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Exam Part I

Valuation and Risk Models

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Options, Futures, and Other Derivatives, Ninth Edition,
by John C. Hull

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Quantifying Volatility in VaR Models

1

Learning Objectives

After completing this reading you should be able to:

- Explain how asset return distributions tend to deviate from the normal distribution.
- Explain reasons for fat tails in a return distribution and describe their implications.
- Distinguish between conditional and unconditional distributions.
- Describe the implications of regime switching on quantifying volatility.
- Evaluate the various approaches for estimating VaR.
- Compare and contrast different parametric and non-parametric approaches for estimating conditional volatility.
- Calculate conditional volatility using parametric and non-parametric approaches.
- Explain the process of return aggregation in the context of volatility forecasting methods.
- Evaluate implied volatility as a predictor of future volatility and its shortcomings.
- Explain long horizon volatility/VaR and the process of mean reversion according to an AR(1) model.
- Calculate conditional volatility with and without mean reversion.
- Describe the impact of mean reversion on long horizon conditional volatility estimation.

Excerpt is Chapter 2 of Understanding Market, Credit and Operational Risk: The Value at Risk Approach, *by Linda Allen, Jacob Boudoukh, and Anthony Saunders.*

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THE STOCHASTIC BEHAVIOR OF RETURNS

Measuring VaR involves identifying the tail of the distribution of asset returns. One approach to the problem is to impose specific distributional assumptions on asset returns. This approach is commonly termed the *parametric approach,* requiring a specific set of distributional assumptions. If we are willing to make a specific parametric distributional assumption, for example, that asset returns are normally distributed, then all we need is to provide two parameters—the mean (denoted μ) and the standard deviation (denoted σ) of returns. Given those, we are able to fully characterize the distribution and comment on risk in any way required; in particular, quantifying VaR, percentiles (e.g., 50 percent, 98 percent, 99 percent, etc.) of a loss distribution.

The problem is that, in reality, asset returns tend to deviate from normality. While many other phenomena in nature are often well described by the Gaussian (normal) distribution, asset returns tend to deviate from normality in meaningful ways. As we shall see below in detail, asset returns tend to be:

- *Fat-tailed:* A fat-tailed distribution is characterized by having more probability weight (observations) in its tails relative to the normal distribution.
- *Skewed:* A skewed distribution in our case refers to the empirical fact that declines in asset prices are more severe than increases. This is in contrast to the symmetry that is built into the normal distribution.
- *Unstable:* Unstable parameter values are the result of varying market conditions, and their effect, for example, on volatility.

All of the above require a risk manager to be able to reassess distributional parameters that vary through time.

In what follows we elaborate and establish benchmarks for these effects, and then proceed to address the key issue of how to adjust our set of assumptions to be able to better model asset returns, and better predict extreme market events. To do this we use a specific dataset, allowing us to demonstrate the key points through the use of an example.

The Distribution of Interest Rate Changes

Consider a series of daily observations of interest rates. In the series described below we plot three-month US Treasury bill (T-bill) rates calculated by the Federal Reserve. We use ten years of data and hence we have approximately 2,500 observations. For convenience let us assume we have 2,501 data points on interest rate levels, and hence 2,500 data points on daily interest rate changes. Figure 1-1 depicts the time series of the yield to maturity, fluctuating between 11 percent p.a. and 4 percent p.a. during the sample period (in this example, 1983–92).

The return on bonds is determined by interest rate *changes,* and hence this is the relevant variable for our discussion. We calculate daily interest changes, that is, the first difference series of observed yields. Figure 1-2 is a histogram of yield changes. The histogram is the result of 2,500 observations of daily interest rate changes from the above data set.

Using this series of 2,500 interest rate changes we can obtain the average interest rate change and the standard deviation of interest rate changes over the period. The mean of the series is zero basis points per day. Note that the average daily change in this case is simply the last yield minus the first yield in the series, divided by the number of days in the series. The series in our case starts at 4 percent and ends at a level of 8 percent, hence we have a 400 basis point (bp) change over the course of

FIGURE 1-1 Three-month treasury rates.

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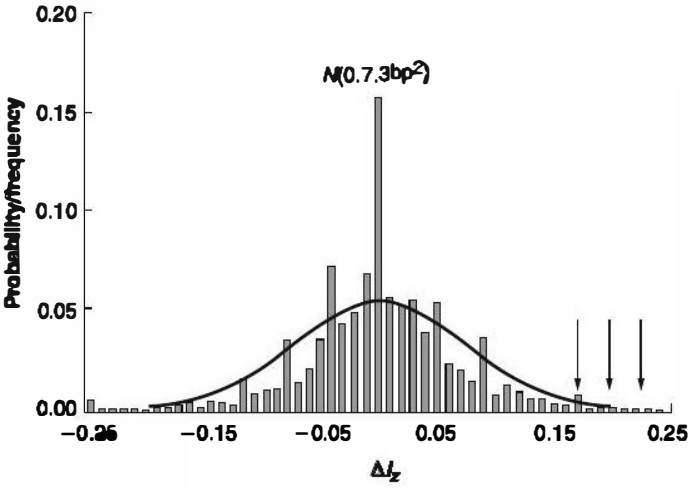

FIGURE 1-2 Three-month treasury rate changes.

2,500 days, for an average change of approximately zero. Zero expected change as a forecast is consistent with the random walk assumption as well. The standard deviation of interest rate changes turns out to be 7.3bp/day.

Using these two parameters, Figure 1-2 plots a normal distribution curve on the same scale of the histogram, with basis point changes on the *X*-axis and probability on the *Y*-axis. If our assumption of normality is correct, then the plot in Figure 1-2 should resemble the theoretical normal distribution. Observing Figure 1-2 we find some important differences between the theoretical normal distribution using the mean and standard deviation from our data, and the empirical histogram plotted by actual interest rate changes. The difference is primarily the result of the "fat-tailed" nature of the distribution.

Fat Tails

The term "fat tails" refers to the tails of one distribution *relative* to another reference distribution. The reference distribution here is the normal distribution. A distribution is said to have "fatter tails" than the normal distribution if it has a similar mean and variance, but different probability mass at the extreme tails of the probability distribution. The critical point is that the first two moments of the distribution, the mean and the variance, are the same.

This is precisely the case for the data in Figure 1-2, where we observe the empirical distribution of interest rate changes. The plot includes a histogram of interest rate changes in different probability buckets. In addition to the histogram, and on the same plot, a normal distribution is also plotted, so as to compare the two distributions. The normal distribution has the same mean (zero) and the same volatility (7.3 basis points) as the empirical distribution.

We can observe "fat tail" effects by comparing the two distributions. There is extra probability mass in the empirical distribution relative to the normal distribution benchmark around zero, and there is a "missing" probability mass in the intermediate portions around the plus ten and minus ten basis point change region of the histogram. Although it is difficult to observe directly in Figure 1-2, it is also the case that at the probability extremes (e.g., around 25bp and higher), there are more observations than the theoretical normal benchmark warrants. A more detailed figure focusing on the tails is presented later in this chapter.

This pattern, more probability mass around the mean and at the tails, and less around plus/minus one standard deviation, is precisely what we expect of a fat tailed distribution. Intuitively, a probability mass is taken from around the one standard deviation region, and distributed to the zero interest rate change and to the two extreme-change regions. This is done in such way so as to preserve the mean and standard deviation. In our case the mean of zero and the standard deviation of 7.3bp, are preserved by construction, because we plot the normal distribution benchmark given these two empirically determined parameters.

To illustrate the impact of fat tails, consider the following exercise. We take the vector of 2,500 observations of interest rate changes, and order this vector not by date but, instead, by the size of the interest rate change, in descending order. This ordered vector will have the larger interest rate increases at the top. The largest change may be, for example, an increase of 35 basis points. It will appear as entry number one of the ordered vector. The following entry will be the second largest change, say 33 basis points, and so on. Zero changes should be found around the middle of this vector, in the vicinity of the 1,250th entry, and large declines should appear towards the "bottom" of this vector, in entries 2,400 to 2,500.

If it were the case that, indeed, the distribution of interest rate changes were normal with a mean of zero and a standard deviation of 7.3 basis points, what would we expect

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of this vector, and, in particular, of the tails of the distribution of interest rate changes? In particular, what should be a one percentile (%) interest rate shock; i.e., an interest rate shock that occurs approximately once in every 100 days? For the standard normal distribution we know that the first percentile is delineated at 2.33 standard deviations from the mean. In our case, though, *losses* in asset values are related to *increases* in interest rates. Hence we examine the +2.33 standard deviation rather than the −2.33 standard deviation event (i.e., 2.33 standard deviations above the mean rather than 2.33 standard deviations below the mean). The +2.33 standard deviations event for the standard normal translates into an increase in interest rates of $\sigma \times 2.33$ or 7.3bp × 2.33 = 17bp. Under the assumption that interest rate changes are normal we should, therefore, see in 1 percent of the cases interest rate changes that are greater or equal to 17 basis points.

What do we get in reality? The empirical first percentile of the distribution of interest rate changes can be found as the 25th out of the 2,500 observations in the ordered vector of interest rate changes. Examining this entry in the vector we find an interest rate increase of 21 basis points. Thus, the empirical first percentile (21bp) does not conform to the theoretical 17 basis points implied by the normality assumption, providing a direct and intuitive example of the fat tailedness of the empirical distribution. That is, we find that the (empirical) tails of the actual distribution are fatter than the theoretical tails of the distribution.

Explaining Fat Tails

The phenomenon of fat tails poses a severe problem for risk managers. Risk measurement, as we saw above, is focused on extreme events, trying to quantify the probability and magnitude of severe losses. The normal distribution, a common benchmark in many cases, seems to fail here. Moreover, it seems to fail precisely where we need it to work best—in the tails of the distributions. Since risk management is all about the tails, further investigation of the tail behavior of asset returns is required.

In order to address this issue, recall that the distribution we examine is the *unconditional distribution* of asset returns. By "unconditional" we mean that on any given day we assume the same distribution exists, regardless of market and economic conditions. This is in spite of the fact that there is information available to market participants about the distribution of asset returns at any given point in time which may be different than on other days. This information is relevant for an asset's *conditional distribution*, as measured by parameters, such as the conditional mean, conditional standard deviation (volatility), conditional skew and kurtosis. This implies two possible explanations for the fat tails: (i) conditional volatility is time-varying; and (ii) the conditional mean is time-varying. Time variations in either could, arguably, generate fat tails in the unconditional distribution, in spite of the fact that the conditional distribution is normal (albeit with different parameters at different points in time, e.g., in recessions and expansions).

Let us consider each of these possible explanations for fat tails. First, is it plausible that the fat tails observed in the unconditional distribution are due to time-varying conditional distributions? We will show that the answer is generally "no." The explanation is based on the implausible assumption that market participants know, or can predict in advance, future changes in asset prices. Suppose, for example, the interest rate changes are, in fact, normal, with a time-varying conditional mean. Assume further that the conditional mean of interest rate changes is known to market participants during the period under investigation, but is unknown to the econometrician. For simplicity, assume that the *conditional* mean can be +5bp/day on some days, and −5bp/day on other days. If the split between high mean and low mean days were 50–50, we would observe an unconditional mean change in interest rates of 0bp/day.

In this case when the econometrician or the risk manager approaches past data without the knowledge of the conditional means, he mistakes variations in interest rates to be due to volatility. Risk is overstated, and changes that are, in truth, distributed normally and are centered around plus or minus five basis points, are mistaken to be normal with a mean of zero. If this were the case we would have obtained a "mixture of normals" with varying means, that would appear to be, unconditionally, fat tailed.

Is this a likely explanation for the observed fat tails in the data? The answer is negative. The belief in efficient markets implies that asset prices reflect all commonly available information. If participants in the marketplace know that prices are due to rise over the next day, prices would have already risen today as traders would have traded

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on this information. Even detractors of market efficiency assumptions would agree that conditional means do not vary enough on a daily basis to make those variations a first order effect.

To verify this point consider the debate over the predictability of market returns. Recent evidence argues that the conditional risk premium, the expected return on the market over and above the risk free rate, varies through time in a predictable manner. Even if we assume this to be the case, predicted variations are commonly estimated to be between zero and 10 percent on an annualized basis. Moreover, variations in the expected premium are slow to change (the predictive variables that drive these variations vary slowly). If at a given point you believe the expected excess return on the market is 10 percent per annum rather than the unconditional value of, say, 5 percent, you predict, on a daily basis, a return which is 2bp different from the market's average premium (a 5 percent per annum difference equals approximately a return of 2bp/day). With the observed volatility of equity returns being around 100bp/day, we may view variations in the conditional mean as a second order effect.

The second possible explanation for the fat tail phenomenon is that volatility (standard deviation) is time-varying. Intuitively, one can make a compelling case against the assumption that asset return volatility is constant. For example, the days prior to important Federal announcements are commonly thought of as days with higher than usual uncertainty, during which interest rate volatility as well as equity return volatility surge. Important political events, such as the turmoil in the Gulf region, and significant economic events, such as the defaults of Russia and Argentina on their debts, are also associated with a spike in global volatility. Time-varying volatility may also be generated by regular, predictable events. For example, volatility in the Federal funds market increases dramatically on the last days of the reserve maintenance period for banks as well as at quarter-end in response to balance sheet window dressing. Stochastic volatility is clearly a candidate explanation for fat tails, especially if the econometrician fails to use relevant information that generates excess volatility.

Effects of Volatility Changes

How does time-varying volatility affect our distributional assumptions, the validity of the normal distribution model and our ability to provide a useful risk measurement system? To illustrate the problem and its potential solution, consider an illustrative example. Suppose interest rate changes do not fit the normal distribution model with a mean of zero and a standard deviation of 7.3 basis points per day. Instead, the true conditional distribution of interest rate changes is normal with a mean of zero but with a time-varying volatility that during some periods is 5bp/day and during other periods is 15bp/day.

This type of distribution is often called a " regime-switching volatility model." The regime switches from low volatility to high volatility, but is never in between. Assume further that market participants are aware of the state of the economy, i.e., whether volatility is high or low. The econometrician, on the other hand, does not have this knowledge. When he examines the data, oblivious to the true regime-switching distribution, he estimates an unconditional volatility of 7.3bp/day that is the result of the mixture of the high volatility and low volatility regimes. Fat tails appear only in the unconditional distribution. The conditional distribution is always normal, albeit with a varying volatility.

Figure 1-3 provides a schematic of the path of interest rate volatility in our regime-switching example. The solid line depicts the true volatility, switching between 5bp/day and 15bp/day. The econometrician observes periods where interest rates change by as much as, say, 30 basis points. A change in interest rates of 30bp corresponds to a change of more than four standard deviations given that the estimated standard deviation is 7.3bp. According to the normal distribution benchmark, a change of four standard deviations or more should be observed very infrequently. More precisely, the probability that a truly random normal variable will deviate from the mean by four standard deviations or more is 0.003 percent. Putting it differently, the odds of seeing such a change are one in 31,560 or once in 121 years. Table 1-1 provides the number of standard deviations, the probability of seeing a random normal being less than or equal to this number of standard deviations, in percentage terms, and the odds of seeing such an event.

The risk manager may be puzzled by the empirical observation of a relatively high frequency of four or more standard deviation moves. His risk model, one could argue, based on an unconditional normal distribution with a standard deviation of 7.3bp, is of little use, since it

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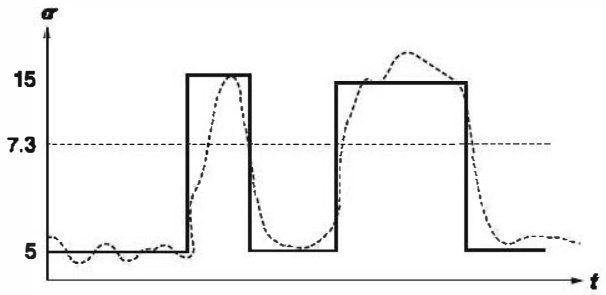

FIGURE 1-3 A schematic of actual and estimated volatility.

under-predicts the odds of a 30bp move. In reality (in the reality of our illustrative example), the change of 30bp occurred, most likely, on a high volatility day. On a high volatility day a 30bp move is only a two standard deviation move, since interest rate changes are drawn from a normal distribution with a standard deviation of 15bp/day. The probability of a change in interest rates of two standard deviations or more, equivalent to a change of 30bp or more on high volatility days, is still low, but is economically meaningful. In particular, the probability of a 30bp move conditional on a high volatility day is 2.27 percent, and the odds are one in 44.

The dotted line in Figure 1-3 depicts the estimated volatility using a volatility estimation model based on historical data. This is the typical picture for common risk measurement engines—the estimated volatility trails true volatility. Estimated volatility rises after having observed an increase, and declines having observed a decrease. The estimation error and estimation lag is a central issue in risk measurement, as we shall see in this chapter.

TABLE 1-1 Tail Event Probability and Odds Under Normality

No. of Deviations Z	Prob ($X < z$) (in %)	Odds (one in . . . days)
−1.50	6.68072	15
−2.00	2.27501	44
−2.50	0.62097	161
−3.00	0.13500	741
−3.50	0.02327	4,298
−4.00	0.00317	31,560
−4.50	0.00034	294,048
−5.00	0.00003	3,483,046

This last example illustrates the challenge of modern dynamic risk measurement. The most important task of the risk manager is to raise a "red flag," a warning signal that volatility is expected to be high in the near future. The resulting action given this information may vary from one firm to another, as a function of strategy, culture, appetite for risk, and so on, and could be a matter of great debate. The importance of the risk estimate as an input to the decision making process is, however, not a matter of any debate. The effort to improve risk measurement engines' dynamic prediction of risk based on market conditions is our focus throughout the rest of the chapter.

This last illustrative example is an extreme case of stochastic volatility, where volatility jumps from high to low and back periodically. This model is in fact quite popular in the macroeconomics literature, and more recently in finance as well. It is commonly known as regime switching.

Can (Conditional) Normality Be Salvaged?

In the last example, we shifted our concept of normality. Instead of assuming asset returns are normally distributed, we now assume that asset returns are *conditionally normally distributed*. Conditional normality, with a time-varying volatility, is an economically reasonable description of the nature of asset return distributions, and may resolve the issue of fat tails observed in unconditional distributions.

This is the focus of the remainder of this chapter. To preview the discussion that follows, however, it is worthwhile to forewarn the reader that the effort is going to be, to an extent, incomplete. Asset returns are generally non-normal, both unconditionally as well as conditionally; i.e., fat tails are exhibited in asset returns regardless of the estimation method we apply. While the use of dynamic risk measurement models capable of adapting model parameters as a function of changing market conditions is important, these models do not eliminate all deviations from the normal distribution benchmark. Asset returns keep exhibiting asymmetries and unexpectedly large movements regardless of the sophistication of estimation models. Putting it more simply—large moves will always occur "out of the blue" (e.g., in relatively low volatility periods).

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One way to examine conditional fat tails is by normalizing asset returns. The process of normalizations of a random normal variable is simple. Consider X, a random normal variable, with a mean of μ and a standard deviation σ,

$$X \sim N(\mu, \sigma^2).$$

A standardized version of X is

$$(X - \mu)/\sigma \sim N(0, 1).$$

That is, given the mean and the standard deviation, the random variable X less its mean, divided by its standard deviation, is distributed according to the standard normal distribution.

Consider now a series of interest rate changes, where the mean is assumed, for simplicity, to be always zero, and the volatility is re-estimated every period. Denote this volatility estimate by σ_t. This is the forecast for next period's volatility based on some volatility estimation model (see the detailed discussion in the next section). Under the normality assumption, interest rate changes are now conditionally normal

$$\Delta i_{t,t+1} \sim N(0, \sigma_t^2).$$

We can standardize the distribution of interest rate changes dynamically using our estimated conditional volatility σ_t, and the actual change in interest rate that followed $\Delta i_{t,t+1}$. We create a series of standardized variables.

$$\Delta i_{t,t+1}/\sigma_t \sim N(0, 1).$$

This series should be distributed according to the standard normal distribution. To check this, we can go back through the data, and with the benefit of hindsight put all pieces of data, drawn under the null assumption of conditional normality from a normal distribution with time-varying volatilities, on equal footing. If interest rate changes are, indeed, conditionally normal with a time-varying volatility, then the unconditional distribution of interest rate changes can be fat tailed. However, the distribution of interest rate changes standardized by their respective conditional volatilities should be distributed as a standard normal variable.

Figure 1-4 does precisely this. Using historical data we estimate conditional volatility. We plot a histogram similar to the one in Figure 1-2, with one exception. The X-axis here is not in terms of interest rate changes, but, instead, in terms of *standardized interest rate changes*. All periods are now adjusted to be comparable, and we may expect

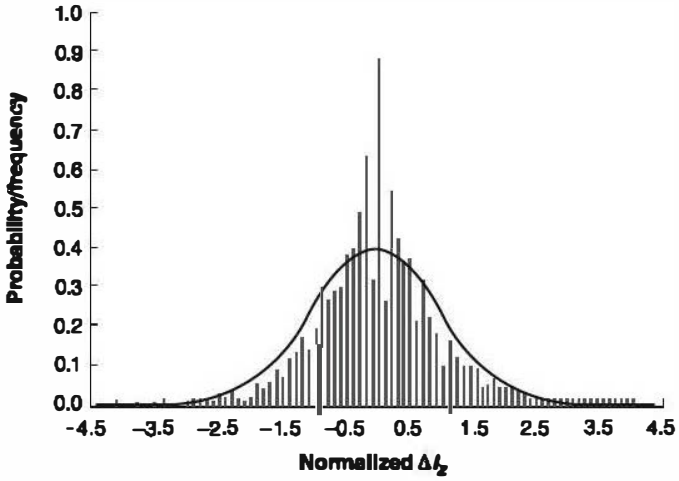

FIGURE 1-4 Standardized interest rate changes—empirical distribution relative to the $N(0, 1)$ benchmark.

to see a "well-behaved" standard normal. Standardized interest rate changes are going to be well behaved on two conditions: (i) that interest rate changes are, indeed, conditionally normal; and (ii) that we accurately estimated conditional volatility, i.e., that we were able to devise a "good" dynamic volatility estimation mechanism. This joint condition can be formalized into a statistical hypothesis that can be tested.

Normalized interest rate changes, plotted in Figure 1-4, provide an informal test. First note that we are not interested in testing for normality *per se*, since we are not interested in the entire distribution. We only care about our ability to capture tail behavior in asset returns—the key to dynamic risk measurement. Casual examination of Figure 1-5, where the picture focuses on the tails of the conditional distribution, vividly shows the failure of the conditional normality model to describe the data. Extreme movements of *standardized* interest rate movements—deviating from the conditional normality model—are still present in the data. Recall, though, that this is a failure of the joint model—conditional normality and the method for dynamic estimation of the conditional volatility. In principle it is still possible that an alternative model of volatility dynamics will be able to capture the conditional distribution of asset returns better and that the conditional returns based on the alternative model will indeed be normal.

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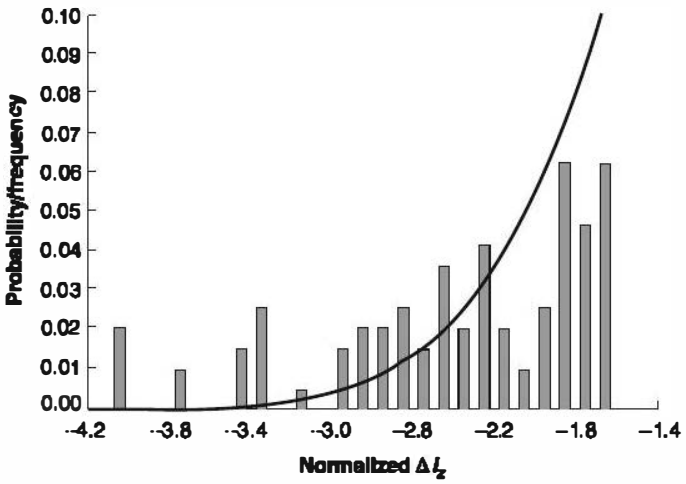

FIGURE 1-5 Tail standardized interest rate changes.

Normality Cannot Be Salvaged

The result apparent in Figure 1-5 holds true, however, to a varying degree, for most financial data series. Sharp movements in asset returns, even on a normalized basis, occur in financial data series no matter how we manipulate the data to estimate volatility. Conditional asset returns exhibit sharp movements, asymmetries. and other difficult-to-model effects in the distribution. This is, in a nutshell, the problem with all extant risk measurement engines. All VaR-based systems tend to encounter difficulty where we need them to perform best—at the tails. Similar effects are also present for the multivariate distribution of portfolios of assets—correlations as well tend to be unstable—hence making VaR engines often too conservative at the worst possible times.

This is a striking result with critical implications for the practice of risk management. The relative prevalence of extreme moves, even after adjusting for current market conditions, is the reason we need additional tools, over and above the standard VaR risk measurement tool. Specifically, the need for *stress testing* and *scenario analysis* is related directly to the failure of VaR-based systems.

Nevertheless, the study of conditional distributions is important. There is still important information in current market conditions, e.g., conditional volatility, that can be exploited in the process of risk assessment. In this chapter we elaborate on risk measurement and VaR methods.

VaR ESTIMATION APPROACHES

There are numerous ways to approach the modeling of asset return distribution in general, and of tail behavior (e.g., risk measurement) in particular. The approaches to estimating VaR can be broadly divided as follows

- *Historical-based approaches.* The common attribute to all the approaches within this class is their use of historical time series data in order to determine the shape of the conditional distribution.
  - *Parametric approach.* The parametric approach imposes a specific distributional assumption on conditional asset returns. A representative member of this class of models is the conditional (log) normal case with time-varying volatility, where volatility is estimated from recent past data.
  - *Nonparametric approach.* This approach uses historical data directly, without imposing a specific set of distributional assumptions. Historical simulation is the simplest and most prominent representative of this class of models.
  - *Hybrid approach.* A combined approach.
- *Implied volatility based approach.* This approach uses derivative pricing models and current derivative prices in order to impute an implied volatility without having to resort to historical data. The use of implied volatility obtained from the Black-Scholes option pricing model as a predictor of future volatility is the most prominent representative of this class of models.

Cyclical Volatility

Volatility in financial markets is not only time-varying, but also sticky, or predictable. As far back as 1963, Mandelbrot wrote:

> large changes tend to be followed by large changes—of either sign—and small changes by small changes. (Mandelbrot 1963)

This is a very useful guide to modeling asset return volatility, and hence risk. It turns out to be a salient feature of most extant models that use historical data. The implication is simple—since the magnitude (but not the sign) of recent changes is informative. The most recent history of returns on a financial asset should be most informative with respect to its volatility in the near future. This intuition is implemented in many simple models by placing

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more weight on recent historical data, and little or no weight on data that is in the more distant past.

Historical Standard Deviation

Historical standard deviation is the simplest and most common way to estimate or predict future volatility. Given a history of an asset's continuously compounded rate of returns we take a specific window of the K most recent returns. The data in hand are, hence, limited by choice to be $r_{t-1,t}, r_{t-2,t-1}, \ldots, r_{t-K,t-K+1}$. This return series is used in order to calculate the current/conditional standard deviation σ_t, defined as the square root of the conditional variance

$$\sigma_t^2 = (r_{t-K,t-K+1}{}^2 + \ldots + r_{t-2,t-1}{}^2 + r_{t-1,t}{}^2)/K.$$

This is the most familiar formula for calculating the variance of a random variable—simply calculating its "mean squared deviation." Note that we make an explicit assumption here, that the conditional mean is zero. This is consistent with the random walk assumption.

The standard formula for standard deviation uses a slightly different formula, first demeaning the range of data given to it for calculation. The estimation is, hence, instead

$$\mu_t = (r_{t-K,t-K+1} + \ldots + r_{t-2,t-1} + r_{t-1,t})/K,$$
$$\sigma_t^2 = ((r_{t-K,t-K+1} - \mu_t)^2 + \ldots + (r_{t-2,t-1} - \mu_t)^2 + (r_{t-1,t} - \mu_t)^2)/(K-1).$$

Note here that the standard deviation is the mean of the squared deviation, but the mean is taken by dividing by $(K-1)$ rather than K. This is a result of a statistical consideration related to the loss of one degree of freedom because the conditional mean, μ_t, has been estimated in a prior stage. The use of $K-1$ in the denominator guarantees that the estimator σ_t^2 is unbiased.

This is a minor variation that makes very little practical difference in most instances. However, it is worthwhile discussing the pros and cons of each of these two methods. Estimating the conditional mean μ_t from the most recent K days of data is risky. Suppose, for example, that we need to estimate the volatility of the stock market, and we decide to use a window of the most recent 100 trading days. Suppose further that over the past 100 days the market has declined by 25 percent. This can be represented as an average decline of 25bp/day (−2,500bp/100 days = −25bp/day). Recall that the econometrician is trying to estimate the conditional mean and volatility that were known to market participants during the period. Using −25bp/day as μ_t, the conditional mean, and then estimating σ_t^2, implicitly assumes that market participants knew of the decline, and that their conditional distribution was centered around minus 25bp/day.

Since we believe that the decline was entirely unpredictable, imposing our priors by using $\mu_t = 0$ is a logical alternative. Another approach is to use the unconditional mean, or an expected change based on some other theory as the conditional mean parameter. In the case of equities, for instance, we may want to use the unconditional average return on equities using a longer period—for example 12 percent per annum, which is the sum of the average risk free rate (approximately 6 percent) plus the average equity risk premium (6 percent). This translates into an average daily increase in equity prices of approximately 4.5bp/day. This is a relatively small number that tends to make little difference in application, but has a sound economic rationale underlying its use.

For other assets we may want to use the forward rate as the estimate for the expected average change. Currencies, for instance, are expected to drift to equal their forward rate according to the expectations hypothesis. If the USD is traded at a forward premium of 2.5 percent p.a. relative to the Euro, a reasonable candidate for the mean parameter would be μ_t = 1bp/day. The difference here between 0bp and 1bp seems to be immaterial, but when VaR is estimated for longer horizons this will become a relevant consideration, as we discuss later.

Implementation Considerations

The empirical performance of historical standard deviation as a predictor of future volatility is affected by statistical error. With respect to statistical error, it is always the case in statistics that "more is better." Hence, the more data available to us, the more precise our estimator will be to the true return volatility. On the other hand, we estimate standard deviation in an environment where we believe, a priori, that volatility itself is unstable. The stickiness of time variations in volatility are important, since it gives us an intuitive guide that recent history is more relevant for the near future than distant history.

In Figure 1-6 we use the series of 2,500 interest rate changes in order to come up with a series of rolling estimates of conditional volatility. We use an estimation window K of different lengths in order to demonstrate

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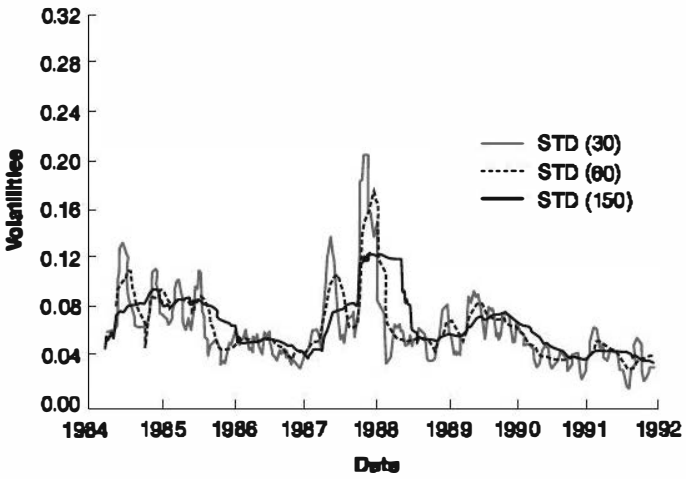

FIGURE 1-6 Time-varying volatility using historical standard deviation with various window lengths.

the tradeoff involved. Specifically, three different window-lengths are used: $K = 30$, $K = 60$, and $K = 150$. On any given day we compare these three lookback windows. That is, on any given day (starting with the 151st day), we look back 30, 60, or 150 days and calculate the standard deviation by averaging the squared interest rate changes (and then taking a square root). The figure demonstrates the issues involved in the choice of K. First note that the forecasts for series using shorter windows are more volatile. This could be the result of a statistical error—30 observations, for example, may provide only a noisy estimate of volatility. On the other hand, variations could be the result of true changes in volatility. The longer window length, $K = 150$ days, provides a relatively smoother series of estimators/forecasts, varying within a tighter range of 4–12 basis points per day. Recall that the unconditional volatility is 7.3bp/day. Shorter window lengths provide extreme estimators, as high as 22bp/day. Such estimators are three times larger than the unconditional volatility.

The effect of the statistical estimation error is particularly acute for small samples, e.g., $K = 30$. The STDEV estimator is particularly sensitive to extreme observations. To see why this is the case, recall that the calculation of STDEV involves an equally weighted average of *squared* deviations from the mean (here zero). Any extreme, perhaps non-normal, observation becomes larger in magnitude by taking it to the power of two. Moreover, with small window sizes each observation receives higher weight by definition. When a large positive or negative return is observed, therefore, a sharp increase in the volatility forecast is observed.

In this context it is worthwhile mentioning that an alternative procedure of calculating the volatility involves averaging absolute values of returns, rather than squared returns. This method is considered more robust when the distribution is non-normal. In fact it is possible to show that while under the normality assumption STDEV is optimal. when returns are non-normal. and, in particular, fat tailed, then the absolute squared deviation method may provide a superior forecast.

This discussion seems to present an argument that longer observation windows reduce statistical error. However, the other side of the coin is that small window lengths provide an estimator that is more adaptable to changing market condition. In the extreme case where volatility does not vary at all, the longer the window length is, the more accurate our estimates. However, in a time varying volatility environment we face a tradeoff—short window lengths are less precise, due to estimation error, but more adaptable to innovations in volatility. Later in this chapter we discuss the issue of benchmarking various volatility estimation models and describe simple optimization procedures that allow us to choose the most appropriate window length. Intuitively, for volatility series that are in and of themselves more volatile, we will tend to shorten the window length, and vice versa.

Finally, yet another important shortcoming of the STDEV method for estimating conditional volatility is the periodic appearance of large decreases in conditional volatility. These sharp declines are the result of extreme observations disappearing from the rolling estimation window. The STDEV methodology is such that when a large move occurs we use this piece of data for K days. Then, on day $K + 1$ it falls off the estimation window. The extreme return carries the same weight of $(100/K)$ percent from day $t - 1$ to day $t - K$, and then disappears. From an economic perspective this is a counterintuitive way to describe memory in financial markets. A more intuitive description would be to incorporate a gradual decline in memory such that when a crisis occurs it is very relevant for the first week, affecting volatility in financial markets to a great extent, and then as time goes by it becomes gradually less important. Using STDEV with equal weights on observations from the most recent K days, and zero thereafter

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(further into the past) is counterintuitive. This shortcoming of STDEV is precisely the one addressed by the exponential smoothing approach, adopted by RiskMetrics™ in estimating volatility.

Exponential Smoothing—RiskMetrics™ Volatility

Suppose we want to use historical data, specifically, squared returns, in order to calculate conditional volatility. How can we improve upon our first estimate, STDEV? We focus on the issue of information decay and on giving more weight to more recent information and less weight to distant information. The simplest, most popular, approach is exponential smoothing. Exponential smoothing places exponentially declining weights on historical data, starting with an initial weight, and then declining to zero as we go further into the past.

The smoothness is achieved by setting a parameter λ, which is equal to a number greater than zero, but smaller than one, raised to a power (i.e., $0 < \lambda < 1$). Any such smoothing parameter λ, when raised to a high enough power, can get arbitrarily small. The sequence of numbers $\lambda^0, \lambda^1, \lambda^2 \ldots \lambda^i, \ldots$ has the desirable property that it starts with a finite number, namely λ^0 (= 1). and ends with a number that could become arbitrarily small (λ^i where i is large). The only problem with this sequence is that we need it to sum to 1 in order for it to be a weighting scheme.

In order to rectify the problem, note that the sequence is geometric, summing up to $1/(1 - \lambda)$. For a smoothing parameter of 0.9 for example, the sum of $0.9^0, 0.9^1, 0.9^2, \ldots, 0.9^i, \ldots$ is $1/(1 - 0.9) = 10$. All we need is to define a new sequence which is the old sequence divided by the sum of the sequence and the new sequence will then sum to 1. In the previous example we would divide the sequence by 10. More generally we divide each of the weights by $1/(1 - \lambda)$, the sum of the geometric sequence. Note that dividing by $1/(1 - \lambda)$ is equivalent to multiplying by $(1 - \lambda)$. Hence, the old sequence $\lambda^0, \lambda^1, \lambda^2 \ldots \lambda^i, \ldots$ is replaced by the new sequence

$$(1-\lambda)\lambda^0, (1-\lambda)\lambda^1, (1-\lambda)\lambda^2, \ldots, (1-\lambda)\lambda^i, \ldots$$

This is a "legitimate" weighting scheme, since by construction it sums to one. This is the approach known as the RiskMetrics™ exponential weighting approach to volatility estimation.

The estimator we obtain for conditional variance is:

$$\sigma_t^2 = (1-\lambda)*(\lambda^0 r_{t-1,t}^{\ 2} + \lambda^1 r_{t-2,t-1}^{\ 2} + \lambda^2 r_{t-3,t-2}^{\ 2} + \cdots + \lambda^N r_{t-N-1,t-N}^{\ 2}),$$

where N is some finite number which is the truncation point. Since we truncate after a finite number (N) of observations the sum of the series is not 1. It is, in fact, λ^N. That is, the sequence of the weights we drop, from the "$N + 1$"th observation and thereafter, sum up to $\lambda^N/(1 - \lambda)$. For example, take $\lambda = 0.94$:

Weight 1	$(1 - \lambda)\lambda^0$	$= (1 - 0.94)$	$= 6.00\%$
Weight 2	$(1 - \lambda)\lambda^1$	$= (1 - 0.94)*0.94$	$= 5.64\%$
Weight 3	$(1 - \lambda)\lambda^2$	$= (1 - 0.94)*0.94^2$	$= 5.30\%$
Weight 4	$(1 - \lambda)\lambda^3$	$= (1 - 0.94)*0.94^3$	$= 4.98\%$
. . .			
Weight 100	$(1 - \lambda)\lambda^{99}$	$= (1 - 0.94)*0.94^{99}$	$= 0.012\%$

The residual sum of truncated weights is $0.94^{100}/(1 - 0.94) = 0.034$.

We have two choices with respect to this residual weight

1. We can increase N so that the sum of residual weight is small (e.g., $0.94^{200}/(1 - 0.94) = 0.00007$);
2. or divide by the truncated sum of weights $(1 - \lambda^N)/(1 - \lambda)$ rather than the infinite sum $1/(1 - \lambda)$. In our previous example this would mean dividing by 16.63 instead of 16.66 after 100 observations.

This is a purely technical issue. Either is technically fine, and of little real consequence to the estimated volatility.

In Figure 1-7 we compare RiskMetrics™ to STDEV. Recall the important commonalities of these methods

- both methods are parametric;
- both methods attempt to estimate conditional volatility;
- both methods use recent historical data;
- both methods apply a set of weights to past squared returns.

The methods differ only as far as the weighting scheme is concerned. RiskMetrics™ poses a choice with respect to the smoothing parameter λ, (in the example above, equal to 0.94) similar to the choice with respect to K in the context of the STDEV estimator. The tradeoff in the case of STDEV was between the desire for a higher precision, consistent with higher K's, and quick adaptability to changes in conditional volatility, consistent with lower K's. Here, similarly, a λ parameter closer to unity exhibits

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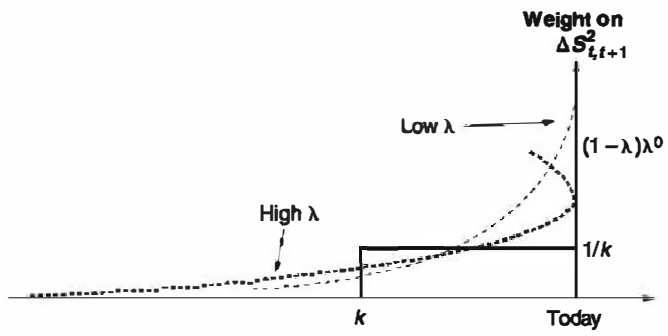

FIGURE 1-7 STDEV and exponential smoothing weighting schemes.

a slower decay in information's relevance with less weight on recent observations (see the dashed-dotted line in Figure 1-7, while lower λ parameters provide a weighting scheme with more weight on recent observations, but effectively a smaller sample (see the dashed line in Figure 1-7).

The Optimal Smoother Lambda

Is there a way to determine an optimal value to the estimation parameter, whether it is the window size *K* or the smoothing parameter λ? As it turns out, one can optimize on the parameters λ or *K*. To outline the procedure, first we must define the mean squared error (MSE) measure, which measures the statistical error of a series of estimates for each specific value of a parameter. We can then search for a minimum value for this MSE error, thereby identifying an optimal parameter value (corresponding with the minimal error).

First, it is important to note that true realized volatility is unobservable. Therefore, it is impossible to directly compare predicted volatility to true realized volatility. It is therefore not immediately clear how to go about choosing between various λ or *K* parameters. We can only "approximate" realized volatility. Specifically, the closest we can get is to take the observed value of $r_{t,t+1}^2$ as an approximate measure of realized volatility. There is no obvious way around the measurement error in measuring true volatility. The MSE measures the deviation between predicted and realized (not true) volatility. We take the squared error between predicted volatility (a function of the smoothing parameter we choose) $\sigma(\lambda)_t^2$ and realized volatility $r_{t,t+1}^2$ such that:

$$MSE(\lambda) = AVERAGE_{t=1,2,\ldots T}\{(\sigma(\lambda)_t^2 - r_{t,t+1}^2)^2\}.$$

We then minimize the MSE(λ) *over different choices of* λ,

$$Min_{\lambda<1}\{MSE(\lambda)\},$$

subject to the constraint that λ is less than one.

This procedure is similar in spirit, although not identical, to the *Maximum Likelihood Method* in statistics. This method attempts to choose the set of parameters given a certain model that will make the observed data the most likely to have been observed. The optimal λ can be chosen for every series independently. The optimal parameter may depend on sample size—for example, how far back in history we choose to extend our data. It also depends critically on the true nature of underlying volatility. As we discussed above, financial time series such as oil prices are driven by a volatility that may exhibit rapid and sharp turns. Since adaptability becomes important in such extremely volatile cases, a low λ will tend to be optimal (minimize MSE). The reverse would hold true for "well-behaved" series.

Variations in optimal λ are wide. The RiskMetrics™ technical document provides optimal λ for some of the 480 series covered. Money market optimal λ are as high as 0.99, and as low as 0.92 for some currencies. The globally optimal λ is derived so as to minimize the weighted average of MSEs with one optimal λ. The weights are determined according to individual forecast accuracy. The optimal overall parameter used by RiskMetrics™ has been $\lambda^{RM} = 0.94$.

Adaptive Volatility Estimation

Exponential smoothing can be interpreted intuitively using a restatement of the formula for generating volatility estimates. Instead of writing the volatility forecast σ_t^2 as a function of a sequence of past returns, it can be written as the sum of last period's forecast σ_{t-1}^2 weighted by λ, and the news between last period and today, $r_{t-1,t}^2$, *weighted by the residual weight* 1 − λ:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1,t}^2.$$

This is a recursive formula. It is equivalent to the previous formulation since the last period's forecast can be now restated as a function of the volatility of the period prior to that and of the news in between − $\sigma_{t-1}^2 = \lambda\sigma_{t-2}^2 + (1-\lambda)r_{t-2,t-1}^2$. Plugging in σ_{t-1}^2 into the original formula, and doing so repeatedly will generate the standard RiskMetrics™ estimator, i.e., current volatility σ_t^2 is an exponentially declining function of past squared returns.

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This model is commonly termed an "adaptive expectations" model. It gives the risk manager a rule that can be used to adapt prior beliefs about volatility in the face of news. If last period's estimator of volatility was low, and extreme news (i.e., returns) occurred, how should the risk manager update his or her information? The answer is to use this formula—place a weight of λ on what you believed yesterday, and a weight of (1 − λ) on the news between yesterday and today. For example, suppose we estimated a conditional volatility of 100bp/day for a portfolio of equities. Assume we use the optimal λ—that is, $\lambda^{RM} = 0.94$. The return on the market today was −300bp. What is the new volatility forecast?

$$\sigma_t = \sqrt{(0.94 \cdot 100^2 + (1 - 0.94) \cdot (-300)^2)} = 121.65.$$

The sharp move in the market caused an increase in the volatility forecast of 21 percent. The change would have been much lower for a higher λ. A higher λ not only means more weight on recent observations, it also means that our current beliefs have not changed dramatically from what we believed to be true yesterday.

The Empirical Performance of RiskMetrics™

The intuitive appeal of exponential smoothing is validated in empirical tests. For a relatively large portion of the reasonable range for lambdas (most of the estimators fall above 0.90), we observe little visible difference between various volatility estimators. In Figure 1-8 we see a series of rolling volatilities with two different smoothing parameters, 0.90 and 0.96. The two series are close to being superimposed on one another. There are extreme spikes using the lower lambda parameter, 0.9, but the choppiness of the forecasts in the back end that we observed with STDEV is now completely gone.

GARCH

The exponential smoothing method recently gained an important extension in the form of a new time series model for volatility. In a sequence of recent academic papers Robert Engel and Tim Bollereslev introduced a new estimation methodology called GARCH, standing for General Autoregressive Conditional Heteroskedasticity. This sequence of relatively sophisticated-sounding technical terms essentially means that GARCH is a statistical time series model that enables the econometrician to model volatility as time varying and predictable. The model is similar in spirit to RiskMetrics™. In a GARCH(1, 1)

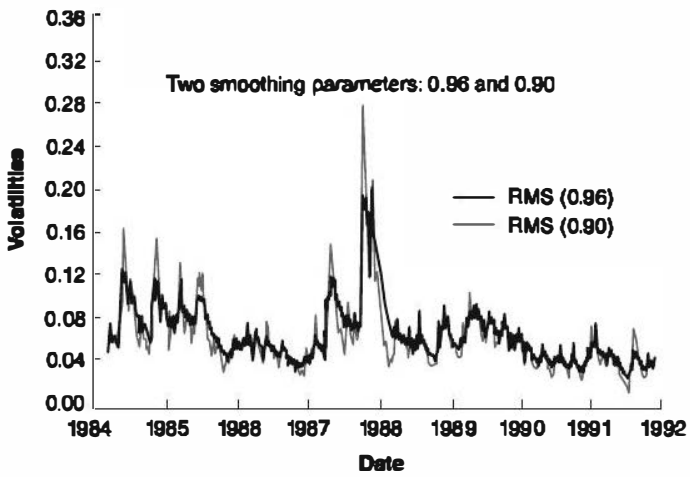

FIGURE 1-8 RiskMetrics™ volatilities.

model the period t conditional volatility is a function of period $t - 1$ conditional volatility and the return from $t - 1$ to t squared,

$$\sigma_t^2 = a + br_{t-1,t}^2 + c\sigma_{t-1}^2,$$

where a, b, and c are parameters that need to be estimated empirically. The general version of GARCH, called GARCH(p,q), is

$$\sigma_t^2 = a + b_1 r_{t-1,t}^2 + b_2 r_{t-2,t-1}^2 + \cdots + b_p r_{t-p+1,t-p}^2 + c_1\sigma_{t-1}^2 + c_2\sigma_{t-2}^2 + \cdots + c_q\sigma_{t-q}^2,$$

allowing for p lagged terms on past returns squared, and q lagged terms on past volatility.

With the growing popularity of GARCH it is worth pointing out the similarities between GARCH and other methods, as well as the possible pitfalls in using GARCH. First note that GARCH(1 , 1) is a generalized case of RiskMetrics™. Put differently, RiskMetrics™ is a restricted case of GARCH. To see this, consider the following two constraints on the parameters of the GARCH(1, 1) process:

$$a = 0, \qquad b + c = 1.$$

Substituting these two restrictions into the general form of GARCH(1, 1) we can rewrite the GARCH model as follows

$$\sigma_t^2 = (1 - c)r_{t-1,t}^2 + c\sigma_{t-1}^2.$$

This is identical to the recursive version of RiskMetrics™.

The two parameter restrictions or constraints that we need to impose on GARCH(1, 1) in order to get the

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RiskMetrics™ exponential smoothing parameter imply that GARCH is more general or less restrictive. Thus, for a given dataset, GARCH should have better explanatory power than the RiskMetrics™ approach. Since GARCH offers more degrees of freedom, it will have lower error or better describe a given set of data. The problem is that this may not constitute a real advantage in practical applications of GARCH to risk management-related situations. In reality, we do not have the full benefit of hindsight. The challenge in reality is to predict volatility out-of-sample, not in-sample. Within sample there is no question that GARCH would perform better, simply because it is more flexible and general. The application of GARCH to risk management requires, however, forecasting ability.

The danger in using GARCH is that estimation error would generate noise that would harm the out-of-sample forecasting power. To see this consider what the econometrician interested in volatility forecasting needs to do as time progresses. As new information arrives the econometrician updates the parameters of the model to fit the new data. Estimating parameters repeatedly creates variations in the model itself, some of which are true to the change in the economic environment, and some simply due to sampling variation. The econometrician runs the risk of providing less accurate estimates using GARCH relative to the simpler RiskMetrics™ model in spite of the fact that RiskMetrics™ is a constrained version of GARCH. This is because while the RiskMetrics™ methodology has just one fixed model—a lambda parameter that is a constant (say 0.94)—GARCH is chasing a moving target. As the GARCH parameters change, forecasts change with it, partly due to true variations in the model and the state variables, and partly due to changes in the model due to estimation error. This can create model risk.

Figure 1-9 illustrates this risk empirically. In this figure we see a rolling series of GARCH forecasts, re-estimated daily using a moving window of 150 observations. The extreme variations in this series relative to a relatively smooth RiskMetrics™ volatility forecast series, that appears on the same graph, demonstrates the risk in using GARCH for forecasting volatility, using a short rolling window.

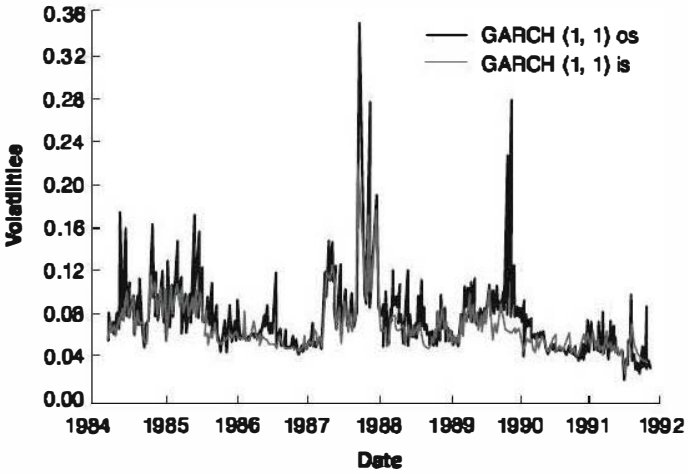

FIGURE 1-9 GARCH in- and out-of-sample.

