2018 Part I

FRM® Exam Prep

Schweser's Secret Sauce®

eBook

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FRM Part I Secret Sauce

FRM 2018 PART I SCHWESER'S SECRET SAUCE®

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How to Use Schweser's Secret Sauce

This review book is a valuable addition to the study tools of any FRM exam candidate. It offers concise coverage of exam topics to enhance your retention of the FRM curriculum.

We suggest that you use this book as a companion to your other, more comprehensive study materials. It is easier to carry with you and will allow you to study these key concepts, definitions, and techniques over and over, which is a crucial part of mastering the material. For a majority of you, there are no shortcuts to learning the broad array of subject matter covered by the FRM curriculum, but this book should be a very valuable tool for learning and reviewing the material as you progress in your studies over the weeks leading up to exam day.

Previous Part I exam pass rates have been slightly below 50%, and many FRM candidates have commented on the high difficulty level of the exam. This is an indication that you should not underestimate the task at hand. Our SchweserNotes, Practice Exams, SchweserPro™ QBank, Online Weekly Class, and Schweser's Secret Sauce are all designed to help you study as efficiently as possible, grasp and retain the material, and apply your knowledge with confidence on exam day.

As a reminder, the 2018 FRM Part I topic coverage and weightings assigned by GARP are as follows:

Book	*Knowledge Domain*	*Exam Weight*	*Exam Questions*
1	Foundations of Risk Management	20%	20
2	Quantitative Analysis	20%	20
3	Financial Markets and Products	30%	30
4	Valuation and Risk Models	30%	30

Foundations of Risk Management

Weight on Exam	20%
SchweserNotes™ Reference	Book 1

Risk Management: A Helicopter View

Cross Reference to GARP Assigned Reading – Crouhy, Galai, and Mark, Chapter 1

The Concept of Risk

Risk arises from the uncertainty regarding an entity's future losses as well as future gains. Therefore, in simplified terms, there is a natural trade-off between risk and return. Risk is not necessarily related to the size of the potential loss. The more important concern is the variability of the loss, especially a loss that could rise to unexpectedly high levels or a loss that suddenly occurs that was not anticipated.

Risk taking refers specifically to the active assumption of incremental risk in order to generate incremental gains. In that regard, risk taking can be thought of in an opportunistic context.

The Risk Management Process

The risk management process involves the following five steps:

Step 1: Identify the risks.
Step 2: Quantify and estimate the risk exposures *or* determine appropriate methods to transfer the risks.
Step 3: Determine the collective effects of the risk exposures *or* perform a cost-benefit analysis on risk transfer methods.
Step 4: Develop a risk mitigation strategy (i.e., avoid, transfer, mitigate, or assume risk).
Step 5: Assess performance and amend risk mitigation strategy as needed.

Two key problems with the process include identifying the correct risk(s) and finding an efficient method of transferring the risk.

One of the challenges in ensuring that risk management will be beneficial to the economy is that risk must be sufficiently dispersed among willing and able participants in the economy.

Another challenge of the risk management process is that it has failed to consistently assist in preventing market disruptions or preventing financial accounting fraud (due to corporate governance failures).

In addition, the use of derivatives as complex trading strategies assisted in overstating the financial position (i.e., net assets on balance sheet) of many entities and understating the level of risk assumed by many entities.

Finally, risk management may not be effective on an overall economic basis because it only involves risk transferring by one party and risk assumption by another party.

Measuring and Managing Risk

Value at risk (VaR) states a certain loss amount and its probability of occurring. For example, a financial institution may have a one-day VaR of $2.5 million at the 95% confidence level. That would be interpreted as having a 5% chance that there will be a loss greater than $2.5 million on any given day. VaR is a useful measure for liquid positions operating under normal market circumstances over a short period of time. It is less useful and potentially dangerous when attempting to measure risk in non-normal circumstances, in illiquid positions, and over a long period of time.

Economic capital refers to holding sufficient liquid reserves to cover a potential loss. For example, if one-day VaR is $2.5 million and the entity holds $2.5 million in liquid reserves, then it is unlikely to go bankrupt that day.

Scenario analysis takes into account potential risk factors with uncertainties that are often non-quantifiable.

Stress testing is a form of scenario analysis that examines a financial outcome based on a given "stress" on the entity.

Enterprise risk management (ERM) takes an integrative approach to risk management within an entire entity, dispensing of the traditional approach of independently managing risk within each department or division of an entity. ERM considers entity-wide risks and tries to integrate risk considerations into key business decisions.

Expected and Unexpected Loss

Expected loss considers how much an entity expects to lose in the normal course of business. It can often be computed in advance (and provided for) with relative ease because of the certainty involved.

Unexpected loss considers how much an entity could lose outside of the normal course of business. Compared to expected loss, it is generally more difficult to predict, compute, and provide for in advance because of the uncertainty involved.

Risk and Reward

There is a trade-off between risk and reward. In very general and simplified terms, the greater the risk taken, the greater the potential reward. However, one must consider the variability of the potential reward. The portion of the variability that is measurable as a probability function could be thought of as risk whereas the portion that is not measurable could be thought of as uncertainty.

In practice, some entities have weak risk management and/or risk governance cultures, which allows for potential returns to be overstated because they are not adjusted for risk. Correlation risks may be ignored, which understate overall risk. Some risk measures may be computed in a misleading manner because the proper computation may result in lower reported profits for the entity.

Risk Classes

There are eight key classes of risk: (1) market risk, (2) credit risk, (3) liquidity risk, (4) operational risk, (5) legal and regulatory risk, (6) business risk, (7) strategic risk, and (8) reputation risk.

Market risk considers how changes in market prices and rates will result in investment losses. There are four subtypes of market risk: (1) interest rate risk, (2) equity price risk, (3) foreign exchange risk, and (4) commodity price risk.

Credit risk refers to a loss suffered by a party whereby the counterparty fails to meet its financial obligations to the party under the contract. Credit risk may also arise if there is an increasing risk of default by the counterparty throughout the duration of the contract. There are four subtypes of credit risk: (1) default risk, (2) bankruptcy risk, (3) downgrade risk, and (4) settlement risk.

Liquidity risk is subdivided into two parts: (1) funding liquidity risk and (2) trading liquidity risk. Funding liquidity risk occurs when an entity is unable to pay down or refinance its debt, satisfy any cash obligations to counterparties, or fund any capital withdrawals. Trading liquidity risk occurs when an entity is unable to buy or sell a security at the market price due to a temporary inability to find a counterparty to transact on the other side of the trade.

Operational risk considers a wide range of non-financial problems such as inadequate computer systems, insufficient internal controls, incompetent management, fraud, human error, and natural disasters.

Legal risk could arise when one party sues the other party in an attempt to nullify or terminate the transaction. **Regulatory risk** could arise from changes in laws and regulations that are unfavorable to the entity (e.g., higher tax rates, higher compliance costs).

Business risk revolves around uncertainty regarding the entity's income statement. Revenues may be uncertain because of the uncertainty surrounding the demand for the products and/or the price that should be set. Production and administration costs may also be uncertain.

Strategic risk can be thought of in the context of large new business investments, which carry a high degree of uncertainty as to ultimate success and profitability. Alternatively, it could be thought of from the perspective of an entity changing its business strategy compared to its competitors.

Reputation risk consists of two parts: (1) the general perceived trustworthiness of an entity (i.e., that the entity is able and willing to meet its obligations to its creditors and counterparties) and (2) the general perception that the entity engages in fair dealing and conducts business in an ethical manner.

Corporate Risk Management: A Primer

Cross Reference to GARP Assigned Reading – Crouhy, Galai, and Mark, Chapter 2

Hedging Risk Exposures

There are some theoretical reasons for a firm not to hedge risk exposures, but most of those reasons make the unrealistic assumption of perfect capital markets, which is not realistic. Also, they ignore the existence of the significant costs of financial distress and bankruptcy. However, in practice, there are some valid reasons not to hedge, including the distraction from focusing on the core business, lack of skills and knowledge, and transaction and compliance costs.

Many reasons exist for a firm to hedge its risk exposures. Key reasons include lowering the cost of capital and reducing volatility of reported earnings, operational improvements, and potential cost savings over traditional insurance products.

Hedging Decisions

As a start, a firm must know its risk and return goals before embarking on a risk management plan. Those goals must be evaluated and approved by the board of directors to ensure the plan is focused and relevant. A major conclusion to consider with the risk/return trade-off is that firms should accept all projects with a positive net present value (NPV), taking into account risk, because it will maximize value for the firm's stakeholders.

The Role of the Board of Directors

Management and the board need to set and communicate the firm's **risk appetite** in a quantitative and/or qualitative manner. There are several possibilities, including:

- Explicitly stating (qualitatively) which risks the firm can tolerate (to be left unhedged) and, therefore, which risks it cannot tolerate (to be hedged).
- Using a quantitative metric such as value at risk (VaR) to convey the maximum loss the firm will tolerate for a given confidence level for a given period of time.
- Using stress testing whereby management considers possible but very severely negative scenarios to determine the level of losses. From there, the board makes the determination of which losses are tolerable (to not mitigate or to leave uninsured) and which ones are not tolerable (to be mitigated or insured).

The Process of Mapping Risks

Mapping risks requires clarification as to which risks are insurable, hedgeable, noninsurable, or nonhedgeable. Mapping risks could be performed for various risks such as market, credit, business, and operational. Essentially, it involves a detailed analysis of the impacts of such risks on the firm's financial position (balance sheet) and financial performance (income statement).

Hedging Operational and Financial Risks

Hedging operational risks covers a firm's activities in production (costs) and sales (revenue), which is essentially the income statement. Financial position risk pertains to a firm's balance sheet. Making the realistic assumption that there are some imperfections in the financial markets, a firm could benefit from hedging financial position risk. Hedging activities should cover both the firm's assets and liabilities in order to fully account for the risks.

Pricing risk could be thought of as a type of operational risk, requiring the hedging of revenues and costs. Foreign currency risk refers to the risk of economic loss due to unfavorable changes in the foreign currency exchange rate; to the extent that there is production and sales activity in the foreign currency, pricing risk would exist simultaneously. Interest rate risk refers to the risk inherent in a firm's net exposure to unfavorable interest rate fluctuations.

Risk Management Instruments

Financial instruments are used to hedge risks and can be classified as exchange traded or over the counter (OTC). **Exchange-traded instruments** cover only certain underlying assets and are quite standardized (e.g., maturities and strike prices) in order to promote liquidity in the marketplace. **OTC instruments** are privately traded between a bank and a firm and thus can be customized to suit the firm's risk management needs. In exchange for the customization, OTC instruments are less liquid and more difficult to price than exchange-traded instruments. In addition, there is credit risk by either of the counterparties (e.g., default risk) that would generally not exist with exchange-traded instruments.

Corporate Governance and Risk Management

Cross Reference to GARP Assigned Reading – Crouhy, Galai, and Mark, Chapter 4

Best Practices

There are numerous best practices in corporate governance, including:

- Board is comprised of a majority of independent members with basic knowledge of the firm's business and industry.
- Board watches out for the interests of all stakeholders, including shareholders and debtholders who may have somewhat differing interests.
- Board is aware of any agency risks and takes steps to reduce them (e.g., compensation committee).
- Board maintains its independence from management (e.g., CEO is not the chairman of the board).
- Board should consider the introduction of a chief risk officer.

There are numerous best practices in risk management, including:

- Board should focus on the firm's economic performance over accounting performance.
- Board should promote a robust risk management process within the firm (e.g., upward mobility for risk management careers).
- Board should set up an ethics committee to uphold high ethical standards within the firm.
- Board should ensure that compensation is based on risk-adjusted performance.
- Board should approve all major transactions.
- Board should always apply professional skepticism to ask probing and relevant questions to management.
- Board should have a risk committee in place.

Risk Governance

In terms of risk governance, the board has some important responsibilities that could be facilitated with the involvement of a risk advisory director.

A **risk advisory director** would be a board member who is a risk specialist who attends risk committee and audit committee meetings and provides advice to increase effectiveness.

More specific duties of the director (and the board in general) would include the review and analysis of the following:

- The firm's risk management policies.
- The firm's periodic risk management reports.
- The firm's risk appetite and its impact on business strategy.
- The firm's internal controls.
- The firm's financial statements and disclosures.
- The firm's related parties and related party transactions.
- Any audit reports from internal or external audits.
- Corporate governance best practices for the industry.
- Risk management practices of competitors and the industry.

The **audit committee** (as part of the board) has traditionally been responsible for the reasonable accuracy of the firm's financial statements and its regulatory reporting requirements. It must ensure that the firm has taken all steps to avoid the risk that the financial statements are materially misstated as a result of undiscovered errors and/or fraud. In addition to the more visible verification duties, the audit committee monitors the underlying systems in place regarding financial reporting, regulatory compliance, internal controls, and risk management. In that respect, the audit committee may be able to rely on some or all of the work of the internal audit team, which usually reports directly to the audit committee.

Risk Appetite and Business Strategy

A firm's risk appetite reflects its tolerance (especially willingness) to accept risk. The subsequent implementation of the risk appetite into defining the firm's risk limits sets some bounds to its business strategy and to its ability to exploit business opportunities. The board needs to develop/approve the firm's risk appetite as well as assist management in developing the firm's overall strategic plan.

Interdependence of Functional Units

The various functional units within a firm are dependent on one another when it comes to risk management and reporting. All transactions must be recorded correctly and in the appropriate period in order to ensure the accuracy of the periodic profit and loss (P&L) statements. Using an investment bank as an example, areas such as valuations, the profit and loss statement, and risk policy require input from more than one of the following units: (1) senior management, (2) risk management, (3) trading room management, (4) operations, and (5) finance.

What is ERM?

Cross Reference to GARP Assigned Reading – Lam, Chapter 4

Enterprise Risk Management

An integrated and centralized approach under **enterprise risk management (ERM)** is significantly more effective in managing a company's risks than under the traditional silo approach of managing and centralizing risks within each risk/business unit. ERM is a comprehensive and integrated framework for managing a firm's key risks to meet business objectives, minimize unexpected earnings volatility, and maximize firm value.

Since the concept of ERM is relatively new and is still evolving, there is a lack of a standard ERM definition. ERM is often defined as a process or activity to manage risks. A more useful definition of ERM is as follows:

> *"Risk is a variable that can cause deviation from an expected outcome. ERM is a comprehensive and integrated framework for managing key risks in order to achieve business objectives, minimize unexpected earnings volatility, and maximize firm value."*[1]

ERM Benefits and Costs

There are three primary motivations for a firm to implement an ERM initiative: (1) integration of risk organization, (2) integration of risk transfer, and (3) integration of business processes. The respective benefits are better organizational effectiveness, better risk reporting, and improved business performance. However, implementation of an integrated firm-wide initiative is costly (both capital and labor intensive) and time-consuming. This process could last several years and requires ongoing senior management and board support.

The Chief Risk Officer

The chief risk officer (CRO) is responsible for all risks facing a company, including market, credit, and operational risks and is responsible for developing and implementing an ERM strategy. The CRO provides overall leadership for ERM and develops policies and standards, including setting the firm's overall risk appetite, measuring and quantifying risks and setting risk limits, and developing risk systems.

The CRO generally reports to the CEO or CFO but could also have a dotted line relationship to both the CEO/CFO and to the board to minimize any potential friction between the CRO and the CEO/CFO (due to excessive risk taking, regulatory issues, or fraud).

An ideal CRO possesses five critical skills: (1) leadership, (2) power of persuasion, (3) ability to protect the firm's assets, (4) technical skills to understand all risks, and (5) consulting skills to educate the board and business functions on risk management.

1. James Lam, *Enterprise Risk Management: From Incentives to Controls*, 2nd Edition, (Hoboken, NJ: John Wiley & Sons, 2014), 53.

ERM Framework Components

There are seven components of a strong ERM framework: (1) corporate governance, (2) line management, (3) portfolio management, (4) risk transfer, (5) risk analytics, (6) data and technology resources, and (7) stakeholder management.

Corporate governance is critical in the implementation of a successful ERM program and ensures that senior management and the board have the requisite organizational practices and processes to adequately control risks.

Line management is the management of activities that relate directly to producing a firm's products and services.

Portfolio management provides a holistic view of the firm's risks if these risks are viewed as individual components of the aggregate risks facing the firm.

Risk transfer reduces or transfers out risks that are either undesirable risks or are desirable but considered concentrated (i.e., excessive risks).

Risk analytics quantifies risk exposures for use in risk analysis, measurement, and reporting.

Data technology and resources improve the quality of data used in evaluating risks.

Stakeholder management facilitates communicating a firm's internal risk management process to external stakeholders, including shareholders, creditors, regulators, and the public.

Risk Management, Governance, Culture, and Risk Taking in Banks

Cross Reference to GARP Assigned Reading – Stulz

Methods to Determine Optimal Level of Risk Exposure

Targeting a certain default probability or targeting a specific credit rating. In general, a bank should not always aim to earn the highest credit rating possible (e.g., AAA). Aiming for AAA may constrain the bank's risk-taking ability and reduce its returns due to lost profitable projects. Aiming for BBB may result in lost customers due to the perception that the bank is engaging in excessive risk-taking activities.

Sensitivity analysis or scenario analysis. Alternatively, a bank could determine its optimal level of risk exposure by the impact of specific shocks.

How the Optimal Level of Risk Can Differ Across Banks

A bank that is focused on deposits, relationship lending customers, or both would usually set the level of risk lower and target a higher credit rating in order to satisfy its customers' desire for safety. In contrast, a bank that is focused more on transactional activities would usually set the level of risk higher and target a lower credit rating.

Risk-Taking Implications

Overall, banks need to take on an optimal amount of risk (usually not zero) in order to maximize shareholder value while satisfying the constraints imposed by bank regulators.

In general terms, if a subject bank takes on too little risk, it may fail to capitalize on enough profitable opportunities and, therefore, may generate suboptimal returns for its shareholders. On the other hand, if a subject bank takes on too much risk, it may become distressed, which could result in losses for other banks that are counterparties to the subject bank that is defaulting on unsecured obligations.

If incremental changes in risk taken do not result in much change in the value of a bank, then investing in risk management is destroying the bank's value due to the fixed cost of having a risk management department. However, if taking on incremental risk would otherwise result in excessive total risk and a significant decrease in the bank's value, then there is added value in having risk management policies to prevent the bank from taking on excessive risk.

Risk Management Challenges and Limitations

Real-time risk measures do not exist for entire banks although they do exist for certain banking activities. Additionally, risk measures are far from perfect and can result in inaccurate computations. Finally, excessive optimism and the effects of group decision making could result in key issues being overlooked.

In practice, many risks are nearly or entirely impossible to hedge (e.g., terrorism risk), and some hedges are imperfect.

Some risk takers within the bank (e.g., traders) are motivated to maximize their compensation by taking excessive risks that may ultimately reduce the value of the bank.

Ideally, effective risk management would require that the risk management function within a bank be independent of the activities of its business lines. There must be a separation between the manager to whom the risk manager reports and the manager of the business line that he is monitoring.

A unit within a bank that uses the value at risk (VaR) measure in setting limits must consider its ability to adjust its VaR by trading efficiently (i.e., quickly and at a low cost). The aggregation of market, credit, and operational risks in arriving at a firmwide risk measure needs to consider the correlation estimates between such risks; the higher the correlations, the higher the firmwide VaR (and vice versa). Unfortunately, there is usually insufficient data available to make such estimates accurately.

Bank Risk Profile and Performance

Governance

In general, it is difficult to demonstrate that a bank's governance has a significant impact on its risk profile and performance. First, there is very limited data on how the risk function operates in banks. Second, risk function characteristics are affected by the bank's risk appetite in addition to governance. Third, it is possible that at the firm level, poor performance will occur even in the presence of strong governance.

Incentive Structure

In general, incentives must be designed so that they do not merely reward managers for the performance of their respective business units alone. Incentives should reward managers for taking risks that create value for the overall bank while at the same time penalize them for taking risks that destroy value.

Risk Culture

Two studies have been conducted that pertained to using employee surveys to determine the attractiveness of working at certain companies. One of these studies concluded that companies where managers were perceived as honest and trustworthy were more profitable and were given higher valuations. The other study concluded that shareholder governance improvements would change a firm's culture from focusing on employee integrity and customer service to focusing on end results.

Financial Disasters

Cross Reference to GARP Assigned Reading – Allen, Chapter 4

Chase Manhattan Bank and Drysdale Securities

In 1976, Drysdale Government Securities was able to borrow $300 million in unsecured funds from Chase Manhattan. The borrowed funds far exceeded Drysdale's capital of $20 million and consequently any amount it would have normally been approved to borrow. The company used the borrowed funds to take bond positions, which eventually declined in value. Given the loss in market value, Drysdale was unable to repay the borrowed funds and was forced into bankruptcy. Almost all of the losses had to be absorbed by Chase Manhattan since it brokered most of Drysdale's borrowings.

Kidder Peabody

The head of the government bond trading desk at Kidder Peabody, Joseph Jett, misreported a series of trades between 1992 and 1994, which allowed him to report substantial artificial profits. After these errors were detected, $350 million in falsely reported gains had to be reversed.

Jett was able to report false profits since the computer system used to report government bond trading activity did not account for a forward contract's present value. This enabled Jett to earn an instant profit when purchasing a bond for cash and delivering the forward contract.

Eventually, Jett's profits came under fire after Kidder realized that no individual trading strategy could produce the substantial profits that were being reported. This misleading reporting case demonstrates the importance of investigating large profits from unknown trading strategies.

Barings

Nick Leeson, a British Barings junior trader in Singapore, took speculative derivative positions in an effort to recoup prior trading losses that he was able to hide fraudulently. The losses went undetected due to inadequate control systems.

Leeson had previously incurred huge trading losses that would have cost him his job if they were revealed. In an effort to recover those losses, he abandoned the hedged posture in the long-short futures arbitrage strategy and initiated a speculative long-long futures position on both exchanges in hope of profiting from an increase in the Nikkei 225. This move exposed the firm to enormous market risk and event risk, which stems from unexpected major events that, while not directly related to markets, can affect markets.

This lack of oversight contributed to Barings' failure as the Nikkei continued to drop. Between 1993 and 1995, Leeson's actions resulted in losses of approximately $1.25 billion and forced Barings into bankruptcy.

Allied Irish Bank

Between 1997 and 2002, a currency trader for Allied Irish Bank (AIB), John Rusnak, hid $691 million in losses from management. Rusnak used a number of deceptive means to hide these losses including bullying back-office workers into not following-up on trade confirmations for imaginary trades. However, in 2001, the back-office supervisor realized that something was amiss when he saw that confirmations were missing for a number of trades. After this problem was corrected, the fraudulent actions were eventually identified.

Management believed that Rusnak was running a small currency arbitrage trading strategy. However, the strategy actually being implemented involved very large currency positions. Rusnak was able to hide these trading activities from management by creating fake trades to offset his real trades. The result was the appearance of a trading strategy that involved small positions.

Union Bank of Switzerland

During 1997, Union Bank of Switzerland's (UBS) equity derivatives business lost between $400 and $700 million. An additional loss of $700 million followed the next year, which was mostly due to its large stake in Long-Term Capital Management (LTCM). Losses at UBS forced the firm to merge with Swiss Bank Corporation (SBC).

It is suspected that the losses in 1997 were due, in some part, to the following four factors: (1) British law tax changes; (2) large Japanese bank warrants, which were inappropriately hedged against a drop in the underlying stocks; (3) incorrect valuation of long-dated options on equity baskets; and (4) inappropriate modeling of other long-dated options.

Société Générale

In January 2008, it was discovered that one of Société Générale's junior traders, Jérôme Kerviel, was involved in unauthorized trading activity that resulted in losses of $7.1 billion. The incident damaged the reputation of Société Générale and required the bank to raise additional funds to meet capital needs.

A number of reasons were cited that explained how Kerviel's unauthorized trading activity went undetected, including the incorrect handling of trade cancellations, the lack of proper supervision, and the inability of the bank's trading system to consider gross positions.

Metallgesellschaft

In 1991, Metallgesellschaft Refining and Marketing (MGRM), an American subsidiary of Metallgesellschaft (MG), an international trading, engineering, and chemicals conglomerate, implemented a marketing strategy designed to insulate customers from price volatility in the petroleum markets for a fee.

MGRM offered customers contracts to buy fixed amounts of heating oil and gasoline at a fixed price over a 5- or 10-year period. The fixed price was set at a $3 to $5 per barrel premium over the average futures price of contracts expiring over the next 12 months. Customers were given the option to exit the contract if the spot price rose above the fixed price in the contract, in which case MGRM would pay the customer half of the difference between the futures price and contract price. A customer might exercise this option if she did not need the product or if she were experiencing financial difficulties. In later contracts, the customer could receive the entire difference in exchange for a higher fixed contract price.

The customer contracts effectively gave MGRM a short position in *long-term forward contracts*. MGRM hedged this exposure with long positions in *near-term futures* using a **stack-and-roll hedging strategy**.

Gains and losses on forward contracts are realized at the agreement's expiration, whereas futures contracts are marked to market such that the gains and losses are realized on a daily basis. In MGRM's case, gains and losses on its customer contracts were realized if and when the customers took delivery, which would occur over a 5- to 10-year period.

During 1993, oil prices dropped from a high of about $21 per barrel to about $14 per barrel, resulting in losses of $900 million on MGRM's long positions, which were realized immediately as the futures contracts were marked to market. The offsetting gains on their customer contracts, however, would not be realized for years to come, which created potential short-term cash outflows, and resulted in **funding liquidity risk**. Declining oil prices also created margin calls that exacerbated the cash flow problem. Due to these losses, MG ordered MGRM to close out of its customer contracts. This forced the firm to unwind its positions at very unfavorable terms.

The cash outflows might have been tolerable and possibly balanced out by cash inflows over the life of the hedge were it not for the sheer size of MGRM's position, which would have taken ten days to liquidate. To liquidate without affecting market prices would have taken from 20 to 55 days. As a result, the company lacked liquidity to unwind its positions, if necessary, without significant market impact, and was therefore subject to **trading liquidity risk**. To make matters worse, MGRM was carrying a heavy debt load and had little equity to withstand losses and cash flow problems on positions of this size.

Long-Term Capital Management

Long-Term Capital Management (LTCM), a hedge fund founded in early 1994, generated stellar returns in its first few years of operation: 43% in 1995 and 41% in 1996. The partners worked together at Salomon Brothers (now Citigroup) and, given their success, decided to start their own fund and proceeded to seek capital from investors. Funding was provided to LTCM despite the secretive nature of its positions. In addition, investors were locked into investments for long periods of time in order to prevent liquidation issues since the fund was focused on long-term investment strategies. In the later years of operations, the partners at LTCM invested a large portion of their net worth in the fund since they believed so strongly in the success of their trading strategies.

Most of LTCM's investment strategies could be classified as relative value, credit spreads, and equity volatility. Their relative value strategies involved arbitraging price differences among similar securities and profiting when the prices converged. One benefit of this convergence strategy is that being long and short similar securities hedges risk exposure and reduces volatility.

LTCM believed that, although yield differences between risky and riskless fixed-income instruments varied over time, the risk premium (or credit spread) tended to revert to average historical levels. Noticing that credit spreads were historically high, they entered into mortgage spreads and international high-yield bond spreads intending to profit when the spreads shrank to more typical historical levels. Similarly, their equity volatility strategy assumed that volatility on equity options tended to revert to long-term average levels. When volatility implied by equity options was abnormally high, LTCM "sold volatility" until it regressed to normal levels.

In August of 1998, Russia unexpectedly defaulted on its debt, sending Russian interest rates soaring to 200% and crushing the value of the ruble. This economic shock triggered investor concern about already faltering economies in the Pacific rim, causing the yields on developing nations' debt to increase and a flight to the quality of government bonds in industrialized countries. Yields on corporate debt—both high and low quality—also increased sharply. In other words, the flight to quality increased, rather than decreased, credit spreads, causing huge losses for LTCM. Shortly thereafter, Brazil also devalued its currency, thereby further increasing interest rates and risk premiums. The general increase in volatility also generated losses in LTCM's equity volatility strategies.

Although prices in relative value arbitrage strategies sometimes diverge and create temporary losses before they ultimately converge, the large increase in yield spread caused huge losses and severe cash flow problems caused by realizing marked to market losses and meeting margin calls. The effect of the losses and the cash flow crisis were compounded by the firm's hyper leverage. LTCM lost 44% of its capital in just one month. The firm's lack of equity capital created a cash flow crisis and made it necessary to liquidate positions to meet margin calls.

As a hedge fund, LTCM's reporting obligation to regulators was limited. Although the size of its positions required financial statement reporting and daily position reporting, these reports were incomplete and lacked disclosure of derivative positions and trading strategies. Ultimately, the Federal Reserve Bank of New York orchestrated a bailout in which 14 leading banks and investment houses invested $3.65 billion for a 90% stake in LTCM.

Bankers Trust

Procter & Gamble (P&G) and Gibson Greetings sought the assistance of Bankers Trust (BT) to help them reduce funding costs. BT used derivative trades, which promised the two companies a high-probability, small reduction in funding costs in exchange for a low-probability, large loss. Unfortunately, the derivative trades only resulted in significant losses for both P&G and Gibson.

The derivative structures developed by BT were intentionally complex and prevented P&G and Gibson from fully understanding the trade values and risks that were involved. In 1994, P&G and Gibson finally realized that they had been misled after discovering that they had suffered huge losses. As a result, the two companies sued BT.

It was common practice for BT to tape phone conversations of its traders and marketers in an effort to resolve possible verbal contract disputes. In some of these conversations, BT's staff bragged about how badly they fooled clients with complex structures and showed how price quotes given to P&G and Gibson were manipulated.

The Bankers Trust scandal severely damaged its reputation and forced its CEO to resign. BT was eventually acquired by Deutsche Bank and ultimately dismantled. The actions at BT led to tighter controls for dealing with clients at other firms.

JPMorgan, Citigroup, and Enron

The Enron scandal revealed the use of questionable accounting practices to disguise the size of borrowings from lenders and investors.

Enron collected cash by selling oil for future delivery, and, in turn, agreed to buy back the delivered oil at a fixed price. Thus, no oil was actually delivered, so the agreement was essentially a loan where the company paid cash at a later date to receive cash at the beginning of the agreement. The advantage for Enron was that the company did not have to account for these transactions as loans on its financial statements, which made the company look healthier to both investors and lenders.

JPMorgan Chase and Citigroup were the main counterparties in these transactions. It was later revealed that the investment banks fully understood Enron's intent when entering into these loan-type transactions. As a result, JPMorgan and Citigroup agreed to pay a $286 million fine for assisting with fraud against Enron investors.

Deciphering the Liquidity and Credit Crunch 2007–2008

Cross Reference to GARP Assigned Reading – Brunnermeir

Key Factors Leading to the Housing Bubble

The housing bubble, which later burst and caused a liquidity squeeze, can be seen as the product of the following two broad factors:

1. *Cheap credit.* Interest rates were low in the lead-up to the housing bubble due to a combination of factors. First, an increase in demand for U.S. securities by foreign governments experiencing trade surpluses put downward pressure on interest rates. Also, the Federal Reserve adopted a lax interest rate policy that promoted low interest rates to fend off deflation after the bursting of the internet bubble.

2. *Decline in lending standards.* In the years leading up to the housing bubble, this traditional banking model was replaced by a new **originate-to-distribute model** of banking, in which loans are collected together, sliced into tranches, and then sold as securities. Since loan-originating institutions could transfer the default risk of borrowers to investors, the originating institutions had little incentive to be rigorous or diligent in their creditworthiness assessments.

Banking Industry Trends and the Liquidity Squeeze

The **liquidity squeeze** was caused by two major banking industry trends:

1. *Risk transference through asset securitization*: through the originate-to-distribute model.

2. *Asset-liability maturity mismatch*: the purchase of long-term assets through the rollover of short-term debt instruments.

Securitization

More recently, instead of retaining loans and carrying the risk of default of borrowers, banks began to transfer this risk to investors through securitization. Banks developed structured products [e.g., collateralized debt obligations (CDOs)] based on an underlying pool of mortgages, bonds, and other loans. Securitization enables the originating institution to detach itself from the default risk of the underlying pool of assets and transfer it to investors, allowing these investors to select a tranche based on their specific risk and return preferences.

Asset/Liability Maturity Management

Sponsoring banks created **structured investment vehicles** (SIVs) that focused on investing in structured products. Enabled by the environment of low interest rates and high ratings, these SIVs raised money by issuing short-term commercial paper to fund purchases of long-term assets, such as mortgage-backed securities (MBSs) and other securitized products. However, due to the maturity mismatch between short-term commercial paper and long-term MBSs, the SIVs were exposed to **funding liquidity risk**.

Collateralized Debt Obligations

A **collateralized debt obligation** (CDO) is a "structured" product that banks can use to unburden themselves of risk.

The creation of a CDO might be thought of as a three-step process:

1. Form a diversified portfolio.

2. Slice the portfolio into tranches.

3. Sell tranches to investors.

Credit Default Swaps

Credit default swaps (CDSs) are insurance contracts that pay off when some reference instrument (such as a bond or a CDO tranche) defaults. Like other insurance contracts, the buyer of a CDS pays a fixed periodic fee for protection and in return receives a payment in the event of a default.

Consequences of the Financial Crisis

The financial crisis that stemmed from rising mortgage delinquencies and falling housing prices led to a worldwide liquidity crisis because institutions had (1) taken on too much leverage, (2) generated large maturity mismatches between assets and liabilities, and (3) become too interconnected.

Funding Liquidity and Market Liquidity

Funding liquidity refers to an institution's ability to immediately settle obligations when they are due. If a bank is not able to settle its obligations on time, the bank is illiquid.

Funding liquidity risk refers to the possibility that an institution will not be able to settle its obligations when they are due. In liquid markets, investors can borrow money with ease. These same investors may experience funding liquidity risk when market conditions deteriorate. Traders may be forced to sell an asset at a substantially lower price in deteriorating market conditions to meet margin requirements.

Market liquidity refers to the ease or difficulty of selling an asset to raise money. Market liquidity is high if it is relatively easy to find a counterparty to buy an asset at the going price (i.e., without lowering the price to attract a buyer).

Funding Liquidity vs. Market Liquidity

Raising money by selling an asset is associated with market liquidity. Market liquidity is low if it is difficult to sell the asset. Funding liquidity is associated with the use of an asset as collateral to borrow money against it. Funding liquidity risk is high if margins or redemptions increase, or if rolling over short-term debt becomes difficult.

Loss Spiral and Margin Spiral

To maintain a particular margin (or leverage ratio) in the face of a market decline, an investor may need to sell a portion of assets to ensure that the overall value of the position aligns with the current smaller amount of equity capital and the lower borrowing capacity. These kinds of forced sales put additional downward pressure on asset prices, further eroding equity capital and necessitating more forced sales. This situation becomes a **loss spiral** when the decline leads to a cycle of further price decreases and further forced sales.

A **margin spiral** refers to the forced sale of an asset as a result of an increase in required margin, or, equivalently speaking, a decline in the permitted leverage ratio. Investors may be required to increase their contributed equity capital in conditions of deteriorating market conditions and tightening of lending. To satisfy the increased margin requirements, an investor may have to sell even more assets than under a loss spiral event in which margins and leverage ratios are held constant.

In summary, with a loss spiral, a trader initiates a sale to maintain the leverage ratio; whereas with a margin spiral, a trader sells an asset to fulfill the requirements of a decline in the leverage ratio (as a result of rising margins). Consequently, all else equal, a margin spiral situation results in a lower overall position value and lower borrowing amounts versus a loss spiral situation.

Network Risk

Network risk arises as a result of an increase in counterparty credit risk, particularly in an environment of market stress. This increased risk causes a web of contracting parties to seek additional protection and liquidity, for example, by purchasing credit default swaps or by demanding increased collateral. In this way, network risk can produce a potential systemic effect. Network risk is exacerbated by the absence of a **clearinghouse**, which is a central regulatory authority that can manage multilateral netting among multiple contracting parties that possess less than full information on one another.

Getting Up to Speed on the Financial Crisis: A One-Weekend-Reader's Guide

Cross Reference to GARP Assigned Reading – Gorton and Metrick

Financial Crisis Overview

The major contributing factor in the financial crisis was the bundling of subprime mortgages into mortgage-backed securities (MBSs) as well as asset-backed securities (ABSs) in the form of asset-backed commercial paper (ABCP). The main *trigger* of the financial crisis was the prospect of losses on subprime mortgages. Before 2007, housing prices in the United States and other developed nations showed an increase in prices. In the first half of 2007, housing prices in the United States started to decline, causing several subprime mortgage lenders to file for bankruptcy and subsequently fail. These losses became amplified as they had a ripple effect that spread to the main *vulnerabilities* of the crisis, ABCP and repurchase agreements (repos).

When housing prices declined and homeowners defaulted on their mortgage loans, the value and prices of ABCP also decreased. These declining prices resulted in bank runs on shadow banks and money market mutual funds (MMFs) and signaled the start of a liquidity crisis.

The liquidity crisis continued to spread into repo agreements, with the average haircut going from near zero at the beginning of 2007 to 25% by September of 2008 (Lehman Brothers bankruptcy).

Lehman Brothers Failure

The Lehman Brothers bankruptcy filing in September 2008 is considered the tipping point in the financial crisis. It eroded confidence and caused a run on MMFs. This lack of confidence spread across markets and countries, amplifying losses in the subprime mortgage market.

Previous Financial Crises

A **banking crisis** can be characterized by either of the following occurring: (1) a run on banks that leads to a merger, takeover by the government, or closure of a financial institution, or (2) a merger, takeover, government assistance, or closure of a financial institution that spreads to other financial institutions.

The financial crisis of 2007–2009 was not unique compared to previous banking crises. It followed a similar pattern of increased public and private debt, increased credit supply, and increased housing prices preceding and leading to the crisis.

Panic Periods

The two main panic periods of the financial crisis were August 2007 and September 2008 through October 2008. The first panic period in August 2007 occurred when there were runs on ABCP. ABCP was sold as an interest paying long-term debt asset that bundled credit card receivables, mortgages, and other loans. As housing prices decreased in early 2007, homeowners started defaulting on their mortgage loans causing the value of ABCP to decrease. Holders of ABCP, namely MMFs, experienced a decrease in value of their assets. At the time, MMFs were thought to be a safe haven by investors.

The second panic period started when Lehman Brothers filed for bankruptcy, which caused a major shock to MMFs. Lehman was the major issuer of MBSs, which contained subprime mortgages, and also owned subprime mortgage lenders, which subsequently went out of business.

Lehman's failure caused a run on a particular MMF called **Reserve Primary**, which contained commercial paper issued by Lehman. The run on Reserve Primary spread to other MMFs, starting a contagion effect that spread to other assets that were falling in price in tandem with rising haircuts. This further spread the contagion to the interbank system and to Europe.

Government Policy Responses

The International Monetary Fund (IMF) studied 13 developed countries and their responses to the financial crisis, which resulted in 153 separate policy actions (the United States accounted for 49 of those actions). These 153 actions were divided into 5 subgroups consisting of interest rate change, liquidity support, recapitalization, liability guarantees, and asset purchases.

To measure the impact of interest rate cuts, they used the **economic stress index** (ESI) and the **financial stress index** (FSI). The ESI is a composite of confidence measures from businesses and consumers, nonfinancial firm stock prices, and credit spreads. The FSI is an index representing a composite of stock prices, spreads, and bank credit. Liquidity support was measured using interbank spreads and the FSI. To measure the impact on recapitalization, liability guarantees, and asset purchases, the IMF study used the FSI and an index of credit default swaps (CDSs) on banks.

The evidence suggests that the most effective measures taken were the liquidity support stabilizing the interbank markets in the pre-Lehman period and recapitalization (i.e., capital injections), which was considered the most effective tool in the post-Lehman periods.

Global Effects on Firms and the Economy

Three different studies were done showing the effects of the global financial crisis on corporations and consumers. All three studies pointed out that as the global recession deepened, the demand for credit decreased. This was a reasonable assumption, yet the studies concluded that even with a decrease in demand for credit, there was reduced supply of credit by banks and financial intermediaries.

Risk Management Failures: What Are They and When Do They Happen?

Cross Reference to GARP Assigned Reading – Stulz

The Role of Risk Management

The role of risk management involves performing the following tasks.

- Assess all risks faced by the firm.
- Communicate these risks to risk-taking decision makers.
- Monitor and manage these risks (make sure that the firm only takes the necessary amount of risk).

The risk management process focuses on the output of a particular risk metric [e.g., the value at risk (VaR) for the firm] and attempts to keep the measure at a specified target amount.

A large loss is not necessarily an indication of a risk management failure. As long as risk managers understood and prepared for the possibility of loss, then the implemented risk management was successful.

Incorrectly Measuring and Managing Risk

The process of risk management can fail if one or more of the following events occur.

- Not measuring known risks correctly.
- Not recognizing some risks.
- Not communicating risks to top management.

