Trabalho 2

Descrição do trabalho:

https://web.tecgraf.puc-rio.br/~mgattass/visao/trb/T2.html (https://web.tecgraf.puc-rio.br/~mgattass/visao/trb/T2.html)

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Imports

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
import torch
import torch.nn as nn
import math
```

Função

y = np.sin(np.pi*x) - x**3

In [2]:

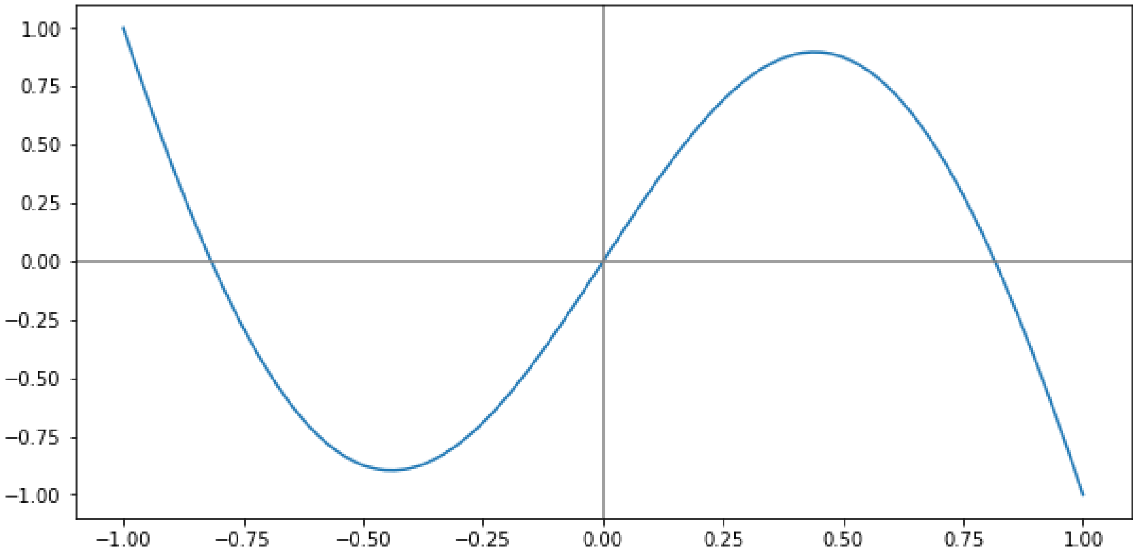
```
num_points = 100
```

In [3]:

```
x = np.linspace( -1, 1, num_points )
y = np.sin( np.pi * x) - x**3
```

In [4]:

```
plt.figure( figsize = ( 10, 5 ) )
plt.plot( x, y )
plt.axhline( 0, color='gray')
plt.axvline( 0, color='gray')
plt.show()
```



Solução por Mínimos Quadrados

Calculando os somatórios

In [5]:

```
sx = sx2 = sx3 = sx4 = sx5 = sx6 = sy = sxy = sx2y = sx3y = 0
n = x.shape[0]

for i in range(n):

    sx += x[i]
    sx2 += x[i]**2
    sx3 += x[i]**3
    sx4 += x[i]**4
    sx5 += x[i]**5
    sx6 += x[i]**6
    sy += y[i]
    sxy += x[i] * y[i]
    sx2y += (x[i]**2) * y[i]
    sx3y += (x[i]**3) * y[i]
```

Calculando os coeficientes

In [6]:

```
# Baseado em https://pythonnumericalmethods.berkeley.edu/notebooks/chapter14.05-Solve-Systems-of-Linear-Equations-in-Python.html

A = np.array([[n, sx, sx2, sx3],
              [sx, sx2, sx3, sx4],
              [sx2, sx3, sx4, sx5],
              [sx3, sx4, sx5, sx6]])
k = np.array([sy, sxy, sx2y, sx3y])