Nonparametric Volatility Forecasting

Historical Simulation

So far we have confined our attention to parametric volatility estimation methods. With parametric models we use all available data, weighted one way or another, in order to estimate parameters of a given distribution. Given a set of relevant parameters we can then determine percentiles of the distribution easily, and hence estimate the VaR of the return on an asset or a set of assets. Nonparametric methods estimate VaR, i.e., percentile of return distribution, directly from the data, without making assumptions about the entire distribution of returns. This is a potentially promising avenue given the phenomena we encountered so far—fat tails, skewness and so forth.

The most prominent and easiest to implement methodology within the class of nonparametric methods is historical simulation (HS). HS uses the data directly. The only thing we need to determine up front is the lookback window. Once the window length is determined, we order returns in descending order, and go directly to the tail of this ordered vector. For an estimation window of 100 observations, for example, the fifth lowest return in a rolling window of the most recent 100 returns is the fifth percentile. The lowest observation is the first percentile. If we wanted, instead, to use a 250 observations window, the fifth percentile would be somewhere between the 12th and the 13th lowest observations (a detailed discussion follows), and the first percentile would be somewhere between the second and third lowest returns.

This is obviously a very simple and convenient method, requiring the estimation of *zero* parameters (window size aside). HS can, in theory, accommodate fat tail skewness and many other peculiar properties of return series. If the "true" return distribution is fat tailed, this will come

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through in the HS estimate since the fifth observation will be more extreme than what is warranted by the normal distribution. Moreover, if the " true" distribution of asset returns is left skewed since market falls are more extreme than market rises, this will surface through the fact that the 5th and the 95th ordered observations will not be symmetric around zero.

This is all true in theory. With an infinite amount of data we have no difficulty estimating percentiles of the distribution directly. Suppose, for example, that asset returns are truly non-normal. and the correct model involves skewness. If we assume normality we also assume symmetry, and in spite of the fact that we have an infinite amount of data we suffer from model specification error—a problem which is insurmountable. With the HS method we could take, say, the 5,000th of 100,000 observations, a very precise estimate of the fifth percentile.

In reality, however, we do not have an infinite amount of data. What is the result of having to use a relatively small sample in practice? Quantifying the precision of percentile estimates using HS in finite samples is a rather complicated technical issue. The intuition is, however, straightforward. Percentiles around the median (the 50th percentile) are easy to estimate relatively accurately even in small samples. This is because every observation contributes to the estimation by the very fact that it is under or over the median.

Estimating extreme percentiles, such as the first or the fifth percentile, is much less precise in small samples. Consider, for example, estimating the fifth percentile in a window of 100 observations. The fifth percentile is the fifth smallest observation. Suppose that a crisis occurs and during the following ten trading days five new extreme declines were observed. The VaR using the HS method grows sharply. Suppose now that in the following few months no new extreme declines occurred. From an economic standpoint this is news—"no news is good news" is a good description here. The HS estimator of the VaR, on the other hand, reflects the same extreme tail for the following few months, until the observations fall out of the 100 day observation window. There is no updating for 90 days, starting from the ten extreme days (where the five extremes were experienced) until the ten extreme days start dropping out of the sample. This problem can become even more acute with a window of one year (250 observations) and a 1 percent VaR, that requires only the second and third lowest observations.

This problem arises because HS uses data very inefficiently. That is, out of a very small initial sample, focus on the tails requires throwing away a lot of useful information. Recall that the opposite holds true for the parametric family of methods. When the standard deviation is estimated, every data point contributes to the estimation. When extremes are observed we update the estimator upwards, and when calm periods bring into the sample relatively small returns (in absolute value), we reduce the volatility forecast. This is an important advantage of the parametric method(s) over nonparametric methods—data arc used more efficiently. Nonparametric methods' precision hinges on large samples, and falls apart in small samples.

A minor technical point related to HS is in place here. With 100 observations the first percentile could be thought of as the first observation. However, the observation itself can be thought of as a random event with a probability mass centered where the observation is actually observed, but with 50 percent of the weight to its left and 50 percent to its right. As such, the probability mass we accumulate going from minus infinity to the lowest of 100 observations is only ½ percent and not the full 1 percent. According to this argument the first percentile is somewhere in between the lowest and second lowest observation. Figure 1-10 clarifies the point.

Finally, it might be argued that we can increase the precision of HS estimates by using more data; say, 10,000 past daily observations. The issue here is one of regime relevance. Consider, for example, foreign exchange rates going back 10,000 trading days—approximately 40 years. Over the last 40 years, there have been a number of different

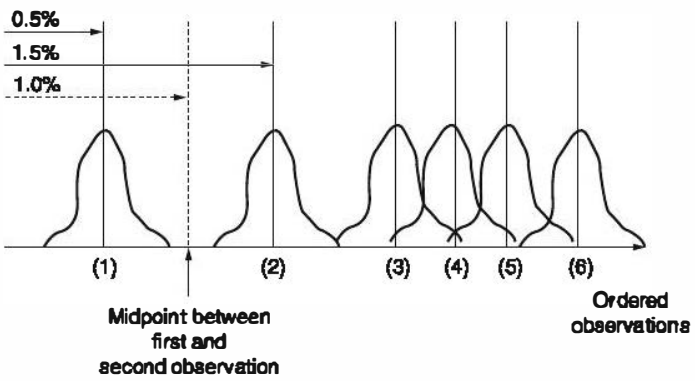

FIGURE 1-10 Historical simulation method.

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exchange rate regimes in place, such as fixed exchange rates under Bretton Woods. Data on returns during periods of fixed exchange rates would have no relevance in forecasting volatility under floating exchange rate regimes. As a result, the risk manager using conventional HS is often forced to rely on the relatively short time period relevant to current market conditions, thereby reducing the usable number of observations for HS estimation.

Multivariate Density Estimation

Multivariate density estimation (MDE) is a methodology used to estimate the joint probability density function of a set of variables. For example, one could choose to estimate the joint density of returns and a set of predetermined factors such as the slope of the term structure, the inflation level, the state of the economy, and so forth. From this distribution, the conditional moments, such as the mean and volatility of returns, conditional on the economic state, can be calculated.

The MDE volatility estimate provides an intuitive alternative to the standard mining volatility forecasts. The key feature of MDE is that the weights are no longer a constant function of time as in RiskMetrics™ or STDEV. Instead, the weights in MDE depend on how the current state of the world compares to past states of the world. If the current state of the world, as measured by the state vector $\boldsymbol{x}_t$, is similar to a particular point in the past, then this past squared return is given a lot of weight in forming the volatility forecast, *regardless of how far back in time it is.*

For example, suppose that the econometrician attempts to estimate the volatility of interest rates. Suppose further that according to his model the volatility of interest rates is determined by the level of rates—higher rates imply higher volatility. If today's rate is, say 6 percent, then the relevant history is any point in the past when interest rates were around 6 percent. A statistical estimate of current volatility that uses past data should place high weight on the magnitude of interest rate changes during such times. Less important, although relevant, are times when interest rates were around 5.5 percent or 6.5 percent, even less important although not totally irrelevant are times when interest rates were 5 percent or 7 percent, and so on. MDE devises a weighting scheme that helps the econometrician decide how far the relevant state variable was at any point in the past from its value today. Note that to the extent that relevant state variables are going to be autocorrelated, MDE weights may look, to an extent, similar to RiskMetrics™ weights.

The critical difficulty is to select the relevant (economic) state variables for volatility. These variables should be useful in describing the economic environment in general, and be related to volatility specifically. For example, suppose that the level of inflation is related to the level of return volatility, then inflation will be a good conditioning variable. The advantages of the MDE estimate are that it can be interpreted in the context of weighted lagged returns, and that the functional form of the weights depends on the true (albeit estimated) distribution of the relevant variables.

Using the MDE method, the estimate of conditional volatility is

$$\sigma_t^2 = \Sigma_{i=1,2,\ldots,K}\,\omega(\boldsymbol{x}_{t-i})r_{t-i}^{\;2}.$$

Here, $\boldsymbol{x}_{t-i}$ is the vector of variables describing the economic state at time $t-i$ (e.g., the term structure), determining the appropriate weight $\omega(\boldsymbol{x}_{t-i})$ to be placed on observation $t-i$, as a function of the "distance" of the state $\boldsymbol{x}_{t-i}$ from the current state $\boldsymbol{x}_t$. The relative weight of "near" relative to "distant" observations from the current state is measured via the kernel function.

MDE is extremely flexible in allowing us to introduce dependence on state variables. For example, we may choose to include past squared returns as conditioning variables. In doing so the volatility forecasts will depend nonlinearly on these past changes. For example, the exponentially smoothed volatility estimate can be added to an array of relevant conditioning variables. This may be an important extension to the GARCH class of models. Of particular note, the estimated volatility is still based directly on past squared returns and thus falls into the class of models that places weights on past squared returns.

The added flexibility becomes crucial when one considers cases in which there are other relevant state variables that can be added to the current state. For example, it is possible to capture: (i) the dependence of interest rate volatility on the level of interest rates; (ii) the dependence of equity volatility on current implied volatilities; and (iii) the dependence of exchange rate volatility on interest rate spreads, proximity to intervention bands, etc.

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There are potential costs in using MDE. We must choose a weighting scheme (a kernel function), a set of conditioning variables, and the number of observations to be used in estimating volatility. For our purposes, the bandwidth and kernel function are chosen objectively (using standard criteria). Though they may not be optimal choices, it is important to avoid problems associated with data snooping and over fitting. While the choice of conditioning variables is at our discretion and subject to abuse, the methodology does provide a considerable advantage. Theoretical models and existing empirical evidence may suggest relevant determinants for volatility estimation, which MDE can incorporate directly. These variables can be introduced in a straightforward way for the class of stochastic volatility models we discuss.

The most serious problem with MDE is that it is data intensive. Many data are required in order to estimate the appropriate weights that capture the joint density function of the variables. The quantity of data that is needed increases rapidly with the number of conditioning variables used in estimation. On the other hand, for many of the relevant markets this concern is somewhat alleviated since the relevant state can be adequately described by a relatively low dimensional system of factors.

As an illustration of the four methodologies put together, Figure 1-11 shows the weights on past squared interest rate changes as of a specific date estimated by each model. The weights for STDEV and RiskMetrics™ are the same in every period, and will vary only with the window length and the smoothing parameter. The GARCH(1,1) weighting scheme varies with the parameters, which are re-estimated every period, given each day's previous 150-day history. The date was selected at random. For that particular day, the GARCH parameter selected is $b = 0.74$. Given that this parameter is relatively low, it is not surprising that the weights decay relatively quickly. Figure 1-11 is particularly illuminating with respect to MDE. As with GARCH, the weights change over time. The weights are high for dates t through $t - 25$ (25 days prior) and then start to decay. The state variables chosen here for volatility arc the level and the slope of the term structure, together providing information about the state of interest rate volatility (according to *our* choice). The weights decrease because the economic environment, as described by the interest rate level and spread, is moving further away from the conditions observed at date t.

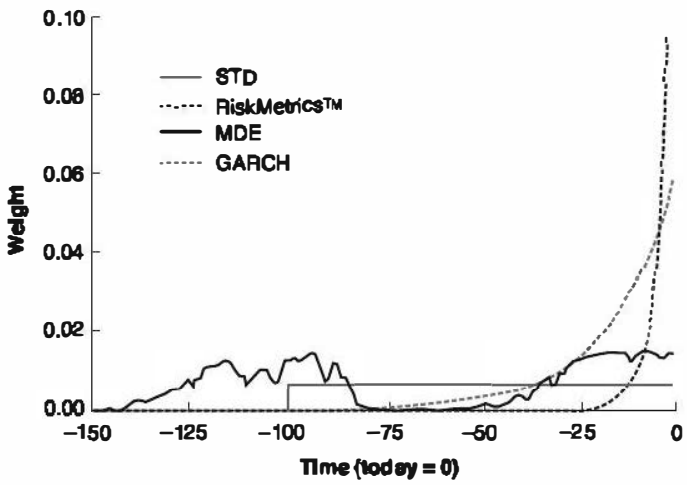

FIGURE 1-11 MDE weights on past returns squared.

However, we observe an increase in the weights for dates $t - 80$ to $t - 120$. Economic conditions in this period (the level and spread) are similar to those at date t. MDE puts high weight on relevant information, regardless of how far in the past this information is.

A Comparison of Methods

Table 1-2 compares, on a period-by-period basis, the extent to which the forecasts from the various models line up with realized future volatility. We define realized daily volatility as the average squared daily changes during the following (trading) week, from day $t + 1$ to day $t + 5$. Recall our discussion of the mean squared error. In order to benchmark various methods we need to test their accuracy vis-à-vis realized volatility—an unknown before and after the fact. If we used the realized squared return during the day following each volatility forecast we run into estimation error problems. On the other hand if we measure realized volatility as standard deviation during the following month, we run the risk of inaccuracy due to over aggregation because volatility may shift over a month's time period. The tradeoff between longer and shorter horizons going forward is similar to the tradeoff discussed earlier regarding the length of the lookback window in calculating STDEV. We will use the realized volatility, as measured by mean squared deviation during the five trading days following each forecast. Interest rate changes are mean-adjusted using the sample mean of the previous 150-day estimation period.

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TABLE 1-2 A Comparison of Methods

	STDEV	RiskMetrics™	MDE	GARCH
Mean	0.070	0.067	0.067	0.073
Std. Dev	0.022	0.029	0.024	0.030
Aurocorr.	0.999	0.989	0.964	0.818
MSE	0.999	0.930	0.887	1.115
Linear regression				
Beta	0.577	0.666	0.786	0.559
(s.e.)	(0.022)	(0.029)	(0.024)	(0.030)
R^2	0.100	0.223	0.214	0.172

The comparison between realized and forecasted volatility is done in two ways. First, we compare the out-of-sample performance over the entire period using the mean-squared error of the forecasts. That is, we take the difference between each model's volatility forecast and the realized volatility, square this difference, and average through time. This is the standard MSE formulation. We also regress realized volatility on the forecasts and document the regression coefficients and R^2s.

The first part of Table 1-2 documents some summary statistics that are quite illuminating. First, while all the means of the volatility forecasts are of a similar order of magnitude (approximately seven basis points per day), the standard deviations are quite different, with the most volatile forecast provided by GARCH(1, 1). This result is somewhat surprising because GARCH(1, 1) is supposed to provide a relatively smooth volatility estimate (due to the moving average term). However, for rolling, out-of-sample forecasting, the variability of the parameter estimates from sample to sample induces variability in the forecasts. These results are, however, upwardly biased, since GARCH would commonly require much more data to yield stable parameter estimates. Here we re-estimate GARCH every day using a 150-day lookback period. From a practical perspective, this finding of unstable forecasts for volatility is a model disadvantage. In particular, to the extent that such numbers serve as inputs in setting time-varying rules in a risk management system (for example, by setting trading limits), smoothness of these rules is necessary to avoid large swings in positions.

Regarding the forecasting performance of the various volatility models, Table 1-2 provides the mean squared error measure (denoted MSE). For this particular sample and window length. MDE minimizes the MSE, with the lowest MSE of 0.887. RiskMetrics™ (using $\lambda = 0.94$ as the smoothing parameter) also performs well, with an MSE of 0.930. Note that this comparison involves just one particular GARCH model (i.e., GARCH(1, 1)), over a short estimation window, and does not necessarily imply anything about other specification and window lengths. One should investigate other window lengths and specifications, as well as other data series, to reach general conclusions regarding model comparisons. It is interesting to note, however, that, nonstationarity aside, exponentially smoothed volatility is a special case of GARCH(1, 1) in sample, as discussed earlier. The results here suggest, however, the potential cost of the error in estimation of the GARCH smoothing parameters on an out-of-sample basis.

An alternative approach to benchmarking the various volatility-forecasting methods is via linear regression of realized volatility on the forecast. If the conditional volatility is measured without error, then the slope coefficient (or beta) should equal one. However, if the forecast is unbiased but contains estimation error, then the coefficient will be biased downwards. Deviations from one reflect a combination of this estimation error plus any systematic over- or underestimation. The ordering in this "horse race" is quite similar to the previous one. In particular, MDE exhibits the beta coefficient closest to one (0.786), and exponentially smoothed volatility comes in second, with a beta parameter of 0.666. The goodness of fit measure, the R^2 of each of the regressions, is similar for both methods.

The Hybrid Approach

The hybrid approach combines the two simplest approaches (for our sample), HS and RiskMetrics™, by estimating the percentiles of the return directly (similar to HS), and using exponentially declining weights on past data (similar to RiskMetrics™). The approach starts with ordering the returns over the observation period just like the HS approach. While the HS approach attributes equal weights to each observation in building the conditional empirical distribution, the hybrid approach attributes exponentially declining weights to historical returns. Hence, while obtaining the 1 percent VaR using 250 daily

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returns involves identifying the third lowest observation in the HS approach, it may involve more or less observations in the hybrid approach. The exact number of observations will depend on whether the extreme low returns were observed recently or further in the past. The weighting scheme is similar to the one applied in the exponential smoothing (EXP hence) approach.

The hybrid approach is implemented in three steps:

Step 1: Denote by $r_{t-1,t}$ the realized return from $t-1$ to t. To each of the most recent K returns $r_{t-1,t}, r_{t-2,t-1}, \ldots, r_{t-K,t-K-1}$ assign a weight $[(1-\lambda)/(1-\lambda^K)], [(1-\lambda)/(1-\lambda^K)]\lambda, \ldots, [(1-\lambda)/(1-\lambda^K)]\lambda^{K-1}$, respectively. Note that the constant $[(1-\lambda)/(1-\lambda^K)]$ simply ensures that the weights sum to one.

Step 2: Order the returns in ascending order.

Step 3: In order to obtain the x percent VaR of the portfolio, start from the lowest return and keep accumulating the weights until x percent is reached. Linear interpolation is used between adjacent points to achieve exactly x percent of the distribution.

TABLE 1-3 The Hybrid Approach—An Example

Order	Return	Periods Ago	Hybrid Weight	Hybrid Cumul. Weight	HS Weight	HS Cumul. Weight
Initial date:						
1	−3.30%	3	0.0221	0.0221	0.01	0.01
2	−2.90%	2	0.0226	0.0447	0.01	0.02
3	−2.70%	65	0.0063	0.0511	0.01	0.03
4	−2.50%	45	0.0095	0.0605	0.01	0.04
5	−2.40%	5	0.0213	0.0818	0.01	0.05
6	−2.30%	30	0.0128	0.0947	0.01	0.06
25 days later:						
1	−3.30%	28	0.0134	0.0134	0.01	0.01
2	−2.90%	27	0.0136	0.0270	0.01	0.02
3	−2.70%	90	0.0038	0.0308	0.01	0.03
4	−2.50%	70	0.0057	0.0365	0.01	0.04
5	−2.40%	30	0.0128	0.0494	0.01	0.05
6	−2.30%	55	0.0077	0.0571	0.01	0.06

Consider the following example, we examine the VaR of a given series at a given point in time, and a month later, assuming that no extreme observations were realized during the month. The parameters are $\lambda = 0.98$, $K = 100$.

The top half of Table 1-3 shows the ordered returns at the initial date. Since we assume that over the course of a month no extreme returns are observed, the ordered returns 25 days later are the same. These returns are, however, further in the past. The last two columns show the equally weighted probabilities under the HS approach. Assuming an observation window of 100 days, the HS approach estimates the 5 percent *VaR* to be 2.35 percent for both cases (note that VaR is the negative of the actual return). This is obtained using interpolation on the actual historical returns. That is, recall that we assume that half of a given return's weight is to the right and half to the left of the actual observation (see Figure 1-10). For example, the −2.40 percent return represents 1 percent of the distribution in the HS approach, and we assume that this weight is split evenly between the intervals from the actual observation to points halfway to the next highest and lowest observations. As a result, under the HS approach, −2.40 percent represents the 4.5th percentile, and the distribution of weight leads to the 2.35 percent VaR (halfway between 2.40 percent and 2.30 percent).

In contrast, the hybrid approach departs from the equally weighted HS approach. Examining first the initial period, Table 1-3 shows that the cumulative weight of the −2.90 percent return is 4.47 percent and 5.11 percent for the −2.70 percent return. To obtain the 5 percent VaR for the initial period, we must interpolate as shown in Figure 1-10. We obtain a cumulative weight of 4.79 percent for the −2.80 percent return. Thus, the 5th percentile VaR under the hybrid approach for the initial period lies somewhere between 2.70 percent and 2.80 percent. We define the

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required VaR level as a linearly interpolated return, where the distance to the two adjacent cumulative weights determines the return. In this case, for the initial period the 5 percent VaR under the hybrid approach is:

$$2.80\% - (2.80\% - 2.70\%) \cdot [(0.05 - 0.0479)/(0.0511 - 0.0479)] = 2.73\%.$$

Similarly, the hybrid approach estimate of the 5 percent VaR 25 days later can be found by interpolating between the −2.40 percent return (with a cumulative weight of 4.94 percent) and −2.35 percent (with a cumulative weight of 5.33 percent, interpolated from the values on Table 1-3). Solving for the 5 percent VaR:

$$2.35\% - (2.35\% - 2.30\%) \cdot [(0.05 - 0.0494)/(0.0533 - 0.0494)] = 2.34\%.$$

Thus, the hybrid approach initially estimates the 5 percent VaR as 2.73 percent. As time goes by and no large returns are observed, the VaR estimate smoothly declines to 2.34 percent. In contrast, the HS approach yields a constant 5 percent VaR over both periods of 2.35 percent, thereby failing to incorporate the information that returns were stable over the two month period. Determining which methodology is appropriate requires backtesting (see the Appendix).

RETURN AGGREGATION AND VaR

Our discussion of the HS and hybrid methods missed one key point so far. How do we aggregate a number of positions into a single VaR number for a portfolio comprised of a number of positions? The answer to this question in the RiskMetrics™ and STDEV approaches is simple—under the assumption that asset returns are jointly normal, the return on a portfolio is also normally distributed. Using the variance-covariance matrix of asset returns we can calculate portfolio volatility and VaR. This is the reason for the fact that the RiskMetrics™ approach is commonly termed the Variance-Covariance approach (VarCov).

The HS approach needs one more step—missing so far from our discussion—before we can determine the VaR of a portfolio of positions. This is the aggregation step. The idea is simply to aggregate each period's historical returns, weighted by the relative size of the position. This is where the method gets its name—"simulation." We calculate returns using historical data, but using today's weights. Suppose for example that we hold today positions in three equity portfolios—indexed to the S&P 500 index, the FTSE index and the Nikkei 225 index—in equal amounts. These equal weights are going to be used to calculate the return we would have gained J days ago if we were to hold this equally weighted portfolio. This is regardless of the fact that our equity portfolio J days ago may have been completely different. That is, we pretend that the portfolio we hold today is the portfolio we held up to K days into the past (where K is our lookback window size) and calculate the returns that would have been earned.

From an implementation perspective this is very appealing and simple. This approach has another important advantage—note that we do not estimate any parameters whatsoever. For a portfolio involving N positions the VarCov approach requires the estimation of N volatilities and $N(N-1)/2$ correlations. This is potentially a very large number, exposing the model to estimation error. Another important issue is related to the estimation of correlation. It is often argued that when markets fall, they fall together. If, for example, we see an abnormally large decline of 10 percent in the S&P index on a given day, we strongly believe that other components of the portfolio, e.g., the Nikkei position and the FTSE position, will also fall sharply. This is regardless of the fact that we may have estimated a correlation of, for example, 0.30 between the Nikkei and the other two indexes under more normal market conditions (see Longin and Solnik (2001)).

The possibility that markets move together at the extremes to a greater degree than what is implied by the estimated correlation parameter poses a serious problem to the risk manager. A risk manager using the VarCov approach is running the risk that his VaR estimate for the position is understated. At the extremes the benefits of diversification disappear. Using the HS approach with the initial aggregation step may offer an interesting solution. First, note that we do not need to estimate correlation parameters (nor do we need to estimate volatility parameters). If, on a given day, the S&P dropped 10 percent, the Nikkei dropped 12 percent and the FTSE dropped 8 percent, then an equally weighted portfolio will show a drop of 10 percent—the average of the three returns. The following step of the HS methods is to order the observations in ascending order and pick the fifth of 100 observations (for the 5 percent VaR, for example). If the tails are

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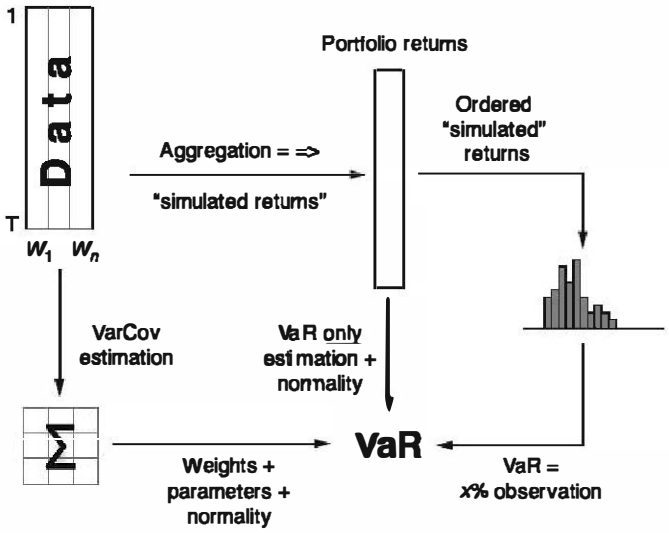

FIGURE 1-12 VaR and aggregation.

extreme, and if markets co-move over and above the estimated correlations, it will be taken into account through the aggregated data itself.

Figure 1-12 provides a schematic of the two alternatives. Given a set of historical data and current weights we can either use the variance-covariance matrix in the VarCov approach, or aggregate the returns and then order them in the HS approach. There is an obvious third alternative methodology emerging from this figure. We may estimate the volatility (and mean) of the vector of aggregated returns and assuming normality calculate the VaR of the portfolio.

Is this approach sensible? If we criticize the normality assumption we should go with the HS approach. If we believe normality we should take the VarCov approach. What is the validity of this intermediate approach of aggregating first, as in the HS approach, and only then assuming normality as in the VarCov approach? The answer lies in one of the most important theorems in statistics, the strong law of large numbers. Under certain assumptions it is the case that an average of a very large number of random variables will end up converging to a normal random variable.

It is, in principle, possible, for the specific components of the portfolio to be non-normal, but for the portfolio as a whole to be normally distributed. In fact, we are aware of many such examples. Consider daily stock returns for example. Daily returns on specific stocks are often far from normal, with extreme moves occurring for different stocks at different times. The aggregate, well-diversified portfolio of these misbehaved stocks, could be viewed as normal (informally, we may say the portfolio is more normal than its component parts—a concept that could easily be quantified and is often tested to be true in the academic literature). This is a result of the strong law of large numbers.

Similarly here we could think of normality being regained, in spite of the fact that the single components of the portfolio are non-normal. This holds only if the portfolio is well diversified. If we hold a portfolio comprised entirely of oil- and gas-related exposures, for example, we may hold a large number of positions that are all susceptible to sharp movements in energy prices.

This last approach—of combining the first step of aggregation with the normality assumption that requires just a single parameter estimate—is gaining popularity and is used by an increasing number of risk managers.

IMPLIED VOLATILITY AS A PREDICTOR OF FUTURE VOLATILITY

Thus far our discussion has focused on various methods that involve using historical data in order to estimate future volatility. Many risk managers describe managing risk this way as similar to driving by looking in the rear-view mirror. When extreme circumstances arise in financial markets an immediate reaction, and preferably even a preliminary indication, are of the essence. Historical risk estimation techniques require time in order to adjust to changes in market conditions. These methods suffer from the shortcoming that they may follow, rather than forecast risk events. Another worrisome issue is that a key assumption in all of these methods is stationarity; that is, the assumption that the past is indicative of the future.

Financial markets provide us with a very intriguing alternative—option-implied volatility. Implied volatility can be imputed from derivative prices using a specific derivative pricing model. The simplest example is the Black-Scholes implied volatility imputed from equity option prices. The implementation is fairly simple, with a few technical issues along the way. In the presence of

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multiple implied volatilities for various option maturities and exercise prices, it is common to take the at-the-money (ATM) implied volatility from puts and calls and extrapolate an average implied; this implied is derived from the most liquid (ATM) options. This implied volatility is a candidate to be used in risk measurement models in place of historical volatility. The advantage of implied volatility is that it is a forward-looking, predictive measure.

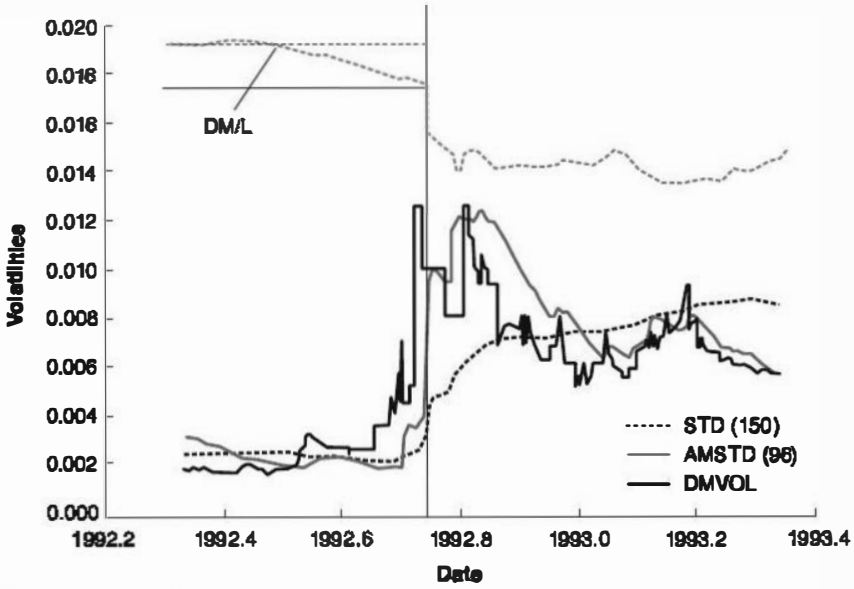

FIGURE 1-13 Implied and historical volatility: the GBP during the ERM crisis of 1992.

A particularly strong example of the advantage obtained by using implied volatility (in contrast to historical volatility) as a predictor of future volatility is the GBP currency crisis of 1992. During the summer of 1992, the GBP came under pressure as a result of the expectation that it should be devalued relative to the European Currency Unit (ECU) components, the deutschmark (DM) in particular (at the time the strongest currency within the ECU). During the weeks preceding the final drama of the GBP devaluation, many signals were present in the public domain. The British Central Bank raised the GBP interest rate. It also attempted to convince the Bundesbank to lower the DM interest rate, but to no avail. Speculative pressures reached a peak toward summer's end, and the British Central Bank started losing currency reserves, trading against large hedge funds such as the Soros fund.

The market was certainly aware of these special market conditions, as shown in Figure 1-13. The top dotted line is the DM/GBP exchange rate, which represents our "event clock." The event is the collapse of the exchange rate. Figure 1-13 shows the Exchange Rate Mechanism (ERM) intervention bands. As was the case many times prior to this event, the most notable predictor of devaluation was already present—the GBP is visibly close to the intervention band. A currency so close to the intervention band is likely to be under attack by speculators on the one hand, and under intervention by the central banks on the other. This was the case many times prior to this event, especially with the Italian lira's many devaluations. Therefore, the market was prepared for a crisis in the GBP during the summer of 1992. Observing the thick solid line depicting option-implied volatility, the growing pressure on the GBP manifests itself in options prices and volatilities. Historical volatility is trailing, "unaware" of the pressure. In this case, the situation is particularly problematic since historical volatility happens to decline as implied volatility rises. The fall in historical volatility is due to the fact that movements close to the intervention band are bound to be smaller by the fact of the intervention bands' existence and the nature of intervention, thereby dampening the historical measure of volatility just at the time that a more predictive measure shows increases in volatility.

As the GBP crashed, and in the following couple of days, RiskMetrics™ volatility increased quickly (thin solid line). However, simple STDEV (K = 50) badly trailed events—it does not rise in time, nor does it fall in time. This is, of course, a particularly sharp example, the result of the intervention band preventing markets from fully reacting to information. As such, this is a unique example. Does it generalize to all other assets? Is it the case that implied volatility is a superior predictor of future volatility, and hence a superior risk measurement tool, relative to historical? It would seem as if the answer must be affirmative, since implied volatility can react immediately to market conditions. As a predictor of future volatility this is certainly an important feature.

Implied volatility is not free of shortcomings. The most important reservation stems from the fact that implied

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volatility is model-dependent. A misspecified model can result in an erroneous forecast. Consider the Black–Scholes option-pricing model. This model hinges on a few assumptions, one of which is that the underlying asset follows a continuous time lognormal diffusion process. The underlying assumption is that the volatility parameter is constant from the present time to the maturity of the contract. The implied volatility is supposedly this parameter. In reality, volatility is not constant over the life of the options contract. Implied volatility varies through time. Oddly, traders trade options in "vol" terms, the volatility of the underlying, fully aware that (i) this vol is implied from a constant volatility model, and (ii) that this very same option will trade tomorrow at a different vol, which will also be assumed to be constant over the remaining life of the contract.

Yet another problem is that at a given point in time, options on the same underlying may trade at different vols. An example is the *smile effect*—deep out of the money (especially) and deep in the money (to a lesser extent) options trade at a higher vol than at the money options.

The key is that the option-pricing model provides a convenient nonlinear transformation allowing traders to compare options with different maturities and exercise prices. The true underlying process is not a lognormal diffusion with constant volatility as posited by the model. The underlying process exhibits stochastic volatility, jumps, and a non-normal conditional distribution. The vol parameter serves as a "kitchen-sink" parameter. The market converses in vol terms, adjusting for the possibility of sharp declines (the smile effect) and variations in volatility.

The latter effect—stochastic volatility, results in a particularly difficult problem for the use of implied volatility as a predictor of future volatility. To focus on this particular issue, consider an empirical exercise repeatedly comparing the 30-day implied volatility with the empirically measured volatility during the following month. Clearly, the forecasts (i.e., implied) should be equal to the realizations (i.e., measured return standard deviation) only on average. It is well understood that forecast series are bound to be smoother series, as expectations series always are relative to realization series. A reasonable requirement is, nevertheless, that implied volatility should be equal, on average, to realized volatility. This is a basic requirement of every forecast instrument—it should be unbiased.

Empirical results indicate, strongly and consistently, that implied volatility is, on average, greater than realized volatility. From a modeling perspective this raises many interesting questions, focusing on this empirical fact as a possible key to extending and improving option pricing models. There are, broadly, two common explanations. The first is a market inefficiency story, invoking supply and demand issues. This story is incomplete, as many market-inefficiency stories are, since it does not account for the presence of free entry and nearly perfect competition in derivative markets. The second, rational markets, explanation for the phenomenon is that implied volatility is greater than realized volatility due to stochastic volatility. Consider the following facts: (i) volatility is stochastic; (ii) volatility is a priced source of risk; and (iii) the underlying model (e.g., the Black–Scholes model) is, hence, misspecified, assuming constant volatility. The result is that the premium required by the market for stochastic volatility will manifest itself in the forms we saw above—implied volatility would be, on average, greater than realized volatility.

From a risk management perspective this bias, which can be expressed as $\sigma_{implied} = \sigma_{true} + Stoch.Vol.Premium$, poses a problem for the use of implied volatility as a predictor for future volatility. Correcting for this premium is difficult since the premium is unknown, and requires the "correct" model in order to measure precisely. The only thing we seem to know about this premium is that it is on average positive, since implied volatility is on average greater than historical volatility.

It is an empirical question, then, whether we are better off with historical volatility or implied volatility as the predictor of choice for future volatility. Many studies have attempted to answer this question with a consensus emerging that implied volatility is a superior estimate. This result would have been even sharper if these studies were to focus on the responsiveness of implied and historical to sharp increases in conditional volatility. Such times are particularly important for risk managers, and are the primary shortcoming associated with models using the historical as opposed to the implied volatility.

In addition to the upward bias incorporated in the measures of implied volatility, there is another more fundamental problem associated with replacing historical volatility with implied volatility measures. It is available for very few assets/market factors. In a covariance matrix

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of 400 by 400 (approximately the number of assets/markets that RiskMetrics™ uses), very few entries can be filled with implied volatilities because of the sparsity of options trading on the underlying assets. The use of implied volatility is confined to highly concentrated portfolios where implied volatilities are present. Moreover, recall that with more than one pervasive factor as a measure of portfolio risk, one would also need an implied correlation. Implied correlations are hard to come by. In fact, the only place where reliable liquid implied correlations could be imputed is in currency markets.

As a result, implied volatility measures can only be used for fairly concentrated portfolios with high foreign exchange rate risk exposure. Where available, implied volatility can always be compared in real time to historical (e.g., RiskMetrics™) volatility. When implied volatilities get misaligned by more than a certain threshold level (say, 25 percent difference), then the risk manager has an objective "red light" indication. This type of rule may help in the decision making process of risk limit readjustment in the face of changing market conditions. In the discussion between risk managers and traders, the comparison of historical to implied can serve as an objective judge.

LONG HORIZON VOLATILITY AND VaR

In many current applications, e.g., such as by mutual fund managers, there is a need for volatility and VaR forecasts for horizons longer than a day or a week. The simplest approach uses the "square root rule." Under certain assumptions, to be discussed below, the rule states that an asset's J-period return volatility is equal to the square root of J times the signal period return volatility

$$\sigma(r_{t,t+J}) = \sqrt{(J)} \times \sigma(r_{t,t+1}).$$

Similarly for VaR this rule is

$$J\text{-period VaR} = \sqrt{(J)} \times \text{1-period VaR}.$$

The rule hinges on a number of key assumptions. It is important to go through the proof of this rule in order to examine its limits. Consider, first, the multiperiod continuously compounded rate of return. For simplicity consider the two-period return:

$$r_{t,t+2} = r_{t,t+1} + r_{t+1,t+2}.$$

The variance of this return is

$$\text{var}(r_{t,t+2}) = \text{var}(r_{t,t+1}) + \text{var}(r_{t+1,t+2}) + 2 \cdot \text{cov}(r_{t,t+1}, r_{t+1,t+2}).$$

Assuming:

$$A1: \quad \text{cov}(r_{t,t+1}, r_{t+1,t+2}) = 0,$$

$$A2: \quad \text{var}(r_{t,t+1}) = \text{var}(r_{t+1,t+2}),$$

we get

$$\text{var}(r_{t,t+2}) = 2 \cdot \text{var}(r_{t,t+1}),$$

and hence

$$\text{STD}(r_{t,t+2}) = \sqrt{(2)} \cdot \text{STD}(r_{t,t+1}).$$

Which is the square root rule for two periods. The rule generalizes easily to the J period rule.

The first assumption is the assumption of non-predictability, or the random walk assumption. The term $\text{cov}(r_{t,t+1}, r_{t+1,t+2})$ is the autocovariance of returns. Intuitively the autocovariance being zero means that knowledge that today's return is, for example, positive, tells us nothing with respect to tomorrow's return. Hence this is also a direct result of the random walk assumption, a standard market efficiency assumption. The second assumption states that the volatility is the same in every period (i.e., on each day).

In order to question the empirical validity of the rule, we need to question the assumptions leading to this rule. The first assumption of non-predictability holds well for most asset return series in financial markets. Equity returns are unpredictable at short horizons. The evidence contrary to this assertion is scant and usually attributed to luck. The same is true for currencies. There is some evidence of predictability at long horizons (years) for both, but the extent of predictability is relatively small. This is not the case, though, for many fixed-income-related series such as interest rates and especially spreads.

Interest rates and spreads are commonly believed to be predictable to varying degrees, and modeling predictability is often done through time series models accounting for autoregression. An autoregressive process is a stationary process that has a long run mean, an average level to which the series tends to revert. This average is often called the "Long Run Mean" (LRM). Figure 1-14 represents a schematic of interest rates and their long run mean. The dashed lines represent the expectations of the interest

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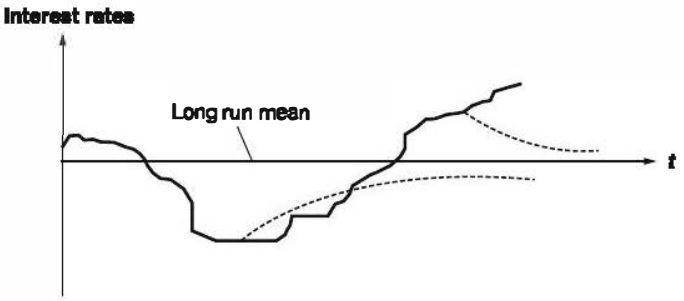

FIGURE 1-14 Mean reverting process.

rate process. When interest rates are below their LRM they are expected to rise and vice versa.

Mean reversion has an important effect on long-term volatility. To understand the effect, note that the autocorrelation of interest rate changes is no longer zero. If increases and decreases in interest rates (or spreads) are expected to be reversed, then the serial covariance is negative. This means that the long horizon volatility is overstated using the zero-autocovariance assumption. *In the presence of mean reversion in the underlying asset's long horizon, volatility is lower than the square root times the short horizon volatility.*

The second assumption is that volatility is constant. As we have seen throughout this chapter, this assumption is unrealistic. Volatility is stochastic, and, in particular, autoregressive. This is true for almost all financial assets. Volatility has a long run mean—a "steady state" of uncertainty. Note here the important difference—most financial series have an unpredictable series of returns, and hence no long run mean (LRM), with the exception of interest rates and spreads. However, most volatility series are predictable, and do have an LRM.

When current volatility is above its long run mean then we can expect a decline in volatility over the longer horizon. Extrapolating long horizon volatility using today's volatility will overstate the true expected long horizon volatility. On the other hand, if today's volatility is unusually low, then extrapolating today's volatility using the square root rule may understate true long horizon volatility. The bias—upwards or downwards, hence, depends on today's volatility relative to the LRM of volatility. The discussion is summarized in Table 1-4.

TABLE 1-4 Long Horizon Volatility

Mean Reversion	$\sqrt{J}$ Rule Using Today's Volatility
In returns	overstates true long horizon volatility
In return volatility	If today's vol. > LRM vol. then overstated If today's vol. < LRM vol. then understated

MEAN REVERSION AND LONG HORIZON VOLATILITY

Modeling mean reversion in a stationary time series framework is called the analysis of autoregression (AR). We present here an AR(1) model, which is the simplest form of mean reversion in that we consider only one lag. Consider a process described by the regression of the time series variable X_t:

$$X_{t+1} = a + bX_t + e_{t+1}.$$

This is a regression of a variable on its own lag. It is often used in financial modeling of time series to describe processes that are mean reverting, such as the real exchange rate, the price/dividend or price/earnings ratio, and the inflation rate. Each of these series can be modeled using an assumption about how the underlying process is predictable. This time series process has a finite long run mean under certain restrictions, the most important of which is that the parameter b is less than one. The expected value of X_t as a function of period t information is

$$E_t[X_{t+1}] = a + bX_t.$$

We can restate the expectations as follows

$$E_t[X_{t+1}] = (1 - b)*[a/(1 - b)] + bX_t.$$

Next period's expectations are a weighted sum of today's value, X_t, and the long run mean $a/(1 - b)$. Here b is the key parameter, often termed "the speed of reversion" parameter. If $b = 1$ then the process is a random walk—a nonstationary process with an undefined (infinite) long run mean, and, therefore, next period's expected value is equal to today 's value. If $b < 1$ then the process is mean reverting. When X_t is above the LRM, it is expected to decline, and vice versa.

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By subtracting X_t from the autoregression formula we obtain the "return," the change in X_t

$$X_{t+1} - X_t = a + bX_t + e_{t+1} - X_t$$
$$= a + (b-1)X_t + e_{t+1}$$

and the two period return is

$$X_{t+2} - X_t = a + ab + b^2X_t + be_{t+1} + e_{t+2} - X_t$$
$$= a(1+b) + (b^2-1)X_t + be_{t+1} + e_{t+2}.$$

The single period conditional variance of the rate of change is

$$\text{var}_t(X_{t+1} - X_t) = \text{var}_t(a + bX_t + e_{t+1} - X_t)$$
$$= \text{var}_t(e_{t+1})$$
$$= \sigma^2.$$

The volatility of e_{t+1} is denoted by σ. The two period volatility is

$$\text{var}_t(X_{t+2} - X_t) = \text{var}_t(a(1+b) + (b^2-1)X_t + be_{t+1} + e_{t+2})$$
$$= \text{var}_t(be_{t+1} + e_{t+2})$$
$$= (1+b^2)*\sigma^2.$$

This is the key point—the single period variance is σ^2. The two period variance is $(1 + b^2)\sigma^2$ which is less than $2\sigma^2$, note that if the process was a random walk, i.e., $b = 1$, then we would get the standard square root volatility result. The square root volatility fails due to mean reversion. That is, with no mean reversion, the two period volatility would be $\sqrt{(2)}\sigma = 1.41\sigma$. With mean reversion, e.g., for $b = 0.9$, the two period volatility is, instead, $\sqrt{(1+0.9^2)}\sigma = 1.34\sigma$.

The insight, that mean reversion effects conditional volatility and hence risk is very important, especially in the context of arbitrage strategies. Risk managers often have to assess the risk of trading strategies with a vastly different view of risk. The trader may view a given trade as a convergence trade. Convergence trades assume explicitly that the spread between two positions, a long and a short, is mean reverting. If the mean reversion is strong, than the long horizon risk is smaller than the square root volatility. This may create a sharp difference of opinions on the risk assessment of a trade. It is common for risk managers to keep a null hypothesis of market efficiency—that is, that the spread underlying the convergence trade is random walk.

CORRELATION MEASUREMENT

Thus far, we have confined our attention to volatility estimation and related issues. There are similar issues that arise when estimating correlations. For example, there is strong evidence that exponentially declining weights provide benefits in correlation estimation similar to the benefits in volatility estimation. There are two specific issues related to correlation estimation that require special attention. The first is correlation breakdown during market turmoil. The second issue is an important technical issue—the problem of using nonsynchronous data.

The problem arises when sampling daily data from market closing prices or rates, where the closing time is different for different series. We use here the example of US and Japanese interest rate changes, where the closing time in the US is 4:00 p.m. EST, whereas the Japanese market closes at 1:00 a.m. EST, fifteen hours earlier. Any information that is relevant for global interest rates (e.g., changes in oil prices) coming out after 1:00 a.m. EST and before 4:00 p.m. EST will influence today's interest rates in the US and tomorrow's interest rates in Japan.

Recall that the correlation between two assets is the ratio or their covariance divided by the product of their standard deviations

$$\text{corr}(\Delta i_{t,t+1}^{US}, \Delta i_{t,t+1}^{Jap})$$
$$= \text{cov}(\Delta i_{t,t+1}^{US}, \Delta i_{t,t+1}^{Jap})/\{\text{STD}(\Delta i_{t,t+1}^{US})*\text{STD}(\Delta i_{t,t+1}^{Jap})\}.$$

Assume that the daily standard deviation is estimated correctly irrespective of the time zone. The volatility of close-to-close equities covers 24 hours in any time zone. However, the covariance term is *underestimated* due to the nonsynchronicity problem.

The problem may be less important for portfolios of few assets, but as the number of assets increase, the problem becomes more and more acute. Consider for example an equally weighted portfolio consisting of n assets, all of which have the same daily standard deviation, denoted σ and the same cross correlation, denoted ρ. The variance of the portfolio would be

$$\sigma_p^2 = (1/n)\sigma^2 + (1 - 1/n)\rho\sigma^2.$$

The first term is due to the own asset variances, and the second term is due to the cross covariance terms. For a

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large n, the volatility of the portfolio is $\rho\sigma^2$, which is the standard deviation of each asset scaled down by the correlation parameter. The bias in the covariance would translate one-for-one into a bias in the portfolio volatility.

For US and Japanese ten year zero coupon rate changes for example, this may result in an understatement of portfolio volatilities by up to 50 percent relative to their true volatility. For a global portfolio of long positions this will result in a severe understatement of the portfolio's risk. Illusionary diversification benefits will result in lower-than-true VaR estimates.

There are a number of solutions to the problem. One solution could be sampling both market open and market close quotes in order to make the data more synchronous. This is, however, costly because more data are required, quotes may not always be readily available and quotes may be imprecise. Moreover, this is an incomplete solution since some nonsynchronicity still remains. There are two other alternative avenues for amending the problem and correcting for the correlation in the covariance term. Both alternatives are simple and appealing from a theoretical and an empirical standpoint.

The first alternative is based on a natural extension of the random walk assumption. The random walk assumption assumes consecutive daily returns are independent. In line with the independence assumption, assume intraday independence—e.g., consecutive hourly returns—are independent. Assume further, for the purpose of demonstration, that the US rate is sampled without a lag, whereas the Japanese rate is sampled with some lag. That is, 4:00 p.m. EST is the "correct" time for accurate and up to the minute sampling, and hence a 1:00 a.m. EST. quote is stale. The true covariance is

$$\begin{aligned}&\mathrm{cov}^{tr}(\Delta i_{t,t+1}^{US}, \Delta i_{t,t+1}^{Jap})\\ &= \mathrm{cov}^{obs}(\Delta i_{t,t+1}^{US}, \Delta i_{t,t+1}^{Jap}) + \mathrm{cov}^{obs}(\Delta i_{t,t+1}^{US}, \Delta i_{t+1,t+2}^{Jap}),\end{aligned}$$

a function of the contemporaneous observed covariance plus the covariance of today's US change with tomorrow's change in Japan.

The second alternative for measuring true covariance is based on another assumption in addition to the independence assumption; the assumption that the intensity of the information flow is constant intraday, and that the Japanese prices/rates are 15 hours behind US prices/rates. In this case

$$\mathrm{cov}^{tr}(\Delta i_{t,t+1}^{US}, \Delta i_{t,t+1}^{Jap}) = [24/(24-15)]*\mathrm{cov}^{obs}(\Delta i_{t,t+1}^{US}, \Delta i_{t,t+1}^{Jap}).$$

The intuition behind the result is that we observe a covariance which is the result of a partial overlap, of only 9 out of 24 hours. If we believe the intensity of news throughout the 24 hour day is constant than we need to inflate the covariance by multiplying it by 24/9 = 2.66. This method may result in a peculiar outcome, that the correlation is greater than one, a result of the assumptions. This factor will transfer directly to the correlation parameter—the numerator of which increases by a factor of 2.66, while the denominator remains the same. The factor by which we need to inflate the covariance term falls as the level of nonsynchronicity declines. With London closing 6 hours prior to New York, the factor is smaller—24/(24 − 6) = 1.33.

Both alternatives rely on the assumption of independence and simply extend it in a natural way from interday to intraday independence. This concept is consistent, in spirit, with the kind of assumptions backing up most extant risk measurement engines. The first alternative relies only on independence, but requires the estimation of one additional covariance moment. The second alternative assumes in addition to independence that the intensity of news flow is constant throughout the trading day. Its advantage is that it requires no further estimation.