- Not monitoring risk adequately.
- Not managing risk adequately.
- Not using the appropriate risk metrics.

Risk mismeasurement can occur when risk managers do not understand the distribution of returns of a single risky position or the relationships of the distributions among different positions.

Failing to take known and unknown risks into account (i.e., ignoring risks) can take three forms:

1. Ignoring a risk that is known.
2. Knowing about a risk, but failing to properly incorporate it into risk models.
3. Failing to discover all risks.

A firm ignores known risks by failing to realize how various position risks can lead to a potential disaster.

Not collecting and entering data into the appropriate risk models is another potential source of disaster. In this case, the firm may make an attempt to recognize the risk. However, not obtaining proper data to measure the risk adequately will have similar consequences to ignoring risks.

Some risks may go completely undetected by risk managers. In some cases, however, unknown risks may not be too severe of a problem.

Properly Communicating Risks

The purpose of risk management is to allow senior managers of the firm to make the optimal strategic decisions to maximize firm value. Thus, risk management efforts are wasted unless the results can be effectively communicated to the appropriate decision makers. This includes timely communication that has not been distorted by intermediaries.

Ongoing Risk Management

Risk managers must recognize how portfolio risk profiles can change even during the absence of trading. The properties of some securities can change for several reasons (e.g., changes in interest rates, embedded derivatives). Also, some securities can have complex relationships with market variables; for example, a security may increase in value when interest rates decline over one particular range and then decline in value as interest rates decline further outside of that range.

The pricing of subprime derivatives serves as an example of changing risk exposures. Previously, the ABX indices (i.e., asset-backed securities indices) showed no variation for AAA-rated tranches of securitization. However, during the recent financial crisis, the values declined suddenly and dramatically, and anyone who had relied on historical values of the ABX indices for allocations incurred large losses.

Firms can fail to monitor and manage risk on an ongoing basis by not having an adequate incentive structure and/or culture that promotes effective risk management. If risk is everyone's concern, then unobserved risks are less likely. In addition, if compensation is a function of risk, then employees will likely take more interest in lowering firm risk.

The Role of Risk Metrics

Risk metrics aid the management process by providing managers a target to achieve (e.g., a particular VaR level). Monitoring these risk metrics allows managers to appropriately manage risk. However, risk metrics may be too narrow in scope, which can make it more difficult to achieve the overall objective of managing risk in an effort to create value.

The Standard Capital Asset Pricing Model

Cross Reference to GARP Assigned Reading – Elton, et al., Chapter 13

The Capital Asset Pricing Model (CAPM)

The CAPM provides a way to calculate an asset's expected return (or "required" return) based on its level of systematic (or market-related) risk, as measured by the asset's beta.

The CAPM has a number of underlying assumptions:

1. Investors face no transaction costs when trading assets.
2. Assets are infinitely divisible.
3. There are no taxes; therefore, investors are indifferent between capital gains and income or dividends.
4. Investors are price takers whose individual buy and sell decisions have no effect on asset prices.
5. Investors' utility functions are based solely on expected portfolio return and risk.
6. Unlimited short-selling is allowed.

7. Investors can borrow and lend unlimited amounts at the risk-free rate.
8. Investors are only concerned about returns and risk over a single period, and the single period is the same for all investors.
9. All investors have the same forecasts of expected returns, variances, and covariances.
10. All assets are marketable, including human capital.

The Capital Market Line (CML)

In the presence of riskless lending and borrowing, the efficient frontier transforms from a curve to a line tangent to the previous curve. Investors will choose to invest in some combination of their tangency portfolio and the risk-free asset. Assuming investors have identical expectations regarding expected returns, standard deviations, and correlations of all assets, there will be only one tangency line, which is referred to as the **capital market line** (CML).

The equation for the CML is:

$$E(R_P) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M}\right]\sigma_P$$

Beta

The sensitivity of an asset's return to the market return is referred to as the asset's **beta.** Beta is a standardized measure of the covariance of the asset's return with the market return. Beta can be calculated as follows:

$$\beta_i = \frac{\text{covariance of Asset i's return with the market return}}{\text{variance of the market return}} = \frac{Cov_{i,M}}{\sigma_M^2}$$

We can use the definition of the correlation between the returns on Asset *i* with the returns on the market to get the covariance equation:

$$\rho_{i,M} = \frac{Cov_{i,M}}{\sigma_i \sigma_M}$$

$$Cov_{i,M} = \rho_{i,M}\sigma_i\sigma_M$$

In addition to individual assets, beta can also be computed for portfolios. The beta of a portfolio is the sum of the weighted individual asset betas within a portfolio.

Calculating Expected Return Using the CAPM

Example: Expected return on a stock

Assume you are assigned the task of evaluating the stock of Sky-Air, Inc. To evaluate the stock, you calculate its required return using the CAPM. The following information is available:

expected market risk premium	5%
risk-free rate	4%
Sky-Air beta	1.5

Using CAPM, **calculate** and **interpret** the expected return for Sky-Air.

Answer:

The expected return for Sky-Air is:

$$E(R_{SA}) = 0.04 + 1.5(0.05) = 0.115 = 11.5\%$$

Applying the CAPM to Performance Measurement: Single-Index Performance Measurement Indicators

Cross Reference to GARP Assigned Reading – Amenc and Le Sourd, Chapter 4

Measures of Performance

The **Treynor measure** is equal to the risk premium divided by beta, or systematic risk:

$$\text{Treynor measure of a portfolio} = \left[\frac{E(R_P) - R_F}{\beta_P}\right]$$

The **Sharpe measure** is equal to the risk premium divided by the standard deviation, or total risk:

$$\text{Sharpe measure of a portfolio} = \left[\frac{E(R_P) - R_F}{\sigma_P}\right]$$

The **Jensen measure** (or Jensen's alpha or just **alpha**), is the asset's excess return over the return predicted by the CAPM:

$$\text{Jensen measure of a portfolio} = \alpha_P = E(R_P) - [R_F + [E(R_M) - R_F]\beta_P]$$

In all three cases, for a given portfolio, the higher, the better. The two that are most similar are the Treynor and Sharpe measures. They both normalize the risk premium by dividing by a measure of risk. Investors can apply the Sharpe measure to all portfolios because it uses total risk, and it is more widely used than the other two measures. The Treynor measure is more appropriate for comparing well-diversified portfolios. Jensen's alpha is the most appropriate for comparing portfolios that have the same beta.

Tracking error is the term used to describe the standard deviation of the difference between the portfolio return and the benchmark return. This source of variability is another source of risk to use in assessing the manager's success.

The **information ratio** is essentially the alpha of the managed portfolio relative to its benchmark divided by the tracking error. If we let R_B denote the return of the benchmark we can write:

$$e_P = R_P - R_B$$

$$\text{tracking error} = \sigma_{eP}$$

$$\text{information ratio} = \left[\frac{E(R_P) - E(R_B)}{\sigma_{eP}}\right] = \frac{\alpha_P}{\sigma_{eP}}$$

The **Sortino ratio** is similar to the Sharpe ratio except for two changes. We replace the risk-free rate with a minimum acceptable return, denoted R_{min}, and we replace the standard deviation with a type of semi-standard deviation. A semi-standard deviation measures the variability of only those returns that fall below the minimum acceptable return.

The Sortino ratio can be interpreted as a variation of the Sharpe ratio that is more appropriate for a case where returns are not symmetric.

Arbitrage Pricing Theory and Multifactor Models of Risk and Return

Cross Reference to GARP Assigned Reading – Bodie, Kane, and Marcus, Chapter 10

The Multifactor Model of Risk and Return

The equation for a multifactor model for stock *i* can be expressed as follows:

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{ik}F_k + e_i$$

where:
R_i = return on stock *i*
$E(R_i)$ = expected return for stock *i*
β_{ij} = j^{th} factor beta for stock *i*
F_j = deviation of macroeconomic factor *j* from its expected value
e_i = firm-specific return for stock *i*

The **factor beta**, β_{ij}, equals the sensitivity of the stock return to a 1-unit change in the factor.

The **firm-specific return**, e_i, is the portion of the stock's return that is unexplained by macro factors (i.e., the F terms in the equation).

Assume the common stock of HealthCare Inc. (HCI) is examined with a single-factor model, using unexpected percent changes in GDP as the single factor. Assume the following data is provided:

Expected return for HCI	= 10%
GDP factor beta	= 2.00
Expected GDP growth	= 3%

Given this data, we can see that the stock return for HCI is strongly impacted by GDP. On average, the stock price changes by two percentage points for every one percentage point change in GDP.

Suppose new macroeconomic information indicates that GDP growth will equal 4% rather than the original consensus forecast of 3%. Also assume there's no new information regarding firm-specific events. The revised expected return for HCI using a single-factor model can be calculated as follows:

$$R_{HCI} = E(R_{HCI}) + \beta_{HCI,GDP}F_{GDP} + e_{HCI}$$

$$R_{HCI} = 0.10 + 2(0.01) = 0.12 = 12\%$$

Therefore, based on the single-factor model, the analyst should revise the expected return for HCI from 10% to 12%, because GDP was revised above its original expected value. The additional two percentage points resulted from the one percentage point deviation of GDP from its expected value, combined with the GDP factor beta of 2: 2 × 0.01 = 0.02 = 2%.

The Law of One Price and Arbitrage Opportunities

According to the **Law of One Price**, identical assets selling in different locations should be priced identically in the different locations.

The action of buying an asset in the cheaper market and simultaneously selling that asset in the more expensive market is called **arbitrage**. The actions of arbitrageurs cause prices to rise in the cheaper market and fall in the expensive market. The simultaneous trades will continue until the asset trades at one price in both markets, at which point the arbitrage opportunity will be fully exploited.

Well-Diversified Portfolios

The part of an individual security's risk that is uncorrelated with the volatility of the market portfolio is that security's **nonsystematic risk** (or *diversifiable risk*). The part of an individual security's risk that arises because of the positive covariance of that security's returns with overall market returns is called its **systematic risk**. As the number of securities in a portfolio becomes large, the portfolio's nonsystematic risk approaches zero. In other words, portfolio risk reduction through diversification comes from reducing nonsystematic risk. Therefore, when a risky security is added to a well-diversified (efficient) portfolio, the portfolio's risk is only affected by the systematic risk of that security.

Hedging Exposures to Multiple Factors

Consider an investor who manages a portfolio with the following factor betas:

GDP beta	= 0.50
Consumer sentiment beta	= 0.30

Assume the investor wishes to pursue strategies to hedge exposure to GDP risk, or to consumer sentiment risk, or to both factor risks. The following explanation makes use of what are called **factor portfolios**, which are well-diversified portfolios with betas equal to one for a single risk factor and betas equal to zero on all remaining factors.

Now, assume the investor wishes to hedge away GDP factor risk, yet maintain the 0.30 exposure to consumer sentiment. To do so, the investor should combine the original portfolio with a 50% short position in the GDP factor portfolio. The GDP factor beta on the 50% short position in the GDP factor portfolio equals –0.50, which perfectly offsets the 0.50 GDP factor beta on the original portfolio. The combined long and short positions hedge away GDP risk but retain the consumer sentiment exposure.

The Arbitrage Pricing Theory

The arbitrage pricing theory (APT) describes expected returns as a linear function of exposures to common (i.e., macroeconomic) risk factors:

$$E(R_i) = R_F + \beta_{i1}[E(R_1) - R_F] + \beta_{i2}[E(R_2) - R_F] + \ldots + \beta_{ik}[E(R_k) - R_F]$$

The assumptions underlying the APT model are as follows:

- Returns follow a *k*-factor process: $R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots + \beta_{ik}F_k + e_i$.
- Well-diversified portfolios can be formed.
- No arbitrage opportunities exist.

Both the arbitrage pricing theory model and the capital asset pricing model describe equilibrium expected returns for assets. In fact, the CAPM can be considered a special restrictive case of the APT in which there is only one risk factor (the market risk factor).

The Fama-French Three-Factor Model

In contrast to the APT model, the Fama-French model identifies the factors. In addition to the market return factor ($R_M - R_F$), the Fama-French three-factor model specifies the following two factors:

- SMB (small minus big) is the firm size factor equal to the difference in returns between portfolios of small and big firms ($R_S - R_B$).
- HML (high minus low) is the book-to-market (i.e., book value per share divided by stock price) factor equal to the difference in returns between portfolios of high and low book-to-market firms ($R_H - R_L$).

The equation for the Fama-French three-factor model is:

$$R_i - R_F = \alpha_i + \beta_{i,M}(R_M - R_F) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + e_i$$

The intercept term (i.e., alpha) equals the abnormal performance of the asset after controlling for its exposures to the market, firm size, and book-to-market factors. In equilibrium, the intercept should equal zero, assuming the Fama-French three factors adequately capture all systematic risks.

Principles for Effective Risk Data Aggregation and Risk Reporting

Cross Reference to GARP Assigned Reading – Basel Committee on Banking Supervision

Benefits of Risk Data Aggregation

According to the Basel Committee on Banking Supervision, **risk data aggregation** means “defining, gathering and processing risk data according to the bank’s risk reporting requirements to enable the bank to measure its performance against its risk tolerance/appetite.” The aggregation process includes breaking down, sorting, and merging data and datasets. Risk management reports should reflect risks in a reliable way.

Benefits that accrue from effective risk data aggregation and reporting include: (1) an increased ability of managers and the board to anticipate problems, (2) enhanced ability to identify alternative routes to restore financial health in times of financial stress, (3) improved resolvability in the event of bank stress or failure, and (4) an enhanced ability to make strategic decisions, increasing the bank’s efficiency, reducing the chance of loss, and ultimately increasing bank profitability.

Governance

The governance principle (Principle 1) suggests that risk data aggregation should be part of the bank's overall risk management framework. The board and senior management should assure that adequate resources are devoted to risk data aggregation and reporting.

Data Architecture and IT Infrastructure

The data architecture and IT infrastructure principle (Principle 2) states that a bank should design, build, and maintain data architecture and IT infrastructure that fully supports its risk data aggregation capabilities and risk reporting practices not only in normal times but also during times of stress or crisis, while still meeting the other principles. It stresses that banks should devote considerable financial and human resources to risk data aggregation and reporting.

Risk Data Aggregation Capabilities

Principles 3–6 specify standards and requirements for effective risk data aggregation. Banks should ensure that the data is accurate and has integrity (Principle 3), is complete (Principle 4), is timely (Principle 5), and is adaptable to the end user (Principle 6). In addition, the bank should not have high standards for one principle at the expense of another. Aggregated risk data should exhibit all of the features together, not in isolation.

Effective Risk Reporting Practices

Principles 7–11 specify standards and requirements for effective risk reporting practices. Risk reports should be accurate (Principle 7), comprehensive (Principle 8), and clear and useful (Principle 9). Principle 10 states that reports should be "appropriately frequent" (i.e., frequency depends on the role of the recipient—board members need reports less frequently than risk committee members). Reports should be distributed to relevant parties in a timely fashion while maintaining confidentially (Principle 11).

GARP Code of Conduct[2]

Cross Reference to GARP Assigned Reading – GARP Board of Trustees

1. Professional Integrity and Ethical Conduct

GARP Members:

1.1. shall act professionally, ethically and with integrity in all dealings with employers, existing or potential clients, the public, and other practitioners in the financial services industry.

1.2. shall exercise reasonable judgment in the provision of risk services while maintaining independence of thought and direction. GARP Members must not offer, solicit, or accept any gift, benefit, compensation, or consideration that could be reasonably expected to compromise their own or another's independence and objectivity.

1.3. must take reasonable precautions to ensure that the Member's services are not used for improper, fraudulent or illegal purposes.

1.4. shall not knowingly misrepresent details relating to analysis, recommendations, actions, or other professional activities.

1.5. shall not engage in any professional conduct involving dishonesty or deception or engage in any act that reflects negatively on their integrity, character, trustworthiness, or professional ability or on the risk management profession.

1.6. shall not engage in any conduct or commit any act that compromises the integrity of GARP, the FRM® designation, or the integrity or validity of the examinations leading to the award of the right to use the FRM designation or any other credentials that may be offered by GARP.

1.7. shall be mindful of cultural differences regarding ethical behavior and customs, and avoid any actions that are, or may have the appearance of being unethical according to local customs. If there appears to be a conflict or overlap of standards, the GARP Member should always seek to apply the highest standard.

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2. Conflict of Interest

GARP Members shall:

2.1. act fairly in all situations and must fully disclose any actual or potential conflict to all affected parties.

2.2. make full and fair disclosure of all matters that could reasonably be expected to impair independence and objectivity or interfere with respective duties to their employer, clients, and prospective clients.

3. Confidentiality

GARP Members:

3.1. shall not make use of confidential information for inappropriate purposes and unless having received prior consent shall maintain the confidentiality of their work, their employer or client.

3.2. must not use confidential information for personal benefit.

4. Fundamental Responsibilities

GARP Members shall:

4.1. comply with all applicable laws, rules, and regulations (including this Code) governing the GARP Members' professional activities and shall not knowingly participate or assist in any violation of such laws, rules, or regulations.

4.2. have ethical responsibilities and cannot outsource or delegate those responsibilities to others.

4.3. understand the needs and complexity of their employer or client, and should provide appropriate and suitable risk management services and advice.

4.4. be diligent about not overstating the accuracy or certainty of results or conclusions.

4.5. clearly disclose the relevant limits of their specific knowledge and expertise concerning risk assessment, industry practices, and applicable laws and regulations.

5. Best Practices

GARP Members shall:

5.1. execute all services with diligence and perform all work in a manner that is independent from interested parties. GARP Members should collect, analyze and distribute risk information with the highest level of professional objectivity.

5.2. be familiar with current generally accepted risk management practices and shall clearly indicate any departure from their use.

5.3. ensure that communications include factual data and do not contain false information.

5.4. make a distinction between fact and opinion in the presentation of analysis and recommendations.

Violations of the Code of Conduct

Violations of the Code of Conduct may result in temporary suspension or permanent removal from GARP membership. In addition, violations could lead to a revocation of the right to use the FRM designation. Sanctions would be issued after a formal investigation is conducted by GARP.

Quantitative Analysis

Weight on Exam	20%
SchweserNotes™ Reference	Book 2

Probabilities

Cross Reference to GARP Assigned Reading – Miller, Chapter 2

Random Variables

- A **random variable** is an uncertain quantity/number.
- An **outcome** is an observed value of a random variable.
- An **event** is a single outcome or a set of outcomes.
- **Mutually exclusive events** are events that cannot happen at the same time.
- **Exhaustive events** are those that include all possible outcomes.

A **probability distribution** describes the probabilities of all the possible outcomes for a random variable. The probabilities of all possible outcomes must sum to 1. A simple probability distribution is that for the roll of one fair die there are six possible outcomes and each one has a probability of 1/6, so they sum to 1.

A **discrete random variable** is one for which the number of possible outcomes can be counted, and for each possible outcome, there is a measurable and positive probability.

A **continuous random variable** is one for which the number of possible outcomes is infinite, even if lower and upper bounds exist.

Distribution Functions

A **probability density function** (pdf) is a function, denoted f(x), that can be used to generate the probability that outcomes of a continuous distribution lie within a particular range of outcomes. For a continuous distribution, it is the equivalent of a *probability function* for a discrete distribution.

A **cumulative distribution function** (cdf), or simply *distribution function*, defines the probability that a random variable, *X*, takes on a value equal to or less than a specific value, *x*. It represents the sum, or *cumulative value*, of the probabilities for the outcomes up to and including a specified outcome. The cumulative distribution function for a random variable, *X*, may be expressed as $F(x) = P(X \leq x)$.

Instead of finding the probability less than or equal to a specific value, x, the **inverse cumulative distribution function** can be used to find the value that corresponds to a specific probability.

A **discrete uniform random variable** is one for which the probabilities for all possible outcomes for a discrete random variable are equal. For example, consider the *discrete uniform probability distribution* defined as $X = \{1, 2, 3, 4, 5\}$, $p(x) = 0.2$. Here, the probability for each outcome is equal to 0.2 [i.e., $p(1) = p(2) = p(3) = p(4) = p(5) = 0.2$]. Also, the cumulative distribution function for the *n*th outcome, $F(x_n) = np(x)$, and the probability for a range of outcomes is $p(x)k$, where *k* is the number of possible outcomes in the range.

Conditional Probabilities

Unconditional probability (i.e., *marginal probability*) refers to the probability of an event regardless of the past or future occurrence of other events.

A **conditional probability** is one where the occurrence of one event affects the probability of the occurrence of another event. For example, we might be concerned with the probability of a recession *given* that the monetary authority increases interest rates. This is a conditional probability. The key word to watch for here is "given." Using probability notation, "the probability of A *given* the occurrence of B" is expressed as $P(A \mid B)$, where the vertical bar (|) indicates "given," or "conditional upon."

Independent and Mutually Exclusive Events

Independent events refer to events for which the occurrence of one has no influence on the occurrence of the others. The definition of independent events can be expressed in terms of conditional probabilities. Events A and B are independent if and only if:

$$P(A \mid B) = P(A), \text{ or equivalently, } P(B \mid A) = P(B)$$

If this condition is not satisfied, the events are **dependent events** (i.e., the occurrence of one is dependent on the occurrence of the other).

Calculating a Joint Probability of Any Number of Independent Events

On the roll of two dice, the joint probability of getting two 4s is calculated as:

P(4 on first die and 4 on second die) = P(4 on first die) × P(4 on second die) = 1/6 × 1/6 = 1/36 = 0.0278

On the flip of two coins, the probability of getting two heads is:

P(heads on first coin and heads on second coin) = 1/2 × 1/2 = 1/4 = 0.25

When dealing with *independent events*, the word *and* indicates multiplication, and the word *or* indicates addition. In probability notation:

P(A or B) = P(A) + P(B), and P(A and B) = P(A) × P(B)

Joint probabilities of independent events can be conveniently summarized using a probability matrix (sometimes known as a probability table). Suppose, for example, that we wanted to view how the state of the economy relates to the direction of interest rates. The probability matrix in Figure 1 shows the joint and unconditional probabilities of these two variables.

Figure 1: Joint and Unconditional Probabilities

		Interest Rates		
		Increase	No Increase	
Economy	Good	14%	6%	**20%**
	Normal	20%	30%	**50%**
	Poor	6%	24%	**30%**
		40%	**60%**	**100%**

From this probability matrix, we see that the joint probability of a poor economy and an increase in interest rates is 6%. Similarly, the joint probability of a normal economy and no increase in interest rates is 30%. Unconditional probabilities are shown as the sum of each column and each row.

Basic Statistics

Cross Reference to GARP Assigned Reading – Miller, Chapter 3

Expectations

The **expected value** is the weighted average of the possible outcomes of a random variable, where the weights are the probabilities that the outcomes will occur. The mathematical representation for the expected value of random variable *X* is:

$$E(X) = \Sigma P(x_i)x_i = P(x_1)x_1 + P(x_2)x_2 + \ldots + P(x_n)x_n$$

If *X* and *Y* are any random variables, then:

$$E(X + Y) = E(X) + E(Y)$$

The mean and variance of a distribution are defined as the first and second moments of the distribution, respectively. **Variance** is defined as:

$$\text{Var}(X) = E[(X - \mu)^2]$$

The square root of the variance is called the **standard deviation**. The variance and standard deviation provide a measure of the extent of the dispersion in the values of the random variable around the mean.

If *X* and *Y* are independent random variables, then:

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$$

$$\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y)$$

Covariance and Correlation

Covariance is the expected value of the product of the deviations of the two random variables from their respective expected values. A common symbol for the covariance between random variables *X* and *Y* is Cov(X,Y). Since we will be mostly concerned with the covariance of asset returns, the following formula has been written in terms of the covariance of the return of asset *i*, R_i, and the return of asset *j*, R_j:

$$\text{Cov}(R_i,R_j) = E(R_i,R_j) - E(R_i) \times E(R_j)$$

If *X* and *Y* are NOT independent, then:

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2 \times \text{Cov}(X,Y)$$

$$\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y) - 2 \times \text{Cov}(X,Y)$$

To make the covariance of two random variables easier to interpret, it may be divided by the product of the random variables' standard deviations. The resulting value is called the correlation coefficient, or simply, **correlation**. The relationship between covariances, standard deviations, and correlations can be seen in the following expression for the correlation of the returns for asset *i* and *j*:

$$\text{Corr}\left(R_i,R_j\right) = \frac{\text{Cov}\left(R_i,R_j\right)}{\sigma\left(R_i\right)\sigma\left(R_j\right)}$$

Moments and Central Moments

The first raw moment is the **mean** of the distribution, which is the expected value of returns.

Central moments are measured relative to the mean (i.e., central around the mean).

The second central moment is the **variance** of the distribution, which measures the dispersion of data.

The third central moment measures the departure from symmetry in the distribution. This moment will equal zero for a symmetric distribution (such as the normal distribution). The **skewness** statistic is the standardized third central moment.

The fourth central moment measures the degree of clustering in the distribution. The **kurtosis** statistic is the standardized fourth central moment of the distribution.

Skewness and Kurtosis

Skewness, or skew, refers to the extent to which a distribution is not symmetrical. Nonsymmetrical distributions may be either positively or negatively skewed and result from the occurrence of outliers in the data set. Outliers are observations with extraordinarily large values, either positive or negative.

- A *positively skewed* distribution is characterized by many outliers in the upper region, or right tail. A positively skewed distribution is said to be skewed right because of its relatively long upper (right) tail.
- A *negatively skewed* distribution has a disproportionately large amount of outliers that fall within its lower (left) tail. A negatively skewed distribution is said to be skewed left because of its long lower tail.

Kurtosis is a measure of the degree to which a distribution is more or less "peaked" than a normal distribution. **Leptokurtic** describes a distribution that is more peaked than a normal distribution, whereas **platykurtic** refers to a distribution that is less peaked (or flatter) than a normal distribution. A distribution is **mesokurtic** if it has the same kurtosis as a normal distribution.

A distribution is said to exhibit **excess kurtosis** if it has either more or less kurtosis than the normal distribution. The computed kurtosis for all normal distributions is three. Statisticians, however, sometimes report excess kurtosis, which is defined as kurtosis minus three. Thus, a normal distribution has excess kurtosis equal to zero, a leptokurtic distribution has excess kurtosis greater than zero, and platykurtic distributions will have excess kurtosis less than zero.

In a similar fashion, we can identify cross central moments for the concept of covariance. The third cross central moment is known as **coskewness** and the fourth cross central moment is known as **cokurtosis**.

The Best Linear Unbiased Estimator

Desirable properties of an estimator are unbiasedness, efficiency, consistency, and linearity.

- An *unbiased* estimator is one for which the expected value of the estimator is equal to the parameter you are trying to estimate.

- An unbiased estimator is also *efficient* if the variance of its sampling distribution is smaller than all the other unbiased estimators of the parameter you are trying to estimate.
- A *consistent* estimator is one for which the accuracy of the parameter estimate increases as the sample size increases.
- A point estimate is a *linear* estimator when it can be used as a linear function of sample data.

If the estimator is the best available (i.e., has the minimum variance), exhibits linearity, and is unbiased, it is said to be the **best linear unbiased estimator** (BLUE).

Distributions

Cross Reference to GARP Assigned Reading – Miller, Chapter 4

The Uniform Distribution

The **continuous uniform distribution** is defined over a range that spans between some lower limit, *a*, and some upper limit, *b*, which serve as the parameters of the distribution. Outcomes can only occur between *a* and *b*, and since we are dealing with a continuous distribution, even if $a < x < b$, $P(X = x) = 0$.

The mean and variance, respectively, of a uniform distribution are:

$$E(x) = \frac{a+b}{2}$$

$$Var(x) = \frac{(b-a)^2}{12}$$

The Bernoulli Distribution

A Bernoulli distributed random variable only has two possible outcomes. The outcomes can be defined as either a "success" or a "failure." Bernoulli distributed random variables are commonly used for assessing whether or not a company defaults during a specified time period.

The Binomial Distribution

A **binomial random variable** may be defined as the number of "successes" in a given number of trials, whereby the outcome can be either "success" or "failure." Think of a trial as a mini-experiment (or Bernoulli trial). The final outcome is the number of successes in a series of *n* trials. Under these conditions, the binomial probability function defines the probability of *x* successes in *n* trials. It can be expressed using the following formula:

$$p(x) = P(X = x) = (\text{number of ways to choose } x \text{ from } n)p^x(1 - p)^{n-x}$$

where:

$$(\text{number of ways to choose x from n}) = \frac{n!}{(n-x)!x!}$$

p = the probability of "success" on each trial

For a given series of *n* trials, the expected number of successes, or E(X), is given by the following formula:

$$\text{expected value of X} = E(X) = np$$

The variance of a binomial random variable is given by:

$$\text{variance of X} = np(1 - p)$$

The Poisson Distribution

The Poisson distribution is another discrete probability distribution with a number of real-world applications. For example, the number of defects per batch in a production process or the number of calls per hour arriving at the 911 emergency switchboard are discrete random variables that follow a Poisson distribution.

While the Poisson random variable *X* refers to the *number of successes per unit*, the parameter lambda (λ) refers to the average or *expected number of successes per unit*. The mathematical expression for the Poisson distribution for obtaining *X* successes, given that λ successes are expected, is:

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

An interesting feature of the Poisson distribution is that both its mean and variance are equal to the parameter, λ.

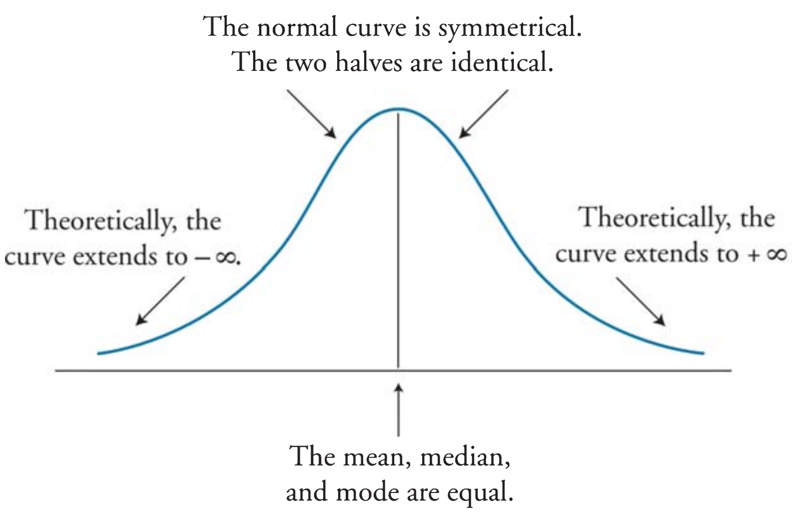
The Normal Distribution

The normal distribution has the following key properties:

- X is normally distributed with mean μ and variance σ^2.
- Skewness = 0, meaning the normal distribution is symmetric about its mean, so that $P(X \leq \mu) = P(\mu \leq X) = 0.5$, and mean = median = mode.
- Kurtosis = 3; this is a measure of how flat the distribution is.
- A linear combination of normally distributed independent random variables is also normally distributed.
- The probabilities of outcomes further above and below the mean get smaller and smaller but do not go to zero (the tails get very thin but extend infinitely).

Many of these properties are evident from examining the graph of a normal distribution's probability density function as illustrated in Figure 2.

Figure 2: Normal Distribution Probability Density Function



The Standard Normal Distribution

A standard normal distribution (i.e., *z*-distribution) is a normal distribution that has been standardized so it has a mean of zero and a standard deviation of 1 [i.e., N~(0,1)]. To standardize an observation from a given normal distribution, the *z-value* of the observation must be calculated. The *z*-value represents the number of standard deviations a given observation is from the population mean. *Standardization* is the process of converting an observed value for a random variable to its *z*-value. The following formula is used to standardize a random variable:

$$z = \frac{\text{observation} - \text{population mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

The Lognormal Distribution

The lognormal distribution is generated by the function e^x, where x is normally distributed. Since the natural logarithm, ln, of e^x is x, the logarithms of lognormally distributed random variables are normally distributed, thus the name.

- The lognormal distribution is skewed to the right.
- The lognormal distribution is bounded from below by zero so that it is useful for modeling asset prices which never take negative values.

The Central Limit Theorem

The central limit theorem states that for simple random samples of size n from a *population* with a mean μ and a finite variance σ^2, the sampling distribution of the sample mean $\bar{x}$ approaches a normal probability distribution with mean μ and variance equal to $\frac{\sigma^2}{n}$ as the sample size becomes large. This is possible because, when the sample size is large, the sums of independent and identically distributed (i.i.d.) random variables (the individual items drawn for the sample) will be normally distributed.

Student's *t*-Distribution

Student's *t*-distribution, or simply the *t*-distribution, is a bell-shaped probability distribution that is symmetrical about its mean. It is the appropriate distribution to use when constructing confidence intervals based on *small samples* ($n < 30$) from populations with *unknown variance* and a normal, or approximately normal, distribution. It may also be appropriate to use the *t*-distribution when the population variance is unknown and the sample size is large enough that the central limit theorem will assure that the sampling distribution is approximately normal.

Student's *t*-distribution has the following properties:

- It is symmetrical.
- It is defined by a single parameter, the degrees of freedom (df), where the degrees of freedom are equal to the number of sample observations minus 1, n – 1, for sample means.
- It has more probability in the tails (fatter tails) than the normal distribution.
- As the degrees of freedom (the sample size) gets larger, the shape of the t-distribution more closely approaches a standard normal distribution.

The Chi-Squared Distribution

Hypothesis testing of the population variance requires the use of a chi-squared distributed test statistic, denoted χ^2. The chi-squared distribution is asymmetrical, bounded below by zero, and approaches the normal distribution in shape as the degrees of freedom increase.

The *F*-Distribution

The hypotheses concerned with the equality of the variances of two populations are tested with an *F*-distributed test statistic. Hypothesis testing using a test statistic that follows an *F*-distribution is referred to as the *F*-test. The *F*-test is used under the assumption that the populations from which samples are drawn are normally distributed and that the samples are independent.

Mixture Distributions

The distributions discussed, as well as others, can be combined to create unique probability density functions. It may be helpful to create a new distribution if the underlying data you are working with does not currently fit a predetermined distribution. In this case, a newly created distribution may assist with explaining the relevant data.

Bayesian Analysis

Cross Reference to GARP Assigned Reading – Miller, Chapter 6

Bayes' Theorem

Bayesian analysis is applied in numerous disciplines and is growing in interest in finance and risk management. The foundation of Bayesian analysis is **Bayes' theorem**. Bayes' theorem for two random variables *A* and *B* is defined as follows:

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

The numerator of the equation just provided [P(B | A) × P(A)] is the joint probability of events *A* and *B*. The joint probability of two events occurring at the same time can also be stated as P(AB). Therefore, another way of expressing Bayes' theorem based on the joint probability of both events occurring is shown as follows:

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

Suppose a bond manager is interested in knowing the probability of Bond A defaulting given that Bond B is already in default. Figure 3 provides a probability matrix defining two events for both bonds, default and no default.

Figure 3: Probability Matrix for Bond A and Bond B

		Bond A No Default	*Bond A* Default	
Bond B	No Default	80%	8%	88%
	Default	8%	4%	12%
		88%	12%	100%

An unconditional probability is a random event that is not contingent on any additional information or events occurring. The unconditional probability of Bond A defaulting is the overall probability of Bond A default given in the example as 12%.

The conditional probability of Bond A defaulting given that Bond B is already in default is defined by: $P(A \mid B) = P(AB) / P(B)$. The numerator is the joint probability of both defaulting, $P(AB) = 4\%$. The denominator is the unconditional probability of Bond B defaulting, $P(B)$. Thus, the conditional probability can be computed as:

$$P(A \mid B) = \frac{P(AB)}{P(B)} = \frac{4\%}{12\%} = \frac{1}{3} \text{ or } 33.3333\%$$

Bayesian Approach vs. Frequentist Approach

The **frequentist approach** involves drawing conclusions from sample data based on the frequency of that data. For example, the approach suggests that the probability of a positive event will be 100% if the sample data consists of only observations that are positive events. The primary difference between the Bayesian approach and the frequentist approach is that the Bayesian approach is instead based on a prior belief regarding the probability of an event occurring.

With small sample sizes, such as three years of historical performance, the Bayesian approach is often used in practice. With larger sample sizes, most analysts tend to use the frequentist approach. The frequentist approach is also often used because it is easier to implement and understand than the Bayesian approach.

Hypothesis Testing and Confidence Intervals

Cross Reference to GARP Assigned Reading – Miller, Chapter 7

Mean and Variance

It is important to recognize that the sample statistic itself is a random variable and, therefore, has a probability distribution. The **sampling distribution** of the sample statistic is a probability distribution of all possible sample statistics computed from a set of equal-size samples that were randomly drawn from the same population. Think of it as the probability distribution of a statistic from many samples.

In general, we can say that for *n* observations:

$$E(\overline{X}) = \mu_X$$

Also, $\text{Var}(\overline{X}) = \frac{\sigma_X^2}{n}$ for *n* observations, and the standard deviation of the sample average is equal to $\frac{\sigma_X}{\sqrt{n}}$. This standard deviation measure is known as the **standard error**.

The **sample mean** is the sum of all the values in a sample of a population, ΣX, divided by the number of observations in the sample, *n*. It is used to make *inferences* about the population mean.

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

The **sample variance**, s^2, is the measure of dispersion that applies when we are evaluating a sample of *n* observations from a population. The sample variance is calculated using the following formula:

$$s^2 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}$$

The **sample standard deviation** can be calculated by taking the square root of the sample variance. The sample standard deviation, *s*, is defined as:

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}}$$

Confidence Intervals

Confidence interval estimates result in a range of values within which the actual value of a parameter will lie, given the probability of $1 - \alpha$. Here, alpha, α, is called the *level of significance* for the confidence interval, and the probability $1 - \alpha$ is referred to as the *degree of confidence*.

Confidence intervals are usually constructed by adding or subtracting an appropriate value from the point estimate. In general, confidence intervals take on the following form:

point estimate ± (reliability factor × standard error)

If the population has a *normal distribution with a known variance*, a confidence interval for the population mean can be calculated as:

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

where:

$\bar{x}$ = point estimate of the population mean (sample mean)

$z_{\alpha/2}$ = reliability factor, a standard normal random variable for which the probability in the right-hand tail of the distribution is $\alpha/2$. In other words, this is the *z*-score that leaves $\alpha/2$ of probability in the upper tail.

$\frac{\sigma}{\sqrt{n}}$ = the standard error of the sample mean where σ is the known standard deviation of the population, and *n* is the sample size

The most commonly used standard normal distribution reliability factors are:

$z_{\alpha/2}$ = 1.65 for 90% confidence intervals (the significance level is 10%, 5% in each tail).

$z_{\alpha/2}$ = 1.96 for 95% confidence intervals (the significance level is 5%, 2.5% in each tail).

$z_{\alpha/2}$ = 2.58 for 99% confidence intervals (the significance level is 1%, 0.5% in each tail).

Hypothesis Testing

A **hypothesis** is a statement about the value of a population parameter developed for the purpose of testing a theory or belief.

The **null hypothesis**, designated H_0, is the hypothesis the researcher wants to reject.

The **alternative hypothesis**, designated H_A, is what is concluded if there is sufficient evidence to reject the null hypothesis.

A test statistic is calculated by comparing the point estimate of the population parameter with the hypothesized value of the parameter (i.e., the value specified in the null hypothesis). With reference to our option return example, this means we are concerned with the difference between the mean return of the sample and the hypothesized mean return. As indicated in the following expression, the test statistic is the difference between the sample statistic and the hypothesized value, scaled by the standard error of the sample statistic.

$$\text{test statistic} = \frac{\text{sample statistic} - \text{hypothesized value}}{\text{standard error of the sample statistic}}$$

One-Tailed and Two-Tailed Tests of Hypotheses

The alternative hypothesis can be one-sided or two-sided. A one-sided test is referred to as a **one-tailed test**, and a two-sided test is referred to as a **two-tailed test**. Whether the test is one- or two-sided depends on the proposition being tested. If a researcher wants to test whether the return on stock options is greater than zero, a one-tailed test should be used. However, a two-tailed test should be used if the research question is whether the return on options is simply different from zero.

When drawing inferences from a hypothesis test, there are two types of errors:

- Type I error: the rejection of the null hypothesis when it is actually true.
- Type II error: the failure to reject the null hypothesis when it is actually false.

The significance level is the probability of making a Type I error (rejecting the null when it is true) and is designated by the Greek letter alpha (α). For instance, a significance level of 5% (α = 0.05) means there is a 5% chance of rejecting a true null hypothesis. When conducting hypothesis tests, a significance level must be specified in order to identify the critical values needed to evaluate the test statistic.

The Relation Between Confidence Intervals and Hypothesis Tests

A confidence interval is a range of values within which the researcher believes the true population parameter may lie.