coeficientes = np.linalg.solve(A, k)
print(coeficientes)
```

```
[-3.81492595e-16  2.67730870e+00  1.07359038e-15 -3.86086038e+00]
```

Mostrando o gráfico da função ajustada com os coeficientes encontrados.

In [7]:

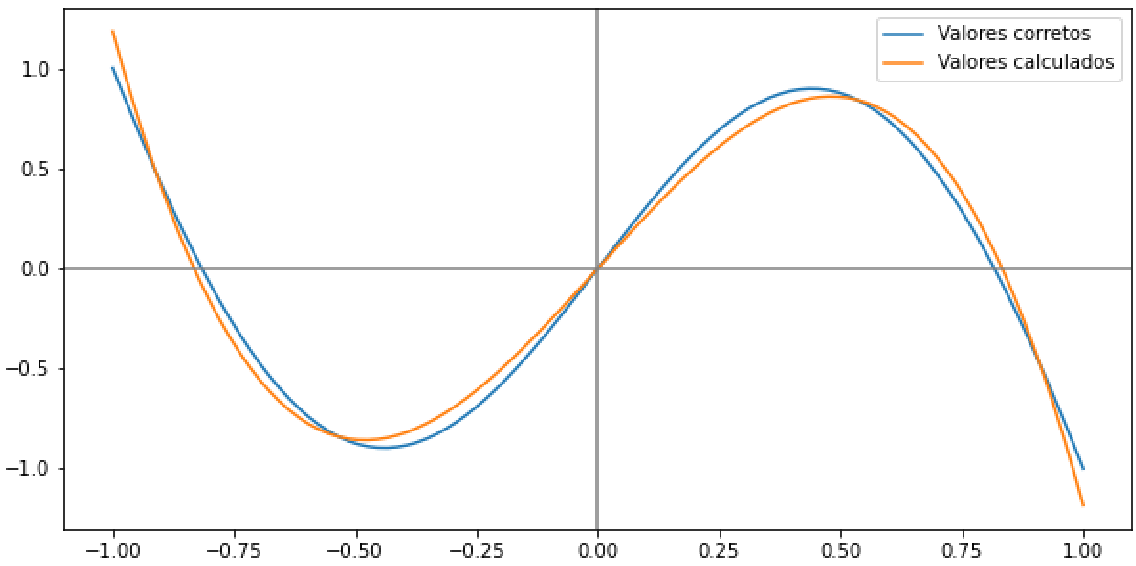
```
a = coeficientes[0]
b = coeficientes[1]
c = coeficientes[2]
d = coeficientes[3]

y_novo = a + b * x + c * (x**2) + d * x**3

print()
print( f'Função aproximada: {a:.3f} + {b:.3f} x + {c:.3f} x^2 + {d:.3f} x^3' )
print()

plt.figure( figsize=(10, 5) )
plt.plot( x, y, label='Valores corretos' )
plt.plot( x, y_novo, label='Valores calculados' )
plt.axhline( 0, color='gray')
plt.axvline( 0, color='gray')
plt.legend()
plt.show()
```

```
Função aproximada: -0.000 + 2.677 x + 0.000 x^2 + -3.861 x^3
```



Gradiente Descendente

Função de erro

In [8]:

```
def calcula_erro( a, b, c, d ):

    soma = 0.0
    n = x.shape[0]
    for i in range(n):

        e = y[i] - ( a + b*x[i] + c*(x[i]**2) + d*(x[i]**3) )
        soma += e * e

    # return soma/n # Sem dividir pelo N (quantidade de elementos) a função é ajustada
 mais rapidamente.
    return soma
```

Inicializando os coeficientes que serão ajustados conforme a aplicação do algoritmo de Gradiente Descendente.

In [9]:

```
# a = np.random.rand()
# b = np.random.rand()
# c = np.random.rand()
# d = np.random.rand()

a = 0.0
b = 0.0
c = 0.0
d = 0.0

print( f'a: {a}' )
print( f'b: {b}' )
print( f'c: {c}' )
print( f'd: {d}' )
```

```
a: 0.0
b: 0.0
c: 0.0
d: 0.0
```

In [10]:

```
err = calcula_erro( a, b, c, d )
err
```

Out[10]:

```
39.97367619260233
```

Rodando o algoritmo de Gradiente Descendente

In [11]:

```
# Baseado em https://pytorch.org/tutorials/beginner/pytorch_with_examples.html

def grad_desc( a0, b0, c0, d0, tol, max_it, learning_rate ):

    erro0 = calcula_erro( a0, b0, c0, d0 )
    erros = [erro0]

    for t in range(max_it):

        # Forward pass: compute predicted y
        y_pred = a0 + b0 * x + c0 * x ** 2 + d0 * x ** 3

        # Compute and print loss
        loss = calcula_erro( a0, b0, c0, d0 )

        # Backprop to compute gradients of a, b, c, d with respect to loss
        grad_y_pred = 2.0 * (y_pred - y)
        grad_a = grad_y_pred.sum()
        grad_b = (grad_y_pred * x).sum()
        grad_c = (grad_y_pred * x ** 2).sum()
        grad_d = (grad_y_pred * x ** 3).sum()

