type(-2j)
```

Out[7]:

```
complex
```

Conferindo se os valores calculados pela função DFT estão corretos em comparação com a biblioteca do Numpy.

In [8]:

```
x = np.random.random(1024)
x
```

Out[8]:

```
array([0.34905452, 0.77770278, 0.16321539, ..., 0.43358237, 0.282365  ,
       0.1177479 ])
```

In [9]:

```
DFT(x)
```

Out[9]:

```
array([ 4.94922303e+02+0.j        ,  6.46716237e+00+7.08353417j,
       -2.81449215e-02-3.91458952j, ..., -6.44242189e+00-0.9344051j ,
       -2.81449215e-02+3.91458952j,  6.46716237e+00-7.08353417j])
```

In [10]:

```
np.allclose(DFT(x), np.fft.fft(x))
```

Out[10]:

```
True
```

Comparando o tempo de execução da implementação da função DFT com a FFT (do Numpy).

In [11]:

```
%timeit DFT(x)
%timeit np.fft.fft(x)
```

```
137 ms ± 17.3 ms per loop (mean ± std. dev. of 7 runs, 10 loops each)
11.2 µs ± 726 ns per loop (mean ± std. dev. of 7 runs, 100000 loops each)
```

FFT

Radix-2 Cooley-Tukey FFT: O tamanho do *array* deve ser uma potência de 2.

In [12]:

```
def FFT(x):
    """A recursive implementation of the 1D Cooley-Tukey FFT"""
    x = np.asarray(x, dtype=float)
    N = x.shape[0]

    if N % 2 > 0:
        raise ValueError("size of x must be a power of 2")
    elif N <= 32:  # this cutoff should be optimized
        return DFT(x)
    else:
        X_even = FFT(x[::2])
        X_odd = FFT(x[1::2])
        factor = np.exp(-2j * np.pi * np.arange(N) / N)
        return np.concatenate([X_even + factor[:N // 2] * X_odd,
                               X_even + factor[N // 2:] * X_odd])
```

Conferindo se os valores calculados pela função FFT estão corretos em comparação com a biblioteca do Numpy.

In [13]:

```
x = np.random.random(1024)
x
```

Out[13]:

```
array([0.34042789, 0.94205502, 0.19831553, ..., 0.95735732, 0.93123015,
       0.83569929])
```

In [14]:

```
fft_x = FFT(x)
fft_x
```

Out[14]:

```
array([526.89784446+0.j        ,  -8.6603198 +2.26833428j,
         2.54143812-4.03399084j, ...,   8.15850166+0.62190577j,
         2.54143812+4.03399084j,  -8.6603198 -2.26833428j])
```

In [15]:

```
np.allclose(FFT(x), np.fft.fft(x))
```

Out[15]:

```
True
```

Comparando o tempo de execução da implementação da função FFT_vectorized com a FFT (do Numpy).

In [16]:

```
%timeit DFT(x)
%timeit FFT(x)
%timeit np.fft.fft(x)
```

```
81.6 ms ± 1.19 ms per loop (mean ± std. dev. of 7 runs, 10 loops each)
4.9 ms ± 700 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
10.9 µs ± 211 ns per loop (mean ± std. dev. of 7 runs, 100000 loops each)
```

Transformada da função *seno*

In [17]:

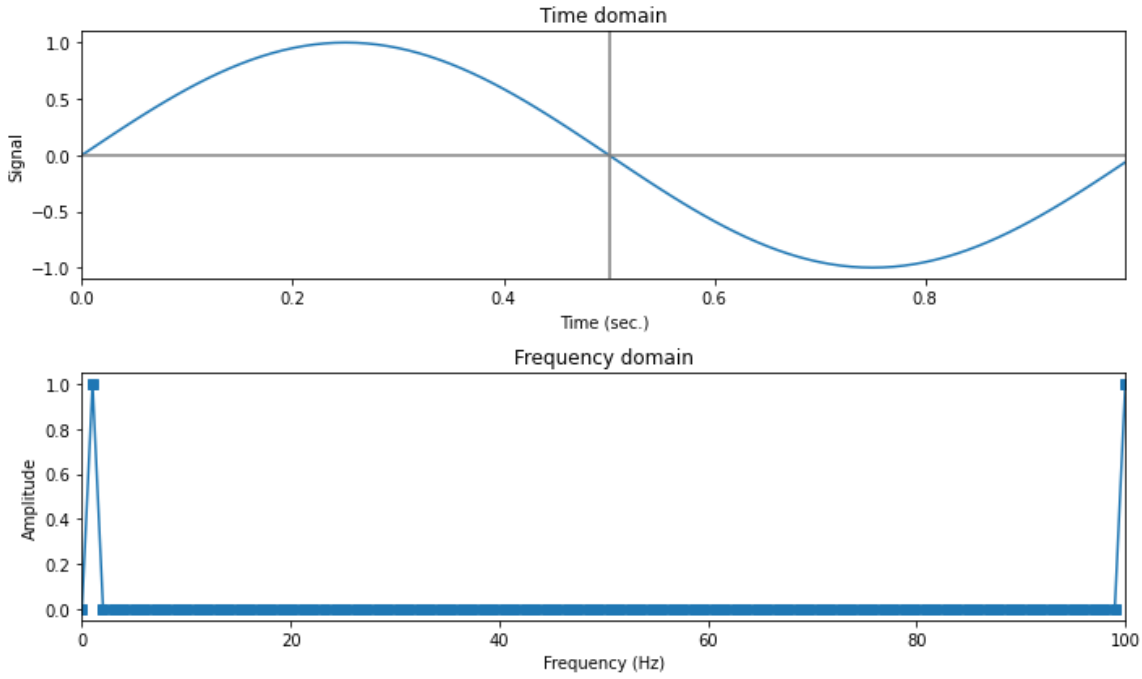
```
# Baseado em https://www.udemy.com/course/fourier-transform-mxc/