A confidence interval is determined as:

$$\left\{\left[\begin{pmatrix}\text{sample}\\ \text{statistic}\end{pmatrix} - \begin{pmatrix}\text{critical}\\ \text{value}\end{pmatrix}\begin{pmatrix}\text{standard}\\ \text{error}\end{pmatrix}\right] \leq \begin{matrix}\text{population}\\ \text{parameter}\end{matrix} \leq \left[\begin{matrix}\text{sample}\\ \text{statistic}\end{matrix} + \begin{pmatrix}\text{critical}\\ \text{value}\end{pmatrix}\begin{pmatrix}\text{standard}\\ \text{error}\end{pmatrix}\right]\right\}$$

The *p*-Value

The *p*-value is the probability of obtaining a test statistic that would lead to a rejection of the null hypothesis, assuming the null hypothesis is true. It is the smallest level of significance for which the null hypothesis can be rejected.

The *t*-Test

The *t*-test is a widely used hypothesis test that employs a test statistic that is distributed according to a *t*-distribution.

The computed value for the test statistic based on the *t*-distribution is referred to as the *t*-statistic. For hypothesis tests of a population mean, a *t*-statistic with n – 1 degrees of freedom is computed as:

$$t_{n-1} = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

The *z*-Test

The *z*-test is the appropriate hypothesis test of the population mean when the *population is normally distributed with known variance*. The computed test statistic used with the *z*-test is referred to as the *z*-statistic. The *z*-statistic for a hypothesis test for a population mean is computed as follows:

$$\text{z-statistic} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Example: The *z*-test

When your company's gizmo machine is working properly, the mean length of gizmos is 2.5 inches. However, from time to time the machine gets out of alignment and produces gizmos that are either too long or too short. When this happens, production is stopped and the machine is adjusted. To check the machine, the quality control department takes a gizmo sample each day. Today, a random sample of 49 gizmos showed a mean length of 2.49 inches. The population standard deviation is known to be 0.021 inches. Using a 5% significance level, **determine** if the machine should be shut down and adjusted.

Answer:

Let μ be the mean length of all gizmos made by this machine, and let $\bar{x}$ be the corresponding mean for the sample.

Statement of hypothesis. For the information provided, the null and alternative hypotheses are appropriately structured as:

H_0: μ = 2.5 (The machine does not need an adjustment.)
H_A: μ ≠ 2.5 (The machine needs an adjustment.)

Note that since this is a two-tailed test, H_A allows for values above and below 2.5.

Select the appropriate test statistic. Since the population variance is known and the sample size is > 30, the *z*-statistic is the appropriate test statistic. The *z*-statistic is computed as:

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Specify the level of significance. The level of significance is given at 5%, implying that we are willing to accept a 5% probability of rejecting a true null hypothesis.

State the decision rule regarding the hypothesis. The ≠ sign in the alternative hypothesis indicates that the test is two-tailed with two rejection regions, one in each tail of the standard normal distribution curve. Because the total area of both rejection regions combined is 0.05 (the significance level), the area of the rejection region in each tail is 0.025. You should know that the critical *z*-values for $\pm z_{0.025}$ are ±1.96. This means that the null hypothesis should not be rejected if the computed z-statistic lies between –1.96 and +1.96 and should be rejected if it lies outside of these critical values. The decision rule can be stated as:

Reject H_0 if: z-statistic < $-z_{0.025}$ or z-statistic > $z_{0.025}$, or equivalently,
Reject H_0 if: z-statistic < –1.96 or z-statistic > + 1.96

Collect the sample and calculate the test statistic. The value of $\bar{x}$ from the sample is 2.49. Since σ is given as 0.021, we calculate the *z*-statistic using σ as follows:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{2.49 - 2.5}{0.021/\sqrt{49}} = \frac{-0.01}{0.003} = -3.33$$

Make a decision regarding the hypothesis. The calculated value of the *z*-statistic is –3.33. Since this value is less than the critical value, $-z_{0.025} = -1.96$, it falls in the rejection region in the left tail of the *z*-distribution. Hence, there is sufficient evidence to reject H_0.

Make a decision based on the results of the test. Based on the sample information and the results of the test, it is concluded that the machine is out of adjustment and should be shut down for repair.

Linear Regression with One Regressor

Cross Reference to GARP Assigned Reading – Stock & Watson, Chapter 4

Regression Analysis

A regression analysis has the goal of measuring how changes in one variable, called a **dependent** or **explained** variable can be explained by changes in one or more other variables called the **independent** or **explanatory** variables. The regression analysis measures the relationship by estimating an equation (e.g., linear regression model). The **parameters** of the equation indicate the relationship.

Population Regression Function

A population regression line indicates the expected value of a dependent variable conditional on one or more independent variables:

$$E(Y_i \mid X_i) = B_0 + B_1 \times (X_i)$$

There is a dispersion of *Y*-values around each conditional expected value. The difference between each *Y* and its corresponding conditional expectation (i.e., the line that fits the data) is the **error term** or **noise component** denoted ε_i.

$$\varepsilon_i = Y_i - E(Y_i \mid X_i)$$

The error term represents effects from independent variables not included in the model. An analyst may need to include several of these variables (e.g., trading style and experience) into the population regression function to reduce the error term by a noticeable amount. Often, it is found that limiting an equation to the one or two independent variables with the most explanatory power is the best choice.

Sample Regression Function

The **sample regression function** is an equation that represents a relationship between the *Y* and *X* variable(s) that is based only on the information in a sample of the population. In almost all cases the slope and intercept coefficients of a sample regression function will be different from that of the population regression function. Here we have denoted the population parameters with capital letters (i.e., B_0 and B_1) and the sample coefficients with small letters as indicated in the following sample regression function:

$$Y_i = b_0 + b_1 \times X_i + e_i$$

The sample regression coefficients are b_0 and b_1, which are the intercept and slope. There is also an extra term on the end called the **residual**: $e_i = Y_i - (b_0 + b_1 \times X_i)$. Since the population and sample coefficients are almost always different, the residual will very rarely equal the corresponding population error term (i.e., generally $e_i \neq \varepsilon_i$).

Properties of Regression

Under certain, basic assumptions, we can use a linear regression to estimate the population regression function. The term "linear" has implications for both the independent variable and the coefficients. One interpretation of the term *linear* relates to the independent variable(s) and specifies that the independent variable(s) enters into the equation without a transformation such as a square root or logarithm.

A second interpretation for the term linear applies to the parameters. It specifies that the dependent variable is a linear function of the parameters, but does not require that there is linearity in the variables.

It would not be appropriate to apply linear regression to estimate the parameters of these functions. The primary concern for linear models is that they display linearity in the parameters. Therefore, when we refer to a linear regression model we generally assume that the equation is linear in the parameters; it may or may not be linear in the variables.

Ordinary Least Squares Regression

Ordinary least squares (OLS) estimation is a process that estimates the population parameters B_i with corresponding values for b_i that minimize the squared residuals (i.e., error terms).

$$\text{minimize } \sum e_i^2 = \Sigma[Y_i - (b_0 + b_1 \times X_i)]^2$$

The estimated **slope coefficient** (b_1) for the regression line describes the change in *Y* for a one unit change in *X*. It can be positive, negative, or zero, depending on the relationship between the regression variables. The slope term is calculated as:

$$b_1 = \frac{\text{Cov}(X, Y)}{\text{Var}(X)}$$

The **intercept** term (b_0) is the line's intersection with the *Y*-axis at X = 0. It can be positive, negative, or zero. A property of the least squares method is that the intercept term may be expressed as:

$$b_0 = \overline{Y} - b_1\overline{X}$$

where:
$\overline{Y}$ = mean of Y
$\overline{X}$ = mean of X

Assumptions Underlying Linear Regression

OLS regression requires a number of assumptions. Most of the major assumptions pertain to the regression model's residual term (i.e., error term). Three key assumptions are as follows:

- The expected value of the error term, conditional on the independent variable, is zero ($E(\varepsilon_i|X_i) = 0$).
- All (X, Y) observations are independent and identically distributed (i.i.d.).
- It is unlikely that large outliers will be observed in the data. Large outliers have the potential to create misleading regression results.

Properties of OLS Estimators

Drawing multiple samples from a population will produce multiple sample means. The distribution of these sample means is referred to as the *sampling distribution of the sample mean*. The mean of this sampling distribution is used as an estimator of the population mean and is said to be an **unbiased estimator** of the population mean.

Given the **central limit theorem**, for large sample sizes, it is reasonable to assume that the sampling distribution will approach the normal distribution. This means that the estimator is also a **consistent estimator**.

OLS Regression Results

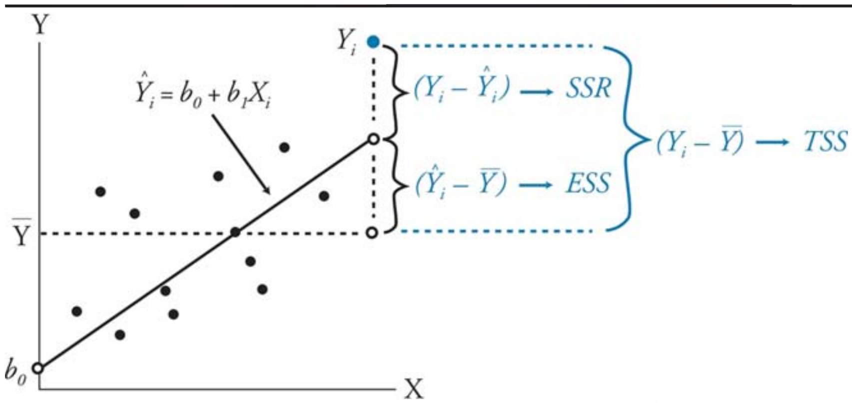
The **sum of squared residuals** (SSR), sometimes denoted SSE, for sum of squared errors, is the sum of squares that results from placing a given intercept and slope coefficient into the equation and computing the residuals, squaring the residuals and summing them. It is represented by $\sum e_i^2$. The sum is an indicator of how well the sample regression function explains the data.

The Coefficient of Determination

The **coefficient of determination**, represented by R^2, is a measure of the "goodness of fit" of the regression. It is interpreted as a percentage of variation in the dependent variable explained by the independent variable. The underlying concept is that for the dependent variable, there is a total sum of squares (TSS) around the sample mean. The regression equation explains some portion of that TSS. Since the explained portion is determined by the independent variables, which are assumed independent of the errors, the total sum of squares can be broken down as follows:

Total sum of squares	=	explained sum of squares	+ sum of squared residuals
$\sum(Y_i - \overline{Y})^2$	=	$\sum(\hat{Y} - \overline{Y})^2$	$+ \sum(Y_i - \hat{Y})^2$
TSS	=	ESS	+ SSR

Figure 4: Components of the Total Variation



The coefficient of determination can be calculated as follows:

$$R^2 = \frac{ESS}{TSS} = \frac{\sum(\hat{Y}_i - \overline{Y})^2}{\sum(Y_i - \overline{Y})^2}$$

$$R^2 = 1 - \frac{SSR}{TSS} = 1 - \frac{\sum(Y_i - \hat{Y}_i)^2}{\sum(Y_i - \overline{Y})^2}$$

The Standard Error of the Regression

The standard error of the regression (SER) measures the degree of variability of the actual Y-values relative to the estimated Y-values from a regression equation. The SER gauges the "fit" of the regression line. The smaller the standard error, the better the fit.

The SER is the standard deviation of the error terms in the regression. As such, SER is also referred to as the standard error of the residual, or the standard error of estimate (SEE).

Regression with a Single Regressor: Hypothesis Tests and Confidence Intervals

Cross Reference to GARP Assigned Reading – Stock & Watson, Chapter 5

Regression Coefficient Confidence Intervals

The confidence interval for the regression coefficient, B_1, is calculated as:

$$b_1 \pm \left(t_c \times s_{b_1}\right), \text{or} \left[b_1 - \left(t_c \times s_{b_1}\right) < B_1 < b_1 + \left(t_c \times s_{b_1}\right)\right]$$

The standard error of the regression coefficient is denoted as s_{b_1}. It is a function of the SER: as SER rises, s_{b_1} also increases, and the confidence interval widens. This makes sense because SER measures the variability of the data about the regression line, and the more variable the data, the less confidence there is in the regression model to estimate a coefficient.

Regression Coefficient Hypothesis Testing

A *t*-test may also be used to test the hypothesis that the true slope coefficient, B_1, is equal to some hypothesized value. Letting b_1 be the point estimate for B_1, the appropriate test statistic with n – 2 degrees of freedom is:

$$t = \frac{b_1 - B_1}{s_{b_1}}$$

Rejection of the null means that the slope coefficient is *different* from the hypothesized value of B_1.

For two-tailed tests, the *p*-value is the probability that lies above the positive value of the computed test statistic *plus* the probability that lies below the negative value of the computed test statistic.

A very small *p*-value provides support for rejecting the null hypothesis. This would indicate a large test statistic that is likely greater than critical values for a common level of significance (e.g., 5%).

What is Heteroskedasticity?

If the variance of the residuals is constant across all observations in the sample, the regression is said to be **homoskedastic.** When the opposite is true, the regression exhibits **heteroskedasticity**, which occurs when the variance of the residuals is not the same across all observations in the sample. This happens when there are subsamples that are more spread out than the rest of the sample.

Unconditional heteroskedasticity occurs when the heteroskedasticity is not related to the level of the independent variables, which means that it doesn't systematically increase or decrease with changes in the value of the independent variable(s). While this is a violation of the equal variance assumption, *it usually causes no major problems with the regression.*

Conditional heteroskedasticity is heteroskedasticity that is related to the level of (i.e., conditional on) the independent variable. Conditional heteroskedasticity *does create significant problems for statistical inference.*

Effect of Heteroskedasticity on Regression Analysis

There are several effects of heteroskedasticity you need to be aware of:

- The standard errors are usually unreliable estimates.
- The coefficient estimates (the b_j) aren't affected.
- If the standard errors are too small, but the coefficient estimates themselves are not affected, the *t*-statistics will be too large and the null hypothesis of no statistical significance is rejected too often. The opposite will be true if the standard errors are too large.

The Gauss-Markov Theorem

The **Gauss-Markov theorem** says that if the linear regression model assumptions are true and the regression errors display homoskedasticity, then the OLS estimators have the following properties.

1. The OLS estimated coefficients have the minimum variance compared to other methods of estimating the coefficients (i.e., they are the most precise).

2. The OLS estimated coefficients are based on linear functions.

3. The OLS estimated coefficients are unbiased, which means that in repeated sampling the averages of the coefficients from the sample will be distributed around the true population parameters [i.e., $E(b_0) = B_0$ and $E(b_1) = B_1$].

4. The OLS estimate of the variance of the errors is unbiased [i.e., $E(\hat{\sigma}^2) = \sigma^2$].

One limitation of the Gauss-Markov theorem is that its conditions may not hold in practice, particularly when the error terms are heteroskedastic, which is sometimes observed in economic data. Another limitation is that alternative estimators, which are not linear or unbiased, may be more efficient than OLS estimators.

Linear Regression with Multiple Regressors

Cross Reference to GARP Assigned Reading – Stock & Watson, Chapter 6

Omitted Variable Bias

Omitted variable bias is present when two conditions are met: (1) the omitted variable is correlated with the movement of the independent variable in the model, and (2) the omitted variable is a determinant of the dependent variable. When relevant variables are absence from a linear regression model, the results will likely lead to incorrect conclusions as the OLS estimators may not accurately portray the actual data.

Multiple regression analysis is therefore used to eliminate omitted variable bias since it can estimate the effect of one independent variable on the dependent variable while holding all other variables constant.

Multiple Regression Basics

Multiple regression is regression analysis with more than one independent variable. It is used to quantify the influence of two or more independent variables on a dependent variable. For instance, simple (or univariate) linear regression explains the variation in stock returns in terms of the variation in systematic risk as measured by beta. With multiple regression, stock returns can be regressed against beta and against additional variables, such as firm size, equity, and industry classification, that might influence returns.

The estimators of these coefficients are known as **ordinary least squares** (OLS) **estimators**. The OLS estimators are typically found with statistical software, but can also be computed using calculus or a trial-and-error method. The result of this procedure is the following regression equation:

$$\hat{Y}_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + \ldots + b_k X_{ki}$$

where the lowercase b_i's indicate an estimate for the corresponding regression coefficient

Interpreting the Multiple Regression Results

The interpretation of the estimated regression coefficients from a multiple regression is the same as in simple linear regression for the intercept term but significantly different for the slope coefficients:

- The **intercept term** is the value of the dependent variable when the independent variables are all equal to zero.
- Each slope coefficient is the estimated change in the dependent variable for a one-unit change in that independent variable, *holding the other independent variables constant.* That's why the slope coefficients in a multiple regression are sometimes called **partial slope coefficients**.

Measures of Fit

The **standard error of the regression** (SER) measures the uncertainty about the accuracy of the predicted values of the dependent variable, $\hat{Y}_i = b_0 + b_1 X_i$. Graphically, the relationship is stronger when the actual x,y data points lie closer to the regression line (i.e., the e_i are smaller).

Formally, SER is the standard deviation of the predicted values for the dependent variable about the regression line.

Coefficient of Determination, R^2

The multiple coefficient of determination, R^2, can be used to test the overall effectiveness of the entire set of independent variables in explaining the dependent variable. Its interpretation is similar to that for simple linear regression: the percentage of variation in the dependent variable that is *collectively* explained by all of the independent variables.

Adjusted R^2

To overcome the problem of overestimating the impact of additional variables on the explanatory power of a regression model, many researchers recommend adjusting R^2 for the number of independent variables. The *adjusted R^2* value is expressed as:

$$R_a^2 = 1 - \left[\left(\frac{n-1}{n-k-1}\right) \times (1 - R^2)\right]$$

Assumptions of Multiple Regression

As with simple linear regression, most of the assumptions made with the multiple regression pertain to ε, the model's error term:

- A linear relationship exists between the dependent and independent variables.
- The independent variables are not random, and there is no exact linear relation between any two or more independent variables.
- The expected value of the error term, conditional on the independent variables, is zero [i.e., $E(\varepsilon | X_1, X_2, \ldots X_k) = 0$].
- The variance of the error terms is constant for all observations [i.e., $E(\varepsilon_i^2) = \sigma_\varepsilon^2$].
- The error term for one observation is not correlated with that of another observation [i.e., $E(\varepsilon_i \varepsilon_j) = 0$, $j \neq i$].
- The error term is normally distributed.

Multicollinearity

Multicollinearity refers to the condition when two or more of the independent variables, or linear combinations of the independent variables, in a multiple regression are highly correlated with each other. This condition distorts the standard error of the regression and the coefficient standard errors, leading to problems when conducting *t*-tests for statistical significance of parameters.

The degree of correlation will determine the difference between perfect and imperfect multicollinearity. If one of the independent variables is a perfect linear combination of the other independent variables, then the model is said to exhibit **perfect multicollinearity**. In this case, it will not be possible to find the OLS estimators necessary for the regression results. **Imperfect multicollinearity** arises when two or more independent variables are highly correlated, but less than perfectly correlated. When conducting regression analysis, we need to be cognizant of imperfect multicollinearity since OLS estimators will be computed, but the resulting coefficients may be improperly estimated.

As a result of multicollinearity, there is a *greater probability that we will incorrectly conclude that a variable is not statistically significant* (e.g., a Type II error). Multicollinearity is likely to be present to some extent in most economic models. The issue is whether the multicollinearity has a significant effect on the regression results.

Hypothesis Tests and Confidence Intervals in Multiple Regression

Cross Reference to GARP Assigned Reading – Stock & Watson, Chapter 7

Hypothesis Testing of Regression Coefficients

Example: Testing the statistical significance of a regression coefficient

Consider the hypothesis that future 10-year real earnings growth in the S&P 500 (EG10) can be explained by the trailing dividend payout ratio of the stocks in the index (PR) and the yield curve slope (YCS). **Test** the statistical significance of the independent variable PR in the real earnings growth example at the 10% significance level. Assume that the number of observations is 46. The results of the regression are reproduced in the following figure.

Coefficient and Standard Error Estimates for Regression of EG10 on PR and YCS

	Coefficient	*Standard Error*
Intercept	–11.6%	1.657%
PR	0.25	0.032
YCS	0.14	0.280

Answer:

We are testing the following hypothesis:

H_0: PR = 0 versus H_A: PR $\neq$ 0

The 10% two-tailed critical *t*-value with 46 – 2 – 1 = 43 degrees of freedom is approximately 1.68. We should reject the null hypothesis if the *t*-statistic is greater than 1.68 or less than –1.68.

The *t*-statistic is:

$$t = \frac{0.25}{0.032} = 7.8$$

Therefore, because the *t*-statistic of 7.8 is greater than the upper critical *t*-value of 1.68, we can reject the null hypothesis and conclude that the PR regression coefficient is statistically significantly different from zero at the 10% significance level.

Confidence Intervals for a Regression Coefficient

The confidence interval for a regression coefficient in multiple regression is calculated and interpreted the same way as it is in simple linear regression. For example, a 95% confidence interval is constructed as follows:

$$b_j \pm (t_c \times s_{b_j})$$

or

estimated regression coefficient ± (critical *t*-value)(coefficient standard error)

The critical *t*-value is a two-tailed value with n – k – 1 degrees of freedom and a 5% significance level, where *n* is the number of observations and *k* is the number of independent variables.

Example: Calculating a confidence interval for a regression coefficient

Calculate the 90% confidence interval for the estimated coefficient for the independent variable PR in the real earnings growth example.

Answer:

The critical *t*-value is 1.68, the same as we used in testing the statistical significance at the 10% significance level (which is the same thing as a 90% confidence level). The estimated slope coefficient is 0.25 and the standard error is 0.032. The 90% confidence interval is:

$$0.25 \pm (1.68)(0.032) = 0.25 \pm 0.054 = 0.196 \text{ to } 0.304$$

Joint Hypothesis Testing

An *F*-test assesses how well the set of independent variables, as a group, explains the variation in the dependent variable. That is, the *F*-statistic is used to test whether *at least one* of the independent variables explains a significant portion of the variation of the dependent variable.

For example, if there are four independent variables in the model, the hypotheses are structured as:

$$H_0: B_1 = B_2 = B_3 = B_4 = 0 \text{ versus } H_A: \text{at least one } B_j \neq 0$$

The *F*-statistic, *which is always a one-tailed test*, is calculated as:

$$\frac{ESS/k}{SSR/(n-k-1)}$$

where:
ESS = explained sum of squares
SSR = sum of squared residuals

Example: Calculating and interpreting the *F*-statistic

An analyst runs a regression of monthly value-stock returns on five independent variables over 60 months. The total sum of squares is 460, and the sum of squared residuals is 170. Test the null hypothesis at the 5% significance level (95% confidence) that all five of the independent variables are equal to zero.

Answer:

The null and alternative hypotheses are:

$$H_0: B_1 = B_2 = B_3 = B_4 = B_5 = 0 \text{ versus } H_A\text{: at least one } B_j \neq 0$$

$$\text{ESS} = \text{TSS} - \text{SSR} = 460 - 170 = 290$$

$$F = \frac{58.0}{3.15} = 18.41$$

The critical *F*-value for 5 and 54 degrees of freedom at a 5% significance level is approximately 2.40. Remember, it's a **one-tailed** test, so we use the 5% *F*-table! Therefore, we can reject the null hypothesis and conclude that at least one of the five independent variables is significantly different than zero.

Interpreting Regression Results

Just as in simple linear regression, the variability of the dependent variable or **total sum of squares** (TSS) can be broken down into **explained sum of squares** (ESS) and **sum of squared residual**s (SSR). As shown previously, the coefficient of determination is:

$$R^2 = \frac{\text{ESS}}{\text{TSS}} = \frac{\sum\left(\hat{Y} - \bar{Y}\right)^2}{\sum\left(Y_i - \bar{Y}\right)^2} = 1 - \frac{\text{SSR}}{\text{TSS}} = 1 - \frac{\sum e_i^2}{\sum\left(Y_i - \bar{Y}\right)^2}$$

Regression results usually provide R^2 and a host of other measures. However, it is useful to know how to compute R^2 from other parts of the results. Figure 5 is an ANOVA table of the results of a regression.

The **coefficient of multiple correlation** is simply the square root of *R*-squared. In the case of a multiple regression, the coefficient of multiple correlation is always positive.

Figure 5: ANOVA Table

R-squared	0.934					
Adj *R*-squared	0.890					
Standard error	1.407					
Observations	6					
	Degrees of Freedom	*SS*	*MS*	*F*		
Explained	2	84.057	42.029	21.217		
Residual	3	5.943	1.981			
Total	5	90				
Variables	*Coeff*	*Std Error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	–4.4511	3.299	–1.349	0.270	–14.950	6.048
Lockup	2.057	0.337	6.103	0.009	0.984	3.130
Experience	2.008	0.754	2.664	0.076	–0.391	4.407

The results in Figure 5 produce the following equation:

$$\hat{Y}_i = -4.451 + 2.057 \times X_{1i} + 2.008 \times X_{2i}$$

The statistics in the ANOVA table also allow for the testing of the joint hypothesis that both slope coefficients equal zero.

H_0: $B_1 = B_2 = 0$

H_A: $B_1 \neq 0$ or $B_2 \neq 0$

The test statistic in this case is the ***F*-statistic** where the degrees of freedom are indicated by two numbers: the number of slope coefficients (2) and the sample size minus the number of slope coefficients minus one (6 – 2 – 1 = 3). The *F*-statistic given the hedge fund data can be calculated as follows:

$$F = \frac{ESS/df}{SSR/df} = \frac{84.057/2}{5.943/3} = \frac{42.029}{1.981} = 21.217$$

The critical *F*-statistic at a 5% significance level is $F_{0.05} = 9.55$. Since the value from the regression results is greater than that value: F = 21.217 > 9.55, a researcher would reject the null hypothesis: H_0: $B_1 = B_2 = 0$. It should be noted that rejecting the null hypothesis indicates one or both of the coefficients are significant.

Modeling and Forecasting Trend

Cross Reference to GARP Assigned Reading – Diebold, Chapter 5

Linear and Nonlinear Trends

A *time series* is a set of observations for a variable over successive periods of time (e.g., monthly stock market returns for the past 10 years). The series has a **trend** if a consistent pattern can be seen by plotting the data (i.e., the individual observations) on a graph.

A *linear trend* is a time series pattern that can be graphed using a straight line. The simplest form of a linear trend is represented by the following model:

$$y_t = \beta_0 + \beta_1(t)$$

where:
y_t = the value of the time series (the dependent variable at time t)
β_0 = regression intercept at the vertical axis
β_1 = regression slope coefficient (or trend coefficient)
t = time trend or time dummy (the independent variable)

A *nonlinear trend* is a time series pattern that can be graphed with a curve. For example, a nonlinear trend would result if a variable increases at an increasing rate.

A possible way to capture nonlinearities is to use a **quadratic trend** as follows:

$$y_t = \beta_0 + \beta_1(t) + \beta_2(t)^2$$

While quadratic trends may be adequate for modeling some nonlinear trends, other trends may be better approximated using an **exponential trend**. In particular, financial time series often display exponential growth (i.e., growth with continuous compounding).

When a series exhibits exponential growth, it can be modeled using an exponential trend as follows:

$$y_t = \beta_0 e^{\beta_1(t)}$$

where:
y_t = the value of the dependent variable at time *t*
β_0 = regression intercept term
β_1 = the constant rate of growth
t = time

This nonlinear trend model defines *y*, the dependent variable, as an exponential function of time, the independent variable. Rather than try to fit the nonlinear data with a linear (straight line) regression, we take the natural log of both sides of the equation and arrive at the **log-linear trend** as follows:

$$\ln(y_t) = \ln(\beta_0) + \beta_1(t)$$

Estimating and Forecasting Trends

Ordinary least squares (OLS) **regression** is used to estimate the coefficients in a trend line.

Recall that with trend models, *t* takes on the value of the time period. For example, in period 2, the equation becomes:

$$\hat{y}_2 = \hat{\beta}_0 + \hat{\beta}_1(2)$$

This means $\hat{y}$ increases by the value of $\hat{\beta}_1$ each period.

Example: Using a linear trend model

Suppose you are given a linear trend model with $\hat{\beta}_0 = 1.7$ and $\hat{\beta}_1 = 3.0$.

Calculate $\hat{y}_t$ for t = 1 and t = 2.

Answer:

When t = 1, $\hat{y}_1 = 1.7 + 3.0(1) = 4.7$.

When t = 2, $\hat{y}_2 = 1.7 + 3.0(2) = 7.7$.

Notice that the difference between $\hat{y}_1$ and $\hat{y}_2$ is 3.0, the value of the trend coefficient $\hat{\beta}_1$.

Model Selection Criteria

Mean Squared Error

Mean squared error (MSE) is a statistical measure computed as the sum of squared residuals divided by the total number of observations in the sample.

$$\text{MSE} = \frac{\sum_{t=1}^{T} e_t^2}{T}$$

where:
T = total sample size
$e_t = y_t - \hat{y}_t$ (the residual for observation *t* or difference between the observed and expected observation)
$\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1(t)$ (i.e., a regression model)

The MSE is based on *in-sample* data. The regression model with the smallest MSE is also the model with the smallest sum of squared residuals.

Model selection is one of the most important criteria in forecasting data. Unfortunately, selecting the best model based on the highest R^2 or smallest MSE is not effective in producing good *out-of-sample* forecasting models.

The s^2 Measure

One way to reduce the bias associated with MSE is to impose a penalty on the degrees of freedom, *k*. The **s^2 measure** is an unbiased estimate of the MSE because it corrects for degrees of freedom as follows:

$$s^2 = \frac{\sum_{t=1}^{T} e_t^2}{T - k}$$

Akaike and Schwarz Criterion

The unbiased MSE estimate, s^2, defined earlier, can be re-written to highlight the penalty for degrees of freedom. In the following equation, the first term (T / T – k) can be thought of as the **penalty factor**.

$$s^2 = \left(\frac{T}{T - k}\right) \frac{\sum_{t=1}^{T} e_t^2}{T}$$

This notation is useful when comparing different selection criteria because it takes the form of a *penalty factor times the MSE*. The **Akaike information criterion** (AIC) and the **Schwarz information criterion** (SIC) use different penalty factors as follows:

$$AIC = e^{\left(\frac{2k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$$

$$SIC = T^{\left(\frac{k}{T}\right)} \frac{\sum_{t=1}^{T} e_t^2}{T}$$

Note that the penalty factors for s^2, AIC, and SIC are (T / T – k), $e^{(2k / T)}$, and $T^{(k / T)}$, respectively.

Evaluating Consistency

Consistency is a key property that is used to compare different selection criteria. Two conditions are required for a model selection criteria to be considered consistent based on whether the *true* model is included among the regression models being considered.

- When the *true* model or *data generating process* (DGP) is one of the defined regression models, then the probability of selecting the *true* model approaches one as the sample size increases.
- When the *true* model is *not* one of the defined regression models being considered, then the probability of selecting the *best approximation* model approaches one as the sample size increases.

Choosing the best forecasting model is an important task and we have discussed four key selection criteria. Adjusting for the degrees of freedom is extremely important and the SIC is the best selection criteria because it is consistent and also has the highest penalty factor. The AIC is also an important measure that is often considered in addition to SIC.

Modeling and Forecasting Seasonality

Cross Reference to GARP Assigned Reading – Diebold, Chapter 6

Sources of Seasonality

Seasonality in a time series is a pattern that tends to repeat from year to year. One example is monthly sales data for a retailer. Because sales data normally varies according to the calendar, we might expect this month's sales (x_t) to be related to sales for the same month last year (x_{t-12}).

There are two approaches for modeling and forecasting a time series impacted by seasonality: (1) using a seasonally adjusted time series and (2) regression analysis with seasonal dummy variables.

A **seasonally adjusted time series** is created by removing the seasonal variation from the data. This type of adjustment is commonly made in macroeconomic forecasting where the goal is to only measure the *nonseasonal fluctuations* of a variable. However, the use of seasonal adjustments in business forecasting is usually inappropriate because seasonality often accounts for large variations in a time series. Financial forecasters should be interested in capturing *all* variation in a time series, not just the nonseasonal portions.

Modeling Seasonality with Regression Analysis

A regression that incorporates seasonal dummies can be an effective technique for modeling seasonality. In this process, **seasonal dummy variables** can take a value of either "1" or "0," to represent the independent variable being "on" or "off."

An important consideration when performing multiple regression and modeling seasonality with dummy variables is the number of dummy variables to include in the model. If we include an intercept in our model and there are *s* seasons, we use s – 1 dummy variables to avoid the problem of (perfect) multicollinearity.

Consider the following regression equation for explaining quarterly earnings per share (EPS) in terms of the quarter of their occurrence:

$$EPS_t = \beta_0 + \beta_1 D_{1,t} + \beta_2 D_{2,t} + \beta_3 D_{3,t} + \varepsilon_t$$

where
EPS_t = a quarterly observation of earnings per share
$D_{1,t}$ = 1 if period *t* is the first quarter of a year, $D_{1,t}$ = 0 otherwise
$D_{2,t}$ = 1 if period *t* is the second quarter of a year, $D_{2,t}$ = 0 otherwise
$D_{3,t}$ = 1 if period *t* is the third quarter of a year, $D_{3,t}$ = 0 otherwise

The intercept term, β_0, represents the average value of EPS for the fourth quarter. The slope coefficient on each dummy variable estimates the *difference* in EPS (on average) between the respective quarter (i.e., quarter 1, 2, or 3) and the omitted quarter (the fourth quarter, in this case).

Seasonality can be extended to account for other types of calendar effects, such as **holiday variations** (which adjust for holidays like Easter that may occur in different months each year) and **trading-day variations** (which reflect the varying number of days each month).

Characterizing Cycles

Cross Reference to GARP Assigned Reading – Diebold, Chapter 7

Covariance Stationary Time Series

A process such as a time series must have certain properties if we want to forecast its future values based on its past values. In particular, it needs the relationships among its present and past values to remain stable over time. We refer to such a time series as being **covariance stationary**.

To be covariance stationary, a time series must exhibit the following three properties:

1. Its mean must be stable over time.

2. Its variance must be finite and stable over time.

3. Its covariance structure must be stable over time.

Covariance structure refers to the covariances among the values of a time series at its various **lags** or **displacements**, which are a given number of periods apart at which we can observe its values. We use the lowercase Greek letter tau, τ, to represent a displacement.

The covariance between the current value of a time series and its value τ periods in the past is referred to as its **autocovariance** at displacement τ. Its autocovariances for all τ make up its **autocovariance function**.

To convert an autocovariance function to an **autocorrelation function**, we divide the autocovariance at each τ by the variance of the time series. This gives us an autocorrelation for each τ that will be scaled between +1 and –1.

Autoregression is a linear regression of a time series against its own past values. The regression coefficient that results is referred to as the partial autocorrelation for that lag. Partial autocorrelations for all lags make up the **partial autocorrelation function** of the time series.

White Noise

A time series might exhibit no correlation among any of its lagged values. Such a time series is said to be **serially uncorrelated**.

A special type of serially uncorrelated series is one that has a mean of zero and a constant variance. This condition is referred to as **white noise**, or zero-mean white noise, and the time series is said to follow a white noise process.

If the observations in a white noise process are independent, as well as uncorrelated, the process is referred to as **independent white noise**. If the process also follows a normal distribution, it is known as **normal white noise** or **Gaussian white noise**. Not all independent white noise processes are normally distributed, but all normal white noise processes are also independent white noise.

Lag Operators

A commonly used notation for time series modeling is the **lag operator**, L. If y_t is the value of a time series at time *t*, and y_{t-1} is its value one period earlier, we can express a lag operator as:

$$y_{t-1} = Ly_t$$

Notation for any degree of lag can be displayed as:

$$L^m y_t = y_{t-m}$$

A **distributed lag** is a model that assigns weights to past values of a time series. A lag operator polynomial of degree *m* is a distributed lag that includes all lags from 1 to *m*.

Wold's Theorem

Wold's theorem, or **Wold's representation**, holds that a covariance stationary process can be modeled as an infinite distributed lag of a white noise process. Such a model would take the following form:

$$\varepsilon_t + b_1\varepsilon_{t-1} + b_2\varepsilon_{t-2} + \ldots = \sum_{i=0}^{\infty} b_i\varepsilon_{t-i}$$

Because this expression can be applied to any covariance stationary series, it is known as a **general linear process**.

In Wold's representation the ε terms are referred to as **innovations**. Innovations can be thought of as the errors that would result from a good forecast of the covariance stationary process. That is, they are a white noise process with an unconditional mean of zero.

Estimating Autocorrelations

The sample mean, $\overline{y}$, is of course the arithmetic average of the observations. The **sample autocorrelation** for displacement τ is estimated by the following formula:

$$\widehat{\rho_\tau} = \frac{\sum_{t=\tau+1}^{T}\left[(y_t - \overline{y})(y_{t-\tau} - \overline{y})\right]}{\sum_{t=1}^{T}(y_t - \overline{y})^2}$$

Earlier we defined *partial autocorrelations* as the results from a linear regression of a time series against its lagged values. Theoretically, this assumed an infinite set of observations. If we perform such a regression with a finite sample of time series data, what we actually get are **sample partial autocorrelations**.

If we have T observations of a time series, the standard deviation of its autocorrelations or partial correlations is $1/\sqrt{T}$. One of the ways to determine whether a time series can be considered white noise is by displaying its autocorrelation and partial autocorrelation functions with bands at $\pm 2/\sqrt{T}$.

A more rigorous way of determining whether a time series is white noise is to test the hypothesis where autocorrelations are jointly equal to zero. A test statistic for this hypothesis is the **Box-Pierce Q-statistic**, which follows a Chi-squared distribution. A similar test statistic that may be more useful with small samples is the **Ljung-Box Q-statistic**.

Modeling Cycles: MA, AR, and ARMA Models

Cross Reference to GARP Assigned Reading – Diebold, Chapter 8

First-Order Moving Average Process

Conceptually, a moving average process is a linear regression of the current values of a time series against both the current and previous unobserved white noise error terms, which are random shocks. The first-order moving average

[MA(1)] process has a mean of zero and a constant variance and can be defined as:

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

where:
y_t = the time series variable being estimated
ε_t = current random white noise shock
ε_{t-1} = one-period lagged random white noise shock
θ = coefficient for the lagged random shock

The **MA(1) process** is considered to be first-order because it only has one lagged error term (ε_{t-1}).

One key feature of moving average processes is called the *autocorrelation (ρ) cutoff*. We would compute the autocorrelation using the following formula:

$$\rho_1 = \frac{\theta_1}{1+\theta_1^2}\text{; where } \rho_\tau = 0 \text{ for } \tau > 1$$

For any value beyond the first lagged error term, the autocorrelation will be zero in an MA(1) process. This is important because it is one condition of being covariance stationary (i.e., mean = 0, variance = σ^2), which is a condition of this process being a useful estimator.

MA(q) Process

The MA(1) process is a subset of a much larger picture. Forecasters can broaden their horizon to a finite-order moving average process of order *q*, which essentially adds lag operators out to the q^{th} observation and potentially improves on the MA(1) process. The **MA(q) process** is expressed in the following formula:

$$y_t = \varepsilon_t + \theta_1\varepsilon_{t-1} + \ldots + \theta_q\varepsilon_{t-q}$$

where:
y_t = the time series variable being estimated
ε_t = current random white noise shock
ε_{t-1} = one-period lagged random white noise shock
ε_{t-q} = q^{th}-period lagged random white noise shock
θ = coefficients for the lagged random shocks

First-Order Autoregressive Process

The **first-order autoregressive** [**AR(1)**] **process** must also have a mean of zero and a constant variance. It is specified in the form of a variable regressed against itself in a lagged form. This relationship can be shown in the following formula:

$$y_t = \phi y_{t-1} + \varepsilon_t$$

where:
y_t = the time series variable being estimated
y_{t-1} = one-period lagged observation of the variable being estimated
ε_t = current random white noise shock
ϕ = coefficient for the lagged observation of the variable being estimated

Just like the moving average process, the predictive ability of this model hinges on it being covariance stationary. In order for an AR(1) process to be covariance stationary, the absolute value of the coefficient on the lagged operator must be less than one (i.e., $|\phi| < 1$).

AR(p) Process

Just as the MA(1) process was described as a subset of the much broader MA(q) process, so is the relationship between the AR(1) process and the AR(p) process. The AR(p) process expands the AR(1) process out to the p^{th} observation as seen in the following formula:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t$$

where:
y_t = the time series variable being estimated
y_{t-1} = one-period lagged observation of the variable being estimated
y_{t-p} = p^{th}-period lagged observation of the variable being estimated
ε_t = current random white noise shock
ϕ = coefficients for the lagged observations of the variable being estimated

The AR(p) process is also covariance stationary if $|\phi| < 1$ and it exhibits the same decay in autocorrelations that was found in the AR(1) process. However, while an AR(1) process only evidences oscillation in its autocorrelations (switching from positive to negative) when the coefficient is negative, an AR(p) process will naturally oscillate as it has multiple coefficients interacting with each other.