SUMMARY

This chapter addressed the motivation for and practical difficulty in creating a dynamic risk measurement methodology to quantify VaR. The motivation for dynamic risk measurement is the recognition that risk varies through time in an economically meaningful and in a predictable manner. One of the many results of this intertemporal volatility in asset returns distributions is that the magnitude and likelihood of tail events changes though time. This is critical for the risk manager in determining prudent risk measures, position limits, and risk allocation.

Time variations are often exhibited in the form of fat tails in asset return distributions. One attempt is to incorporate the empirical observation of fat tails to allow volatility to vary through time. Variations in volatility can create deviations from normality, but to the extent that we can measure and predict volatility through time we may be able to recapture normality in the conditional versions, i.e., we may be able to model asset returns as conditionally normal with time-varying distributions.

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As it turns out, while indeed volatility is time-varying, it is not the case that extreme tails events disappear once we allow for volatility to vary through time. It is still the case that asset returns are, even conditionally, fat tailed. This is the key motivation behind extensions of standard VaR estimates obtained using historical data to incorporate scenario analysis and stress testing.

APPENDIX

Backtesting Methodology and Results

Earlier, we discussed the MSE and regression methods for comparing standard deviation forecasts. Next, we present a more detailed discussion of the methodology for backtesting VaR methodologies. The dynamic VaR estimation algorithm provides an estimate of the x percent VaR for the sample period for each of the methods. Therefore, the probability of observing a return lower than the calculated VaR should be x percent:

$$\text{prob}[r_{t-1t} < -VaR_t] = x\%.$$

There are a few attributes which are desirable for VaR_t. We can think of an indicator variable I_t, which is 1 if the VaR is exceeded, and 0 otherwise. There is no direct way to observe whether our VaR estimate is precise; however, a number of different indirect measurements will, together, create a picture of its precision.

The first desirable attribute is *unbiasedness*. Specifically, we require that the VaR estimate be the x percent tail. Put differently, we require that the average of the indicator variable I_t should be x percent:

$$\text{avg}[I_t] = x\%.$$

This attribute alone is an insufficient benchmark. To see this, consider the case of a VaR estimate which is constant through time, but is also highly precise unconditionally (i.e., achieves an average VaR probability which is close to x percent). To the extent that tail probability is cyclical, the occurrences of violations of the VaR estimate will be "bunched up" over a particular state of the economy. This is a very undesirable property, since we require dynamic updating which is sensitive to market conditions.

Consequently, the second attribute which we require of a VaR estimate is that extreme events do not "bunch up." Put differently, a VaR estimate should increase as the tail of the distribution rises. If a large return is observed today, the VaR should rise to make the probability of another tail event exactly x percent tomorrow. In terms of the indicator variable, I_t, we essentially require that I_t be independently and identically distributed (i.i.d.). This requirement is similar to saying that the VaR estimate should provide a filter to transform a serially dependent return volatility and tail probability into a serially independent I_t series.

The simplest way to assess the extent of independence here is to examine the empirical properties of the tail event occurrences, and compare them to the theoretical ones. Under the null that I_t is independent over time

$$\text{corr}[I_{t-s}, I_t] = 0 \quad \forall s,$$

that is, the indicator variable should not be autocorrelated at any lag. Since the tail probabilities that are of interest tend to be small, it is very difficult to make a distinction between pure luck and persistent error in the above test for any individual correlation. Consequently, we consider a joint test of whether the first five daily autocorrelations (one trading week) are equal to zero.

Note that for both measurements the desire is essentially to put all data periods on an equal footing in terms of the tail probability. As such, when we examine a number of data series for a given method, we can aggregate across data series, and provide an average estimate of the unbiasedness and the independence of the tail event probabilities. While the different data series may be correlated, such an aggregate improves our statistical power.

The third property which we examine is related to the first property—the biasedness of the VaR series, and the second property—the autocorrelation of tail events. We calculate a rolling measure of the absolute percentage error. Specifically, for any given period, we look forward 100 periods and ask how many tail events were realized. If the indicator variable is both unbiased and independent, this number is supposed to be the VaR's percentage level, namely x. We calculate the average absolute value of the difference between the actual number of tail events and the expected number across all 100-period windows within the sample. Smaller deviations from the expected value indicate better VaR measures.

The data we use include a number of series, chosen as a representative set of "interesting" economic series. These series are interesting since we *a priori* believe that their high order moments (skewness and kurtosis) and, in particular,

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their tail behavior, pose different degrees of challenge to VaR estimation. The data span the period from January 1, 1991 to May 12, 1997, and include data on the following:

- DEM the dollar/DM exchange rate;
- OIL the spot price for Brent crude oil;
- S&P the S&P 500 Index;
- BRD a general Brady bond index (JP Morgan Brady Broad Index).

We have 1,663 daily continuously compounded returns for each series.

In the tables, in addition to reporting summary statistics for the four series, we also analyze results for:

- EQW an equally weighted portfolio of the four return series
- AVG statistics for tail events averaged across the four series.

The EQW results will give us an idea of how the methods perform when tail events are somewhat diversified (via aggregation). The AVG portfolio simply helps us increase the effective size of our sample. That is, correlation aside, the AVG statistics may be viewed as using four times more data. Its statistics are therefore more reliable, and provide a more complete picture for general risk management purposes. Therefore, in what follows, we shall refer primarily to AVG statistics, which include 6,656 observations.

In the tables we use a 250-trading day window throughout. This is, of course, an arbitrary choice. which we make in order to keep the tables short and informative. The statistics for each of the series include 1,413 returns, since 250 observations are used as back data. The AVG statistics consist of 5,652 data points, with 282 tail events expected in the 5 percent tail, and 56.5 in the 1 percent tail.

In Table 1-5 we document the percentage of tail events for the 5 percent and the 1 percent VaR. There is no apparent strong preference among the models for the 5 percent VaR. The realized average varies across methods, between 4.62 percent and 5.65 percent. A preference is observed, however, when examining the empirical performance for the 1 percent VaR across methods. That is, HS and Hybrid ($\lambda = 0.99$) appear to yield results that are closer to 1 percent than the other methods. Thus, the nonparametric methods, namely HS and Hybrid, appear to outperform the parametric methods for these data series, perhaps because nonparametric methods, by design, are better suited to addressing the well known tendency of financial return series to be fat tailed. Since the estimation of the 1 percent tail requires a lot of data, there seems to be an expected advantage to high smoothers ($\lambda = 0.99$) within the hybrid method.

TABLE 1-5 Comparison of Methods—Results for Empirical Tail Probabilities

	Historical STD	Historical Simulation	EXP		Hybrid	
			0.97	0.99	0.97	0.99
5% Tail						
DEM	5.18	5.32	5.74	5.18	5.25	5.04
OIL	5.18	4.96	5.60	5.39	5.18	5.18
S&P	4.26	5.46	4.68	4.18	6.17	5.46
BRD	4.11	5.32	4.47	4.40	5.96	5.46
EQW	4.40	4.96	5.04	4.26	5.67	5.39
AVG	**4.62**	**5.21**	**5.11**	**4.68**	**5.65**	**5.30**
1% Tail						
DEM	1.84	1.06	2.20	1.63	1.84	1.28
OIL	1.84	1.13	1.77	1.77	1.70	1.35
S&P	2.06	1.28	2.20	2.13	1.84	1.42
BRD	2.48	1.35	2.70	2.41	1.63	1.35
EQW	1.63	1.49	1.42	1.42	1.63	1.21
AVG	**1.97**	**1.26**	**2.06**	**1.87**	**1.73**	**1.32**

In Table 1-6 we document the mean absolute error (MAE) of the VaR series. The MAE is a conditional version of the previous statistic (percentage in the tail from Table 1-4). The MAE uses a rolling 100-period window. Here again, we find an advantage in favor of the nonparametric methods, HS and Hybrid, with the hybrid method performing best for high λ ($\lambda = 0.99$) (note, though, that this is not always true: $\lambda = 0.97$ outperforms for the 5 percent for both the hybrid and the EXP). Since a statistical error is inherent in

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TABLE 1-6 Rolling Mean Absolute Percentage Error of VaR

	Historical STD	Historical Simulation	EXP		Hybrid	
			0.97	0.99	0.97	0.99
5% Tail						
DEM	2.42	2.42	1.58	2.11	1.08	1.77
OIL	2.84	2.62	2.36	2.67	1.93	2.44
S&P	1.95	1.91	1.52	1.85	1.72	1.68
BRD	3.41	3.53	3.01	3.34	2.54	2.97
EQW	2.43	2.36	2.48	2.33	1.50	2.20
AVG	**2.61**	**2.57**	**2.19**	**2.46**	**1.76**	**2.21**
1% Tail						
DEM	1.29	0.87	1.50	1.12	1.02	0.88
OIL	1.71	0.96	1.07	1.39	0.84	0.80
S&P	1.45	1.14	1.40	1.42	0.99	0.82
BRD	2.15	1.32	1.98	2.06	1.03	1.12
EQW	1.57	1.52	1.25	1.25	0.72	0.87
AVG	**1.63**	**1.16**	**1.44**	**1.45**	**0.92**	**0.90**

this statistic we cannot possibly expect a mean absolute error of zero. As such, the 38 percent improvement of the hybrid method with λ of 0.99 (with MAE of 0.90 percent for the AVG series' 1 percent tail) relative to the EXP method with the same λ (with MAE of 1.45), is an understatement of the level of improvement. A more detailed simulation exercise would be needed in order to determine how large this improvement is. It is worthwhile to note that this improvement is achieved very persistently across the different data series.

The adaptability of a VaR method is one of the most critical elements in determining the best way to measure VaR. When a large return is observed, the VaR level should increase. It should increase, however, in a way that will make the next tail event's probability precisely x percent. We can therefore expect these tail event realizations to be i.i.d. (independent) events with x percent probability. This independence can be examined using the autocorrelation of tail events, with the null being that autocorrelation is zero. As we see in Table 1-7, the hybrid method's autocorrelation for the AVG series is closest to zero. Interestingly, this is especially true for the more fat tailed series, such as BRD and OIL. As such, the hybrid method is very well suited for fat tailed, possibly skewed series.

In Tables 1-8A and B we test the statistical significance of the autocorrelations in Table 1-7. Specifically, we examine the first through fifth autocorrelations of the tail event series, with the null being that all of these autocorrelations should be zero. The test statistic is simply the sum of the squared autocorrelations, appropriately adjusted to the sample size. Under the null this statistic is distributed as $\chi^2(5)$. These test statistics are generally lower for the hybrid method relative to the EXP. For the specific series four rejections out of a possible eight are obtained with the hybrid method, relative to seven out of eight for the EXP method.

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TABLE 1-7 First-Order Autocorrelation of the Tail Events

	Historical STD	Historical Simulation	EXP		Hybrid	
			0.97	0.99	0.97	0.99
5% Tail						
DEM	0.39	0.09	−2.11	−1.06	−2.63	−2.28
OIL	1.76	2.29	2.11	1.25	3.20	0.31
S&P	0.77	1.09	−0.15	0.94	0.77	2.46
BRD	11.89	12.69	13.60	12.27	10.12	12.08
EQW	5.52	2.29	3.59	4.26	−2.04	−0.14
AVG	**4.07**	**3.69**	**3.41**	**3.53**	**1.88**	**2.49**
1% Tail						
DEM	2.04	−1.08	1.05	2.76	−1.88	−1.29
OIL	−1.88	−1.15	2.27	2.27	−1.73	−1.37
S&P	4.94	9.96	7.65	8.04	2.04	8.70
BRD	15.03	9.30	10.75	12.60	−1.66	3.97
EQW	2.76	3.12	3.63	3.63	2.76	4.73
AVG	**4.58**	**4.07**	**5.07**	**5.86**	**−0.09**	**2.95**

TABLE 1-8A Test Statistic for Independence (autocorrelations 1–5)

	Historical STD	Historical Simulation	EXP		Hybrid	
			0.97	0.99	0.97	0.99
5% Tail						
DEM	7.49	10.26	3.80	8.82	3.73	6.69
OIL	9.58	12.69	5.82	4.90	4.71	3.94
S&P	8.09	8.32	0.88	4.31	0.81	3.87
BRD	66.96	87.80	88.30	78.00	46.79	69.29
EQW	16.80	6.30	11.66	14.75	4.87	12.10
AVG	**21.78**	**25.07**	**22.09**	**22.18**	**12.18**	**19.18**
1% Tail						
DEM	3.34	5.33	4.56	4.39	7.58	3.83
OIL	33.98	8.29	3.82	18.89	8.53	3.54
S&P	14.67	36.15	22.68	25.18	3.26	24.10
BRD	88.09	29.37	41.60	82.77	11.26	11.36
EQW	41.55	14.69	16.85	16.85	5.08	13.05
AVG	**36.32**	**18.77**	**17.90**	**29.61**	**7.14**	**11.18**

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TABLE 1-8B *p*-Value for Independence (autocorrelations 1–5)

	Historical STD	Historical Simulation	EXP		Hybrid	
			0.97	0.99	0.97	0.99
5% Tail						
DEM	0.19	0.07	0.58	0.12	0.59	0.24
OIL	0.09	0.03	0.32	0.43	0.45	0.56
S&P	0.15	0.14	0.97	0.51	0.98	0.57
BRD	0.00	0.00	0.00	0.00	0.00	0.00
EQW	0.00	0.28	0.04	0.01	0.43	0.03
AVG	**0.09**	**0.10**	**0.38**	**0.21**	**0.49**	**0.28**
1% Tail						
DEM	0.65	0.38	0.47	0.49	0.18	0.57
OIL	0.00	0.14	0.58	0.00	0.13	0.62
S&P	0.01	0.00	0.00	0.00	0.66	0.00
BRD	0.00	0.00	0.00	0.00	0.05	0.04
EQW	0.00	0.01	0.00	0.00	0.41	0.02
AVG	**0.13**	**0.11**	**0.21**	**0.10**	**0.28**	**0.25**

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Putting VaR to Work

2

Learning Objectives

After completing this reading you should be able to:

- Explain and give examples of linear and non-linear derivatives.
- Describe and calculate VaR for linear derivatives.
- Describe the delta-normal approach for calculating VaR for non-linear derivatives.
- Describe the limitations of the delta-normal method.
- Explain the full revaluation method for computing VaR.
- Compare delta-normal and full revaluation approaches for computing VaR.
- Explain structured Monte Carlo, stress testing, and scenario analysis methods for computing VaR, and identify strengths and weaknesses of each approach.
- Describe the implications of correlation breakdown for scenario analysis.
- Describe worst-case scenario (WCS) analysis and compare WCS to VaR.

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Excerpt is Chapter 3 of Understanding Market, Credit and Operational Risk: The Value at Risk Approach, *by Linda Allen, Jacob Boudoukh, and Anthony Saunders.*

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THE VaR OF DERIVATIVES—PRELIMINARIES

The pricing and risk management of derivatives are intimately related. Since a derivative's price depends on an underlying asset, they both share the same risk factors. For example, a call option on the S&P 100 index changes in value as a function of the underlying factor—the S&P 100 index. The value of a convertible bond depends on two factors—interest rates and the value of the asset into which the bond is convertible.

In order to analyze the risk of a derivative one needs a *pricing model* that specifies the value of the derivative as a function of the underlying factor(s). In addition, one must specify how the risk factor may vary through time; that is, what are reasonable scenarios for the underlying factor? In the case where there are a few relevant underlying factors, one must specify how the underlying factors may co-vary.

In reality, some complex derivatives (e.g., mortgage-backed securities) cannot be priced with a reasonable level of precision of the relevant pricing factors. Therefore, even though we may know some of the relevant factors, some of the variation is asset-specific or asset-class-specific. We can break down derivatives' return volatility along these lines into risk factor-related volatility and asset-specific volatility. Asset-specific or asset-class-specific risk can be attributed to factors that are unknown to the financial economist or the trader, but are known to the market. Asset-specific risk can also be viewed as being a result of modeling errors.

In this chapter, we initially focus on factor-related risk, assuming that derivatives' returns are fully attributable to variations in known risk factors. This assumption is exact only in a theoretical world, for example, when we price an option in a Black-Scholes world using the Black-Scholes option pricing formula. In reality, pricing models do not describe the world perfectly. As a result, actual derivatives prices incorporate some element of asset-specific risk. Later on in the chapter, we will discuss asset-specific and asset class risk.

Linear Derivatives

We distinguish, broadly, between two types of derivatives, *linear derivatives* and *nonlinear derivatives*. A linear derivative is linear in the sense that the relationship between the derivative and the underlying pricing factor(s) is linear. It does not need to be one-for-one, but the "transmission parameter," the delta, needs to be constant for all levels of the underlying factor. This is (approximately) the case, for example, for a futures contract on the S&P 500 index, as we explain below. This is not the case for an option on the S&P 500—a given change in the underlying factor will result in a change in the value of the option that depends on the option's "moneyness," i.e., the degree to which an option is in or out of the money.

A futures contract on the S&P 500 is defined as a dollar multiple of the index level. The S&P 500 option traded on the Chicago Mercantile Exchange is defined as a $250 index. An increase (decrease) of one point in the S&P 500 index will result in a gain of $250 on the long (short) futures contract, regardless of the level of the S&P 500. That is, the sensitivity parameter, the delta, is not a function of the level of the index:

$$F_t = \$250 * S_t$$

where F_t is the futures contract and S_t is the S&P index. If the S&P rises by one point, the futures contract rises by $250 (e.g., a margin account with a long position in one futures contract receives $250). This is regardless of the *level* of the index.

Many so-called linear derivatives are only approximately linear. We often ignore the fact that there may be other underlying factors, whose relevance is much lower, and the linearity of the derivative with respect to those factors may not hold true. Consider, for example, a foreign currency forward. The standard pricing formula of a forward is

$$F_{t,T} = S_t(1 + i_{t,T})/(1 + i^*_{t,T})$$

where $F_{t,T}$ is the $T - t$ period forward rate at t forward rate, S_t is the spot exchange rate, $i_{t,T}$ is the domestic and interest rate, and $i^*_{t,T}$ is the foreign interest rate.

The formula is derived by arbitrage, using the fact that the following positions are equivalent:

- purchase an FX forward;
- short a dollar-denominated bond at $i_{t,T}$, convert the proceeds into foreign currency, and long a foreign currency-denominated bond at $i^*_{t,T}$.

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The synthetic forward (the latter position) has the same payoff as the forward, hence the arbitrage formula.

The VaR of a forward is, therefore, related to the spot rate and the two interest rates. If interest rates were fixed and we were looking at very short intervals the following would be a good approximation:

$$F_{t,T} = (1 + i_{t,T})/(1 + i^*_{t,T})S_t \approx KS_t$$

That is, the interest rate differential is a constant K which is not a function of time. The continuously compounded return on the forward, $\Delta f_{t,t+1}$, is approximately equal to the return on the spot, $\Delta s_{t,t+1}$.

$$\begin{aligned}\Delta f_{t,t+1} &= \ln(F_{t+1,T-1}/F_{t,T}) \\ &= \ln(S_{t+1}/S_t) + \ln(\text{change in the interest rate differential}) \\ &\approx \ln(S_{t+1}/S_t)\end{aligned}$$

Thus, if to a first approximation the only relevant factor is the exchange rate, then the VaR of a spot position and a forward position (notional amount) are similar. It is not unreasonable to focus on exchange rate fluctuations to the exclusion of interest rate fluctuations because the typical exchange rate volatility is about 80bp/day, ten times larger than the typical interest rate volatility of about 8bp/day.

In principle, though, accounting for the change in the two interest rates is more precise, and this would result in a nonlinear relationship. The nonlinearity can be viewed in light of the arbitrage pricing relationship as a result of the nonlinear relation between bond prices and interest rates. Since the forward position can be thought of as a short/long position in domestic/foreign bonds, as we showed above, the nonlinearity would carry through.

It is important to note that linearity or nonlinearity depends on the definition of the underlying risk factors. An interest rate swap contract can be thought of as equivalent to holding a long position in a floating rate note and a short position in a fixed-rate bond. It is hence linear with respect to these underlying assets. These underlying assets, in turn, are nonlinear in interest rates.

Another such example is a currency swap. A currency swap can be thought of as a portfolio of foreign exchange forward contracts. Being a sum of forwards, a currency swap is, hence, linear in the underlying forward contracts. Forwards are linear in the underlying exchange rate, as we saw above, but are also sensitive to interest rate changes. For short maturity forwards the interest rate sensitivity is second order to the exchange rate dependence. Linearity falls apart, however, for long dated forwards that involve longer-term interest rates. As a result, currency swaps are nonlinear in interest rates, since some of the underlying forwards are long dated, and are hence affected by interest rate changes in a meaningful way.

The duration effect plays the role of a magnifying glass. Consider, for example, a ten-year swap. The last exchange on the swap is similar to a ten-year currency forward contract. Interest rate fluctuations are magnified by the duration effect since a ten-year bond underlies the synthetic ten-year currency forward. Thus, even relatively small interest rate fluctuations represent large potential price movements for long duration bonds (see the Appendix for a more detailed discussion of duration and its effect on prices). To conclude, thinking of a foreign exchange swap with a medium to long maturity as exposed to exchange rates alone may be a bad categorization. It may be a reasonable approximation, though, for a short-dated forward or swap.

Nonlinear Derivatives

The primary example for a nonlinear derivative is an option. Consider for example an at-the-money (ATM) call option with six months to expiration written on a non-dividend-paying stock worth $100, with a volatility of 20 percent per annum. The value of the call option is $6.89 according to the Black–Scholes option pricing formula. If the underlying were to fall by $1.00 to $99.00, the option would fall by $0.59 to $6.30. In percentage terms a decline of 1 percent in the underlying would cause a decline of 8.5 percent in the option. The "$Delta" here is $0.59—a decline of $0.59 in the option of a $1.00 decline in the underlying. The "Delta" is 8.5—a 1 percent decline in the underlying generates an 8.5 percent decline in the option.

Consider now an option with a higher exercise price, $110, on the same underlying asset. The Black–Scholes value of this option is $2.91, and if the underlying fell by 1 percent to $99, the option value would decline to $2.58, a decline of 11 percent, hence a Delta of 11. For the same percentage decline in the underlying, we see a larger percentage decline for the more levered out-of-the-money option. This difference exemplifies the nonlinearity of options.

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More generally, the change in the value of the derivative as a function of the change in the value of the underlying is state dependent. In our case the state can be summarized as a function of S/X, the level of moneyness of the option.

Approximating the VaR of Derivatives

Calculating the VaR of a linear derivative is straightforward. Consider, again, the futures example:

$$F_t = \$250*S_t$$

Then the VaR of the futures contract is at the VaR of the underlying index. To see this, assume the underlying does move by its VaR during the trading day t to $t + 1$, then the VaR of the futures contract is

$$\begin{aligned}\text{VaR}(F_t) &= \\ &= F_{t+1} - F_t \\ &= \$250*(S_{t+1} - S_t) \\ &= \$250*(S_t + \text{VaR}(S_t) - S_t) \\ &= \$250*\text{VaR}(S_t)\end{aligned}$$

In words, the VaR of the futures is the number of index point movements in the underlying index, times the contract's multiple – \$250.

More generally, the VaR of the underlying factor (denoted as VaR_f) is defined as a movement in the factor that is related to its current volatility times some multiple that is determined by the desired VaR percentile. The VaR of a linear derivative on this underlying factor would then be the factor VaR times the sensitivity of the derivative's price to fluctuations in the underlying factor. The latter term is simply the derivative's delta. Thus, for the case of linear derivative, we can express the derivative's VaR (denoted VaR_p) as:

$$VaR_p = Delta*VaR_f \tag{2.1}$$

That is, the VaR of the derivative is delta times the VaR of the underlying risk factor.

An important caveat should be noted here. Our derivation assumes implicitly that the delta is positive. A positive delta implies a long position or a positive exposure to the underlying. If the delta is negative, a loss of *VaR* on the underlying generates a *gain* of *Delta*VaR* on the derivative. It is hence the case that one needs to look for cases of extreme gain in the underlying in order to find extreme cases of loss in the derivative when the delta is negative.

Turning to nonlinear derivatives we should first note that every asset is locally linear. That is, for small enough moves we could extrapolate given the *local delta* of the derivative, where the local delta is taken to mean the percentage change in the derivative for a 1 percent change in the underlying factor.

Consider for example, the at-the-money call option shown in Table 2-1. As we saw, the delta of the option is 8.48: a decline of 1 percent in the underlying will generate a decline of 8.48 percent in the option. Suppose now that we wish to calculate the one-day VaR of this option. Recall that the underlying asset has an annualized volatility of 20 percent. This annual volatility corresponds to, roughly, 1.25 percent per day. The 5 percent VaR of the underlying asset corresponds, under normality, to 1.65 standard deviation move, where the standard deviation on a daily basis is 1.25 percent. Assuming a zero mean return,

TABLE 2-1 Call Option Prices and Deltas*

Stock Price	\$90	\$99	\$99.9	\$100	\$100.1	\$101	\$110
Call	\$2.35	\$6.30	\$6.83	\$6.89	\$6.95	\$7.50	\$14.08
Change in Stock Price DS(%)	−10.0%	−1.0%	−0.1%		0.1%	1.0%	10.0%
Change in Call Value DC(%)	−65.9%	−8.5%	−0.9%		0.9%	8.9%	104.3%
DC(%)/DS(%)	6.59	8.48	8.66		8.70	8.87	10.43

* Assume a strike price of $X = 100$, time to expiration of ½ year $t = 0.5$, a riskfree rate $r = 5\%$, and stock price volatility $\sigma = 20\%$.

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this implies that the 5 percent VaR of the underlying is 0 − 1.25*1.65 = −2.06%. This, in turn, implies a decline in the value of the call option of:

$$5\%VaR(call) = -2.06\% * delta = -2.06\% * 8.48 = -17.47\%$$

That is, there is a 5 percent probability that the option value will decline by 17.47 percent or more. Recall that this is only an approximation, because as the asset declines in value the delta changes. The precise change can be calculated using the Black-Scholes formula (assuming that is the correct model to price the option exactly) to evaluate the option when the underlying declines from a current value of $100 by 2.06 percent, to $97.94. The precise value of the option is $5.72, implying a decline in value of 17.0 percent. While there is some imprecision, the extent of imprecision could be thought of as relatively small.

Consider the case where we want the VaR of the option for the one week horizon. The weekly volatility of the underlying is $20\%/\sqrt{(52)}$ = 2.77%. Still assuming normality and a mean of zero, the 1 percent VaR is calculated as 0 − 2.33*2.77% = −6.46%. That is, a decline of 6.46 percent in the underlying corresponds, using our delta-linear approximation, to (8.48)(−6.46) = −54.78%. That is, given a one week 1 percent VaR of 6.46 percent for the underlying, the one-week 1 percent VaR of the call is 54.78 percent. In order to evaluate the precision of the linear approximation in this case, we need to price the option given a decline in the underlying of 6.46 percent. That is, we should reprice the option with an exercise price of $100 assuming that the underlying asset falls in value to $93.54. The value of the option in this case would decline from $6.83, the at-the-money value, to $3.62. This is a decline of 47.4 percent.

The level of imprecision in the one-day VaR can be quantified by taking the ratio of the linear VaR to the correct, full revaluation, VaR. For the 5 percent daily VaR, this ratio is 17.4%/17% = 1.023. The bias resulting from the linear approximation is 2.3 percent. We can compare this ratio to the accuracy ratio for the one-week 99th percentile VaR. This ratio is 54.78%/47.4% = 1.15. The linear VaR is much more biased for a larger move relative to the "true" full revaluation VaR. The bias grows from 2.3 percent to 15 percent as the VaR percentile goes from 5 percent to 1 percent and as the time period increases from one day to one week.

Figures 2-1 and 2-2 provide a schematic of this effect. Figure 2-1 graphs the value of the call option on the *Y*-axis as a function of the option's moneyness on the *X*-axis. The option is convex in the value of the underlying. For small enough moves, though, the linear approximation should work well. The slope of the call's price as a function of the underlying is a close approximation to the changes in

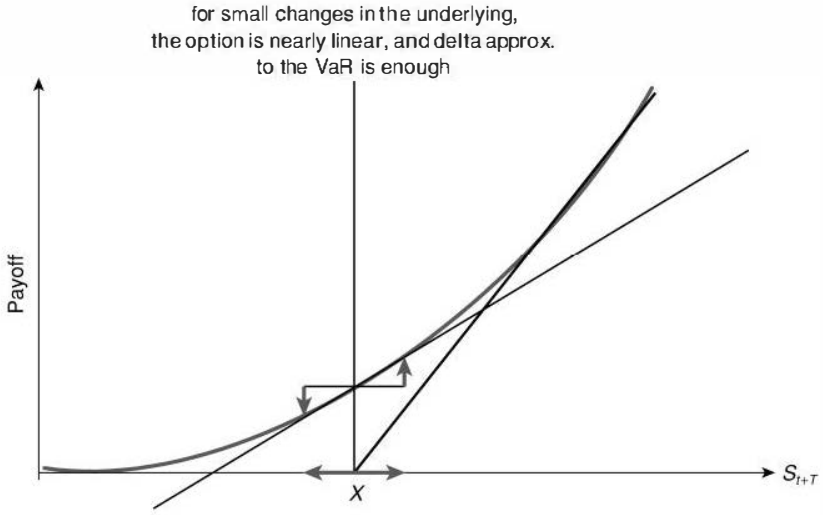

FIGURE 2-1 The VaR of options: small moves.

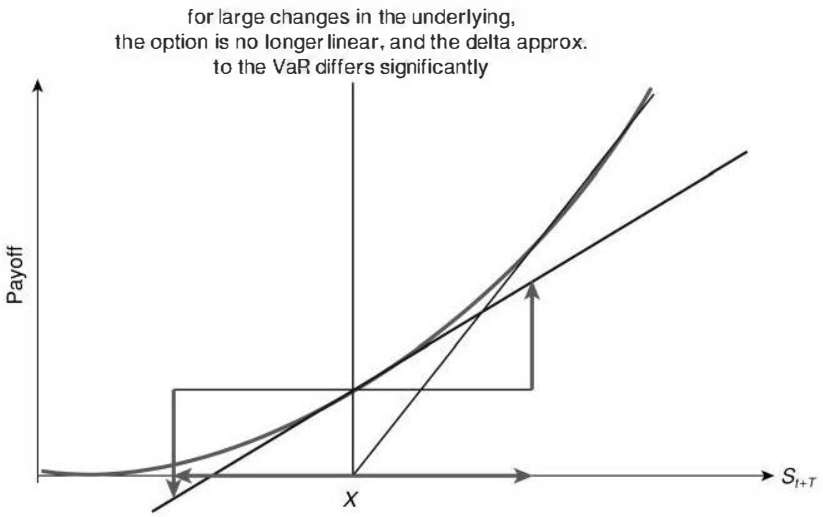

FIGURE 2-2 The VaR of options: large moves.

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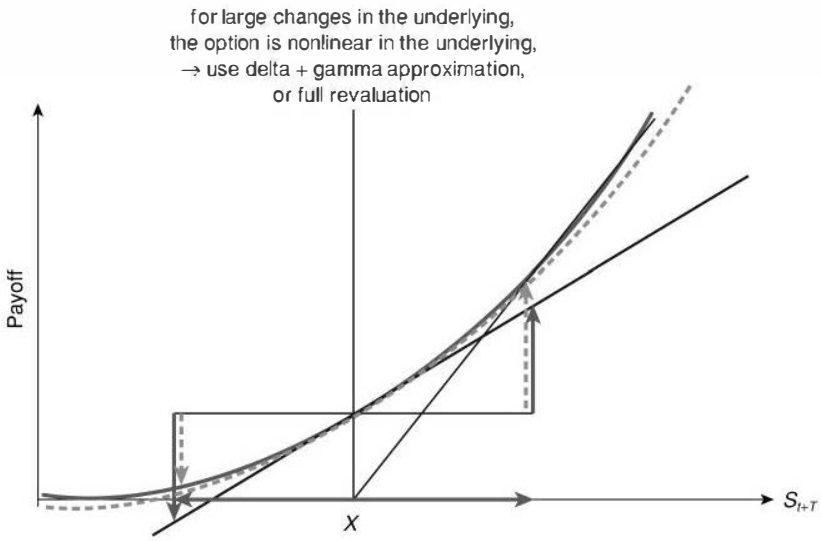

FIGURE 2-3 The VaR of options: convexity correction.

value we would see for small enough moves in the underlying. The nonlinearity results in the slope changing for different level of moneyness.

The change in the slope coefficient is the result of the option's nonlinearity in the underlying. This nonlinearity is also the culprit for the imprecision in the linear approximation, and the increasing degree of imprecision as we consider larger moves of the underlying. In particular, as Figure 2-2 shows, for larger moves we can see a clear bias in the linear approximation of the change in the value of the call option. In fact, the bias will always be positive; that is, whether the underlying rises or falls, the linear approximation will underestimate the true value of the option. In other words, the required correction term is always positive. In Figure 2-2 this is visible from the fact that the straight line, corresponding to the linear approximation, lies underneath the true option value (using the Black–Scholes option pricing formula).

The bias is a result of the fact that the slope changes as the underlying changes. The further the underlying asset moves away from its original value, where the slope was correct, the more the slope changes, and the worse the linear approximation with the given fixed slope becomes. Since the value of the option is convex in the underlying (i.e., the second derivative is positive), we will get a positive bias assuming linearity.

Figure 2-3 describes one way to mitigate the problem, by approximating the curvature of the pricing function using a convexity correction. The premise is related to what is known in mathematics as a Taylor Series approximation. Simply put, the approximation says that any "well behaved" function can be approximated by a polynomial of order two (i.e., a quadratic) as follows:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) + 1/2 f''(x_0)(x - x_0)^2 \quad (2.2)$$

This means that the value of a function $f(x)$ for any value of x is approximated by starting with the value of the function at a given point x_0 (which is assumed to be known), and then approximating the change in the value of the function from $f(x_0)$ to $f(x)$ by accounting for:

- the slope of the function at x_0, $f'(x_0)$, times the change in the x-variable, $(x - x_0)$;
- plus the curvature of the function at x_0, $f''(x_0)$, times the change squared divided by two;

the first term is exactly the linear approximation, while the second term is the convexity correction.

The improvement in the convexity correction turns out to be important in application. This correction is implemented by calculating the option's second derivative with respect to the underlying factor and the option's gamma.

One particularly well-known example of the need to approximate a nonlinear relation is, in fact, not from the derivative securities area. The example is the nonlinear relation between interest rates and the value of a pure discount bond. The value of a zero coupon bond with one year to maturity as a function of the one-year rate y is:

$$d = 1/r$$

This is clearly a nonlinear relation. The graph of this function is a hyperbola. Figure 2-4 describes the relationship. The straight line marked by "duration" is the linear approximation to the price change as a function of the interest rate (see Appendix for a detailed discussion of duration). Duration apparently falls apart as an approximation for the bond's price change for large moves in interest rates. To this end we have the convexity correction to the duration approximation.

The idea behind the duration-convexity approximation of the impact of interest rate fluctuations on bond prices is

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identical to the idea behind the delta-gamma approximation of the impact of fluctuations in the underlying on the value of an option. Both rely on:

- the knowledge of the pricing model and the existence of an explicit pricing formula;
- the ability to provide a first and second derivative to this pricing formula as a function of the underlying;
- the use of the Taylor Series approximation.

This approach is not unique. There are many types of derivatives where a pricing relation can be derived analytically or via computations. Examples include:

- convertible bonds which are nonlinear in the value of the underlying asset into which the bonds are convertible;
- defaultable bonds that are nonlinear in changes in the default probability;
- mortgage-backed securities, which are nonlinear in the refinancing incentive (the difference between the mortgage pool's rate and the current refinancing rate).

Fixed Income Securities with Embedded Optionality

The Taylor Series approximation described earlier does not perform well in the case of derivatives with extreme nonlinearities. For example, mortgage-backed securities (MBS) represent fixed income securities with embedded options; that is, the mortgagor (the borrower) can choose to prepay the mortgage at any time, particularly, when mortgage rates are declining. Figure 2-5 depicts the sensitivity of three different mortgage pools, Government National Mortgage Association (GNMA) 8 percent, 9 percent and 10 percent, as a function of the long rate. The *Y*-axis is the conditional regression beta—regressing MBS prices on interest rate changes. The coefficient is conditional in the sense that the estimate is for the local sensitivity of prices to rates in the vicinity of a given interest rate (i.e., small rate changes). For all three mortgage pools the regression beta is negative. This is intuitive—as interest rates rise the value of all three mortgage pools drop, and vice versa. The beta can be interpreted as the duration of the pools (with the reverse sign).

The interesting point is that the beta/duration is not constant and changes in a remarkable way for different long-term rates. As rates fall from higher levels of, say, 9.5 percent, duration increases (i.e., the beta becomes more negative). This is an effect common to many ordinary fixed income securities such as bonds—that duration rises as interest rates fall. As interest rates fall further duration starts to fall. This is the result of an actual and

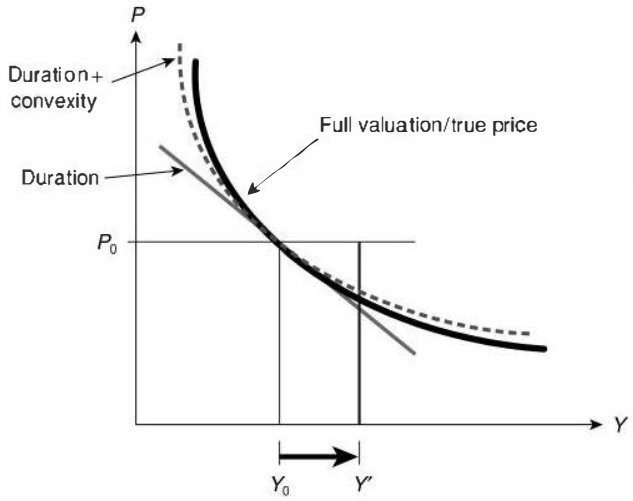

FIGURE 2-4 Duration and convexity in bond pricing.

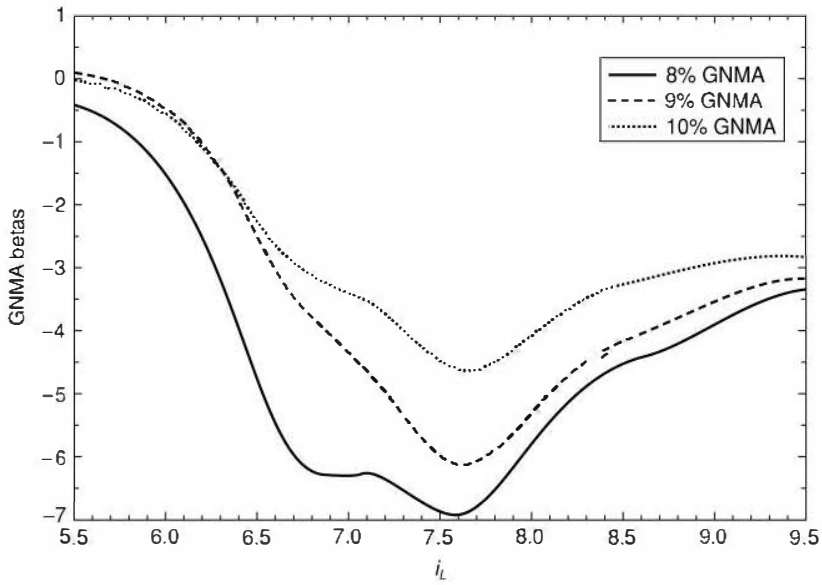

FIGURE 2-5 Empirical interest rate sensitivity of MBSs.

Source: Boudoukh, Richardson, Stanton, and Whitelaw (1997)

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anticipated rise in prepayment activity. As rates fall a high coupon mortgage is likely to be refinanced by a homeowner. This causes the life of the MBS to shorten—a security that was supposed to pay fixed interest payments for a long period now pays down the full face amount, exercising the option to prepay.

Such shifts in duration result in a security that is not simple to price or hedge. A similar effect is also observed in other fixed income securities with embedded derivatives such as callable debt. Callable debt will also exhibit an initial rise in duration as rates fall, but then, as the option to call back the debt enters into the money the expected call will shorten the life of the callable bond.

These securities pose a problem for risk managers. First, it is clearly the case that such securities require fairly sophisticated models for pricing, and hence for risk management. These may not be compatible with simple risk measurement techniques that may be suitable for linear assets. Moreover, the sharp changes in duration may make the duration-convexity approximation weaker. For these securities the rate of change in duration changes for different interest rates, making the convexity correction much less accurate. Thus, convexity alone cannot be used to correct for the change in duration.

"Delta-Normal" vs. Full Revaluation

There are two primary approaches to the measurement of the risk of nonlinear securities. The first is the most straightforward approach—the full revaluation approach. The approach is predicated on the fact that the derivative moves one-for-one, or one-for-delta with the underlying factor. Assuming a positive delta, i.e., that the derivative moves in the same direction as the factor, we use a valuation expression to price the derivative at the VaR tail of the underlying factor. For example, the 1 percent VaR of an option on the S&P 500 index can be calculated by first finding out the 1 percent VaR of the index. This step can be done using any approach—be it parametric (e.g., assuming normality) or nonparametric (e.g., historical simulation). The VaR of the option is just the value of the option evaluated at the value of the index after reducing it by the appropriate percentage decline that was calculated as the 1 percent VaR of the index itself.

This approach has the great advantage of accuracy. It does not involve any approximations. However, this approach can be computationally very burdensome. Specifically, we may be able to reprice a bond or an option easily, but repricing a portfolio of complex derivatives of MBSs, swaptions, exotic options and so on can require many computations. In particular, as we will see later on, we may want to evaluate thousands of different scenarios. Thousands of revaluations of a portfolio consisting of hundreds of exotic securities using simulations or binomial trees may require computing power that takes days to generate the results, thereby rendering them useless.

The alternative is the approach known as the "delta-normal" approach, which involves the delta (linear) approximation, or the delta-gamma (Taylor Series) approximation. The approach is known as "delta-normal" because the linear approximation shown in Equation (2.1) is often used in conjunction with a normality assumption for the distribution of fluctuations in the underlying factor value. The approach can be implemented relatively simply. First we calculate the VaR of the underlying. Then we use Equation (2.1) to revalue the derivative according to its delta with respect to the underlying times the change in the underlying. Clearly the first step—finding out the VaR of the underlying—does not need to be calculated necessarily using the normality assumption. We could just as well use historical simulation for example. The key is that the approach uses the delta approximation.

This approach is extremely inexpensive computationally. Calculating the risk of a complex security can be almost "free" as far as computational time is concerned. In particular, consider a fixed income derivative that is priced today, for simplicity, at $100 for $100 of par. Suppose we used a binomial interest rate tree to price and hedge this derivative given a current interest rate of 6 percent p.a. Assume further that the security matures in 10 years, and that our binomial interest rate tree is built with a time step of one month. There are, hence, 120 one-month periods to maturity. Suppose the first one-month step involves a change in interest rates of 10bp up or down. That is, the binomial tree that we use for pricing takes today's rate of 6 percent and after the first time step rates can be either 6.1 percent or 5.9 percent. Binomial pricing involves spanning a complete tree of interest rates to the maturity of the derivative, discounting back through the tree the derivative's cash flows.

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As we work our way back through the tree when pricing this security we can note the prices the security can take next period, i.e., in one month. Suppose that the bond prices were $101 for the downstate of interest rates, 5.9 percent, and $99.2 for the up-state of 6.1 percent. If we are willing to ignore that one-month time value, these two numbers give us an approximate interest rate sensitivity measure. Specifically, we know that the following is approximately true:

$$Y = 5.9\% \rightarrow P = \$99.2,\ Y = 6\% \rightarrow P = \$100,$$

$$Y = 6.1\% \rightarrow P = \$101$$

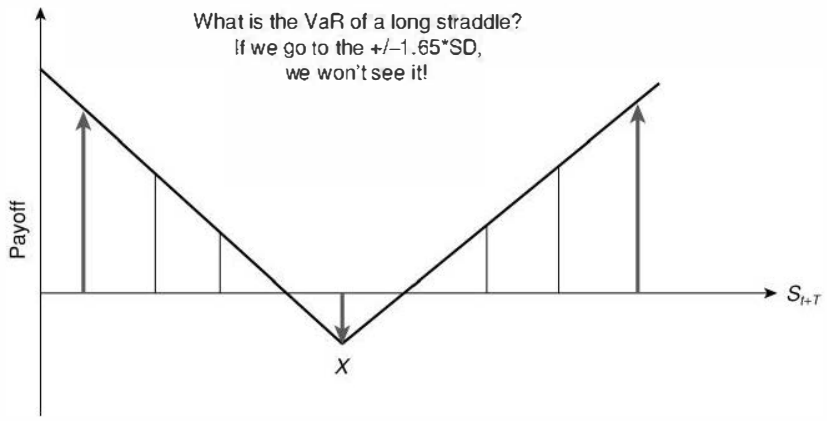

FIGURE 2-6 The VaR of options: a straddle.

This information provides us with an estimate of the derivative's interest rate sensitivity. In particular, for a difference in rates of 20bp (6.1% – 5.9%) we know that the price of the security would fall by $1.80, the difference between the up-state price of $99.2, and the down-state price of $101. A linear approximation would imply that a rise of 100bp in rates would result in a change in value of $9. Given a par value of $100, this means that a rise of 1 percent would result in approximately $9 drop in price.

Note that this calculation did not require full revaluation. In the full revaluation approach if we wanted to price the security for a 100bp shift up in rates, we would have to rebuild the binomial interest rate tree starting from a current rate of 7 percent instead of 6 percent. The empirical duration method presented here provides us with a linear approximation to the price change. In particular, we would expect a drop in value of 9 percent for a rise of 1 percent in rates. In our case this also corresponds to a drop in value of $9, since we assumed the security trades at par value.

STRUCTURED MONTE CARLO, STRESS TESTING, AND SCENARIO ANALYSIS

Motivation

The calculation of VaR can be an easy task if the portfolio consists of linear securities. Practical issues remain with respect to the implementation and accuracy of the VaR estimate, but conceptually there are no obstacles left. As we have seen in this chapter, this is certainly not the case for nonlinear derivatives. Moreover, portfolios of nonlinear derivatives will pose a special challenge to the risk manager. To see the problem consider an options straddle position—a long position in a call and a put with the same exercise price. The cash flow diagram of the straddle appears in Figure 2-6.

How can we calculate the VaR of this option position? Since this is a portfolio of derivatives, we need to first come up with the VaR of the underlying, and then either revalue the derivative at this underlying value or use a delta approximation approach. If the derivative involves an implicit short position, then we need to examine an extreme rise in the underlying as the relevant VaR event rather than an extreme decline. Suppose our example involves a straddle on the S&P 500 index. Suppose further that the standard deviation of the index is 100bp/day, and that the 1 percent one-day VaR under normality is a decline of 233bp. The mirror image case assuming that returns are symmetric would be an increase of 233bp. With an at-the-money straddle it is clearly the case that we make money in either case. Straddles, being a bullish bet on volatility, pay off when the underlying moves sharply. Loss scenarios for straddles, precisely what VaR is supposed to deliver, involve the underlying remaining close to its current value. How do we generalize our derivative approach to VaR calculation to accommodate such a complication?

Structured Monte Carlo

For the straddle, large loss scenarios involve small, not large, moves in the underlying. The methodology described so far clearly cannot handle this situation.

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There is, however, a distribution of possible values for the portfolio given possible values for the underlying. By definition, there exists a VaR. One way to derive this VaR is to simulate thousands of possible values for the underlying given its distribution (e.g., under the normality assumption).

Suppose that we generate, say, 10,000 values for the S&P 500 index tomorrow based on a standard deviation of 100bp/day. Then we re-evaluate the straddle for each of these 10,000 values of the underlying. As a result we have 10,000 values that the straddle might take by the end of tomorrow's trading, based, of course, on our statistical assumptions about the price distribution of the underlying S&P 500. Ordering the 10,000 simulated values for the straddle from smallest to largest would generate a distribution for the straddle, and the 9,900th value would be the simulated 1st percentile. This value corresponds to the 1 percent VaR.

More generally, suppose that one needs to generate scenarios for an asset whose returns are distributed normally with a mean of μ and a standard deviation of σ^2. The simulation requires a random-number generator that generates draws from a normal distribution with a mean of zero and a standard deviation of one. Denote these $N(0,1)$ draws by $z_1, z_2, \ldots, z_{NSIM}$. The NSIM scenarios are, hence $\mu + \sigma z_1, \mu + \sigma z_2, \ldots, \mu + \sigma z_{NSIM}$. Since we use continuously compounded returns, the index's simulated value for a given random normal draw z_i is denoted $S_{t+1,i}$ and can be expressed as:

$$S_{t+1,i} = S_t^* exp\{\mu + \sigma^* z_i\}$$

For each of these values we revalue the entire derivative portfolio. Next, we order the NSIM simulated value and pick the $(1 - X/100)^*NSIMth$ value as the X% VaR.

We can extend the Monte Carlo approach to the more relevant case facing real world risk managers in financial institutions—the case of multiple assets with multiple risk exposures. The extension is conceptually straightforward, although some technical issues arise. Briefly, for K risk factors and $NSIM$ simulations we need to generate K^*NSIM independent variables distributed as $N(0,1)$. These can be stacked as $NSIM$ vectors of size K. Each such vector is distributed as multivariate normal with a mean vector which is a K^*1 vector of zeros, and a variance covariance matrix which is an identity matrix of size K^*K. Similar to the one dimensional, single factor case we generate $NSIM$ scenarios for K underlying factors

$$Ln(\mathbf{S}_{t+1}/\mathbf{S}_t) = exp\{\boldsymbol{\mu} + A'^*\mathbf{Z}_t\}$$

where

$Ln(\mathbf{S}_{t+1}/\mathbf{S}_t)$ is a K^*1 vector of lognormal returns;

$\boldsymbol{\mu}$ is a K^*1 vector of mean returns;

$\mathbf{Z}_t$ is a K^*1 vector of $N(0,1)$'s;

and A' is the Cholesky decomposition of the factor return covariance matrix Σ, that is $\mathbf{A'A} = \Sigma$.

Simulated factor returns are distributed with a mean and a covariance matrix that can be estimated from live market data, or postulated based on a model or theory.

The main advantage of the use of structured Monte Carlo (SMC) simulation is that we can generate correlated scenarios based on a statistical distribution. To see this advantage one needs to compare this approach to the standard scenario analysis approach, of, say, revaluing the portfolio given a 100bp rise in rates. Analyzing the effect of a parallel shift of 100bp on the portfolio's value tells us something about its interest rate risk, but nothing about the overall risk of the portfolio. The SMC approach to portfolio risk measurement addresses most of the relevant issues.

The first issue is that while a 100bp parallel shift in rates is a possible scenario, there is no guidance as to how likely this event is. There is no probability attached to scenario analysis that is performed based on scenarios that are pulled from thin air. As such, it is not clear what to do about the result. It is clearly the case that an institution would want to protect itself from a 1:100 event, but it is not clear what it is supposed to do about a 1:1,000,000 event, if anything. What are the odds of a 100bp move, then?

