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(1) a) $Z \sim N(\mu, \sigma^2)$

sample mean $Z_{avg} = \sum_{i=1}^{n} \frac{z_i}{n}$ how close is z_{avg} to μ?

$\mu = 0, \sigma = 1$

$Z \sim N(0,1)$ $n = 10{,}000$

$Pr(z_{avg} \geq 0.1)$, $Pr(z_{avg} \geq 0.01)$, $Pr(z_{avg} \geq 0.001)$

$C = [0.1, 0.01, 0.001]$

$Pr(\bar{z} > c_i)$

$= Pr\left(\frac{\bar{z} - \mu}{\sigma_{\bar{z}}} \geq \frac{c_i - \mu}{\sigma_{\bar{z}}}\right)$ and $\sigma_{\bar{z}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{n}}$

$= Pr(z_{avg} \geq c_i \sqrt{n})$

$= 1 - Pr(z_{avg} < c_i \sqrt{n})$

$c_0: 1 - \Phi(10) = 1 - 1 = 0$

$c_1: 1 - \Phi(1) = 0.15866$

$c_2: 1 - \Phi(0.1) = 0.46017$

b) $\bar{z} \sim N(\mu, \sigma^2)$

$$Pr\left(\frac{\bar{z}-\mu}{\sigma_{\bar{z}}} \geq \frac{n^c-\mu}{\sigma_{\bar{z}}}\right)$$

$$\sigma_{\bar{z}} = \frac{\sigma}{\sqrt{n}}$$

$$P_1\left(z_{avg} \geq \frac{n^c-\mu}{\sigma}\sqrt{n}\right) = \boxed{1-\Phi\left(\frac{n^c-\mu}{\sigma}\sqrt{n}\right)}$$

2 a) $\frac{1}{n}\sum_{i=1}^{n}(x_i\beta - y_i)^2$

$$= \frac{1}{n}\sum_{i=1}^{n}\left((x_i\beta)^2 - 2x_i\beta y_i + y_i^2\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i^2\beta^2 - 2(x_iy_i)\beta + y_i^2\right)$$

b) $A(x_i, y_i) = \frac{1}{n}\sum_{i=1}^{n} x_i^2$

$$B(x_i, y_i) = -\frac{2}{n}\sum_{i=1}^{n} x_iy_i$$

$$C(x_i, y_i) = \frac{1}{n}\sum_{i=1}^{n} y_i^2$$

c) $\min_{\beta} : \frac{1}{n}\sum_{i=1}^{n}(x_i\beta - y_i)^2$

$$\frac{\partial}{\partial\beta}\left(\frac{1}{n}\sum_{i=1}^{n}(x_i\beta - y_i)^2\right) = 0 \qquad 2(x_i\beta - y_i)x_i$$

$$\frac{2}{n}\sum_{i=1}^{n}x_i^2\beta - \frac{2}{n}\sum_{i=1}^{n}x_i y_i = 0$$

$$\sum_{i=1}^{n}x_i^2\beta = \sum_{i=1}^{n}x_i y_i$$

$$\hat{\beta} = \frac{\sum_{i=1}^{n}x_i y_i}{\sum_{i=1}^{n}x_i^2}$$

$$\boxed{\hat{\beta} = \frac{\sum_{i=1}^{n}x_i y_i}{\|\vec{x}\|_2^2}}$$

d) $\hat{\beta} = \frac{\sum_{i=1}^{n} x_i(x_i\beta + e_i)}{\sum_{i=1}^{n} x_i^2} = \beta + \frac{\sum_{i=1}^{n} x_i e_i}{\sum_{i=1}^{n} x_i^2}$

$$\hat{\beta} = \beta + \frac{\vec{x}^T \cdot e}{\vec{x}^T \cdot \vec{x}}$$

$$\boxed{Z = \frac{\vec{x}^T}{\|\vec{x}\|_2^2}}$$

Ze is a normalized error of our estimate $\hat{\beta}$ with respect to β. Z is a matrix that normalizes the error vector with respect to the observed data points.