Autoregressive Moving Average Process

An **autoregressive moving average** (**ARMA**) **process** and is expressed by the following formula:

$$y_t = \phi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$$

where:
y_t = the time series variable being estimated
ϕ = coefficient for the lagged observations of the variable being estimated
y_{t-1} = one-period lagged observation of the variable being estimated
ε_t = current random white noise shock
θ = coefficient for the lagged random shocks
ε_{t-1} = one-period lagged random white noise shock

You can see that the ARMA formula merges the concepts of an AR process and an MA process. In order for the ARMA process to be covariance stationary, which is important for forecasting, we must still observe $|\theta| < 1$.

Application of AR and ARMA Processes

A forecaster might begin by plotting the autocorrelations for a data series and find that the autocorrelations decay gradually rather than cut off abruptly. In this case, the forecaster should rule out using a moving average process. If the autocorrelations instead decay gradually, he should consider specifying either an autoregressive (AR) process or an autoregressive moving average (ARMA) process. The forecaster should especially consider these alternatives if he notices periodic spikes in the autocorrelations as they are gradually decaying. For example, if every 12th autocorrelation jumps upward, this observation indicates a possible seasonality effect in the data and would heavily point toward using either an AR or ARMA model.

Volatility

Cross Reference to GARP Assigned Reading – Hull, Chapter 10

Volatility, Variance, and Implied Volatility

The **volatility** of a variable, σ, is represented as the standard deviation of that variable's continuously compounded return. With option pricing, volatility is typically expressed as the standard deviation of return over a one-year period. This differs from risk management, where volatility is typically expressed as the standard deviation of return over a one-day period.

By assuming daily returns are independent with the same level of variation, daily volatility can be extended over a number of days, T, by multiplying the standard deviation of the return by the square root of T. This is known as the *square root of time rule*. For example, if the daily volatility is 1.5%, the standard deviation of the return (compounded continuously) over a 10-day period would be computed as $1.5\% \times \sqrt{10} = 4.74\%$.

Risk managers may also compute a variable's **variance rate**, which is simply the square of volatility (i.e., standard deviation squared: σ^2). In contrast to volatility, which increases with the square root of time, the variance of an asset's return will increase in a linear fashion over time. For example, if the daily volatility is 1.5%, the variance rate is $1.5\%^2 = 0.0225\%$. Thus, over a 10-day period, the variance will be 0.225% (i.e., $0.0225\% \times 10$).

The **implied volatility** of an option is computed from an option pricing model, such as the Black-Scholes-Merton (BSM) model. The volatility of an asset is not directly observed in the BSM model, so we compute implied volatility as the volatility level that will result when equating an option's market price to its model price.

The Power Law

In practice, the distribution of asset price changes is more likely to exhibit fatter tails than the normal distribution. An alternative approach to assuming a normal distribution is to apply the power law.

The power law states that when X is large, the value of a variable V has the following property:

$$P(V > X) = K \times X^{-\alpha}$$

where:
V = the variable
X = large value of V
K and α = constant

Example: The power law

Assume that data on asset price changes determines the constants in the power law equation to be the following: K = 10 and α = 5. **Calculate** the probability that this variable will be greater than a value of 3 and a value of 5.

Answer:

$$P(V > 3) = 10 \times 3^{-5} = 0.0412 \text{ or } 4.12\%$$
$$P(V > 5) = 10 \times 5^{-5} = 0.0032 \text{ or } 0.32\%$$

Estimating Volatility

In simplest terms, historical data is used to generate returns in an asset-pricing series. This historical return information is then used to generate a volatility parameter, which can be used to infer expected realizations of risk. However, straightforward approaches weight each observation equally in that more distant past returns have the same influence on estimated volatility as observations that are more recent. If the goal is to estimate the current level of volatility, we may want to weight recent data more heavily.

The Exponentially Weighted Moving Average Model

The exponentially weighted moving average (EWMA) model is a specific case of the general weighting model presented in the previous section. The main difference is that the weights are assumed to decline exponentially back through time. This assumption results in a specific relationship for variance in the model:

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$

where:
λ = weight on previous volatility estimate (λ between zero and one)

The simplest interpretation of the EWMA model is that the day-*n* volatility estimate is calculated as a function of the volatility calculated as of day $n - 1$ and the most recent squared return. Depending on the weighting term λ, which ranges between zero and one, the previous volatility and most recent squared returns will have differential impacts. High values of λ will minimize the effect of daily percentage returns, whereas low values of λ will tend to increase the effect of daily percentage returns on the current volatility estimate.

The GARCH(1,1) Model

One of the most popular methods of estimating volatility is the **generalized autoregressive conditional heteroskedastic** (GARCH)(1,1) model. A GARCH(1,1) model not only incorporates the most recent estimates of variance and squared return, but also a variable that accounts for a long-run average level of variance.

The best way to describe a GARCH(1,1) model is to take a look at the formula representing its determination of variance, which can be shown as:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

where:
α = weighting on the previous period's return
β = weighting on the previous volatility estimate
ω = weighted long-run variance $= \gamma V_L$
V_L = long-run average variance $= \dfrac{\omega}{1-\alpha-\beta}$
$\alpha + \beta + \gamma = 1$
$\alpha + \beta < 1$ for stability so that γ is not negative

The EWMA is nothing other than a special case of a GARCH(1,1) volatility process, with $\omega = 0$, $\alpha = 1 - \lambda$, and $\beta = \lambda$. Similar to the EWMA model, β represents the exponential decay rate of information. The GARCH(1,1) model adds to the information generated by the EWMA model in that it also assigns a weighting to the average long-run variance estimate. An additional characteristic of a GARCH(1,1) estimate is the implicit assumption that variance tends to revert to a long-term average level.

The sum of $\alpha + \beta$ is called the **persistence**, and if the model is to be stationary over time (with reversion to the mean), the sum must be less than one. The persistence describes the rate at which the volatility will revert to its long-term value following a large movement.

One of the useful features of GARCH models is that they do a very good job at modeling volatility clustering when periods of high volatility tend to be followed by other periods of high volatility and periods of low volatility tend to be followed by subsequent periods of low volatility. Thus, there is autocorrelation in u_i^2. If GARCH models do a good job of explaining volatility changes, there should be very little autocorrelation in u_i^2 / σ_i^2. GARCH models appear to do a very good job of explaining volatility.

Correlations and Copulas

Cross Reference to GARP Assigned Reading – Hull, Chapter 11

Covariance Using EWMA and GARCH Models

Covariance is a statistical measure that is calculated over historical time periods. Conventional wisdom suggests that more recent observations should carry more weight because they more accurately reflect the current market environment. The following equation calculates a new covariance on day *n* using an **exponentially weighted moving average (EWMA) model**. This model is designed to vary the weight given to more recent observations (by adjusting λ).

$$\text{cov}_n = \lambda \,\text{cov}_{n-1} + (1 - \lambda) X_{n-1} Y_{n-1}$$

where:
λ = the weight for the most recent covariance on day $n - 1$
X_{n-1} = the percentage change for variable X on day $n - 1$
Y_{n-1} = the percentage change for variable Y on day $n - 1$

An alternative method for updating the covariance rate for two variables *X* and *Y* uses the **generalized autoregressive conditional heteroskedasticity (GARCH)**

model. The GARCH(1,1) model for updating covariance rates is defined as follows:

$$\text{cov}_n = \omega + \alpha X_{n-1} Y_{n-1} + \beta \text{cov}_{n-1}$$

GARCH(1,1) applies a weight of α to the most recent observation on covariance ($X_{n-1}Y_{n-1}$) and a weight of β to the most recent covariance estimate (cov_{n-1}). In addition, a weight of ω is given to the long-term average covariance rate.

Evaluating Consistency for Covariances

A **variance-covariance matrix** can be constructed using the calculated estimates of variance and covariance rates for a set of variables. The diagonal of the matrix represents the variance rates where $i = j$. The covariance rates are all other elements of the matrix where $i \neq j$.

A matrix is known as *positive-semidefinite* if it is internally consistent. An example of a variance-covariance matrix that is not internally consistent is shown as follows:

$$\begin{pmatrix} 1 & 0 & 0.8 \\ 0 & 1 & 0.8 \\ 0.8 & 0.8 & 1 \end{pmatrix}$$

A method for testing for consistency is:

$$\rho_{12}^2 + \rho_{13}^2 + \rho_{23}^2 - 2\rho_{12}\rho_{13}\rho_{23} \leq 1$$

When computing the formula, if the left side of the expression is greater than the right side, the matrix is not internally consistent.

Copulas

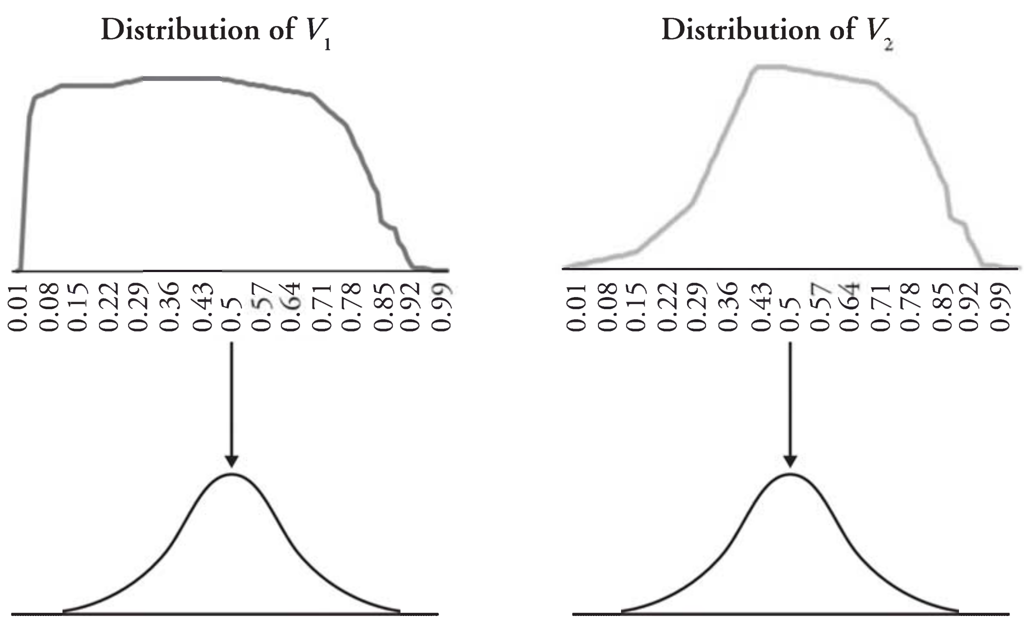
A **copula** creates a joint probability distribution between two or more variables while maintaining their individual marginal distributions. This is accomplished by mapping the marginal distributions to a new known distribution. For example, a Gaussian copula maps the marginal distribution of each variable to the standard normal distribution, which, by definition, has a mean of zero and a standard deviation of one. The mapping of each variable to the new distribution is done based on percentiles.

The key property of a copula correlation model is the *preservation of the original marginal distributions while defining a correlation between them*. A correlation copula is created by converting two distributions that may be unusual or have unique shapes and mapping them to known distributions with well-defined properties, such as the normal distribution.

Types of Copulas

A **Gaussian copula** maps the marginal distribution of each variable to the standard normal distribution. Figure 6 illustrates that V_1 and V_2 have unique marginal distributions. The observations of each distribution is mapped to the standard normal distribution on a percentile-to-percentile basis to create a Gaussian copula as follows:

Figure 6: Mapping Gaussian Copula to Standard Normal Distribution



The **Student's *t*-copula** is similar to the Gaussian copula. However, variables are mapped to distributions of U_1 and U_2 that have a bivariate Student's *t*-distribution rather than a normal distribution.

Tail Dependence

There is greater **tail dependence** in a bivariate Student's *t*-distribution than a bivariate normal distribution. This suggests that the Student's *t*-copula is better than a Gaussian copula in describing the correlation structure of assets that historically have extreme outliers in the distribution tails at the same time.

Simulation Methods

Cross Reference to GARP Assigned Reading – Brooks

Monte Carlo Simulation

Monte Carlo simulations are often used to model complex problems or to estimate variables when there are small sample sizes.

There are four basic steps required to conduct a Monte Carlo simulation.

Step 1: Specify the data generating process (DGP)
Step 2: Estimate an unknown variable or parameter
Step 3: Save the estimate from step 2
Step 4: Go back to step 1 and repeat this process *N* times

Reducing Monte Carlo Sampling Error

The sampling variation for a Monte Carlo simulation is quantified as the standard error estimate. The standard error of the true expected value is computed as $s/\sqrt{N}$, where *s* is the standard deviation of the output variables and *N* is the number of scenarios or replications in the simulation. Based on this equation, it intuitively follows that in order to reduce the standard error estimate by a factor of 10, the analyst must increase *N* by a factor of 100. (Because the square root of 100 is 10, if we increase the sample size 100 times it will reduce the standard error estimate by dividing by 10.)

However, increasing the number of generated scenarios can become costly for more complex multi-period simulations. Variance reduction techniques offer an alternative way to reduce the sampling error of a Monte Carlo simulation. The two most commonly used techniques for reducing the standard error estimate are antithetic variates and control variates.

Antithetic Variates

Increasing the number of samples drawn may be too costly and time consuming. As an alternative approach, the **antithetic variate technique** can reduce Monte Carlo sampling error by rerunning the simulation using a *complement* set of the original set of random variables.

If the original set of random draws is denoted u_t for each replication, then the simulation is rerun with the complement set of random numbers denoted $-u_t$.

The use of antithetic variates results in a negative covariance between the original random draws and their complements (i.e., antithetic variates). Thus, the use of antithetic variates causes the error terms to be independent for the two sets, which results in a negative covariance term in the variance equation. This negative relationship means that the Monte Carlo sampling error must always be smaller using this approach.

Control Variates

The **control variate technique** is a widely used method to reduce the sampling error in Monte Carlo simulations. A control variate involves replacing a variable *x* (under simulation) that has unknown properties with a similar variable y that has known properties.

A practical financial example of applying control variates is the use of Monte Carlo simulations in pricing Asian options. The use of a similar derivative, such as a European option, with known statistical properties can be used as a control variate.

Reusing Sets of Random Numbers

Reusing sets of random number draws across Monte Carlo experiments reduces the estimate variability across experiments by using the same set of random numbers for each simulation. Normally, a user would not desire to reuse the same random draws. However, in certain situations this technique is useful. Two examples of reusing sets of random numbers are for testing the power of the Dickey-Fuller test (used to determine whether a time series is covariance stationary) or for different experiments with options using time series data.

Bootstrapping Method

The bootstrapping approach draws random return data from a sample of historical data. Under traditional Monte Carlo simulation, data sets are created by selecting random variables drawn from a pre-determined probability distribution. The bootstrapping method uses actual historical data instead of random data from a probability distribution. In addition, bootstrapping repeatedly draws data from a historical data set and replaces the data so it can be drawn again.

An obvious advantage of the bootstrapping approach is that no assumptions are made regarding the true distribution of the parameter estimate that is being examined.

Two situations that cause the bootstrapping method to be ineffective are *outliers* in the data and *non-independent data*.

If outliers exist in the data, the inferences drawn from parameter estimates may not be accurate depending on how many times the outliers are included in the bootstrapped sample.

If autocorrelation exists in the original sample data, then the original historical data are not independent of one another.

Random Number Generation

A good random number generator has the ability to reproduce a random sequence and analyze characteristics of random numbers. Simulation software programs are able to reproduce the same sequence of iterations by starting sequences with a seed random number. The algorithms used to generate these random sequences are referred to as **pseudo-random number generators**. These number generators are advantageous because risk managers can improve models by reducing the estimate variance or debugging computer codes if the same sequence of random numbers is reproduced when programming the model.

The term *pseudo* implies that these computer-generated numbers are *not truly random*, because they are actually generated from a formula.

The choice of seed value will influence the properties of the random number distribution that is generated. Therefore, a good random number generator uses sequences with long cycles that require numerous iterations before a sequence is repeated.

Disadvantages of Simulation Approaches

Disadvantages of the simulation approach to financial problem solving include:

- High computation costs
- Results are imprecise
- Results are difficult to replicate
- Results are experiment-specific

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Banks

Cross Reference to GARP Assigned Reading – Hull, Chapter 2

Major Risks Faced by Banks

The main risks faced by a bank include credit risk, market risk, and operational risk.

- **Credit risk** refers to the risk that borrowers may default on loans or that counterparties to contracts such as derivatives may default on their obligations.
- **Market risk** refers to the risk of losses from a bank's trading activities, such as declines in the value of securities the bank owns.
- **Operational risk** refers to the possibility of losses arising from external events or failures of a bank's internal controls.

Economic Capital vs. Regulatory Capital

Regulatory capital refers to the amount determined by bank regulators. In terms of bank regulation, equity is referred to as "Tier 1 capital" and subordinated long-term debt is referred to as "Tier 2 capital."

Economic capital refers to the amount of capital that a bank believes is adequate based on its own risk models. Even if economic capital is less than regulatory capital, as is often the case, a bank must maintain its capital at the regulatory minimum or greater.

Deposit Insurance and Moral Hazard

To increase public confidence in the banking system and prevent runs on banks, most countries have established systems of **deposit insurance**. Typically, a depositor's funds are guaranteed up to some maximum amount if a bank fails.

Like other forms of insurance, deposit insurance brings an element of **moral hazard**. Moral hazard is the observed phenomenon that insured parties take greater risks than they would normally take if they were not insured.

Investment Banking Financing Arrangements

In a **private placement**, securities are sold directly to qualified investors with substantial wealth and investment knowledge.

If the securities are sold to the investing public at large, the issuance is referred to as a **public offering**. Investment banks have two methods of assisting with a public offering. With a **firm commitment**, the investment bank agrees to purchase the entire issue at a price that is negotiated between the issuer and bank. An investment bank can also agree to distribute an issue on a **best efforts** basis rather than agreeing to purchase the whole issue. If only part of the issue can be sold, the bank is not obligated to buy the unsold portion.

First-time issues of stock by firms whose shares are not currently publicly traded are called **initial public offerings** (IPOs). An IPO price may also be discovered through a **Dutch auction** process. A Dutch auction begins with a price greater than what any bidder will pay, and this price is reduced until a bidder agrees to pay it.

Potential Conflicts of Interest

If a bank or a bank holding company provides commercial banking, investment banking, and securities services, several conflicts of interest may arise. Information may be acquired in a commercial banking or investment banking transaction that would give the other units an unfair advantage. An investment bank's task of selling newly issued stocks and bonds may conflict with a securities unit's duties to act in the best interests of its clients and recommend trading actions independently.

Where banking firms are permitted to have commercial banking, securities, and investment banking units, the firms must implement **Chinese walls,** which are internal controls to prevent information from being shared among these units.

Banking Book vs. Trading Book

The **banking book** refers to loans made, which are the primary assets of a commercial bank. Normally, the balance sheet value of a loan includes the principal amount to be repaid and accrued interest on the loan. However, for a **nonperforming loan** the value does not include accrued interest.

The **trading book** refers to assets and liabilities related to a bank's trading activities. Unlike other assets and liabilities, trading book items are marked to market daily.

The Originate-to-Distribute Model

In contrast to a bank making loans and keeping them as assets, the **originate-to-distribute model** involves making loans and selling them to other parties. Many mortgage lenders in the United States operate on the originate-to-distribute model.

The benefit of the originate-to-distribute model is that it increases liquidity in the sectors of the lending market where it is used. A drawback of this model is that, in some cases, it has led banks to loosen lending standards.

Insurance Companies and Pension Plans

Cross Reference to GARP Assigned Reading – Hull, Chapter 3

Categories of Insurance Companies

Life Insurance

Life insurance companies usually provide long-term coverage and will make a specified payment to the policyholder's beneficiaries upon the death of the policyholder during the policy term. **Term** (temporary) **life insurance** provides a specified amount of insurance coverage for a fixed period of time. **Whole** (permanent) **life insurance** provides a specified amount of insurance coverage for the life of the policyholder so payment will occur upon death, but there is uncertainty as to the timing.

Property and Casualty (P&C) Insurance

P&C insurance companies usually provide annual and renewable coverage against loss events. **Property insurance** covers property losses such as fire and theft. **Casualty** (liability) **insurance** covers third-party liability for injuries sustained while on a policyholder's premises or caused by the policyholder's use of a vehicle, for example.

Health Insurance

Health insurance companies provide coverage to policyholders for medical services that are not covered under a publicly funded health care system.

Major risks facing insurance companies include the following:

- Insufficient funds to satisfy policyholders' claims.
- Poor return on investments.
- Liquidity risk of investments.
- Credit risk.
- Operational risk.

Mortality Tables

Mortality tables can be used to compute life insurance premiums. The tables include information related to the probability of an individual dying within the next year, the probability of an individual surviving to a specific age, and the remaining life expectancy of an individual of a specific age.

With the information in the mortality tables, we can calculate the breakeven premium payment by equating the present value of the expected payout to the present value of the expected premium payments.

Example: Breakeven premium payments

The relevant interest rate for insurance contracts is 3% per annum (semiannual compounding applies), and all premiums are paid annually at the beginning of the year. A $500,000 term insurance contract is being proposed for a 60-year-old male in average health (the probability of death within one year is 0.011197). Assuming that payouts occur halfway throughout the year, **calculate** the insurance company's breakeven premium for a one-year term.

Answer:

The expected payout for a one-year term is 0.011197 × $500,000 = $5,598.50. Assuming the payout occurs in six months, the breakeven premium is: $5,598.50 / 1.015 = $5,515.76.

P&C Insurance Ratios

Property and casualty insurance companies compute the following ratios:

- The **loss ratio** for a given year is the percentage of payouts versus premiums generated.
- The **expense ratio** for a given year is the percentage of expenses versus premiums generated.
- The **combined ratio** for a given year is equal to the sum of the loss ratio and the expense ratio.
- The **combined ratio after dividends** for a given year is equal to the combined ratio plus the payment of dividends to policyholders as a percentage of premiums (if applicable).
- The **operating ratio** for a given year is the combined ratio (after dividends) less investment income as a percentage of premiums.

Moral Hazard and Adverse Selection

Moral hazard describes the risk to the insurance company that having insurance will lead the policyholder to act more recklessly than if the policyholder did not have insurance. Methods to mitigate against moral hazard include: deductibles, coinsurance provisions, and policy limits.

Adverse selection describes the situation where an insurer is unable to differentiate between a good risk and a bad risk. Methods to mitigate against adverse selection include: (1) greater initial due diligence and (2) ongoing due diligence.

Mortality Risk vs. Longevity Risk

Mortality risk refers to the risk of policyholders dying earlier than expected due to illness or disease, for example. From the perspective of the insurance company, the risk of losses increases due to the earlier-than-expected life insurance payout.

Longevity risk refers to the risk of policyholders living longer than expected due to better healthcare and healthier lifestyle choices, for example. From the perspective of the insurance company, the risk of losses increases due to the longer-than-expected annuity payout period.

There is a natural hedge (or offset) for insurance companies that deal with both life insurance products and annuity products. For example, longevity risk is bad for the annuity business but is good for the life insurance business due to the delayed payout (or no payout if the policyholder has term insurance and dies after the policy expires). Mortality risk is bad for the life insurance business but is good for the annuity business because of the earlier-than-expected termination of payouts.

Capital Requirements for Insurance Companies

Under an asset-liability management approach, the life insurance company attempts to equate asset duration with liability duration. There is risk associated with both sides of the balance sheet. Equity capital represents contributed capital plus retained earnings and serves as a protection barrier if payouts are larger than loss reserves.

For P&C insurance companies, assets typically comprise of highly liquid bonds with shorter maturities than those used by life insurance companies. On the liability side, there are unearned premiums (non-existent with life insurance companies) that represent prepaid insurance contracts whereby amounts are received but the coverage applies to future time periods. Finally, there is substantially more equity capital than for a life insurance company because of the highly unpredictable nature of claims for P&C insurance contracts.

Guaranty System for Insurance Companies

For insurance companies, every insurer must be a member of the guaranty association in the state(s) in which it operates. If an insurance company becomes insolvent in a state, each of the other insurance companies must contribute an amount to the state guaranty fund based on the amount of premium income it earns in that state.

In contrast, the guaranty system for banks is a permanent fund to protect depositors and consists of amounts remitted by banks to the Federal Deposit Insurance Corporation (FDIC). No such permanent fund generally exists for insurance companies.

Pension Funds

Defined benefit plans explicitly state the amount of the pension that the employee will receive upon retirement. It is usually calculated as a fixed percentage times the number of years of employment times the annual salary for a specific period of time. There is significant risk borne by the employer because it is obligated to fund the benefit to the employee.

Defined contribution plans involve both employer and employee contributions being invested in one or more investment options selected by the employee. There is virtually no risk borne by the employer because it is obligated simply to make a set contribution and no more. The risk of underperformance of the plan's investments is borne solely by the employee.

Mutual Funds and Hedge Funds

Cross Reference to GARP Assigned Reading – Hull, Chapter 4

Types of Mutual Funds

Mutual funds are pooled investment vehicles that offer instant diversification for their investors. There are three primary types of commingled pools of investments that are available to investors: open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs).

Open-End Mutual Funds

Open-end mutual funds, which are often simply called *mutual funds*, are the most common pooled investment vehicle. These investors begin their investment by purchasing a set dollar amount of an open-end mutual fund and then they receive a proportional ownership interest in the mutual fund. This means that the number of shares goes up as new investors arrive and goes down as investors withdraw assets. When investors decide that they want to exit their investment in an open-end mutual fund, they can redeem their shares directly from the fund company. Open-end funds trade at the fund's **net asset value** (NAV).

Closed-End Mutual Funds

Closed-end funds transact throughout the trading day, but shares cannot be redeemed at the fund company and their price may differ substantially from their NAV—the shares must be bought or sold by other investors.

Exchange-Traded Funds

Exchange-traded funds (ETFs) enable instant diversification like an open-end fund, but they are exchange-traded, which means they trade throughout the day on the open market just like a closed-end fund does. Because they trade throughout the day, investors can utilize stop orders, limit orders, and even short selling in some cases.

Net Asset Value

In order to calculate the net asset value (NAV), the fund needs to know the current value of all investment holdings (including cash positions), any liabilities like management fees payable, and the total number of shares outstanding. Calculation of the NAV is shown as follows:

$$\text{NAV} = \frac{\text{fund assets} - \text{fund liabilities}}{\text{total shares outstanding}}$$

Hedge Funds vs. Mutual Funds

Both mutual funds and hedge funds offer professional management, instant diversification, and the ability to commingle funds with other investors. However, there are some notable differences between mutual funds and hedge funds. Hedge funds are only marketed to wealthy and sophisticated investors. Because of this, hedge funds escape certain regulatory oversight, which enables them to avoid allowing investors to redeem shares at any time they want, calculating the NAV daily, and disclosing investment policies and strategies. They are also permitted to use leverage and short selling, which are commonly not permitted for mutual funds. In addition, hedge funds use lock-up periods to prevent investor withdrawals at the wrong time for the fund.

Hedge Fund Expected Returns and Fee Structures

Incentive fees are engineered to give hedge fund managers significant payouts based on their performance. The typical hedge fund fee structure is known as "**2 plus 20%**," which means that they charge a flat 2% of all assets that they manage plus an additional 20% of all profits above a specified benchmark.

Hedge funds do soften the incentive fee structure with a few safeguards for investors. The first safeguard is the **hurdle rate**, which is the benchmark that must be beaten before incentive fees can be charged.

The second safeguard is a **high-water mark clause**, which essentially states that previous losses must first be recouped and hurdle rates surpassed before incentive fees once again apply.

The third safeguard for investors is a **clawback clause**, which enables investors to retain a portion of previously paid incentive fees in an escrow account that is used to offset investment losses should they occur.

Hedge Fund Strategies

Long/Short Equity

Long/short equity hedge funds endeavor to find mispriced securities. They will buy (go long) a stock that they believe to be undervalued, and they will short sell (go short) a stock that they believe to be overvalued.

Dedicated Short

Dedicated short hedge funds are focused exclusively on finding a company that they think is overvalued and then short selling the stock.

Distressed Securities

Distressed securities hedge funds are searching for distressed bonds with the potential to turn things around. Many of these distressed companies are in or close to being in bankruptcy proceedings.

Merger Arbitrage

Merger arbitrage hedge funds try to find arbitrage opportunities after mergers are announced. These are primarily positive deals where the managers are planning on the deal going through. There are two different types of mergers: *cash deals* and *stock deals*.

Convertible Arbitrage

A convertible arbitrage hedge fund develops a sophisticated model to value convertible bonds that factors everything from default risk to interest rate risk. Sometimes they offset investment risk by shorting the issuer's stock or by using more sophisticated assets like credit default swaps and interest rate swaps.

Fixed Income Arbitrage

Fixed income arbitrage hedge funds attempt to exploit perceived mispricings in the realm of fixed-income securities.

Emerging Market

Emerging market hedge funds focus on investments in developing countries. If managers decide to invest using emerging market debt, then they need to consider default risk because several countries have defaulted multiple times.

Global Macro

In this strategy, hedge fund managers attempt to profit from a global macroeconomic trend that they feel is not in equilibrium (priced correctly and rationally).

Managed Futures

Managed futures hedge funds attempt to predict future movements in commodity prices based on either technical analysis or fundamental analysis.

Hedge Fund Performance and Measurement Bias

Participation in hedge fund indices is voluntary. If the fund had good performance, then they will report their results to the index vendor. If they did not have good results, then they simply do not report their results to the index. This is known as the **measurement bias** of hedge fund index reporting. When returns are reported by a hedge fund, the database is then backfilled with the fund's previous returns. This is known as **backfill bias** and it creates an issue with reliability for hedge fund benchmarks. It is very common for a hedge fund to have a string of several good years and then have a meltdown. Over the last 10 years, reported hedge fund performance suggests that they have only beaten the S&P 500 Index in two of those years.

Introduction (Options, Futures, and Other Derivatives)

Cross Reference to GARP Assigned Reading – Hull, Chapter 1

Derivative Markets

An **open outcry system** and **electronic trading system** are different forms of trading securities (matching buyers with sellers). The open outcry system (e.g., CBOT) is the more traditional system, which involves traders actually indicating their trades through hand signals and shouting. Electronic trading does not involve an actual "physical" exchange location, but rather involves matching buyers and sellers electronically via computers (e.g., NASDAQ).

An **over-the-counter (OTC) market** differs from a traditional exchange. It is a customized trading market which utilizes telephone and computers to make trades. Since terms are not specified by an "exchange," participants have more flexibility to negotiate the most mutually agreeable or attractive trade.

Hedging Strategies

Hedgers use forward contracts and options to reduce or eliminate financial exposure. An investor or business with a long exposure to an asset can hedge exposure by either entering into a short futures contract or by buying a put option. An investor or business with a short exposure to an asset can hedge exposure by either entering into a long futures contract or by buying a call option.

Speculative Strategies

Speculators have a different motivation for using derivatives than hedgers. They use derivatives to make bets on the market, while hedgers try to eliminate exposures.

The motivation for using futures in speculation is that the limited amount of initial investment creates significant **leverage**. The amount of investment required for futures is the amount of the initial margin required by the exchange. This is generally a small percentage of the notional value of the underlying, and Treasury securities can typically be posted as margin. Futures contracts can result in large gains or large losses, and contract payoffs are symmetrical.

Arbitrage Opportunities

Arbitrageurs are also frequent users of derivatives. Arbitrageurs seek to earn a risk-free profit in excess of the risk-free rate through the discovery and manipulation of mispriced securities. They earn a riskless profit by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities typically do not last long as supply and demand forces will adjust prices to quickly eliminate the arbitrage situation.

Risk From Derivatives

Derivatives are versatile and can be used for hedging, arbitrage, and pure speculation. If, however, the "bet" one makes starts going in the wrong direction, the results can be catastrophic. Risk limits should be set, and adherence to risk limits should be monitored.

Futures Markets and Central Counterparties

Cross Reference to GARP Assigned Reading – Hull, Chapter 2

Features of Futures Contracts

The purchaser of a futures contract is said to have gone long or taken a **long position**, while the seller of a futures contract is said to have gone short or taken a **short position**. For each contract traded, there is a buyer and a seller. The long has contracted to buy the asset at the contract price at contract expiration, and the short has an obligation to sell at that price. Futures contracts are used by **speculators** to gain exposure to changes in the price of the asset underlying a futures contract. A **hedger**, in contrast, will use futures contracts to reduce exposure to price changes in the asset (i.e., hedge their asset price risk). An example is a wheat farmer who sells wheat futures to reduce the uncertainty about the price of wheat at harvest time.

Margin Requirements

Margin is cash or highly liquid collateral placed in an account to ensure that any trading losses will be met. Marking to market is the daily procedure of adjusting the margin account balance for daily movements in the futures price. The amount required to open a futures position is called the **initial margin**. The **maintenance margin** is the minimum margin account balance required to retain the futures position. When the margin account balance falls below the maintenance margin, the investor gets a margin call, and he must bring the margin account back to the initial margin amount. The amount necessary to do this is called the **variation margin**.

Clearinghouses in Futures Transactions

Each exchange has a **clearinghouse**. The clearinghouse guarantees that traders in the futures market will honor their obligations. The clearinghouse does this by splitting each trade once it is made and acting as the opposite side of each position. The clearinghouse acts as the buyer to every seller and the seller to every buyer.

Central Counterparties in Over-the-Counter Transactions

The **over-the-counter** (OTC) **market** includes the trading in all securities not listed on one of the registered exchanges. This market is subject to a good deal of credit risk since the party on the other side of an OTC contract could default on its payments. In an effort to reduce credit risk, the OTC market has adopted some practices from futures exchanges.

Clearinghouses for standard OTC transactions are referred to as **central counterparties** (CCPs). These CCPs operate in a similar fashion to clearinghouses on futures exchanges. After two parties (X and Y) negotiate an OTC agreement, it is submitted to the CCP for acceptance. Assuming the transaction is accepted, the CCP will become the counterparty to both parties X and Y. Thus, it assumes the credit risk of both parties in an OTC transaction. This risk is managed by requiring the parties to post initial margin and any variation margins on a daily basis.

Arguments for the use of CCPs in OTC markets include (1) collateralized positions with reserves and margins, (2) reduction of financial system credit risk, and (3) increased transparency of OTC trades.

Futures Market Quotes

Futures quotes can be found from exchanges as well as various online websites.

Figure 1: Gold Futures Quotes

Month	*Last Trade*	*Change*	*Prior Settlement*	*Open*	*High*	*Low*	*Volume*
Dec-17	1280.6	+2.9	1277.7	1278.0	1281.8	1274.9	245,277
Feb-18	1285.1	+3.0	1282.1	1282.8	1286.1	1279.4	13,715
Apr-18	1289.1	+2.8	1286.3	1287.4	1289.7	1284.0	1,983
Jun-18	1292.4	+1.8	1290.6	1290.4	1294.1	1288.2	2,075
Dec-18	1306.5	+3.1	1303.4	1304.1	1306.5	1304.1	526

The current trading price of a given futures contract is shown in the second column of Figure 1. The change between the previous day's settlement price and the last trade is reflected in the third column. The **settlement price** is typically computed as the price right before the end of the previous trading day. This price is used for computing daily gains and losses as well as determining margin requirements.

Figure 1 also shows the pattern of futures prices as a function of contract maturity. Depending on the direction of futures settlement prices, the market may be normal or inverted. In this case, gold futures contract prices are moving higher with increasing time horizons. Increasing settlement prices over time would indicate a **normal futures market**. Conversely, decreasing settlement prices over time would indicate an **inverted futures market**.

The Delivery Process

There are four ways to terminate a futures contract:

1. A short can terminate the contract by delivering the goods. When the long accepts this delivery, he pays the contract price to the short. This is called **delivery**.

2. In a **cash-settlement contract**, delivery is not an option. The futures account is marked to market based on the settlement price on the last day of trading.

3. You may make a **reverse**, or **offsetting**, trade in the futures market. With futures, the other side of your position is held by the clearinghouse—if you make an exact opposite trade (maturity, quantity, and good) to your current position, the clearinghouse will net your positions out, leaving you with a zero balance. This is how most futures positions are settled.

4. A position may also be settled through an **exchange for physicals**. Here you find a trader with an opposite position to your own and deliver the goods and settle up between yourselves, off the floor of the exchange.

Types of Orders

Market orders are orders to buy or sell at the best price available. A **discretionary order** is a market order where the broker has the option to delay transaction in search of a better price.

Limit orders are orders to buy or sell away from the current market price.

Stop-loss orders are used to prevent losses or to protect profits.

Variations on these order types also exist. **Stop-limit orders** are a combination of a stop and limit order. **Market-if-touched orders**, or MIT orders, are orders that would become market orders once a specified price is reached in the marketplace.

For those orders that remain outstanding until the designated price range is reached, the trader making the order needs to indicate the time period for the order (**time-of-day order**). **Good-till-canceled** (GTC) **orders** (a.k.a. **open orders**) are orders that remain open until they either transact or are canceled. A popular method of submitting a limit order is to have it automatically canceled at the end of the trading day in which it was submitted. **Fill-or-kill orders** must be executed immediately or the trade will not take place.

Forwards and Futures Contracts

Futures contracts are similar to forward contracts in that both allow for a transaction to take place at a future date at a price agreed upon today. Forward and futures contracts differ in the following ways:

- Forwards are private transactions between two parties; futures are traded on organized exchanges.
- Forwards are customizable to satisfy both parties; futures are standardized for underlying asset, size, and maturity.
- Forwards are bilateral agreements with counterparty risk; futures trade with clearinghouses and have no counterparty risk.
- Forwards are usually not regulated; futures are regulated by securities regulators.
- Forwards transaction costs are embedded in the contract prices set by dealers; futures transaction costs are commissions paid to brokers and are relatively low.
- It is difficult to offset or cancel a forward contract because trading and liquidity are low; it is easy to offset or cancel futures because the market is active and provides good liquidity.
- Forwards settle at expiration; futures are marked to market and settle daily.

Hedging Strategies Using Futures

Cross Reference to GARP Assigned Reading – Hull, Chapter 3

Hedging With Futures

A **short hedge** occurs when the hedger shorts (sells) a futures contract to hedge against a price decrease in the existing long position. When the price of the hedged asset decreases, the short futures position realizes a positive return, offsetting the decline in asset value. Therefore, a short hedge is appropriate when you have a long position and expect prices to decline.

A **long hedge** occurs when the hedger buys a futures contract to hedge against an increase in the value of the asset that underlies a short position. In this case, an increase in the value of the shorted asset will result in a loss to the short seller. The objective of the long hedge is to offset the loss in the short position with a gain from the long futures position. A long hedge is therefore appropriate when you have a short position and expect prices to rise.

Advantages and Disadvantages of Hedging

It is easy to see that the benefit from hedging leads to less uncertainty regarding future profitability. However, there are some arguments against hedging. The main issue is that hedging can lead to less profitability if the asset being hedged ends up increasing in value. The increase in value will be offset by a corresponding loss in the futures contract used for the hedge.

Another argument against hedging is the questionable benefit that accrues to shareholders. Clearly, hedging reduces risk for a company and its shareholders, but there is reason to believe that shareholders can more easily hedge risk on their own. A third argument deals with the nature of the hedging company's industry. For example, assume that prices in an industry frequently adjust for changes in input prices and exchange rates. If competitors do not hedge, then there is an incentive to keep the status quo. In this way, the company ensures that profitability will remain more stable than if it were to hedge frequent changes.

Basis

Basis is defined as the cash price less the futures price. In equation form, basis is represented as follows:

$$\text{basis} = S_t - F_0$$

where:
S_t = cash (or spot) price of the underlying asset at time *t*
F_0 = current price of the futures contract

Basis Risk

When the spot price increases faster than the futures price over the hedging horizon, basis increases and a strengthening of the basis is said to occur. When the futures price increases faster than the spot price and the basis decreases, a weakening of the basis occurs. When hedging, a change in basis is unavoidable. The change in basis over the hedge horizon is termed *basis risk*, and it can work either for or against a hedger.

Three sources of basis risk are:

1. *Interruption in the convergence of the futures and spot prices.* Normally, spot prices and futures prices will converge as the time to maturity decreases, and basis reduces to zero at maturity. However, if the position is unwound prior to maturity, the return to the futures position could be different from the return to the cash position.

2. *Changes in the cost of carry.* Significant basis risk can arise due to changes in the components of the cost of carry.

3. *Imperfect matching between the cash asset and the hedge asset.* Sometimes it may be more efficient to **cross hedge** or hedge a cash position with a hedge asset that is closely related but different from the cash asset.