srate = 100
time = np.arange(0, 1, 1.0 / srate) # [0, 1)
n = len( time )
hz = np.linspace(0, srate, n) # [0, 100)
# sinal = np.sin( 2 * np.pi * 5 * time) # pure sine wave
sinal = np.sin( 2 * np.pi * 1 * time) # pure sine wave
dft_sinal = DFT( sinal )
ampl = 2 * np.abs( dft_sinal / n )

fig,ax = plt.subplots(2,1,figsize=(10,6))
ax[0].plot( time, sinal )
ax[0].set_xlabel('Time (sec.)')
ax[0].set_ylabel('Signal')
ax[0].set_title('Time domain')
ax[0].set_xlim(time[[0,-1]])
ax[0].axhline( 0, color='gray')
ax[0].axvline( 0.5, color='gray')

ax[1].plot( hz, ampl, 's-')
ax[1].set_xlabel('Frequency (Hz)')
ax[1].set_ylabel('Amplitude')
ax[1].set_title('Frequency domain')
ax[1].set_xlim([0,100])

plt.tight_layout()
plt.show()
```



In [18]:

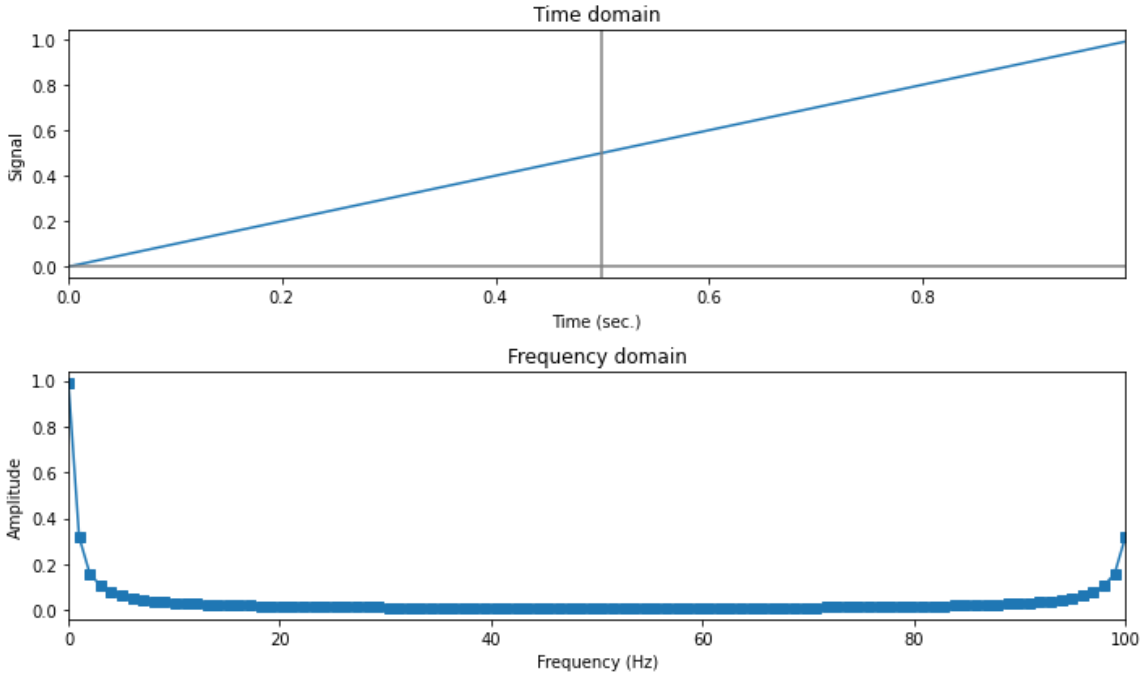
```
# Baseado em https://www.udemy.com/course/fourier-transform-mxc/

