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Exercise assignment (Modeling of a Quadrotor)

Download the exercise files here: **http://www.asl.ethz.ch/education/master/aircraft/2013-L7-Exercises.pdf**

The performance and the dynamics of a Quadrotor shall be evaluated. The Quadrotor under consideration has the following parameters.

Parameter	**Value**
Mass	m = 0.58Kg
Inertia	I = diag(0.0086,0.0086,0.0172) Kgm^2
Arm length	L = 0.225 m
Hub length	h = 0.007 m
Propeller radius	R = 0.08 m
Blade pitch angle	Θ(r) = 25°-25°*(r/R-0.2)
Blade chord	c/R(r) = -0.6641*$(r/R)^2$+0.6969* r/R+0.0172
Lift coefficient	$C_{l\alpha}$ = 5.7
Drag coefficient	C_d = 0
Zero lift angle of attack	α_0 = -2°
Number of Blades	N_b = 2
Motor and Propeller inertia	I = 12*10^-6 Kgm^2
Motor resistance	R = 0.128 Ohm
Motor constant	K = 1900 RPM/Volt
Air density	ρ = 1.2 Kg/m^3
Gravity constant	g = 9.81 m/s^2

Additionally the Quadrotor is equipped with a LIPO battery pack with the following properties (capacity = 1200mAh, Voltage level = 12V).

Using the momentum theory, estimate the time, the Quadrotor is able to hover given battery pack, assuming a figure of merit of 0.8.

1. Calculate the thrust of one propeller in hover.

   Solution: During hovering, the total thrust has to be equal to the gravity force. Since no moments shall be created, each of the propellers shall produce a thrust equal to a quarter of the gravity force.

   $$T_{prop} = \frac{1}{4} mg = 1.42N$$

2. Calculate the ideal power consumption of one propeller using the momentum theory.

   Solution: In general, the induced velocity at the rotor disc can be calculated using $T=2\rho A(V+v_1)v_1$ and then calculating the Power by $P=T(V+v_1)$. In the case of hover, the Power can directly be calculated by

   $$P = \frac{T_{prop}^{3/2}}{\sqrt{2\rho R^2 \pi}} = 7.72W$$

3. Calculate the real power of the Quadrotor using the figure of merit and all propellers.
   Solution: The figure of merit is defined by

$$FOM = \frac{P_{ideal}}{P_{real}}$$

   Thus the real power used by the propeller can be calculated with

$$P_{real} = \frac{P_{ideal}}{FOM} = 9.65W$$

   Finally, all the four propellers will give a total power consumption of

$$P_T = 4P_{real} = 38.61W$$

4. Calculate the maximum flight time with the given battery.
   a. Calculate the energy in the battery.
      Solution: The energy of the Battery is its capacity times the voltage

$$E_{Batt} = C_{Batt} U_{Batt} = 14.40Wh$$

   b. Calculate the time until the energy from the battery is used.
      Solution: Since the power used is assumed to be constant, the energy up to time t from the propellers can be calculated by

$$E_{\Pr} = P_T t$$

      Maximum flight time is given when the propeller use all the energy in the battery

$$E_{\Pr} = E_{Batt} \Rightarrow t = \frac{E_{Batt}}{P_T} = 0.27h = 22\min$$

Now a model of the Quadrotor shall be designed in Matlab for further control design purposes. Assume hover condition for the Quadrotor. Since no propeller thrust and drag data are available, the thrust and the drag coefficients shall be calculated using the BEMT theory.

1. Implement the rigid body dynamics in simulink.
   a. What are the states of the model?
      Solution: The system can move in space with 6DoF. Thus we need to have 6 states for the position and the orientation. Since the model is driven by forces and moment, 6 additional states are needed for the corresponding velocities. This gives a total of 12 states for position, Euler angles, velocity and rotational velocity.
   b. Write down the dynamic equations in the respective block in the simulink model
      Solution: Inspect the rigid body subsystem block of the Simulink model Quadrotor.slx.
2. Model the aerodynamics.
   a. Which kind of forces and moments are acting on the Quadrotor? Only list the relevant forces near hover.
      Solution: Near hover, the assumption can be made that the advance ratio μ is small. Thus, for one propeller only the thrust force and the drag torque are relevant. For the CoG of the Quadrotor, the relevant forces and moments are:

Forces	Moments
Gravity	Sum of drag torques
Sum of thrust forces	Thrust induced moments

   b. We need to figure out the thrust and drag constant. We use the BEMT method to approximate those constants. Using BEMT, calculate the thrust and the drag moment for various motor rpms from 0 up to 5000rpm. Then fit a quadratic curve on the data. Use the Bemt() function template.

i. Separate the propeller blade into 20 section equally distributed across the blade starting from the inner radius r/R = 0.2 (Everything below is considered as the root of the blade to the shaft and will not generate an aerodynamic force).