The Optimal Hedge Ratio

A hedge ratio is the ratio of the size of the futures position relative to the spot position. The *optimal hedge ratio*, which minimizes the variance of the combined hedge position, is defined as follows:

$$\mathrm{HR} = \rho_{S,F}\frac{\sigma_S}{\sigma_F}$$

Example: Minimum variance hedge ratio

Suppose a currency trader computed the correlation between the spot and futures to be 0.925, the annual standard deviation of the spot price to be \$0.10, and the annual standard deviation of the futures price to be \$0.125. **Compute** the hedge ratio.

Answer:

$$\mathrm{HR} = 0.925 \times \frac{0.100}{0.125} = 0.74$$

The ratio of the size of the futures to the spot should be 0.74.

Hedging With Stock Index Futures

A common hedging application is the hedging of equity portfolios using futures contracts on stock indices (index futures). In this application, it is important to remember that the hedged portfolio's beta serves as a hedge ratio when determining the correct number of contracts to purchase or sell. The number of futures contracts required to completely hedge an equity position is determined with the following formula:

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{value of futures contract}}\right)$$
$$= \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

Tailing the Hedge

A hedger may actually over-hedge the underlying exposure if daily settlement is not properly accounted for. To correct for the possibility of over-hedging, a hedger can implement a **tailing the hedge** strategy. The extra step needed to carry out this strategy is to multiply the hedge ratio by the daily spot price to futures price ratio. In practice, it is not efficient to adjust the hedge for every daily change in the spot-to-futures ratio.

Adjusting the Portfolio Beta

Hedging an existing equity portfolio with index futures is an attempt to reduce the *systematic risk* of the portfolio. If the beta of the capital asset pricing model is used as the systematic risk measure, then hedging boils down to a reduction of the portfolio beta. Let β be our portfolio beta, β^* be our target beta after we implement the strategy with index futures, P be our portfolio value, and A be the value of the underlying asset (i.e., the stock index futures contract). To compute the appropriate number of futures, we use the following equation:

$$\text{number of contracts} = (\beta^* - \beta)\frac{P}{A}$$

Rolling a Hedge Forward

When the hedging horizon is long relative to the maturity of the futures used in the hedging strategy, hedges have to be rolled forward as the futures contracts in the hedge come to maturity or expiration. Typically, as a maturity date approaches, the hedger must close out the existing position and replace it with another contract with a later maturity. This is called **rolling the hedge forward**.

When rolling a hedge forward, hedgers are not only exposed to the basis risk of the original hedge, they are also exposed to the basis risk of a new position each time the hedge is rolled forward. This is referred to as rollover basis risk, or simply **rollover risk**.

Interest Rates

Cross Reference to GARP Assigned Reading – Hull, Chapter 4

Types of Rates

Three interest rates play a key role in interest rate derivatives:

- **Treasury rates.** Treasury rates are the rates that correspond to government borrowing in its own currency. They are considered risk-free rates.
- **LIBOR.** The London Interbank Offered Rate (LIBOR) is the rate at which large international banks fund their activities. Some credit risk exists with LIBOR.
- **Repo rates.** The "repo" or repurchase agreement rate is the implied rate on a repurchase agreement. In a repo agreement, one party agrees to sell a security to another with the understanding that the selling party will buy it back later at a specified higher price.

Compounding

If we have an initial investment of A that earns an annual rate R, compounded m times a year for n years, then it has a future value of:

$$FV_1 = A\left(1+\frac{R}{m}\right)^{m\times n}$$

If our same investment is continuously compounded over that period, it has a future value of:

$$FV_2 = Ae^{R \times n}$$

For any rate, R, FV_2 will always be greater than FV_1. The difference will decrease as m increases. In fact, as m becomes infinitely large, the difference goes to zero.

Spot (Zero) Rates and Bond Pricing

Spot rates are the rates that correspond to zero-coupon bond yields. They are the appropriate discount rates for a single cash flow at a particular future time or maturity. Spot rates are also often called zero rates. Most interest rates that are observed in the market, such as coupon bond yields, are not spot rates.

A coupon bond makes a series of cash flows. Each cash flow considered in isolation is equivalent to a zero-coupon bond. Using this interpretation, a coupon bond is a series of zero-coupon bonds, and its value, assuming continuous compounding and semiannual coupons, is:

$$B = \left(\frac{c}{2} \times \sum_{j=1}^{N} e^{-\frac{z_j}{2} \times j} \right) + \left(FV \times e^{-\frac{z_N}{2} \times N} \right)$$

where:
c = the annual coupon
N = the number of semiannual payment periods
z_j = the bond equivalent spot rate that corresponds to j periods (j/2 years) on a continuously compounded basis
FV = the face value of the bond

Example: Calculating bond price

Compute the price of a \$100 face value, 2-year, 4% semiannual coupon bond using the annualized spot rates in Figure 2.

Figure 2: Spot Rates

Maturity (Years)	*Spot Rate (%)*
0.5	2.5
1.0	2.6
1.5	2.7
2.0	2.9

Answer:

$$B = \left(\$2 \times e^{-\frac{0.025}{2}\times 1}\right) + \left(\$2 \times e^{-\frac{0.026}{2}\times 2}\right) + \left(\$2 \times e^{-\frac{0.027}{2}\times 3}\right) + \left(\$102 \times e^{-\frac{0.029}{2}\times 4}\right) = \$102.10$$

Bond Yield

The yield of a bond is the single discount rate that equates the present value of a bond to its market price. You can use a financial calculator to compute bond yield, as in the following example.

Example: Calculating bond yield

Compute the yield for the bond in the previous example.

Answer:

PMT = 2; N = 4; PV = –102.10; FV = 100; CPT → I/Y = 1.456;
Y = 1.456% × 2 ≈ 2.91%

Bootstrapping Spot Rates

The theoretical spot curve is derived by interpreting each Treasury bond (T-bond) as a package of zero-coupon bonds. Using the prices for each bond, the spot curve is computed using the bootstrapping methodology.

For example, suppose there is a T-bond maturing on a coupon date in exactly six months. Further assume that the bond is priced at 102.2969% of par and has a semiannual coupon of 6.125% How is the corresponding spot rate computed? In this case, this is truly a zero-coupon bond, since there is only one cash flow, which occurs in six months. Simply solve for z_1 in the bond valuation equation, given the price, as follows:

$$\$102.2969 = \left(\$100 + \frac{\$6.125}{2}\right) \times e^{-\frac{z_1}{2}}$$

Solving this for z_1:

$$z_1 = -2 \times \ln\left[\frac{\$102.2969}{\left(\$100 + \frac{\$6.125}{2}\right)}\right] = 1.491\%$$

Forward Rates

Forward rates are interest rates implied by the spot curve for a specified future period.

1. Invest for two years at 2.915%.
2. Invest for a year at 2.136% and then roll over that investment for another year at the forward rate.

It does not matter which investment is chosen if they both offer the same return at the end of two years. This is the same as stating that both strategies give the same future value at the end of two years. Equating the two future values:

$$e^{\frac{0.02915}{2} \times 4} = e^{\frac{0.02136}{2} \times 2} \times e^{\frac{R_{Forward}}{2} \times 2}$$

where:
$R_{Forward}$ = the 1-year forward rate one year from now

We can simplify this calculation by using the following equation:

$$R_{\text{Forward}} = \frac{R_2T_2 - R_1T_1}{T_2 - T_1} = R_2 + (R_2 - R_1) \times \left(\frac{T_1}{T_2 - T_1}\right)$$

where:
R_i = the spot rate corresponding with T_i periods
R_{Forward} = the forward rate between T_1 and T_2

For example, if the 1-year rate is 2.136% and the 2-year rate is 2.915%, the 1-year forward rate one year from now is:

$$R_{\text{Forward}} = 0.02915 + (0.02915 - 0.02136) \times \left(\frac{1}{2-1}\right) = 0.03694 = 3.694\%$$

Forward Rate Agreements

A **forward rate agreement** (FRA) is a forward contract obligating two parties to agree that a certain interest rate will apply to a principal amount during a specified future time. Obviously, forward rates play a crucial role in the valuation of FRAs. The T_2 cash flow of an FRA that promises the receipt or payment of R_K is:

$$\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$$
$$\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$$

where:
L = principal
R_K = annualized rate on L, expressed with compounding period $T_1 - T_2$
R = annualized actual rate, expressed with compounding period $T_1 - T_2$
T_i = time i, expressed in years

Calculating Bond Price With Duration and Convexity

Example: Estimating price changes with the duration/convexity approach

Estimate the effect of a 100 basis point increase and decrease on a 10-year, 5%, option-free bond currently trading at par, using the duration/convexity approach. The bond has a duration of 7 and a convexity of 90.

Answer:

Using the duration/convexity approach:

percentage bond price change ≈ duration effect + convexity effect

$$\Delta B_{+\Delta y} \approx [-7 \times 0.01] + [(1/2) \times 90 \times (0.01^2)]$$

$$\approx -0.07 + 0.0045 = -0.0655 = -6.55\%$$

$$\Delta B_{-\Delta y} \approx [-7 \times -0.01] + [(1/2) \times 90 \times (-0.01^2)]$$

$$\approx 0.07 + 0.0045 = 0.0745 = 7.45\%$$

Theories of the Term Structure

The **expectations theory** suggests that forward rates correspond to expected future spot rates. That is, forward rates are good predictors of expected future spot rates. In reality, the expectations theory fails to explain all future spot rate expectations. The **market segmentation theory** states that the bond market is segmented into different maturity sectors and that supply and demand for bonds in each maturity range dictate rates in that maturity range. The **liquidity preference theory** suggests that most depositors prefer short-term liquid deposits. In order to coax them to lend longer term, the intermediary will raise longer-term rates by adding a liquidity premium.

Determination of Forward and Futures Prices

Cross Reference to GARP Assigned Reading – Hull, Chapter 5

Investment and Consumption Assets

An **investment asset** is an asset that is held for the purpose of investing. This type of asset is held by many different investors for the sake of investment. A **consumption asset** is an asset that is held for the purpose of consumption.

Short-Selling and Short Squeeze

Short sales are orders to sell securities that the seller does not own. Short selling is also known as "shorting" and is possible with some investment assets. For a short sale, the short seller (1) simultaneously borrows and sells securities through a broker, (2) must return the securities at the request of the lender or when the short sale is closed out, and (3) must keep a portion of the proceeds of the short sale on deposit with the broker.

The short seller may be forced to close his position if the broker runs out of securities to borrow. This is known as a **short squeeze,** and the seller will need to close his short position immediately.

Forward Pricing

The forward price may be written as:

$$F_0 = S_0 e^{rT}$$

The right-hand side of the equation is the cost of borrowing funds to buy the underlying asset and carrying it forward to time *T*. This cost must equal the forward price. If $F_0 > S_0e^{rT}$, then arbitrageurs will profit by selling the forward and buying the asset with borrowed funds. If $F_0 < S_0e^{rT}$, arbitrageurs will profit by selling the asset, lending out the proceeds, and buying the forward. Hence, the equality in the forward price equation must hold. Note that this model assumes perfect markets.

Example: Computing a forward price with no interim cash flows

Suppose we have an asset currently worth $1,000. The current continuously compounded rate is 4% for all maturities. **Compute** the price of a 6-month forward contract on this asset.

Answer:

$$F_0 = \$1{,}000e^{0.04(0.5)} = \$1{,}020.20$$

If the underlying pays a known amount of cash over the life of the forward contract, a simple adjustment is made to the previous equation. Since the owner of the forward contract does not receive any of the cash flows from the underlying asset between contract origination and delivery, the present value of these cash flows must be deducted from the spot price when calculating the forward price. This is most easily seen when the underlying asset makes a periodic payment. With this in mind, we let *I* represent the *present value* of the cash flows over *T* years.

$$F_0 = (S_0 - I)\, e^{rT}$$

When the underlying asset for a forward contract pays a dividend, we assume that the dividend is paid continuously. Letting *q* represent the continuously compounded dividend yield paid by the underlying asset expressed on a per annum basis, the forward price becomes:

$$F_0 = S_0 e^{(r-q)T}$$

Currency Futures

Interest rate parity (IRP) states that the forward exchange rate, *F* [measured in domestic currency (DC) per unit of foreign currency (FC)], must be related to the spot exchange rate, *S*, and to the interest rate differential between the domestic and the foreign country, $r_{DC} - r_{FC}$.

$$F_0 = S_0 e^{(r_{DC} - r_{FC})T}$$

Note that this is equivalent to the previous equation with r_{FC} replacing *q*. Just as the continuous dividend yield *q* was used to adjust the cost of carry, we use the continuous yield on a foreign currency deposit here.

Forward Prices vs. Futures Prices

The most significant difference between forward contracts and futures contracts is the daily marking to market requirement on futures contracts. When interest rates are known over the life of a contract, T, forward and futures prices can be shown to be the same. Various relationships can be derived, depending on the assumptions made between the value of the underlying and the level of change in interest rates. In general, when T is small, the price differences are usually very small and can be ignored. Empirical research comparisons of forwards and futures prices are mixed. Some studies conclude a significant difference and others do not. The important concept to understand here is that assuming the two are the same is an approximation, and under certain circumstances the approximation can be inaccurate.

Delivery Options in the Futures Market

Some futures contracts grant **delivery options** to the short—options on what, where, and when to deliver. Some Treasury bond contracts give the short a choice of several bonds that are acceptable to deliver and options as to when to deliver during the expiration month. Physical assets, such as gold or corn, may offer a choice of delivery locations to the short. These options can be of significant value to the holder of the short position in a futures contract.

Futures and Expected Future Spot Prices

The cost of carry model is a widely used method for estimating the appropriate price of a futures contract, but other theories exist for explaining the futures price. One intuitively appealing model expresses the futures price as a function of the expected spot price (S_T).

$$F_0 = E(S_T)$$

For obvious reasons, this is called the **expectations model** and states that the current futures price for delivery at time T is equal to the expected spot price at time T.

Contango and Backwardation

Backwardation refers to a situation where the futures price is below the spot price. For this to occur, there must be a significant benefit to holding the asset. Backwardation might occur if there are benefits to holding the asset that offset the opportunity cost of holding the asset (the risk-free rate) and additional net holding costs.

Contango refers to a situation where the futures price is above the spot price. If there are no benefits to holding the asset (e.g., dividends, coupons, or convenience yield), contango will occur because the futures price will be greater than the spot price.

Interest Rate Futures

Cross Reference to GARP Assigned Reading – Hull, Chapter 6

Day Count Conventions

Day count conventions play a role when computing the interest that accrues on a fixed income security. When a bond is purchased, the buyer must pay any **accrued interest** earned through the settlement date.

$$\text{accrued interest} = \text{coupon} \times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$$

In the United States, there are three commonly used day count conventions.

1. U.S. Treasury bonds use **actual/actual.**
2. U.S. corporate and municipal bonds use **30/360**.
3. U.S. money market instruments (Treasury bills) use **actual/360**.

Quotations for T-Bonds

The cash price (a.k.a. **invoice price** or **dirty price**) is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond (a.k.a. **quoted price** or **clean price**) plus the accrued interest. Conversely, the clean price is the cash price less accrued interest:

$$\text{quoted price} = \text{cash price} - \text{accrued interest}$$

Quotations for T-Bills

T-bills and other money-market instruments use a discount rate basis and an actual/360 day count. A T-bill with a $100 face value with *n* days to maturity and a cash price of *Y* is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

This is referred to as the discount rate in annual terms.

Treasury Bond Futures

The conversion factor defines the price received by the short position of the contract (i.e., the short position is delivering the contract to the long). Specifically, the cash received by the short position is computed as follows:

cash received = (QFP × CF) + AI

where:
QFP = quoted futures price (most recent settlement price)
CF = conversion factor for the bond delivered
AI = accrued interest since the last coupon date on the bond delivered

Conversion factors are supplied by the CBOT on a daily basis. Conversion factors are calculated as: (discounted price of a bond – accrued interest) / face value.

Cheapest-to-Deliver Bond

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver.

The CTD bond minimizes the following: quoted bond price – (QFP × CF). This expression calculates the cost of delivering the bond.

Eurodollar Futures

The 3-month **eurodollar futures** contract trades on the Chicago Mercantile Exchange (CME) and is the most popular interest rate futures in the United States. This contract settles in cash and the minimum price change is one "tick," which is a price change of one basis point, or $25 per $1 million contract. Eurodollar futures are based on a eurodollar deposit (a eurodollar is a U.S. dollar deposited outside the United States) with a face amount of $1 million. The interest rate underlying this contract is essentially the 3-month (90-day) forward LIBOR.

Convexity Adjustment

The daily marking to market aspect of the futures contract can result in differences between actual forward rates and those implied by futures contracts. This difference is reduced by using the convexity adjustment. In general, long-dated eurodollar futures contracts result in implied forward rates larger than actual forward rates. The two are related as follows:

actual forward rate =

forward rate implied by futures – $(\frac{1}{2} \times \sigma^2 \times T_1 \times T_2)$

where:
T_1 = the maturity on the futures contract
T_2 = the time to the maturity of the rate underlying the contract (T_1 + 90 days)
σ = the annual standard deviation of the change in the rate underlying the futures contract, or 90-day LIBOR

Notice that as T_1 increases, the convexity adjustment will need to increase. So as the maturity of the futures contract increases, the necessary convexity adjustment increases. Also, note that the σ and the T_2 are largely dictated by the specifications of the futures contract.

Duration-Based Hedging

The objective of a **duration-based hedge** is to create a combined position that does not change in value when yields change by a small amount. In other words, a position that has a duration of zero needs to be produced. The combined position consists of our portfolio with a hedge horizon value of P and a futures position with a contract value of F. Denote the duration of the portfolio at the hedging horizon as D_P and the corresponding duration of the futures contract as D_F. Using this notation, the duration-based hedge ratio can be expressed as follows:

$$N = -\frac{P \times D_P}{F \times D_F}$$

where:
N = number of contracts to hedge

The minus sign suggests that the futures position is the opposite of the original position.

Limitations of Duration

The price/yield relationship of a bond is convex, meaning it is nonlinear in shape. Duration measures are linear approximations of this relationship. Therefore, as the change in yield increases, the duration measures become progressively less accurate. Moreover, duration implies that all yields are perfectly correlated. Both of these assumptions place limitations on the use of duration as a single risk measurement tool. When changes in interest rates are both large and nonparallel (i.e., not perfectly correlated), duration-based hedge strategies will perform poorly.

Swaps

Cross Reference to GARP Assigned Reading – Hull, Chapter 7

Mechanics of Interest Rate Swaps

The most common interest rate swap is the **plain vanilla interest rate swap.** In this swap arrangement, Company X agrees to pay Company Y a periodic fixed rate on a notional principal over the tenor of the swap. In return, Company Y agrees to pay Company X a periodic floating rate on the same notional principal. Both payments are in the same currency. Therefore, only the net payment is exchanged. Most interest rate swaps use the London Interbank Offered Rate (LIBOR) as the reference rate for the floating leg of the swap. Finally, since the payments are based in the same currency, there is no need for the exchange of principal at the inception of the swap. This is why it is called notional principal.

For example, companies X and Y enter into a 2-year plain vanilla interest rate swap. The swap cash flows are exchanged semiannually, and the reference rate is 6-month LIBOR. The LIBOR rates are shown in Figure 3. The fixed rate of the swap is 3.784%, and the notional principal is $100 million. We will compute the cash flows for Company X, the fixed payer of this swap.

Figure 3: 6-Month LIBOR

Beginning of Period	*LIBOR*
1	3.00%
2	3.50%
3	4.00%
4	4.50%
5	5.00%

The gross cash flows for the end of the first period for both parties are calculated in the following manner:

$$\text{floating} = \$100 \text{ million} \times 0.03 \times 0.5 = \$1.5 \text{ million}$$

$$\text{fixed} = \$100 \text{ million} \times 0.03784 \times 0.5 = \$1.892 \text{ million}$$

The net payment for Company X is an outflow of $0.392 million.

Let's continue with companies X and Y. Suppose that X has a 2-year floating-rate liability, and Y has a 2-year fixed-rate liability. After they enter into the swap, interest rate risk exposure from their liabilities has completely changed for each party. X has transformed the floating-rate liability into a fixed-rate liability, and Y has transformed the fixed-rate liability to a floating-rate liability. Note that X pays fixed and receives floating, so X's liability becomes fixed.

Financial Intermediaries

Financial intermediaries, such as banks, will typically earn a spread of about 3 to 4 basis points for bringing two nonfinancial companies together in a swap agreement. This fee is charged to compensate the intermediary for the risk involved. If one of the parties defaults on its swap payments, the intermediary is responsible for making the other party whole.

Confirmations, as drafted by the International Swaps and Derivatives Association (ISDA), outline the details of each swap agreement.

Comparative Advantage

Let's return to companies X and Y and assume that they have access to borrowing for two years as specified in Figure 4.

Figure 4: Borrowing Rates for X and Y

Company	*Fixed Borrowing*	*Floating Borrowing*
Y	5.0%	LIBOR + 10 bps
X	6.5%	LIBOR + 100 bps

Company Y has an **absolute advantage** in both markets but a comparative advantage in the fixed market. Notice that the differential between X and Y in the fixed market is 1.5%, or 150 basis points (bps), and the corresponding differential in the floating market is only 90 basis points. When this is the case, Y has a comparative advantage in the fixed market, and X has a comparative advantage in the floating market. When a **comparative advantage** exists, a swap arrangement will reduce the costs of both parties. In this example, the net potential borrowing savings by entering into a swap is the difference between the differences, or 60 bps. In other words, by entering into a swap, the total savings shared between X and Y is 60 bps.

Valuing an Interest Rate Swap With Bonds

From X's perspective, there are two series of cash flows—one fixed going out and one floating coming in. Essentially, X has a long position in a floating-rate note (since it is an inflow) and a short position in a fixed-rate note (since it is an outflow). From Y's perspective, it is exactly the opposite—Y has a short position in a floating-rate note (since it is an outflow) and a long position in a fixed-rate note (since it is an inflow).

If we denote the present value of the fixed-leg payments as B_{fix} and the present value of the floating-leg payments as B_{flt}, the value of the swap can be written for both X and Y as:

$$V_{swap}(X) = B_{flt} - B_{fix}$$

$$V_{swap}(Y) = B_{fix} - B_{flt}$$

Note that $V_{swap}(X) + V_{swap}(Y) = 0$. This is by design since the two positions are mirror images of one another. At inception of the swap, it is convention to select the fixed payment so that $V_{swap}(X) = V_{swap}(Y) = 0$. As expected floating rates in the future change, the swap value for each party is no longer zero.

Example: Valuing an interest rate swap

Consider a $1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. **Calculate** the value of the swap to the fixed-rate receiver using the bond methodology.

Answer:

Time	*Fixed Cash Flow*	*Floating Cash Flow*	*Present Value Factor*	*PV Fixed CF*	*PV Floating CF*
0.25 (3 months)	30,000	1,025,000	0.9866*	29,598	1,011,255
0.75 (9 months)	30,000		0.9589*	28,766	
1.25 (15 months)	1,030,000		0.9301*	957,968	
Total				1,016,332	1,011,255

* Note that some rounding has occurred.

The value of the swap = 1,016,332 – 1,011,255 = $5,077.

Currency Swaps

A **currency swap** exchanges both principal and interest rate payments with payments in different currencies. The exchange rate used in currency swaps is the spot exchange rate. The valuation and application of currency swaps is similar to the interest rate swap. However, since the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in the interest rate swap.

Example: Calculate the value of a currency swap

Suppose we have two companies (A and B) that enter into a fixed-for-fixed currency swap with periodic payments annually. Company A pays 6% (in Great Britain pounds, or GBP) to Company B and receives 5% (in U.S. dollars, or USD) from Company B. Company A pays a principal amount to Company B of USD175 million, and Company B pays GBP100 million to Company A at the outset of the swap.

The yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1. **Value** the currency swap just discussed assuming the swap will last for three more years.

Answer:

$$B_{USD} = 8.75e^{-0.02\times1} + 8.75e^{-0.02\times2} + 183.75e^{-0.02\times3} = \text{USD190.03 million}$$

$$B_{GBP} = 6e^{-0.04\times1} + 6e^{-0.04\times2} + 106e^{-0.04\times3} = \text{GBP105.32 million}$$

$$V_{swap} \text{ (to A in USD)} = 190.03 - (1.5 \times 105.32) = \text{USD32.05 million}$$

Swap Credit Risk

Because $V_{swap}(A) + V_{swap}(B) = 0$, whenever one side of a swap has a positive value, the other side must be negative. For example, if $V_{swap}(A) > 0$, $V_{swap}(B) < 0$. As $V_{swap}(A)$ increases in value, $V_{swap}(B)$ must become more negative. This results in increased credit risk to A since the likelihood of default increases as B has larger and larger payments to make to A. However, the potential losses in swaps are generally much smaller than the potential losses from defaults on debt with the same principal. This is because the value of swaps is generally much smaller than the value of the debt.

Other Types of Swaps

In an **equity swap**, the return on a stock, a portfolio, or a stock index is paid each period by one party in return for a fixed-rate or floating-rate payment. The return can be the capital appreciation or the total return including dividends on the stock, portfolio, or index.

A **swaption** is an option which gives the holder the right to enter into an interest rate swap. Swaptions can be American- or European-style options. Like any option, a swaption is purchased for a premium that depends on the strike rate (the fixed rate) specified in the swaption.

Firms may enter into **commodity swap** agreements where they agree to pay a fixed rate for the multi-period delivery of a commodity and receive a corresponding floating rate based on the average commodity spot rates at the time of delivery. Although many commodity swaps exist, the most common use is to manage the costs of purchasing energy resources such as oil and electricity.

A **volatility swap** involves the exchanging of volatility based on a notional principal. One side of the swap pays based on a pre-specified volatility while the other side pays based on historical volatility.

Swaps are also sometimes created for exotic structures. An example of an **exotic swap** was between Procter and Gamble and Banker's Trust where P&G's payments were based on the commercial paper rate.

Mechanics of Options Markets

Cross Reference to GARP Assigned Reading – Hull, Chapter 10

Option Types

Option contracts have asymmetric payoffs. The buyer of an option has the right to exercise the option but is not obligated to exercise. Therefore, the maximum loss for the buyer of an option contract is the loss of the price (premium) paid to acquire the position, while the potential gains in some cases are theoretically infinite. Because option contracts are a zero-sum game, the seller of the option contract could incur substantial losses, but the maximum potential gain is the amount of the premium received for writing the option. *American options* may be exercised at any time up to and including the contract's expiration date, while *European options* can be exercised only on the contract's expiration date.

Call Options

A *call option* gives the *owner* the right, but not the obligation, to buy the stock from the seller of the option. The owner is also called the *buyer* or the holder of the *long position*. The buyer benefits, at the expense of the option *seller*, if the underlying stock price is greater than the exercise price. The option *seller* is also called the *writer* or holder of the *short position*.

$$C_T = \max(0, S_T - X)$$

Put Options

A put option gives the owner the right to sell a stock to the seller of the put at a specific price. At expiration, the buyer benefits if the price of the underlying is less than the exercise price *X*:

$$P_T = \max(0, X - S_T)$$

Underlying Assets

Stock options. Stock options are typically exchange-traded, American-style options. Each option contract is normally for 100 shares of stock.

Currency options. Investors holding currency options receive the right to buy or sell an amount of foreign currency based on a domestic currency amount. Note that the unit size for currency options is considerably larger than stock options.

Index options. Options on stock indices are typically European-style options and are cash settled. Index options can be found on both the over-the-counter markets and the exchange-traded markets.

Futures options. American-style, exchange-traded options are most often utilized for futures contracts. Typically, the futures option expiration date is set to a date shortly before the expiration date of the futures contract.

Stock Options Specifications

Options can be either American or European style. As mentioned previously, American options can be exercised throughout the life of the option, while European options can only be exercised on the expiration date of the option. For this reason, American options are always at least as valuable as corresponding European options.

Strike prices are dictated by the value of the stock. Low-value stocks have smaller strike increments than higher-value stocks.

An option price (or premium) prior to expiration has two components: the time value and the intrinsic value. The *intrinsic value* is the maximum of zero or the difference between the underlying asset and the strike price [i.e., intrinsic value of a call option = max(0, S – X) and intrinsic value of a put option = max(0, X – S)]. The *time value* is the difference between the option premium and the intrinsic value.

Nonstandard option products include flexible exchange (FLEX) options, exchange-traded fund (ETF) options, weekly options, binary options, credit event binary options (CEBOs), and deep out-of-the-money (DOOM) options.

Option Trading

Market makers will quote bid and offer (or ask) prices whenever necessary. They profit on the bid-offer spread and add liquidity to the market. Floor brokers represent a particular firm and execute trades for the general public. The order book official enters limit orders relayed from the floor broker. An offsetting trade takes place when a long (short) option position is offset with a sale (purchase) of the same option. If a trade is not an offsetting trade, then open interest increases by one contract.

Commissions

Commission costs often vary based on trade size and broker type (discount vs. full service). Brokers typically structure commission rates as a fixed amount plus a percentage of the trade amount.

Margin Requirements

Options with maturities nine months or fewer cannot be purchased on margin. This is because the leverage would become too high. For options with longer maturities, investors can borrow a maximum of 25% of the option value.

Investors who engage in writing options must have a margin account due to the high potential losses and potential default. The required margin for option writers is dependent on the amount and position of option contracts written.

Exercising an Option

When an investor decides to exercise an option prior to contract expiration, her broker contacts the assigned **Options Clearing Corporation** (OCC) member responsible for clearing that broker's trades. This OCC member then submits an exercise order to the OCC which matches it with a clearing member who identifies an investor who has written a stock option. This assigned investor then must sell (if a call option) or buy (if a put option) the underlying at the specified strike price on the third business day after the order to exercise is received. Exercising an option results in the open interest being reduced by one. At contract expiration, unexercised options that are in the money after accounting for transaction costs will be exercised by brokers.

Properties of Stock Options

Cross Reference to GARP Assigned Reading – Hull, Chapter 11

Six Factors That Affect Option Prices

The following six factors will impact the value of an option:

1. S_0 = current stock price.
2. X = strike price of the option.
3. T = time to expiration of the option.
4. r = short-term risk-free interest rate over *T*.
5. D = present value of the dividend of the underlying stock.
6. σ = expected volatility of stock prices over *T*.

Figure 5 summarizes the factors' effects on option prices: "+" indicates a positive effect on option price from an increase in the factor, and "–" is a negative effect on option price.

Figure 5: Summary of Effects of Increasing a Factor on the Price of an Option

Factor	*European Call*	*European Put*	*American Call*	*American Put*
S	+	–	+	–
X	–	+	–	+
T	?	?	+	+
σ	+	+	+	+
r	+	–	+	–
D	–	+	–	+

Upper and Lower Pricing Bounds

In addition to those previously introduced, consider the following variables:

- c = value of a European call option.
- C = value of an American call option.
- p = value of a European put option.
- P = value of an American put option.
- S_T = value of the stock at expiration.

Call option upper bound:

$$c \leq S_0 \text{ and } C \leq S_0$$

Put option upper bound:

$$p \leq X \text{ and } P \leq X$$

For a European put option, we can further reduce the upper bound. Since it cannot be exercised early, it can never be worth more than the present value of the strike price:

$$p \leq Xe^{-rT}$$

Call option lower bound:

$$c \geq \max\,(S_0 - Xe^{-rT}, 0)$$

Put option lower bound:

$$p \geq \max\,(Xe^{-rT} - S_0, 0)$$

Computing Option Values Using Put-Call Parity

The derivation of **put-call parity** is based on the payoffs of two portfolio combinations, a fiduciary call and a protective put.

A *fiduciary call* is a combination of a pure-discount (i.e., zero coupon), riskless bond that pays *X* at maturity and a call with exercise price *X*. The payoff for a fiduciary call at expiration is *X* when the call is out of the money, and $X + (S - X) = S$ when the call is in the money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is $(X - S) + S = X$ when the put is in the money, and *S* when the put is out of the money.

When the put is in the money, the call is out of the money, and both portfolios pay *X* at expiration. Similarly, when the put is out of the money and the call is in the money, both portfolios pay *S* at expiration.

Put-call parity holds that portfolios with identical payoffs must sell for the same price to prevent arbitrage. We can express the put-call parity relationship as:

$$c + Xe^{-rT} = S + p$$

Lower Pricing Bounds for an American Call Option on a Nondividend-Paying Stock

Since the American option can be exercised early, American options can always be used to replicate their corresponding European options simply by choosing not to exercise them until expiration.

Lower Pricing Bounds for an American Put Option on a Nondividend-Paying Stock

While it is never optimal to exercise an American call on a nondividend-paying stock, American puts are optimally exercised early if they are sufficiently in-the-money. If an option is sufficiently in-the-money, it can be exercised, and the payoff $(X - S_0)$ can be invested to earn interest. In the extreme case when S_0 is close to zero, the future value of the exercised cash value, Xe^{rT}, is always worth more than a later exercise, *X*. We know that:

$P \geq p \geq \max(Xe^{-rT} - S_0, 0)$ for the same reasons that $C \geq c$

However, we can place an even stronger bound on an American put since it can always be exercised early:

$$P \geq \max(X - S_0, 0)$$

Relationship Between American Call Options and Put Options

Put-call parity only holds for European options. For American options, we have an inequality. This inequality places upper and lower bounds on the difference between the American call and put options.

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

Trading Strategies Involving Options

Cross Reference to GARP Assigned Reading – Hull, Chapter 12

Covered Calls and Protective Puts

When an at-the-money long put position is combined with the underlying stock, we have created a **protective put** strategy. A protective put (also called *portfolio insurance* or a *hedged portfolio*) is constructed by holding a long position in the underlying security and buying a put option. You can use a protective put to limit the downside risk at the cost of the put premium, P_0.

Another common strategy is to sell a call option on a stock that is owned by the option writer. This is called a **covered call** position. By writing an out-of-the-money call option, the combined position caps the upside potential at the strike price.

Spread Strategies

In a *bull call spread,* the buyer of the spread purchases a call option with a low exercise price, X_L, and subsidizes the purchase price of the call by selling a call with a higher exercise price, X_H. The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

A *bear call spread* is the sale of a bull spread. That is, the bear spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a "bear" strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call's cost. The purpose of the long call is to protect from sharp increases in stock prices.

A *butterfly spread* involves the purchase or sale of *three* different call options. Here, the investor buys one call with a low exercise price, buys another call with a high exercise price, and sells *two* calls with an exercise price in between. The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is limited.

A *calendar spread* is created by transacting in two options that have the same strike price but different expirations.

A *diagonal spread* is similar to a calendar spread except that instead of using options with the same strike price and different expirations, the options in a diagonal spread can have different strike prices in addition to different expirations.

A *box spread* is a combination of a bull call spread and a bear put spread on the same asset. This strategy will produce a constant payoff that is equal to the high exercise price (X_H) minus the low exercise price (X_L). Under a no arbitrage assumption, the present value of the payoff will equal the net premium paid (i.e., profit will equal zero).

Combination Strategies

A long *straddle* (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Both options have the same exercise price and expiration. Note that this strategy is profitable when the stock price moves strongly in either direction. This strategy bets on volatility. A short straddle (top straddle or straddle write) sells both options and bets on little movement in the stock. A short straddle bets on the same thing as the butterfly spread or the calendar spread, except the losses are not limited. It is a bet that will profit more if correct but also lose more if it is incorrect. Straddles are symmetric around the strike price.

A *strangle* (or bottom vertical combination) is similar to a straddle except that the options purchased are slightly out-of-the-money, so it is cheaper to implement than the straddle. The payoff is similar to the straddle except for a flat section between the strike prices. Because it is cheaper, the stock will have to move more relative to the straddle before the strangle pays off. Strangles are also symmetric around the strikes.

A *strip* involves purchasing two puts and one call with the same strike price and expiration. Notice the asymmetry of the payoff. A strip is betting on volatility but is more bearish since it pays off more on the downside.

A *strap* involves purchasing two calls and one put with the same strike price and expiration. A strap is betting on volatility but is more bullish since it pays off more on the upside.

A *collar* is the combination of a protective put and covered call. The usual goal is for the owner of the underlying asset to buy a protective put and then sell a call to pay for the put. If the premiums of the two are equal, it is called a **zero-cost collar**.

Exotic Options

Cross Reference to GARP Assigned Reading – Hull, Chapter 26

Evaluating Exotic Options

Plain vanilla derivatives include listed futures contracts and commonly used forwards and other over-the-counter (OTC) derivatives that are traded in fairly liquid markets. Exotic derivatives are customized to fit a specific firm need for hedging that cannot be met by plain vanilla derivatives. With plain vanilla derivatives, there is little uncertainty about the cost, the current market value, when they will pay, how much they will pay, and the cost of exiting the position. With exotic derivatives, some or all of these may be in question.

Exotic derivatives are developed for several reasons. The main purpose is to provide a unique hedge for a firm's underlying assets. Other reasons include addressing tax and regulatory concerns as well as speculating on the expected future direction of market prices.

Using Packages to Formulate a Zero-Cost Product

A package is defined as some combination of standard European options, forwards, cash, and the underlying asset. Bull, bear, and calendar spreads, as well as straddles and strangles, are examples of packages. Packages usually consist of selling one instrument with certain characteristics and buying another with somewhat different characteristics. Because packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero.

Exotic Option Payoff Structures

Gap Options

A gap option has two strike prices, X_1 and X_2. (X_2 is sometimes referred to as the trigger price.) If the two strike prices differ and the payoff for a gap option is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices.

Forward Start Options

Forward start options are options that begin their existence at some time in the future. Employee incentive plans commonly incorporate forward start options in which at-the-money options will be created after some period of employment has passed.

Compound Options

Compound options consist of two strike prices and two exercise dates. The first strike price and exercise date are used by the holder to evaluate whether to exercise the first option to receive the second option, where the second option is an option on the underlying asset, or just let the compound option expire. For example, a call on a call would be exercised if the price of the call on the underlying for the second call option were greater than the strike price of the initial option. The strike price and exercise date on the second call, however, are related to the value of the underlying asset.

Chooser Options

This option allows the owner, after a certain period of time has elapsed, to choose whether the option is a call or a put. The option with the greater value after the requisite time has elapsed will determine whether the owner will choose the option to be a put or a call.

Barrier Options

Barrier options are options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option. These options are usually less expensive than standard options, and essentially come in either *knock-out* or *knock-in* flavors.

Barrier options have characteristics that can be very different from those of standard options. For example, vega, the sensitivity of an option's price to changes in volatility, is always positive for a standard option but may be negative for a barrier option. Increased volatility on a down-and-out option and an up-and-out option does not increase value because the closer the underlying gets to the barrier price, the greater the chance the option will expire.

Binary Options

Binary options generate discontinuous payoff profiles because they pay only one price at expiration if the asset value is above the strike price. The term binary means that the option payoff has one of two states: the option pays a set dollar amount at expiration if the option is above the strike price, or the option pays nothing if the price is below the strike price.

Lookback Options

Lookback options are options whose payoffs depend on the maximum or minimum price of the underlying asset during the life of the option. A **floating lookback call** pays the difference between the expiration price and the minimum price of the stock over the horizon of the option. This essentially allows the owner to purchase the security at its lowest price over the option's life. On the other hand, a **floating lookback put** pays the difference between the expiration and maximum price of the stock over the time period of the option. This translates into allowing the owner of the option to sell the security at its highest price over the life of the option.

Shout Options

A shout option allows the owner to pick a date when he "shouts" to the option seller, which then translates into an intrinsic value of the option at the time of the shout. At option expiration, the owner receives the maximum of the shout intrinsic value or the option expiration intrinsic value.

Asian Options

Asian options have payoff profiles based on the average price of the security over the life of the option. *Average price* calls and puts pay off the difference between the average stock price and the strike price. Note that the average price will be much less volatile than the actual price. This means that the price for an Asian average price option will be lower than the price of a comparable standard option. *Average strike* calls and average strike puts pay off the difference between the stock expiration price and average price, which essentially represents the strike price in a typical intrinsic value calculation.

Exchange Options

A common use of an option to exchange one asset for another, often called an exchange option, is to exchange one currency with another. For example, consider a U.S. investor who holds an option to purchase euros with yen at a specified exchange rate. In this particular case, the option will be exercised if euros are more valuable to the U.S. investor than yen.

Basket Options

Basket options are simply options to purchase or sell baskets of securities. These baskets may be defined specifically for the individual investor and may be composed of specific stocks, indices, or currencies. Any exotic options that involve several different assets are more generally referred to as **rainbow options**.

Volatility and Variance Swaps

A **volatility swap** involves the exchange of volatility based on a notional principal. One side of the swap pays based on a pre-specified fixed volatility while the other side pays based on realized volatility.

Much like a volatility swap, a **variance swap** involves exchanging a pre-specified fixed variance rate for a realized variance rate.

Issues in Hedging Exotic Options

When the replication portfolio requires frequent adjustments to the holdings in the underlying assets, the hedging procedure is referred to as dynamic options replication. **Dynamic options replication** requires frequent trading, which makes it costly to implement.