srate = 100
time = np.arange(0, 1, 1.0 / srate) # [0, 1)
n = len( time )
hz = np.linspace(0, srate, n) # [0, 100)
sinal = time
dft_sinal = DFT( sinal )
ampl = 2 * np.abs( dft_sinal / n )

fig,ax = plt.subplots(2,1,figsize=(10,6))
ax[0].plot( time, sinal )
ax[0].set_xlabel('Time (sec.)')
ax[0].set_ylabel('Signal')
ax[0].set_title('Time domain')
ax[0].set_xlim(time[[0,-1]])
ax[0].axhline( 0, color='gray')
ax[0].axvline( 0.5, color='gray')

ax[1].plot( hz, ampl, 's-')
ax[1].set_xlabel('Frequency (Hz)')
ax[1].set_ylabel('Amplitude')
ax[1].set_title('Frequency domain')
ax[1].set_xlim([0,100])

plt.tight_layout()
plt.show()
```



2. Implemente a filtragem de uma imagem tanto com o uso de Transformada de Fourier quanto por Correlação (Convolução).

In [19]:

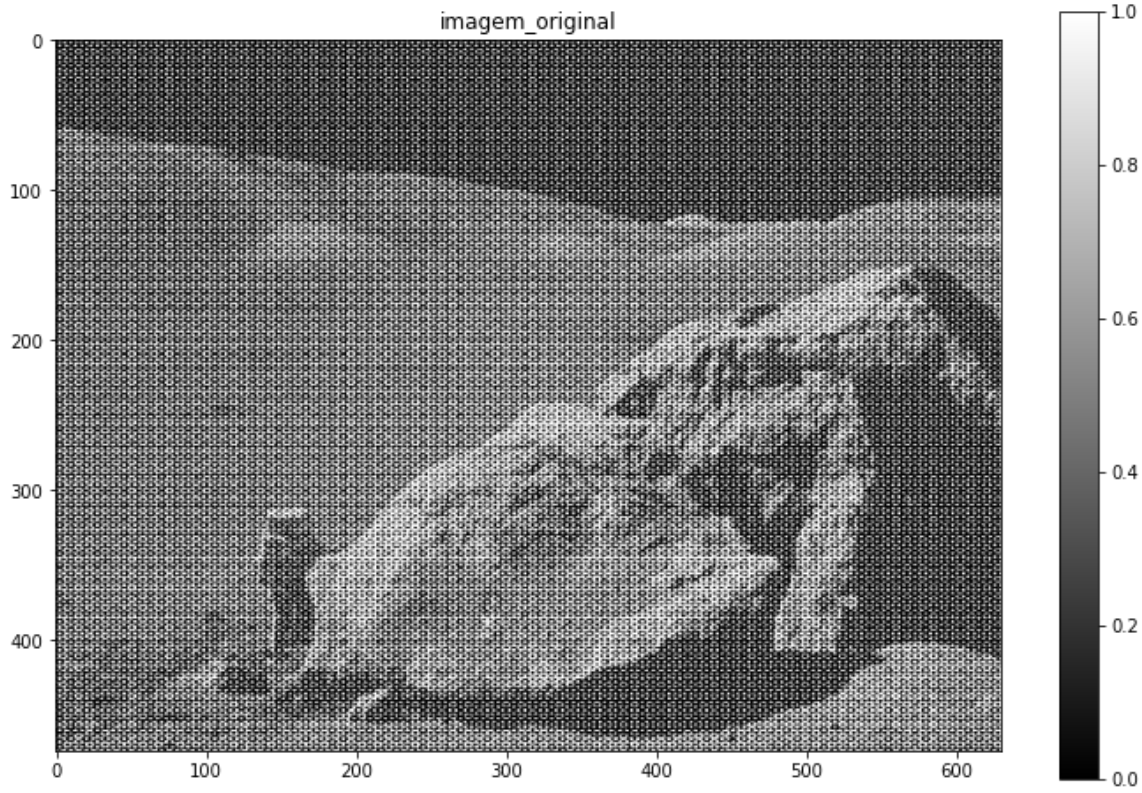
```
path = './images/'