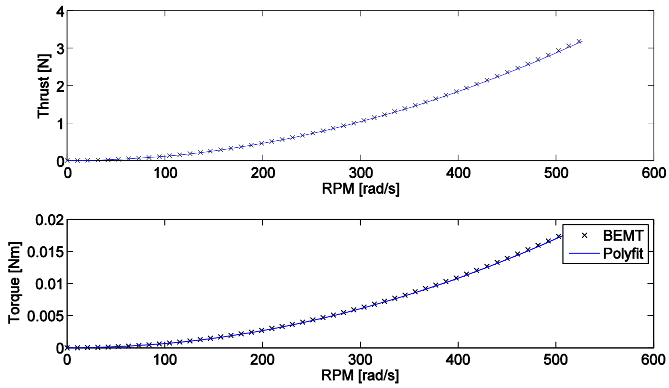
ii. Calculate the vertical velocity V'p at hover using the BEMT formula.

iii. Calculate and integrate the thrust and the drag moment at each section.

iv. Fit a quadratic curve on the results using the polyfit command.

v. Calculate the aerodynamic forces and moment using the thrust and drag constant evaluated with the BEMT.

Solution: The solution can be found in the Matlab script Bemt.m. The solution of the BEMT analysis will give a purely quadratic relation. Since the dimensionless lift coefficient C_l is modeled as only dependent on the angle of attack, the relation V'p/Vt at given radius r will be constant for different rotational velocities and thus, the differential thrust element depends quadratic on the rotational speed.



3. Model the motor dynamics
   a. The motor is modeled as a second order system with an electrical and a mechanical system. Since we can assume that the electrical dynamics are fast with respect to the mechanical dynamics, the electrical system can be simplified.
      i. Simplify the electrical dynamics by assuming they will be always in steady state (di/dt=0) and solve for the current i.
      ii. Implement the mechanical system with the steady state current on the Quadrotor model.

      Solution: The electrical system in the model is often simplified for numerical reasons in the simulation since the electrical system is generally much faster with respect to the mechanical system. The electrical system can be simplified to

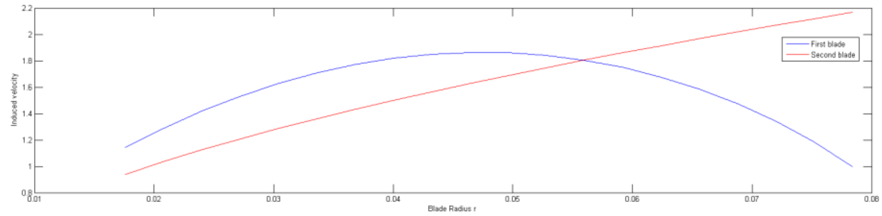
$$L\frac{di}{dt} = U - Ri - U_{ind} = 0 \Rightarrow i = \frac{U - K_t\omega_m}{R}$$

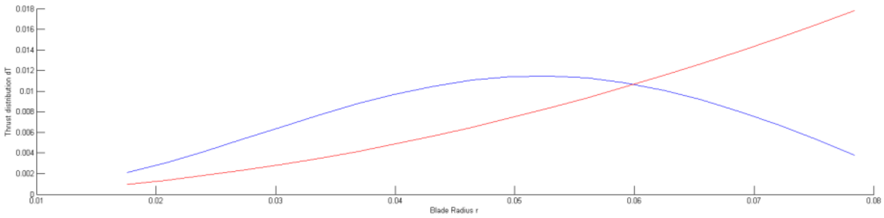
Inserting the current into the mechanical system of the motor leads to

$$I_m\frac{d\omega_m}{dt} = \frac{K_t}{R}U - \frac{K_t^2}{R}\omega_m - Q$$

4. Play with the model
   a. Change the motor speed input into the model. You can add a generator block to give arbitrary input.
5. Play with Bemt
   a. Visualize the induced airflow and thrust distribution calculated through Bemt for the propeller

      Solution: The distribution can be seen in the figure below





Due to the blade twist and tapering a better induced velocity and force distribution can be realized. The distribution is much more uniform over the blade. Additionally, the distribution decreases at the blade tip to reduce the blade tip vortex.

b. Calculate and visualize the induced airflow, thrust distribution and lift and drag constant for a simple propeller with the following properties

Blade pitch angle	Θ(r) = 13.5° = const.
Blade chord	c (r) = 0.015 m = const.

Please keep the model for next week's control exercise.