Second, the 100bp parallel shift scenario is a test of the effect of a single risk factor—the level of domestic rates. It is not clear what is the relevance of such a scenario, especially in the context of a globally diversified portfolio. A more complete risk model would recognize that statistically the likelihood of a 100bp rise in rates in isolation is a remote likelihood scenario, relative to a scenario in which rates rise across many countries. This is a critical point for a global fixed income portfolio.

Consider a simple example of a speculative position that is long US Bonds and short UK bonds. An increase in US interest rates will make the position look very risky. While the long side of the position—US bonds, will fall

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in value, there will be no commensurate fall in the short-gilts UK side of the portfolio. However, chances are that a sharp increase in US interest rates will coincide with a rise in UK rates. Under those circumstances, the resulting decline in UK bond prices on the short side would generate a profit that will compensate for the loss in the long US bond position. Of course, there is no absolute certainty that interest rates will move in tandem across countries, but a sharp increase in interest rates is often attributable to a global rise in inflationary expectations and political instability that make such co-movement more likely to occur.

SMC copes effectively with these two issues. The scenarios that will be generated using the estimated variance-covariance matrix will be generated based on a set of correlations estimated from real data. As such, the scenarios will be as likely in the simulation as they are in reality. With respect to the first point, regarding the likelihood of a given scenario, in an SMC we have a clear interpretation of the distribution of the *NSIM* simulations we generate. Each is as likely. It is hence the case that the 1 percent VaR according to the SMC is the first percentile of the distribution of the simulated scenario portfolio values.

With respect to the manner in which various risk factors co-vary with one another, the use of the covariance matrix of the scenarios as an input guarantees that the economic nature of the events driving the simulation is plausible. It is important to note, though, that the economic content is only as sound as our ability to model co-movements of the factors through the covariance matrix (see Boyle, Broadie, and Glasserman, 1997).

Scenario Analysis

As we have seen in the previous discussion, structured Monte Carlo simulation may be used to solve the special problems associated with estimating the VaR of a portfolio of nonlinear derivatives. However, the approach is not free of shortcomings. In particular, generating scenarios in simulation and claiming that their distribution is relevant going forward is as problematic as estimating past volatility and using it as a forecast for future volatility. Generating a larger number of simulations cannot remedy the problem. As we will see in the remainder of this section, scenario analysis may offer an alternative that explicitly considers future events.

Correlation Breakdown

Consider the case of a global bond portfolio investment. Such a portfolio is often predicated on the notion of diversification. Specifically, global bond portfolios often achieve excess risk-adjusted performance by taking on sovereign risk, taking advantage of the "fact" that sovereign risk premiums are too high. Historical estimation of the co-movement of portfolios of sovereign bonds, e.g., Brady Bonds, generate unusually low correlations. These correlations, in turn, imply strong risk-reduction due to diversification.

It is possible to use these risk estimates to demonstrate that the yield enhancement of taking on sovereign credit risk comes at little cost as far as pervasive risk is concerned. However, low cross-country correlations are likely to be exhibited by the data, as long as there were no global crises within the estimation window. However, during times of crisis a *contagion* effect is often observed. Consider two specific examples.

First, consider the correlation of the rates of return on Brady Bonds issued by Bulgaria and the Philippines. These two countries are, loosely speaking, as unrelated as can be as far as their creditworthiness. Their geographic regions are distinct, their economic strengths and weaknesses rely on unrelated factors, and so forth. Indeed, the correlation of the return series of bonds issued by these countries is low—around 0.04. A portfolio comprised of bonds issued by these two countries would show little pervasive volatility. However, during the economic crisis in east Asia in 1997/8 the correlation between these bonds rose dramatically, from 0.04 to 0.84!

The second example is the statistical link between the yield change series of US government bonds and JGBs (Japanese Government Bonds). These two bonds tend to exhibit low correlation during normal times. In particular, while the correlation of the two series may vary through time, prior to August 1990 the estimated correlation was 0.20. During the war in the Gulf region in the summer of 1990 and early 1991 the correlation increased to 0.80. A common factor of global unrest and inflationary fears due to rising oil prices may have caused a global rise in yields.

These events of breakdown in historic correlation matrices occur when the investor needs the diversification benefit the most. In particular, note that the increase in volatility that occurs during such crises would require an even

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TABLE 2-2 Correlation Breakdown

Event	Date	Correlations between Variables	Prior to	During
ERM crisis	Sep92	GBP/USD, GBP LIBOR	−0.10	0.75
Mexican crisis	Dec94	Peso/USD, 1mo Cetes	0.30	0.80
Crash of 1987	Oct 87	Junk yield, 10yr Treasury	0.80	−0.70
Gulf War	Aug90	10yr JGBs, 10yr Treasury	0.20	0.80
Asian crisis	1997/8	Brady debt of Bulgaria and the Philippines	0.04	0.84

stronger diversification effect in order not to generate extreme returns. The opposite is true in the data. Spikes in volatility occur at the same time that correlations approach one because of an observed contagion effect. A rise in volatility and correlation generates an entirely different return generating process. Table 2-2 includes a few examples of stress events and the "before" and "during" correlations between relevant variables.

In addition to the Asian crisis of 1997/8 and the Gulf War of 1990/1 there are a few other examples of correlation breakdown:

- the GBP/USD exchange rate and the GBP LIBOR rate before and during the collapse of the period that the UK dropped out of the ERM;
- the Peso/USD and the Peso rate during the 1994/5 Mexican crisis;
- yield changes (returns) on low grade and high grade debt before and during the 1987 stock market crash.

Generating Reasonable Stress

Our discussion of correlation breakdown carries strong implications for our interpretation of results from an SMC simulation. A simulation using SMC based on a covariance matrix that was estimated during normal times cannot generate scenarios that are economically relevant for times of crisis. A common, but potentially erroneous, remedy is to increase the number of simulations and go further out in the tail to examine more "severe stress." For example, it is common to see a mention of the 0.01 percentile of the simulation, e.g., the 10th out of 100,000 simulations. It is not uncommon for financial institutions to run an especially long portfolio simulation over the weekend to come up with such numbers. Unfortunately the 10th out of 100,000 simulations, while strictly speaking is indeed the 0.01 percent VaR given the covariance matrix, has little to do with the 1 in 10,000 event. The 1 in 10,000 event on a daily basis is an event that we are likely to see only once in 40 years (the odds of 1:10,000 divided by 250 trading days per year). It would be dangerous to assert that an SMC as described above can provide a reasonable assessment of losses on such a day (or week, or month, for that matter).

TABLE 2-3 Empirical Stress

SDs	Normal Distribution	S&P 500	Yen/$	10-yr Rate
2	4500	3700	5600	5700
3	270	790	1300	1300
4	6.4	440	310	240
5	0	280	78	79
6	0	200	0	0

Table 2-3 demonstrates the problem further. A four or more standard deviation event should occur, according to the normal benchmark in expectation, 6.4 times in 100,000. It is a 1 in 15,625 event, or an event that is expected to occur once every 62 years. Consider now the S&P 500 index. This broad well-diversified index exhibits daily returns that are four or more standard deviation events at a rate that is equivalent to 440 in 100,000 (over a smaller post-WWII sample, or course). The true likelihood of a four or more standard deviation event is once every 0.9 years. An extreme move that a normal-distribution-based SMC would predict should occur once in a blue moon is, in reality, a relatively common event.

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We obtain results similar to those shown in Table 2-3 for common risk factors such as the ten-year interest rate and the USD/JPY exchange rate. This is a simple extension of the fat tails effect for single assets and risk factors. The difficulty here is twofold—the spikes in returns and the collapse of low correlations during financial crises. While there is a lively debate in the literature on whether the contagion effect is an example of irrationality or a result of rational behavior in the presence of a spike in the volatility of global risk factors, the fact remains that covariance matrices cannot generate stress in SMC simulations, regardless of the number of simulations.

One approach is to stress the correlation matrix that generates the SMC scenarios. Stressing a correlation matrix is an attempt, intuitively, to model the contagion effect that may occur, and how it may affect volatilities and correlations. The exercise of stressing a correlation matrix is not straightforward in practice. A covariance matrix should have certain properties that may be lost when tinkering with the correlations of this matrix. In particular, the variance-covariance matrix needs to be invertible, for example. Significant work has been done on the subject of how to increase contagion given an estimated covariance matrix without losing the desirable properties of the covariances that were estimated using historical data.

Stress Testing in Practice

It is safe to say that stress testing is an integral component of just about every financial institution's risk management system. The common practice is to provide two independent sections to the risk report: (i) a VaR-based risk report; and (ii) a stress testing-based risk report. The VaR-based analysis includes a detailed top-down identification of the relevant risk generators for the trading portfolio as a whole. The stress testing-based analysis typically proceeds in one of two ways: (i) it examines a series of historical stress events; and (ii) it analyzes a list of predetermined stress scenarios. In both cases we need to assess the effect of the scenarios on the firm's current portfolio position.

Historical stress events may include such events as the crash of 1987, the 1990/1 Gulf War, the Mexican crisis of 1994, the east Asian crisis of 1996 and the near collapse of LTCM in 1998. The alternative approach is to examine predetermined stress scenarios as described above. For example, we may ask what is the effect of such extreme scenarios as a 200bp shift up in rates, an increase in volatility to 25 percent p.a. and so on, on the portfolio's value. The latter approach is a standard requirement in many regulatory risk reports (e.g., the Office of Thrift Supervision's requirement for savings banks to report periodically the effect of parallel shifts in the yield curve on the institution's asset–liability portfolio).

These approaches to stress testing provide valuable information. The analysis of past extreme events can be highly informative about the portfolio's points of weakness. The analysis of standard scenarios can illuminate the relative riskiness of various portfolios to standard risk factors and as such may allow the regulator to get a notion of the financial system's aggregate exposure to, say, interest rate risk. Nevertheless, the approach of analyzing pre-prescribed scenarios may generate unwarranted red flags on the one hand, and create dangerous loopholes on the other.

In particular, consider the analysis of specific term structure scenarios. While the analysis of parallel shift scenarios and perhaps a more elaborate analysis of both parallel shift as well as tilt scenarios may give us an idea of the interest rate risk exposure of the bank's portfolio with respect to changes in domestic interest rates, this risk measure may be deceiving for a number of reasons.

The first was discussed in detail earlier when structured Monte Carlo was introduced and motivated. Briefly, to the extent that interest rates move in tandem around the world, at least when it comes to large moves, then large losses in the domestic bond portfolio are likely to occur together. This effect may cause a severe error in risk estimation—with a long and short position risk may be overstated, with a long-only portfolio risk may be understated.

Another problem with this approach is the issue of asset-class-specific risk. It is often argued that some asset classes may have asset-class-specific risks. For example, emerging market debt could suffer from contagion risk—the complete loss of investor sentiment for investment in sovereign debt. Another example is the mortgage-backed securities market, where interest rate risk factors explain only a portion of the total risk. There are, apparently, other pervasive risk factors governing returns in this sector. These factors are not well understood or modeled (see Boudoukh, Richardson, Stanton, and Whitelaw, 1997).

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From an academic perspective it is important to sort out whether such co-movements within an asset class are rational or irrational. Using terms such as "investor sentiment" and "contagion" as reasons for co-movements within an asset class may allude to some form of market irrationality. Alternatively, however, co-movements within the asset class may be rational and attributable to explanatory variables that are erroneously omitted from our models. Moreover, the models may be misspecified—that is, that the right functional form or structural model was not correctly identified. Which one of these possible explanations is correct is probably unimportant from a risk management perspective. What is important is that these pricing errors can undermine the accuracy of stress tests.

Note that asset-specific risk is extremely important for financial institutions that are not well diversified across many lines of business. Specialized financial institutions may carry large inventory positions relative to their capital in their area of specialization or focus. Such institutions may be able to assess, report, and risk-manage their known risk (e.g., interest rate risk), but often cannot measure and manage their total risk exposure.

Total risk is rightfully thought of as unimportant in asset pricing. Asset pricing theory states that the asset or firm specific risks are not priced—it is only systematic risk that is priced (whether it is market risk, interest risk or any other systematic form of risk). However, from the perspective of the risk manager both systematic risk and asset specific risk may matter, in particular if asset specific risk is not diversified through a large portfolio of different assets that are drawn from different asset classes.

Stress Testing and Historical Simulation

As discussed above, the common approach in stress testing is to choose past events that are known to be periods of financial market volatility as the relevant stress tests. This approach could be problematic since it might miss the true relevant risk sources that may be specific to an institution. An alternative approach is an approach based on the same intuition of historical simulation. The idea is to let the data decide which scenarios fit the definition "extreme stress."

In particular, consider, for example, examining the returns on all the factors that are relevant for our trading portfolio over one-week horizons for the last 20 years. The last 20 years provide a sample of 1,040 weekly periods, including a few of the well-known crises that are often used as stress tests. However, a fixed income portfolio, for example, may experience extreme movements during different periods than an equity portfolio, and that may differ from a currency portfolio. Thus, the definition of stress periods may differ from asset to asset. It is always inferior to base the analysis on a prespecified set of events rather than examining all possible events in order to identify those with extreme returns.

Unlike the case of historical simulation as a counterpart to VaR, here we are not interested in the 5 percent VaR—the 5th of 100 trading periods or the 52nd of 1,040 trading weeks. We are going to focus our attention on the five or ten worst weeks of trading, given today's portfolio. These will help us determine the true risk exposures of our portfolio. To the extent the LTCM crash is the relevant stress event, this will show up in the data. To the extent the east Asian crisis is relevant, this will show up as an extreme move. But it is also possible that an entirely different period may become the focal point through this examination. The difference here is that the extreme events are assumed to be extreme valuations, as opposed to extreme movements in underlying risk factors.

The decline in rates during the 1994–5 period that resulted in extreme refinancing activity may not be thought of as a major stress event. Consider, however, a mortgage portfolio's risk. For a holder of a portfolio of CMOs this may be the most relevant stress event. It will show up as such only using the historical simulation-based approach we discussed in this section.

Asset Concentration

No discussion of risk management would be complete without reiterating the first rule of prudent risk management—diversification. The effect of diversification is a mathematical fact, not a theory. The question is how do we achieve "true" diversification? Long Term Capital Management, for example, may have had reasons to believe that the different trades run by the different "desks" in the firm had little in common. After the fact, it is clear that there was a strong pervasive factor to all the trades; that is, they were all exposed to liquidity crises. This is a factor that is difficult to quantify or forecast. In particular, unlike most other factors where live quotes exist and risk estimates can hence be provided, liquidity is hard to measure, and spikes in liquidity occur seemingly "out of the blue."

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The trigger event of stress is hard to predict. Worse than that, financial markets find a way of discovering different triggers for each new crisis; thus, the list of triggers keeps getting longer. The inflation spiral of 1973–4 was triggered by the war in the Middle East in October 1973, the crash of 1987 had no apparent trigger, the Asian crisis was triggered by sharp currency moves in Thailand, the collapse of the internet bubble, some would argue, was the March 2000 verdict in the Microsoft case, and the list goes on.

The only solution to the problem may seem rather simple minded and arbitrary. The solution comes in the form of explicit dollar limits on specific counterparty exposure and limits on total notional amount exposure per asset or asset class. For example, it is standard practice for financial institutions to set limits on the maximal amount of outstanding credit exposure to any single counterparty. These limits may be a function of the total loan amount and/or the total notional outstanding and/or the total mark to market of positions. The limit would often be quoted as a percentage of both the counterparty as well as the institution's capital.

This solution may, at first, seem arbitrary, and even overly simplistic. For example, one might argue that while setting limits on total mark-to-market exposure may make sense, setting limits on aggregate notional outstanding makes no sense at all. Consider, for example writing at-the-money vs. out-of-the-money options. If we want to fix the mark-to-market exposure and compare across exercise prices it is clearly the case that the notional amount of out-of-the-money options would be much greater, since their value is smaller. This is, however, the point of this approach. The limit on the notional amount makes sense as the only possible indicator of an extreme exposure.

As an example, consider the liability of two put option writers shown in Table 2-4. One writes at the money (ATM) put options and the other out-of-the-money (OTM) put options. Consider options with one year to maturity on an underlying with a volatility of 16 percent p.a. and a risk free rate of 5 percent p.a. Today's value of the underlying is 100. The value of an ATM put with an exercise price of 100 is 4.08, while a deep OTM put has a value of 0.25.

The daily standard deviation is approximately 1 percent, and for simplicity we will consider a 2 percent decline in the underlying to 98 as the VaR, corresponding to the 2.5 percent VaR tail. A decline of the underlying to

TABLE 2-4 Theoretical Stress and Position Limits*

	X	S_0	P_0	S_{VaR}	P_{VaR}	S_{Xtrm}	P_{Xtrm}
ATM	100	100	4.08	98	4.82	80	16.11
OTM	80	100	0.25	98	0.34	80	3.27

* S = 16%p.a., r = 5%p.a.; T = 365 days.

98 would increase the ATM liability by 18 percent from 4.08 to 4.82, while the OTM liability would rise by 36 percent, from 0.25 to 0.34. It is clearly the case that the OTM option is riskier in percentage terms for an equal size move in the underlying. A VaR-sensitive risk limit system would be sensitive to that effect.

VaR limits are often set in terms of dollar amounts. Suppose we fix the "quality" of the counterparty and normalize by assuming that the ATM counterparty is allowed to write one put option, and hence a VaR of 4.82 − 4.08 = 0.74. The per-unit VaR of the OTM put writer is 0.34 − 0.25 = 0.09. The OTM writer may, hence, write 8.22 OTM options that will generate a VaR of:

8.22 options*0.09 VaR per option = 0.74 total VaR

Now consider an extreme decline in the underlying, from 100 to 80. The liability of the ATM writer would rise from 4.08 to 16.11, a rise of 295 percent. The OTM writer would see his liability rising from 0.25 to 3.27, a rise of 1,200 percent. When we add to this the fact that the OTM position was allowed to be 8.22 times larger due to equating VaR limits across the two positions, we would get a liability that rises from 0.25 × 8.22 = 2.06 to 3.27 × 8.22 =26.87. The rise in percentage terms is still 1,200 percent, of course, but the risk should be measured in monetary, not percentage units. The loss, defined as the increase in the liability, in the extreme stress scenario, of the ATM writer is 16.11 − 4.08 = 12.03. The loss in the case of the OTM writer is 26.87 − 2.06 = 24.81.

The stress event loss inherent in the two seemingly equal risk (from a VaR perspective) positions is vastly different. The OTM writer has a stress event risk approximately twice as large as that of the ATM writer. This loophole in the VaR limit system may be caught by setting limits per counterparty. Recall, the OTM put writer was allowed the same VaR as the ATM writer. As a result he was allowed to have a position in 8.22 options rather than just one that the ATM writer was allowed. The idea carries through to other types of derivatives and levered positions.

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WORST-CASE SCENARIO (WCS)

WCS vs. VaR

In this section a complementary measure to VaR is offered which is related to stress testing. It is the "worst-case scenario" (WCS) measure. WCS asks the following question "What is the worst that can happen to the value of the firm's trading portfolio over a given period (e.g., the next 20 or 100 trading days)?" This is to be compared with VaR's focus on the 5th or 1st *percentile* of the distribution.

To understand why WCS may be a more appropriate risk measure than VaR, consider the example above, where the firm's portfolio return is normally distributed with a mean μ_p and volatility σ_p. VaR tells us that losses greater than $\mu_p - 2.33\sigma_p$ will occur, on average, once over the next 100 trading periods, and that losses greater than $\mu_p - 1.65\sigma_p$ will occur, on average, once over the next 20 trading periods. From a risk management perspective, however, managers care more about the magnitude of losses given that a large loss occurs (WCS), rather than the number of times they should expect to face a loss of a given amount or greater (VaR).

In contrast to VaR, WCS focuses on the distribution of the loss during the worst trading period ("period" being, e.g., one day or two weeks), over a given horizon ("horizon" being, e.g., 100 days or one year). *The key point is that a worst period will occur with probability one. The question is how bad will it be?* As shown in Figure 2-7, WCS analysis will show that the expected loss during the worst period is far greater than the corresponding VaR. Of more importance, there is a substantial probability of a much more severe loss.

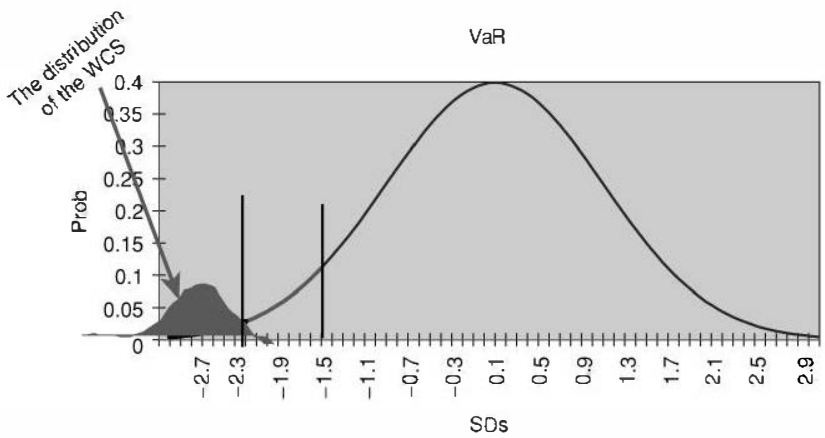

FIGURE 2-7 "The worst will happen."

A Comparison of VaR to WCS

We assume that the firm's portfolio return is normally distributed with a mean of 0 and a volatility of 1. This is without loss of generality because we can always scale the portfolio up and down, both as far as mean as well as variance is concerned. Over N of these intervals, VaR states how many times one might expect to exceed a particular loss. In contrast, WCS states what the distribution of the maximum loss will be. That is, it focuses on $F(min[Z_1, Z_2, \ldots, Z_N])$, denoted $F(Z)$, where $F(\cdot)$ denotes the distribution function and Z_i denotes the normalized return series, corresponding to the change in the portfolio's value over interval i.

Table 2-5 shows the expected number of trading periods in which VaR will be exceeded. For example, the 5 percent VaR corresponds to 1.65 in the normalized units in the table and is expected to be exceeded once over a horizon of length 20, and five times over a horizon of length 100. This is the "classical" notion of VaR.

Table 2-5 also provides information regarding the WCS measures over different horizons. The distribution is obtained via a simulation of 10,000 random normal vectors (using antithetic variates) of lengths N, corresponding to the various horizons. The WCS distribution indicates that the expected worst loss over the next 20 periods is 1.86, while over the next 100 periods it is 2.51. More importantly, over the next 20 periods there is a 5 percent and a 1 percent probability of losses exceeding 2.77 and 3.26 respectively. The corresponding losses for a 100-period horizon are 3.28 and 3.72 respectively.

Looking at the results from a different perspective, for the 1 percent, 100-period VaR measure, the VaR is 2.33 while the expected WCS is 2.51 and the first percentile of the WCS distribution is 3.72. If the fraction of capital invested throughout the 100 periods is maintained, then WCS is the appropriate measure in forming risk management policies regarding financial distress. If the firm maintains capital at less than 160 percent of its VaR, there is a 1 percent

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TABLE 2-5 The Distribution of the Minimum*

	H = 5	H = 20	H = 100	H = 250
E[number of $Z_i < -2.33$]	.05	.20	1.00	2.50
E[number of $Z_i < -1.65$]	.25	1.00	5.00	12.50
Expected WCS	−1.16	−1.86	−2.51	−2.82
Percentile of Z				
1%	−2.80	−3.26	−3.72	−3.92
5%	−2.27	−2.77	−3.28	−3.54
10%	−2.03	−2.53	−3.08	−3.35
50%	−1.13	−1.82	−2.47	−2.78

* The WCS denoted Z is the lowest observation of a vector of $N(0,1)$'s of size H.

chance that the firm will face financial distress over the 100 periods.

To summarize, consider a horizon of $H = 100$. The expected number of events where Z is less than −2.33 is 1 out of 100 (1 percent VaR). The distribution of the worst case, Z, is such that its average is −2.51, and its 5th and 1st percentiles are −3.28 and −3.72 respectively. That is, over the next 100 trading periods a return worse than −2.33 is expected to occur once, when it does, it is expected to be of size −2.51, but with probability 1 percent it might be −3.72 or worse (i.e., we focus on the 1% of the Z's).

Extensions

Our analysis indicates the importance of the information in the WCS over and above VaR. In practice, the WCS analysis has some natural extensions and caveats, which also pertain to VaR.

First, our analysis was developed in the context of a specific model of the firm's investment behavior; that is, we assumed that the firm, in order to remain "capital efficient," increases the level of investment when gains are realized. There are alternative models of investment behaviour, which suggest other aspects of the distribution of returns should be investigated. For example, we might be interested in the distribution of "bad runs," corresponding to partial sums of length J periods for a given horizon of H.

Second, the effect of time-varying volatility has been ignored. Assuming that the risk capital measures are adjusted to reflect this, e.g., via RiskMetrics, GARCH, density estimation, implied volatility, or another method, there is the issue of model risk. That is, to the extent that volatility is not captured perfectly, there may be times when we understate it. Consequently, the probability of exceeding the VaR and the size of the 1 percent tail of the WCS will be understated.

Third, and related to model risk, there is the issue of the tail behavior of financial series. It is well established that volatility-forecasting schemes tend to understate the likelihood and size of extreme moves. This holds true for currencies, commodities, equities, and interest rates (to varying degrees). This aspect will also tend to understate the frequency and size of extreme losses. For a specific case, one could infer a distribution from the historical series in order to obtain a better description of the relevant distribution and so capture the tails. This caveat extends naturally to the issue of correlations, where the most important question is whether extreme moves have the same correlation characteristics as the rest of the data. Of course, if correlations in the extremes are higher, we face the risk of understating the WCS risk.

In conclusion, the analysis of the WCS, and further investigation of the caveats discussed above, is important for the study of some of the more recent proposals on the use of internal models and the more lenient capital requirements imposed on "sophisticated" banks and major dealers. These issues are even more critical when it comes to estimating credit risk and operational risk exposures.

SUMMARY

The first section of this chapter focused on the calculation of VaR for derivatives. While linear derivatives are relatively easy to price and hedge, nonlinear derivatives pose a challenge. There are two approaches to calculating the VaR of nonlinear derivatives—the Greeks approach, and the full revaluation approach. The Greeks approach relies on approximating the valuation function of the

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derivative. The approximation can be rough (linear or delta approximation) or more precise (nonlinear or delta-gamma approximation). The full revaluation approach calls for the revaluation of the derivative at the VaR value of the underlying. That is, in order to assess the risk in the derivative position, the derivatives need to be revalued at an extreme value (e.g., the VaR value) of the underlying.

Difficulty arises in generalizing this approach since some derivative positions may "hide" loss scenarios. For example, an ATM straddle does not lose at extreme values of the underlying but, rather, at current levels. Motivated by this issue, we turned to a discussion of structured Monte Carlo (SMC). SMC is an approach that facilitates the generation of a large number of economically meaningful scenarios. In comparison to scenario analysis, SMC-generated scenarios are drawn from the variance-covariance matrix of underlying risk factors. As such, risk factors will be as correlated in SMC scenarios as they are in reality. Moreover, SMC generates a large number of scenarios, thereby giving a probabilistic meaning to extreme scenarios.

In spite of these clear advantages SMC can generate scenarios only as informative as the variance–covariance matrix that was used to generate such scenarios. To the extent that this matrix is not fully representative of risk factor co-movements under extreme market stress, then SMC will fail to generate realistic scenarios. We provide anecdotal evidence that correlations do seem to fall apart during extreme market conditions, motivating a historical-simulation-based approach to stress testing.

We conclude with a discussion of the worst-case scenario measure for risk, an alternative to the standard VaR approach. The pros and cons of the two approaches are discussed.

APPENDIX

Duration

Consider first a t-period zero coupon bond. For simplicity we will discuss annual compounding, although the convention is often semi-annual compounding. The price-rate relation can be written as follows

$$d_t(r_t) = \frac{1}{(1+r_t)^t}$$

For example, compare the value (per $1 of face value) of a one-year vs. a five-year zero, where rates are assumed to be in both cases 5 percent. The value of the one-year zero is $0.9524, and the value of the five-year zero is $0.7835.

In order to discuss the price sensitivity of fixed income securities as a function of changes in interest rates, we first introduce dollar duration, the price sensitivity, and then duration, the percentage sensitivity. We define dollar duration as the change in value in the zero for a given change in interest rates of 1 percent. This is approximately the derivative of the zero with respect to interest rates, or:

$$d_t'(r_t) = \frac{-t}{(1+r_t)^{t+1}}$$

therefore

$$\$dur_t = -d_t'(r_t) = \frac{t}{(1+r_t)^{t+1}}$$

The dollar duration of the one-year zero is $1/(1.05)^2 = 0.9070$ whereas the dollar duration of the five-year zero is $5/(1.05)^6 = 3.73$. What this means is that an increase in rates from 5 percent to 6 percent should generate a loss of $0.00907 in the value of the one-year zero, as compared to a loss of $0.0373 in the value of the five-year zero coupon bond. Thus, the five-year zero is more sensitive to interest rate changes. Its sensitivity is close to being five times as large (if interest rates were 0 percent, then this comparison would be precise).

The expression for duration is actually an approximation. In contrast, the precise calculation would show that if interest rates increased 1 percent from 5 percent to 6 percent, then the new price of the one-year zero would be $1/(1.06) = \$0.9434$, and $1/(1.06)^5 = \$0.7473$ for the five-year. Comparing these new prices to the original prices before the interest rate increase (i.e., $0.9524 for the one-year and $0.7835 for the five-year), we can obtain a precise calculation of the price losses due to the interest rate increase. For the one-year zero, the precise calculation of price decline is $0.9524 − 0.9434 = $0.0090 and for the five-year zero, $0.7835 − 0.7472 = $0.0363. Comparing these results to the duration approximation above, we see that the duration approximation overstated price declines for the one-year zero by $0.00007 = 0.00907 − 0.0090. The overstatement was higher for the five-year zero; $0.0010 = 0.0373 − 0.0363. Duration is an overly pessimistic approximation of price changes resulting from unanticipated interest rate fluctuations. That is, duration

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TABLE 2-6 Duration Example

t	d(r = 5%)	$Dur	Dur	D-loss	d(r = 6%)	True Loss	Duration Error
1	**$0.9524**	**$0.9070**	**0.9524**	**$0.0091**	**$0.9434**	**$0.0090**	**−$0.0001**
2	$0.9070	$1.7277	1.9048	$0.0173	$0.8900	$0.0170	−$0.0002
3	$0.8638	$2.4681	2.8571	$0.0247	$0.8396	$0.0242	−$0.0005
4	$0.8227	$3.1341	3.8095	$0.0313	$0.7921	$0.0306	−$0.0007
5	**$0.7635**	**$3.7311**	**4.7619**	**$0.0373**	**$0.7473**	**$0.0363**	**−$0.0010**
6	$0.7462	$4.2641	5.7143	$0.0426	$0.7050	$0.0413	−$0.0014
7	$0.7107	$4.7379	6.6667	$0.0474	$0.6651	$0.0456	−$0.0018
8	$0.6768	$5.1569	7.6190	$0.0516	$0.6274	$0.0494	−$0.0021
9	$0.6446	$5.5252	8.5714	$0.0553	$0.5919	$0.0527	−$0.0025
10	$0.6139	$5.8468	9.5238	$0.0585	$0.5584	$0.0555	−$0.0029
1&5	**$1.7359**	**$4.6381**	**2.6719**	**$0.0464**	**$1.6907**	**$0.0453**	**−$0.0011**

overstates the price decline in the event of interest rate increases and understates the price increase in the event of an interest rate decline. Table 2-6 summarizes our example.

It is easy to generalize this price–rate relationship to coupon bonds and all other fixed cash flow portfolios. Assuming all interest rates change by the same amount (a parallel shift), it is easy to show that

$$\text{portfolio } \$dur = k_1 \times \$dur_{t_1} + k_2 \times \$dur_{t_2} + \ldots$$

where $k_1, k_2, \ldots$ are the dollar cash flows in periods $t_1, t_2, \ldots$

Duration is easy to define now as:

duration

≈ [percent change in value] per [100 bp change in rates]

$$= \left[\frac{\text{dollar change in value per 100 bp}}{\text{initial value}}\right] / [1/100]$$

$$= \left[\frac{\text{dollar duration} \times 0.01}{\text{initial value}}\right] \times 100$$

$$= \frac{\text{dollar duration}}{\text{initial value}}$$

Therefore we get

$$\text{duration} = \frac{\dfrac{t}{(1+r_t)^{t+1}}}{\dfrac{1}{(1+r_t)^t}} = \frac{t}{(1+r_t)}$$

and for a portfolio we get

$$\text{duration} = \frac{\text{dollar duration}}{\text{value}} = \frac{k_1 \times \$dur_{t_1} + k_2 \times \$dur_{t_2} + \cdots}{k_1 \times d_{t_1} + k_2 \times d_{t_2} + \cdots}$$

but since

$$\$dur_t = d_t \times dur_t$$

we get

$$\text{portfolio dur} = \frac{k_1 \times d_{t_1} \times dur_{t_1} + k_2 \times d_{t_2} \times dur_{t_2} + \cdots}{k_1 \times d_{t_1} + k_2 \times d_{t_2} + \cdots}$$

$$\Rightarrow \text{portfolio dur} = w_1 \times dur_{t_1} + w_2 \times dur_{t_2} + \cdots$$

where $w_i = \dfrac{k_i \times d_{t_i}}{k_1 \times d_{t1} + k_2 \times d_{t_2} + \cdots}$ is the pv weight of cash flow i.

That is, the duration, or interest rate sensitivity, of a portfolio, under the parallel shift in rates assumption, is just the weighted sum of all the portfolio sensitivities of the portfolio cash flow components, each weighted by its

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present value (i.e., its contribution to the present value of the entire portfolio).

Going back to our example, consider a portfolio of cash flows consisting of $1 in one year and $1 in five years. Assuming 5 percent p.a. interest rates, the value of this portfolio is the sum of the two bonds, $0.9524 + 0.7835 = $1.7359, and the sum of the dollar durations, $0.9070 + 3.73 = $4.6370. However, the duration of the portfolio is:

$$\$2.18116 = \frac{0.9524}{1.7359}(0.907) + \frac{0.7835}{1.7359}(3.73)$$

This tells us that a parallel shift upwards in rates from a flat term structure at 5 percent to a flat term structure at 6 percent would generate a loss of 2.18116 percent. Given a portfolio value of $1.7359, the dollar loss is 2.18116% × $1.7359 = $0.03786.

The way to incorporate duration into VaR calculations is straightforward. In particular, duration is a portfolio's percentage loss for a 1 percent move in interest rates. The percentage VaR of a portfolio is, hence, its duration multiplied by interest rate volatility. For example, suppose we are interested in the one-month VaR of the portfolio of one-year and five-year zeros, whose value is $1.7359 and duration is 2.18116. Suppose further that the volatility of interest rates is 7bp/day, and there are 25 trading days in a month. The monthly volatility using the square root rule is $\sqrt{(25)} \times 7 = 35$bp/month. The %VaR is therefore 2.18116 × 0.35 = 0.7634%, and the $VaR = 0.007634 × $1.7359 = $0.01325.

There is clearly no question of aggregation. In particular, since we assume a single factor model for interest rates, the assumed VaR shift in rates of 35bp affects all rates along the term structure. The correlation between losses of all cash flows of the portfolio is one. It is, therefore, the case that the VaR of the portfolio is just the sum of the VaRs of the two cash flows, the one-year zero and the five-year zero.

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Measures of Financial Risk

3

Learning Objectives

After completing this reading you should be able to:

- Describe the mean-variance framework and the efficient frontier.
- Explain the limitations of the mean-variance framework with respect to assumptions about return distributions.
- Define the Value-at-Risk (VaR) measure of risk, describe assumptions about return distributions and holding period, and explain the limitations of VaR.
- Define the properties of a coherent risk measure and explain the meaning of each property.
- Explain why VaR is not a coherent risk measure.
- Explain and calculate expected shortfall (ES), and compare and contrast VaR and ES.
- Describe spectral risk measures, and explain how VaR and ES are special cases of spectral risk measures.
- Describe how the results of scenario analysis can be interpreted as coherent risk measures.

Excerpt is Chapter 2 of Measuring Market Risk, *Second Edition, by Kevin Dowd.*

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This chapter deals with measures of financial risk. As we have already seen, work on financial risk management over the last decade or so has tended to focus on the VaR, but there are many other risk measures to choose from, and it is important to choose the 'right' one. To put our discussion into a specific context, suppose we are working to a daily horizon period. At the end of day $t-1$, we observe that the value of our portfolio is P_{t-1} but, looking forward, the value of our portfolio at the end of tomorrow, P_t, is uncertain. Ignoring any intra-day returns or intra-day interest, if P_t turns out to exceed P_{t-1}, we will make a profit equal to the difference, $P_t - P_{t-1}$; and if P_t turns out to be less than P_{t-1}, we will make a loss equal to $P_{t-1} - P_t$. Since P_t is uncertain, as viewed from the end of $t-1$, then so too is the profit or loss (P/L) (or return). Thus, our next-period P/L (or return) is risky, and we want a framework or paradigm to measure this risk.

This chapter examines three such measurement frameworks, the first based on the mean–variance or portfolio-theory approach, the second based on VaR, and the third based on the newer coherent risk measures. . . . We will discuss these in their chronological order, but before discussing them in any detail, it is worth highlighting the themes underlying the ways in which these frameworks have evolved. Three themes in particular stand out:

- The first is the drive to extend the range of P/L or return distributions that can be handled. The mean–variance framework is quite limited in this regard, as it only applies if we are dealing with normal or near-normal distributions—or, more precisely, if we are dealing with data that are (or can be transformed to become) elliptically distributed. By contrast, the later frameworks are very general and can accommodate any form of distribution (although some distributions are much easier to handle than others). So a key theme is the desire to escape from the confines of a framework that can only handle normal or near-normal distributions, and this is very important because many of the empirical distributions we might encounter are very non-normal.
- A second and related theme is to improve the usefulness of the resulting risk measure. In the mean–variance framework, the measure of risk is the standard deviation (of P/L or returns) or some simple transformation of it. In the normal (or near-normal) circumstances assumed in that framework, this standard deviation is also an ideal risk measure, and we can use it to identify our risk-expected return trade-off and make decisions accordingly. For its part, the VaR framework gives us a risk measure, the VaR, that is more or less equivalent in usefulness to the standard deviation if we are dealing with normal (or near-normal) distributions. The VaR also has the advantage that it can be estimated for any distribution, but it has major problems as a *usable* risk measure in the presence of seriously non-normal distributions. The VaR framework therefore liberates us from the confines of near normality in the sense that it provides a risk measure that can be estimated for any distribution we like, but this turns out to be an empty victory, because the *usefulness* of VaR as a measure of risk is highly questionable outside the confines of near-normal distributions. This problem motivated the development of the third and latest framework, the coherent framework: this provides risk measures that have the benefits of the VaR (i.e., they apply to any distribution) but, unlike the VaR, can be used more reliably for decision-making in the presence of seriously non-normal distributions. So, in short, the second theme is the drive to produce risk measures that can be useful outside the confines of near-normality.
- There is also a third theme. Each framework allows us to aggregate individual risks in an intellectually respectable way, but the portfolio theory approach is rather limited in its range of application—essentially, it applies to equity and similar types of risks—whereas the VaR and coherent approaches are much more general in their ranges of application. However, this greater range of application comes at a cost: we have to deal with problems of valuation and market illiquidity that do not usually arise in the more limited cases considered by classical portfolio theory, and a considerable amount of effort has gone into dealing with these sorts of problems. The importance of being able to 'generalise' the range of applicability of our risk measures has been further reinforced by the emergence of enterprise-wide risk management (ERM; sometimes also known as integrated risk management) as a major theme of financial risk management since the late 1990s. ERM seeks to measure and manage risks across different categories in a holistic and integrated way across the firm as a whole, and in doing so take account of the ways in which different risk categories interact with each other.

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ERM is a hugely important theme of modern risk management and we don't have space to say much about it in this book, but it suffices for the moment to note that successful ERM *presupposes* a risk measurement framework that is capable of extension across the major risk categories that a firm has to deal with, and the basic portfolio-theory framework is far from adequate to this task.

We should keep these three themes in mind as we now proceed to examine each of these frameworks in turn.

THE MEAN–VARIANCE FRAMEWORK FOR MEASURING FINANCIAL RISK

The traditional approach used to measure financial risks is the mean–variance framework: we model financial risk in terms of the mean and variance (or standard deviation, the square root of the variance) of P/L (or returns).[1] As a convenient (although oversimplified) starting point, we can regard this framework as underpinned by the assumption that daily P/L (or returns) obeys a normal distribution.[2] A random variable X is normally distributed with mean μ and variance σ^2 (or standard deviation σ) if the probability that X takes the value x, $f(x)$, obeys the following probability density function (pdf):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \tag{3.1}$$

where x is defined over $-\infty < x < \infty$. A normal pdf with mean 0 and standard deviation 1, known as a standard normal, is illustrated in Figure 3-1.

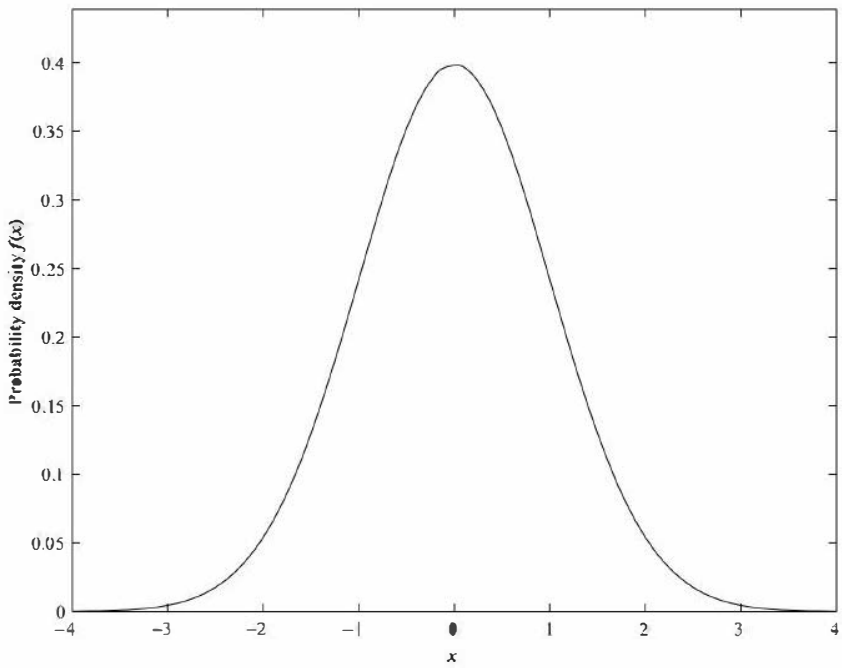

FIGURE 3-1 The normal probability density function.

A pdf gives a complete representation of possible random outcomes: it tells us what outcomes are possible, and how likely these outcomes are. This particular pdf is the classic bell curve. It tells us that outcomes (or x-values) are more likely to occur close to the mean μ; it also tells us that the spread of the probability mass around the mean depends on the standard deviation σ: the greater the standard deviation, the more dispersed the probability mass. The pdf is also symmetric around the mean: X is as likely to take a particular value $\mu + x$ as to take the corresponding negative value $\mu - x$. The pdf falls as we move further away from the mean, and outcomes well away from the mean are very unlikely, because the tail probabilities diminish exponentially as we go further out into the tail. In risk management, we are particularly concerned about outcomes in the left-hand tail, which corresponds to high negative returns—or big losses, in plain English.

The assumption of normality is attractive for various reasons. One reason is that it often has some, albeit limited, plausibility in circumstances where we can appeal to the central limit theorem. Another attraction is that it provides

[1] For a good account of portfolio theory and how it is used, see, e.g., Elton and Gruber (1995).

[2] To simplify the text, we shall sometimes talk 'as if' the mean–variance framework requires normality. However, the mean–variance approach in fact only requires that we assume ellipticality and elliptical distributions are more general. Nonetheless, the mean–variance framework is most easily understood in terms of an underlying normality assumption, and non-normal elliptical distributions are harder to understand, less tractable and in any case share many of the same features as normality.

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us with straightforward formulas for both cumulative probabilities and quantiles, namely:

$$\Pr[X \le x] = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dX \qquad \textbf{(3.2a)}$$

$$q_\alpha = \mu + \sigma z_\alpha \qquad \textbf{(3.2b)}$$

where α is the chosen confidence level (e.g., 95%), and z_α is the standard normal variate for that confidence level (e.g., so $z_{0.95} = 1.645$). z_α can be obtained from standard statistical tables or from spreadsheet functions (e.g., the 'normsinv' function in Excel or the 'norminv' function in MATLAB). Equation (3.2a) is the normal distribution (or cumulative density) function: it gives the normal probability of X being less than or equal to x, and enables us to answer probability questions. Equation (3.2b) is the normal quantile corresponding to the confidence level α (i.e., the lowest value we can expect at the stated confidence level) and enables us to answer quantity questions.

A related attraction of particular importance in the present context is that the normal distribution requires only two parameters—the mean and the standard deviation (or variance), and these parameters have ready financial interpretations: the mean is the expected return on a position, and the standard deviation can be interpreted as the risk associated with that position. This latter point is perhaps the key characteristic of the mean-variance framework: it tells us that we can use the standard deviation (or some function of it, such as the variance) as our measure of risk. And conversely, the use of the standard deviation as our risk measure indicates that we are buying into the assumptions—normality or, more generally, ellipticality—on which that framework is built.

To illustrate how the mean-variance approach works, suppose we wish to construct a portfolio from a particular universe of financial assets. We are concerned about the expected return on the portfolio, and about the variance or standard deviation of its returns. The expected return and standard deviation of return depend on the composition of the portfolio, and assuming that there is no risk-free asset for the moment, the various possibilities are shown by the curve in Figure 3-2: any point inside this region (i.e., below or on the curve) is attainable by a suitable asset combination. Points outside this region are not attainable. Since the investor regards a higher expected return as 'good' and a higher standard deviation of returns (i.e., in this context, higher risk) as 'bad', the investor wants to achieve the highest possible expected return for any given level of risk; or equivalently, wishes to minimise the level of risk associated with any given expected return. This implies that the investor will choose some point along the upper edge of the feasible region, known as the efficient frontier. The point chosen will depend on their risk-expected return preferences (or utility or preference function): an investor who is more risk-averse will choose a point on the efficient frontier with a low risk and a low expected return, and an investor who is less risk-averse will choose a point on the efficient frontier with a higher risk and a higher expected return.

Figure 3-2 is one example of the mean-variance approach. However, the mean-variance approach is often presented in a slightly different form. If we assume a risk-free asset

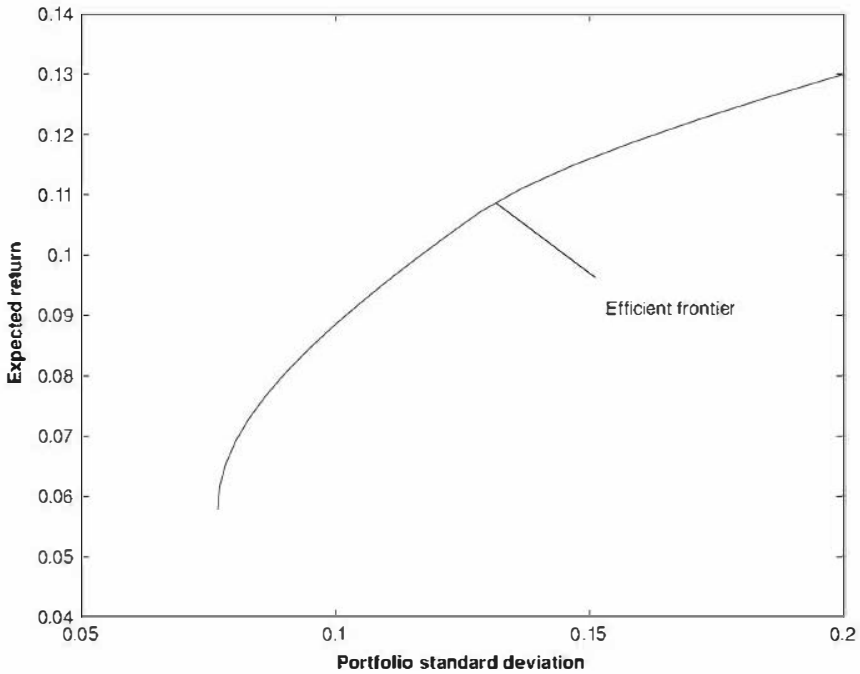

FIGURE 3-2 The mean-variance efficient frontier without a risk-free asset.

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and (for simplicity) assume there are no short-selling constraints of any kind, then the attainable set of outcomes can be expanded considerably—and this means a considerable improvement in the efficient frontier. Given a risk-free rate equal to 4.5% in Figure 3-3, the investor can now achieve any point along a straight line running from the risk-free rate through to, and beyond, a point or portfolio *m* just touching the top of the earlier attainable set. *m* is also shown in the figure, and is often identified with the 'market portfolio' of the CAPM. As the figure also shows, the investor now faces an expanded choice set (and can typically achieve a higher expected return for any given level of risk).

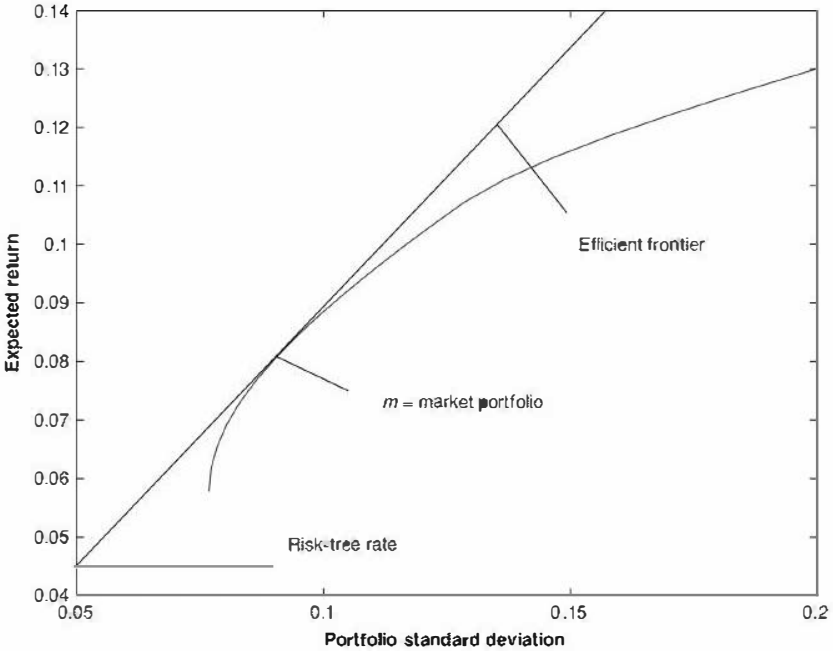

FIGURE 3-3 The mean–variance efficient frontier without a risk-free asset.

