Daniel Diamont

dd28977

Lab1 Written Questions

(1)

	$X=0$	$X=1$
$Y=0$	$1/4$	$1/4$
$Y=1$	$1/6$	$1/3$

$Pr(X=0) = \sum_y p(0,y) = \frac{6}{24} + \frac{4}{24} = \frac{5}{12}$

(a) $Pr(X=1) = ?$

$Pr(X=1) = \sum_y p(1,y) = \frac{8}{24} + \frac{6}{24} = \frac{7}{12}$

$$P_X(x) = \begin{cases} \frac{7}{12} & \text{if } X=1 \\ \frac{5}{12} & \text{if } X=0 \end{cases}$$

$$\boxed{P(X=1) = \frac{7}{12}}$$

(b) $Pr(X=1 \mid Y=1) = ?$

$P_Y(y=0) = \sum_x p(x,0) = \frac{1}{2}$ $\quad P_Y(y=1) = \sum_x p(x,1) = \frac{1}{2}$

$$p_{X|Y}(1|1) = \frac{p_{X,Y}(1,1)}{P_Y(1)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \boxed{\frac{2}{3}}$$

(c) $var(x) = ?$

$\mathbb{E}[X] = \frac{7}{12}$

$$\mathbb{E}\left[\left(X - \mathbb{E}[X]\right)^2\right] = \left(0 - \frac{7}{12}\right)^2 \cdot \frac{5}{12} + \left(1 - \frac{7}{12}\right)^2 \cdot \frac{7}{12} = \frac{35}{144} = \boxed{0.243}$$

(d) $var(X|Y=1) = ?$

$$\mathbb{E}[X|Y=1] = ?$$

$$p_{X|Y}(X|Y=1) = \begin{cases} \frac{p_{X,Y}(0,1)}{p_Y(1)} = \frac{1}{3}, & x=0 \\ \frac{p_{X,Y}(1,1)}{p_Y(1)} = \frac{2}{3}, & x=1 \end{cases}$$

$$\mathbb{E}[X|Y=1] = \frac{1}{3}\cdot 0 + \frac{2}{3}\cdot 1 = \frac{2}{3}$$

$$\cancel{var(X|Y=1) = \mathbb{E}[(X|Y=1) - \mathbb{E}[X|Y=1])^2] =}$$

$$var(X|Y=1) = \mathbb{E}[(X - \mathbb{E}[X|Y=1])^2] = (0-\tfrac{2}{3})^2\cdot\tfrac{1}{3} + (1-\tfrac{2}{3})^2\cdot\tfrac{2}{3}$$

$$\boxed{var(X|Y=1) = \frac{2}{9}}$$

(e) $\mathbb{E}[X^3 + X^2 + 3Y^7 | Y=1] = ?$

$= \mathbb{E}[X^3|Y=1] + \mathbb{E}[X^2|Y=1] + 3\,\mathbb{E}[\cancel{Y^7|Y=1}]$ $3\cdot 1 = 3$

~~Let A = (X|Y=1)~~

$$\mathbb{E}[X^3|Y=1] = \frac{2}{3}$$

$$\mathbb{E}[X^2|Y=1] = \frac{2}{3}$$

$$p(x^2|1) = \begin{cases} \frac{1}{3}, & x=0 \\ \frac{2}{3}, & x=1 \end{cases}$$

$$p(x^3|1) = \begin{cases} \frac{1}{3}, & x=0 \\ \frac{2}{3}, & x=1 \end{cases}$$

$$\boxed{\mathbb{E}[X^3 + X^2 + 3Y^7 | Y=1] = \frac{2}{3} + \frac{2}{3} + 1 = \frac{7}{3}}$$

(2) $\vec{v}_1 = [1,1,1]$ $\quad P_1=(3,3,3)$, $P_2=(1,2,3)$, $P3=(0,0,1)$

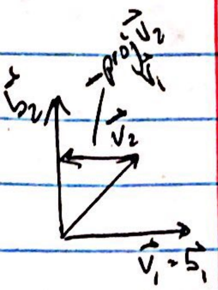
$\vec{v}_2 = [1,0,0]$

Subspace $X \subseteq \mathbb{R}^3$ spanned by $\vec{v}_1$ and $\vec{v}_2$.

Are v_1 and v_2 orthogonal?

$V_1 \cdot V_2 = 1\cdot 1 + 1\cdot 0 + 1\cdot 0 = 1$ No they are not orthogonal.

We must find an orthogonal basis before making a projection onto the ~~plane~~ subspace. Let's have our ~~basis~~ orthogonal basis be defined by $\vec{b}_1$ and $\vec{b}_2$.



Let $\vec{v}_1 = \vec{b}_1$.

Then, $\vec{b}_2 = \vec{v}_2 - \text{proj}_{\vec{v}_1}\vec{v}_2 = \vec{v}_2 - \frac{\vec{v}_2 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1}\vec{v}_1$

$$\vec{b}_2 = [1,0,0] - \frac{1}{3}[1,1,1] = [2/3, -1/3, -1/3]$$

Then, we can project P_1, P_2, and P_3 onto X. Let the projections be defined by y_1, y_2, and y_3, respectively. Then,

$$y_i = \frac{p_i \cdot b_1}{b_1 \cdot b_1} b_1 + \frac{p_i \cdot b_2}{b_2 \cdot b_2} b_2$$

Using a calculator we get

$$y_1 = [3,3,3] + [0,0,0] = [3,3,3]$$

$$y_2 = [2,2,2] + [-1, \tfrac{1}{2}, \tfrac{1}{2}] = [1, 2.5, 2.5]$$

$$y_3 = [1/3, 1/3, 1/3] + [-\tfrac{1}{3}, \tfrac{1}{6}, \tfrac{1}{6}] = [0, \tfrac{1}{2}, \tfrac{1}{2}]$$

(3) Let H be the event a coin is heads and let T be the event a coin is tails. Then, let the probability that the coin is H or T be as follows:

$$Pr(H) = \frac{2}{3} \qquad Pr(T) = \frac{1}{3}.$$

Since we want to know Pr(number of H ≤ 50) we can model the 100 coin tosses with a binomial distribution, and we can sum the probabilities that we get 1 head up to 50 heads.

$$Pr(\text{number of } H \le 50) = \sum_{i=1}^{50} \binom{100}{i} \left(\frac{2}{3}\right)^{i} \left(\frac{1}{3}\right)^{100-i}$$

Using Wolfram Alpha as my calculator,

$$\boxed{Pr(\text{number of } H \le 50) = 0.000419}$$