As an alternative, a **static options replication** approach may be used to hedge positions in exotic options. In this case, a short portfolio of actively traded options that approximates the option position to be hedged is constructed. This short replication options portfolio is created once, which drastically reduces the transaction costs associated with dynamic rebalancing.

Commodity Forwards and Futures

Cross Reference to GARP Assigned Reading – McDonald, Chapter 6

Storage Costs

The forward price of a commodity reflects the cost of carrying the commodity until the futures expiration date. The cost of carry includes interest cost as well as any storage costs. **Storage costs** are the cost of storing a commodity and include incidental costs such as insurance and spoilage. The markets for those commodities that are storable are called as **carry markets.**

In carry markets, the forward price relationship includes storage costs (λ) as follows:

$$F_{0,T} = S_0 e^{(r + \lambda)T}$$

A **cash-and-carry arbitrage** consists of buying the commodity, storing/holding the commodity, and selling the commodity at the futures price when the contract expires. The steps in a cash-and-carry arbitrage are as follows:

At the initiation of the contract:

- Borrow money for the term of the contract at market interest rates.
- Buy the underlying commodity at the spot price.
- Sell a futures contract at the current futures price.

At contract expiration:

- Deliver the commodity and receive the futures contract price.
- Repay the loan plus interest.

If the futures price is too low (which presents a profitable arbitrage opportunity), the opposite of each step should be executed to earn a riskless profit.

The steps in a **reverse cash-and-carry arbitrage** are as follows.

At the initiation of the contract:

- Sell commodity short.
- Lend short sale proceeds at market interest rates.
- Buy futures contract at market price.

At contract expiration:

- Collect loan proceeds.
- Take delivery of the commodity for the futures price and cover the short sale commitment.

Lease Rates

A **lease rate** is the amount of interest a lender of a commodity requires. The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. From the commodity borrower's perspective, the lease rate represents the cost of borrowing the commodity. The lease rate and risk-free rate are important inputs to determine the commodity forward price.

The commodity forward price for time *T* with an active lease market is expressed as:

$$F_{0,T} = S_0 e^{(r - \delta)T}$$

where:
S_0 = current spot price
$r - \delta$ = risk-free rate less the lease rate

Note that the lease rate, δ, is income earned only if the commodity is loaned out.

Example: Pricing a commodity forward with a lease payment

Calculate the 12-month forward price for a bushel of corn that has a spot price of $5 and an annual lease rate of 7%. The appropriate continuously compounding annual risk-free rate for the commodity is equal to 9%.

Answer:

We can determine the 12-month forward price as follows:

$$F_{0,T} = (S_0)e^{(r - \delta)T} = \$5 \times e^{(0.09 - 0.07)} = \$5.101$$

Contango and Backwardation

The market is described as being in **contango** with an upward-sloping forward curve. A contango commodity market occurs when the lease rate is less than the risk-free rate. Based on the commodity forward formula, $F_{0,T} = S_0 e^{(r - \delta)T}$, if $r > \delta$, the forward price must be greater than the spot price.

The market is described as being in **backwardation** with a downward-sloping forward curve. A backwardation commodity market occurs when the lease rate is greater than the risk-free rate. Based on the commodity forward formula, $F_{0,T} = S_0 e^{(r - \delta)T}$, if $r < \delta$, the forward price must be less than the spot price.

Convenience Yields

If the owners of the commodity need the commodity for their business, holding physical inventory of the commodity creates value.

The non-monetary benefit of holding excess inventory is referred to as the **convenience yield.** The convenience yield is only relevant when a commodity is stored (i.e., in a carry market). A convenience yield *cannot* be earned by the average investor who does not have a business reason for holding the commodity.

The forward price including a convenience yield is calculated as follows:

$$F_{0,T} \geq S_0 e^{(r + \lambda - c)T}$$

where:
c = continuously compounded convenience yield

Commodity Characteristics

Certain commodities exhibit unique properties that impact their forward price. For example, gold, corn, electricity, natural gas, and oil are all commodities with characteristics that differ with respect to storage costs, the ability to store, production costs, and demand seasonality. These differences are reflected in lease rates, storage costs, and convenience yields that influence the commodity forward prices and the shape of the forward curves.

Gold forward price factors: Gold can earn a return by being loaned out. The value of gold is also influenced by the cost of production.

Corn forward price factors: There is seasonal production and a constant demand.

Electricity price factors: Electricity is not a storable commodity. Once it is produced, it must be used or it will likely go to waste. In addition, demand for electricity is not constant and will vary with time of day, day of the week, and season.

Natural gas forward price factors: There is constant production but seasonal demand.

Oil forward price factors: The price of oil is comparable worldwide. In addition, demand is high in one hemisphere when it is low in the other. Supply and demand adjust to price changes in the long run.

Commodity Spread

A **commodity spread** results from a commodity that is an input in the production process of other commodities. For example, soybeans are used in the production of soybean meal and soybean oil. A trader creates a **crush spread** by holding a long (short) position in soybeans and a short (long) position in soybean meal and soybean oil.

Similarly, oil can be refined to produce different types of petroleum products (distillates) such as heating oil, kerosene, or gasoline. This process is known as **cracking**, and thus the difference in prices of crude oil, heating oil, and gasoline is known as a **crack spread**.

Example: Pricing a crack (commodity) spread

Suppose we plan on buying crude oil in one month to produce gasoline and kerosene for sale in two months. The 1-month futures price for crude oil is currently \$30/barrel. The 2-month futures prices for gasoline and heating oil are \$41/barrel and \$31.50/barrel, respectively. **Calculate** the 5-3-2 crack (commodity) spread.

Answer:

The 5-3-2 spread tells us the amount of gross margin that can be locked in by buying five barrels of oil and producing three barrels of gasoline and two barrels of heating oil.

gross margin for a 5-3-2 spread =

$(3 \times \$41) + (2 \times \$31.50) - (5 \times \$30) = \$123 + \$63 - \$150 = \$36$ for five barrels, or \$7.20/barrel

Strip Hedge vs. Stack Hedge

A **strip hedge** is created by buying futures contracts that match the maturity and quantity for every month of the obligation. A **stack hedge** is created by buying a futures contract with a single maturity based on the present value of the future obligations. A strategy of continually rolling into the next near-term contract is referred to as **stack and roll**.

A stack hedge has the advantage when near-term contracts are more liquid (and hence the bid-ask spread is lower) and, therefore, lower transaction costs. In addition, an oil producer may prefer a stack hedge in order to speculate on the shape of the forward curve.

Cross Hedging

In many cases, a futures contract with an underlying instrument that is exactly the same as the position to be hedged will not exist. For example, there are no contracts for jet fuel futures in the United States. Therefore, hedging a jet fuel exposure requires use of a **cross hedge**. In a cross hedge, a futures contract that is highly correlated with the underlying exposure is selected. Some firms hedge the cost of jet fuel with crude oil futures while others use a combination of crude oil and heating oil futures. Three factors are relevant when making a cross hedge decision:

- The liquidity of the futures contract (since physical delivery may not be an option).
- The correlation between the underlying for the futures contract and the asset(s) being hedged.
- The maturity of the futures contract.

A cross hedge is also applied when firms use **weather derivatives**. Weather risk is a business risk that is faced by many agricultural firms as well as by firms providing recreational services. A cross hedge is used to mitigate losses caused by changes in weather.

Exchanges, OTC Derivatives, DPCs and SPVs

Cross Reference to GARP Assigned Reading – Gregory, Chapter 2

Exchange Functions

Market participants can trade derivatives bilaterally or through exchanges. An **exchange** is a central market where standardized futures, options, and other derivatives contracts can be traded.

Exchange functions fall into three primary categories: product standardization, trading venue, and reporting services.

Forms of Clearing

Clearing is the process of reconciling and matching contracts between counterparties from the time the commitments are made until settlement. Clearing, along with the mechanisms of margining and netting, are important counterparty risk mitigants.

The three forms of clearing include direct clearing, clearing rings, and complete clearing (i.e., central clearing). Direct clearing is a mechanism for bilaterally reconciling commitments between two counterparties. A clearing ring is a mechanism to reduce counterparty exposure between members by allowing for counterparty substitution. Complete clearing is clearing through a CCP, where the CCP assumes the obligations of clearing exchange members.

Exchange-Traded vs. OTC Derivatives

The following table provides a comparison of the differences between exchange-traded and OTC derivatives.

	Exchange-Traded Derivatives	*OTC Derivatives*
Terms	Standardized	Custom, negotiable
Maturity	Standardized	Negotiable, non-standard
Liquidity	Strong	Weak
Credit risk	Little (CCP guarantee)	High (bilateral)

The clearing process is also different for exchange-traded and OTC derivatives. Whereas exchange-traded derivatives are typically shorter term and are settled within a few days, OTC derivatives are longer term with later settlements. This makes clearing more challenging for OTC derivatives.

Classes of OTC Derivatives

OTC derivatives comprise of five broad classes: interest rate, foreign exchange, equity, commodity, and credit derivatives. Interest rate derivatives dominate the five classes and comprise nearly three quarters of the total gross notional outstanding. The second and third largest categories were foreign exchange derivatives and credit default swaps, respectively, followed by equity and commodity derivatives.

Mitigating Risks of OTC Derivatives

The default of a large market participant can create a ripple effect leading to systemic risk through the failure of many counterparties. Various mitigants exist to contain or reduce the risk of the initial default, including capital requirements, regulation, netting, and margining.

In addition to these mitigants, other mechanisms also exist for controlling counterparty risk. These include special purpose vehicles (SPVs), derivatives product companies (DPCs), monolines, and credit derivative product companies (CDPCs).

SPVs are bankruptcy remote legal entities set up by a parent firm to shield the SPV from any financial distress of the firm. SPVs essentially alter bankruptcy rules and transform counterparty risk into legal risk. The legal risk is consolidation, or the risk that the courts view the SPV and the originating firm as the same legal entity.

DPCs are bankruptcy remote subsidiaries of firms set up to originate derivatives products sold to investors. DPCs are separately capitalized and have restrictions on their activities and margin. They are generally AAA rated, where the rating depends on three criteria: (1) market risk minimization, (2) parent support, and (3) credit risk and operational risk management.

Monolines are highly rated insurance companies that provide financial guarantees or "credit wraps" to investors. CDPCs are akin to DPCs, but with a business model closer to that of a monoline.

Basic Principles of Central Clearing

Cross Reference to GARP Assigned Reading – Gregory, Chapter 3

The Role of a Central Counterparty

A **central counterparty** (CCP) plays an important role in the clearing and settlement of transactions following the initial trade execution. **Clearing** refers to the processes (including margining and netting) between the period from trade execution until settlement. **Settlement** of a trade occurs when the trade is completed and all payments have been made and legal obligations satisfied.

Key functions of a CCP related to the clearing process include: margining, novation, netting, managing the auction process, and loss mutualization.

When a central clearing member defaults, rather than closing out the trades at market value, the CCP typically auctions off the trades to the surviving members through an **auctioning** process.

Loss mutualization is a form of insurance and refers to members' contributions to a default fund to cover future losses from member defaults.

Other aspects and mechanics of CCPs include:

- *Categories of OTC derivatives products*: (1) long history of central clearing (e.g., interest rate swaps), (2) short history of central clearing (e.g., index credit default swaps), (3) soon to be centrally cleared (e.g., interest rate swaptions, credit default swaps), and (4) not suitable for central clearing (e.g., exotic derivatives).
- *Conditions needed for central clearing*: product standardization, lower complexity, and high liquidity.
- *Participants*: Transacting with CCPs is restricted to clearing members only. Member criteria include admission criteria, financial commitment, and operational criteria.
- *Number of CCPs*: It is generally not feasible to have a single CCP due to regional differences in trades and requirements, differences in product types, and regulatory reasons.
- *Types of CCPs*: CCPs could be utility-driven (i.e., focused on long-term stability) or profit-driven (i.e., focused on bottom line). Arguments generally support profit-driven CCPs.
- *Failure of a CCP*: The potential failure of a large CCP could create a catastrophic event. CCPs must, therefore, ensure sufficient loss absorption capacity.

Central Clearing

The following table illustrates the primary differences between the OTC derivatives markets and CCPs and exchanges.

	OTC Derivatives	*CCP/Exchanges*
Trading	Bilateral	Bilateral / Centralized
Counterparty	Original trade counterparty	CCP (replaces counterparty)
Participants	All	Clearing members (dealers)
Products	All (including non-standard, exotic)	Standard, vanilla
Margining	Bilateral, custom	Full margining set by CCP (initial, variation)
Loss buffers	Margin, regulatory capital	Initial margin, default fund, CCP capital

Advantages of CCPs include transparency, offsetting, loss mutualization, legal and operational efficiency, liquidity, and default management.

Disadvantages of CCPs include moral hazard, adverse selection, separation of cleared and non-cleared products, and procyclicality of margin requirements.

Margining

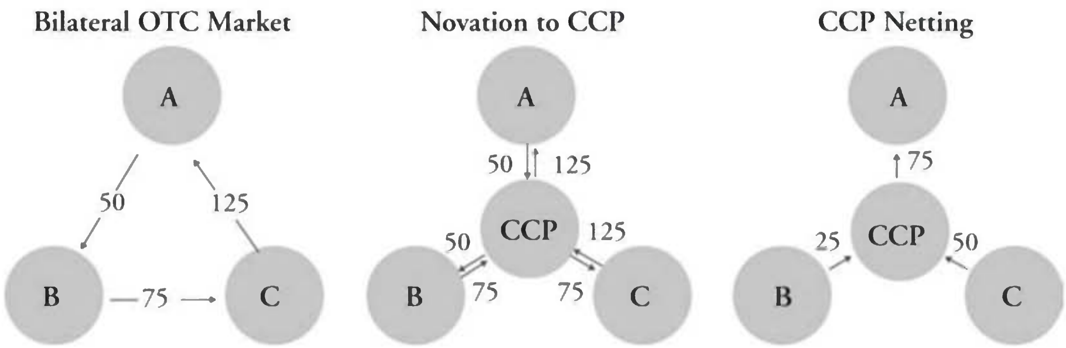
Margining by CCPs is stricter than in the OTC derivatives markets and it involves posting cash or marketable security collateral for initial margin and variation margin requirements. Initial margin represents cash or liquid assets transferred by a member at trade inception to cover a worst-case loss in the event of a member default. Variation margin is typically cash posted by a member to cover the daily net change of the member's position.

Novation and Netting

The legal process of interposing the CCP between the seller and the buyer is called **novation**. Through novation, one contract (the bilateral contract between OTC participants) is replaced with another contract (or contracts) with the CCP.

Multilateral offset, or netting, refers to creating a single net obligation between each participant and the CCP from the various bilateral OTC trades (which typically include redundant trades). Netting reduces total risk and minimizes contagion from a member default.

Figure 6: Multilateral Offsetting



Impact of Central Clearing

When a CCP is included in the clearing process, systemic risk is reduced because CCPs reduce counterparty risk by offsetting positions (novation and netting), provide transparency for the market, and improve liquidity. However, the potential requirement that members post higher initial margin during times of increased market volatility could increase systemic risk.

Risks Caused by CCPs

Cross Reference to GARP Assigned Reading – Gregory, Chapter 14

Risks Faced by Central Counterparties

Default Risk

The default of a clearing member and its flow through effects is the most significant risk for a CCP. Because of a default, there may be the default or distress of other clearing members given that default correlation is likely to be high among over-the-counter (OTC) derivatives market participants.

Model Risk

OTC derivatives are not priced by the market but are instead priced using valuation models that perform the mark-to-market function, which subjects CCPs to model risk. Especially sensitive to model risk would be a CCP's determination of initial margins.

Liquidity Risk

There are large amounts of cash inflows and outflows flowing through the CCP due to initial margins and margin calls. As a result, CCPs are exposed to liquidity risk.

There is the risk that the CCP's investments are not always quickly and easily convertible to cash, which may require some liquidity support from a central bank.

Operational Risk

CCPs face operational risks that are common to all entities such as business interruption due to information systems failures and internal or external fraud.

Legal Risk

Legal risks in the form of litigation or claims may arise due to differing laws in different jurisdictions or laws that are inconsistent with the CCP's regulations.

Other Risks

Investment risk refers to the risk of losses of margin funds resulting from investment actions performed within or outside of the stated investment policy.

Settlement and payment risk refers to the risk that a bank no longer provides cash settlement services between a CCP and its members.

Foreign exchange risk refers to the risk of mismatches between margin payments and cash inflows or outflows in different currencies.

Custody risk refers to the risk of loss of securities, margins, or both by a custodian due to its failure, fraud, or negligence.

Concentration risk refers to the risk of clearing members, margins, or both that are located in a single geographic area. Essentially, it is a lack of diversification.

Sovereign risk refers to the risk that a foreign government could default on its debt obligations, thereby causing members to fail.

Wrong-way risk refers to the risk that exposure to a counterparty is negatively correlated with the credit quality of the counterparty.

Risks to Clearing Non-Members

Non-members face exposure from CCPs, clearing members, and other non-members. If a CCP fails, a non-member may be able to avoid losses so long as its counterparty (a clearing member) is solvent. Unlike clearing members, non-members are not required to contribute to default funds so, therefore, non-members are not exposed to losses that result from CCP failures.

Lessons Learned from CCP Failures

- Operational risk must be controlled to the maximum extent possible.
- Variation margins should be recalculated often and collected quickly.
- CCPs should have an information system that allows for automated payments.
- There should be cross-margining linkage arrangements between CCPs.
- Initial margins and default funds should be sufficiently large.
- CCPs must actively monitor positions.
- CCPs must have one or more external sources of liquidity.

Foreign Exchange Risk

Cross Reference to GARP Assigned Reading – Saunders, Chapter 13

Sources of Foreign Exchange Risk

A bank's actual exposure to any given currency can be measured by the **net position exposure**. Net exposure is the extent to which a bank is net long (or *positive*) or net short (or *negative*) in a given currency. For example, a bank's net euro (EUR) exposure would be:

$$\text{net EUR exposure} = (\text{EUR assets} - \text{EUR liabilities}) + (\text{EUR bought} - \text{EUR sold})$$

$$\text{net EUR exposure} = \text{net EUR assets} + \text{net EUR bought}$$

A **positive net exposure** position means that the bank is *net long in a currency*. In this instance, the financial institution faces the risk that the foreign currency will *fall* in value against the domestic currency.

A **negative net exposure** position means that the bank is *net short in a currency*. The financial institution faces the risk that the foreign currency will *rise* in value against the domestic currency.

The potential gain/loss exposure to a foreign currency (FC) is a function of the size of the position and the potential change in the value of the foreign currency:

$$\text{dollar gain/loss in EUR} = \text{net EUR exposure (measured in \$)} \times \text{\% change in the \$/FC rate}$$

Note that a positive exposure and a positive change in the FC yield a gain; a negative exposure and negative change in the FC also yield a gain. Similarly, a positive exposure and negative change or a negative exposure and a positive change yield a loss.

Foreign Trading Activities

A financial institution's buying and selling of foreign currencies, and hence the institution's position in the FX market, reflects four key trading activities:

1. Enabling customers to participate in international commercial business transactions.
2. Enabling customers to take positions in real or financial foreign investments. Note that a financial institution may also transact in foreign currencies to take positions in real or financial foreign investments for its own portfolio.
3. Offsetting exposure in a given currency for hedging purposes.
4. Speculating on foreign currencies in search of profit by forecasting and/or anticipating futures FX rate movements.

Sources of Profits and Losses on Foreign Exchange Trading

Most returns on FX trading arise from speculation in currencies or taking an unhedged position in a particular currency. Financial institutions also earn fees as a secondary source of revenues. These revenues are earned from market-making activities, acting as agents for retail or wholesale customers, or a combination of both.

A financial institution can also have foreign exchange exposure due to mismatches between foreign financial asset and liability portfolios.

Balance Sheet Hedging

There are two principle methods of better controlling the impact of FX exposure:

1. **On-balance-sheet hedging** is achieved when a financial institution has a matched maturity and foreign currency balance sheet.

2. **Off-balance-sheet hedging** occurs through the purchase of forwards for institutions that choose to remain unhedged on the balance sheet.

Interest Rate Parity

Interest rate parity (IRP) suggests that the discounted spread between domestic and foreign interest rates equals the percentage spread between forward and spot exchange rates. For example, the hedged dollar return on foreign investments should be equal to the return on domestic investments. IRP implies that in a competitive market, a firm should not be able to make excess profits from foreign investments (i.e., a higher domestic currency return from lending in a foreign currency and locking in the forward rate of exchange).

The exact IRP equation using direct quotes is:

$$\text{forward} = \text{spot}\left[\frac{(1+r_{DC})}{(1+r_{FC})}\right]^T$$

where:
r_{DC} = domestic currency rate
r_{FC} = foreign currency rate

IRP can also be stated using continuously compounded rates as follows:

$$\text{forward} = \text{spot} \times e^{(r_{DC}-r_{FC})T}$$

Diversification in Multicurrency Foreign Asset-Liability Positions

Diversification across several asset and liability markets can potentially reduce portfolio risk as well as the cost of funds. Domestic and foreign interest rates and stock returns generally do not move together perfectly over time. This means that the risks from mismatching one-currency positions may be offset by potential gains from asset-liability portfolio diversification.

The **nominal interest rate**, *r*, is the compounded sum of the real interest rate, *real r*, and the expected rate of inflation, *E*(i), over an estimation horizon. This relationship is often called the Fisher equation:

$$\text{exact methodology: } (1 + \text{r}) = (1 + \text{real r})[1 + \text{E(i)}]$$

$$\text{linear approximation: } \text{r} \approx \text{real} + \text{E(i)}$$

Corporate Bonds

Cross Reference to GARP Assigned Reading – Fabozzi, Chapter 13

Bond Indenture and Role of Corporate Trustee

The **bond indenture** is a document that sets forth the obligation of the issuer and the rights of the investors in the bonds (i.e., the bondholders). It is usually a detailed document filled with legal language. One of the roles of the **corporate trustee** is to interpret this language and represent the interests of the bondholders.

Maturity Date

The maturity date of a bond is when the bond issuer's obligations are fulfilled. At maturity, the issuer pays the principal and any accrued interest or premium. The longer the maturity of the bond, the more time a company has to retire the bond issue.

Interest Payment Classifications

Straight-coupon bonds, also called fixed-rate bonds, have a fixed interest rate set for the entire life of the issue. In the United States, fixed-rate bonds typically pay interest every six months. In Europe and some other countries, bonds make annual interest rate payments.

Participating bonds pay at least the specified interest rate but may pay more if the company's profits increase. **Income bonds** pay at most the specified interest, but they may pay less if the company's income is not sufficient.

Floating-rate bonds are also known as variable rate bonds. The interest paid is generally linked to some widely used reference rate such as LIBOR or the Federal Funds rate.

Zero-coupon bonds pay the face value or principal at maturity. There is not a cash interest payment; instead, the bondholder earns a return by purchasing the bond at a discount to face value and receiving the full face value at maturity. Variations of the zero-coupon bond include the **deferred-interest bond** (DIB) and the **payment-in-kind bond** (PIK).

One advantage of zero-coupon bonds is zero **reinvestment risk**. The bondholder does not have to make an effort to reinvest cash interest payments or worry about the available rates at which to reinvest them. A disadvantage is that the bondholder must pay taxes each year on the accrued interest even though no cash is received from the bond issuer.

Bond Types

Mortgage bonds can be issued in a series in a blanket arrangement. In this case, one group of bonds is issued under the mortgage, and then others are issued later. When earlier issues mature, additional bonds are then issued in their place.

Collateral trust bonds are backed by stocks, notes, bonds, or other similar obligations that the company owns. The underlying assets are called the collateral or personal property.

Equipment trust certificates (ETCs) are a variation of a mortgage bond where a particular piece of equipment underlies the bond. The usual arrangement is that the borrower does not actually purchase the equipment. Instead, the trustee purchases the equipment and leases it to the user of the equipment (the effective borrower), who pays rent on the equipment, and that rent is passed through to the holders of the ETCs.

Debentures are unsecured bonds (i.e., they do not have any assets underlying the issue). Most corporate bonds are debentures and usually pay a higher interest rate for that reason.

Subordinated debenture bonds have a claim that is at the bottom of the list of creditors if the issuer goes into default. They are bonds that are unsecured and have another unsecured bond with a higher claim above them.

Issuers may choose to issue **convertible debentures**, which give the bondholder the right to convert the bond into common stock. A variation of convertible debentures is **exchangeable debentures** that are convertible into the common stock of a corporation other than that of the issuer.

Bonds issued by one company may also be guaranteed by other companies. These bonds are known as **guaranteed bonds**.

Methods for Retiring Bonds

Call and refunding provisions are essentially call options on the bonds that the issuer owns and give the issuer the right to purchase at a fixed price either in whole or in part prior to maturity. These provisions allow a firm to call back debt that has a high coupon and reissue debt with a lower coupon.

A **sinking fund provision** generally means the issuing firm retires a specified portion of the debt each year as outlined in the indenture. The bonds can either be retired by use of a lottery where the owners of the selected bonds must redeem them, or the bonds are purchased in the open market.

A **maintenance and replacement fund** (M&R) has the same goal as a sinking fund provision, which is to maintain the credibility of the property backing the bonds. The provisions differ in that the M&R provision is more complex since it requires valuation formulas for the underlying assets.

Tender offers are usually a means for retiring debt for most firms. The firm openly indicates an interest in buying back a certain dollar amount of bonds or, more often, all of the bonds at a set price. The goal is to eliminate restrictive covenants or to use excess cash.

Credit Risk

Credit risk includes credit default risk and credit spread risk. **Credit default risk** is the uncertainty concerning the issuer making timely payments of interest and principal as prescribed by the bond's indenture. The most widely-used indicators of this risk are bond ratings that major rating agencies assign when those agencies perform credit analysis of a firm. Fitch Ratings, Moody's, and Standard & Poor's are the main rating agencies in the United States.

Credit spread risk focuses on the difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security. This difference is known as the **credit spread**. It should be noted that other factors such as embedded options and liquidity factors can affect this spread; therefore, it is not only a function of credit risk.

High-Yield Bonds

There are many types of high-yield bonds. One type includes companies who issue bonds with a non-investment grade rating. Such issuers include young and growing companies that do not have strong financial statements but who have promising prospects. Firms may issue such bonds to raise venture capital, and their prospects are tied to a particular project or story, which gives them the name "story bonds."

Established firms that have had a deteriorating financial situation may need to raise debt capital as well, and they would issue bonds that reflect their situation. Also, an established firm who already has unsecured debt issued with an investment-grade credit rating may be able to issue subordinated debt, but that debt would be non-investment grade.

Fallen angels are another type of high-yield bond. They are bonds that were issued with an investment-grade rating, but then events led to the ratings agencies lowering the rating to below investment grade.

Mortgages and Mortgage-Backed Securities

Cross Reference to GARP Assigned Reading – Tuckman, Chapter 20

Residential Mortgage Products

A **mortgage** is a loan that is collateralized with a specific piece of real property. Within the past few decades, it is more common for mortgage lenders to sell the loans in the **secondary market** through a process known as **securitization**.

In the secondary market, mortgages are pooled together and packaged to investors in the form of a **mortgage-backed security** (MBS). The payments of an MBS can follow a **pass-through structure** where the interest and principal collected from the borrower pass through the banks and ultimately end up with the MBS investor.

Prime (A-grade) loans constitute most of the outstanding loans. They have low rates of delinquency and default as a result of low **loan-to-value (LTV) ratios,** borrowers with stable and sufficient income, and a strong history of repayments.

Subprime (B-grade) loans have higher rates of delinquency and default compared to prime loans.

Alternative-A loans are the loans in between prime and subprime.

Fixed-rate mortgages have a set rate of interest for the term of the mortgage. Payments are constant for the term and consist of blended amounts of interest and principal.

Adjustable-rate mortgages (ARMs) have rate changes throughout the term of the mortgage. The rate is usually based on a base rate (e.g., prime rate, LIBOR) plus a spread.

Government loans are those that are backed by federal government agencies (e.g., Government National Mortgage Association or GNMA).

Conventional loans could be securitized by either government-sponsored enterprises (GSEs): Federal Home Loan Mortgage Corporation (FHLMC) or Federal National Mortgage Association (FNMA).

Agency (or conforming) MBSs are those that are guaranteed by any of three government-sponsored entities (GSEs): GNMA, FNMA, and FHLMC.

The GSEs have restrictions on what mortgages they can guarantee/securitize, which opened up the private label market for those participants willing to take on the risks inherent in nonconventional loans—**jumbo loans** (mortgage principal balance over the limit) and/or loans with high LTVs.

Fixed-Rate, Level-Payment Mortgages

There are four important features of fixed-rate, level payment, fully amortized mortgage loans:

1. The amount of the principal payment increases as time passes.
2. The amount of interest decreases as time passes.
3. The servicing fee also declines as time passes.
4. The ability of the borrower to repay results in **prepayment risk**.

Prepayment

Mortgage prepayments come in two forms: (1) increasing the frequency or amount of payments (where permitted) and (2) repaying/refinancing the entire outstanding balance. Prepayments are much more likely to occur when market interest rates fall and borrowers wish to refinance their existing mortgages at a new and lower rate.

Other Factors That Influence Prepayments

Seasonality. The summertime is a popular time for individuals to move (and mortgages must be paid out prior to the sale of a home), so it is the period of time with the greatest prepayment risk.

Age of mortgage pool. Refinancing often involves penalties and administrative charges, so borrowers tend not to do so until several years into the mortgage.

Personal. Marital breakdown, loss of employment, family emergencies, and destruction of property are commonly cited reasons for prepayments based on personal reasons.

Housing prices. Property value increases may spur an increase in prepayments caused by borrowers wanting to take out some of the increased equity for personal use.

Refinancing burnout. To the extent that there has been a significant amount of prepayment or refinancing activity in the mortgage pool in the past, the risk of prepayment in the future decreases.

Pass-Through Securities

A **mortgage pass-through security** represents a claim against a pool of mortgages. The mortgages in the pool have different maturities and different mortgage rates. The **weighted average maturity** (WAM) of the pool is equal to the weighted average of all mortgage ages in the pool, each weighted by the relative outstanding mortgage balance to the value of the entire pool. The **weighted average coupon** (WAC) of the pool is the weighted average of the mortgage rates in the pool.

Prepayments cause the timing and amount of cash flows from mortgage loans and MBSs to be uncertain; they speed up principal repayments and reduce the amount of interest paid over the life of the mortgage. Thus, it is necessary to make specific assumptions about the rate at which prepayment of the pooled mortgages occurs when valuing pass-through securities. Two industry conventions have been adopted as benchmarks for prepayment rates: the **conditional prepayment rate** (CPR) and the **Public Securities Association** (PSA) prepayment benchmark.

The *CPR* is the annual rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool. A mortgage pool's CPR is a function of past prepayment rates and expected future economic conditions.

We can convert the CPR into a monthly prepayment rate called the **single monthly mortality rate** (SMM) (also referred to as constant maturity mortality) using the following formula:

$$SMM = 1 - (1 - CPR)^{1/12}$$

An SMM of 10% implies that 10% of a pool's beginning-of-month outstanding balance, less scheduled payments, will be prepaid during the month.

The *PSA prepayment benchmark* assumes that the monthly prepayment rate for a mortgage pool increases as it ages or becomes seasoned. The PSA benchmark is expressed as a monthly series of CPRs.

Dollar Roll Transaction

MBS trading requires the same securities to be priced for different settlement dates. A **dollar roll transaction** occurs when an MBS market maker buys positions for one settlement month and, at the same time, sells those same positions for another month.

Empirical evidence suggests that the most likely outcome is that a price drop between the two settlement dates makes purchasing the security in the back month more attractive.

When the price difference/drop is large enough to result in financing at less than the implied cost of funds, then the dollar roll is trading *special*.

Prepayment Modeling

Refinancing a mortgage involves using the proceeds of a new mortgage to pay off the principal from an existing mortgage. If a homeowner is holding a high interest rate mortgage and the current mortgage rates fall, the incentive to refinance is large (given that rates decline enough to cover the transaction costs of refinancing).

Because most borrowers sell their homes without regard for the path of mortgage rates, MBS investors will be subjected to a degree of **housing turnover** that does not correlate with the behavior of rates. One factor that slows the degree of housing turnover is known as the *lock-in effect*. This essentially means that borrowers may wish to avoid the costs of a new mortgage, which likely consists of a higher mortgage rate.

When a borrower **defaults**, mortgage guarantors pay the interest and principal outstanding. These payments act as a source of prepayment. Modeling prepayments from default requires an analysis of loan-to-value (LTV) ratios and FICO scores, as well as an overall analysis of the housing market.

Partial payments by the borrower are referred to as **curtailments**. These partial payments tend to occur when a mortgage is older or has a relatively low balance. Thus, prepayment modeling due to curtailment typically takes into account the age of the mortgage.

Dynamic Valuation

Prepayments on mortgage pass-through securities are interest rate path-dependent. This means that a given month's prepayment rate depends on whether there were prior opportunities to refinance since the origination of the underlying mortgages.

The **Monte Carlo methodology** is a simulation approach for valuing MBSs. Monte Carlo is actually a process of steps rather than a specific model. It is extremely useful when there are numerous variables with multiple outcomes. Monte Carlo is used to provide a probability distribution of the value of an MBS. The valuation of an MBS is influenced by future interest rates, the shape of the yield curve, future interest rate volatility, prepayment rates, default rates, and recovery rates.

The Monte Carlo approach provides a range of possible outcomes with a probability distribution for the value of a mortgage security. The mean or average value of this range of outcomes is then taken as the estimated value of the MBS. The other information, such as the range of possible outcomes and percentile information, is useful in gauging the value of the security.

The following steps are required to value a mortgage security using the Monte Carlo methodology:

Step 1: Simulate the interest rate path and refinancing path.
Step 2: Project cash flows for each interest rate path.
Step 3: Calculate the present value of cash flows for each interest rate path.
Step 4: Calculate the theoretical value of the mortgage security.

Option-Adjusted Spread

The **option-adjusted spread** (OAS) is defined as the spread, that, when added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest.

The OAS is determined with an iterative process. If the average theoretical value determined by the model is higher (lower) than the MBS market value, the spread is increased (decreased).

Cash flows for MBSs are monthly annuity payments, while Treasury securities pay semiannual interest-only payments and a large bullet payment. The **zero-volatility spread** (*z*-spread) is a spread measure that an investor realizes over the entire Treasury spot rate curve, assuming the mortgage security is held to maturity. The zero-volatility spread is the yield that equates the present value of the cash flows from the MBS to the price of the MBS discounted at the Treasury spot rate plus the spread. Thus, an iterative process is required to determine the zero-volatility spread.

The option cost measures the prepayment (or option) risk. It is the implied cost of the option embedded in the MBS. The option cost is calculated as the difference between the OAS at the assumed volatility of interest rates and the zero-volatility spread as follows:

$$\boxed{\text{option cost} = \text{zero-volatility spread} - \text{OAS}}$$

Valuation and Risk Models

Weight on Exam	30%
SchweserNotes™ Reference	Book 4

Quantifying Volatility in VaR Models

Cross Reference to GARP Assigned Reading – Allen et al., Chapter 2

Three common deviations from normality that are problematic in modeling risk result from asset returns that are fat-tailed, skewed, or unstable.

Fat-tailed refers to a distribution with a higher probability of observations occurring in the tails relative to the normal distribution.

A distribution is **skewed** when the distribution is not symmetrical. A risk manager is more concerned when there is a higher probability of a large negative return than a large positive return.

In modeling risk, a number of assumptions are necessary. If the parameters of the model are **unstable**, they are not constant but vary over time. For example, if interest rates, inflation, and market premiums are changing over time, this will affect the volatility of the returns going forward.

"Fat tails" are most likely the result of the volatility and/or the mean of the distribution changing over time. If the mean and standard deviation are the same for asset returns for any given day, the distribution of returns is referred to as an **unconditional distribution** of asset returns. However, different market or economic conditions may cause the mean and variance of the return distribution to change over time. In such cases, the return distribution is referred to as a **conditional distribution**.

Assume we separate the full data sample into two normally distributed subsets based on market environment with **conditional** means and variances. Pulling a data sample at different points of time from the full sample could generate fat tails in the unconditional distribution even if the conditional distributions are normally distributed with similar means but different volatilities. If markets are efficient and all available information is reflected in stock prices, it is not likely that the first moments or conditional means of the distribution vary enough to make a difference over time.

The second possible explanation for “fat tails” is that the second moment or volatility is time-varying. This explanation is much more likely given observed changes in interest rate volatility (e.g., prior to a much-anticipated Federal Reserve announcement). Increased market uncertainty following significant political or economic events results in increased volatility of return distributions.

Market Regimes and Conditional Distributions

A **regime-switching volatility model** assumes different market regimes exist with high or low volatility.

The probability of large deviations from normality occurring are much less likely under the regime-switching model. The regime-switching model captures the conditional normality and may resolve the fat-tail problem and other deviations from normality.

Despite efforts to more accurately model financial data, extreme events do still occur. The model (or distribution) used may not capture these extreme movements. For example, value at risk (VaR) models are typically utilized to model the risk level apparent in asset prices. VaR assumes asset returns follow a normal distribution, but as we have just discussed, asset return distributions tend to exhibit fat tails. As a result, VaR may underestimate the actual loss amount.

However, some tools exist that serve to complement VaR by examining the data in the tail of the distribution. For example, stress testing and scenario analysis can examine extreme events by testing how hypothetical and/or past financial shocks will impact VaR.

Value at Risk

Historical-based approaches typically fall into three sub-categories: parametric, nonparametric, and hybrid.

1. The **parametric approach** requires specific assumptions regarding the asset returns distribution. A parametric model typically assumes asset returns are normally or lognormally distributed with time-varying volatility.

2. The **nonparametric approach** is less restrictive in that there are no underlying assumptions of the asset returns distribution. The most common nonparametric approach models volatility using the historical simulation method.

3. As the name suggests, the **hybrid approach** combines techniques of both parametric and nonparametric methods to estimate volatility using historical data.

Parametric Approaches for VaR

The RiskMetrics® [i.e., exponentially weighted moving average (EWMA) model] and GARCH approaches are both exponential smoothing weighting methods. RiskMetrics® is actually a special case of the GARCH approach. Both exponential smoothing methods are similar to the historical standard deviation approach because all three methods:

- Are parametric.
- Attempt to estimate conditional volatility.
- Use recent historical data.
- Apply a set of weights to past squared returns.

The only major difference between the historical standard deviation approach and the two exponential smoothing approaches is with respect to the weights placed on historical returns that are used to estimate future volatility. The historical standard deviation approach assumes all *K* returns in the window are equally weighted. Conversely, the exponential smoothing methods place a higher weight on more recent data, and the weights decline exponentially to zero as returns become older.

Nonparametric vs. Parametric VaR Methods

Three common types of nonparametric methods used to estimate VaR are: (1) historical simulation, (2) multivariate density estimation, and (3) hybrid. These nonparametric methods exhibit the following advantages and disadvantages over parametric approaches.

Advantages of nonparametric methods compared to parametric methods:

- Nonparametric models do not require assumptions regarding the entire distribution of returns to estimate VaR.
- Fat tails, skewness, and other deviations from some assumed distribution are no longer a concern in the estimation process for nonparametric methods.
- Multivariate density estimation (MDE) allows for weights to vary based on how relevant the data is to the current market environment, regardless of the timing of the most relevant data.
- MDE is very flexible in introducing dependence on economic variables (called *state variables* or *conditioning variables*).
- Hybrid approach does not require distribution assumptions because it uses a historical simulation approach with an exponential weighting scheme.

Disadvantages of nonparametric methods compared to parametric methods:

- Data is used more efficiently with parametric methods than nonparametric methods. Therefore, large sample sizes are required to precisely estimate volatility using historical simulation.
- Separating the full sample of data into different market regimes reduces the amount of usable data for historical simulations.
- MDE may lead to data snooping or over-fitting in identifying required assumptions regarding the weighting scheme identification of relevant conditioning variables and the number of observations used to estimate volatility.
- MDE requires a large amount of data that is directly related to the number of conditioning variables used in the model.

Return Aggregation

When a portfolio is comprised of more than one position using the RiskMetrics® or historical standard deviation approaches, a single VaR measurement can be estimated by assuming asset returns are all normally distributed.

The historical simulation approach requires an additional step that aggregates each period's historical returns weighted according to the relative size of each position. The weights are based on the market value of the portfolio positions today, regardless of the actual allocation of positions *K* days ago in the estimation window.

A third approach to calculating VaR estimates the volatility of the vector of aggregated returns and assumes normality based on the strong law of large numbers.

Putting VaR to Work

Cross Reference to GARP Assigned Reading – Allen et al., Chapter 3

Linear vs. Non-Linear Derivatives

A derivative is described as *linear* when the relationship between an underlying factor and the derivative is linear in nature. For example, an equity index futures contract is a linear derivative, while an option on the same index is non-linear. The delta for a linear derivative must be constant for all levels of the underlying factor, but not necessarily equal to one.