# file = 'moonlanding.png'
file = 'moonlanding_scipy.png'
# file = 'balloons_noisy.png'

img = plt.imread( path + file ).astype(float)

def show_img( img, title ):
  plt.figure( figsize=(12,8) )
  plt.title( title )
  plt.imshow( img, plt.cm.gray )
  plt.colorbar()
  # plt.savefig( title + '.png' )
  plt.show()

show_img( img, 'imagem_original' )
```



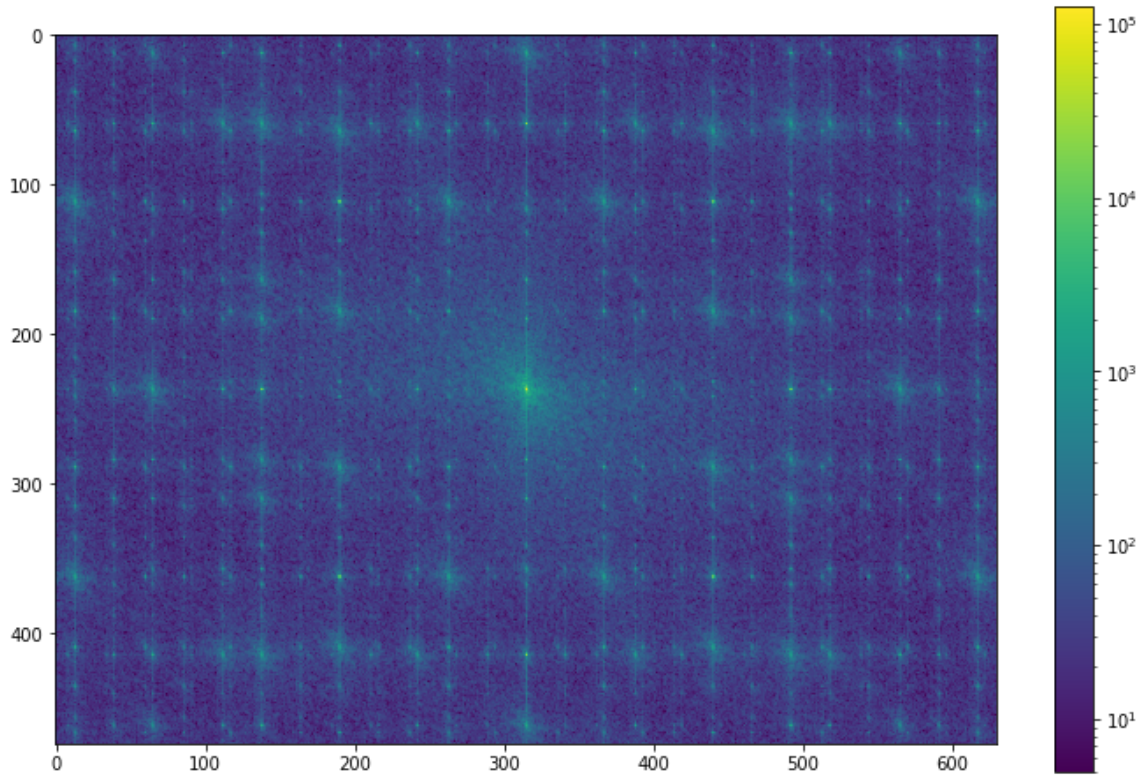
In [20]:

```
IMG = fftpack.fft2(img)
IMGS = fftpack.fftshift(IMG)
print(img.shape)

# Show the results
def show_spectrum(im_fft, title):
  # A logarithmic colormap
  plt.figure(figsize=(12,8))
  plt.imshow(np.abs(im_fft), norm=LogNorm(vmin=5))
  plt.colorbar()
  # plt.savefig(title+'png')
  plt.show()