So the mean–variance framework gives us a nice approach to the twin problems of how to measure risks *and* how to choose between risky alternatives. On the former question, our primary concern for the moment, it tells us that we can measure risk by the standard deviation of returns. Indeed, it goes further and tells us that the standard deviation of returns is in many ways an *ideal* risk measure in circumstances where risks are normally (or elliptically) distributed.

However, the standard deviation can be a very unsatisfactory risk measure when we are dealing with seriously non-normal distributions. Any risk measure at its most basic level involves an attempt to capture or summarise the shape of an underlying density function, and although the standard deviation does that very well for a normal (and up to a point, more general elliptical) distribution, it does not do so for others. Recall that any statistical distribution can be described in terms of its moments or moment-based parameters such as mean, standard deviation, skewness and kurtosis. In the case of the normal distribution, the mean and standard deviation can be anything (subject only to the constraint that the standard deviation can never be negative), and the skewness and kurtosis are 0 and 3. However, other distributions can have quite different skewness and/or kurtosis, and therefore have quite different shapes than the normal distribution, and this is true even if they have the same mean and standard deviation.

To illustrate this for the skewness, Figure 3-4 compares a normal distribution with a skewed one (which is in fact a Gumbel distribution). The parameters of these are chosen so that both distributions have the same mean and standard deviation. As we can see, the skew alters the whole distribution, and tends to pull one tail in while pushing the other tail out. A portfolio theory approach would suggest that these distributions have equal risks, because they have equal standard deviations, and yet we can see clearly that the distributions (and hence the 'true' risks, whatever they might be) must be quite different. The implication is that the presence of skewness makes portfolio theory unreliable, because it undermines the normality assumption on which it is (archetypically) based.

To illustrate this point for excess kurtosis, Figure 3-5 compares a normal distribution with a heavy-tailed one (i.e., a *t* distribution with 5 degrees of freedom). Again, the parameters are chosen so that both distributions have the same mean and standard deviation. As the name suggests, the heavy-tailed distribution has a longer tail, with much more mass in the extreme tail region. Tail heaviness—kurtosis in excess of 3—means that we are more likely to

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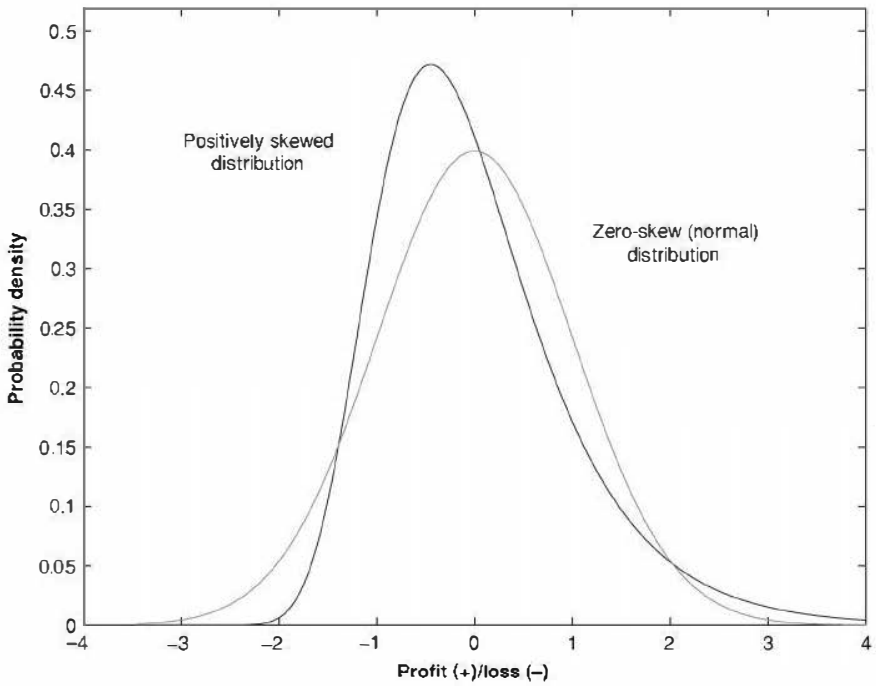

FIGURE 3-4 A skewed distribution.

Note: The symmetric distribution is standard normal, and the skewed distribution is a Gumbel with location and scale equal to $-0.57722\sqrt{6/\pi}$ and $\sqrt{6/\pi}$.

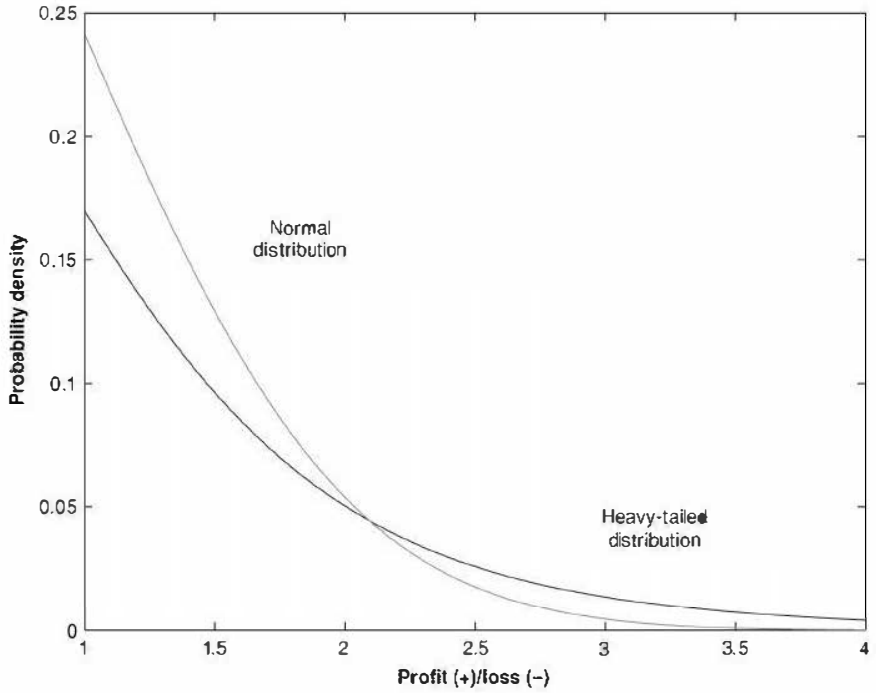

FIGURE 3-5 A heavy-tailed distribution.

Note: The symmetric distribution is standard normal, and the heavy-tailed distribution is a t with mean 0, std 1 and 5 degrees of freedom.

lose (or gain) a lot, and these losses (or gains) will tend to be larger, relative to normality. A portfolio theory approach would again suggest that these distributions have equal risks, so the presence of excess kurtosis can also make portfolio theory unreliable.

Thus, the normality assumption is only strictly appropriate if we are dealing with a symmetric (i.e., zero-skew) distribution with a kurtosis of 3. If our distribution is skewed or has heavier tails—as is typically the case with financial returns—then the normality assumption is inappropriate and the mean–variance framework can produce misleading estimates of risk. This said, more general elliptical distributions share many of the features of normality and with suitable reparameterisations can be tweaked into a mean–variance framework. The mean–variance framework can also be (and often is) applied conditionally, rather than unconditionally, meaning that it might be applied conditionally on sets of parameters that might themselves be random. Actual returns would then typically be quite non-normal (and often skewed and heavy tailed) because they are affected by the randomness of the parameters as well as by the randomness of the conditional elliptical distribution. But even with their greater flexibility, it is still doubtful whether conditionally elliptical distributions can give sufficiently good 'fits' to many empirical return processes. And, there again, we can use the mean–variance framework more or less regardless of the underlying distribution if the user's utility (or preference) function is a quadratic function that depends only on the mean and variance of return (i.e., so the user only *cares* about mean and standard deviation). However, such a utility function has undesirable properties and would itself be difficult to justify.

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BOX 3-1 Traditional Dispersion Risk Measures

There are a number of traditional measures of risk based on alternative measures of dispersion. The most widely used is the standard deviation (or its square, the variance), but the standard deviation has been criticised for the arbitrary way in which deviations from the mean are squared and for giving equal treatment to upside and downside outcomes. If we are concerned about these, we can use the mean absolute deviation or the downside semi-variance instead: the former replaces the squared deviations in the standard deviation formula with absolute deviations and gets rid of the square root operation; the latter can be obtained from the variance formula by replacing upside values (i.e., observations above the mean) with zeros. We can also replace the standard deviation with other simple dispersion measures such as the entropy measure or the Gini coefficient.

A more general approach to dispersion is provided by a Fishburn (or lower partial moment) measure, defined as $\int_{-\infty}^{t}(t-x)^{\alpha} f(x)dx$. This measure is defined on two parameters: α, which describes our attitude to risk (and which is not to be confused with the confidence level!), and t, which specifies the cut-off between the downside that we worry about and the upside that we don't worry about. Many risk measures are special cases of the Fishburn measure or are closely related to it. These include: the downside semi-variance, which is very closely related to the Fishburn measure with $\alpha = 2$ and t equal to the mean; Roy's safety-first criterion, which corresponds to the Fishburn measure where $\alpha \to 0$; and the expected shortfall (ES), which is a multiple of the Fishburn measure with $\alpha = 1$. In addition, the Fishburn measure encompasses the stochastic dominance rules that are sometimes used for ranking risky alternatives:[4] the Fishburn measure with $\alpha = n + 1$ is proportional to the nth order distribution function, so ranking risks by this Fishburn measure is equivalent to ranking by nth order stochastic dominance.

So the bottom line is that the mean–variance framework tells us to use the standard deviation (or some function of it) as our risk measure, but even with refinements such as conditionality, this is justified only in limited cases (discussed elsewhere), which are often too restrictive for many of the empirical distributions we are likely to meet.

VALUE-AT-RISK

Basics of VaR[3]

We turn now to our second framework. As we have seen already, the mean–variance framework works well with elliptical distributions, but is not reliable where we have serious non-normality. We therefore seek an alternative framework that will give us risk measures that are valid in the face of more general distributions. We now allow the P/L or return distribution to be less restricted, but focus on the tail of that distribution—the worst p% of outcomes, and this brings us back to the VaR. More formally, if we have a confidence level α and set $p = 1 - \alpha$, and if q_p is the p-quantile of a portfolio's prospective profit/loss (P/L) over some holding period, then the VaR of the portfolio at that confidence level and holding period is equal to:

$$VaR = -q_p \tag{3.3}$$

[3] The roots of the VaR risk measure go back to Baumol (1963, p. 174), who suggested a risk measure equal to $\mu + k\sigma$, where μ and σ are the mean and standard deviation of the distribution concerned, and k is a subjective confidence-level parameter that reflects the user's attitude to risk. As we shall see, this risk measure is comparable to the VaR under the assumption that P/L is normal or t distributed.

[4] An nth order distribution function is defined as $F^{(n)}(x) = 1/(n-1)!\int_{-\infty}^{x}(x-u)^{n-1} f(u)du$, and X_1 is said to be nth order stochastically dominant over X_2 if $F_1^{(n)}(x) \le F_2^{(n)}(x)$, where $F_1^{(n)}(x)$ and $F_2^{(n)}(x)$ are the nth degree distribution functions of X_1 and X_2 (see Yoshiba and Yamai (2001, p. 8)). First-order stochastic dominance implies that the distribution function for X_1 is never above the distribution function for X_2, second-order stochastic dominance implies that their second-degree distribution functions do not cross, and so on. Since a risk measure with nth degree stochastic dominance is also consistent with lower degrees of stochastic dominance, first-order stochastic dominance implies second and higher orders of stochastic dominance, but not the reverse. First-order stochastic dominance is a very implausible condition that will hardly ever hold (as it implies that one distribution always gives higher values than the other, in which case choosing between the two is trivial), second-order stochastic dominance is less unreasonable, but will often not hold; third-order stochastic dominance is more plausible, and so on: higher orders of stochastic dominance are more plausible than lower orders of stochastic dominance.

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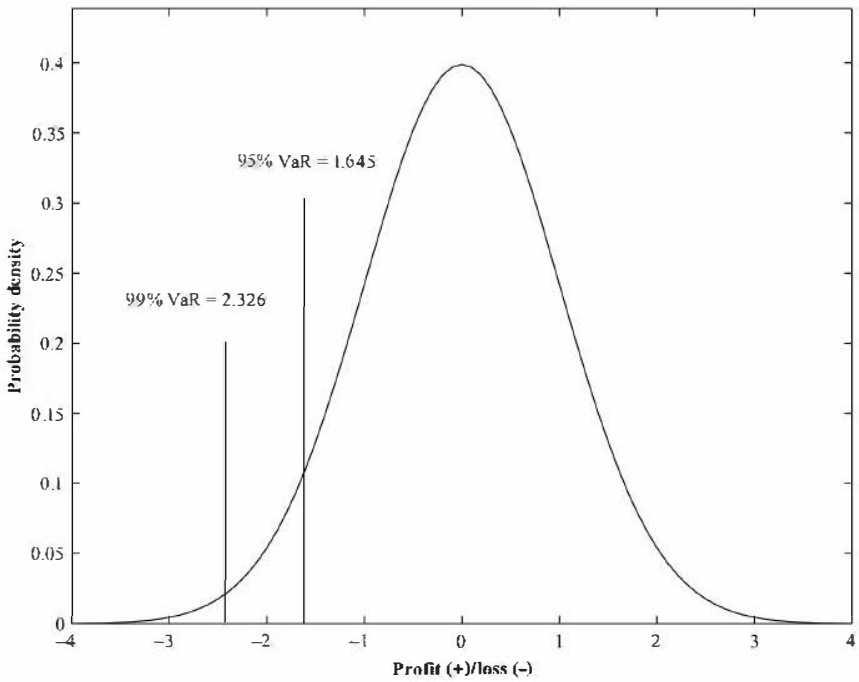

FIGURE 3-6 Value-at-risk.

Note: Produced using the 'normalvarfigure' function.

The VaR is simply the negative of the q_p quantile of the P/L distribution.[5] Thus, the VaR is defined contingent on two arbitrarily chosen parameters—a confidence level α, which indicates the likelihood that we will get an outcome no worse than our VaR, and which might be any value between 0 and 1; and a holding or horizon period, which is the period of time until we measure our portfolio profit or loss, and which might be a day, a week, a month, or whatever.

Some VaRs are illustrated in Figure 3-6, which shows a common probability density function (pdf) of profit/loss over a chosen holding period.[6] Positive values correspond to profits, and negative observations to losses, and positive values will typically be more common than negative ones. If $\alpha = 0.95$, the VaR is given by the negative of the point on the x-axis that cuts off the top 95% of P/L observations from the bottom 5% of tail observations. In this case, the relevant x-axis value (or quantile) is −1.645, so the VaR is 1.645. The negative P/L value corresponds to a positive VaR, indicating that the worst outcome at this level of confidence is a loss of 1.645.[7] Let us refer to this VaR as the 95% VaR for convenience. Alternatively, we could set $\alpha = 0.99$ and in this case the VaR would be the negative of the cut-off between the bottom 1% tail and everything else. The 99% VaR here is 2.326.

Since the VaR is contingent on the choice of confidence level, Figure 3-6 suggests that it will usually increase when the confidence level changes.[8] This point is further illustrated in the next figure (Figure 3-7), which shows how the VaR varies as we change the confidence level. In this particular case, which is also quite common in practice, the VaR not only rises with the confidence level, but also rises at an increasing rate—a point that risk managers might care to note.

As the VaR is also contingent on the holding period, we should consider how the VaR varies with the holding period as well. This behaviour is illustrated in Figure 3-8, which plots 95% VaRs based on two alternative μ values

[5] It is obvious from the figure that the VaR is unambiguously defined when dealing with a continuous P/L distribution. However, the VaR can be ambiguous when the P/L distribution is discontinuous (e.g., as it might be if the P/L distribution is based on historical experience). To see this, suppose there is a gap between the lowest 5% of the probability mass on the left of a figure otherwise similar to Figure 3-4, and the remaining 95% on the right. In this case, the VaR could be the negative of any value between the left-hand side of the 95% mass and the right-hand side of the 5% mass: discontinuities can make the VaR ambiguous. However, in practice, this issue boils down to one of approximation, and won't make much difference to our results given any reasonable sample size.

[6] The figure is constructed on the assumption that P/L is normally distributed with mean 0 and standard deviation 1 over a holding period of 1 day.

[7] In practice, the point on the x-axis corresponding to our VaR will usually be negative and, where it is, will correspond to a (positive) loss and a positive VaR. However, this x-point can sometimes be positive, in which case it indicates a profit rather than a loss and, hence, a negative VaR. This also makes sense: if the worst outcome at this confidence level is a particular profit rather than a loss, then the VaR, the likely loss, must be negative.

[8] Strictly speaking, the VaR is non-decreasing with the confidence level, which means that the VaR can sometimes remain the same as the confidence level rises. However, the VaR cannot fall as the confidence level rises.

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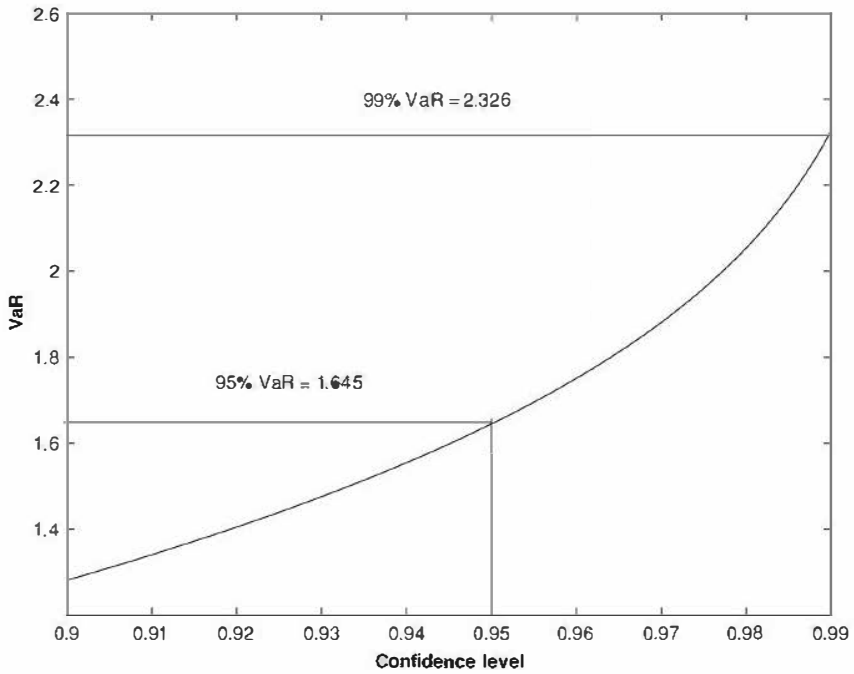

FIGURE 3-7 VaR and confidence level.

Note: Produced using the 'normalvarplot2D_cl' function.

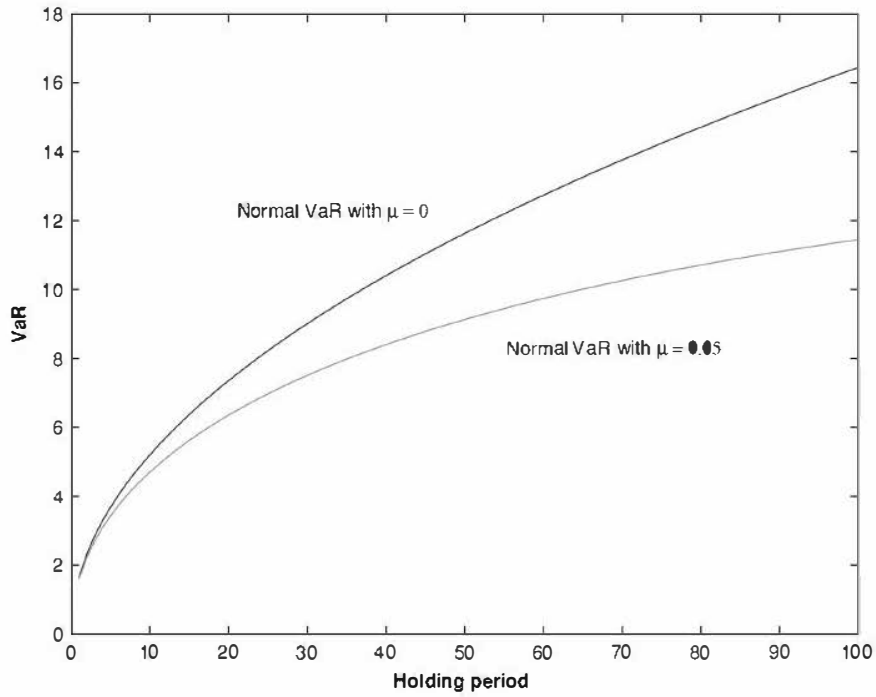

FIGURE 3-8 VaR and holding period.

against a holding period that varies from 1 day to 100 days. With $\mu = 0$, the VaR rises with the square root of the holding period, but with $\mu > 0$, the VaR rises at a lower rate and would in fact eventually turn down. Thus, the VaR varies with the holding period, and the way it varies with the holding period depends significantly on the μ parameter.

Of course, each of the last two figures only gives a partial view of the relationship between the VaR and the parameters on which it depends: the first takes the holding period as given and varies the confidence level, and the second varies the holding period while taking the confidence level as given. To form a more complete picture, we need to see how VaR changes as we allow both parameters to change. The result is a VaR surface—as shown in Figure 3-9, based here on a hypothetical assumption that $\mu = 0$—that enables us to read off the VaR for any given combination of these two parameters. The shape of the VaR surface shows how VaR changes as underlying parameters change, and conveys a great deal of risk information. In this case, which is typical of many, the surface rises with both confidence level and holding period to culminate in a spike—indicating where our portfolio is most vulnerable—as both parameters approach their maximum values.

Determination of the VaR Parameters

The use of VaR involves two arbitrarily chosen parameters—the confidence level and the holding period—but how do we choose the values of these parameters?

The choice of confidence level depends on the purposes to which our risk measures are put. For example, we would want a high confidence level if we were using our risk measures to set firmwide capital requirements, but for backtesting, we

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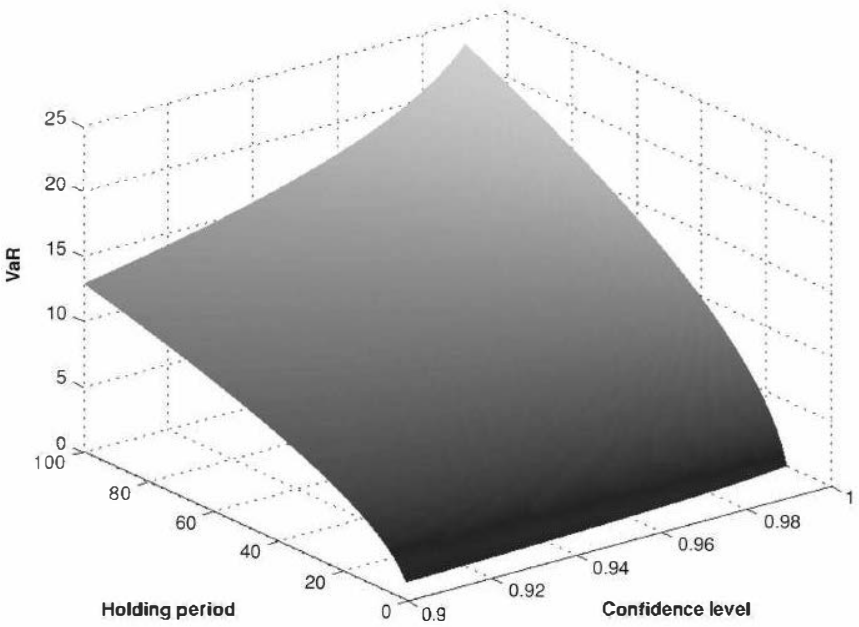

FIGURE 3-9 A VaR surface.

Note: Produced using the 'normalvarplot3D' function. This plot is based on illustrative assumptions that $\mu = 0$ and $\sigma = 1$.

often want lower confidence levels to get a reasonable proportion of excess-loss observations. The same goes if we were using VaR to set risk limits: many institutions prefer to use confidence levels in the region of 95% to 99%, as this is likely to produce a small number of excess losses and so force the people concerned to take the limit seriously. And when using VaRs for reporting or comparison purposes, we would probably wish to use confidence levels that are comparable to those used for similar purposes by other institutions, which are again typically in the range from 95% to 99%.

The usual holding periods are one day or one month, but institutions can also operate on other holding periods (e.g., one quarter or more), depending on their investment and/or reporting horizons. The holding period can also depend on the liquidity of the markets in which an institution operates: other things being equal, the ideal holding period appropriate in any given market is the length of time it takes to ensure orderly liquidation of positions in that market. The holding period might also be specified by regulation: for example, BIS capital adequacy rules stipulate that banks should operate with a holding period of two weeks (or 10 business days). The choice of holding period can also depend on other factors:

- The assumption that the portfolio does not change over the holding period is more easily defended with a shorter holding period.
- A short holding period is preferable for model validation or backtesting purposes: reliable validation requires a large data set, and a large data set requires a short holding period.

Thus, the 'best' choice of these parameters depends on the context. However, it is a good idea to work with ranges of parameter values rather than particular point values: a VaR surface is much more informative than a single VaR number.

Limitations of VaR as a Risk Measure

There are several advantages of VaR—it is a common, holistic, probabilistic risk measure. However, the VaR also has its drawbacks. Some of these we have met before—that VaR estimates can be subject to error, that VaR systems can be subject to model risk (i.e., the risk of errors arising from models being based on incorrect assumptions) or implementation risk (i.e., the risk of errors arising from the way in which systems are implemented). On the other hand, such problems are common to many if not all risk measurement systems, and are not unique to VaR ones.

Yet the VaR also has its own distinctive limitations as a risk measure. One important limitation is that the VaR only tells us the most we can lose if a tail event does *not* occur (e.g., it tells us the most we can lose 95% of the time); if a tail event *does* occur, we can expect to lose more than the VaR, but the VaR itself gives us no indication of how much that might be. The failure of VaR to take account of the magnitude of losses in excess of itself implies that two positions can have the same VaR—and therefore appear

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to have the same risk if we use the VaR to measure risk—and yet have very different risk exposures.

This can lead to some very undesirable outcomes. For instance, if a prospective investment has a high expected return but also involves the possibility of a very high loss, a VaR-based decision calculus might suggest that the investor should go ahead with the investment if the higher loss does not affect the VaR (i.e. because it exceeds the VaR), regardless of the size of the higher expected return and regardless of the size of the possible loss. Such a categorical acceptance of any investment that increases expected return—regardless of the possible loss, provided only that it is insufficiently probable—undermines sensible risk-return analysis, and can leave the investor exposed to very high losses.[9]

If the VaR can lead an investor working on his/her own behalf to make perverse decisions, it creates even more scope for problems when there are principal-agent (or delegation) issues. This would be the case where decision-making is decentralised and traders or asset managers work to VaR-defined risk targets or remuneration packages. The classic example is where traders who face a VaR-defined risk target have an incentive to sell out-of-the-money options that lead to higher income in most states of the world and the occasional large hit when the firm is unlucky. If the options are suitably chosen, the bad outcomes will have probabilities low enough to ensure that there is no effect on the VaR, and the trader benefits from the higher income (and hence higher bonuses) earned in 'normal' times when the options expire out of the money. Thus the fact that VaR does not take account of what happens in 'bad' states can distort incentives and encourage traders to 'game' a VaR target (and/or a VaR-defined remuneration package) and promote their own interests at the expense of the interests of the institutions that employ them.[10]

So the VaR has a number of serious limitations as a risk measure, and we will have more to say on some of these presently. There are some nice ironies here. We have seen that there is one important class of distributions where VaR is in many ways a very good measure of risk: these distributions are of course the elliptical distributions. In such circumstances the VaR works well, but in such circumstances we do not really need it: the VaR is then merely a simple transformation of the standard deviation, and a VaR framework tells us nothing that we could not have found out from a basic mean-variance framework. Thus, in the face of elliptical distributions, the mean-variance framework works well and the value of upgrading to a VaR framework is negligible. Yet the whole point of upgrading from the mean-variance framework to something more general is to be able to measure the risks associated with seriously non-normal distributions. The VaR enables us to do this, but it is in exactly these circumstances that the VaR is not a reliable (and perhaps not even useful) risk measure. The bottom line is a delightful irony: *where the VaR is reliable, we don't need it; and where we do need it, it isn't reliable*. We therefore need an alternative framework that can give us useful risk measures in a seriously non-normal environment.

COHERENT RISK MEASURES

The Coherence Axioms and Their Implications

We therefore turn to our third risk measurement paradigm: the theory of coherent risk measures proposed by Artzner *et al.* (1997, 1999). This approach provides the first

[9] To elaborate further: a VaR-based risk-return analysis only makes intuitive sense if returns are elliptically distributed. If returns are non-elliptical, then a VaR-based risk-return analysis is inconsistent with classical (von Neumann-Morgenstern) expected utility theory. Indeed, it appears that unless we assume ellipticality (which we usually cannot) then a VaR-based risk-return analysis can only be justified if preferences are quadratic (i.e., more specifically, if agents don't care about higher moments, which is weird) or lexicographic, and lexicographic preferences are highly implausible because they allow no substitutability in utility between risk and expected return. (For more on some of these issues, see Grootveld and Hallerbach (2004).) A VaR-based risk-return analysis can only be justified under conditions that are empirically usually too restrictive and/or a priori implausible.

[10] We can sometimes ameliorate these problems by using more VaR information. For example, the trader who spikes his firm might be detected if the VaR of his position is estimated at a higher confidence level as well. A (partial) solution to our earlier problems is, therefore, to look at more than one point on the VaR-confidence level curve and not just to look at a single VaR figure. However, such 'solutions' are often not practically feasible and, in any case, fail to address the root problem.

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formal (i.e., mathematically grounded) theory of financial risk. Their starting point is a simple but profound one: that although we all have an intuitive sense of what financial risk entails, it is difficult to give a quantitative assessment of financial risk unless we specify what we actually mean by a measure of risk. For example, we all have a vague notion of temperature, but it is hard to conceptualise it clearly without the notion of a thermometer, which tells us how temperature should be measured. In much the same way, the notion of risk itself is hard to conceptualise without a clear idea of what we mean by a measure of risk. To clarify these issues, Artzner *et al.* postulated a set of axioms—the axioms of coherency—and began to work out their implications.

Let X and Y represent any two portfolios' P/L (or future values, or more loosely, the two portfolios themselves), and let $\rho(.)$ be a measure of risk over a chosen horizon.[11] The risk measure $\rho(.)$ is said to be coherent if it satisfies the following properties:

I. *Monotonicity*: $Y \geq X \Rightarrow \rho(Y) \leq \rho(X)$.

II. *Subadditivity*: $\rho(X + Y) \leq \rho(X) + \rho(Y)$.

III. *Positive homogeneity*: $\rho(hX) = h\rho(X)$ for $h > 0$.

Iv. *Translational invariance*: $\rho(X + n) = \rho(X) - n$ for some certain amount n.

Properties i, iii and iv are essentially 'well-behavedness' conditions intended to rule out awkward outcomes.[12]

The most important property is ii, subadditivity. This tells us that a portfolio made up of subportfolios will risk an amount which is no more than, and in some cases less than, the sum of the risks of the constituent subportfolios. Subadditivity is the most important criterion we would expect a 'reasonable' risk measure to satisfy. It reflects an expectation that when we aggregate individual risks, they diversify or, at worst, do not increase: the risk of the sum is always less than or equal to the sum of the risks. Subadditivity means that aggregating risks does not increase overall risk.[13]

Subadditivity is more than just a matter of theoretical 'tidiness' and has important practical implications. For example, non-subadditivity is treacherous because it suggests that diversification might be a bad thing, which would suggest the laughable conclusion that putting all your eggs into one basket might be good risk management practice! It also means that in adding risks together we might create an extra 'residual' risk that someone has to bear, and that didn't exist before. This would have some awkward consequences:

- Non-subadditive risk measures can tempt agents trading on an organised exchange to break up their accounts, with separate accounts for separate risks, in order to reduce their margin requirements. This would concern the exchange because the margin requirements on the separate accounts would no longer cover the combined risks, and so leave the exchange itself exposed to possible loss.
- If regulators use non-subadditive risk measures to set capital requirements, then a financial firm might be tempted to break itself up to reduce its regulatory capital requirements, because the sum of the capital requirements of the smaller units would be less than the capital requirement of the firm as a whole.
- If risks are subadditive, adding risks together would give us an overestimate of combined risk, and this means that we can use the sum of risks as a conservative estimate of combined risk. This facilitates decentralised decision-making within a firm, because

[11] At a deeper level, we can also start with the notion of an acceptance set, the set of all positions acceptable to some stakeholder (e.g., a financial regulator). We can then interpret the risk measure $\rho(.)$ as the minimum extra cash that has to be added to the risky position and invested prudently in some reference asset to make the risky position acceptable. If $\rho(.)$ is negative, its negativity can be interpreted as the maximum amount that can be safely withdrawn, and still leave the position acceptable.

[12] The other conditions can be understood from the last footnote. Monotonicity means that a random cash flow or future value Y that is always greater than X should have a lower risk: this makes sense, because it means that less has to be added to Y than to X to make it acceptable, and the amount to be added is the risk measure. Positive homogeneity implies that the risk of a position is proportional to its scale or size, and makes sense if we are dealing with liquid positions in marketable instruments. Translational invariance requires that the addition of a sure amount reduces *pari passu* the cash needed to make our position acceptable, and is obviously valid when one appreciates that the cash needed is our risk measure.

[13] However, the coherence axioms can run into a problem relating to liquidity risk. If a position is 'large' relative to the market, then doubling the size of this position can more than double the risk of the position, because bid prices will depend on the position size. This raises the possibility of liquidity-driven violations of homogeneity and subadditivity. Perhaps the best way to resolve this difficulty, suggested by Acerbi (2004, p. 150), is to add a liquidity charge to a coherent risk measure. This charge would take account of relative size effects, but also have the property of going to zero as size/illiquidity effects become negligible.

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a supervisor can always use the sum of the risks of the units reporting to him or her as a conservative back-of-the-envelope risk measure. But if risks are not subadditive, adding them together gives us an underestimate of combined risks, which makes the sum of risks treacherous and therefore effectively useless as a back-of-the-envelope measure.

The bottom line is that subadditivity matters.

This spells trouble for the VaR, because VaR is not subadditive. Recall that for a risk measure to be subadditive, the subadditivity condition $\rho(X + Y) \leq \rho(X) + \rho(Y)$ must apply for *all* possible X and Y. We can therefore prove that VaR is not subadditive if we can find a *single* counterexample where VaR violates this condition. Now consider the following:

> We have two identical bonds, A and B. Each defaults with probability 4%, and we get a loss of 100 if default occurs, and a loss of 0 if no default occurs. The 95% VaR of each bond is therefore 0, so $\text{VaR}(A) = \text{VaR}(B) = \text{VaR}(A) + \text{VaR}(B) = 0$. Now suppose that defaults are independent. Elementary calculations then establish that we get a loss of 0 with probability $0.96^2 = 0.9216$, a loss of 200 with probability $0.04^2 = 0.0016$, and a loss of 100 with probability $1 - 0.9216 - 0.0016 = 0.0768$. Hence $\text{VaR}(A + B) = 100$. Thus, $\text{VaR}(A + B) = 100 > 0 = \text{VaR}(A) + \text{VaR}(B)$, and the VaR violates subadditivity. Hence, the VaR is not subadditive. QED

We can only 'make' the VaR subadditive if we impose restrictions on the form of the P/L distribution. It turns out, in fact, that we can only 'make' the VaR subadditive by imposing the severe restriction that the P/L distribution is elliptically distributed,[14] and this is of limited consolation because in the real world non-elliptical distributions are the norm rather than the exception.

The failure of VaR to be subadditive is a fundamental problem because it means that VaR has no claim to be regarded as a 'proper' risk measure at all. A VaR is merely a quantile. It has its uses as a quantile, but it is very unsatisfactory as a risk measure. There is also a deeper problem:

> from an epistemologic point of view the main problem with VaR is not its lack of subadditivity, but rather the very fact that no set of axioms for a risk measure and therefore no unambiguous definition of financial risk has ever been associated with this statistic. So, despite the fact that some VaR supporters still claim that subadditivity is not a necessary axiom, none of them, to the best of our knowledge, has ever tried to write an alternative meaningful and consistent set of axioms for a risk measure which are fulfilled also by VaR.[15]

Given these problems with the VaR, we seek alternative, coherent, risk measures that retain the benefits of the VaR—in terms of providing a common, aggregative, holistic, etc. measure of risk—while avoiding its drawbacks. If they are to retain the benefits of the VaR, we might also expect that any such risk measures will be 'VaR-like' in the sense that they will reflect the quantiles of the P/L or loss distribution, but will be non-trivial functions of those quantiles rather than a single 'raw' quantile taken on its own.

The Expected Shortfall

A good candidate is the expected shortfall (ES). The ES is the average of the worst $100(1 - \alpha)\%$ of losses:[16]

$$ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 q_p\,dp \qquad (3.4)$$

[14] Artzner *et al.* (1999), p. 217.

[15] Acerbi (2004), p. 150.

[16] The ES is one of a family of closely related risk measures, members of which have been variously called the expected tail loss, tail conditional expectation (TCE), tail VaR, conditional VaR, tail conditional VaR and worst conditional expectation, as well as expected shortfall. Different writers have used these terms in inconsistent ways, and there is an urgent need to cut through the confusion created by all this inconsistent terminology and agree on some consensus nomenclature. This said, the substantive point is that this family of risk measures has two significant substantially distinct members. The first is the measure we have labelled the ES, as defined in Equation (3.4); this is defined in terms of a probability threshold. The other is its quantile-delimited cousin, most often labelled as the TCE, which is the average of losses exceeding VaR, i.e., $TCE_\alpha = -E[X|X > q_\alpha(X)]$. The ES and TCE will always coincide when the loss distribution is continuous, but the TCE can be ambiguous when the distribution is discrete, whereas the ES is always uniquely defined (see Acerbi (2004, p. 158)). We therefore ignore the TCE in what follows, because it is not an interesting statistic except where it coincides with the ES.

It is also interesting to note that the ES risk measure has been familiar to insurance practitioners for a long time: it is very similar to the measures of conditional average claim size that have long been used by casualty insurers. Insurers are also very familiar with the notion of the conditional coverage of a loss in excess of a threshold (e.g., in the context of reinsurance treaties). For more on ES and its precursors, see Artzner *et al.* (1999, pp. 223–224).

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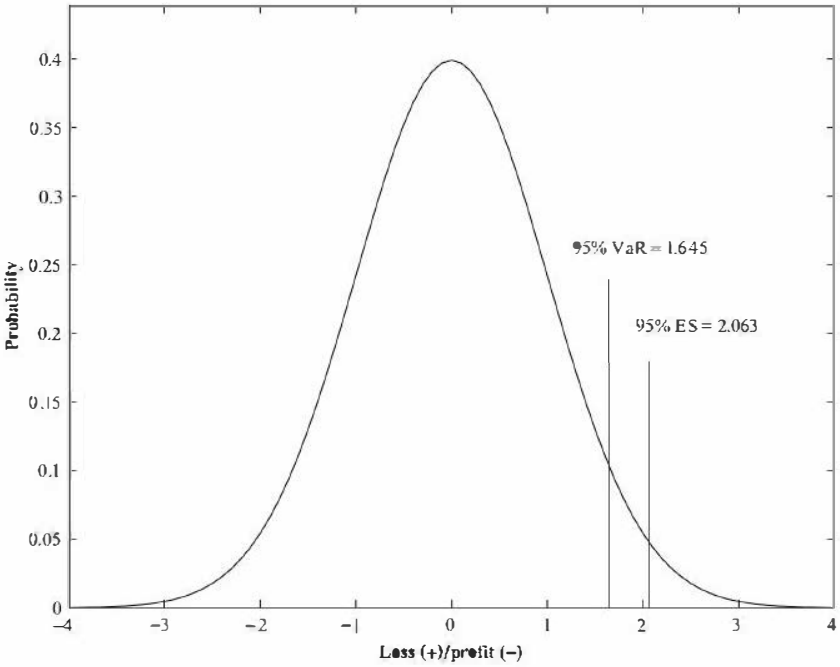

FIGURE 3-10 Expected shortfall.

Note: Produced using the 'normalvaresfigure' function.

If the loss distribution is discrete, then the ES is the discrete equivalent of Equation (3.4):

$$ES_\alpha = \frac{1}{1-\alpha}\sum_{p=0}^{\alpha} [p\text{th highest loss}] \times [\text{probability of } p\text{th highest loss}] \quad \textbf{(3.5)}$$

The subadditivity of ES follows naturally. If we have N equal-probability quantiles in a discrete P/L distribution, then:

$$ES_\alpha(X) + ES_\alpha(Y) \quad \textbf{(3.6)}$$

$$= [\text{mean of } N\alpha \text{ highest losses of } X] + [\text{mean of } N\alpha \text{ highest losses of } Y]$$

$$\geq [\text{mean of } N\alpha \text{ highest losses of } (X+Y)]$$

$$= ES_\alpha(X+Y)$$

A continuous loss distribution can be regarded as the limiting case as N gets large. In general, the mean of the $N\alpha$ worst cases of X and the mean of the $N\alpha$ worst cases of Y will be bigger than the mean of the $N\alpha$ worst cases of $(X + Y)$, except in the special case where the worst X and Y occur in the same $N\alpha$ events, and in this case the sum of the means will equal the mean of the sum. It is easy to show that the ES also satisfies the other properties of coherence, and is therefore coherent (Acerbi (2004, proposition 2.16)).

An illustrative ES is shown in Figure 3-10. If we express our data in loss terms, the VaR and ES are shown on the right-hand side of the figure: the VaR is 1.645 and the ES is 2.063. Both VaR and ES depend on the underlying parameters and distributional assumptions, and these particular figures are based on a 95% confidence level and 1-day holding period, and on the assumption that daily P/L is distributed as standard normal (i.e., with mean 0 and standard deviation 1).

Since the ES is conditional on the same parameters as the VaR itself, it is immediately obvious that any given ES figure is only a point on an ES curve or ES surface. The ES—confidence level curve is shown in Figure 3-11. This curve is similar to the earlier VaR curve shown in Figure 3-7 and, like it, tends to rise with the confidence level. There is also an ES—holding period curve corresponding to the VaR–holding period curve shown in Figure 3-8.

There is also an ES surface, illustrated for the $\mu = 0$ case in Figure 3-12, which shows how ES changes as both confidence level and holding period change. In this case, as with its VaR equivalent in Figure 3-9, the surface rises with both confidence level and holding period, and spikes as both parameters approach their maximum values.

Like the VaR, the ES provides a common consistent risk measure across different positions, it takes account of correlations in a correct way, and so on. It also has many of the same uses as the VaR. However, the ES is also a better risk measure than the VaR for a number of reasons:

- The ES tells us what to expect in bad states—it gives an idea of how bad bad might be—while the VaR tells us nothing other than to expect a loss higher than the VaR itself.
- An ES-based risk-expected return decision rule is valid under more general conditions than a VaR-based risk-expected return decision rule: in particular, the ES-based rule is consistent with expected utility maximisation if risks are rankable by a second-order stochastic dominance rule, while a VaR-based rule is only

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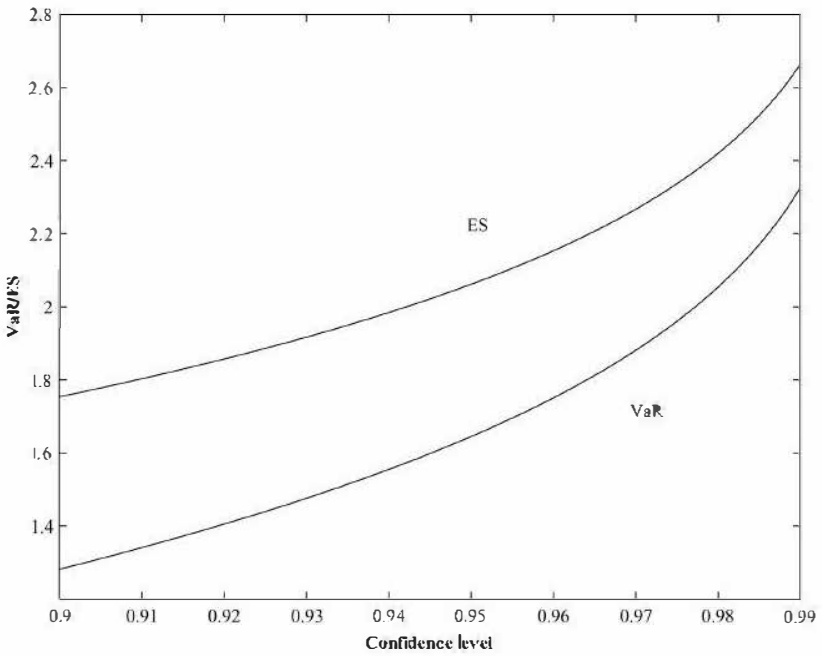

FIGURE 3-11 ES and the confidence level.

Note: Produced using the 'normalvaresplot2D_cl' function.

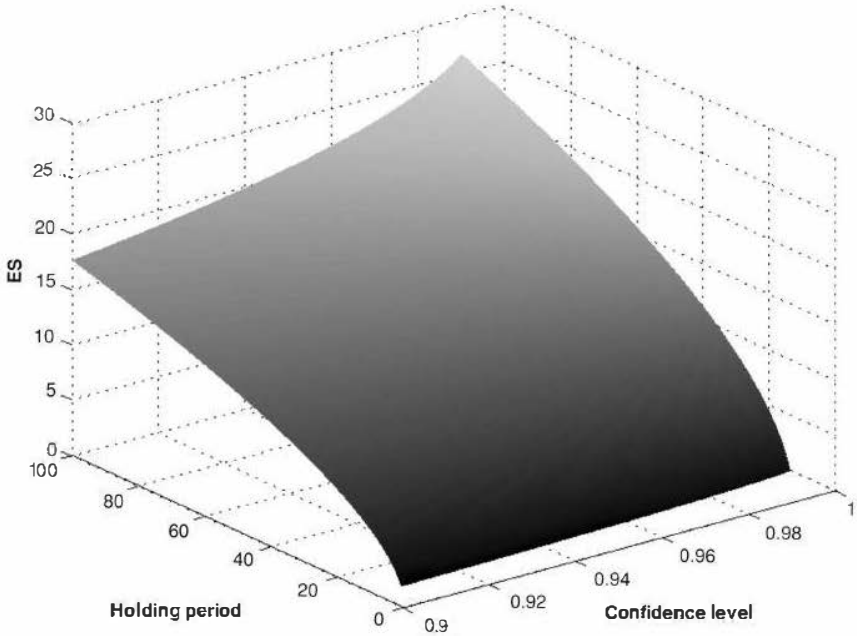

FIGURE 3-12 The ES surface.

Note: Produced using the 'normalesplot3D' function. This plot is based on illustrative assumptions that $\mu = 0$ and $\sigma = 1$.

consistent with expected utility maximisation if risks are rankable by a (much) more stringent first-order stochastic dominance rule.[17]

- Because it is coherent, the ES always satisfies subadditivity, while the VaR does not. The ES therefore has the various attractions of subadditivity, and the VaR does not.
- Finally, the subadditivity of ES implies that the portfolio risk surface will be convex, and convexity ensures that portfolio optimisation problems using ES measures, unlike ones that use VaR measures, will always have a unique well-behaved optimum.[18] In addition, this convexity ensures that portfolio optimisation problems with ES risk measures can be handled very efficiently using linear programming techniques.[19]

In short, the ES easily dominates the VaR as a risk measure.

Spectral Risk Measures

However, the ES is also rarely, if ever, the 'best' coherent risk measure. Going back to first principles, suppose we define more general risk measures M_ϕ that are weighted averages of the quantiles of the loss distribution:

$$M_\phi = \int_0^1 \phi(p) q_p \, dp \tag{3.7}$$

where the weighting function $\phi(p)$ remains to be determined. This function is also known as the risk spectrum or risk-aversion function.

It is interesting to note that both the ES and the VaR are special cases of Equation (3.7).

[17] See Yoshiba and Yamai (2001), pp. 21–22.

[18] See, e.g., Uryasev (2000) and Acerbi and Tasche (2002).

[19] See Rockafellar and Uryasev (2002) and Uryasev (2000).

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The ES is a special case of M_ϕ obtained by setting $\phi(p)$ to the following:

$$\phi(p) = \begin{cases} 0 & \text{if } p < \alpha \\ 1/(1-\alpha) & \phantom{\text{if }} p \geq \alpha \end{cases} \tag{3.8}$$

The ES gives tail losses an equal weight of $1/(1-\alpha)$, and other quantiles a weight of 0. The VaR is also a special case—albeit a highly degenerate one—of $M\phi$. Because the VaR is just a single quantile, the spectral risk measure is the VaR if $\phi(p)$ takes the form of a Dirac delta function, which assigns a probability 1 to the event $p = \alpha$, and a probability of 0 to $p \neq \alpha$. This is degenerate because it gives an infinite value to the pdf at $p = \alpha$ and a zero value to the pdf everywhere else. So one measure places equal weight on tail losses, and the other places no weight at all on them.

However, we are concerned for the moment with the broader class of coherent risk measures. In particular, we want to know the conditions that $\phi(p)$ must satisfy in order to make M_ϕ coherent. The answer is the class of (non-singular) spectral risk measures, in which $\phi(p)$ takes the following properties (Acerbi (2004, proposition 3.4)):[20]

- *Non-negativity:* $\phi(p) \geq 0$ for all p in the range [0,1].
- *Normalization*: $\int_0^1 \phi(p)dp = 1$.
- *Weakly increasing*: If some probability p_2 exceeds another probability p_1, then p_2 must have a weight bigger than or equal to that of p_1.

The first two conditions are fairly obvious as they require that weights should be positive and sum to 1. The critical condition is the third one. This condition reflects the risk-aversion, requiring that the weights attached to higher losses should be bigger than, or certainly no less than, the weights attached to lower losses. Given that it ensures coherence, this condition suggests that *the key to coherence is that a risk measure must give higher losses at least the same weight as lower losses*.

The weights attached to higher losses in spectral risk measures are thus a direct reflection of the user's risk-aversion. If a user has a 'well-behaved' risk-aversion function, then the weights will rise smoothly, and the rate at which the weights rise will be related to the degree of risk aversion: the more risk-averse the user, the more rapidly the weights will rise. This is exactly as it should be.