The value of a *nonlinear* derivative is a function of the change in the value of the underlying asset and is dependent on the state of the underlying asset. A call option is a good example of a nonlinear derivative. The value of the call option does not increase (decrease) at a constant rate when the underlying asset increases (decreases) in value.

In general, the VaR of a long position in a linear derivative is $VaR_p = \Delta VaR_f$, where VaR_f is the VaR of the underlying factor and the derivative's delta, Δ, is the sensitivity of the derivative's price to changes in the underlying factor. Delta is assumed to be positive because we're modeling a long position. The local delta is defined as the percentage change in the derivative's price for a 1% change in the underlying asset. For small changes in the underlying price of the asset the change in price of the derivative can be extrapolated based on the local delta. This local delta is, in essence, the slope of the line or the change in the call price divided by the change in the underlying asset price.

Taylor Approximation

The slope of the line is only useful in estimating the call value with small changes in the underlying stock value. The gap between the tangency line representing the delta or slope of the line at the tangency point widens the further away the estimate is from the point of tangency. The first derivative of a function tells us the slope of the line at any given point. The second derivative tells us the rate of change. This information is summarized mathematically in the **Taylor Series approximation** of the function f(x) as follows:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$$

The Delta-Normal and Full Revaluation Methods

Both the delta-normal and full revaluation methods measure the risk of nonlinear securities. The **full revaluation approach** calculates the VaR of the derivative by valuing the derivative based on the underlying value of the index after the decline corresponding to an *x*% VaR of the index. This approach is accurate, but can be highly computational.

The **delta-normal approach** calculates the risk using the delta approximation ($VaR_p = \Delta VaR_f$), which is linear or the delta-gamma approximation, $f(x) \approx f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$, which adjusts for the curvature of the underlying relationship. This approach simplifies the calculation of more complex securities by approximating the changes based on linear relationships (delta).

The Monte Carlo Approach

The **structured Monte Carlo (SMC) approach** simulates thousands of valuation outcomes for the underlying assets based on the assumption of normality. The VaR for the portfolio of derivatives is then calculated from the simulated outcomes. The general equation assumes the underlying asset has normally distributed returns with a mean of μ and a standard deviation of σ.

An *advantage* of the SMC approach is that it is able to address multiple risk factors by assuming an underlying distribution and modeling the correlations among the risk factors.

A *disadvantage* of the SMC approach is that in some cases it may not produce an accurate forecast of future volatility and increasing the number of simulations will not improve the forecast.

Correlations During Crisis

In times of crisis, correlations increase (some substantially) and strategies that rely on low correlations fall apart in those times. Certain economic or crisis events can cause diversification benefits to deteriorate in times when the benefits are most needed. A contagion effect occurs with a rise in volatility and correlation causing a different return generating process. Some specific examples of events leading to the breakdown of historical correlation matrices are the Asian crisis, the U.S. stock market crash of October 1987, the events surrounding the failure of Long-Term Capital Management (LTCM), and the recent global credit crisis.

A simulation using the SMC approach is not capable of predicting scenarios during times of crisis if the covariance matrix was estimated during normal times. Unfortunately, increasing the number of simulations does not improve predictability in any way.

Stress Testing

During times of crisis, a contagion effect often occurs where volatility and correlations both increase, thus mitigating any diversification benefits. *Stressing* the correlation is a method used to model the contagion effect that could occur in a crisis event.

Worst Case Scenario Measure

The **worst case scenario** (WCS) assumes that an unfavorable event will occur with certainty. The focus is on the distribution of worst possible outcomes given an unfavorable event. An expected loss is then determined from this worst case distribution analysis. Thus, the WCS information extends the VaR analysis by estimating the extent of the loss given an unfavorable event occurs.

Measures of Financial Risk

Cross Reference to GARP Assigned Reading – Dowd, Chapter 2

Value at Risk

Value at risk (VaR) is interpreted as the worst possible loss under normal conditions over a specified period. Another way to define VaR is as an estimate of the maximum loss that can occur with a given confidence level. If an analyst says, "for a given month, the VaR is $1 million at a 95% level of confidence," then this translates to mean "under normal conditions, in 95% of the months (19 out of 20 months), we expect the fund to either earn a profit or lose no more than $1 million." Analysts may also use other standard confidence levels (e.g., 90% and 99%). Delta-normal VaR can be computed using the following expression: $[\mu - (z)(\sigma)]$.

VaR measurements work well with elliptical return distributions, such as the normal distribution. VaR is also able to calculate the risk for non-normal distributions; however, VaR estimates may be unreliable in this case. Limitations in implementing the VaR model for determining risk result from the underlying return distribution, arbitrary confidence level, arbitrary holding period, and the inability to calculate the magnitude of losses. The measure of VaR also violates the coherent risk measure property of subadditivity when the return distribution is not elliptical.

Coherent Risk Measures

If we allow *R* to be a set of random events and $\rho(R)$ to be the risk measure for the random events, then **coherent risk measures** should exhibit the following properties:

1. **Monotonicity**: a portfolio with greater future returns will likely have less risk: $R_1 \geq R_2$, then $\rho(R_1) \leq \rho(R_2)$

2. **Subadditivity**: the risk of a portfolio is at most equal to the risk of the assets within the portfolio: $\rho(R_1 + R_2) \leq \rho(R_1) + \rho(R_2)$

3. **Positive homogeneity**: the size of a portfolio, β, will impact the size of its risk: for all $\beta > 0$, $\rho(\beta R) = \beta\rho(R)$

4. **Translation invariance**: the risk of a portfolio is dependent on the assets within the portfolio: for all constants *c*, $\rho(c + R) = \rho(R) - c$

Expected Shortfall

Value at risk is the minimum percent loss, equal to a pre-specified worst case quantile return (typically the 5th percentile return). **Expected shortfall** (ES) is the expected loss given that the portfolio return already lies below the pre-specified worst case quantile return (i.e., below the 5th percentile return). In other words, expected shortfall is the mean percent loss among the returns falling below the *q*-quantile. Expected shortfall is also known as **conditional VaR** or **expected tail loss** (ETL).

Scenario Analysis

The results of scenario analysis can be interpreted as coherent risk measures by first assigning probabilities to a set of loss outcomes. These losses can be thought of as tail drawings of the relevant distribution function. The expected shortfall for the distribution can then be computed by finding the arithmetic average of the losses. Therefore, the outcomes of scenario analysis must be coherent risk measurements, because ES is a coherent risk measurement.

Binomial Trees

Cross Reference to GARP Assigned Reading – Hull, Chapter 13

One-Step Binomial Model

A **one-step binomial model** is best described within a two-state world where the price of a stock will either go up once or down once, and the change will occur one step ahead at the end of the holding period.

The one-step binomial model can also be expressed in terms of probabilities and call prices. The sizes of the upward and downward movements are defined as functions of the volatility and the length of the "steps" in the binomial model:

$$U = \text{size of the up-move factor} = e^{\sigma\sqrt{t}}$$

$$D = \text{size of the down-move factor} = e^{-\sigma\sqrt{t}} = \frac{1}{e^{\sigma\sqrt{t}}} = \frac{1}{U}$$

where:
σ = annual volatility of the underlying asset's returns
t = the length of the step in the binomial model

The risk-neutral probabilities of upward and downward movements are then calculated as follows:

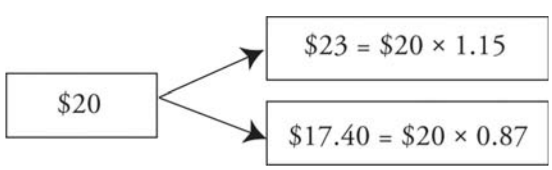
$$\pi_u = \text{probability of an up move} = \frac{e^{rt} - D}{U - D}$$

$$\pi_d = \text{probability of a down move} = 1 - \pi_u$$

where:
r = continuously compounded annual risk-free rate

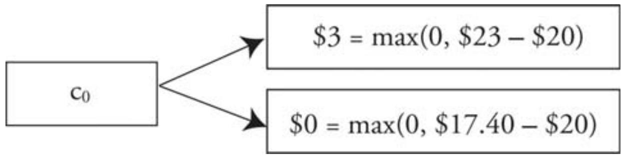
An example of a binomial tree is as follows:

Figure 1: Binomial Tree—Stock



The binomial tree for the option with a strike price of $20 is shown as follows:

Figure 2: Binomial Tree—Option

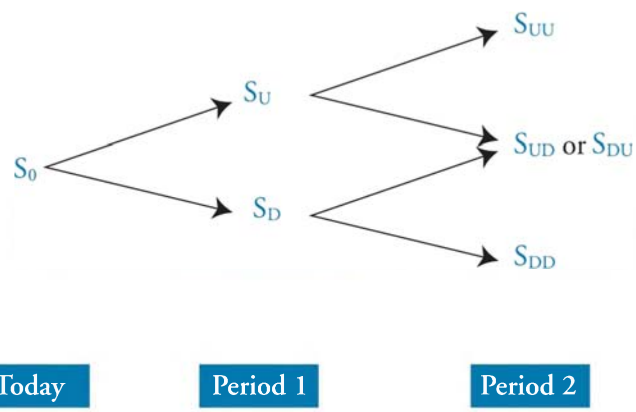


Notice that the call option is in-the-money in the "up" state, so its value is \$3. It is out-of-the-money in the "down" state, so its value is zero.

Two-Step Binomial Model

In the two-period and multi-period models, the *tree* is expanded to provide for a greater number of potential outcomes.

Figure 3: Two-Step Binomial Model Stock Price Tree



Modifying the Binomial Model

The binomial option pricing model can be altered to value a stock that pays a continuous **dividend yield**, *q*. Since the total return in a risk-neutral setting is the risk-free rate, *r*, and dividends provide a positive yield, capital gains must be equal to r – q. The risk-neutral probabilities of upward and downward movements incorporate a dividend yield as follows:

$$\pi_u = \frac{e^{(r-q)t} - D}{U - D}$$

$$\pi_d = 1 - \pi_u$$

The equations for the size of the up-move and down-move factors will be the same. Options on stock indices are valued in a similar fashion to stocks with dividends, because it is assumed that stocks underlying the index pay a dividend yield equal to q.

American Options

Valuing American options with a binomial model requires the consideration of the ability of the holder to exercise early. In the case of a two-step model, that means determining whether early exercise is optimal at the end of the first period. If the payoff from early exercise (the intrinsic value of the option) is greater than the option's value (the present value of the expected payoff at the end of the second period), then it is optimal to exercise early.

Increasing the Number of Time Periods

If we shorten the length of the intervals in a binomial model, there are more intervals over the same time period, more branches to consider, and more possible ending values. If we continue to shrink the length of intervals in the model until they are what mathematicians call "arbitrarily small," we approach continuous time as the limiting case of the binomial model. The Black-Scholes-Merton model is a continuous time model. The binomial model "converges" to this continuous time model as we make the time periods arbitrarily small.

The Black-Scholes-Merton Model

Cross Reference to GARP Assigned Reading – Hull, Chapter 15

Lognormal Stock Prices

The model used to develop the **Black-Scholes-Merton** (BSM) **model** assumes stock prices are lognormally distributed.

$$\ln S_T \sim N\left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right]$$

where:
S_T = stock price at time T
S_0 = stock price at time 0
μ = expected return on stock per year
σ = volatility of the stock price per year
$N[m, s]$ = normal distribution with mean = m and standard deviation = s

Since ln S_T is normally distributed, S_T has a lognormal distribution.

Dividing the mean and standard deviation results in the continuously compounded annual return of a stock price. Specifically, the continuously compounded annual returns are *normally distributed* with a mean of:

$$\left(\mu - \frac{\sigma^2}{2}\right)$$

and a standard deviation of:

$$\frac{\sigma}{\sqrt{T}}$$

Notice that the BSM model assumes stock prices are lognormally distributed, but stock returns are normally distributed. Also, notice in the standard deviation formula that volatility will be lower for longer periods of time.

Using the properties of a *lognormal distribution*, we can show that the expected value of S_T , $E(S_T)$, is:

$$E(S_T) = S_0 e^{\mu T}$$

where:
μ = expected rate of return

Black-Scholes-Merton Model Assumptions

In addition to the no-arbitrage condition, the assumptions underlying the BSM model are the following:

- The price of the underlying asset follows a lognormal distribution.

- The (continuous) risk-free rate is constant and known.
- The volatility of the underlying asset is constant and known.
- Markets are "frictionless."
- The underlying asset has no cash flow, such as dividends or coupon payments.
- The options valued are European options, which can only be exercised at maturity.

Black-Scholes-Merton Option Pricing Model

The formulas for the BSM model are:

$$c_0 = \left[S_0 \times N(d_1)\right] - \left[X \times e^{-R_f^c \times T} \times N(d_2)\right]$$

$$p_0 = \left\{X \times e^{-R_f^c \times T} \times \left[1 - N(d_2)\right]\right\} - \left\{S_0 \times \left[1 - N(d_1)\right]\right\}$$

where:

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left[R_f^c + \left(0.5 \times \sigma^2\right)\right] \times T}{\sigma \times \sqrt{T}}$$

$$d_2 = d_1 - \left(\sigma \times \sqrt{T}\right)$$

T = time to maturity (as % of a 365-day year)
S_0 = asset price
X = exercise price
R_f^c = continuously compounded risk-free rate
σ = volatility of continuously compounded returns on the stock
$N(\bullet)$ = cumulative normal probability

Black-Scholes-Merton Model With Dividends

Just as we subtracted the present value of expected cash flows from the asset price when valuing forwards and futures, we can subtract it from the asset price in the BSM model. Since the BSM model is in continuous time, in practice $S_0 \times e^{-qT}$ is substituted for S_0 in the BSM formula, where q is equal to the continuously compounded rate of dividend payment. Over time, the asset price is discounted by a greater amount to account for the greater amount of cash flows. Cash flows will increase put values and decrease call values.

Valuation of Warrants

Warrants are attachments to a bond issue that give the holder the right to purchase shares of a security at a stated price. After purchasing the bond, warrants can be exercised separately or stripped from the bond and sold to other investors. Hence, warrants can be valued as a separate call option on the firm's shares.

Volatility Estimation

The steps in computing **historical volatility** for use as an input in the BSM continuous-time options pricing model are:

- Convert a time series of *N* prices to returns:

$$R_i = \frac{P_i - P_{i-1}}{P_{i-1}}, \; i = 1 \text{ to } N$$

- Convert the returns to continuously compounded returns:

$$R_i^c = \ln(1 + R_i), \; i = 1 \text{ to } N$$

- Calculate the variance and standard deviation of the continuously compounded returns.

Implied volatility is the value for standard deviation of continuously compounded rates of return that is "implied" by the market price of the option. Of the five inputs into the BSM model, four are observable: (1) stock price, (2) exercise price, (3) risk-free rate, and (4) time to maturity. If we use these four inputs in the formula and set the BSM formula equal to market price, we can solve for the volatility that satisfies the equality.

Greek Letters

Cross Reference to GARP Assigned Reading – Hull, Chapter 19

Naked and Covered Call Options

A **naked position** occurs when one party sells a call option without owning the underlying asset. A **covered position** occurs when the party selling a call option owns the underlying asset.

A Stop-Loss Strategy

Stop-loss strategies with call options are designed to limit the losses associated with short option positions (i.e., those taken by call writers). The strategy requires purchasing the underlying asset for a naked call position when the asset rises above the option's strike price. The asset is then sold as soon as it goes below the strike price. The objective here is to hold a naked position when the option is out-of-the-money and a covered position when the option is in-the-money.

Delta Hedging

The **delta** of an option, Δ, is the ratio of the change in price of the call option, *c*, to the change in price of the underlying asset, *s*, for small changes in *s*. Mathematically:

$$\text{delta} = \Delta = \frac{\partial c}{\partial s}$$

where:
∂c = change in the call option price
∂s = change in the stock price

Option Delta

A call delta equal to 0.6 means that the price of a call option on a stock will change by approximately \$0.60 for a \$1.00 change in the value of the stock. To completely hedge a long stock or short call position, an investor must purchase the number of shares of stock equal to delta times the number of options sold. Another term for being completely hedged is **delta-neutral**. For example, if an investor is short 1,000 call options, he will need to be long 600 (0.6 × 1,000) shares of the underlying.

Forward Delta

When the underlying asset pays a dividend, q, the delta of an option or forward must be adjusted. If a dividend yield exists, the delta for a call option equals $e^{-qT} \times N(d_1)$, the delta of a put option equals $e^{-qT} \times [N(d_1) - 1]$, and the delta of a forward contract equals e^{-qT}.

Futures Delta

Unlike forward contracts, the delta of a futures position is not ordinarily one because of the spot-futures parity relationship. For example, the delta of a futures position is e^{rT} on a stock or stock index that pays no dividends, where r is the risk-free rate and T is the time to maturity. Assets that pay a dividend yield, q, would generate a delta equal to $e^{(r-q)T}$. An investor would hedge short futures positions by going long the amount of the deliverable asset.

Dynamic Aspects of Delta Hedging

The delta of an option is a function of the underlying stock price. That means when the stock price changes, so does the delta. When the delta changes, the portfolio will no longer be hedged (i.e., the number of options and underlying stocks will no longer be in balance), and the investor will need to either purchase or sell the underlying asset. This rebalancing must be done on a continual basis to maintain the delta-neutral hedged position.

The goal of a **delta-neutral portfolio** (or delta-neutral hedge) is to combine a position in an asset with a position in an option *so that the value of the portfolio does not change with changes in the value of the asset*. In referring to a stock position, a delta-neutral portfolio can be made up of a risk-free combination of a long stock position and a short call position where the number of calls to short is given by $1/\Delta_c$.

$$\text{number of options needed to delta hedge} = \frac{\text{number of shares hedged}}{\text{delta of call option}}$$

The delta of a portfolio of options on a single underlying asset can be calculated as the weighted average delta of each option position in the portfolio:

$$\text{portfolio delta} = \Delta_p = \sum_{i=1}^{n} w_i \Delta_i$$

where:
w_i = the portfolio weight of each option position
Δ_i = the delta of each option position

Therefore, portfolio delta represents the expected change of the overall option portfolio value given a small change in the price of the underlying asset.

Theta, Gamma, Vega, and Rho

Theta, Θ, measures the option's sensitivity to a decrease in time to expiration. Theta is also termed the "time decay" of an option. Theta varies as a function of both time and the price of the underlying asset.

Gamma, Γ, represents the expected change in the delta of an option. It measures the curvature of the option price function not captured by delta.

Delta-neutral positions can hedge the portfolio against small changes in stock price, while gamma can help hedge against relatively large changes in stock price. Therefore, it is not only desirable to create a delta-neutral position but also to create one that is **gamma-neutral**. In that way, neither small nor large stock price changes adversely affect the portfolio's value.

Since underlying assets and forward instruments generate linear payoffs, they have zero gamma and, hence, cannot be employed to create gamma-neutral positions. Gamma-neutral positions have to be created using instruments that are not linearly related to the underlying instrument, such as options. The specific relationship that determines the number of options that must be added to an existing portfolio to generate a gamma-neutral position is $-(\Gamma_P / \Gamma_T)$, where Γ_P is the gamma of the existing portfolio position, and Γ_T is the gamma of a traded option that can be added.

Relationship Among Delta, Theta, and Gamma

Stock option prices are affected by delta, theta, and gamma as indicated in the following relationship:

$$r\Pi = \Theta + rS\Delta + 0.5\sigma^2 S^2 \Gamma$$

This equation shows that the change in the value of an option position is directly affected by its sensitivities to the Greeks.

For a delta-neutral portfolio, $\Delta = 0$, so the preceding equation reduces to:

$$r\Pi = \Theta + 0.5\sigma^2 S^2 \Gamma$$

The left side of the equation is the dollar risk-free return on the option (risk-free rate times option value). Assuming the risk-free rate is small, this demonstrates that for large positive values of theta, gamma tends to be large and negative, and vice versa, which explains the common practice of using theta as a proxy for gamma.

Vega

Vega measures the sensitivity of the option's price to changes in the volatility of the underlying stock. For example, a vega of 8 indicates that for a 1% increase in volatility, the option's price will increase by 0.08. For a given maturity, exercise price, and risk-free rate, the vega of a call is equal to the vega of a put.

Rho

Rho, ρ, measures an option's sensitivity to changes in the risk-free rate. Keep in mind, however, that equity options are not as sensitive to changes in interest rates as they are to changes in the other variables (e.g., volatility and stock price). Large changes in rates have only small effects on equity option prices. Rho is a much more important risk factor for fixed-income derivatives.

In-the-money calls and puts are more sensitive to changes in rates than out-of-the-money options. Increases in rates cause larger *increases* for in-the-money call prices (versus out-of-the-money calls) and larger *decreases* for in-the-money puts (versus out-of-the-money puts).

Hedging in Practice

Large financial institutions usually adjust to a delta-neutral position and then monitor exposure to the other Greeks. Two offsetting situations assist in this monitoring activity. First, institutions that have sold options to their clients are exposed to negative gamma and vega, which tend to become more negative as time passes. In contrast, when the options are initially sold at-the-money, the level of sensitivity to gamma and vega is highest, but as time passes, the options tend to go either in-the-money or out-of-the-money. The farther in- or out-of-the-money an option becomes, the less the impact of gamma and vega on the delta-neutral position.

Scenario analysis involves calculating expected portfolio gains or losses over desired periods using different inputs for underlying asset price and volatility. In this way, traders can assess the impact of changing various factors individually, or simultaneously, on their overall position.

Prices, Discount Factors, and Arbitrage

Cross Reference to GARP Assigned Reading – Tuckman and Serrat, Chapter 1

Fundamentals of Bond Valuation

The value (or price) of any financial asset, such as a bond, can be determined by summing the asset's discounted cash flows. There are three steps in the bond valuation process:

Step 1: **Estimate the cash flows** over the life of the security. For a bond, there are two types of cash flows: (1) the coupon payments and (2) the return of principal.

Step 2: **Determine the appropriate discount rate** based on the risk of (uncertainty about) the receipt of the estimated cash flows.

Step 3: **Calculate the present value of the estimated cash flows** by multiplying the bond's expected cash flows by the appropriate discount factors.

Discount Factors

Discount factors are used to determine present values. The discount function is expressed as *d(t)*, where *t* denotes time in years.

Example: Calculating bond value using discount factors

Suppose that the discount factor for the first 180-day coupon period is as follows:

$$d(0.5) = 0.92432$$

Calculate the price of a bond that pays \$108 six months from today.

Answer:

Since \$1 to be received in six months is worth \$0.92432 today, \$108 received in six months is worth $0.92432 \times \$108 = \$99.83$ today.

Example: Calculating discount factors given bond prices

Figure 4 shows selected T-bond prices for semiannual coupon \$100 face value bonds.

Figure 4: Selected Treasury Bond Prices

Prices are from 5/14/06, with t + 1 settlement			
Bond	*Coupon*	*Maturity*	*Price*
1	4.25%	11/15/06	101-16
2	7.25%	5/15/07	105-31+
3	2.00%	11/15/07	101-07
4	12.00%	5/15/08	120-30
5	5.75%	11/15/08	110-13+

Generate the discount factors for the dates indicated.

Answer:

Bond 1:

When this bond matures on 11/15/06, it makes its last interest payment of $2.125 = \left(\frac{0.0425}{2} \times \$100\right)$ plus the principal repayment of 100. The present value of the 102.125 is given as the price of 101-16, or 101.50.

$$\text{price(PV)} = \text{CF}(0.5) \times d(0.5)$$

$$101.5 = 102.125\ d(0.5)$$

Solving for the discount function yields:

$$d(0.5) = 0.9939$$

Figure 5: Discount Factors

Time to Maturity	*Discount Factor*
0.5	0.9939
1.0	0.9880
1.5	0.9825
2.0	0.9731
2.5	0.9633

Determining Value Using Discount Functions

The discount functions previously mentioned can be used to estimate the value of a bond. Since investors do not care about the origin of a cash flow, all else equal, a cash flow from one bond is just as good as a cash flow from another bond. This phenomenon is commonly referred to as the **law of one price**. If investors are able to exploit a mispricing because of the law of one price, it is referred to as an arbitrage opportunity.

Example: Identifying arbitrage opportunities

Suppose you observe the annual coupon bonds shown in Figure 6.

Figure 6: Observed Bond Yields and Prices

Maturity	*YTM*	*Coupon (annual payments)*	*Price (% of par)*
1 year	4%	0%	96.154
2 years	8%	0%	85.734
2 years	8%	8%	100.000

The 2-year spot rate is 8.167%. Is there an arbitrage opportunity? If so, **describe** the trades necessary to exploit the arbitrage opportunity.

Answer:

The answer is yes, an arbitrage profit may be realized because the YTM on the 2-year zero coupon is too low (8% versus 8.167%), which means the bond is trading *rich* (the bond price is too high). To exploit this violation of the law of one price, buy the 2-year, 8% coupon bond, strip the coupons, and short sell them separately. The discount factors are derived from the prices of the zero-coupon bonds.

Figure 7: Discount Factors

Time to Maturity	*Discount Factor*
1.0	0.96154
2.0	0.85734

To demonstrate the process of exploiting the arbitrage opportunity here, consider the following 3-step process (the dollar amounts given are arbitrary):

Step 1: Buy $1 million of the 2-year, 8% coupon bonds because they are trading *cheap*.
Step 2: Short sell $80,000 of the 1-year, zero-coupon bonds at 96.154.
Step 3: Short sell $1.08 million of the 2-year, zero-coupon bonds at 85.734.

Figure 8: Cash Flow Diagram

Time = 0		*1 year*		*2 years*	
–1,000,000.00	(cost of 2-year, 8% coupon bonds)	+80,000	(coupon, interest)	+1,080,000	(coupon, interest)
+76,923.20*	(proceeds 1-year, 0% bonds)	–80,000	(maturity)		
+925,927.20**	(proceeds 2-year, 0% bonds)			–1,080,000	(maturity)
+2,850.40	Net	0		0	

*76,923.20 = 0.96154 × 80,000

**925,927.20 = 0.85734 × 1,080,000

The result is receiving *positive income today* in return for *no future obligation*, which is an *arbitrage opportunity*. The selling of the 2-year STRIPS would force the price down to 85.469 (the price at which the YTM = 8.167%), at which point the arbitrage opportunity would disappear.

Treasury Coupon Bonds and Treasury STRIPS

Zero-coupon bonds issued by the Treasury are called **STRIPS** (separate trading of registered interest and principal securities). STRIPS are created by request when a coupon bond is presented to the Treasury. The bond is "stripped" into two components: principal and coupon (**P-STRIPS** and **C-STRIPS**, respectively).

Spot, Forward, and Par Rates

Cross Reference to GARP Assigned Reading – Tuckman and Serrat, Chapter 2

Annual Compounding vs. Semiannual Compounding

Use the following formula to find the future value of a bond using different compounding methods:

$$FV_n = PV_0 \times \left[1 + \frac{r}{m}\right]^{m \times n}$$

where:
r = annual rate
m = number of compounding periods per year
n = number of years

Deriving Discount Factors from Swap Rates

Example: Computing discount rates from swap rates

Given the following swap rates, **compute** the discount factors for maturities ranging from six months to two years assuming a notional swap amount of $100.

Figure 9: Swap Rates

Maturity (Years)	*Swap Rates*
0.5	0.65%
1.0	0.80%
1.5	1.02%
2.0	1.16%

Answer:

The six-month discount rate is computed as:

$$\left(100+\frac{0.65}{2}\right)d(0.5)=100$$

d(0.5) = 0.9968

The 1-year discount rate, given the six-month discount rate, is then computed as:

$$\left(\frac{0.8}{2}\right)d(0.5)+\left(100+\frac{0.8}{2}\right)d(1.0)=100$$

$$\left(\frac{0.8}{2}\right)(0.9968)+\left(100+\frac{0.8}{2}\right)d(1.0)=100$$

d(1.0) = 0.9920

Figure 10 shows all discount factors for maturities ranging from six months to two years.

Figure 10: Discount Factors

Maturity (Years)	*Discount Factor*
0.5	0.9968
1.0	0.9920
1.5	0.9848
2.0	0.9771

The Spot Rate Curve

A *t*-period **spot rate**, denoted as *z(t)*, is the yield to maturity on a zero-coupon bond that matures in *t*-years (assuming semiannual compounding). The **spot rate curve** is the graph of the relationship between spot rates and maturity. The spot rate curve can be derived from either a series of STRIPS prices, or the comparable discount factors.

Forward Rates

Forward rates are interest rates that span future periods. Given the spot rates, it is possible to compute forward rates implied by that spot curve. The spot rates are the appropriate rates that an investor should expect to realize for various periods for a risk-free investment starting today. Should the investor be concerned whether the investment is composed of a single instrument or a series of shorter investments rolled over consecutively? No, because if the risk is the same, the realized return must be the same, regardless of how the investment is packaged. This concept is at the core of forward rate analysis.

Example: Computing a forward rate

Compute the 6-month forward rate in six months, given the following spot rates:

$z(0.5) = 1.50\%$

$z(1.0) = 2.15\%$

Answer:

In order for strategies 1 and 2 to realize the same return, the 6-month forward rate, f(1.0), on an investment that matures in one year must solve the following equation:

$$\left(1+\frac{0.0215}{2}\right)^2 = \left(1+\frac{0.0150}{2}\right)^1 \times \left(1+\frac{f(1.0)}{2}\right)^1$$

$$\Rightarrow f(1.0) = 0.028 = 2.80\%$$

Par Rates

The **par rate** at maturity is the rate at which the present value of a bond equals its par value. By assuming a 2-year bond pays semiannual coupons and has a par value of $100, the 2-year par rate can be computed by incorporating bond discount factors from each semiannual period as follows:

$$\frac{\text{par rate}}{2}\left[d(0.5)+d(1.0)+d(1.5)+d(2.0)\right]+100\times d(2.0)=100$$

Pricing a Bond Using Spot, Forward, and Par Rates

Example: Calculating the price of a bond

Suppose a 1-year Treasury bond (T-bond) pays a 4% coupon semiannually. **Compute** its price using the discount factors, spot rates, forward rates, and par rates from Figure 11.

Figure 11: Discount Factors, Spot Rates, Forward Rates, and Par Rates

Maturity (Years)	*Discount Factor*	*Spot Rate*	*6-Month Forward Rate*	*Par Rates*
0.5	0.992556	1.50%	1.50%	1.5000%
1.0	0.978842	2.15%	2.80%	2.1465%
1.5	0.962990	2.53%	3.29%	2.5225%
2.0	0.943299	2.94%	4.18%	2.9245%
2.5	0.921205	3.31%	4.80%	3.2839%
3.0	0.897961	3.62%	5.18%	3.5823%

Answer:

Using discount factors:

$$\text{bond price} = (\$2 \times 0.992556) + (\$102 \times 0.978842) = \$101.83$$

Using annuity and discount factors:

$$\text{bond price} = [\$2 \times (0.992556 + 0.978842)] + (\$100 \times 0.978842) = \$101.83$$

Using spot rates:

$$\text{bond price} = \frac{\$2}{\left(1 + \frac{0.0150}{2}\right)^1} + \frac{\$102}{\left(1 + \frac{0.0215}{2}\right)^2} = \$101.83$$

Using forward rates:

$$\text{bond price} = \frac{\$2}{\left(1+\frac{0.0150}{2}\right)^1} + \frac{\$102}{\left(1+\frac{0.0150}{2}\right)^1 \times \left(1+\frac{0.0280}{2}\right)^1} = \$101.83$$

Using par rates:

$$\text{bond price} = \$100 + \left[\left(\$2 - \frac{2.1465}{2}\right) \times (0.992556 + 0.978842)\right] = \$101.83$$

Effect of Maturity on Bond Prices and Returns

In general, bond prices will tend to increase with maturity when coupon rates are above the relevant forward rates. The opposite holds when coupon rates are below the relevant forward rates (i.e., bond prices will tend to decrease with maturity in this scenario).

Yield Curve Shapes

When the yield curve undergoes a **parallel shift**, the yields on all maturities change in the same direction and by the same amount. The slope of the yield curve remains unchanged following a parallel shift.

When the yield curve undergoes a **nonparallel shift**, the yields for the various maturities change by differing amounts. The slope of the yield curve after a nonparallel shift is not the same as it was prior to the shift. Nonparallel shifts fall into two general categories: twists and butterfly shifts.

Yield curve twists refer to yield curve changes when the slope becomes either flatter or steeper. With an upward-sloping yield curve, a *flattening* of the yield curve means that the spread between short- and long-term rates has narrowed.

Yield curve butterfly shifts refer to changes in the degree of curvature. A positive butterfly means that the yield curve has become less curved. A negative butterfly means that there is more curvature to the yield curve.

Returns, Spreads, and Yields

Cross Reference to GARP Assigned Reading – Tuckman and Serrat, Chapter 3

Realized Return

Example: Calculating gross realized return

What is the gross realized return for a bond that is currently selling for $112 if it was purchased exactly six-months ago for $105 and paid a $2 coupon today?

Answer:

Substituting the appropriate values into the realized return equation, we get:

$$R_{t-1,t} = \frac{\$112 + \$2 - \$105}{\$105}$$

$$R_{t-1,t} = 8.57\%$$

The **net realized return** for a bond is its gross realized return minus per period financing costs.

Bond Spread

The market price of a bond may differ from the computed price of a bond using spot rates or forward rates. Any difference between bond market price and bond price according to the term structure of interest rates is known as the **spread** of a bond. A bond's spread is a relative measure of value which helps investors identify whether investments are trading cheap or rich relative to the yield curve (i.e., the term structure of rates).

Yield to Maturity

The **yield to maturity**, or YTM, of a fixed-income security is equivalent to its internal rate of return. The YTM is the discount rate that equates the present value of all cash flows associated with the instrument to its price.

For a security that pays a series of known annual cash flows, the computation of yield uses the following relationship:

$$P = \frac{C_1}{(1+y)^1} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \ldots + \frac{C_N}{(1+y)^N}$$

where:
P = the price of the security
C_k = the annual cash flow in year k
N = term to maturity in years
y = the annual yield or YTM on the security

Spot Rates and YTM

Example: Spot rates and YTM

A bond with a $100 par value pays a 5% coupon annually for 4 years. The spot rates corresponding to the payment dates are as follows:

Year 1: 4.0%
Year 2: 4.5%
Year 3: 5.0%
Year 4: 5.5%

Assume the price of the bond is $98.47. **Show** the calculation of the price of the bond using spot rates and **determine** the YTM for the bond.

Answer:

The formula for the price of the bond using the spot rates is as follows:

$$P = \frac{5}{(1.04)} + \frac{5}{(1.045)^2} + \frac{5}{(1.05)^3} + \frac{105}{(1.055)^4}$$

$$\$98.47 = 4.81 + 4.58 + 4.32 + 84.76$$

Now compute the YTM:

$$\$98.47 = \frac{5}{(1+\text{YTM})} + \frac{5}{(1+\text{YTM})^2} + \frac{5}{(1+\text{YTM})^3} + \frac{105}{(1+\text{YTM})^4}$$

$$\text{FV} = \$100;\ \text{PV} = -\$98.47;\ \text{PMT} = 5;\ \text{N} = 4;\ \text{CPT} \rightarrow \text{I / Y} = 5.44\%$$

$$\text{YTM} = 5.44\%$$

We see from this example that the YTM is closest to the 4-year spot rate. This is because the largest cash flow occurs at year 4 as the bond matures. If the spot curve is upward sloping, as in this example, the YTM will be less than the 4-year spot (i.e., the last spot rate). If the spot curve is flat, the YTM will be equal to the 4-year spot, and if the spot curve is downward sloping, the YTM will be greater than the 4-year spot.

The Relationship Between YTM, Coupon Rate, and Price

Suppose that after the bond was issued, market interest rates declined substantially. Investors in the bond would receive coupon rates substantially higher than what the market currently offers. Because of this, the price of the bond would adjust upward. This bond is a **premium bond**. If interest rates were to increase substantially after the bond was issued, investors would have to be compensated for the fact that the coupon rate of the bond is substantially lower than those offered currently in the market. The price would adjust downward as a consequence. The bond would be referred to as a **discount bond**.

Therefore:

- If coupon rate > YTM, the bond will sell for more than par value, or at a **premium**.
- If coupon rate < YTM, the bond will sell for less than par value, or at a **discount**.
- If coupon rate = YTM, the bond will sell for **par value**.

If two bonds are identical in all respects except their coupon, *the bond with the smaller coupon will be more sensitive to interest rate changes.* That is, for any given change in yield, the smaller-coupon bond will experience a bigger percentage change in price than the larger-coupon bond. All else being equal:

- The lower the coupon rate, the greater the interest-rate risk.
- The higher the coupon rate, the lower the interest-rate risk.

Return Decomposition

Return decomposition for a bond breaks down bond profitability or loss (P&L) into component parts. This decomposition of P&L helps bond investors understand how their investments are making or losing money. A bond's profitability or loss is generated through price appreciation and explicit cash flows (i.e., cash-carry), such as coupons and financing costs. Bond total price appreciation can be broken down into three component parts for price effect analysis: carry-roll-down, rate changes, and spread change. Note that the sum of all component parts will equal total price appreciation.

Carry-Roll-Down Scenarios

The carry-roll-down component of total price appreciation considers movement to an expected term structure. Traders make investment return calculations based on their expectations, and many traders will consider scenarios where rates do not change. Given this expectation, term structure choices for no change scenarios include: realized forwards, unchanged term structure, and unchanged yields.

One-Factor Risk Metrics and Hedges

Cross Reference to GARP Assigned Reading – Tuckman and Serrat, Chapter 4

Interest Rate Factors

Measures of interest rate sensitivity allow investors to evaluate bond price changes as a result of interest rate changes. Being able to properly measure price sensitivity can be useful in the following situations:

1. Hedgers must understand how the bond being hedged as well as the hedging instrument used will respond to interest rate changes.

2. Investors need to determine the optimal investment to make in the event that expected changes in rates do in fact occur.

3. Portfolio managers would like to know the portfolio level of volatility for expected changes in rates.

4. Asset/liability managers need to match the interest rate sensitivity of their assets with the interest rate sensitivity of their liabilities.

Dollar Value of a Basis Point

The DV01 is the "dollar value of an 01," meaning the change in a fixed income security's value for every one basis point change in interest rates. The "01" refers to one basis point (i.e., 0.0001).

DV01 is computed using the following formula:

$$\text{DV01} = -\frac{\Delta \text{BV}}{10{,}000 \times \Delta y}$$

where:
ΔBV = change in bond value
Δy = change in yield

DV01 Application to Hedging

Example: Computing the amount of bonds needed to hedge

An investor takes a long position in an option worth \$100 million. The option has a DV01 of 0.141. The investor wishes to hedge this option position with a 15-year zero-coupon bond which increases in price from \$56.40 to \$56.58 when yields drop by one basis point. **Calculate** the face amount of the bond required to hedge this option position.

Answer:

First compute the DV01 of the bond position:

$$\text{DV01}^{\text{B}} = -\frac{\$56.58 - \$56.40}{10{,}000 \times -0.0001} = 0.18$$

To determine the face amount of the bond required to hedge this option exposure, we use the following approach:

$$\text{face value} = \text{option position} \times \frac{\text{DV01}^{0}}{\text{DV01}^{\text{B}}}$$

$$\text{face value} = 100\text{M} \times \frac{0.141}{0.18} = \$78.33\text{M}$$

So in order to hedge this \$100 million option position, the investor must short \$78.33 million in face value of the bond.

Duration

Duration is the most widely used measure of bond price volatility. A bond's price volatility is a function of its coupon, maturity, and initial yield. Duration captures the impact of all three of these variables in a single measure. Just as important, a bond's duration and its price volatility are directly related (i.e., the longer the duration, the more price volatility there is in a bond). Of course, such a characteristic greatly facilitates the comparative evaluation of alternative bond investments.

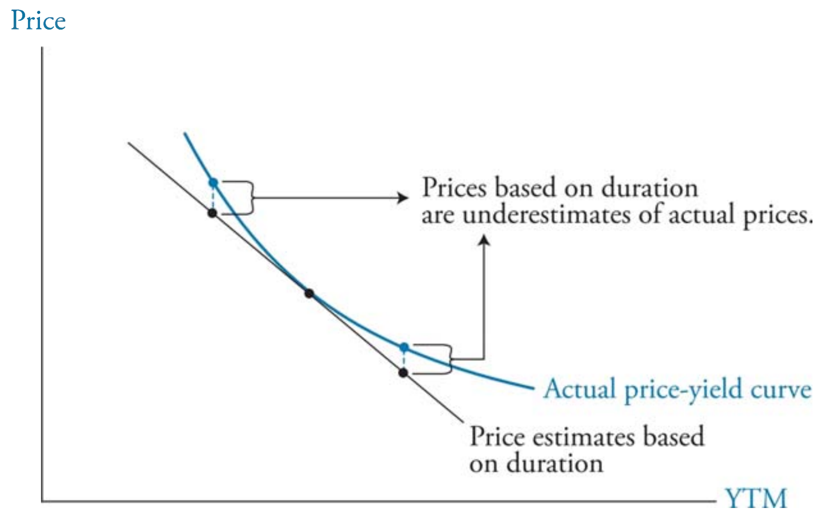
$$\text{effective duration} = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

where:
$BV_{-\Delta y}$ = estimated price if yield decreases by a given amount, Δy
$BV_{+\Delta y}$ = estimated price if yield increases by a given amount, Δy
BV_0 = initial observed bond price
Δy = change in required yield, in decimal form

Convexity

Duration is a good approximation of price changes for relatively small changes in interest rates. Like DV01, duration is a linear estimate since it assumes that the price change will be the same regardless of whether interest rates go up or down. As rate changes grow larger, the curvature of the bond price-yield relationship becomes more important.