        # Update weights
        a1 = a0 - learning_rate * grad_a
        b1 = b0 - learning_rate * grad_b
        c1 = c0 - learning_rate * grad_c
        d1 = d0 - learning_rate * grad_d

        a0 = a1
        b0 = b1
        c0 = c1
        d0 = d1

        erro1 = calcula_erro( a1, b1, c1, d1 )
        erros.append( erro1 )

    print( f'Solução a = {a1:.3f}' )
    print( f'Solução b = {b1:.3f}' )
    print( f'Solução c = {c1:.3f}' )
    print( f'Solução d = {d1:.3f}' )

    return a1, b1, c1, d1, np.array(erros)
```

Exibindo o gráfico do erro ao longo da execução do algoritmo de Gradiente Descendente

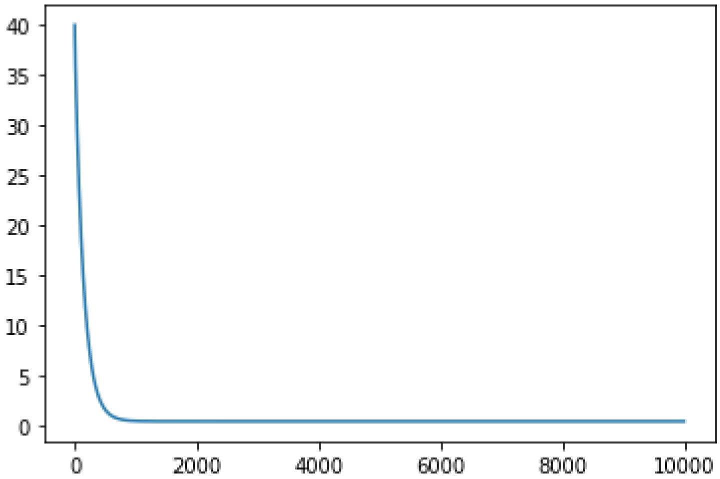
In [12]:

```
%%time

a_gd, b_gd, c_gd, d_gd, erros = grad_desc(a0 = a,
                                          b0 = b,
                                          c0 = c,
                                          d0 = d,
                                          tol = 1e-5,
                                          max_it = 10000,
                                          learning_rate = 0.001)

interacao = np.linspace( 0, erros.shape[0] - 1, erros.shape[0] )
plt.plot( interacao, erros )
plt.show()
```

```
Solução a = -0.000
Solução b = 2.677
Solução c = 0.000
Solução d = -3.861
```




```
CPU times: user 6.29 s, sys: 54.8 ms, total: 6.34 s
Wall time: 7.54 s
```

Mostrando o gráfico da função ajustada com os coeficientes encontrados.

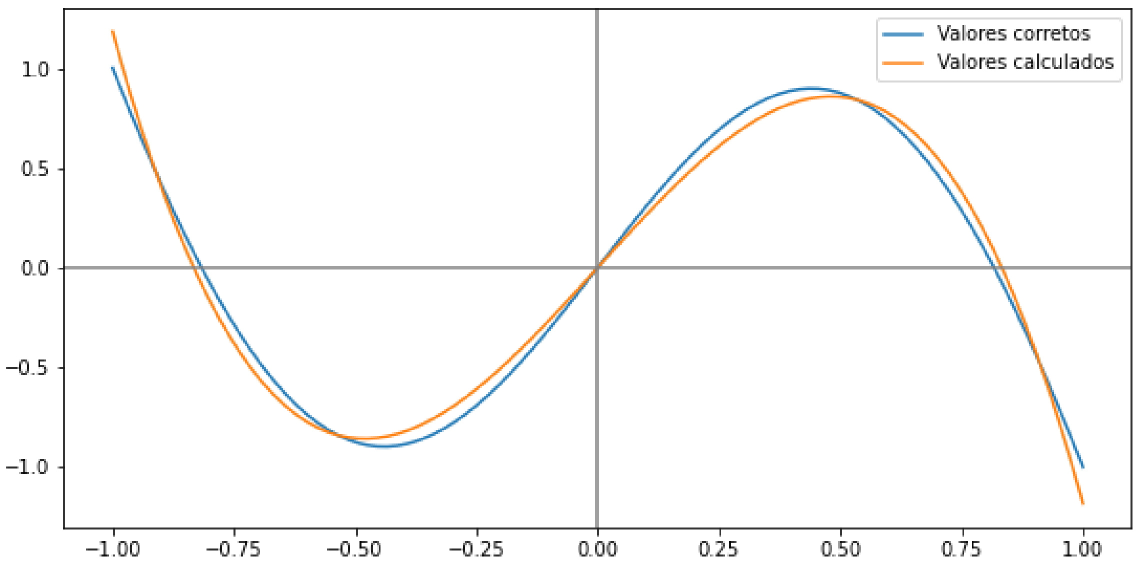
In [13]:

```
y_novo = a_gd + b_gd * x + c_gd * (x**2) + d_gd * x**3

print()
print( f'Função aproximada: {a_gd:.3f} + {b_gd:.3f} x + {c_gd:.3f} x^2 + {d_gd:.3f} x^
3' )
print()

plt.figure( figsize=(10, 5) )
plt.plot( x, y, label='Valores corretos' )
plt.plot( x, y_novo, label='Valores calculados' )
plt.axhline( 0, color='gray')
plt.axvline( 0, color='gray')
plt.legend()
plt.show()
```

```
Função aproximada: -0.000 + 2.677 x + 0.000 x^2 + -3.861 x^3
```



PyTorch (usando o autograd)

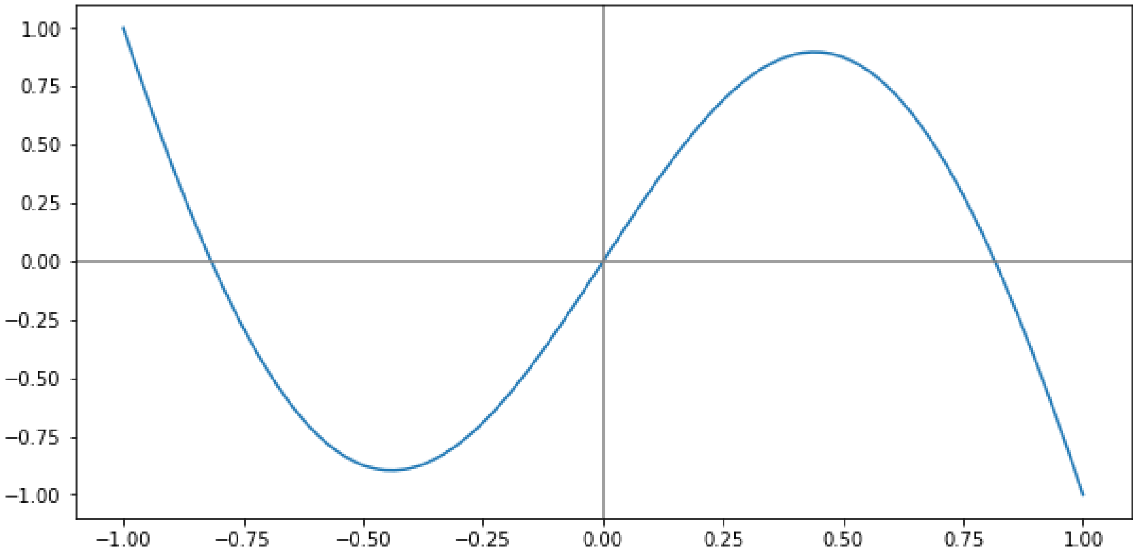
In [14]:

```
x = torch.linspace( -1, 1, num_points, dtype=torch.float )
y = torch.sin( np.pi * x) - x**3
```

Conferir se o gráfico continua o mesmo apesar da mudança do formato do dado para o formato do PyTorch.

In [15]:

```
plt.figure( figsize = ( 10, 5 ) )
plt.plot( x, y )
plt.axhline( 0, color='gray')
plt.axvline( 0, color='gray')
plt.show()
```



Inicializando os coeficientes que serão ajustados conforme a aplicação do algoritmo de Gradiente Descendente.

In [16]:

```
# a = torch.rand( (), dtype=torch.float, requires_grad=True )
# b = torch.rand( (), dtype=torch.float, requires_grad=True )
# c = torch.rand( (), dtype=torch.float, requires_grad=True )
# d = torch.rand( (), dtype=torch.float, requires_grad=True )

a = torch.tensor(0.0, dtype=torch.float32, requires_grad=True)
b = torch.tensor(0.0, dtype=torch.float32, requires_grad=True)
c = torch.tensor(0.0, dtype=torch.float32, requires_grad=True)
d = torch.tensor(0.0, dtype=torch.float32, requires_grad=True)

print( f'a (inicial): {a:.3f}' )
print( f'b (inicial): {b:.3f}' )
print( f'c (inicial): {c:.3f}' )
print( f'd (inicial): {d:.3f}' )
```

```
a (inicial): 0.000
b (inicial): 0.000
c (inicial): 0.000
d (inicial): 0.000
```

Calculando os coeficientes usando o autograd do PyTorch. O autograd é executado durante a chamada de *l.backward()*.

In [17]:

```
%%time

learning_rate = 0.001
iteracoes = 10000

def forward(x):
    return a + b*x + c*(x**2) + d*(x**3)

def loss(y, y_pred):
    return (y - y_pred).pow(2).sum()
    # return (y - y_pred).pow(2).sum() / len(y) # Sem dividir pelo N (quantidade de ele
mentos) a função é ajustada mais rapidamente.

for iteracao in range(0, iteracoes):

    y_pred = forward(x)

    l = loss(y, y_pred)

    # Calcular os gradientes de forma automática usando o PyTorch (autograd)
    l.backward()

    with torch.no_grad():

        a -= learning_rate * a.grad
        b -= learning_rate * b.grad
        c -= learning_rate * c.grad
        d -= learning_rate * d.grad

    a.grad.zero_()
    b.grad.zero_()
    c.grad.zero_()
    d.grad.zero_()

    # if iteracao % 20 == 0:
    #     print(f'iteracao {iteracao}')
    #     print(f'loss = {l.item():.8f}')
    #     print()
    #     print(f'a = {a.item():.3f}')
    #     print(f'b = {b.item():.3f}')
    #     print(f'c = {c.item():.3f}')
    #     print(f'd = {d.item():.3f}')
    #     print()
    #     print( '------------------------------' )
    #     print()

print()
print( f'Solução a = {a.item():.3f}' )
print( f'Solução b = {b.item():.3f}' )
print( f'Solução c = {c.item():.3f}' )
print( f'Solução d = {d.item():.3f}' )
print()
```

```
Solução a = -0.000
Solução b = 2.677
Solução c = 0.000
Solução d = -3.861

CPU times: user 3.71 s, sys: 30.4 ms, total: 3.74 s
Wall time: 5.09 s
```

Mostrando o gráfico da função ajustada com os coeficientes encontrados.

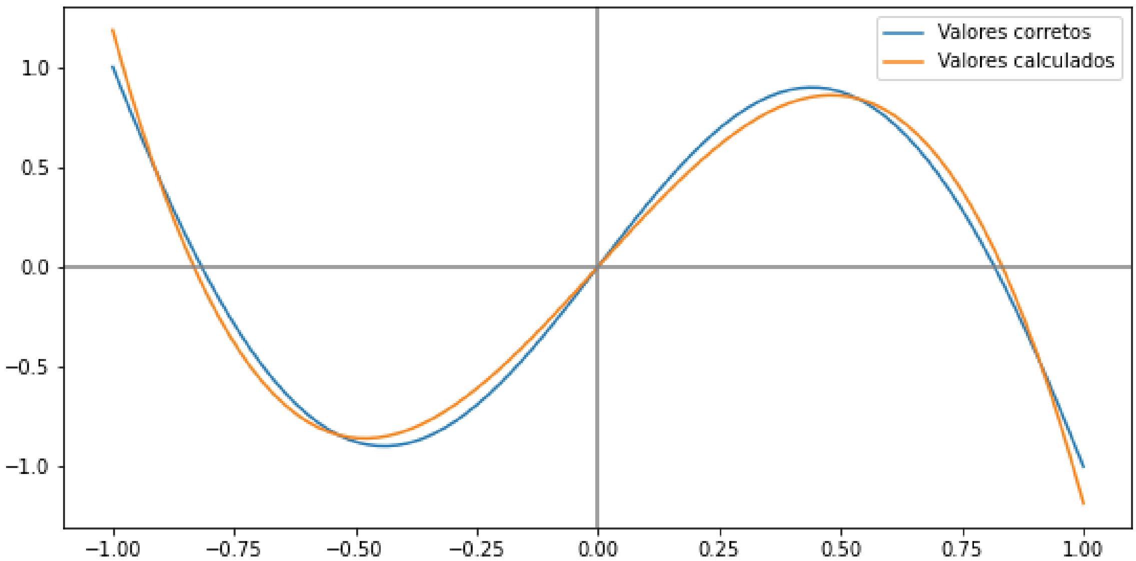
In [18]:

```
y_novo = a.item() + b.item() * x + c.item() * (x**2) + d.item() * x**3

print()
print( f'Função aproximada: {a.item():.3f} + {b.item():.3f} x + {c.item():.3f} x^2 + {d
.item():.3f} x^3' )
print()

plt.figure( figsize=(10, 5) )
plt.plot( x, y, label='Valores corretos' )
plt.plot( x, y_novo, label='Valores calculados' )
plt.axhline( 0, color='gray')
plt.axvline( 0, color='gray')
plt.legend()
plt.show()
```

```
Função aproximada: -0.000 + 2.677 x + 0.000 x^2 + -3.861 x^3
```



Para exportar para HTML

Alterando o diretório no Google Drive para o local onde está este notebook

In [19]:

```
from google.colab import drive
drive.mount('/content/drive/')
```

```
Drive already mounted at /content/drive/; to attempt to forcibly remount,
call drive.mount("/content/drive/", force_remount=True).
```

In [20]:

```
!cd "drive/MyDrive/Doutorado/Disciplinas/[2022.2] [PUC-Rio] Visão Computacional - Profe
ssor Marcelo Gattass/Trabalhos/Trabalho 2/"
```

In [21]:

```
!pwd
```

```
/content
```

In [22]:

```
%%time

!jupyter nbconvert --to html ./T2_DanielCosta.ipynb
```

```
[NbConvertApp] WARNING | pattern './T2_DanielCosta.ipynb' matched no files
This application is used to convert notebook files (*.ipynb)
        to various other formats.

        WARNING: THE COMMANDLINE INTERFACE MAY CHANGE IN FUTURE RELEASES.

Options
=======
The options below are convenience aliases to configurable class-options,
as listed in the "Equivalent to" description-line of the aliases.
To see all configurable class-options for some <cmd>, use:
    <cmd> --help-all

--debug
    set log level to logging.DEBUG (maximize logging output)
    Equivalent to: [--Application.log_level=10]
--show-config
    Show the application's configuration (human-readable format)
    Equivalent to: [--Application.show_config=True]
--show-config-json
    Show the application's configuration (json format)
    Equivalent to: [--Application.show_config_json=True]
--generate-config
    generate default config file
    Equivalent to: [--JupyterApp.generate_config=True]
-y
    Answer yes to any questions instead of prompting.
    Equivalent to: [--JupyterApp.answer_yes=True]
--execute
    Execute the notebook prior to export.
    Equivalent to: [--ExecutePreprocessor.enabled=True]
--allow-errors
    Continue notebook execution even if one of the cells throws an error a
nd include the error message in the cell output (the default behaviour is
to abort conversion). This flag is only relevant if '--execute' was specif
ied, too.
    Equivalent to: [--ExecutePreprocessor.allow_errors=True]
--stdin
    read a single notebook file from stdin. Write the resulting notebook w
ith default basename 'notebook.*'
    Equivalent to: [--NbConvertApp.from_stdin=True]
--stdout
    Write notebook output to stdout instead of files.
    Equivalent to: [--NbConvertApp.writer_class=StdoutWriter]
--inplace
    Run nbconvert in place, overwriting the existing notebook (only
            relevant when converting to notebook format)
    Equivalent to: [--NbConvertApp.use_output_suffix=False --NbConvertApp.
export_format=notebook --FilesWriter.build_directory=]
--clear-output
    Clear output of current file and save in place,
            overwriting the existing notebook.
    Equivalent to: [--NbConvertApp.use_output_suffix=False --NbConvertApp.
export_format=notebook --FilesWriter.build_directory= --ClearOutputPreproc
essor.enabled=True]
--no-prompt
    Exclude input and output prompts from converted document.
    Equivalent to: [--TemplateExporter.exclude_input_prompt=True --Templat
eExporter.exclude_output_prompt=True]
--no-input
    Exclude input cells and output prompts from converted document.
```