The connection between the $\phi(p)$ weights and risk-aversion sheds further light on the inadequacies of the ES and the VaR. We saw earlier that the ES is characterised by all losses in the tail region having the same weight. If we interpret the weights as reflecting the user's attitude toward risk, these weights imply that the user is risk-*neutral* between tail-region outcomes. Since we usually assume that agents are risk-*averse*, this would suggest that the ES is not, in general, a good risk measure to use, notwithstanding its coherence. If a user is risk-averse, it should have a weighting function that gives *higher* losses a *higher* weight.[21]

The implications for the VaR are much worse, and we can see that the VaR's inadequacies are related to its failure to satisfy the increasing-weight property. With the VaR, we give a large weight to the loss associated with a p-value equal to α, and we give a lower (indeed, zero) weight to any *greater* loss. The implication is that the user is actually risk-*loving* (i.e., has *negative* risk-aversion) in the tail loss region.[22] To make matters worse, since the weight

[20] Strictly speaking, the set of spectral risk measures is the convex hull (or set of all convex combinations) of all ESs for all α belonging to [0,1]. There is also an 'if and only if' connection here: a risk measure $M\phi$ is coherent if and only if $M\phi$ is spectral and $\phi(p)$ satisfies the conditions indicated in the text. Moreover, there is also a good argument that the spectral measures so defined are the only really interesting coherent risk measures. Acerbi (2004, pp. 180–182) goes on to show that all coherent risk measures that satisfy the two additional properties of comonotonic additivity and law invariance are also spectral measures. The former condition is that if two random variables X and Y are comonotonic (i.e., always move in the same direction), then $\rho(X + Y) = \rho(X) + \rho(Y)$; comonotonic additivity is an important aspect of subadditivity, and represents the limiting case where diversification has no effect. Law invariance boils down to the (for practical purposes essential) requirement that a measure is estimable from empirical data. Both conditions are very important, and coherent risk measures that do not satisfy them—that is to say, non-spectral coherent risk measures—are seriously questionable.

[21] The claim that the selection of the ES as the preferred risk measure indicates risk-neutrality is confirmed from the perspective of the downside risk or lower partial moment literature (see, e.g., Fishburn (1977)). The parameter α reflects the degree of risk aversion, and the user is risk-averse if $\alpha > 1$, risk-neutral if $\alpha = 1$, and risk-loving if $0 < \alpha < 1$. However, we would only choose the ES as our preferred risk measure if $\alpha = 1$ (Grootveld and Hallerbach (2004, p. 36)). Hence, the use of the ES implies that we are risk-neutral.

[22] Following on from the last footnote, the expected utility-downside risk literature also indicates that the VaR is the preferred risk measure if $\alpha = 0$. From the perspective of this framework, $\alpha = 0$ indicates an extreme form of risk-loving (Grootveld and Hallerbach (2004, p. 35)). Thus, two very different approaches both give the same conclusion that VaR is only an appropriate risk measure if preferences exhibit extreme degrees of risk-loving.

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drops to zero, we are also talking about risk-loving of a very aggressive sort. The blunt fact is that with the VaR weighting function, we give a large weight to a loss equal to the VaR itself, and we don't *care at all* about any losses exceeding the VaR! It is therefore hardly surprising that the VaR has its problems.

To obtain a spectral risk measure, the user must specify a particular form for their risk-aversion function. This decision is subjective, but can be guided by the economic literature on utility-function theory. An example is an exponential risk-aversion function:

$$\phi_\gamma(p) = \frac{e^{-(1-p)/\gamma}}{\gamma(1-e^{-1/\gamma})} \qquad (3.9)$$

where $\gamma \in (0, \infty)$ reflects the user's degree of risk-aversion: a smaller γ reflecting a greater degree of risk-aversion. This function satisfies the conditions required of a spectral risk measure, but is also attractive because it is a simple function that depends on a single parameter γ, which gets smaller as the user becomes more risk-averse.

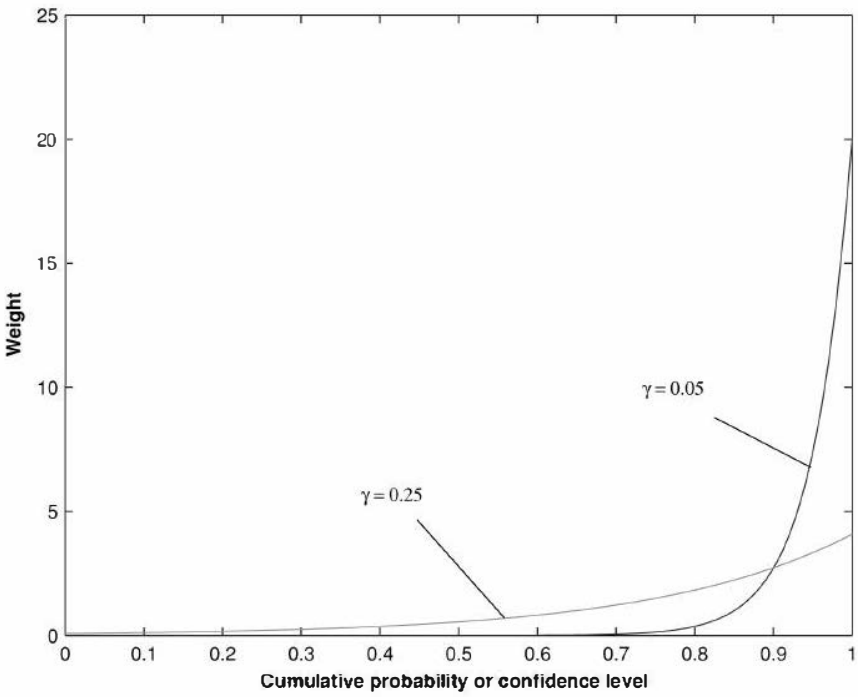

FIGURE 3-13 Exponential-spectral weights.

A spectral risk-aversion function is illustrated in Figure 3-13. This shows how the weights rise with the cumulative probability p, and the rate of increase depends on γ. The more risk-averse the user, the more rapidly the weights rise as losses increase.

To obtain our spectral measure M_ϕ using the exponential weighting function, we choose a value of γ and substitute $\phi(p)$ (or Equation (3.9)) into Equation (3.7) to get:

$$M_\phi = \int_0^1 \phi(p) q_p \, dp = \int_0^1 \frac{e^{-(1-p)/\gamma}}{\gamma(1-e^{-1/\gamma})} q_p \, dp \qquad (3.10)$$

The spectral-exponential measure is therefore a weighted average of quantiles, with the weights given by the exponential risk-aversion function (Equation (3.9)). It can be estimated using a suitable numerical integration method.

We can also see how the risk measure itself varies with the degree of risk-aversion from the plot of M_ϕ against γ given in Figure 3-14. As we can see, M_ϕ rises as γ gets smaller. The risk measure rises as the user becomes more risk-averse. It is also curious to note that the shape of this curve is reminiscent of the curves describing the way the VaR changes with the confidence level (see Figure 3-7), and this suggests that the spectral parameter γ plays a similar role in spectral measures as the confidence level plays in the VaR.

All this indicates that there is an optimal risk measure for each user, and the optimal measure depends on the user's risk-aversion function. Two users might have identical portfolios, but their risks—in the spectral-coherent sense of the term—will only be guaranteed to be the same if they also have exactly the same risk-aversion. From a methodological or philosophical perspective, this means that 'risk' necessarily has a subjective element, even if one subscribes to a frequentist view of probability that maintains that 'probability is objective'. When it comes to risk measures, there is no 'one size that fits all'. This also implies that (true) risk would be very difficult to regulate effectively, if only because regulators could not anticipate the impact of such regulations without subjective information that they are hardly likely to have.[23]

[23] There are also other important implications. Any convex combination of two coherent risk measures is also a coherent risk measure, so a manager presiding over two different business units might take the overall risk measure to be some convex combination of the risks of the two subsidiary units. Furthermore, there is no requirement that the risks of the business units will be predicated on the same confidence level or risk-aversion parameters.

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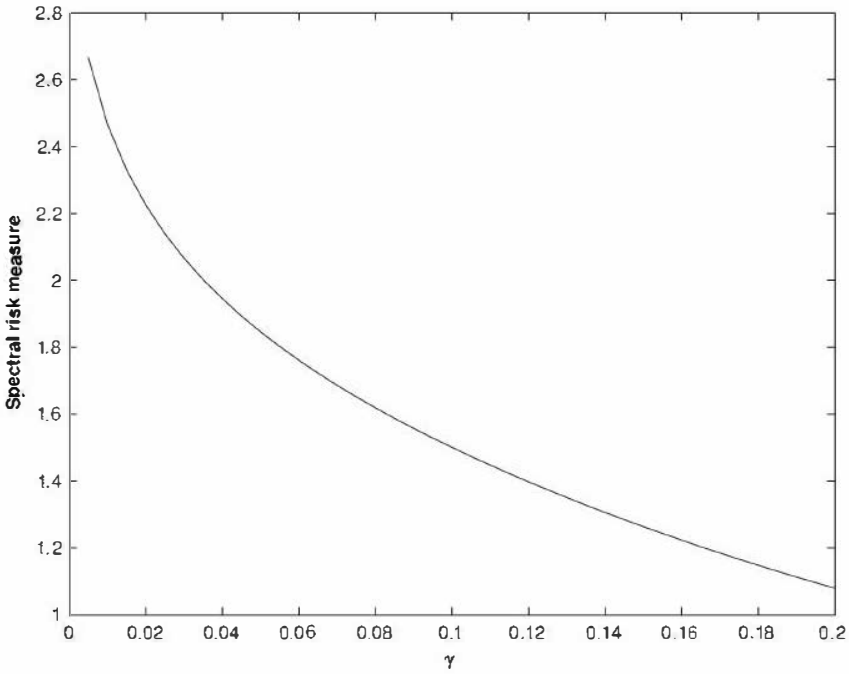

FIGURE 3-14 Plot of exponential spectral risk measure against γ.

Note: Obtained using the 'normal_spectral_risk measure_plot' function in the MMR Toolbox.

Scenarios as Coherent Risk Measures

The theory of coherent risk measures also sheds some interesting light on usefulness of scenario analyses, as it turns out that the results of scenario analyses can be interpreted as coherent risk measures. Suppose we consider a set of loss outcomes combined with a set of associated probabilities. The losses can be regarded as tail drawings from the relevant distribution function, and their expected (or average) value is the ES associated with this distribution function. Since the ES is a coherent risk measure, this means that the outcomes of scenario analyses are also coherent risk measures. The outcomes of scenario analyses are therefore 'respectable' risk measures, and this means that the theory of coherent risk measures provides a solid risk-theoretical justification for stress testing!

This argument can be extended in some interesting ways. Consider a set of 'generalised scenarios'—a set of n loss outcomes and a *family* of distribution functions from which the losses are drawn. Take any one of these distributions and obtain the associated ES. Now do the same again with another distribution function, leading to an alternative ES. Now do the same again and again. It turns out that the maximum of these ESs is itself a coherent risk measure: if we have a set of m comparable ESs, each of which corresponds to a different loss distribution function, then the maximum of these ESs is a coherent risk measure.[24] Furthermore, if we set $n = 1$, then there is only one tail loss in each scenario and each ES is the same as the probable maximum loss or likely worst-case scenario outcome. If we also set $m = 1$, then it immediately follows that the highest expected loss from a single scenario analysis is a coherent risk measure; and if $m > 1$, then the highest expected of m worst case outcomes is also a coherent risk measure. In short, the ES, the highest expected loss from a set of possible outcomes (or loss estimates from scenario analyses), the highest ES from a set of comparable ESs based on different distribution functions, and the highest expected loss from a set of highest losses, are all coherent risk measures.

Thus, the outcomes of (simple or generalised) scenarios can be interpreted as coherent risk measures. However, the reverse is also true as well: coherent risk measures can be interpreted as the outcomes of scenarios associated with particular density functions. This

[24] An example of a scenario-based coherent risk measure is given by the outcomes of worst-case scenario analyses (WCSA) suggested by Boudoukh *et al.* (1995) and Bahar *et al.* (1997): in essence, these take a large number of sample drawings from a chosen distribution, and the risk measure is the mean of the sample highest losses. Another example of a standard stress testing framework whose outcomes qualify as coherent risk measures is the Standard Portfolio Analysis of Risk (SPAN) system used by the Chicago Mercantile Exchange to calculate margin requirements. This system considers 16 specific scenarios, consisting of standardised movements in underlying risk factors. Fourteen of these are fairly moderate scenarios, and two are extreme. The measure of risk is the maximum loss incurred across all scenarios, using the full loss from the first 14 scenarios and 35% of the loss from the two extreme ones. (Taking 35% of the losses on the extreme scenarios can be regarded as allowing for the extreme losses to be less probable than the others.) The calculations involved can be interpreted as producing the maximum expected loss under 16 distributions. The SPAN risk measures are coherent because the margin requirement is equal to the shortfall from this maximum expected loss.

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BOX 3-2 Distortion Risk Measures

Distortion risk measures are closely related to coherent measures, but emerged from the actuarial/insurance literature rather than the mainstream financial risk literature. They were proposed by Wang (1996) and have been applied to a wide variety of insurance problems, most particularly to the determination of insurance premiums.

A distortion risk measure is the expected loss under a transformation of the cumulative density function known as a distortion function, and the choice of distortion function determines the particular risk measure. More formally, if $F(x)$ is some cdf, the transformation $F^*(x) = g(F(x))$ is a distortion function if $g:[0,1] \rightarrow [0,1]$ is an increasing function with $g(0) = 0$ and $g(1) = 1$. The distortion risk measure is then the expectation of the random loss X using probabilities obtained from $F^*(x)$ rather than $F(x)$. Like coherent risk measures, distortion risk measures have the properties of homogeneity, positive homogeneity, and translational invariance; they also share with spectral risk measures the property of comonotonic additivity. To make good use of distortion measures, we would choose a 'good' distortion function, and there are many distortion functions to choose from. The properties we might look for in a 'good' distortion function include continuity, concavity, and differentiability; of these, continuity is necessary and sufficient for the distortion risk measure to be coherent, and concavity is sufficient (Wang *et al.* (1997)).

Of the various distortion functions the best-known is the renowned Wang transform (Wang (2000)):

$$g(u) = \phi[\phi^{-1}(u) - \lambda]$$

where λ can be taken to be equal to $\phi^{-1}(\alpha)$ or to the market price of risk. This distortion function is everywhere continuous and differentiable. The continuity of this distortion function also means that it produces coherent risk measures, and these measures are superior to the ES because they take account of the losses below VaR, and also take better account of extreme losses.

is very useful, because it means that we can always estimate coherent risk measures by specifying the relevant scenarios and then taking (as relevant) their (perhaps probability-weighted) averages or maxima: all we need to know are the loss outcomes (which are quantiles from the loss distribution), the density functions to be used (which give us our probabilities), and the type of coherent risk measure we are seeking.[25,26]

SUMMARY

This chapter has reviewed three alternative risk measurement frameworks. The first, the mean–variance framework, is adequate in the face of a limited set of situations (i.e., if we assume returns or losses are elliptical or if we impose unreasonable restrictions on the utility function). This leaves us with the problem of finding a 'good' risk measure that can be used in less restrictive conditions. The answer often proposed is to use the VaR. Because the VaR is simply a quantile, we can estimate it for any distribution we like. However, the VaR has serious flaws as a measure of risk, and there are good grounds to say that it should not be regarded as a 'proper' risk measure at all. A better answer is to use coherent risk measures. These give us 'respectable' measures of risk that are valid for all possible return or loss distributions, and they are manifestly superior to the VaR as a risk measure. The solution is therefore to upgrade further from VaR to coherent (or distortion) risk measures. These better risk measures are straightforward to estimate if one already has a VaR calculation engine in place, as the costs of upgrading from a VaR calculation engine to a coherent (or distortion) risk measure engine are very small. Perhaps the key lesson in all of this is that it is much less important *how* we estimate risk measures; it is much more important that we estimate the *right* risk measure.

[25] Coherent risk measures produce other surprises too. There is an intimate link between coherent risk measures and the generalised arbitrage bounds or 'good deal bounds' of Cerny and Hodges (1999). This leads to some interesting and profound interrelationships between coherent risk measures, arbitrage valuation, valuation bounds, portfolio optimisation and utility maximisation. For more on these, see Jaschke and Küchler (2000).

[26] Another important related family of risk measures are the dynamic or multi-period risk measures. Multi-period measures take account of interim cash flows, and allow us to look at risk measures over a period rather than just at the *end* of it. Dynamic risk measures are also more satisfactory in dynamic situations where, for example, 10-day risk measures are rolled forward from one day to the next. When used in the context of larger optimisation problems, dynamic risk measures are less prone to consistency issues over time. For more on these measures, see, e.g., Wang (1999) and Cvitanic and Karatzas (1999).

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Binomial Trees

4

Learning Objectives

After completing this reading you should be able to:

- Calculate the value of an American and a European call or put option using a one-step and two-step binomial model.
- Describe how volatility is captured in the binomial model.
- Describe how the value calculated using a binomial model converges as time periods are added.
- Explain how the binomial model can be altered to price options on: stocks with dividends, stock indices, currencies, and futures.

Excerpt is Chapter 13 of Options, Futures, and Other Derivatives, *Ninth Edition, by John C. Hull.*

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A useful and very popular technique for pricing an option involves constructing a *binomial tree*. This is a diagram representing different possible paths that might be followed by the stock price over the life of an option. The underlying assumption is that the stock price follows a *random walk*. In each time step, it has a certain probability of moving up by a certain percentage amount and a certain probability of moving down by a certain percentage amount. In the limit, as the time step becomes smaller, this model is the same as the Black–Scholes–Merton model we will be discussing in Chapter 5. Indeed, in the appendix to this chapter, we show that the European option price given by the binomial tree converges to the Black–Scholes–Merton price as the time step becomes smaller.

The material in this chapter is important for a number of reasons. First, it explains the nature of the no-arbitrage arguments that are used for valuing options. Second, it explains the binomial tree numerical procedure that is widely used for valuing American options and other derivatives. Third, it introduces a very important principle known as risk-neutral valuation.

The general approach to constructing trees in this chapter is the one used in an important paper published by Cox, Ross, and Rubinstein in 1979.

A ONE-STEP BINOMIAL MODEL AND A NO-ARBITRAGE ARGUMENT

We start by considering a very simple situation. A stock price is currently \$20, and it is known that at the end of 3 months it will be either \$22 or \$18. We are interested in valuing a European call option to buy the stock for \$21 in 3 months. This option will have one of two values at the end of the 3 months. If the stock price turns out to be \$22, the value of the option will be \$1; if the stock price turns out to be \$18, the value of the option will be zero. The situation is illustrated in Figure 4-1.

It turns out that a relatively simple argument can be used to price the option in this example. The only assumption needed is that arbitrage opportunities do not exist. We set up a portfolio of the stock and the option in such a way that there is no uncertainty about the value of the portfolio at the end of the 3 months. We then argue that, because the portfolio has no risk, the return it earns must equal the risk-free interest rate. This enables us to work out the cost of setting up the portfolio and therefore the

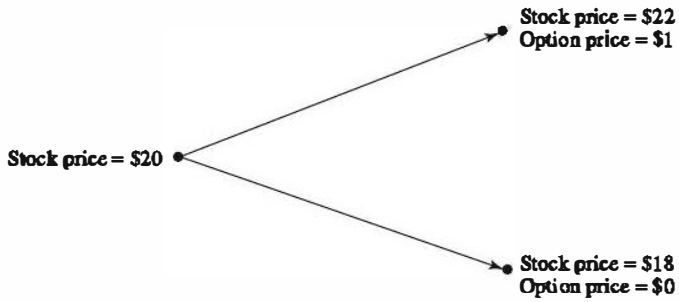

FIGURE 4-1 Stock price movements for numerical example in this section.

option's price. Because there are two securities (the stock and the stock option) and only two possible outcomes, it is always possible to set up the riskless portfolio.

Consider a portfolio consisting of a long position in Δ shares of the stock and a short position in one call option (Δ is the Greek capital letter "delta"). We calculate the value of Δ that makes the portfolio riskless. If the stock price moves up from \$20 to \$22, the value of the shares is 22Δ and the value of the option is 1, so that the total value of the portfolio is $22\Delta - 1$. If the stock price moves down from \$20 to \$18, the value of the shares is 18Δ and the value of the option is zero, so that the total value of the portfolio is 18Δ. The portfolio is riskless if the value of Δ is chosen so that the final value of the portfolio is the same for both alternatives. This means that

$$22\Delta - 1 = 18\Delta$$

or

$$\Delta = 0.25$$

A riskless portfolio is therefore

Long: 0.25 shares

Short: 1 option.

If the stock price moves up to \$22, the value of the portfolio is

$$22 \times 0.25 - 1 = 4.5$$

If the stock price moves down to \$18, the value of the portfolio is

$$18 \times 0.25 = 4.5$$

Regardless of whether the stock price moves up or down, the value of the portfolio is always 4.5 at the end of the life of the option.

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Riskless portfolios must, in the absence of arbitrage opportunities, earn the risk-free rate of interest. Suppose that, in this case, the risk-free rate is 12% per annum. It follows that the value of the portfolio today must be the present value of 4.5, or

$$4.5e^{-0.12\times 3/12} = 4.367$$

The value of the stock price today is known to be $20. Suppose the option price is denoted by f. The value of the portfolio today is

$$20 \times 0.25 - f = 5 - f$$

It follows that

$$5 - f = 4.367$$

or

$$f = 0.633$$

This shows that, in the absence of arbitrage opportunities, the current value of the option must be 0.633. If the value of the option were more than 0.633, the portfolio would cost less than 4.367 to set up and would earn more than the risk-free rate. If the value of the option were less than 0.633, shorting the portfolio would provide a way of borrowing money at less than the risk-free rate.

Trading 0.25 shares is, of course, not possible. However, the argument is the same if we imagine selling 400 options and buying 100 shares. In general, it is necessary to buy Δ shares for each option sold to form a riskless portfolio. The parameter Δ (delta) is important in the hedging of options. It is discussed further later in this chapter and in Chapter 6.

A Generalization

We can generalize the no-arbitrage argument just presented by considering a stock whose price is S_0 and an option on the stock (or any derivative dependent on the stock) whose current price is f. We suppose that the option lasts for time T and that during the life of the option the stock price can either move up from S_0 to a new level, S_0u, where $u > 1$, or down from S_0 to a new level, S_0d, where $d < 1$. The percentage increase in the stock price when there is an up movement is $u - 1$; the percentage decrease when there is a down movement is $1 - d$. If the stock price moves up to S_0u, we suppose that the payoff from the option is f_u; if the stock price moves down to S_0d, we suppose the payoff from the option is f_d. The situation is illustrated in Figure 4-2.

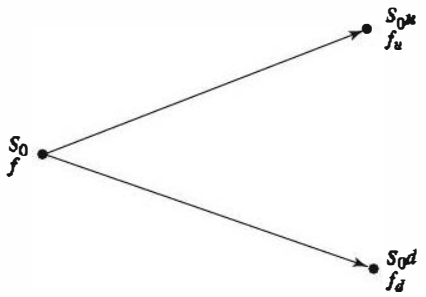

FIGURE 4-2 Stock and option prices in a general one-step tree.

As before, we imagine a portfolio consisting of a long position in Δ shares and a short position in one option. We calculate the value of Δ that makes the portfolio riskless. If there is an up movement in the stock price, the value of the portfolio at the end of the life of the option is

$$S_0u\Delta - f_u$$

If there is a down movement in the stock price, the value becomes

$$S_0d\Delta - f_d$$

The two are equal when

$$S_0u\Delta - f_u = S_0d\Delta - f_d$$

or

$$\Delta = \frac{f_u - f_d}{S_0u - S_0d} \tag{4.1}$$

In this case, the portfolio is riskless and, for there to be no arbitrage opportunities, it must earn the risk-free interest rate. Equation (4.1) shows that Δ is the ratio of the change in the option price to the change in the stock price as we move between the nodes at time T.

If we denote the risk-free interest rate by r, the present value of the portfolio is

$$(S_0u\Delta - f_u)e^{-rT}$$

The cost of setting up the portfolio is

$$S_0\Delta - f$$

It follows that

$$S_0\Delta - f = (S_0u\Delta - f_u)e^{-rT}$$

or

$$f = S_0\Delta(1 - ue^{-rT}) + f_ue^{-rT}$$

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Substituting from Equation (4.1) for Δ, we obtain

$$f = S_0\left(\frac{f_u - f_d}{S_0u - S_0d}\right)(1 - ue^{-rT}) + f_ue^{-rT}$$

or

$$f = \frac{f_u(1 - de^{-rT}) + f_d(ue^{-rT} - 1)}{u - d}$$

or

$$f = e^{-rT}[pf_u + (1 - p)f_d] \tag{4.2}$$

where

$$p = \frac{e^{rT} - d}{u - d} \tag{4.3}$$

Equations (4.2) and (4.3) enable an option to be priced when stock price movements are given by a one-step binomial tree. The only assumption needed for the equation is that there are no arbitrage opportunities in the market.

In the numerical example considered previously (see Figure 4-1), $u = 1.1$, $d = 0.9$, $r = 0.12$, $T = 0.25$, $f_u = 1$, and $f_d = 0$. From Equation (4.3), we have

$$p = \frac{e^{0.12\times 3/12} - 0.9}{1.1 - 0.9} = 0.6523$$

and, from Equation (4.2), we have

$$f = e^{-0.12\times 0.25}(0.6523 \times 1 + 0.3477 \times 0) = 0.633$$

The result agrees with the answer obtained earlier in this section.

Irrelevance of the Stock's Expected Return

The option pricing formula in Equation (4.2) does not involve the probabilities of the stock price moving up or down. For example, we get the same option price when the probability of an upward movement is 0.5 as we do when it is 0.9. This is surprising and seems counterintuitive. It is natural to assume that, as the probability of an upward movement in the stock price increases, the value of a call option on the stock increases and the value of a put option on the stock decreases. This is not the case.

The key reason is that we are not valuing the option in absolute terms. We are calculating its value in terms of the price of the underlying stock. The probabilities of future up or down movements are already incorporated into the stock price: we do not need to take them into account again when valuing the option in terms of the stock price.

RISK-NEUTRAL VALUATION

We are now in a position to introduce a very important principle in the pricing of derivatives known as *risk-neutral valuation*. This states that, when valuing a derivative, we can make the assumption that investors are *risk-neutral*. This assumption means investors do not increase the expected return they require from an investment to compensate for increased risk. A world where investors are risk-neutral is referred to as a *risk-neutral world*. The world we live in is, of course, not a risk-neutral world. The higher the risks investors take, the higher the expected returns they require. However, it turns out that assuming a risk-neutral world gives us the right option price for the world we live in, as well as for a risk-neutral world. Almost miraculously, it finesses the problem that we know hardly anything about the risk aversion of the buyers and sellers of options.

Risk-neutral valuation seems a surprising result when it is first encountered. Options are risky investments. Should not a person's risk preferences affect how they are priced? The answer is that, when we are pricing an option in terms of the price of the underlying stock, risk preferences are unimportant. As investors become more risk averse, stock prices decline, but the formulas relating option price to stock prices remain the same.

A risk-neutral world has two features that simplify the pricing of derivatives:

1. The expected return on a stock (or any other investment) is the risk-free rate.
2. The discount rate used for the expected payoff on an option (or any other instrument) is the risk-free rate.

Returning to Equation (4.2), the parameter p should be interpreted as the probability of an up movement in a risk-neutral world, so that $1 - p$ is the probability of a down movement in this world. We assume $u > e^{rT}$, so that $0 < p < 1$. The expression

$$pf_u + (1 - p)f_d$$

is the expected future payoff from the option in a risk-neutral world and Equation (4.2) states that the value of the option today is its expected future payoff in a

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risk-neutral world discounted at the risk-free rate. This is an application of risk-neutral valuation.

To prove the validity of our interpretation of p, we note that, when p is the probability of an up movement, the expected stock price $E(S_T)$ at time T is given by

$$E(S_T) = pS_0u + (1 - p)S_0d$$

or

$$E(S_T) = pS_0(u - d) + S_0d$$

Substituting from Equation (4.3) for p gives

$$E(S_T) = S_0e^{rT} \tag{4.4}$$

This shows that the stock price grows, on average, at the risk-free rate when p is the probability of an up movement. In other words, the stock price behaves exactly as we would expect it to behave in a risk-neutral world when p is the probability of an up movement.

Risk-neutral valuation is a very important general result in the pricing of derivatives. It states that, when we assume the world is risk-neutral, we get the right price for a derivative in all worlds, not just in a risk-neutral one. We have shown that risk-neutral valuation is correct when a simple binomial model is assumed for how the price of the stock evolves. It can be shown that the result is true regardless of the assumptions we make about the evolution of the stock price.

To apply risk-neutral valuation to the pricing of a derivative, we first calculate what the probabilities of different outcomes would be if the world were risk-neutral. We then calculate the expected payoff from the derivative and discount that expected payoff at the risk-free rate of interest.

The One-Step Binomial Example Revisited

We now return to the example in Figure 4-1 and illustrate that risk-neutral valuation gives the same answer as no-arbitrage arguments. In Figure 4-1, the stock price is currently \$20 and will move either up to \$22 or down to \$18 at the end of 3 months. The option considered is a European call option with a strike price of \$21 and an expiration date in 3 months. The risk-free interest rate is 12% per annum.

We define p as the probability of an upward movement in the stock price in a risk neutral world. We can calculate p from Equation (4.3). Alternatively, we can argue that the expected return on the stock in a risk-neutral world must be the risk-free rate of 12%. This means that p must satisfy

$$22p + 18(1 - p) = 20e^{0.12\times3/12}$$

or

$$4p = 20e^{0.12\times3/12} - 18$$

That is, p must be 0.6523.

At the end of the 3 months, the call option has a 0.6523 probability of being worth 1 and a 0.3477 probability of being worth zero. Its expected value is therefore

$$0.6523 \times 1 + 0.3477 \times 0 = 0.6523$$

In a risk-neutral world this should be discounted at the risk-free rate. The value of the option today is therefore

$$0.6523e^{-0.12\times3/12}$$

or \$0.633. This is the same as the value obtained earlier, demonstrating that no-arbitrage arguments and risk-neutral valuation give the same answer.

Real World vs. Risk-Neutral World

It should be emphasized that p is the probability of an up movement in a risk-neutral world. In general, this is not the same as the probability of an up movement in the real world. In our example $p = 0.6523$. When the probability of an up movement is 0.6523, the expected return on both the stock and the option is the risk-free rate of 12%. Suppose that, in the real world, the expected return on the stock is 16% and p^* is the probability of an up movement in this world. It follows that

$$22p^* + 18(1 - p^*) = 20e^{0.16\times3/12}$$

so that $p^* = 0.7041$.

The expected payoff from the option in the real world is then given by

$$p^* \times 1 + (1 - p^*) \times 0$$

or 0.7041. Unfortunately, it is not easy to know the correct discount rate to apply to the expected payoff in the real world. The return the market requires on the stock is 16% and this is the discount rate that would be used for the expected cash flows from an investment in the stock. A position in a call option is riskier than a position in the stock. As a result the discount rate to be applied to the payoff from a call option is greater than 16%, but

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we do not know how much greater than 16% it should be.[1] Using risk-neutral valuation solves this problem because we know that in a risk-neutral world the expected return on all assets (and therefore the discount rate to use for all expected payoffs) is the risk-free rate.

TWO-STEP BINOMIAL TREES

We can extend the analysis to a two-step binomial tree such as that shown in Figure 4-3. Here the stock price starts at $20 and in each of two time steps may go up by 10% or down by 10%. Each time step is 3 months long and the risk-free interest rate is 12% per annum. We consider a 6-month option with a strike price of $21.

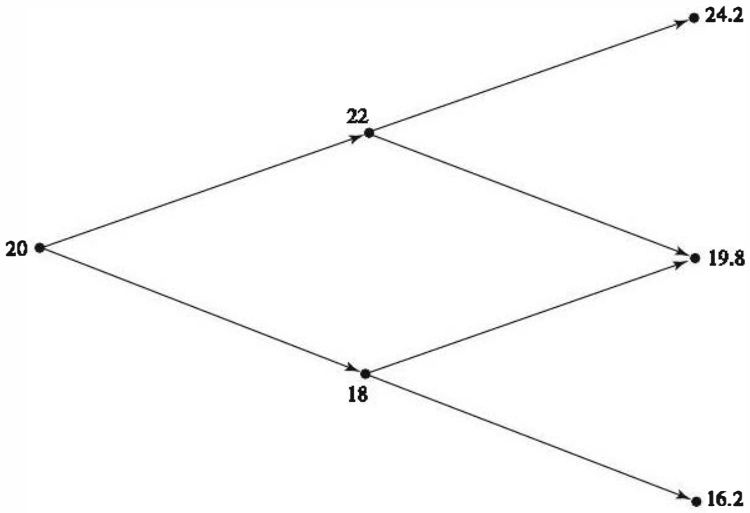

FIGURE 4-3 Stock prices in a two-step tree.

The objective of the analysis is to calculate the option price at the initial node of the tree. This can be done by repeatedly applying the principles established earlier in the chapter. Figure 4-4 shows the same tree as Figure 4-3, but with both the stock price and the option price at each node. (The stock price is the upper number and the option price is the lower number.) The option prices at the final nodes of the tree are easily calculated. They are the payoffs from the option. At node D the stock price is 24.2 and the option price is 24.2 − 21 = 3.2; at nodes E and F the option is out of the money and its value is zero.

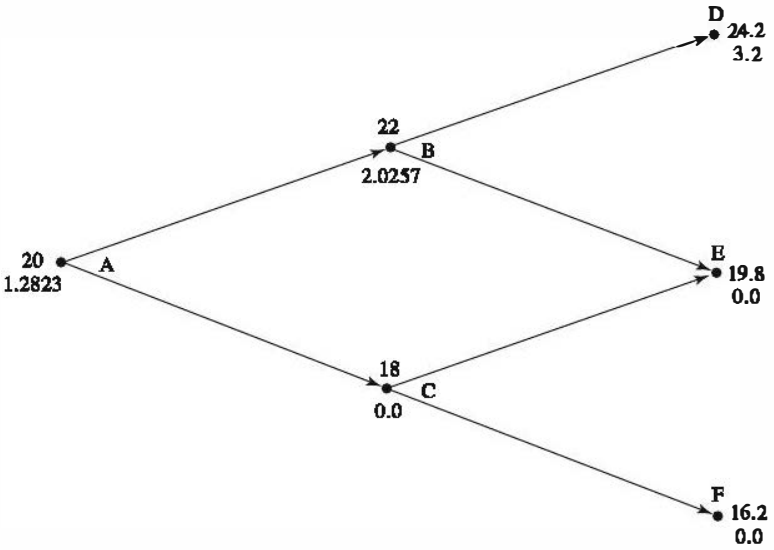

FIGURE 4-4 Stock and option prices in a two-step tree. The upper number at each node is the stock price and the lower number is the option price.

At node C the option price is zero, because node C leads to either node E or node F and at both of those nodes the option price is zero. We calculate the option price at node B by focusing our attention on the part of the tree shown in Figure 4-5. Using the notation introduced earlier in the chapter, $u = 1.1$, $d = 0.9$, $r = 0.12$, and $T = 0.25$, so that $p = 0.6523$, and Equation (4.2) gives the value of the option at node B as

$$e^{-0.12\times3/12}(0.6523 \times 3.2 + 0.3477 \times 0) = 2.0257$$

It remains for us to calculate the option price at the initial node A. We do so by focusing on the first step of the tree. We know that the value of the option at node B is 2.0257 and that at node C it is zero. Equation (4.2) therefore gives the value at node A as

$$e^{-0.12\times3/12}(0.6523 \times 2.0257 + 0.3477 \times 0) = 1.2823$$

The value of the option is $1.2823.

Note that this example was constructed so that u and d (the proportional up and down movements) were the

[1] Since we know the correct value of the option is 0.633, we can deduce that the correct real-world discount rate is 42.58%. This is because $0.633 = 0.7041e^{-0.4258\times3/12}$.

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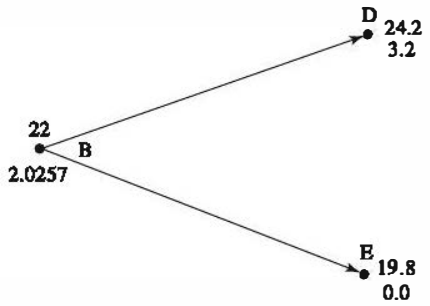

FIGURE 4-5 Evaluation of option price at node B of Figure 4-4.

same at each node of the tree and so that the time steps were of the same length. As a result, the risk-neutral probability, p, as calculated by Equation (4.3) is the same at each node.

A Generalization

We can generalize the case of two time steps by considering the situation in Figure 4-6. The stock price is initially S_0. During each time step, it either moves up to u times its initial value or moves down to d times its initial value. The notation for the value of the option is shown on the tree. (For example, after two up movements the value of the option is f_{uu}.) We suppose that the risk-free interest rate is r and the length of the time step is Δt years.

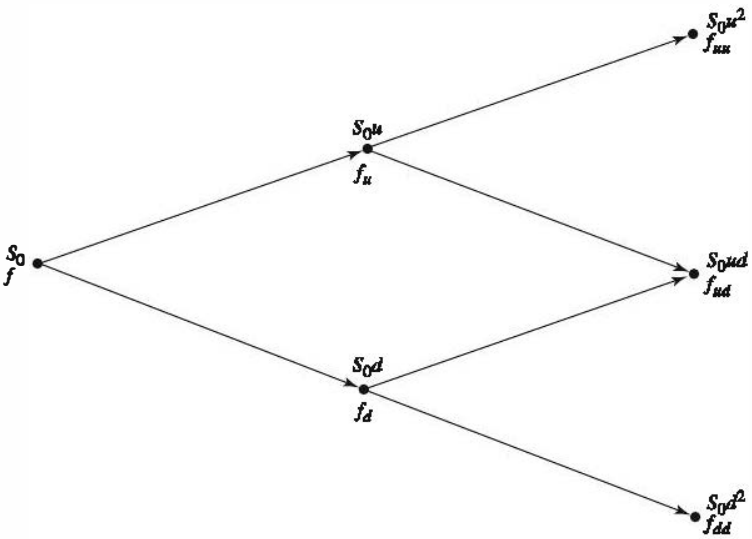

FIGURE 4-6 Stock and option prices in general two-step tree.

Because the length of a time step is now Δt rather than T, Equations (4.2) and (4.3) become

$$f = e^{-r\Delta t}[pf_u + (1-p)f_d] \tag{4.5}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{4.6}$$

Repeated application of Equation (4.5) gives

$$f_u = e^{-r\Delta t}[pf_{uu} + (1-p)f_{ud}] \tag{4.7}$$

$$f_d = e^{-r\Delta t}[pf_{ud} + (1-p)f_{dd}] \tag{4.8}$$

$$f = e^{-r\Delta t}[pf_u - (1-p)f_d] \tag{4.9}$$

Substituting from Equations (4.7) and (4.8) into (4.9), we get

$$f = e^{-2r\Delta t}[p^2f_{uu} + 2p(1-p)f_{ud} + (1-p)^2f_{dd}] \tag{4.10}$$

This is consistent with the principle of risk-neutral valuation mentioned earlier. The variables p^2, $2p(1-p)$, and $(1-p)^2$ are the probabilities that the upper, middle, and lower final nodes will be reached. The option price is equal to its expected payoff in a risk-neutral world discounted at the risk-free interest rate.

As we add more steps to the binomial tree, the risk-neutral valuation principle continues to hold. The option price is always equal to its expected payoff in a risk-neutral world discounted at the risk-free interest rate.

A PUT EXAMPLE

The procedures described in this chapter can be used to price puts as well as calls. Consider a 2-year European put with a strike price of \$52 on a stock whose current price is \$50. We suppose that there are two time steps of 1 year, and in each time step the stock price either moves up by 20% or moves down by 20%. We also suppose that the risk-free interest rate is 5%.

The tree is shown in Figure 4-7. In this case $u = 1.2$, $d = 0.8$, $\Delta t = 1$, and $r = 0.05$. From Equation (4.6) the value of the risk-neutral probability, p, is given by

$$p = \frac{e^{0.05\times 1} - 0.8}{1.2 - 0.8} = 0.6282$$

The possible final stock prices are: \$72, \$48, and \$32. In this case, $f_{uu} = 0$, $f_{ud} = 4$, and $f_{dd} = 20$. From Equation (4.10),

$$f = e^{-2\times 0.05\times 1}(0.6282^2 \times 0 + 2 \times 0.6282 \times 0.3718 \times 4 + 0.3718^2 \times 20) = 4.1923$$

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The value of the put is $4.1923. This result can also be obtained using Equation (4.5) and working back through the tree one step at a time. Figure 4-7 shows the intermediate option prices that are calculated.

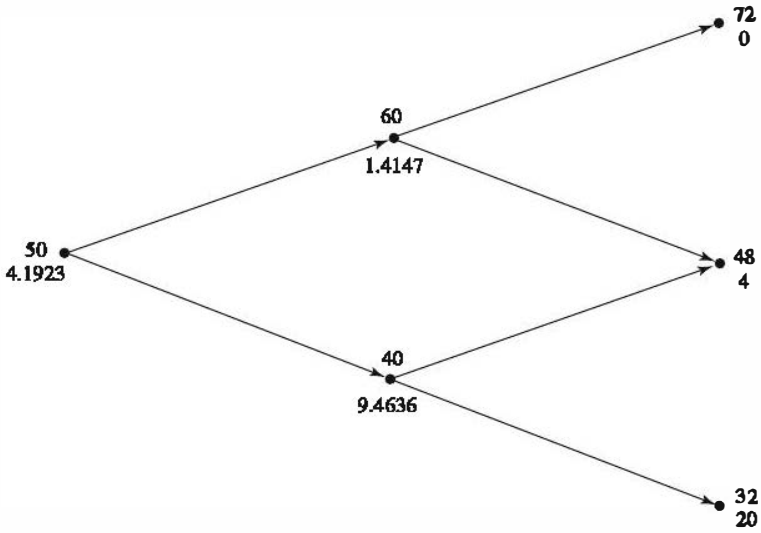

FIGURE 4-7 Using a two-step tree to value a European put option. At each node, the upper number is the stock price and the lower number is the option price.

AMERICAN OPTIONS

Up to now all the options we have considered have been European. We now move on to consider how American options can be valued using a binomial tree such as that in Figure 4-4 or 4-7. The procedure is to work back through the tree from the end to the beginning, testing at each node to see whether early exercise is optimal. The value of the option at the final nodes is the same as for the European option. At earlier nodes the value of the option is the greater of

1. The value given by Equation (4.5)
2. The payoff from early exercise.

Figure 4-8 shows how Figure 4-7 is affected if the option under consideration is American rather than European. The stock prices and their probabilities are unchanged. The values for the option at the final nodes are also unchanged. At node B, Equation (4.5) gives the value of the option as 1.4147, whereas the payoff from early exercise is negative (= −8). Clearly early exercise is not optimal at node B, and the value of the option at this node is 1.4147. At node C, Equation (4.5) gives the value of the option as 9.4636, whereas the payoff from early exercise is 12. In this case, early exercise is optimal and the value of the option at the node is 12. At the initial node A, the value given by Equation (4.5) is

$$e^{-0.05\times 1}(0.6282 \times 1.4147 + 0.3718 \times 12.0) = 5.0894$$

and the payoff from early exercise is 2. In this case early exercise is not optimal. The value of the option is therefore $5.0894.

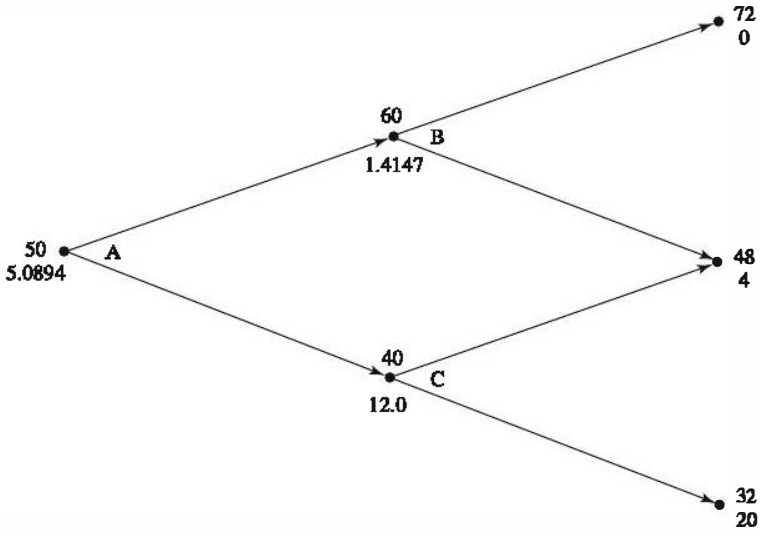

FIGURE 4-8 Using a two-step tree to value an American put option. At each node, the upper number is the stock price and the lower number is the option price.

DELTA

At this stage, it is appropriate to introduce *delta*, an important parameter (sometimes referred to as a "Greek letter" or simply a "Greek") in the pricing and hedging of options.

The delta (Δ) of a stock option is the ratio of the change in the price of the stock option to the change in the price of the underlying stock. It is the number of units of the stock we should hold for each option shorted in order to create a riskless portfolio. It is the same as the Δ introduced earlier in this chapter. The construction of a riskless portfolio is sometimes referred to as *delta hedging*. The delta of a call option is positive, whereas the delta of a put option is negative.

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From Figure 4-1, we can calculate the value of the delta of the call option being considered as

$$\frac{1-0}{22-18} = 0.25$$

This is because when the stock price changes from \$18 to \$22, the option price changes from \$0 to \$1. (This is also the value of Δ calculated earlier.)

In Figure 4-4 the delta corresponding to stock price movements over the first time step is

$$\frac{2.0257-0}{22-18} = 0.5064$$

The delta for stock price movements over the second time step is

$$\frac{3.2-0}{24.2-19.8} = 0.7273$$

if there is an upward movement over the first time step, and

$$\frac{0-0}{19.8-16.2} = 0$$

if there is a downward movement over the first time step.

From Figure 4-7, delta is

$$\frac{1.4147-9.4636}{60-40} = -0.4024$$

at the end of the first time step, and either

$$\frac{0-4}{72-48} = -0.1667 \text{ or } \frac{4-20}{48-32} = -1.0000$$

at the end of the second time step.

The two-step examples show that delta changes over time. (In Figure 4-4, delta changes from 0.5064 to either 0.7273 or 0; and, in Figure 4-7, it changes from −0.4024 to either −0.1667 or −1.0000.) Thus, in order to maintain a riskless hedge using an option and the underlying stock, we need to adjust our holdings in the stock periodically. We will return to this feature of options in Chapter 6.

MATCHING VOLATILITY WITH *u* AND *d*

The three parameters necessary to construct a binomial tree with time step Δt are u, d, and p. Once u and d have been specified, p must be chosen so that the expected return is the risk-free rate r. We have already shown that

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{4.11}$$

The parameters u and d should be chosen to match volatility. The volatility of stock (or any other asset), σ, is defined so that the standard deviation of its return in a short period of time Δt is $\sigma\sqrt{\Delta t}$ (see Chapter 5 for a further discussion of this). Equivalently the variance of the return in time Δt is $\sigma^2\Delta t$. The variance of a variable X is defined as $E(X^2) - [E(X)]^2$, where E denotes expected value. During a time step of length Δt, there is a probability p that the stock will provide a return of $u - 1$ and a probability $1 - p$ that it will provide a return of $d - 1$. It follows that volatility is matched if

$$p(u-1)^2 + (1-p)(d-1)^2 - [p(u-1) + (1-p)(d-1)]^2 = \sigma^2\Delta t \tag{4.12}$$

Substituting for p from Equation (4.11), this simplifies to

$$e^{r\Delta t}(u+d) - ud - e^{2r\Delta t} = \sigma^2\Delta t \tag{4.13}$$

When terms in Δt^2 and higher powers of Δt are ignored, a solution to Equation (4.13) is[2]

$$u = e^{\sigma\sqrt{\Delta t}} \text{ and } d = e^{-\sigma\sqrt{\Delta t}}$$

These are the values of u and d used by Cox, Ross, and Rubinstein (1979).

In the analysis just given we chose u and d to match volatility in the risk-neutral world. What happens if instead we match volatility in the real world? As we will now show, the formulas for u and d are the same.

Suppose that p^* is the probability of an up-movement in the real world while p is as before the probability of an up-movement in a risk-neutral world. This is illustrated in Figure 4-9. Define μ as the expected return in the real world. We must have

$$p^*u + (1-p^*)d = e^{\mu\Delta t}$$

or

$$p^* = \frac{e^{\mu\Delta t} - d}{u - d} \tag{4.14}$$

Suppose that σ is the volatility in the real world. The equation matching the variance is the same as Equation (4.12) except that p is replaced by p^*. When this equation is combined with Equation (4.14), we obtain

$$e^{\mu\Delta t}(u+d) - ud - e^{2\mu\Delta t} = \sigma^2\Delta t$$

[2] We are here using the series expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

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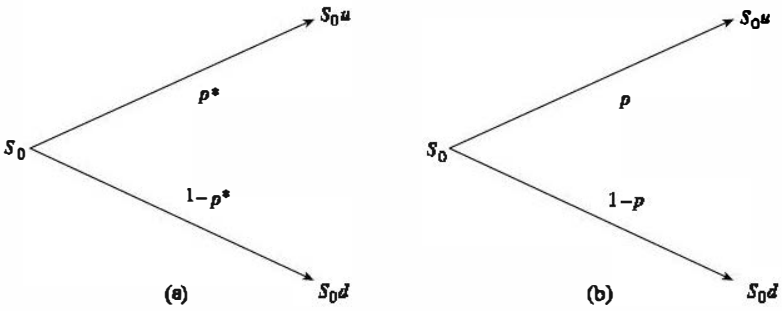

FIGURE 4-9 Change in stock price in time Δt in (a) the real world and (b) the risk-neutral world.

This is the same as Equation (4.13) except the r is replaced by μ.. When terms in Δt^2 and higher powers of Δt are ignored, it has the same solution as Equation (4.13):

$$u = e^{\sigma\sqrt{\Delta t}} \text{ and } d = e^{-\sigma\sqrt{\Delta t}}$$

Girsanov's Theorem

The results we have just produced are closely related to an important result known as *Girsanov's theorem*. When we move from the risk-neutral world to the real world, the expected return from the stock price changes, but its volatility remains the same. More generally, when we move from a world with one set of risk preferences to a world with another set of risk preferences, the expected growth rates in variables change, but their volatilities remain the same. Moving from one set of risk preferences to another is sometimes referred to as *changing the measure*. The real-world measure is sometimes referred to as the *P-measure*, while the risk-neutral world measure is referred to as the *Q-measure*.[3]

THE BINOMIAL TREE FORMULAS

The analysis in the previous section shows that, when the length of the time step on a binomial tree is Δt, we should match volatility by setting

$$u = e^{\sigma\sqrt{\Delta t}} \tag{4.15}$$

and

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{4.16}$$

[3] With the notation we have been using, p is the probability under the Q-measure, while p^* is the probability under the P-measure.

Also, from Equation (4.6),

$$p = \frac{a-d}{u-d} \tag{4.17}$$

where

$$a = e^{r\Delta t} \tag{4.18}$$

Equations (4.15) to (4.18) define the tree.