Figure 12: Duration-Based Price Estimates vs. Actual Bond Prices



Convexity is a measure of the curvature in the relationship between bond yield and price. An understanding of convexity can illustrate how a bond's duration changes as interest rates change.

$$\text{convexity} = \frac{BV_{-\Delta y} + BV_{+\Delta y} - 2 \times BV_0}{BV_0 \times \Delta y^2}$$

Negative Convexity

With **callable debt**, the upside price appreciation in response to decreasing yields is limited (sometimes called price compression). Consider the case of a bond that is currently callable at 102. The fact that the issuer can call the bond at any time for $1,020 per $1,000 of face value puts an effective upper limit on the value of the bond. As yields fall and the price approaches $1,020, the price-yield curve rises more slowly than that of an identical but noncallable bond. When the price begins to *rise at a decreasing rate* in response to further decreases in yield, the price-yield curve "bends over" to the left and exhibits **negative convexity**.

Constructing a Barbell Portfolio

A **barbell strategy** is typically used when an investment manager uses bonds with short and long maturities, thus forgoing any intermediate-term bonds. A **bullet strategy** is used when an investment manager buy bonds concentrated in the intermediate maturity range.

The advantages and disadvantages of a barbell versus a bullet portfolio are dependent on the investment manager's view on interest rates. If the manager believes that rates will be especially volatile, the barbell portfolio would be preferred over the bullet portfolio.

Multi-Factor Risk Metrics and Hedges

Cross Reference to GARP Assigned Reading – Tuckman and Serrat, Chapter 5

Weaknesses of Single-Factor Approaches

The major weakness of a single-factor model approach to hedging or asset/liability management is the assumption that the entire yield curve can be described by one interest rate factor. A single-factor-based approach to hedging or asset/liability management will not protect against changes in nonparallel shifts in the shape of the yield curve. A multiple-factor approach is needed when changes in the shape of the yield curve occur.

Key Rate Exposures

Key rate exposures help describe how the risk of a bond portfolio is distributed along the term structure, and they assist in setting up a proper hedge for a bond portfolio. Key rate exposures are utilized for measuring and hedging risk in bond portfolios using rates from the most liquid bonds available, which are generally government bonds that have been issued recently and are selling at or near par.

Key Rate Shift Technique

Key rate shift analysis makes the simplifying assumption that all rates can be determined as a function of a few "key rates." To cover risk across the entire term structure, a small number of key rates are used, pertaining only to the most liquid government securities.

The key rate shift technique is an approach to nonparallel shifts in the yield curve, which allows for changes in all rates to be determined by changes from selected key rates.

This is obviously a somewhat limiting and simplistic approach. However, the key rate approach is appealing because (1) key rates are affected by a combination of rates closest to them; (2) key rates are mostly affected by the closest key rate; (3) key rate effects are smooth (do not jump across maturity); and (4) a parallel shift across the yield curve results.

Key Rate '01 and Key Rate Duration

Key rate '01s are calculated as follows:

$$\mathrm{DV01}^{k} = -\frac{1}{10{,}000}\frac{\Delta \mathrm{BV}}{\Delta y^{k}}$$

Key rate durations are calculated as follows:

$$D^{k} = -\frac{1}{\mathrm{BV}}\frac{\Delta \mathrm{BV}}{\Delta y^{k}}$$

Hedging Applications

For every basis point shift in a key rate, the corresponding key rate '01 provides the *dollar change* in the value of the bond. Similarly, key rate duration provides the approximate *percentage change* in the value of the bond. Key rate duration works off 100 basis point changes, so it is the percentage of price movement for every 100 basis point change in rates.

Hedging positions can be created in response to shifts in key rates by equating individual key rate exposures adjacent to key rate shifts to the overall key rate exposure for that particular key rate change. The resulting positions indicate either long or short positions in securities to protect against interest rate changes surrounding key rate shifts.

Partial '01s and Forward-Bucket '01s

With more complex portfolios that contain swaps, partial '01s and forward-bucket '01s are often used instead of key rates. These approaches are similar to the key rate approach, but instead divide the term structure into more parts. Risk along the yield curve is thus measured more frequently, in fact daily.

A partial '01 will measure the change in the value of the portfolio from a one basis point decrease in the fitted rate and subsequent refitting of the curve. In other words, with partial '01s, yield curve shifts are able to be fitted more precisely because we are constantly fitting securities.

The forward-bucket '01 approach is a more direct and mechanical approach for looking at exposures. Forward-bucket '01s are computed by shifting the forward rate over several regions of the term structure, one region at a time, after the term structure is divided into various buckets.

Estimating Portfolio Volatility

Key rates and bucket analysis allow a manager to use more than a single factor to manage interest rate risk effects on a portfolio. These multi-factor approaches work well not only in estimating changes in the level of the portfolio, but also in the estimation of portfolio volatility because it incorporates correlation effects between various interest rate assumptions.

Country Risk: Determinants, Measures and Implications

Cross Reference to GARP Assigned Reading – Damodaran

Sources of Country Risk

Key sources of country risk include: (1) where the country is in the economic growth life cycle, (2) political risks, (3) the legal systems of countries, including both the structure and the efficiency of legal systems, and (4) the disproportionate reliance of a country on one commodity or service.

Economic Growth Life Cycle

More mature markets and companies within those markets are less risky than those firms and countries in the early stages of growth.

Political Risk

There are at least four components of political risk, including the level of corruption in the country, the occurrences of physical violence due to wars or civil unrest, the possibility of nationalization and expropriations, and continuous versus discontinuous risks.

Legal Risk

The protection of property rights (i.e., the structure of the legal system) and the speed with which disputes are settled (i.e., the efficiency of the legal system) affect risk.

Economic Structure

A disproportionate reliance on a single commodity or service in an economy increases a country's risk exposure.

Investors must assess all of the sources of country risk not only in isolation but also in conjunction with each other, when analyzing non-domestic investment opportunities.

Evaluating Country Risk

There are numerous services that attempt to evaluate country risk in its entirety, including:

- **Political Risk Services** (**PRS**). This for-profit firm (i.e., the risk assessments are available to paying members of the service) evaluates more than 100 countries on the key areas of country risk.
- ***Euromoney.*** The magazine surveys 400 economists who assess country risk factors and rank countries from 0 to 100 with higher numbers indicating lower risk.
- ***The Economist.*** Currency risk, sovereign debt risk, and banking risks are assessed by the magazine to develop country risk scores.
- **The World Bank**. The World Bank compiles risk measures that assess six areas, including the level of corruption, government effectiveness, political stability, rule of law, voice and accountability, and regulatory quality.

There are many limitations associated with these risk services that may diminish their value to businesses and investors. First, there is no standardization across the information providers. Second, the methodologies used to generate scores are often developed by non-businesses and may have more relevance to economists and policymakers than to businesses and investors. Third, the scores are better used as rankings than as true scores.

Sovereign Defaults

Throughout history, some governments have relied on debt borrowed from other countries or banks in those countries. The debt, denominated in the foreign currency, is called **foreign currency debt**. Countries often default on foreign currency debt. Countries may be without the foreign currency to meet the obligation and are unable to print money in a foreign currency to repay the debt. A large proportion of sovereign defaults are foreign currency defaults.

Many of the countries that defaulted on foreign currency debt over the last several decades were simultaneously defaulting on local (i.e., domestic) country debt. There are three reasons that help explain local currency defaults.

1. **The Gold Standard.** The gold standard limited the amount of currency a country could print, reducing its flexibility in terms of printing currency to repay debt.

2. **Shared Currency** (**e.g., the euro**). A shared currency limits the abilities of individual countries to print money.

3. **Currency Debasement.** Printing money may devalue and debase the currency. It also leads to higher inflation, sometimes exponentially higher inflation.

Consequences of Sovereign Default

- **GDP growth.** Gross domestic product (GDP) growth falls between 0.5% and 2.0% following a sovereign default. However, the decline is short-lived.
- **Sovereign ratings and borrowing costs**. One study finds that ratings of countries that have defaulted at least once since 1970 are one to two grades lower than the ratings of similar countries that have not defaulted.
- **Trade retaliation**. Sovereign default can cause trade retaliation.
- **Fragile banking systems**. One study finds, based on 149 countries between 1975 and 2000, that there is a 14% probability of a banking crisis following a sovereign default, which is 11% higher than for non-defaulting countries.
- **Political change**. Sharp currency devaluations often follow defaults.

Factors Influencing Sovereign Default Risk

1. **The country's level of indebtedness**. The level of indebtedness is the most fundamental factor used to determine the risk of default. One must consider not only the country's debts to foreign banks and investors, but also the amount the country owes its own citizens (e.g., for social safety nets such as welfare and universal health care).

2. **Pension funds and social services**. Countries with greater pension commitments and health care commitments (e.g., Medicare in the United States) have higher default risk.

3. **Tax receipts**. The greater the tax receipts, the more able a country is to make debt payments.

4. **Stability of tax receipts**. Governments must pay debt obligations in both good and bad economic times. This means the revenue stream must be stable to meet these fixed obligations.

5. **Political risk**. Autocracies may be more likely to default than democracies because defaults put pressure on, and may cause a change in, the leadership of the country.

6. **Backing from other countries/entities**. It is assumed that stronger countries will help weaker countries and not allow them to default.

In summary, sovereign default risk is multi-faceted and must be analyzed from many perspectives.

How Rating Agencies Measure Risk

The three main rating agencies use similar processes to determine sovereign ratings. The ratings process includes:

- **Evaluating factors that may contribute to default.** These factors are related to the economic, political, and institutional detail of the country with respect to its ability to repay debt.
- **Ratings recommendation.** An analyst prepares a draft report, recommending the rating. A committee debates each category and then votes on a score.
- **Foreign currency versus local currency ratings.** The local currency rating is generally equal to or better than the foreign currency rating.
- **Ratings review process.** Ratings are reviewed on a periodic basis.

Rating agencies have been criticized on a number of counts. These include:

- **Ratings are biased upward**. Some argue that rating agencies are too optimistic when it comes to rating sovereigns and corporations.
- **Herd behavior**. When one agency upgrades or downgrades a country, the other agencies tend to follow suit.
- **Not timely enough**. Investors need rating agencies to update ratings in a timely fashion.
- **Overreaction leads to a vicious cycle**. Some argue that the agencies overreact to crises, lowering ratings too much in response to a crisis, creating a feedback effect that worsens the crisis.
- **Ratings failures**. Multiple ratings changes imply that the agencies were incorrect in their initial assessments of the country.

The Sovereign Default Spread

The **sovereign default spread** is generated by the market and is continuously updated as sovereign bonds are traded. It is in essence a comparison of the sovereign bond yield compared to a riskless investment in the same currency and maturity.

Advantages of Default Risk Spreads

1. **Changes occur in real time**. Market-based spreads are more dynamic than ratings. As bonds trade and bond yields rise and fall, default risk spreads change, revealing information about the market's perception of risk.

2. **Granularity**. Default risk spreads are more granular.

3. **Adjust quickly to new information**. Yield spreads adjust more quickly to new information regarding the sovereign relative to bond ratings. This means investors are signaled earlier of impending threats and can adjust portfolios accordingly.

Disadvantages of Default Risk Spreads

1. **Need for a risk-free security.** In order to calculate a default risk spread, there must be a risk-free security in the currency in which the bonds are issued.

2. **Cannot compare local currency bonds.** Local currency bonds do not have a risk-free security with which to compare.

3. **Greater volatility.** Default risk spreads are volatile and changes in spreads may be affected by variables that are unrelated to the default risk of the sovereign.

External and Internal Ratings

Cross Reference to GARP Assigned Reading – de Servigny & Renault, Chapter 2

External Credit Ratings

External rating scales are designed to convey information about either a specific instrument, called an **issue-specific credit rating**, or information about the entity that issued the instrument, which is called an **issuer credit rating**, or both.

The **ratings process** usually consists of the following steps:

1. Conducting qualitative analysis (e.g., competition and quality of management).

2. Conducting quantitative analysis, which would include financial ratio analysis.

3. Meeting with the firm's management.

4. Meeting of the committee in the rating agency assigned to rating the firm.

5. Notifying the rated firm of the assigned rating.

6. Opportunity for the firm to appeal or offer new information.

7. Disseminating the rating to the public via the news media.

Researchers have composed tables (known as a transition matrices) that show the frequency of default, as a percent, over given time horizons for bonds that began the time horizon with a given rating. These tables use historical data to report that for bonds that began a 5-year period with an Aa rating, for example, a certain percent defaulted during the five years. These tables demonstrate that the higher the credit rating, the lower the default frequency.

The probability of default given any rating at the beginning of a cycle *increases with the horizon*. The increase in the default rates, or cumulative default rate, is much more dramatic for non-investment grade bonds. In addition to the condition of the firm, forecasted events in the horizon will affect the probabilities. Ratings agencies try to give *a rating that incorporates the effect of an average cycle*. This practice leads to the ratings being relatively stable over an economic or industrial cycle. Unfortunately, this averaging practice may lead to an over- or underestimate during periods when the economic conditions deviate too far from an average cycle.

For a *given rating category, default rates can vary from industry to industry* (e.g., a higher percentage of banks with a given rating will default when compared with firms in other industries with the same rating). However, *geographic location does not seem to cause a similar variation of default* for a given rating class.

The evidence supporting the impact of ratings changes on bonds is not surprising:

- *A rating downgrade* is likely to make the *bond price decrease* (stronger evidence).
- *A rating upgrade* is likely to make the *bond price increase* (weaker evidence).

For stocks, the change in bond ratings has an even more asymmetric effect on the stock prices than it does on bond prices:

- *A rating downgrade* is likely to lead to a *stock price decrease* (moderate evidence).
- *A rating upgrade* is somewhat likely to lead to a *stock price increase* (evidence is mixed).

Internal Credit Ratings

Banks have increasingly been formulating their own internal ratings systems, which can vary from bank to bank. A given bank may have more than one system, such as an **at-the-point approach**, to score a company. This approach's goal is to predict the credit quality over a relatively short horizon of a few months or, more generally, a year. Banks use this approach and employ quantitative models (e.g., logit models) to determine the credit score.

A bank may also use a **through-the-cycle approach**, which focuses on a longer time horizon and includes the effects of forecasted cycles. The approach uses more qualitative assessments. Given the stability of the ratings over an economic cycle, when using through-the-cycle approaches, high-rated firms may be underrated during growth periods and overrated during the decline of a cycle.

Capital Structure in Banks

Cross Reference to GARP Assigned Reading – Schroeck, Chapter 5

Credit Risk Factors

The **probability of default (PD)** is the likelihood that a borrower will default.

The **exposure amount (EA)**, sometimes referred to as exposure at default (EAD), is the loss exposure stated as a dollar amount (e.g., the loan balance outstanding).

The **loss rate (LR)**, sometimes referred to as loss given default (LGD), represents the likely percentage loss if the borrower defaults. Note that, by definition, LR = 1 – recovery rate (RR). Therefore, the factors that affect the loss rate will also impact the recovery rate.

Expected Loss

Expected loss (**EL**) is defined as the anticipated deterioration in the value of a risky asset that the bank has taken onto its balance sheet. EL is calculated as the product of EA, PD, and LR:

$$EL = EA \times PD \times LR$$

Unexpected Loss

Unexpected loss (UL) represents the variation in expected loss. The observation that the unexpected loss represents the variability of potential losses can be modeled using the typical definition of standard deviation.

$$UL = EA \times \sqrt{PD \times \sigma^2_{LR} + LR^2 \times \sigma^2_{PD}}$$

Portfolio Expected and Unexpected Loss

Expected loss on the portfolio, EL_P, is the sum of the expected losses of each asset:

$$EL_P = \sum_i EL_i = \sum_i (EA_i \times LR_i \times PD_i)$$

The calculation of portfolio unexpected loss (UL_P) is more complicated from the cross-terms in the variance formula for an N-asset portfolio:

$$UL_P = \sqrt{\sum_i \sum_j \rho_{ij} UL_i UL_j}$$

where each individual unexpected loss follows the unexpected loss equation discussed previously. In the special case where each $\rho_{ij} = 1$ for $i \neq j$, UL_P = sum of individual unexpected losses. In most cases, UL_P will be significantly less than the sum of individual UL_i.

This equation demonstrates that the risk of the portfolio is much less than the sum of the individual risk levels and illustrates that each asset contributes to only a portion of its unexpected loss in the portfolio. This effect is captured by the partial derivative of UL_P with respect to UL_i. Hence, the **risk contribution** (RC), also known as the **unexpected loss contribution** (ULC), is defined as:

$$RC_i = \frac{UL_i \sum_j UL_j \rho_{ij}}{UL_P}$$

Economic Capital

Banks set aside credit reserves in preparation for expected losses. However, for unexpected losses, banks need to estimate the excess capital reserves needed to cover any unexpected losses. The excess capital needed to match the bank's estimate of unexpected loss is referred to as **economic capital**.

The amount of economic capital needed to absorb credit losses is the distance between the unexpected (negative) outcome and the expected outcome for a given confidence level.

Modeling Credit Risk

When estimating economic capital for credit risk, we are largely concerned with the tail of the chosen loss distribution. Credit risks are not normally distributed and tend to be highly skewed, because maximum gains are limited to receiving promised payments while extreme losses are very rare events. Therefore, in practice, a beta distribution is commonly applied in credit risk modeling.

The bottom-up risk measurement framework that attempts to quantify credit risk has several limitations:

1. Credits are presumed to be illiquid assets.
2. Credit risk models used in practice only use a one-year estimation horizon.
3. Other risk components (such as operational risk and market risk) are separated from credit risk and, thus, managed and measured in different departments within the bank.

Operational Risk

Cross Reference to GARP Assigned Reading – Hull, Chapter 23

Operational Risk Capital Requirements

The Basel Committee has proposed three approaches for determining the operational risk capital requirement (i.e., the amount of capital needed to protect against the possibility of operational risk losses): (1) the basic indicator approach, (2) the standardized approach, and (3) the advanced measurement approach. The **basic indicator approach** and the **standardized approach** determine capital requirements as a multiple of gross income at either the business line or institution level. The **advanced measurement approach** (AMA) offers institutions the possibility to lower capital requirements in exchange for investing in risk assessment and management technologies.

With the basic indicator approach, operational risk capital is based on 15% of the bank's annual gross income over a 3-year period. Gross income in this case includes both net interest income and noninterest income. For the standardized approach, the bank uses eight business lines with different **beta factors** to calculate the capital charge. With this approach, the beta factor of each business line is multiplied by the annual gross income amount over a 3-year period. The results are then summed to arrive at the total operational risk capital charge under the standardized approach.

Operational Risk Categories

The Basel Committee on Banking Supervision disaggregates operational risk into seven types. A majority of the operational risk losses result from clients, products, and business practices.

1. Clients, products, and business practices.
2. Internal fraud.
3. External fraud.
4. Damage to physical assets.
5. Execution, delivery, and process management.
6. Business disruption and system failures.
7. Employment practices and workplace safety.

Loss Frequency and Loss Severity

Operational risk losses can be classified along two dimensions: loss frequency and loss severity. **Loss frequency** is defined as the number of losses over a specific time period (typically one year), and **loss severity** is defined as the value of financial loss suffered (i.e., the size of the loss). It can be reasonably assumed that these two dimensions are independent.

Loss frequency is most often modeled with a **Poisson distribution** (a distribution that models random events).

Loss severity is often modeled with a **lognormal distribution**. This distribution is asymmetrical (the frequency of high-impact, low-frequency losses is not equal to the frequency of low-impact, high-frequency losses) and fat-tailed (rare events occur more often than would be indicated by a normal distribution).

Loss frequency and loss severity are combined in an effort to simulate an expected loss distribution (known as **convolution**). The best technique to accomplish this simulation is to use a **Monte Carlo simulation** process. With this process, we make random draws from the loss frequency data and then draw the indicated number of draws from the loss severity data. Each combination of frequency and severity becomes a potential loss event in our loss distribution. This process is continued several thousand times to create the potential loss distribution.

Data Limitations

The historical record of operational risk loss data is currently inadequate. This creates challenges when trying to accurately estimate frequency and severity. Given the extreme risk that operational problems create, firms are beginning to build a database of potential loss events. Compared to credit risk losses, the data available for operational risk losses is clearly lacking. For example, firms can rely on credit rating agencies to get a clear view of default probabilities and expected losses when assessing credit risk.

It is recommended that banks use internal data when estimating the frequency of losses and utilize both internal and external data when estimating the severity of losses. Regarding external data, there are two data sources available to firms: sharing agreements with other banks and public data.

Another method for obtaining additional operational risk data points is to use **scenario analysis**. Regulators encourage the use of scenarios since this approach allows management to incorporate events that have not yet occurred. This has a positive effect on the firm since management is actively seeking ways to immunize against potential operational risk losses. The drawback is the amount of time spent by management developing scenarios and contingency plans.

Forward-Looking Approaches

One of the most frequently used tools in operational risk identification and measurement is the **risk and control self assessment** (RCSA) program. The basic approach of an RCSA is to survey those managers directly responsible for the operations of the various business lines. It is presumed that they are the closest to the operations and are, therefore, in the best position to evaluate the risks. The problem with this assumption is that you cannot reasonably expect managers to disclose risks that are out of control. Also, a manager's perception of an appropriate risk-return tradeoff may be different than that of the institution. A sound risk management program requires that risk identification and measurement be independently verified.

The identification of appropriate **key risk indicators** (KRIs) may also be very helpful when attempting to identify operational risks. Examples of KRIs include employee turnover and the number of transactions that ultimately fail. In order to be valuable as risk indicators, the factors must (1) have a predictive relationship to losses and (2) be accessible and measurable in a timely fashion. The idea of utilizing KRIs is to provide the firm with a system that warns of possible losses before they happen.

Scorecard Data

One method for allocating capital is the **scorecard approach**. This approach involves surveying each manager regarding the key features of each type of risk. Questions are formulated, and answers are assigned scores in an effort to quantify responses. The total score for each business unit represents the total amount of risk. Scores are compared across business units and validated by comparison with historical losses.

The objective of the scorecard approach is to make business line managers more aware of operational risks and the potential for losses from those risks. It also encourages senior management to become more involved with the risk management process.

Governance Over Stress Testing

Cross Reference to GARP Assigned Reading – Siddique and Hasan, Chapter 1

Effective governance and controls are critical to ensuring that stress tests are conducted appropriately and are subject to adequate oversight. Key elements of effective governance and controls over stress testing include the governance structure, policies and procedures, documentation, validation and independent review, and internal audit.

Responsibilities of the Board and Senior Management

Board of Directors

The board of directors has oversight for an organization's key strategies and decisions and is responsible and accountable for the entire organization.

Stress testing results are important because they are used to inform the board of the institution's risk appetite and risk profile, as well as its operating and strategic decisions. However, boards should view results with some degree of skepticism and should not rely on a single stress test exercise, but rather supplement it with other tests and quantitative and qualitative information.

Senior Management

Senior management is accountable to the board and is responsible for the satisfactory implementation of the stress testing activities authorized by the board. This entails establishing robust policies and procedures to ensure compliance with these activities, reviewing and coordinating stress test activities, and remedying any issues.

Senior management should ensure that there is a sufficient range of stress testing activities to evaluate risks. Results should be benchmarked and regularly updated given that risks, data sources, and the operating environment can change. An independent auditor should verify test results.

Senior management should regularly report back to the board on stress testing results and developments and on the adequacy of compliance procedures. Reports should be clear and concise, and they should explain the main elements of the stress testing activities, their assumptions, and any limitations.

Policies, Procedures, and Documentation

Clear and comprehensive policies, procedures, and documentation are critical in codifying an institution's risk practices, including its stress testing activities. Policies and procedures should be clear and concise and reviewed and approved annually.

Appropriate documentation of stress testing policies and procedures is also critical. Documentation should include a description of the stress tests used, their results, main assumptions, any limitations and constraints, and the appropriate remedies. Appropriate documentation allows senior management to both track and analyze results over time and is useful to stress test developers.

Stress test policies, procedures, and documentation should address a range of issues, including describing the purpose and process of stress test activities, establishing consistent and adequate practices, defining roles and responsibilities, determining the frequency and priority of stress testing activities, and describing proposed remedies.

Validation and Independent Review

Ongoing validation and independent review of stress testing activities is an important component of an institution's governance. Validation and independent review should be unbiased and critical, and they should form part of the institution's overall validation and review processes.

Results should be compared with appropriate benchmarks that reflect the institution's risks and exposures.

Stress tests for nonstress periods (i.e., "good times") should be incorporated into an institution's model to test their predictive power and usefulness. Models used in nonstress periods may require a different set of assumptions given changing risks, correlations, and behavior by market participants.

Institutions do not need to fully validate stress tests, but limitations, challenges, and proposed remedies should be communicated and disclosed in a transparent manner.

Validation and independent review should challenge the review process and the qualitative components of the stress test, implement development standards, validate and implement stress tests, and monitor performance.

Role of Internal Audits

The internal audit is intended to assess the integrity and reliability of an institution's policies and procedures, including those pertaining to stress tests.

The internal audit should verify that stress tests are conducted thoroughly and as intended, by staff with the relevant expertise. An internal audit should also review the procedures pertaining to the documentation, review, and approval of stress tests. Any deficiencies in stress tests should also be identified.

Key Aspects of Stress Testing Governance

As part of its overall stress testing governance framework, institutions should also consider stress testing coverage, stress testing types and approaches, and capital and liquidity stress testing. While these aspects will differ by institutions, senior management should review and evaluate them and present them for board review.

Stress testing coverage should incorporate the relationship between risks and exposures in the short term and long term, and detect risk concentrations that could negatively impact an institution. Coverage can be applied to individual exposures, the entire institution, or various sublevels within an institution.

Stress testing types and approaches should consider the firm-specific and system-wide impacts of stresses from historical and hypothetical scenarios and should include stresses that challenge the entire institution's viability. When stress testing is complemented with scenario analysis, scenarios should be robust and credible.

Capital and liquidity stress testing should be harmonized with overall strategy and planning and updated for all material results and events. The impact of multiple simultaneous risks should be considered, including capital and liquidity problems that could arise simultaneously and magnify risks. Objectives for capital and liquidity funding costs should be clearly articulated. Excess exposures and areas should be identified where liquidity and capital positions can be strengthened.

Stress Testing and Other Risk Management Tools

Cross Reference to GARP Assigned Reading – Siddique and Hasan, Chapter 2

The Role of Stress Testing

Stress testing, specifically supervisory stress tests for regulatory purposes, assists in obtaining an enterprise-wide understanding of risk. With stress testing and value at risk (VaR) measures, using different data and scenarios could result in different results.

Both stress tests and VaR measures [including economic capital (EC) measures] attempt to transform scenarios into loss estimates. The loss estimates' distributions provide the basis to compute VaR at a very high confidence level. Stress tests tend to look at far fewer scenarios compared to VaR measures.

Additionally, stress tests usually develop scenarios that are conditional. In contrast, VaR/EC measures tend to develop unconditional scenarios.

Complementing Stress Tests with VaR Models

VaR/EC models compute expected losses using the following formula:

$$\text{expected loss} = PD \times LGD \times EAD$$

where:
PD = probability of default
LGD = loss given default
EAD = exposure at default

In an attempt to assign a probability to a hypothetical (i.e., prospective) or historical stress scenario, one could determine where the stress test losses fall within the VaR/EC loss distribution. By assigning probabilities to outcomes, the calculated probability from the loss distribution facilitates the implementation of stress test results.

Stressed Inputs and Stressed VaR

Stressed inputs have been used in analyzing market risk and for both supervisory and internal purposes within financial institutions. The revised Basel market risk capital framework mandates the use of stressed inputs—for example, a **stressed value at risk** (SVaR) measure.

Stressed VaR can be used to analyze the potential losses for an investment portfolio. In addition, it can be used to compute the capital charge for credit valuation adjustments (CVAs). The CVA represents the expected value or price of counterparty credit risk.

Stressed Risk Metrics Advantages and Disadvantages

A key advantage of using stressed risk metrics is that they are conservative. A key disadvantage is that risk metrics will not necessarily respond to current market conditions.

Principles for Sound Stress Testing Practices and Supervision

Cross Reference to GARP Assigned Reading – Basel Committee on Banking Supervision

Stress Testing in Risk Management

Stress testing is an important risk management tool that enables a bank to identify the potential sources of risk, evaluate the magnitude of risk, develop tolerance levels for risk, and generate strategies to mitigate risk.

Recent financial turmoil has substantially enhanced the significance of comprehensive, flexible, and forward-looking stress testing.

Stress Testing and Risk Governance

Major weaknesses and recommendations for stress testing and integration in risk governances are as follows. Weaknesses: lack of involvement of board and senior management, lack of overall organizational view, lack of fully developed stress testing, and lack of adequate response to crisis. Recommendations: Stress testing should form an essential ingredient of overall governance of risk management plan, encompass multiple techniques and perspectives, involve a sound infrastructure and regular assessment, produce written policies and recommendations, and generate comprehensive firm- and market-wide scenario testing.

Stress Testing Methodologies

Stress testing methodologies were based on inadequate infrastructure, inadequate risk assessment approaches, inadequate recognition of correlation, and inadequate firm-wide perspectives. Given these weaknesses, recommendations for improvement include development of a comprehensive stress testing approach, identification and control of risk concentrations, and multiple measurements of stress impact.

Stress Testing Scenarios

Stress testing scenarios lacked depth and breadth because they were based on mild shocks, shorter duration, and smaller correlation effects among various markets, portfolios, and positions.

Stress Test Handling

Banks evaluated the risk of complex structured products based on the credit rating of similar cash instruments. However, the nature, magnitude, and sources of risk for these products are different from non-structured products. In order to identify, assess, monitor, and control risk exposure of complex structured products, stress testing plans should utilize all the relevant information about the underlying asset pool, market conditions, contractual obligations, and subordination levels.

Recommendations to Supervisors

Principles for sound stress testing supervision include assessing stress testing methods, taking corrective actions, challenging firm-wide scenarios, evaluating capital and liquidity needs, applying additional stress scenarios, and consulting additional resources.

Essential Exam Strategies

The level of review contained in this section is different from our other FRM review materials. As always, our objective is to enhance your chances of passing the FRM exam. Unlike the previous part of this book, which covers *what* you need to know to pass the FRM Part I exam, this section provides you with important guidance on *how* to pass the exam. By this time, you have likely studied the entire Part I curriculum and have a solid grasp on the content, so we won't spend any time here reviewing or quizzing you on material you already know. Instead, we provide insights about how to successfully apply your hard-earned knowledge on exam day.

First, we provide some proven approaches to mastering the FRM Part I curriculum. Next, we present a structured plan for the last weeks before the exam. Following this plan ensures that you will be sharp on exam day, and your performance will not be adversely affected by your nerves.

A Formidable Task

Over the past few months, you have studied an enormous amount of material. The FRM Part I curriculum covers 69 readings and almost 500 learning objectives (LOs). This is a huge amount of material. Realistically speaking, it is virtually impossible to remember every detail within the curriculum. However, the good news is that you don't have to know every detail. From this guide, you will learn how to get the most benefit from the short time remaining until the exam.

As you prepare for the FRM exam, try to focus on the exam itself. Don't add to your stress level by worrying about whether you'll pass or what might happen if you don't. If you worry about it before the exam, or especially during the exam, your performance will likely suffer. There is ample stress in remembering the material, let alone worrying whether you'll pass. Many of the tips we provide here are proven stress reducers on exam day. Your grasp of the content, combined with the tips we provide, will have you well prepared for the exam experience.

Basic strategies you should follow while learning the FRM curriculum include being aware of the big picture and knowing the main concepts. With these broad strategies, you will be able to apply more specific strategies related to helping you recall information on exam day, such as how to handle formulas and lists.

The Big Picture

Being aware of the big picture means you should know at least a little about every concept. For example, even if you don't know the formula for effective duration, at least know that effective duration is a measure of interest rate risk. By remembering basic information on exam day, you will be able to narrow your answer choices. You probably won't answer many questions correctly with only a basic grasp of the concepts, but you can improve your odds on a multiple-choice question from 25% to 33% by eliminating one wrong answer choice. With this technique, you will also be able to better distinguish between relevant and irrelevant information in a question. Continuing with the duration theme, you would know that bond rating information provided in a duration question is not relevant because bond ratings reflect credit risk, not interest rate risk.

Know the Main Concepts

It is important to identify those concepts that will most likely be covered on the exam. In any given year, some concepts might be omitted, but if you can answer most of the questions concerning these main/core concepts, you will dramatically increase your chance of passing. Generally, the idea is to be correct on most of the questions dealing with the important concepts and then rely on your "big picture" knowledge to get points on the remaining material.

Formulas

In some cases, you will be given a question where the answer can be obtained by using a formula and a fairly lengthy calculation. However, you may also be able to identify the correct answer without a calculation, if you truly understand the concept or relationship being tested. With any formula you encounter in the assigned readings, you should try to gain a clear understanding of what it is telling you (when it is appropriate to use it) and the relationship among the various input variables.

Think of a formula as just a shorthand way of expressing a relation or concept you need to understand. For example, the population variance is the average squared deviation from the mean. Approaching formulas in this way will reduce your chances of missing a problem if your memory fails you under the stress of exam day.

"Characteristic" Lists

A common source of specific questions is identifying the characteristics of various securities, models, and valuation methods. A common question format would be, "Which of the following most accurately describes . . . ?" Here, a big-picture approach can help you weed out wrong answers. Also, some candidates use mnemonics to help them remember lists of characteristics or lists of pros and cons.

Problem Areas

In addition to these strategies, be aware of your problem areas regarding certain topics. For example, if you have always struggled with Quantitative Analysis, look for ways to improve your grasp of the quantitative material by spending more time with it or attending an instructor-led training session. Do not expect that you can ignore an entire topic area and make up for the lost points by excelling in another area. Similarly, do not skip an area just because you think you already know it. You need to be reasonably comfortable with all assigned material in the FRM curriculum in order to pass the FRM Exam.

Final Preparation

Have a well-defined strategy for the last weeks before the FRM exam. If at all possible, it is best to take some leave from your job. You should save at least one FRM practice exam for the last week. To simulate the real exam, you should avoid looking at this exam or studying questions from it until you are ready to sit down and take it for the first time. Take this exam early in the final weeks. Time yourself so you can get a feel for the time constraints and pressure of exam day. Remember, you have an average of about 2.5 minutes per question. When you have completed the entire exam, grade your answers and use these results to identify areas where you need to focus your study efforts over the last few days. You should devote most of your time to areas where you performed poorly, but you should also spend enough time keeping your stronger topics fresh in your mind.

At some point during the week of the exam, it is a good idea to visit the actual exam center. Figure out how long it will take to get to your test center and where you can park. The fewer surprises and distractions you have on exam day, the better.

During the evening before the FRM exam, it is best to avoid cramming. Try to relax and make a concerted effort to get a good night's sleep. Tired candidates tend to make silly mistakes on the exam. If you are not rested, you will more than likely miss easy points. This seems like an obvious and trite point, but it is difficult to overemphasize the importance of going into the exam well-rested. If, for some unfortunate reason, you do not sleep well the night before, do not panic! It happens to the best of us. Sometimes your brain cannot stop thinking about the pressures of the upcoming day. Keep in mind that you can still function and give a solid effort on exam day with just a little sleep (even though it is not recommended).

Expect problems on exam day—not major ones, but be prepared for things like cold or hot exam rooms, excessive noise, long lines, and so on. Some of these problems you cannot control, but if you are prepared for them, they are less likely to affect your exam performance.

Tips for Answering Exam Questions

Here are some tips to keep in mind as you practice working through multiple-choice questions for the FRM exam.

1. Read the question carefully!

   Watch for double negatives like, "Which of the following is least likely a disadvantage?" It is very important to not miss words, or parts of words, by reading too quickly (e.g., reading "most likely" instead of "least likely," or "advantages" instead of "disadvantages").

2. For nonnumerical questions, read all answer choices.

   Don't just stop when you get to one that sounds right. There may be a better choice.

3. For long questions, dissect the bits of information that are provided.

   What information is relevant? What is most specifically related to the question? Often, a wrong answer looks good because it is consistent with information in the question that is actually irrelevant.

4. After you read the question, determine what you think the question is asking.

   This can help you filter out extraneous information and focus quickly on appropriate answer choices. Similarly, after you read the question, it is a good idea to formulate your own answer before reading the answer choices. Develop an expectation of what the answer should be. This may make the correct answer sound better to you when you read it.

5. On calculation problems, after you have selected an answer, pause for a moment.

   Think about whether the answer makes sense, is the sign of the result correct, or does the direction of change make sense?

6. Do not look for patterns in answers.

   Just because the last three questions all had "C" for an answer, do not expect the next answer not to be "C." There is no reason to expect that GARP has a preference for how many questions are answered with the same letter.

7. Be very sure that you are marking your answer in the right place on the answer sheet.

   If you skip questions, be especially careful to check your answers. Obviously, mismarking can be devastating if you do not catch it right away. Trust your first impressions. You will find that you are often correct.

8. Finally, and probably most importantly, do not lose confidence.

   To our knowledge, no one has ever received a perfect score on the FRM exam. 70% has always been a good approximation of the passing score needed. This means you can miss 30% of the questions and still pass. Even if you have struggled on a few questions, maybe even five or six in a row, do not lose confidence. The worst thing you can do is second-guess yourself. Once you start second-guessing, you may take longer on every question and start changing correct answers.

What to Do With Difficult Exam Questions

There will undoubtedly be questions on the FRM exam that give you trouble. You might not understand the question, may think that none of the answers make sense, or simply may not know the concept being tested. The following tips will prove to be useful if you find yourself facing a difficult question on the FRM exam:

- If the question does not make sense, or if none of the answers look remotely correct, reread the question to see if you missed something. If you are still unsure, mark an answer choice and move on. Don't agonize over it and waste precious time that can be allocated to easier questions.
- Never leave an answer blank. A blank answer has a maximum point value of zero. A randomly marked answer has an expected value of 0.25 points, and if you can eliminate one bad answer, this value increases to 0.33 points. You will not be penalized for wrong answers.

- If you are unable to determine the "best" answer, you still should be able to help your odds. Try to eliminate one answer choice and then take an educated guess.
- Take some comfort in the fact that the FRM exams are likely graded on a curve. If a question gave you trouble, it is quite possible that it was troublesome for many other candidates as well.
- Do not lose your confidence over one, or even several, tough questions.

It is very difficult to generalize FRM exam questions. Some are straightforward, some look straightforward but have a trick to them, and some may be worded in a confusing way. Keep in mind that GARP's objective is to evaluate your grasp of the FRM curriculum, not to confuse or frustrate you with difficult questions.

Time Management

Candidates who fail the FRM exam often cite time management as their biggest downfall. The following tips will help you manage your time more wisely on exam day:

- Take at least one practice exam where you time yourself. This will give you some indication of whether you will have problems on exam day. However, do not let your positive results on practice exams lull you into overconfidence. The stress of exam day, plus possible distractions like noise or a cold exam room, can make a big difference on how fast you work.
- Monitor your progress. Keep an eye on the time as you work through the exam. You may deviate some from the average time per question as you work through easier or more difficult questions, but be careful not to let yourself fall too far behind. Note that some test centers may not have clocks, but your exam proctors will notify you of how much time is remaining.
- One way to alleviate time pressure is to bank a few minutes by doing easy questions first. Select a question you feel comfortable with and start there. This strategy will help you gain confidence as you progress through the exam and will also allow you to get ahead with your time allocation. Be very careful if you jump around to make sure you are marking the correct blanks on the answer sheet.
- Have a game plan before you walk into the exam center. We like the idea of doing short/easy questions first to get going, but we do not recommend skipping around too much as you work through the exam. Skipping back and forth may break your concentration and consume valuable time as you try to figure out what you have and have not answered.
- Finally, never panic! Even if you fall behind, panicking will only make things worse. You won't think clearly, and you'll miss easy questions. If you need a short break, put your pencil down and take a few deep breaths. The 30 seconds or so that this takes may very well help you think clearly enough to answer several additional questions correctly.

Index

D

Q

R

S

T

U

V

W

Y

Z