```
            This mode is ideal for generating code-free reports.
    Equivalent to: [--TemplateExporter.exclude_output_prompt=True --Templa
teExporter.exclude_input=True]
--log-level=<Enum>
    Set the log level by value or name.
    Choices: any of [0, 10, 20, 30, 40, 50, 'DEBUG', 'INFO', 'WARN', 'ERRO
R', 'CRITICAL']
    Default: 30
    Equivalent to: [--Application.log_level]
--config=<Unicode>
    Full path of a config file.
    Default: ''
    Equivalent to: [--JupyterApp.config_file]
--to=<Unicode>
    The export format to be used, either one of the built-in formats
            ['asciidoc', 'custom', 'html', 'latex', 'markdown', 'noteboo
k', 'pdf', 'python', 'rst', 'script', 'slides']
            or a dotted object name that represents the import path for an
            `Exporter` class
    Default: 'html'
    Equivalent to: [--NbConvertApp.export_format]
--template=<Unicode>
    Name of the template file to use
    Default: ''
    Equivalent to: [--TemplateExporter.template_file]
--writer=<DottedObjectName>
    Writer class used to write the
                                    results of the conversion
    Default: 'FilesWriter'
    Equivalent to: [--NbConvertApp.writer_class]
--post=<DottedOrNone>
    PostProcessor class used to write the
                                    results of the conversion
    Default: ''
    Equivalent to: [--NbConvertApp.postprocessor_class]
--output=<Unicode>
    overwrite base name use for output files.
                can only be used when converting one notebook at a time.
    Default: ''
    Equivalent to: [--NbConvertApp.output_base]
--output-dir=<Unicode>
    Directory to write output(s) to. Defaults
                                  to output to the directory of each noteb
ook. To recover
                                  previous default behaviour (outputting t
o the current
                                  working directory) use . as the flag val
ue.
    Default: ''
    Equivalent to: [--FilesWriter.build_directory]
--reveal-prefix=<Unicode>
    The URL prefix for reveal.js (version 3.x).
            This defaults to the reveal CDN, but can be any url pointing t
o a copy
            of reveal.js.
            For speaker notes to work, this must be a relative path to a l
ocal
            copy of reveal.js: e.g., "reveal.js".
            If a relative path is given, it must be a subdirectory of the
            current directory (from which the server is run).
            See the usage documentation
```

```
            (https://nbconvert.readthedocs.io/en/latest/usage.html#reveal-
js-html-slideshow)
            for more details.
    Default: ''
    Equivalent to: [--SlidesExporter.reveal_url_prefix]
--nbformat=<Enum>
    The nbformat version to write.
            Use this to downgrade notebooks.
    Choices: any of [1, 2, 3, 4]
    Default: 4
    Equivalent to: [--NotebookExporter.nbformat_version]

Examples
--------

    The simplest way to use nbconvert is

            > jupyter nbconvert mynotebook.ipynb

            which will convert mynotebook.ipynb to the default format (pro
bably HTML).

            You can specify the export format with `--to`.
            Options include ['asciidoc', 'custom', 'html', 'latex', 'markd
own', 'notebook', 'pdf', 'python', 'rst', 'script', 'slides'].

            > jupyter nbconvert --to latex mynotebook.ipynb

            Both HTML and LaTeX support multiple output templates. LaTeX i
ncludes
            'base', 'article' and 'report'.  HTML includes 'basic' and 'fu
ll'. You
            can specify the flavor of the format used.

            > jupyter nbconvert --to html --template basic mynotebook.ipyn
b

            You can also pipe the output to stdout, rather than a file

            > jupyter nbconvert mynotebook.ipynb --stdout

            PDF is generated via latex

            > jupyter nbconvert mynotebook.ipynb --to pdf

            You can get (and serve) a Reveal.js-powered slideshow

            > jupyter nbconvert myslides.ipynb --to slides --post serve

            Multiple notebooks can be given at the command line in a coupl
e of
            different ways:

            > jupyter nbconvert notebook*.ipynb
            > jupyter nbconvert notebook1.ipynb notebook2.ipynb

            or you can specify the notebooks list in a config file, contai
ning::

                c.NbConvertApp.notebooks = ["my_notebook.ipynb"]
```

```
            > jupyter nbconvert --config mycfg.py

To see all available configurables, use `--help-all`.

CPU times: user 16.1 ms, sys: 12.9 ms, total: 29 ms
Wall time: 643 ms
```