Consider again the American put option in Figure 4-8, where the stock price is \$50, the strike price is \$52, the risk-free rate is 5%, the life of the option is 2 years, and there are two time steps. In this case, $\Delta t = 1$. Suppose that the volatility σ is 30%. Then, from Equations (4.15) to (4.18), we have

$$u = e^{0.3\times1} = 1.3499,\ d = \frac{1}{1.3499} = 0.7408,\ a = e^{0.05\times1} = 1.0513$$

and

$$p = \frac{1.053 - 0.7408}{1.3499 - 0.7408} = 0.5097$$

The tree is shown in Figure 4-10. The value of the put option is 7.43. (This is different from the value obtained in Figure 4-8 by assuming $u = 1.2$ and $d = 0.8$.) Note that the option is exercised at the end of the first time step if the lower node is reached.

INCREASING THE NUMBER OF STEPS

The binomial model presented above is unrealistically simple. Clearly, an analyst can expect to obtain only a very rough approximation to an option price by assuming that stock price movements during the life of the option consist of one or two binomial steps.

When binomial trees are used in practice, the life of the option is typically divided into 30 or more time steps. In each time step there is a binomial stock price movement. With 30 time steps there are 31 terminal stock prices and 2^{30}, or about 1 billion, possible stock price paths are implicitly considered.

The equations defining the tree are Equations (4.15) to (4.18), regardless of the number of time steps. Suppose, for example, that there are five steps instead of two in the example we considered in Figure 4-10. The parameters would be $\Delta t = 2/5 = 0.4$, $r = 0.05$, and $\sigma = 0.3$. These values give $u = e^{0.3\times\sqrt{0.4}} = 1.289$, $d = 1/1.2089 = 0.8272$, $a = e^{0.05\times0.4} = 1.0202$, and $p = (1.0202 - 0.8272) / (1.2089 - 0.8272) = 0.5056$.

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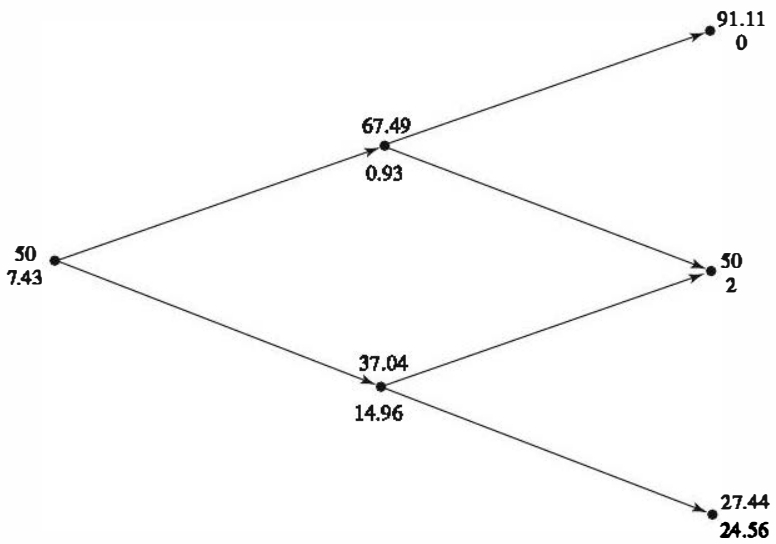

FIGURE 4-10 Two-step tree to value a 2-year American put option when the stock price is 50, strike price is 52, risk-free rate is 5%, and volatility is 30%.

As the number of time steps is increased (so that Δt becomes smaller), the binomial tree model makes the same assumptions about stock price behavior as the Black–Scholes–Merton model, which will be presented in Chapter 5. When the binomial tree is used to price a European option, the price converges to the Black–Scholes–Merton price, as expected, as the number of time steps is increased. This is proved in the appendix to this chapter.

USING DerivaGem

The software program, DerivaGem 2.01, is a useful tool for becoming comfortable with binomial trees. After downloading the software,[4] go to the Equity_FX_Index_Futures_Options worksheet. Choose Equity as the Underlying Type and select Binomial American as the Option Type. Enter the stock price, volatility, risk-free rate, time to expiration, exercise price, and tree steps, as 50, 30%, 5%, 2, 52, and 2, respectively. Click on the *Put* button and then on *Calculate*. The price of the option is shown as 7.428 in the box labeled Price. Now click on *Display Tree* and you will see the equivalent of Figure 4-10. (The red numbers in the software indicate the nodes where the option is exercised.)

[4] The software can be downloaded at http://www-2.rotman.utoronto.ca/~hull/software/index.html.

Return to the Equity_FX_Index_Futures_Options worksheet and change the number of time steps to 5. Hit *Enter* and click on *Calculate*. You will find that the value of the option changes to 7.671. By clicking on *Display Tree* the five-step tree is displayed, together with the values of u, d, a, and p calculated above.

DerivaGem can display trees that have up to 10 steps, but the calculations can be done for up to 500 steps. In our example, 500 steps gives the option price (to two decimal places) as 7.47. This is an accurate answer. By changing the Option Type to Binomial European, we can use the tree to value a European option. Using 500 time steps, the value of a European option with the same parameters as the American option is 6.76. (By changing the Option Type to Black–Scholes European, we can display the value of the option using the Black–Scholes–Merton formula that will be presented in Chapter 5. This is also 6.76.)

By changing the Underlying Type, we can consider options on assets other than stocks. These will now be discussed.

OPTIONS ON OTHER ASSETS

It turns out that we can construct and use binomial trees for these options in exactly the same way as for options on stocks except that the equations for p change. As in the case of options on stocks, Equation (4.2) applies so that the value at a node (before the possibility of early exercise is considered) is p times the value if there is an up movement plus $1 - p$ times the value if there is a down movement, discounted at the risk-free rate.

Options on Stocks Paying a Continuous Dividend Yield

Consider a stock paying a known dividend yield at rate q. The total return from dividends and capital gains in a risk-neutral world is r. The dividends provide a return of q. Capital gains must therefore provide a return of $r - q$. If the stock starts at S_0, its expected value after one time step of length Δt must be $S_0 e^{(r-q)\Delta t}$. This means that

$$pS_0u + (1 - p)S_0d = S_0e^{(r-q)\Delta t}$$

so that

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}$$

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As in the case of options on non-dividend-paying stocks, we match volatility by setting $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$. This means that we can use Equations (4.15) to (4.18), except that we set $a = e^{(r-q)\Delta t}$ instead of $a = e^{r\Delta t}$.

Options on Stock Indices

When calculating a futures price for a stock index we assumed that the stocks underlying the index provided a dividend yield at rate q. We make a similar assumption here. The valuation of an option on a stock index is therefore very similar to the valuation of an option on a stock paying a known dividend yield.

Example 4.1

A stock index is currently 810 and has a volatility of 20% and a dividend yield of 2%. The risk-free rate is 5%. Figure 4-11 shows the output from DerivaGem for valuing a European 6-month call option with a strike price of 800 using a two-step tree. In this case,

$$\Delta t = 0.25, \qquad u = e^{0.20\times\sqrt{0.25}} = 1.1052,$$

$$d = 1/u = 0.9048, \qquad a = e^{(0.05-0.02)\times 0.25} = 1.0075$$

$$p = (1.0075 - 0.9048) / (1.1052 - 0.9048) = 0.5126$$

The value of the option is 53.39.

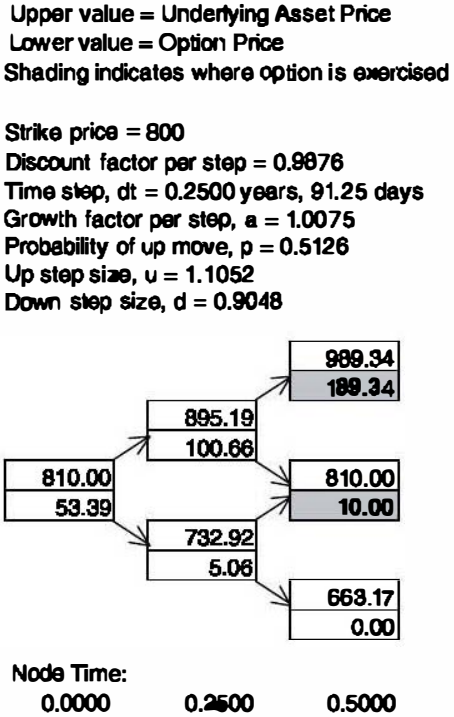

FIGURE 4-11 Two-step tree to value a European 6-month call option on an index when the index level is 810, strike price is 800, risk-free rate is 5%, volatility is 20%, and dividend yield is 2% (DerivaGem output).

Options on Currencies

A foreign currency can be regarded as an asset providing a yield at the foreign risk-free rate of interest, r_f. By analogy with the stock index case we can construct a tree for options on a currency by using Equations (4.15) to (4.18) and setting $a = e^{(r-r_f)\Delta t}$.

Example 4.2

The Australian dollar is currently worth 0.6100 US dollars and this exchange rate has a volatility of 12%. The Australian risk-free rate is 7% and the US risk-free rate is 5%. Figure 4-12 shows the output from DerivaGem for valuing a 3-month American call option with a strike price of 0.4000 using a three-step tree. In this case,

$$\Delta t = 0.08333, \qquad u = e^{0.12\times\sqrt{0.08333}} = 1.0352$$

$$d = 1/u = 0.9440, \qquad a = e^{(0.05-0.07)\times 0.08333} = 0.9983$$

$$p = (0.9983 - 0.9440)/(1.0352 - 0.9440) = 0.4473$$

The value of the option is 0.019.

Options on Futures

It costs nothing to take a long or a short position in a futures contract. It follows that in a risk-neutral world a futures price should have an expected growth rate of zero. As above, we define p as the probability of an up movement in the futures price, u as the percentage up movement, and d as the percentage down movement. If F_0 is the initial futures price, the expected futures price at the end of one time step of length Δt should also be F_0. This means that

$$pF_0u + (1-p)F_0d = F_0$$

so that

$$p = \frac{1-d}{u-d}$$

and we can use Equations (4.15) to (4.18) with $a = 1$.

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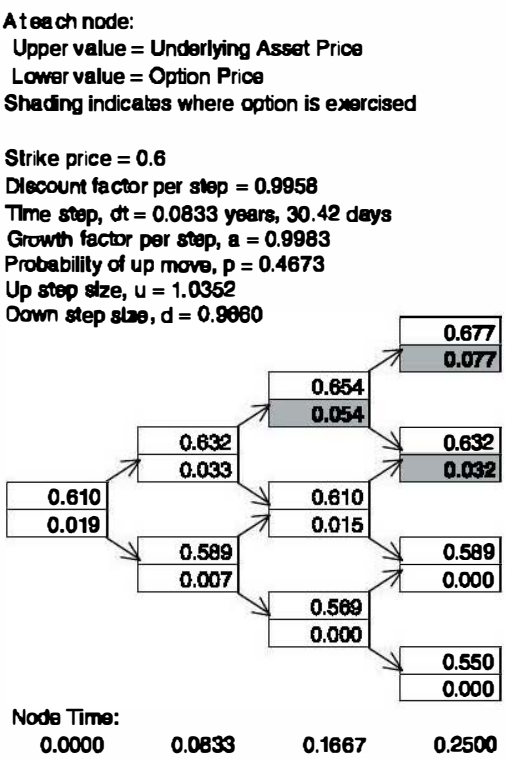

FIGURE 4-12 Three-step tree to value an American 3-month call option on a currency when the value of the currency is 0.6100, strike price is 0.6000, risk-free rate is 5%, volatility is 12%, and foreign risk-free rate is 7% (DerivaGem output).

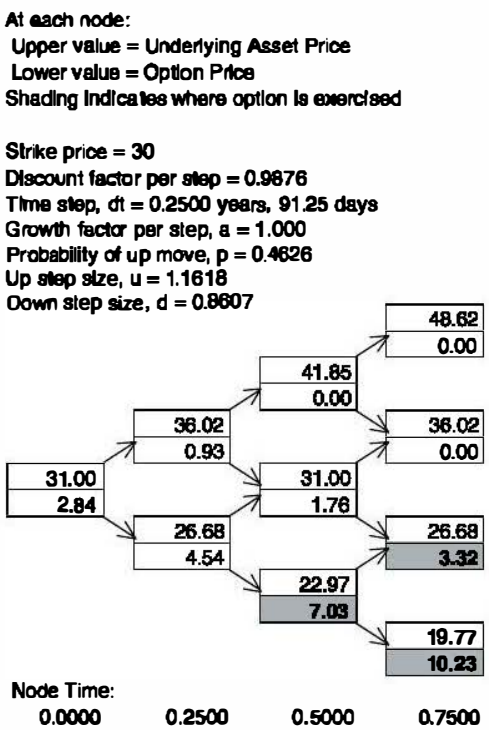

FIGURE 4-13 Three-step tree to value an American 9-month put option on a futures contract when the futures price is 31, strike price is 30, risk-free rate is 5%, and volatility is 30% (DerivaGem output).

Example 4.3

A futures price is currently 31 and has a volatility of 30%. The risk-free rate is 5%. Figure 4-13 shows the output from DerivaGem for valuing a 9-month American put option with a strike price of 30 using a three-step tree. In this case,

$$\Delta t = 0.25, \qquad u = e^{0.3\sqrt{0.25}} = 1.1618$$

$$d = 1/u = 1/1.1618 = 0.8607, \qquad a = 1,$$

$$p = (1 - 0.8607)/(1.1618 - 0.8607) = 0.4626$$

The value of the option is 2.84.

SUMMARY

This chapter has provided a first look at the valuation of options on stocks and other assets using trees. In the simple situation where movements in the price of a stock during the life of an option are governed by a one-step binomial tree, it is possible to set up a riskless portfolio consisting of a position in the stock option and a position in the stock. In a world with no arbitrage opportunities, riskless portfolios must earn the risk-free interest. This enables the stock option to be priced in terms of the stock. It is interesting to note that no assumptions are required about the probabilities of up and down movements in the stock price at each node of the tree.

When stock price movements are governed by a multistep binomial tree, we can treat each binomial step separately and work back from the end of the life of the option to the beginning to obtain the current value of the option. Again only no-arbitrage arguments are used, and no assumptions are required about the probabilities of up and down movements in the stock price at each node.

A very important principle states that we can assume the world is risk-neutral when valuing an option. This chapter

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has shown, through both numerical examples and algebra, that no-arbitrage arguments and risk-neutral valuation are equivalent and lead to the same option prices.

The delta of a stock option, Δ, considers the effect of a small change in the underlying stock price on the change in the option price. It is the ratio of the change in the option price to the change in the stock price. For a riskless position, an investor should buy Δ shares for each option sold. An inspection of a typical binomial tree shows that delta changes during the life of an option. This means that to hedge a particular option position, we must change our holding in the underlying stock periodically.

Constructing binomial trees for valuing options on stock indices, currencies, and futures contracts is very similar to doing so for valuing options on stocks.

Further Reading

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APPENDIX

Derivation of the Black–Scholes–Merton Option-Pricing Formula from a Binomial Tree

One way of deriving the famous Black–Scholes–Merton result for valuing a European option on a non-dividend-paying stock is by allowing the number of time steps in a binomial tree to approach infinity.

Suppose that a tree with n time steps is used to value a European call option with strike price K and life T. Each step is of length T/n. If there have been j upward movements and $n-j$ downward movements on the tree, the final stock price is $S_0u^jd^{n-j}$, where u is the proportional up movement, d is the proportional down movement, and S_0 is the initial stock price. The payoff from a European call option is then

$$\max(S_0u^jd^{n-j} - K, 0)$$

From the properties of the binomial distribution, the probability of exactly j upward and $n-j$ downward movements is given by

$$\frac{n!}{(n-j)!j!}p^j(1-p)^{n-j}$$

It follows that the expected payoff from the call option is

$$\sum_{j=0}^{n}\frac{n!}{(n-j)!j!}p^j(1-p)^{n-j}\max(S_0u^jd^{n-j} - K, 0)$$

As the tree represents movements in a risk-neutral world, we can discount this at the risk-free rate r to obtain the option price:

$$c = e^{-rT}\sum_{j=0}^{n}\frac{n!}{(n-j)!j!}p^j(1-p)^{n-j}\max(S_0u^jd^{n-j} - K, 0) \quad \textbf{(4.19)}$$

The terms in Equation (4.19) are nonzero when the final stock price is greater than the strike price, that is, when

$$S_0u^jd^{n-j} > K$$

or

$$\ln(S_0/K) > -j\ln(u) - (n-j)\ln(d)$$

Since $u = e^{\sigma\sqrt{T/n}}$ and $d = e^{-\sigma\sqrt{T/n}}$, this condition becomes

$$\ln(S_0/K) > n\sigma\sqrt{T/n} - 2j\sigma\sqrt{T/n}$$

or

$$j > \frac{n}{2} - \frac{\ln(S_0/K)}{2\sigma\sqrt{T/n}}$$

Equation (4.19) can therefore be written

$$c = e^{-rT}\sum_{j>\alpha}\frac{n!}{(n-j)!j!}p^j(1-p)^{n-j}(S_0u^jd^{n-j} - K)$$

where

$$\alpha = \frac{n}{2} - \frac{\ln(S_0/K)}{2\sigma\sqrt{T/n}}$$

For convenience, we define

$$U_1 = \sum_{j>\alpha}\frac{n!}{(n-j)!j!}p^j(1-p)^{n-j}u^jd^{n-j} \quad \textbf{(4.20)}$$

and

$$U_2 = \sum_{j>\alpha}\frac{n!}{(n-j)!j!}p^j(1-p)^{n-j} \quad \textbf{(4.21)}$$

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so that

$$c = e^{-rT}(S_0U_1 - KU_2) \tag{4.22}$$

Consider first U_2. As is well known, the binomial distribution approaches a normal distribution as the number of trials approaches infinity. Specifically, when there are n trials and p is the probability of success, the probability distribution of the number of successes is approximately normal with mean np and standard deviation $\sqrt{np(1-p)}$. The variable U_2 in Equation (4.21) is the probability of the number of successes being more than α. From the properties of the normal distribution, it follows that, for large n,

$$U_2 = N\left(\frac{np-\alpha}{\sqrt{np(1-p)}}\right) \tag{4.23}$$

where N is the cumulative probability distribution function for a standard normal variable. Substituting for α, we obtain

$$U_2 = N\left(\frac{\ln(S_0/K)}{2\sigma\sqrt{T}\sqrt{p(1-p)}} + \frac{\sqrt{n}\left(p-\frac{1}{2}\right)}{\sqrt{p(1-p)}}\right) \tag{4.24}$$

From Equations (4.15) to (4.18), we have

$$p = \frac{e^{rT/n} - e^{-\sigma\sqrt{T/n}}}{e^{\sigma\sqrt{T/n}} - e^{-\sigma\sqrt{T/n}}}$$

By expanding the exponential functions in a series, we see that, as n tends to infinity, $p(1-p)$ tends to ¼ and $\sqrt{n}(p-½)$ tends to

$$\frac{(r-\sigma^2/2)\sqrt{T}}{2\sigma}$$

so that in the limit, as n tends to infinity, Equation (4.24) becomes

$$U_2 = N\left(\frac{\ln(S_0/K)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right) \tag{4.25}$$

We now move on to evaluate U_1. From Equation (4.20), we have

$$U_1 = \sum_{j>\alpha}\frac{n!}{(n-j)!j!}(pu)^j[(1-p)d]^{n-j} \tag{4.26}$$

Define

$$p^* = \frac{pu}{pu+(1-p)d} \tag{4.27}$$

It then follows that

$$1-p^* = \frac{(1-p)d}{pu+(1-p)d}$$

and we can write Equation (4.26) as

$$U_1 = [pu+(1-p)d]^n\sum_{j>\alpha}\frac{n!}{(n-j)!j!}(p^*)^j(1-p^*)^{n-j}$$

Since the expected rate of return in the risk-neutral world is the risk-free rate r, it follows that $pu + (1-p)d = e^{rT/n}$ and

$$U_1 = e^{rT}\sum_{j>\alpha}\frac{n!}{(n-j)!j!}(p^*)^j(1-p^*)^{n-j}$$

This shows that U_1 involves a binomial distribution where the probability of an up movement is p^* rather than p. Approximating the binomial distribution with a normal distribution, we obtain, similarly to Equation (4.23),

$$U_1 = e^{rT}N\left(\frac{np^*-\alpha}{\sqrt{np^*(1-p^*)}}\right)$$

and substituting for α gives, as with Equation (4.24),

$$U_1 = e^{rT}N\left(\frac{\ln(S_0/K)}{2\sigma\sqrt{T}\sqrt{p^*(1-p^*)}} + \frac{\sqrt{n}\left(p^*-\frac{1}{2}\right)}{\sqrt{p^*(1-p^*)}}\right)$$

Substituting for u and d in Equation (4.27) gives

$$p^* = \left(\frac{e^{rT/n} - e^{-\sigma\sqrt{T/n}}}{e^{\sigma\sqrt{T/n}} - e^{-\sigma\sqrt{T/n}}}\right)\left(\frac{e^{\sigma\sqrt{T/n}}}{e^{rT/n}}\right)$$

By expanding the exponential functions in a series we see that, as n tends to infinity, $p^*(1-p^*)$ tends to ¼ and $\sqrt{n}(p^*-½)$ tends to

$$\frac{(r+\sigma^2/2)\sqrt{T}}{2\sigma}$$

with the result that

$$U_1 = e^{rT}N\left(\frac{\ln(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}\right) \tag{4.28}$$

From Equations (4.22), (4.25), and (4.28), we have

$$c = S_0N(d_1) - Ke^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}$$

and

$$d_2 = \frac{\ln(S_0/K)+(r-\sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

This is the Black–Scholes–Merton formula for the valuation of a European call option. It will be discussed in Chapter 5. An alternative derivation is given in the appendix to that chapter.

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The Black-Scholes-Merton Model

5

Learning Objectives

After completing this reading you should be able to:

- Explain the lognormal property of stock prices, the distribution of rates of return, and the calculation of expected return.
- Compute the realized return and historical volatility of a stock.
- Describe the assumptions underlying the Black-Scholes-Merton option pricing model.
- Compute the value of a European option using the Black-Scholes-Merton model on a non-dividend-paying stock.
- Compute the value of a warrant and identify the complications involving the valuation of warrants.
- Define implied volatilities and describe how to compute implied volatilities from market prices of options using the Black-Scholes-Merton model.
- Explain how dividends affect the decision to exercise early for American call and put options.
- Compute the value of a European option using the Black-Scholes-Merton model on a dividend-paying stock.

Excerpt is Chapter 15 of Options, Futures, and Other Derivatives, *Ninth Edition, by John C. Hull.*

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In the early 1970s, Fischer Black, Myron Scholes, and Robert Merton achieved a major breakthrough in the pricing of European stock options.[1] This was the development of what has become known as the Black-Scholes-Merton (or Black-Scholes) model. The model has had a huge influence on the way that traders price and hedge derivatives. In 1997, the importance of the model was recognized when Robert Merton and Myron Scholes were awarded the Nobel prize for economics. Sadly, Fischer Black died in 1995; otherwise he too would undoubtedly have been one of the recipients of this prize.

How did Black, Scholes, and Merton make their breakthrough? Previous researchers had made the similar assumptions and had correctly calculated the expected payoff from a European option. However, as explained in Chapter 4, it is difficult to know the correct discount rate to use for this payoff. Black and Scholes used the capital asset pricing model to determine a relationship between the market's required return on the option and the required return on the stock. This was not easy because the relationship depends on both the stock price and time. Merton's approach was different from that of Black and Scholes. It involved setting up a riskless portfolio consisting of the option and the underlying stock and arguing that the return on the portfolio over a short period of time must be the risk-free return. This is similar to what we did in Chapter 4—but more complicated because the portfolio changes continuously through time. Merton's approach was more general than that of Black and Scholes because it did not rely on the assumptions of the capital asset pricing model.

This chapter covers Merton's approach to deriving the Black-Scholes-Merton model. It explains how volatility can be either estimated from historical data or implied from option prices using the model. It shows how the risk-neutral valuation argument introduced in Chapter 4 can be used. It also shows how the Black-Scholes-Merton model can be extended to deal with European call and put options on dividend-paying stocks and presents some results on the pricing of American call options on dividend-paying stocks.

LOGNORMAL PROPERTY OF STOCK PRICES

The model of stock price behavior used by Black, Scholes, and Merton assumes that percentage changes in the stock price in a very short period of time are normally distributed. Define

μ: Expected return on stock per year

σ: Volatility of the stock price per year.

The mean and standard deviation of the return in time Δt are approximately $\mu\,\Delta t$ and $\sigma\sqrt{\Delta t}$ so that

$$\frac{\Delta S}{S} \sim \phi(\mu\Delta t,\ \sigma^2\Delta t) \tag{5.1}$$

where ΔS is the change in the stock price S in time Δt, and $\phi(m, v)$ denotes a normal distribution with mean m and variance v.

The model implies that

$$\ln S_T - \ln S_0 \sim \phi\left[\left(\mu - \frac{\sigma^2}{2}\right)T,\ \sigma^2 T\right]$$

so that

$$\ln\frac{S_T}{S_0} \sim \phi\left[\left(\mu - \frac{\sigma^2}{2}\right)T,\ \sigma^2 T\right] \tag{5.2}$$

and

$$\ln S_T \sim \phi\left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T,\ \sigma^2 T\right] \tag{5.3}$$

where S_T is the stock price at a future time T and S_0 is the stock price at time 0. There is no approximation here. The variable $\ln S_T$ is normally distributed, so that S_T has a lognormal distribution. The mean of $\ln S_T$ is $\ln S_0 + (\mu - \sigma^2/2)T$ and the standard deviation of $\ln S_T$ is $\sigma\sqrt{T}$.

Example 5.1

Consider a stock with an initial price of \$40, an expected return of 16% per annum, and a volatility of 20% per annum. From Equation (5.3), the probability distribution of the stock price S_T in 6 months' time is given by

$$\ln S_T \sim \phi[\ln 40 + (0.16 - 0.2^2/2) \times 0.5,\ 0.2^2 \times 0.5]$$

$$\ln S_T \sim \phi(3.759,\ 0.02)$$

[1] See F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81 (May/June 1973): 637-59; R. C. Merton, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, 4 (Spring 1973): 141-83.

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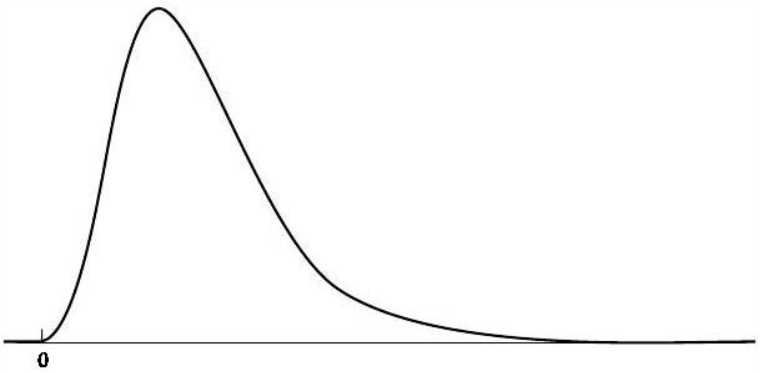

FIGURE 5-1 Lognormal distribution.

There is a 95% probability that a normally distributed variable has a value within 1.96 standard deviations of its mean. In this case, the standard deviation is $\sqrt{0.02} = 0.141$. Hence, with 95% confidence,

$$3.759 - 1.96 \times 0.141 < \ln S_T < 3.759 + 1.96 \times 0.141$$

This can be written

$$e^{3.759-1.96\times0.141} < S_T < e^{3.759+1.96\times0.141}$$

or

$$32.55 < S_T < 56.56$$

Thus, there is a 95% probability that the stock price in 6 months will lie between 32.55 and 56.56.

A variable that has a lognormal distribution can take any value between zero and infinity. Figure 5-1 illustrates the shape of a lognormal distribution. Unlike the normal distribution, it is skewed so that the mean, median, and mode are all different. From Equation (5.3) and the properties of the lognormal distribution, it can be shown that the expected value $E(S_T)$ of S_T is given by

$$E(S_T) = S_0 e^{\mu T} \tag{5.4}$$

This fits in with the definition of μ as the expected rate of return. The variance $\text{var}(S_T)$ of S_T, can be shown to be given by[2]

$$\text{var}(S_T) = S_0^2 e^{2\mu T}(e^{\sigma^2 T} - 1) \tag{5.5}$$

[2] See Technical Note 2 at www.rotman.utoronto.ca/~hull/TechnicalNotes for a proof of the results in Equations (5.4) and (5.5). For a more extensive discussion of the properties of the lognormal distribution, see J. Aitchison and J. A. C. Brown, *The Lognormal Distribution*. Cambridge University Press, 1966.

Example 5.2

Consider a stock where the current price is \$20, the expected return is 20% per annum, and the volatility is 40% per annum. The expected stock price, $E(S_T)$, and the variance of the stock price, $\text{var}(S_T)$, in 1 year are given by

$$E(S_T) = 20e^{0.2\times1} = 24.43 \quad \text{and}$$
$$\text{var}(S_T) = 400e^{2\times0.2\times1}(e^{0.4^2\times1} - 1) = 103.54$$

The standard deviation of the stock price in 1 year is $\sqrt{103.54}$, or 10.18.

THE DISTRIBUTION OF THE RATE OF RETURN

The lognormal property of stock prices can be used to provide information on the probability distribution of the continuously compounded rate of return earned on a stock between times 0 and T. If we define the continuously compounded rate of return per annum realized between times 0 and T as x, then

$$S_T = S_0 e^{xT}$$

so that

$$x = \frac{1}{T}\ln\frac{S_T}{S_0} \tag{5.6}$$

From Equation (5.2), it follows that

$$x \sim \phi\left(\mu - \frac{\sigma^2}{2}, \frac{\sigma^2}{T}\right) \tag{5.7}$$

Thus, the continuously compounded rate of return per annum is normally distributed with mean $\mu - \sigma^2/2$ and standard deviation $\sigma/\sqrt{T}$. As T increases, the standard deviation of x declines. To understand the reason for this, consider two cases: $T = 1$ and $T = 20$. We are more certain about the average return per year over 20 years than we are about the return in any one year.

Example 5.3

Consider a stock with an expected return of 17% per annum and a volatility of 20% per annum. The probability distribution for the average rate of return (continuously compounded) realized over 3 years is normal, with mean

$$0.17 - \frac{0.2^2}{2} = 0.15$$

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or 15% per annum, and standard deviation

$$\sqrt{\frac{0.2^2}{3}} = 0.1155$$

or 11.55% per annum. Because there is a 95% chance that a normally distributed variable will lie within 1.96 standard deviations of its mean, we can be 95% confident that the average return realized over 3 years will be between $15 - 1.96 \times 11.55 = -7.6\%$ and $15 + 1.96 \times 11.55 = +37.6\%$ per annum.

THE EXPECTED RETURN

The expected return, μ, required by investors from a stock depends on the riskiness of the stock. The higher the risk, the higher the expected return. It also depends on the level of interest rates in the economy. The higher the level of interest rates, the higher the expected return required on any given stock. Fortunately, we do not have to concern ourselves with the determinants of μ in any detail. It turns out that the value of a stock option, when expressed in terms of the value of the underlying stock, does not depend on μ at all. Nevertheless, there is one aspect of the expected return from a stock that frequently causes confusion and needs to be explained.

Our model of stock price behavior implies that, in a very short period of time, the mean return is $\mu\,\Delta t$. It is natural to assume from this that μ is the expected continuously compounded return on the stock. However, this is not the case. The continuously compounded return, x, actually realized over a period of time of length T is given by Equation (5.6) as

$$x = \frac{1}{T}\ln\frac{S_T}{S_0}$$

and, as indicated in Equation (5.7), the expected value $E(x)$ of x is $\mu - \sigma^2/2$.

The reason why the expected continuously compounded return is different from μ is subtle, but important. Suppose we consider a very large number of very short periods of time of length Δt. Define S_i as the stock price at the end of the ith interval and ΔS_i as $S_{i+1} - S_i$. Under the assumptions we are making for stock price behavior, the average of the returns on the stock in each interval is close to μ. In other words, $\mu\,\Delta t$ is close to the arithmetic mean of the $\Delta S_i/S_i$. However, the expected return over the whole period covered by the data, expressed with a compounding interval of Δt, is close to $\mu - \sigma^2/2$, not μ.[3] Box 5-1 provides a numerical example concerning the mutual fund industry to illustrate why this is so.

BOX 5-1 Mutual Fund Returns Can Be Misleading

The difference between μ and $\mu - \sigma^2/2$ is closely related to an issue in the reporting of mutual fund returns. Suppose that the following is a sequence of returns per annum reported by a mutual fund manager over the last five years (measured using annual compounding): 15%, 20%, 30%, −20%, 25%.

The arithmetic mean of the returns, calculated by taking the sum of the returns and dividing by 5, is 14%. However, an investor would actually earn less than 14% per annum by leaving the money invested in the fund for 5 years. The dollar value of $100 at the end of the 5 years would be

$$100 \times 1.15 \times 1.20 \times 1.30 \times 0.80 \times 1.25 = \$179.40$$

By contrast, a 14% return with annual compounding would give

$$100 \times 1.14^5 = \$192.54$$

The return that gives $179.40 at the end of five years is 12.4%. This is because

$$100 \times (1.124)^5 = 179.40$$

What average return should the fund manager report? It is tempting for the manager to make a statement such as: "The average of the returns per year that we have realized in the last 5 years is 14%." Although true, this is misleading. It is much less misleading to say: "The average return realized by someone who invested with us for the last 5 years is 12.4% per year." In some jurisdictions, regulations require fund managers to report returns the second way.

This phenomenon is an example of a result that is well known in mathematics. The geometric mean of a set of numbers is always less than the arithmetic mean. In our example, the return multipliers each year are 1.15, 1.20, 1.30, 0.80, and 1.25. The arithmetic mean of these numbers is 1.140, but the geometric mean is only 1.124 and it is the geometric mean that equals 1 plus the return realized over the 5 years.

[3] The arguments in this section show that the term "expected return" is ambiguous. It can refer either to μ or to $\mu - \sigma^2/2$. Unless otherwise stated, it will be used to refer to μ throughout this book.

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For another explanation of what is going on, we start with Equation (5.4):

$$E(S_T) = S_0 e^{\mu T}$$

Taking logarithms, we get

$$\ln[E(S_T)] = \ln(S_0) + \mu T$$

It is now tempting to set $\ln[E(S_T)] = E[\ln(S_T)]$, so that $E[\ln(S_T)] - \ln(S_0) = \mu T$, or $E[\ln(S_T/S_0)] = \mu T$, which leads to $E(x) = \mu$. However, we cannot do this because ln is a nonlinear function. In fact, $\ln[E(S_T)] > E[\ln(S_T)]$, so that $E[\ln(S_T/S_0)] < \mu T$, which leads to $E(x) < \mu$. (As pointed out above, $E(x) = \mu - \sigma^2/2$.)

VOLATILITY

The volatility, σ, of a stock is a measure of our uncertainty about the returns provided by the stock. Stocks typically have a volatility between 15% and 60%.

From Equation (5.7), the volatility of a stock price can be defined as the standard deviation of the return provided by the stock in 1 year when the return is expressed using continuous compounding.

When Δt is small, Equation (5.1) shows that $\sigma^2 \Delta t$ is approximately equal to the variance of the percentage change in the stock price in time Δt. This means that $\sigma\sqrt{\Delta t}$ is approximately equal to the standard deviation of the percentage change in the stock price in time Δt. Suppose that $\sigma = 0.3$, or 30%, per annum and the current stock price is \$50. The standard deviation of the percentage change in the stock price in 1 week is approximately

$$30 \times \sqrt{\frac{1}{52}} = 4.16\%$$

A 1-standard-deviation move in the stock price in 1 week is therefore $50 \times 0.0416 = 2.08$.

Uncertainty about a future stock price, as measured by its standard deviation, increases—at least approximately—with the square root of how far ahead we are looking. For example, the standard deviation of the stock price in 4 weeks is approximately twice the standard deviation in 1 week.

Estimating Volatility from Historical Data

To estimate the volatility of a stock price empirically, the stock price is usually observed at fixed intervals of time (e.g., every day, week, or month). Define:

$n + 1$: Number of observations

S_i: Stock price at end of ith interval, with $i = 0, 1, \ldots, n$

τ: Length of time interval in years

and let

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \quad \text{for } i = 1, 2, \ldots, n$$

The usual estimate, s, of the standard deviation of the u_i is given by

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2}$$

or

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}u_i\right)^2}$$

where $\bar{u}$ is the mean of the u_i.[4]

From Equation (5.2), the standard deviation of the u_i is $\sigma\sqrt{\tau}$. The variable s is therefore an estimate of $\sigma\sqrt{\tau}$. It follows that σ itself can be estimated as $\hat{\sigma}$, where

$$\hat{\sigma} = \frac{s}{\sqrt{\tau}}$$

The standard error of this estimate can be shown to be approximately $\hat{\sigma}/\sqrt{2n}$.

Choosing an appropriate value for n is not easy. More data generally lead to more accuracy, but σ does change over time and data that are too old may not be relevant for predicting the future volatility. A compromise that seems to work reasonably well is to use closing prices from daily data over the most recent 90 to 180 days. Alternatively, as a rule of thumb, n can be set equal to the number of days to which the volatility is to be applied. Thus, if the volatility estimate is to be used to value a 2-year option, daily data for the last 2 years are used.

[4] The mean $\bar{u}$ is often assumed to be zero when estimates of historical volatilities are made.

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Example 5.4

Table 5-1 shows a possible sequence of stock prices during 21 consecutive trading days. In this case, $n = 20$, so that

$$\sum_{i=1}^{n} u_i = 0.09531 \quad \text{and} \quad \sum_{i=1}^{n} u_i^2 = 0.00326$$

and the estimate of the standard deviation of the daily return is

$$\sqrt{\frac{0.00326}{19} - \frac{0.09531^2}{20 \times 19}} = 0.01216$$

or 1.216%. Assuming that there are 252 trading days per year, $\tau = 1/252$ and the data give an estimate for the volatility per annum of $0.01216\sqrt{252} = 0.193$ or 19.3%. The standard error of this estimate is

$$\frac{0.193}{\sqrt{2 \times 20}} = 0.031$$

or 3.1% per annum.

The foregoing analysis assumes that the stock pays no dividends, but it can be adapted to accommodate

TABLE 5-1 Computation of Volatility

Day i	Closing Stock Price (dollars), S_i	Price Relative S_i/S_{i-1}	Daily Return $u_i = \ln(S_i/S_{i-1})$
0	20.00		
1	20.10	1.00500	0.00499
2	19.90	0.99005	−0.01000
3	20.00	1.00503	0.00501
4	20.50	1.02500	0.02469
5	20.25	0.98780	−0.01227
6	20.90	1.03210	0.03159
7	20.90	1.00000	0.00000
8	20.90	1.00000	0.00000
9	20.75	0.99282	−0.00720
10	20.75	1.00000	0.00000
11	21.00	1.01205	0.01198
12	21.10	1.00476	0.00475
13	20.90	0.99052	−0.00952
14	20.90	1.00000	0.00000
15	21.25	1.01675	0.01661
16	21.40	1.00706	0.00703
17	21.40	1.00000	0.00000
18	21.25	0.99299	−0.00703
19	21.75	1.02353	0.02326
20	22.00	1.01149	0.01143

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BOX 5-2 What Causes Volatility?

It is natural to assume that the volatility of a stock is caused by new information reaching the market. This new information causes people to revise their opinions about the value of the stock. The price of the stock changes and volatility results. This view of what causes volatility is not supported by research. With several years of daily stock price data, researchers can calculate:

1. The variance of stock price returns between the close of trading on one day and the close of trading on the next day when there are no intervening nontrading days
2. The variance of the stock price returns between the close of trading on Friday and the close of trading on Monday

The second of these is the variance of returns over a 3-day period. The first is a variance over a 1-day period. We might reasonably expect the second variance to be three times as great as the first variance. Fama (1965), French (1980), and French and Roll (1986) show that this is not the case. These three research studies estimate the second variance to be, respectively, 22%, 19%, and 10.7% higher than the first variance.

At this stage one might be tempted to argue that these results are explained by more news reaching the market when the market is open for trading. But research by Roll (1984) does not support this explanation. Roll looked at the prices of orange juice futures. By far the most important news for orange juice futures prices is news about the weather and this is equally likely to arrive at any time. When Roll did a similar analysis to that just described for stocks, he found that the second (Friday-to-Monday) variance for orange juice futures is only 1.54 times the first variance.

The only reasonable conclusion from all this is that volatility is to a large extent caused by trading itself. (Traders usually have no difficulty accepting this conclusion!)

dividend-paying stocks. The return, u_i, during a time interval that includes an ex-dividend day is given by

$$u_i = \ln\frac{S_i + D}{S_{i-1}}$$

where D is the amount of the dividend. The return in other time intervals is still

$$u_i = \ln\frac{S_i}{S_{i-1}}$$

However, as tax factors play a part in determining returns around an ex-dividend date, it is probably best to discard altogether data for intervals that include an ex-dividend date.

Trading Days vs. Calendar Days

An important issue is whether time should be measured in calendar days or trading days when volatility parameters are being estimated and used. As shown in Box 5-2, research shows that volatility is much higher when the exchange is open for trading than when it is closed. As a result, practitioners tend to ignore days when the exchange is closed when estimating volatility from historical data and when calculating the life of an option. The volatility per annum is calculated from the volatility per trading day using the formula

$$\text{Volatility per annum} = \text{Volatility per trading day} \times \sqrt{\text{Number of trading days per annum}}$$

This is what we did in Example 5.4 when calculating volatility from the data in Table 5-1. The number of trading days in a year is usually assumed to be 252 for stocks.

The life of an option is also usually measured using trading days rather than calendar days. It is calculated as T years, where

$$T = \frac{\text{Number of trading days until option maturity}}{252}$$

THE IDEA UNDERLYING THE BLACK-SCHOLES-MERTON DIFFERENTIAL EQUATION

The Black-Scholes-Merton differential equation is an equation that must be satisfied by the price of any derivative dependent on a non-dividend-paying stock. The equation is derived in the next section. Here we consider the nature of the arguments we will use.

These are similar to the no-arbitrage arguments we used to value stock options in Chapter 4 for the situation where stock price movements were assumed to be binomial. They involve setting up a riskless portfolio consisting of a position in the derivative and a position in the stock. In the absence of arbitrage opportunities, the return from the portfolio must be the risk-free interest rate, r. This leads to the Black-Scholes-Merton differential equation.

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The reason a riskless portfolio can be set up is that the stock price and the derivative price are both affected by the same underlying source of uncertainty: stock price movements. In any short period of time, the price of the derivative is perfectly correlated with the price of the underlying stock. When an appropriate portfolio of the stock and the derivative is established, the gain or loss from the stock position always offsets the gain or loss from the derivative position so that the overall value of the portfolio at the end of the short period of time is known with certainty.

Suppose, for example, that at a particular point in time the relationship between a small change ΔS in the stock price and the resultant small change Δc in the price of a European call option is given by

$$\Delta c = 0.4\,\Delta S$$

This means that the slope of the line representing the relationship between c and S is 0.4, as indicated in Figure 5-2. A riskless portfolio would consist of:

1. A long position in 40 shares
2. A short position in 100 call options.

Suppose, for example, that the stock price increases by 10 cents. The option price will increase by 4 cents and the 40 × 0.1 = \$4 gain on the shares is equal to the 100 × 0.04 = \$4 loss on the short option position.

There is one important difference between the Black-Scholes-Merton analysis and our analysis using a binomial model in Chapter 4. In Black-Scholes-Merton, the position in the stock and the derivative is riskless for only a very short period of time. (Theoretically, it remains riskless only for an instantaneously short period of time.) To remain riskless, it must be adjusted, or *rebalanced*, frequently.[5] For example, the relationship between Δc and ΔS in our example might change from $\Delta c = 0.4\,\Delta S$ today to $\Delta c = 0.5\,\Delta S$ tomorrow. This would mean that, in order to maintain the riskless position, an extra 10 shares would have to be purchased for each 100 call options sold. It is nevertheless true that the return from the riskless portfolio in any very short period of time must be the risk-free interest rate. This is the key element in the Black-Scholes-Merton analysis and leads to their pricing formulas.

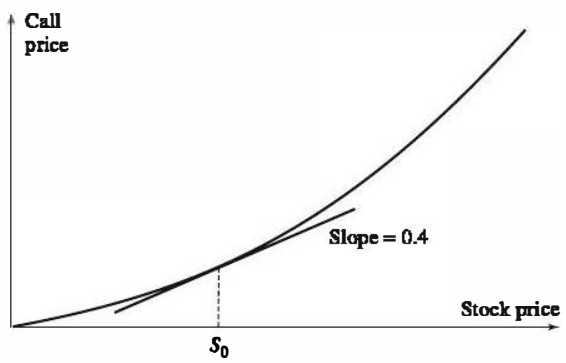

FIGURE 5-2 Relationship between call price and stock price. Current stock price is S_0.

Assumptions

The assumptions we use to derive the Black-Scholes-Merton differential equation are as follows:

1. The stock price follows the process with μ and σ constant.
2. The short selling of securities with full use of proceeds is permitted.
3. There are no transaction costs or taxes. All securities are perfectly divisible.
4. There are no dividends during the life of the derivative.
5. There are no riskless arbitrage opportunities.
6. Security trading is continuous.
7. The risk-free rate of interest, r, is constant and the same for all maturities.

As we discuss in later chapters, some of these assumptions can be relaxed. For example, σ and r can be known functions of t. We can even allow interest rates to be stochastic provided that the stock price distribution at maturity of the option is still lognormal.

DERIVATION OF THE BLACK-SCHOLES-MERTON DIFFERENTIAL EQUATION

In this section, the notation is different from elsewhere in the book. We consider a derivative's price at a general time t (not at time zero). If T is the maturity date, the time to maturity is $T - t$.

[5] We discuss the rebalancing of portfolios in more detail in Chapter 6.

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The stock price process we are assuming is:

$$dS = \mu S\,dt + \sigma S\,dz \tag{5.8}$$

Suppose that f is the price of a call option or other derivative contingent on S. The variable f must be some function of S and t. Hence,

$$df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial f}{\partial S}\sigma S\,dz \tag{5.9}$$

The discrete versions of Equations (5.8) and (5.9) are

$$\Delta S = \mu S\,\Delta t + \sigma S\,\Delta z \tag{5.10}$$

and

$$\Delta f = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t + \frac{\partial f}{\partial S}\sigma S\,\Delta z \tag{5.11}$$

where Δf and ΔS are the changes in f and S in a small time interval Δt. The Wiener processes underlying f and S are the same. In other words, the $\Delta z\left(=\epsilon\sqrt{\Delta t}\right)$ in Equations (5.10) and (5.11) are the same. It follows that a portfolio of the stock and the derivative can be constructed so that the Wiener process is eliminated. The portfolio is

-1: derivative

$+\partial f/\partial S$: shares.

The holder of this portfolio is short one derivative and long an amount $\partial f/\partial S$ of shares. Define Π as the value of the portfolio. By definition

$$\Pi = -f + \frac{\partial f}{\partial S}S \tag{5.12}$$

The change $\Delta\Pi$ in the value of the portfolio in the time interval Δt is given by

$$\Delta\Pi = -\Delta f + \frac{\partial f}{\partial S}\Delta S \tag{5.13}$$

Substituting Equations (5.10) and (5.11) into Equation (5.13) yields

$$\Delta\Pi = \left(-\frac{\partial f}{\partial t} - \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t \tag{5.14}$$

Because this equation does not involve Δz, the portfolio must be riskless during time Δt. The assumptions listed in the preceding section imply that the portfolio must instantaneously earn the same rate of return as other short-term risk-free securities. If it earned more than this return, arbitrageurs could make a riskless profit by borrowing money to buy the portfolio; if it earned less, they could make a riskless profit by shorting the portfolio and buying risk-free securities. It follows that

$$\Delta\Pi = r\Pi\,\Delta t \tag{5.15}$$

where r is the risk-free interest rate. Substituting from Equations (5.12) and (5.14) into (5.15), we obtain

$$\left(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t = r\left(f - \frac{\partial f}{\partial S}S\right)\Delta t$$

so that

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2} = rf \tag{5.16}$$

Equation (5.16) is the Black–Scholes–Merton differential equation. It has many solutions, corresponding to all the different derivatives that can be defined with S as the underlying variable. The particular derivative that is obtained when the equation is solved depends on the *boundary conditions* that are used. These specify the values of the derivative at the boundaries of possible values of S and t. In the case of a European call option, the key boundary condition is

$$f = \max(S - K, 0) \qquad \text{when } t = T$$

In the case of a European put option, it is

$$f = \max(K - S, 0) \qquad \text{when } t = T$$

Example 5.5

A forward contract on a non-dividend-paying stock is a derivative dependent on the stock. As such, it should satisfy Equation (5.16). We know that the value of the forward contract, f, at a general time t is given in terms of the stock price S at this time by

$$f = S - Ke^{-r(T-t)}$$

where K is the delivery price. This means that

$$\frac{\partial f}{\partial t} = -rKe^{-r(T-t)}, \qquad \frac{\partial f}{\partial S} = 1, \qquad \frac{\partial^2 f}{\partial S^2} = 0$$

When these are substituted into the left-hand side of Equation (5.16), we obtain

$$-rKe^{-r(T-t)} + rS$$

This equals rf, showing that Equation (5.16) is indeed satisfied.

A Perpetual Derivative

Consider a perpetual derivative that pays off a fixed amount Q when the stock price equals H for the first time. In this case, the value of the derivative for a particular S has no dependence on t, so the $\partial f/\partial t$ term vanishes and

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the partial differential Equation (5.16) becomes an ordinary differential equation.

Suppose first that $S < H$. The boundary conditions for the derivatives are $f = 0$ when $S = 0$ and $f = Q$ when $S = H$. The simple solution $f = QS/H$ satisfies both the boundary conditions and the differential equation. It must therefore be the value of the derivative.

Suppose next that $S > H$. The boundary conditions are now $f = 0$ as S tends to infinity and $f = Q$ when $S = H$. The derivative price

$$f = Q\left(\frac{S}{H}\right)^{-\alpha}$$

where α is positive, satisfies the boundary conditions. It also satisfies the differential equation when

$$-r\alpha + \frac{1}{2}\sigma^2\alpha(\alpha+1) - r = 0$$

or $\alpha = 2r/\sigma^2$. The value of the derivative is therefore

$$f = Q\left(\frac{S}{H}\right)^{-2r/\sigma^2} \tag{5.17}$$

The Prices of Tradeable Derivatives

Any function $f(S, t)$ that is a solution of the differential Equation (5.16) is the theoretical price of a derivative that could be traded. If a derivative with that price existed, it would not create any arbitrage opportunities. Conversely, if a function $f(S, t)$ does not satisfy the differential Equation (5.16), it cannot be the price of a derivative without creating arbitrage opportunities for traders.

To illustrate this point, consider first the function e^S. This does not satisfy the differential Equation (5.16). It is therefore not a candidate for being the price of a derivative dependent on the stock price. If an instrument whose price was always e^S existed, there would be an arbitrage opportunity. As a second example, consider the function

$$\frac{e^{(\sigma^2-2r)(T-t)}}{S}$$

This does satisfy the differential equation, and so is, in theory, the price of a tradeable security. (It is the price of a derivative that pays off $1/S_T$ at time T.).