show_spectrum(IMGS, 'Fourier transform (shifted)')
```

```
(474, 630)
```



Fazendo uma convolução(correlação) com uma Gaussiana discreta de σ = 1

In [21]:

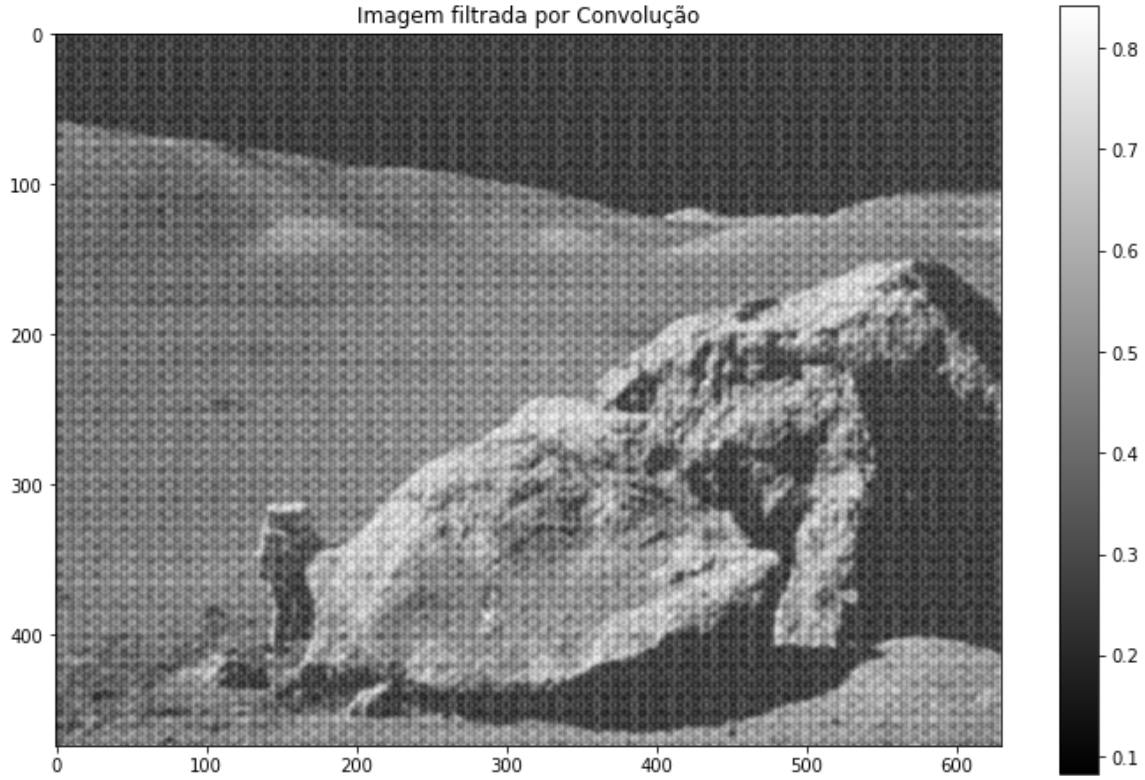
```
g = np.array([[0.0002, 0.0061, 0.0612, 0.2400, 0.3816, 0.2400, 0.0612, 0.0061, 0.0002
]])
g2d = g*g.T

np.set_printoptions(precision=3, suppress=True, floatmode='fixed')
for i in range(9):
  print(f'{g2d[i]}')
```

```
[0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000]
[0.000 0.000 0.000 0.001 0.002 0.001 0.000 0.000 0.000]
[0.000 0.000 0.004 0.015 0.023 0.015 0.004 0.000 0.000]
[0.000 0.001 0.015 0.058 0.092 0.058 0.015 0.001 0.000]
[0.000 0.002 0.023 0.092 0.146 0.092 0.023 0.002 0.000]
[0.000 0.001 0.015 0.058 0.092 0.058 0.015 0.001 0.000]
[0.000 0.000 0.004 0.015 0.023 0.015 0.004 0.000 0.000]
[0.000 0.000 0.000 0.001 0.002 0.001 0.000 0.000 0.000]
[0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000]
```

In [22]:

```
img_conv = signal.correlate2d(img, g2d, boundary='symm', mode='same')
show_img(img_conv, 'Imagem filtrada por Convolução')
```

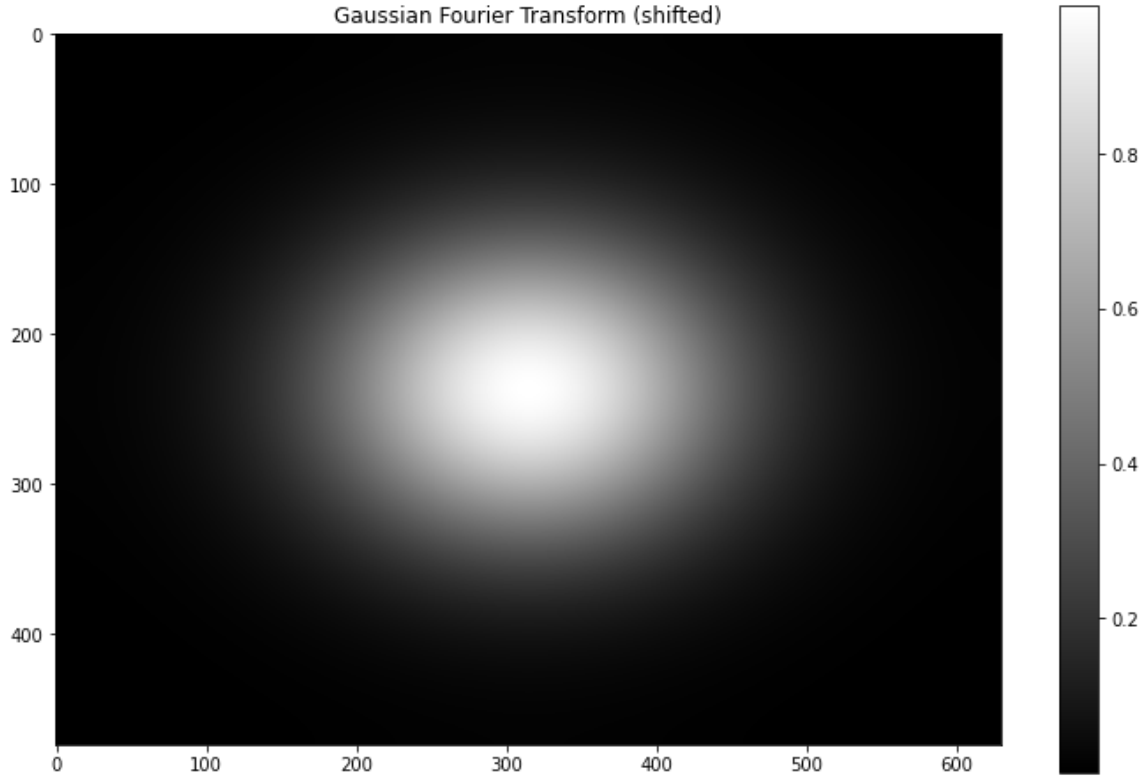


In [23]:

```
gauss2d = np.zeros(IMGS.shape)
h, w = IMGS.shape[:2]
yc = h//2
xc = w//2
gauss2d[yc-4:yc+5, xc-4:xc+5]=g2d
GAUSS2D = fftpack.fft2(gauss2d).real
show_img(np.abs(GAUSS2D), 'Gaussian Fourier Transform')
```

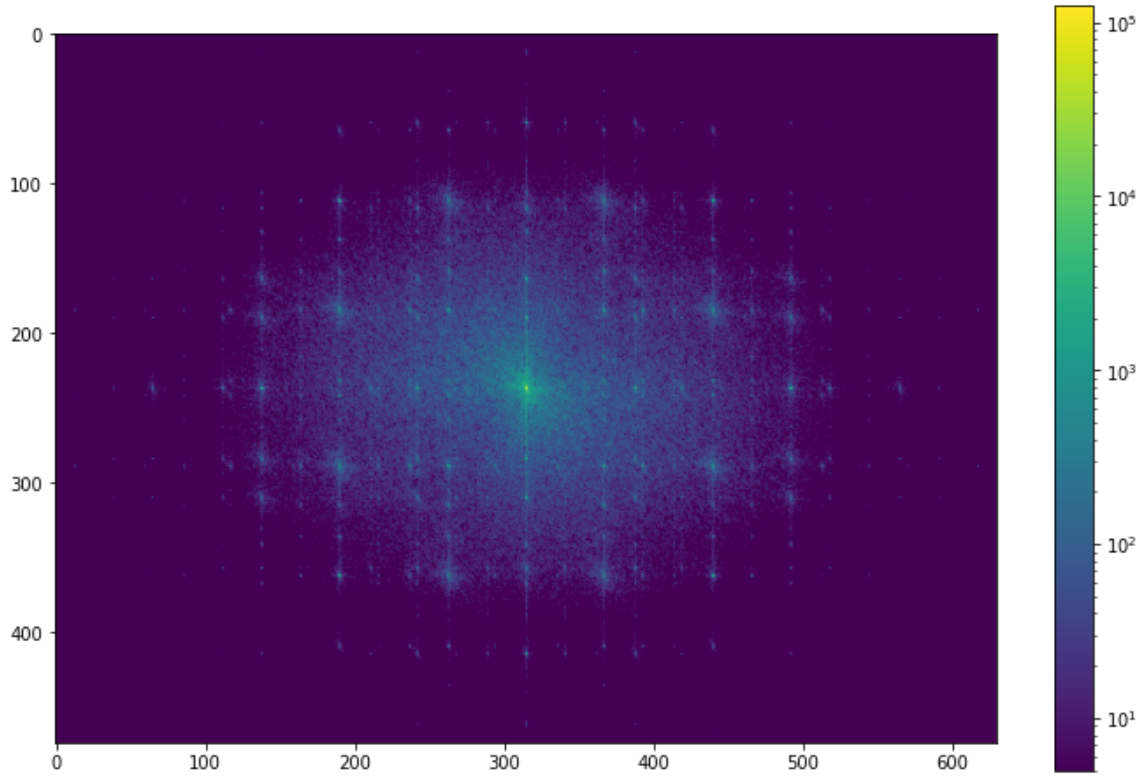
In [24]:

```
GAUSS2DS = fftpack.fftshift(GAUSS2D)
show_img(np.abs(GAUSS2DS), 'Gaussian Fourier Transform (shifted)')
```



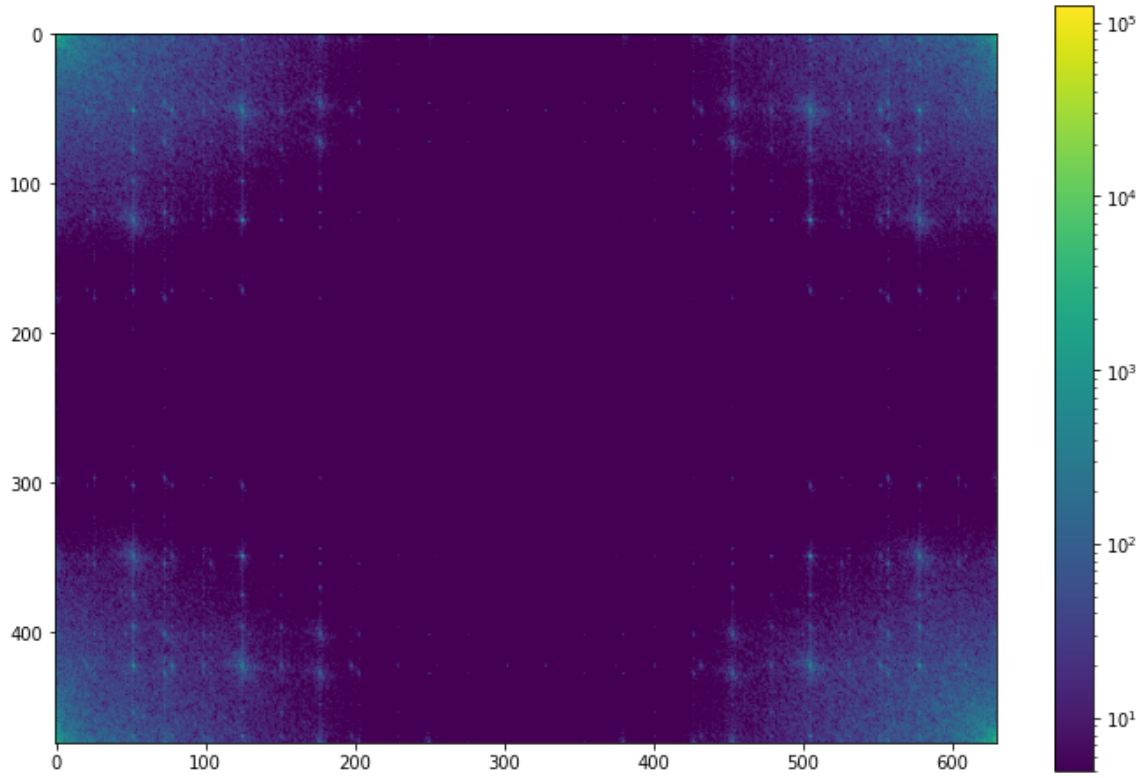
In [25]:

```
IMGFS = GAUSS2DS*IMGS
show_spectrum(IMGFS, 'Fourier tranform (shifted)')
```



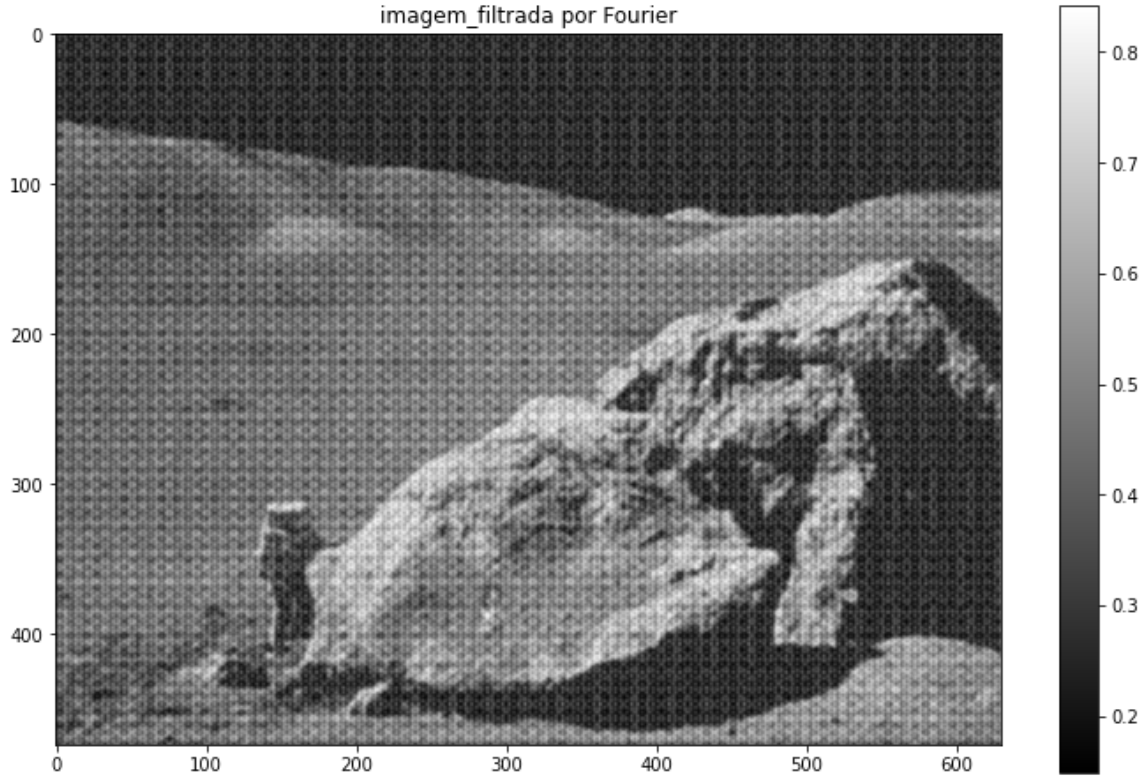
In [26]:

```
IMGF = fftpack.ifftshift(IMGFS)
show_spectrum(IMGF, 'Fourier tranform')
```



In [27]:

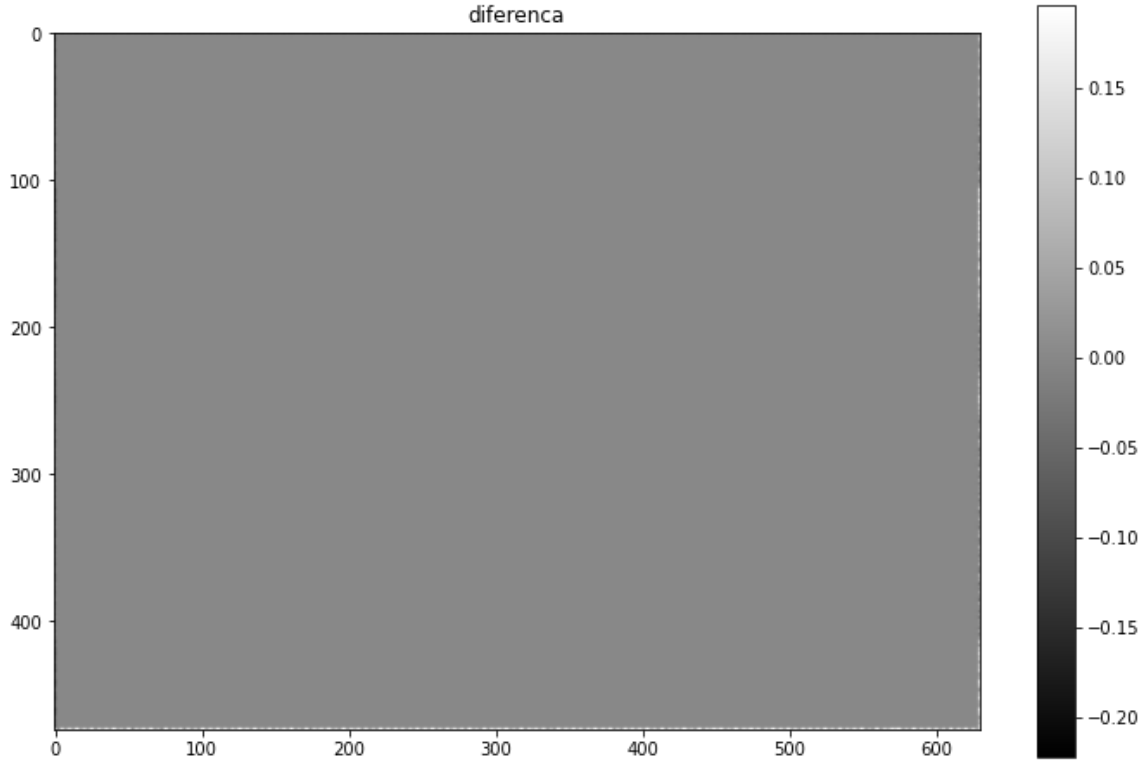
```
imgf = fftpack.ifft2(IMGF).real
imgfs = fftpack.ifftshift(imgf)
show_img(imgfs, 'imagem_filtrada por Fourier')
```



3. Mostre que a filtragem por convolução é equivalente a filtragem por Transformada de Fourier.

In [28]:

```
dif = img_conv - imgfs
show_img(dif, 'diferenca')
```



In [29]:

```
print(np.average(dif))
```

```
9.650932075021655e-17
```

Exporting to HTML file

Changing Google Drive folder.

In [30]:

```
!pwd
```

```
/content/drive/MyDrive/Doutorado/Disciplinas/[2022.2] [PUC-Rio] Visão Comp
utacional - Professor: Marcelo Gattass/Trabalhos/Trabalho 4 - Transformada
```

In [31]:

```
%%time

!jupyter nbconvert --to html ./T4_Transformada_DanielCosta.ipynb
```

```
[NbConvertApp] Converting notebook ./T4_Transformada_DanielCosta.ipynb to
html
[NbConvertApp] Writing 340737 bytes to ./T4_Transformada_DanielCosta.html
CPU times: user 22.4 ms, sys: 8.65 ms, total: 31.1 ms
Wall time: 1.23 s
```