RISK-NEUTRAL VALUATION

We introduced risk-neutral valuation in connection with the binomial model in Chapter 4. It is without doubt the single most important tool for the analysis of derivatives. It arises from one key property of the Black-Scholes-Merton differential Equation (5.16). This property is that the equation does not involve any variables that are affected by the risk preferences of investors. The variables that do appear in the equation are the current stock price, time, stock price volatility, and the risk-free rate of interest. All are independent of risk preferences.

The Black-Scholes-Merton differential equation would not be independent of risk preferences if it involved the expected return, μ, on the stock. This is because the value of μ does depend on risk preferences. The higher the level of risk aversion by investors, the higher μ will be for any given stock. It is fortunate that μ happens to drop out in the derivation of the differential equation.

Because the Black-Scholes-Merton differential equation is independent of risk preferences, an ingenious argument can be used. If risk preferences do not enter the equation, they cannot affect its solution. Any set of risk preferences can, therefore, be used when evaluating f. In particular, the very simple assumption that all investors are risk neutral can be made.

In a world where investors are risk neutral, the expected return on all investment assets is the risk-free rate of interest, r. The reason is that risk-neutral investors do not require a premium to induce them to take risks. It is also true that the present value of any cash flow in a risk-neutral world can be obtained by discounting its expected value at the risk-free rate. The assumption that the world is risk neutral does, therefore, considerably simplify the analysis of derivatives.

Consider a derivative that provides a payoff at one particular time. It can be valued using risk-neutral valuation by using the following procedure:

1. Assume that the expected return from the underlying asset is the risk-free interest rate, r (i.e., assume $\mu = r$).
2. Calculate the expected payoff from the derivative.
3. Discount the expected payoff at the risk-free interest rate.

It is important to appreciate that risk-neutral valuation (or the assumption that all investors are risk neutral) is merely

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an artificial device for obtaining solutions to the Black–Scholes–Merton differential equation. The solutions that are obtained are valid in all worlds, not just those where investors are risk neutral. When we move from a risk-neutral world to a risk-averse world, two things happen. The expected growth rate in the stock price changes and the discount rate that must be used for any payoffs from the derivative changes. It happens that these two changes always offset each other exactly.

Application to Forward Contracts on a Stock

In Example 5.5, we verified that the pricing formula satisfies the Black–Scholes–Merton differential equation. In this section we derive the pricing formula from risk-neutral valuation. We make the assumption that interest rates are constant and equal to r.

Consider a long forward contract that matures at time T with delivery price, K. The value of the contract at maturity is

$$S_T - K$$

where S_T is the stock price at time T. From the risk-neutral valuation argument, the value of the forward contract at time 0 is its expected value at time T in a risk-neutral world discounted at the risk-free rate of interest. Denoting the value of the forward contract at time zero by f, this means that

$$f = e^{-rT}\hat{E}(S_T - K)$$

where $\hat{E}$ denotes the expected value in a risk-neutral world. Since K is a constant, this equation becomes

$$f = e^{-rT}\hat{E}(S_T) - Ke^{-rT} \tag{5.18}$$

The expected return μ on the stock becomes r in a risk-neutral world. Hence, from Equation (5.4), we have

$$\hat{E}(S_T) = S_0e^{rT} \tag{5.19}$$

Substituting Equation (5.19) into Equation (5.18) gives

$$f = S_0 - Ke^{-rT}$$

BLACK–SCHOLES–MERTON PRICING FORMULAS

The most famous solutions to the differential Equation (5.16) are the Black–Scholes–Merton formulas for the prices of European call and put options. These formulas are:

$$c = S_0N(d_1) - Ke^{-rT}N(d_2) \tag{5.20}$$

and

$$p = Ke^{-rT}N(-d_2) - S_0N(-d_1) \tag{5.21}$$

where

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The function $N(x)$ is the cumulative probability distribution function for a variable with a standard normal distribution. In other words, it is the probability that a variable with a standard normal distribution will be less than x. It is illustrated in Figure 5-3. The remaining variables should be familiar. The variables c and p are the European call and European put price, S_0 is the stock price at time zero, K is the strike price, r is the continuously compounded risk-free rate, σ is the stock price volatility, and T is the time to maturity of the option.

One way of deriving the Black–Scholes–Merton formulas is by solving the differential Equation (5.16) subject to the boundary condition mentioned earlier.[6] Another approach is to use risk-neutral valuation. Consider a European call

[6] The differential equation gives the call and put prices at a general time t. For example, the call price that satisfies the differential equation is $c = SN(d_1) - Ke^{-r(T-t)}N(d_2)$, where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}$$

and $d_2 = d_1 - \sigma\sqrt{T - t}$

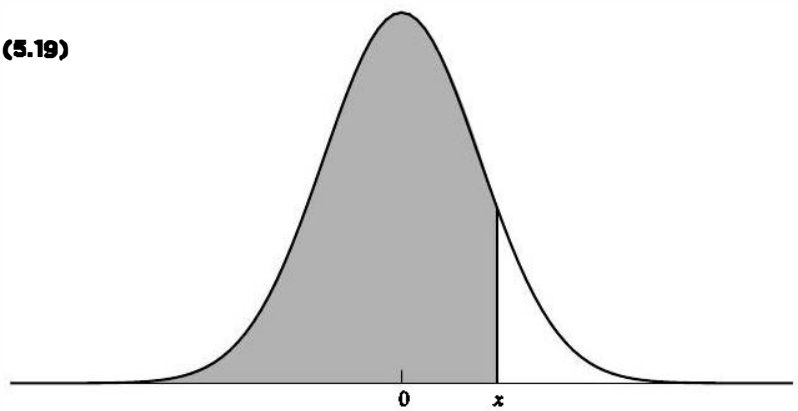

FIGURE 5-3 Shaded area represents $N(x)$.

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option. The expected value of the option at maturity in a risk-neutral world is

$$\hat{E}[\max(S_T - K, 0)]$$

where, as before, $\hat{E}$ denotes the expected value in a risk-neutral world. From the risk-neutral valuation argument, the European call option price c is this expected value discounted at the risk-free rate of interest, that is,

$$c = e^{-rT}\hat{E}[\max(S_T - K, 0)] \quad \textbf{(5.22)}$$

The appendix at the end of this chapter shows that this equation leads to the result in Equation (5.20).

Since it is never optimal to exercise early an American call option on a non-dividend-paying stock, Equation (5.20) is the value of an American call option on a non-dividend-paying stock. Unfortunately, no exact analytic formula for the value of an American put option on a non-dividend-paying stock has been produced.

When the Black–Scholes–Merton formula is used in practice the interest rate r is set equal to the zero-coupon risk-free interest rate for a maturity T. As we show in later chapters, this is theoretically correct when r is a known function of time. It is also theoretically correct when the interest rate is stochastic provided that the stock price at time T is lognormal and the volatility parameter is chosen appropriately. As mentioned earlier, time is normally measured as the number of trading days left in the life of the option divided by the number of trading days in 1 year.

Understanding $N(d_1)$ and $N(d_2)$

The term $N(d_2)$ in Equation (5.20) has a fairly simple interpretation. It is the probability that a call option will be exercised in a risk-neutral world. The $N(d_1)$ term is not quite so easy to interpret. The expression $S_0N(d_1)e^{rT}$ is the expected stock price at time T in a risk-neutral world when stock prices less than the strike price are counted as zero. The strike price is only paid if the stock price is greater than K and as just mentioned this has a probability of $N(d_2)$. The expected payoff in a risk-neutral world is therefore

$$S_0N(d_1)e^{rT} - KN(d_2)$$

Present-valuing this from time T to time zero gives the Black–Scholes–Merton equation for a European call option:

$$c = S_0N(d_1) - Ke^{-rT}N(d_2)$$

For another interpretation, note that the Black–Scholes–Merton equation for the value of a European call option can be written as

$$c = e^{-rT}N(d_2)[S_0e^{rT}N(d_1)/N(d_2) - K]$$

The terms here have the following interpretation:

e^{-rT}: Present value factor

$N(d_2)$: Probability of exercise

$e^{rT}N(d_1)/N(d_2)$: Expected percentage increase in stock price in a risk-neutral world if option is exercised

K: Strike price paid if option is exercised.

Properties of the Black–Scholes–Merton Formulas

We now show that the Black–Scholes–Merton formulas have the right general properties by considering what happens when some of the parameters take extreme values.

When the stock price, S_0, becomes very large, a call option is almost certain to be exercised. It then becomes very similar to a forward contract with delivery price K.

We expect the call price to be

$$S_0 - Ke^{-rT}$$

This is, in fact, the call price given by Equation (5.20) because, when S_0 becomes very large, both d_1 and d_2 become very large, and $N(d_1)$ and $N(d_2)$ become close to 1.0. When the stock price becomes very large, the price of a European put option, p, approaches zero. This is consistent with Equation (5.21) because $N(-d_1)$ and $N(-d_2)$ are both close to zero in this case.

Consider next what happens when the volatility σ approaches zero. Because the stock is virtually riskless, its price will grow at rate r to S_0e^{rT} at time T and the payoff from a call option is

$$\max(S_0e^{rT} - K, 0)$$

Discounting at rate r, the value of the call today is

$$e^{-rT}\max(S_0e^{rT} - K, 0) = \max(S_0 - Ke^{-rT}, 0)$$

To show that this is consistent with Equation (5.20), consider first the case where $S_0 > Ke^{-rT}$. This implies that $\ln(S_0/K) + rT > 0$. As σ tends to zero, d_1 and d_2 tend to $+\infty$, so that $N(d_1)$ and $N(d_2)$ tend to 1.0 and Equation (5.20) becomes

$$c = S_0 - Ke^{-rT}$$

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When $S_0 < Ke^{-rT}$, it follows that $\ln(S_0/K) + rT < 0$. As σ tends to zero, d_1 and d_2 tend to $-\infty$, so that $N(d_1)$ and $N(d_2)$ tend to zero and Equation (5.20) gives a call price of zero. The call price is therefore always $\max(S_0 - Ke^{-rT}, 0)$ as σ tends to zero. Similarly, it can be shown that the put price is always $\max(Ke^{-rT} - S_0, 0)$ as σ tends to zero.

CUMULATIVE NORMAL DISTRIBUTION FUNCTION

When implementing Equations (5.20) and (5.21), it is necessary to evaluate the cumulative normal distribution function $N(x)$. Tables for $N(x)$ are provided at the end of the FRM Exam Part 1 Quantitative Analysis book. The NORMSDIST function in Excel also provides a convenient way of calculating $N(x)$.

Example 5.6

The stock price 6 months from the expiration of an option is \$42, the exercise price of the option is \$40, the risk-free interest rate is 10% per annum, and the volatility is 20% per annum. This means that $S_0 = 42$, $K = 40$, $r = 0.1$, $\sigma = 0.2$, $T = 0.5$,

$$d_1 = \frac{\ln(42/40) + (0.1 + 0.2^2/2) \times 0.5}{0.2\sqrt{0.5}} = 0.7693$$

$$d_2 = \frac{\ln(42/40) + (0.1 - 0.2^2/2) \times 0.5}{0.2\sqrt{0.5}} = 0.6278$$

and

$$Ke^{-rT} = 40e^{-0.05} = 38.049$$

Hence, if the option is a European call, its value c is given by

$$c = 42N(0.7693) - 38.049N(0.6278)$$

If the option is a European put, its value p is given by

$$p = 38.049N(-0.6278) - 42N(-0.7693)$$

Using the NORMSDIST function in Excel gives

$$N(0.7693) = 0.7791, \qquad N(-0.7693) = 0.2209$$

$$N(0.6278) = 0.7349, \qquad N(-0.6278) = 0.2651$$

so that

$$c = 4.76, \qquad p = 0.81$$

Ignoring the time value of money, the stock price has to rise by \$2.76 for the purchaser of the call to break even. Similarly, the stock price has to fall by \$2.81 for the purchaser of the put to break even.

WARRANTS AND EMPLOYEE STOCK OPTIONS

The exercise of a regular call option on a company has no effect on the number of the company's shares outstanding. If the writer of the option does not own the company's shares, he or she must buy them in the market in the usual way and then sell them to the option holder for the strike price. Warrants and employee stock options are different from regular call options in that exercise leads to the company issuing more shares and then selling them to the option holder for the strike price. As the strike price is less than the market price, this dilutes the interest of the existing shareholders.

How should potential dilution affect the way we value outstanding warrants and employee stock options? The answer is that it should not! Assuming markets are efficient the stock price will reflect potential dilution from all outstanding warrants and employee stock options. This is explained in Box 5-3.[7]

Consider next the situation a company is in when it is contemplating a new issue of warrants (or employee stock options). We suppose that the company is interested in calculating the cost of the issue assuming that there are no compensating benefits. We assume that the company has N shares worth S_0 each and the number of new options contemplated is M, with each option giving the holder the right to buy one share for K. The value of the company today is NS_0. This value does not change as a result of the warrant issue. Suppose that without the warrant issue the share price will be S_T at the warrant's maturity. This means that (with or without the warrant issue) the total value of the equity and the warrants at time T will be NS_T. If the warrants are exercised, there is a cash inflow from the strike price increasing this to $NS_T + MK$. This value is distributed among $N + M$ shares, so that the share price immediately after exercise becomes

$$\frac{NS_T + MK}{N + M}$$

[7] Analysts sometimes assume that the sum of the values of the warrants and the equity (rather than just the value of the equity) is lognormal. The result is a Black–Scholes type of equation for the value of the warrant in terms of the value of the warrant. See Technical Note 3 at www.rotman.utoronto.ca/~hull/TechnicalNotes for an explanation of this model.

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BOX 5-3 Warrants, Employee Stock Options, and Dilution

Consider a company with 100,000 shares each worth $50. It surprises the market with an announcement that it is granting 100,000 stock options to its employees with a strike price of $50. If the market sees little benefit to the shareholders from the employee stock options in the form of reduced salaries and more highly motivated managers, the stock price will decline immediately after the announcement of the employee stock options. If the stock price declines to $45, the dilution cost to the current shareholders is $5 per share or $500,000 in total.

Suppose that the company does well so that by the end of three years the share price is $100. Suppose further that all the options are exercised at this point. The payoff to the employees is $50 per option. It is tempting to argue that there will be further dilution in that 100,000 shares worth $100 per share are now merged with 100,000 shares for which only $50 is paid, so that (a) the share price reduces to $75 and (b) the payoff to the option holders is only $25 per option. However, this argument is flawed. The exercise of the options is anticipated by the market and already reflected in the share price. The payoff from each option exercised is $50.

This example illustrates the general point that when markets are efficient the impact of dilution from executive stock options or warrants is reflected in the stock price as soon as they are announced and does not need to be taken into account again when the options are valued.

Therefore the payoff to an option holder if the option is exercised is

$$\frac{NS_T + MK}{N+M} - K$$

or

$$\frac{N}{N+M}(S_T - K)$$

This shows that the value of each option is the value of

$$\frac{N}{N+M}$$

regular call options on the company's stock. Therefore the total cost of the options is M times this. Since we are assuming that there are no benefits to the company from the warrant issue, the total value of the company's equity will decline by the total cost of the options as soon as the decision to issue the warrants becomes generally known. This means that the reduction in the stock price is

$$\frac{M}{N+M}$$

times the value of a regular call option with strike price K and maturity T.

Example 5.7

A company with 1 million shares worth $40 each is considering issuing 200,000 warrants each giving the holder the right to buy one share with a strike price of $60 in 5 years. It wants to know the cost of this. The interest rate is 3% per annum, and the volatility is 30% per annum. The company pays no dividends. From Equation (5.20), the value of a 5-year European call option on the stock is $7.04. In this case, N = 1,000,000 and M = 200,000, so that the value of each warrant is

$$\frac{1{,}000{,}000}{1{,}000{,}000 + 200{,}000} \times 7.04 = 5.87$$

or $5.87. The total cost of the warrant issue is 200,000 × 5.87 = $1.17 million. Assuming the market perceives no benefits from the warrant issue, we expect the stock price to decline by $1.17 to $38.83.

IMPLIED VOLATILITIES

The one parameter in the Black–Scholes–Merton pricing formulas that cannot be directly observed is the volatility of the stock price. Earlier, we discussed how this can be estimated from a history of the stock price. In practice, traders usually work with what are known as *implied volatilities*. These are the volatilities implied by option prices observed in the market.[8]

To illustrate how implied volatilities are calculated, suppose that the value of a European call option on a nondividend-paying stock is 1.875 when S_0 = 21; K = 20, r = 0.1, and T = 0.25. The implied volatility is the value of σ that, when substituted into Equation (5.20), gives c = 1.875. Unfortunately, it is not possible to invert Equation (5.20) so that σ is expressed as a function of S_0, K,

[8] Implied volatilities for European and American options can be calculated using DerivaGem.

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r, T, and c. However, an iterative search procedure can be used to find the implied σ. For example, we can start by trying $\sigma = 0.20$. This gives a value of c equal to 1.76, which is too low. Because c is an increasing function of σ, a higher value of σ is required. We can next try a value of 0.30 for σ. This gives a value of c equal to 2.10, which is too high and means that σ must lie between 0.20 and 0.30. Next, a value of 0.25 can be tried for σ. This also proves to be too high, showing that σ lies between 0.20 and 0.25. Proceeding in this way, we can halve the range for σ at each iteration and the correct value of σ can be calculated to any required accuracy.[9] In this example, the implied volatility is 0.235, or 23.5%, per annum. A similar procedure can be used in conjunction with binomial trees to find implied volatilities for American options.

Implied volatilities are used to monitor the market's opinion about the volatility of a particular stock. Whereas historical volatilities are backward looking, implied volatilities are forward looking. Traders often quote the implied volatility of an option rather than its price. This is convenient because the implied volatility tends to be less variable than the option price. The implied volatilities of actively traded options are used by traders to estimate appropriate implied volatilities for other options.

The VIX Index

The CBOE publishes indices of implied volatility. The most popular index, the SPX VIX, is an index of the implied volatility of 30-day options on the S&P 500 calculated from a wide range of calls and puts.[10] It is sometimes referred to as the "fear factor." An index value of 15 indicates that the implied volatility of 30-day options on the S&P 500 is estimated as 15%. Trading in futures on the VIX started in 2004 and trading in options on the VIX started in 2006. One contract is on 1,000 times the index.

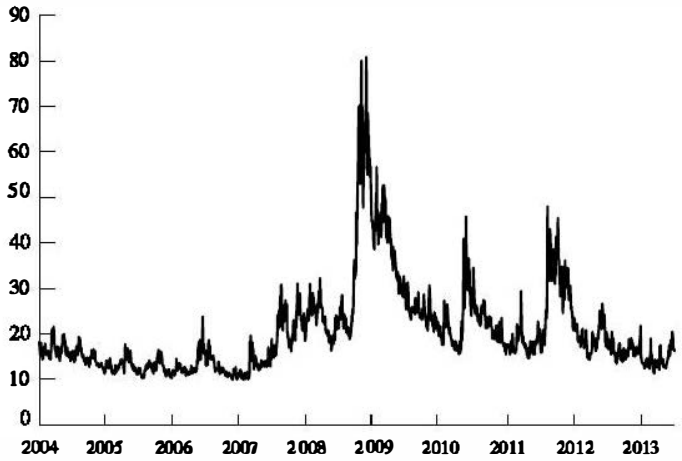

FIGURE 5-4 The VIX Index, January 2004 to June 2013.

Example 5.8

Suppose that a trader buys an April futures contract on the VIX when the futures price is 18.5 (corresponding to a 30-day S&P 500 volatility of 18.5%) and closes out the contract when the futures price is 19.3 (corresponding to an S&P 500 volatility of 19.3%). The trader makes a gain of $800.

A trade involving futures or options on the S&P 500 is a bet on both the future level of the S&P 500 and the volatility of the S&P 500. By contrast, a futures or options contract on the VIX is a bet only on volatility. Figure 5-4 shows the VIX index between January 2004 and June 2013. Between 2004 and mid-2007 it tended to stay between 10 and 20. It reached 30 during the second half of 2007 and a record 80 in October and November 2008 after Lehman's bankruptcy. By early 2010, it had declined to a more normal level, but it spiked again in May 2010 and the second half of 2011 because of stresses and uncertainties in financial markets.

DIVIDENDS

Up to now, we have assumed that the stock on which the option is written pays no dividends. In this section, we modify the Black-Scholes-Merton model to take account of dividends. We assume that the amount and timing of the dividends during the life of an option can be predicted

[9] This method is presented for illustration. Other more powerful methods, such as the Newton-Raphson method, are often used in practice.

[10] Similarly, the VXN is an index of the volatility of the NASDAQ 100 index and the VXD is an index of the volatility of the Dow Jones Industrial Average.

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with certainty. When options last for relatively short periods of time, this assumption is not too unreasonable. (For long-life options it is usual to assume that the dividend yield rather than the dollar dividend payments are known. Options can then be valued.) The date on which the dividend is paid should be assumed to be the ex-dividend date. On this date the stock price declines by the amount of the dividend.[11]

European Options

European options can be analyzed by assuming that the stock price is the sum of two components: a riskless component that corresponds to the known dividends during the life of the option and a risky component. The riskless component, at any given time, is the present value of all the dividends during the life of the option discounted from the ex-dividend dates to the present at the risk-free rate. By the time the option matures, the dividends will have been paid and the riskless component will no longer exist. The Black–Scholes–Merton formula is therefore correct if S_0 is equal to the risky component of the stock price and σ is the volatility of the process followed by the risky component.[12]

Operationally, this means that the Black–Scholes–Merton formulas can be used provided that the stock price is reduced by the present value of all the dividends during the life of the option, the discounting being done from the ex-dividend dates at the risk-free rate. As already mentioned, a dividend is counted as being during the life of the option only if its ex-dividend date occurs during the life of the option.

[11] For tax reasons the stock price may go down by somewhat less than the cash amount of the dividend. To take account of this phenomenon, we need to interpret the word 'dividend' in the context of option pricing as the reduction in the stock price on the ex-dividend date caused by the dividend. Thus, if a dividend of $1 per share is anticipated and the share price normally goes down by 80% of the dividend on the ex-dividend date, the dividend should be assumed to be $0.80 for the purpose of the analysis.

[12] This is not quite the same as the volatility of the whole stock price. (In theory, they cannot both follow geometric Brownian motion.) At time zero, the volatility of the risky component is approximately equal to the volatility of the whole stock price multiplied by $S_0/(S_0 - D)$, where D is the present value of the dividends.

Example 5.9

Consider a European call option on a stock when there are ex-dividend dates in two months and five months. The dividend on each ex-dividend date is expected to be $0.50. The current share price is $40, the exercise price is $40, the stock price volatility is 30% per annum, the risk-free rate of interest is 9% per annum, and the time to maturity is six months. The present value of the dividends is

$$0.5e^{-0.09\times 2/12} + 0.5e^{-0.09\times 5/12} = 0.9742$$

The option price can therefore be calculated from the Black–Scholes–Merton formula, with $S_0 = 40 - 0.9742 = 39.0258$, $K = 40$, $r = 0.09$, $\sigma = 0.3$, and $T = 0.5$:

$$d_1 = \frac{\ln(39.0258/40) + (0.09 + 0.3^2/2)\times 0.5}{0.3\sqrt{0.5}} = 0.2020$$

$$d_2 = \frac{\ln(39.0258/40) + (0.09 - 0.3^2/2)\times 0.5}{0.3\sqrt{0.5}} = -0.0102$$

Using the NORMSDIST function in Excel gives

$$N(d_1) = 0.5800, \qquad N(d_2) = 0.4959$$

and, from Equation (5.20), the call price is

$$39.0258 \times 0.5800 - 40e^{-0.09\times 0.5} \times 0.4959 = 3.67$$

or $3.67.

Some researchers have criticized the approach just described for calculating the value of a European option on a dividend-paying stock. They argue that volatility should be applied to the stock price, not to the stock price less the present value of dividends. A number of different numerical procedures have been suggested for doing this.[13] When volatility is calculated from historical data, it might make sense to use one of these procedures. However, in practice the volatility used to price an option is nearly always implied from the prices of other options. If an analyst uses the same model for both implying and applying volatilities, the resulting prices should be accurate and not highly model dependent. Another important point is that in practice, practitioners usually value a European option in terms of the forward price of the underlying asset. This avoids the need to

[13] See, for example, N. Areal and A. Rodrigues, "Fast Trees for Options with Discrete Dividends," *Journal of Derivatives*, 21, 1 (Fall 2013), 49–63.

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estimate explicitly the income that is expected from the asset. The volatility of the forward stock price is the same as the volatility of the stock price minus the present value of dividends.

The model we have proposed where the stock price is divided into two components is internally consistent and widely used in practice.

American Call Options

Consider next American call options. In the absence of dividends American options should never be exercised early. An extension to the argument shows that, when there are dividends, it can only be optimal to exercise at a time immediately before the stock goes ex-dividend. We assume that n ex-dividend dates are anticipated and that they are at times $t_1, t_2, \ldots, t_n$, with $t_1 < t_2 < \cdots < t_n$. The dividends corresponding to these times will be denoted by $D_1, D_2, \ldots, D_n$, respectively.

We start by considering the possibility of early exercise just prior to the final ex-dividend date (i.e., at time t_n). If the option is exercised at time t_n, the investor receives

$$S(t_n) - K$$

where $S(t)$ denotes the stock price at time t. If the option is not exercised, the stock price drops to $S(t_n) - D_n$. The value of the option is then greater than

$$S(t_n) - D_n - Ke^{-r(T-t_n)}$$

It follows that, if

$$S(t_n) - D_n - Ke^{-r(T-t_n)} \geq S(t_n) - K$$

that is,

$$D_n \leq K[1 - e^{-r(T-t_n)}] \qquad \textbf{(5.23)}$$

it cannot be optimal to exercise at time t_n. On the other hand, if

$$D_n > K[1 - e^{-r(T-t_n)}] \qquad \textbf{(5.24)}$$

for any reasonable assumption about the stochastic process followed by the stock price, it can be shown that it is always optimal to exercise at time t_n for a sufficiently high value of $S(t_n)$. The inequality in Equation (5.24) will tend to be satisfied when the final ex-dividend date is fairly close to the maturity of the option (i.e., $T - t_n$ is small) and the dividend is large.

Consider next time t_{n-1}, the penultimate ex-dividend date. If the option is exercised immediately prior to time t_{n-1}, the investor receives $S(t_{n-1}) - K$. If the option is not exercised at time t_{n-1}, the stock price drops to $S(t_{n-1}) - D_{n-1}$ and the earliest subsequent time at which exercise could take place is t_n. Hence, a lower bound to the option price if it is not exercised at time t_{n-1} is

$$S(t_{n-1}) - D_{n-1} - Ke^{-r(t_n - t_{n-1})}$$

It follows that if

$$S(t_{n-1}) - D_{n-1} - Ke^{-r(t_n - t_{n-1})} \geq S(t_{n-1}) - K$$

or

$$D_{n-1} \leq K[1 - e^{-r(t_n - t_{n-1})}]$$

it is not optimal to exercise immediately prior to time t_{n-1}. Similarly, for any $i < n$, if

$$D_i \leq K[1 - e^{-r(t_{i+1} - t_i)}] \qquad \textbf{(5.25)}$$

it is not optimal to exercise immediately prior to time t_i.

The inequality in Equation (5.25) is approximately equivalent to

$$D_i \leq Kr(t_{i+1} - t_i)$$

Assuming that K is fairly close to the current stock price, this inequality is satisfied when the dividend yield on the stock is less than the risk-free rate of interest. This is often the case.

We can conclude from this analysis that, in many circumstances, the most likely time for the early exercise of an American call is immediately before the final ex-dividend date, t_n. Furthermore, if inequality in Equation (5.25) holds for $i = 1, 2, \ldots, n - 1$ and inequality in Equation (5.23) holds, we can be certain that early exercise is never optimal, and the American option can be treated as a European option.

Black's Approximation

Black suggests an approximate procedure for taking account of early exercise in call options.[14] This involves calculating, as described earlier in this section, the prices of European options that mature at times T and t_n, and then setting the American price equal to the greater of the two.[15] This is an approximation because it in effect

[14] See F. Black, "Fact and Fantasy in the Use of Options," *Financial Analysts Journal*, 31 (July/August 1975): 36–41, 61–72.

[15] For an exact formula, suggested by Roll, Geske, and Whaley, for valuing American calls when there is only one ex-dividend date, see Technical Note 4 at www.rotman.utoronto.ca/~hull/TechnicalNotes. This involves the cumulative bivariate normal distribution function. A procedure for calculating this function is given in Technical Note 5 and a worksheet for calculating the cumulative bivariate normal distribution can be found on the author's website.

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assumes the option holder has to decide at time zero whether the option will be exercised at time T or t_n.

SUMMARY

We started this chapter by examining the properties of the process for stock prices. The process implies that the price of a stock at some future time, given its price today, is lognormal. It also implies that the continuously compounded return from the stock in a period of time is normally distributed. Our uncertainty about future stock prices increases as we look further ahead. The standard deviation of the logarithm of the stock price is proportional to the square root of how far ahead we are looking.

To estimate the volatility σ of a stock price empirically, the stock price is observed at fixed intervals of time (e.g., every day, every week, or every month). For each time period, the natural logarithm of the ratio of the stock price at the end of the time period to the stock price at the beginning of the time period is calculated. The volatility is estimated as the standard deviation of these numbers divided by the square root of the length of the time period in years. Usually, days when the exchanges are closed are ignored in measuring time for the purposes of volatility calculations.

The differential equation for the price of any derivative dependent on a stock can be obtained by creating a riskless portfolio of the derivative and the stock. Because the derivative's price and the stock price both depend on the same underlying source of uncertainty, this can always be done. The portfolio that is created remains riskless for only a very short period of time. However, the return on a riskless portfolio must always be the risk-free interest rate if there are to be no arbitrage opportunities.

The expected return on the stock does not enter into the Black–Scholes–Merton differential equation. This leads to an extremely useful result known as risk-neutral valuation. This result states that when valuing a derivative dependent on a stock price, we can assume that the world is risk neutral. This means that we can assume that the expected return from the stock is the risk-free interest rate, and then discount expected payoffs at the risk-free interest rate. The Black–Scholes–Merton equations for European call and put options can be derived by either solving their differential equation or by using risk-neutral valuation.

An implied volatility is the volatility that, when used in conjunction with the Black–Scholes–Merton option pricing formula, gives the market price of the option. Traders monitor implied volatilities. They often quote the implied volatility of an option rather than its price. They have developed procedures for using the volatilities implied by the prices of actively traded options to estimate volatilities for other options.

The Black–Scholes–Merton results can be extended to cover European call and put options on dividend-paying stocks. The procedure is to use the Black–Scholes–Merton formula with the stock price reduced by the present value of the dividends anticipated during the life of the option, and the volatility equal to the volatility of the stock price net of the present value of these dividends.

In theory, it can be optimal to exercise American call options immediately before any ex-dividend date. In practice, it is often only necessary to consider the final ex-dividend date. Fischer Black has suggested an approximation. This involves setting the American call option price equal to the greater of two European call option prices. The first European call option expires at the same time as the American call option; the second expires immediately prior to the final ex-dividend date.

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APPENDIX

Proof of the Black–Scholes–Merton Formula Using Risk-Neutral Valuation

We will prove the Black–Scholes result by first proving another key result.

Key Result

If V is lognormally distributed and the standard deviation of ln V is w, then

$$E[\max(V-K,0)] = E(V)N(d_1) - KN(d_2) \qquad \textbf{(5.26)}$$

where

$$d_1 = \frac{\ln[E(V)/K] + w^2/2}{w}$$

$$d_2 = \frac{\ln[E(V)/K] - w^2/2}{w}$$

and E denotes the expected value.

Proof of Key Result

Define $g(V)$ as the probability density function of V. It follows that

$$E[\max(V-K,0)] = \int_K^\infty (V-K)g(V)dV \qquad \textbf{(5.27)}$$

The variable ln V is normally distributed with standard deviation w. From the properties of the lognormal distribution, the mean of ln V is m, where[16]

$$m = \ln[E(V)] - w^2/2 \qquad \textbf{(5.28)}$$

Define a new variable

$$Q = \frac{\ln V - m}{w} \qquad \textbf{(5.29)}$$

This variable is normally distributed with a mean of zero and a standard deviation of 1.0. Denote the density function for Q by $h(Q)$ so that

$$h(Q) = \frac{1}{\sqrt{2\pi}} e^{-Q^2/2}$$

Using Equation (5.29) to convert the expression on the right-hand side of Equation (5.27) from an integral over V to an integral over Q, we get

$$E[\max(V-K,0)] = \int_{(\ln K-m)/w}^\infty (e^{Qw+m} - K)h(Q)dQ$$

or

$$E[\max(V-K,0)] = \int_{(\ln K-m)/w}^\infty e^{Qw+m}h(Q)dQ - K\int_{(\ln K-m)/w}^\infty h(Q)dQ \qquad \textbf{(5.30)}$$

Now

$$e^{Qw+m}h(Q) = \frac{1}{\sqrt{2\pi}}e^{(-Q^2+2Qw+2m)/2} = \frac{1}{\sqrt{2\pi}}e^{[-(Q-w)^2+2m+w^2]/2}$$

$$= \frac{e^{m+w^2/2}}{\sqrt{2\pi}}e^{[-(Q-w)^2]/2} = e^{m+w^2/2}h(Q-w)$$

[16] For a proof of this, see Technical Note 2 at www.rotman.utoronto.ca/~hull/TechnicalNotes.

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This means that Equation (5.30) becomes

$$E[\max(V-K,0)] = e^{m+w^2/2}\int_{(\ln K-m)/w}^{\infty} h(Q-w)dQ - K\int_{(\ln K-m)/w}^{\infty} h(Q)dQ \qquad (5.31)$$

If we define $N(x)$ as the probability that a variable with a mean of zero and a standard deviation of 1.0 is less than x, the first integral in Equation (5.31) is

$$1 - N[(\ln K - m)/w - w] = N[(-\ln K + m)/w + w]$$

Substituting for m from Equation (5.28) leads to

$$N\left(\frac{\ln[E(V)/K] + w^2/2}{w}\right) = N(d_1)$$

Similarly the second integral in Equation (5.31) is $N(d_2)$. Equation (5.31), therefore, becomes

$$E[\max(V-K,0)] = e^{m-w^2/2}N(d_1) - KN(d_2)$$

Substituting for m from Equation (5.28) gives the key result.

The Black-Scholes-Merton Result

We now consider a call option on a non-dividend-paying stock maturing at time T. The strike price is K, the risk-free rate is r, the current stock price is S_0, and the volatility is σ. As shown in Equation (5.22), the call price c is given by

$$c = e^{-rT}\hat{E}[\max(S_T - K, 0)] \qquad (5.32)$$

where S_T is the stock price at time T and $\hat{E}$ denotes the expectation in a risk-neutral world. Under the stochastic process assumed by Black-Scholes-Merton, S_T is log-normal. Also, from Equations (5.3) and (5.4), $\hat{E}(S_T) = S_0e^{rT}$ and the standard deviation of $\ln S_T$ is $\sigma\sqrt{T}$.

From the key result just proved, Equation (5.32) implies

$$c = e^{-rT}[S_0e^{rT}N(d_1) - KN(d_2)] = S_0N(d_1) - Ke^{-rT}N(d_2)$$

where

$$d_1 = \frac{\ln[\hat{E}(S_T)/K] + \sigma^2T/2}{\sigma\sqrt{T}} = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln[\hat{E}(S_T)/K] - \sigma^2T/2}{\sigma\sqrt{T}} = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}$$

This is the Black-Scholes-Merton result.

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The Greek Letters

6

Learning Objectives

After completing this reading you should be able to:

- Describe and assess the risks associated with naked and covered option positions.
- Explain how naked and covered option positions generate a stop loss trading strategy.
- Describe delta hedging for an option, forward, and futures contracts.
- Compute the delta of an option.
- Describe the dynamic aspects of delta hedging and distinguish between dynamic hedging and hedge-and-forget strategy.
- Define the delta of a portfolio.
- Define and describe theta, gamma, vega, and rho for option positions.
- Explain how to implement and maintain a delta-neutral and a gamma-neutral position.
- Describe the relationship between delta, theta, gamma, and vega.
- Describe how hedging activities take place in practice, and describe how scenario analysis can be used to formulate expected gains and losses with option positions.
- Describe how portfolio insurance can be created through option instruments and stock index futures.

Excerpt is Chapter 19 of Options, Futures, and Other Derivatives, *Ninth Edition, by John C. Hull.*

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A financial institution that sells an option to a client in the over-the-counter markets is faced with the problem of managing its risk. If the option happens to be the same as one that is traded on an exchange, the financial institution can neutralize its exposure by buying on the exchange the same option as it has sold. But when the option has been tailored to the needs of a client and does not correspond to the standardized products traded by exchanges, hedging the exposure is far more difficult.

In this chapter we discuss some of the alternative approaches to this problem. We cover what are commonly referred to as the "Greek letters", or simply the "Greeks". Each Greek letter measures a different dimension to the risk in an option position and the aim of a trader is to manage the Greeks so that all risks are acceptable. The analysis presented in this chapter is applicable to market makers in options on an exchange as well as to traders working in the over-the-counter market for financial institutions.

Toward the end of the chapter, we will consider the creation of options synthetically. This turns out to be very closely related to the hedging of options. Creating an option position synthetically is essentially the same task as hedging the opposite option position. For example, creating a long call option synthetically is the same as hedging a short position in the call option.

ILLUSTRATION

In the next few sections we use as an example the position of a financial institution that has sold for $300,000 a European call option on 100,000 shares of a non-dividend-paying stock. We assume that the stock price is $49, the strike price is $50, the risk-free interest rate is 5% per annum, the stock price volatility is 20% per annum, the time to maturity is 20 weeks (0.3846 years), and the expected return from the stock is 13% per annum.[1] With our usual notation, this means that

$$S_0 = 49,\ K = 50,\ r = 0.05,\ \sigma = 0.20,\ T = 0.3846,\ \mu = 0.13$$

The Black-Scholes-Merton price of the option is about $240,000. (This is because the value of an option to buy one share is $2.40.) The financial institution has therefore sold a product for $60,000 more than its theoretical value. But it is faced with the problem of hedging the risks.[2]

NAKED AND COVERED POSITIONS

One strategy open to the financial institution is to do nothing. This is sometimes referred to as a *naked position*. It is a strategy that works well if the stock price is below $50 at the end of the 20 weeks. The option then costs the financial institution nothing and it makes a profit of $300,000. A naked position works less well if the call is exercised because the financial institution then has to buy 100,000 shares at the market price prevailing in 20 weeks to cover the call. The cost to the financial institution is 100,000 times the amount by which the stock price exceeds the strike price. For example, if after 20 weeks the stock price is $60, the option costs the financial institution $1,000,000. This is considerably greater than the $300,000 charged for the option.

As an alternative to a naked position, the financial institution can adopt a *covered position*. This involves buying 100,000 shares as soon as the option has been sold. If the option is exercised, this strategy works well, but in other circumstances it could lead to a significant loss. For example, if the stock price drops to $40, the financial institution loses $900,000 on its stock position. This is considerably greater than the $300,000 charged for the option.[3]

Neither a naked position nor a covered position provides a good hedge. If the assumptions underlying the Black-Scholes-Merton formula hold, the cost to the financial institution should always be $240,000 on average for both approaches.[4] But on any one occasion the cost is liable to range from zero to over $1,000,000. A good hedge would ensure that the cost is always close to $240,000.

[1] As shown in Chapters 4 and 5, the expected return is irrelevant to the pricing of an option. It is given here because it can have some bearing on the effectiveness of a hedging scheme.

[2] A call option on a non-dividend-paying stock is a convenient example with which to develop our ideas. The points that will be made apply to other types of options and to other derivatives.

[3] Put-call parity shows that the exposure from writing a covered call is the same as the exposure from writing a naked put.

[4] More precisely, the present value of the expected cost is $240,000 for both approaches assuming that appropriate risk-adjusted discount rates are used.

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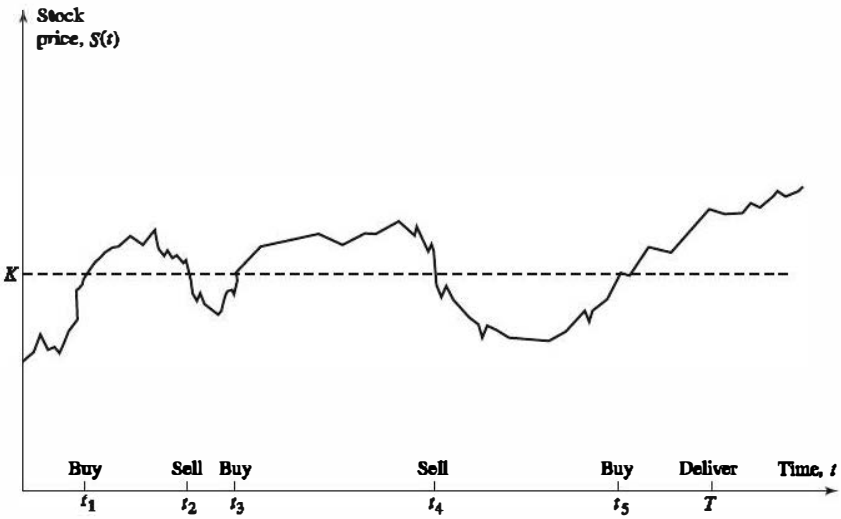

FIGURE 6-1 A stop-loss strategy.

A STOP-LOSS STRATEGY

One interesting hedging procedure that is sometimes proposed involves a *stop-loss strategy*. To illustrate the basic idea, consider an institution that has written a call option with strike price K to buy one unit of a stock. The hedging procedure involves buying one unit of the stock as soon as its price rises above K and selling it as soon as its price falls below K. The objective is to hold a naked position whenever the stock price is less than K and a covered position whenever the stock price is greater than K. The procedure is designed to ensure that at time T the institution owns the stock if the option closes in the money and does not own it if the option closes out of the money. In the situation illustrated in Figure 6-1, it involves buying the stock at time t_1, selling it at time t_2, buying it at time t_3, selling it at time t_4, buying it at time t_5, and delivering it at time T.

As usual, we denote the initial stock price by S_0. The cost of setting up the hedge initially is S_0 if $S_0 > K$ and zero otherwise. It seems as though the total cost, Q, of writing and hedging the option is the option's initial intrinsic value:

$$Q = \max(S_0 - K, 0) \tag{6.1}$$

This is because all purchases and sales subsequent to time 0 are made at price K. If this were in fact correct, the hedging procedure would work perfectly in the absence of transaction costs. Furthermore, the cost of hedging the option would always be less than its Black–Scholes–Merton price. Thus, an investor could earn riskless profits by writing options and hedging them.

There are two key reasons why Equation (6.1) is incorrect. The first is that the cash flows to the hedger occur at different times and must be discounted. The second is that purchases and sales cannot be made at exactly the same price K. This second point is critical. If we assume a risk-neutral world with zero interest rates, we can justify ignoring the time value of money. But we cannot legitimately assume that both purchases and sales are made at the same price. If markets are efficient, the hedger cannot know whether, when the stock price equals K, it will continue above or below K.

As a practical matter, purchases must be made at a price $K + \epsilon$ and sales must be made at a price $K - \epsilon$, for some small positive number ϵ. Thus, every purchase and subsequent sale involves a cost (apart from transaction costs) of 2ϵ. A natural response on the part of the hedger is to monitor price movements more closely, so that ϵ is reduced. Assuming that stock prices change continuously, ϵ can be made arbitrarily small by monitoring the stock prices closely. But as ϵ is made smaller, trades tend to occur more frequently. Thus, the lower cost per trade is offset by the increased frequency of trading. As $\epsilon \to 0$, the expected number of trades tends to infinity.[5]

A stop-loss strategy, although superficially attractive, does not work particularly well as a hedging procedure. Consider its use for an out-of-the-money option. If the stock price never reaches the strike price K, the hedging procedure costs nothing. If the path of the stock price crosses the strike price level many times, the procedure is quite expensive. Monte Carlo simulation can be used to assess the overall performance of stop-loss hedging. This involves randomly sampling paths for the stock price and observing the results of using the procedure. Table 6-1 shows the results for the option considered in the first section. It assumes that the stock price is observed at the end of time intervals of length Δt.[6] The hedge performance

[5] The expected number of times a Wiener process equals any particular value in a given time interval is infinite.

[6] The precise hedging rule used was as follows. If the stock price moves from below K to above K in a time interval of length Δt, it is bought at the end of the interval. If it moves from above K to below K in the time interval, it is sold at the end of the interval; otherwise, no action is taken.

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TABLE 6-1 Performance of Stop-Loss Strategy (The performance measure is the ratio of the standard deviation of the cost of writing the option and hedging it to the theoretical price of the option)

Δt (weeks)	5	4	2	1	0.5	0.25
Hedge performance	0.98	0.93	0.83	0.79	0.77	0.76

measure in Table 6-1 is the ratio of the standard deviation of the cost of hedging the option to the Black–Scholes–Merton price. (The cost of hedging was calculated as the cumulative cost excluding the impact of interest payments and discounting.) Each result is based on one million sample paths for the stock price. An effective hedging scheme should have a hedge performance measure close to zero. In this case, it seems to stay above 0.7 regardless of how small Δt is. This emphasizes that the stop-loss strategy is not a good hedging procedure.

DELTA HEDGING

Most traders use more sophisticated hedging procedures than those mentioned so far. These involve calculating measures such as delta, gamma, and vega. In this section we consider the role played by delta.

The *delta* (Δ) of an option was introduced in Chapter 4. It is defined as the rate of change of the option price with respect to the price of the underlying asset. It is the slope of the curve that relates the option price to the underlying asset price. Suppose that the delta of a call option on a stock is 0.6. This means that when the stock price changes by a small amount, the option price changes by about 60% of that amount. Figure 6-2 shows the relationship between a call price and the underlying stock price. When the stock price corresponds to point A, the option price corresponds to point B, and Δ is the slope of the line indicated. In general,

$$\Delta = \frac{\partial c}{\partial S}$$

where c is the price of the call option and S is the stock price.

Suppose that, in Figure 6-2, the stock price is \$100 and the option price is \$10. Imagine an investor who has sold call options to buy 2,000 shares of a stock (i.e., he or she has sold 20 call option contracts). The investor's position could be hedged by buying 0.6 × 2,000 = 1,200 shares. The gain (loss) on the stock position would then tend to offset the loss (gain) on the option position. For

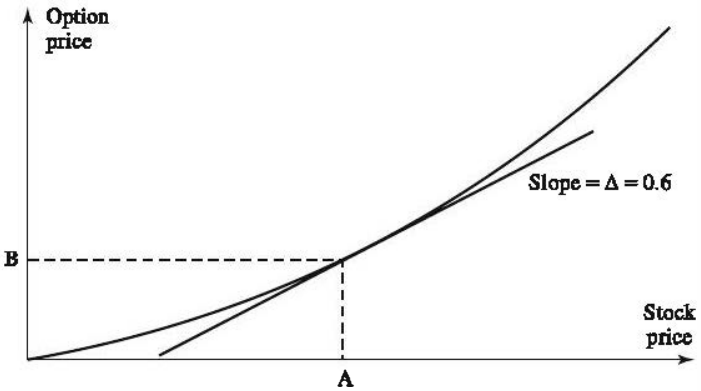

FIGURE 6-2 Calculation of delta.

example, if the stock price goes up by \$1 (producing a gain of \$1,200 on the shares purchased), the option price will tend to go up by 0.6 × \$1 = \$0.60 (producing a loss of \$1,200 on the options written); if the stock price goes down by \$1 (producing a loss of \$1,200 on the shares purchased), the option price will tend to go down by \$0.60 (producing a gain of \$1,200 on the options written).

In this example, the delta of the trader's short position in 2,000 options is

$$0.6 \times (-2{,}000) = -1{,}200$$

This means that the trader loses $1{,}200\Delta S$ on the option position when the stock price increases by ΔS. The delta of one share of the stock is 1.0, so that the long position in 1,200 shares has a delta of +1,200. The delta of the trader's overall position is, therefore, zero. The delta of the stock position offsets the delta of the option position. A position with a delta of zero is referred to as *delta neutral*.

It is important to realize that, since the delta of an option does not remain constant, the trader's position remains delta hedged (or delta neutral) for only a relatively short period of time. The hedge has to be adjusted periodically. This is known as *rebalancing*. In our example, by the end of 1 day the stock price might have increased to \$110. As indicated by Figure 6-2, an increase in the stock price leads to an increase in delta. Suppose that delta rises from 0.60 to 0.65. An extra 0.05 × 2,000 =

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100 shares would then have to be purchased to maintain the hedge. A procedure such as this, where the hedge is adjusted on a regular basis, is referred to as *dynamic hedging*. It can be contrasted with *static hedging*, where a hedge is set up initially and never adjusted. Static hedging is sometimes also referred to as "hedge-and-forget."

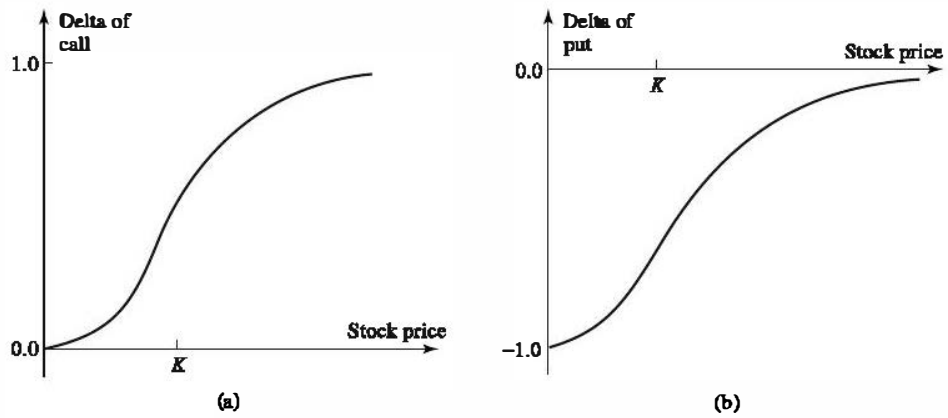

FIGURE 6-3 Variation of delta with stock price for (a) a call option and (b) a put option on a non-dividend-paying stock.

Delta is closely related to the Black–Scholes–Merton analysis. As explained in Chapter 5, the Black–Scholes–Merton differential equation can be derived by setting up a riskless portfolio consisting of a position in an option on a stock and a position in the stock. Expressed in terms of Δ, the portfolio is

−1: option

+Δ: shares of the stock.

Using our new terminology, we can say that options can be valued by setting up a delta-neutral position and arguing that the return on the position should (instantaneously) be the risk-free interest rate.

Delta of European Stock Options

For a European call option on a non-dividend-paying stock, it can be shown that

$$\Delta(\text{call}) = N(d_1)$$

where d_1 is defined as in Equation (5.20) and $N(x)$ is the cumulative distribution function for a standard normal distribution. The formula gives the delta of a long position in one call option. The delta of a short position in one call option is $-N(d_1)$. Using delta hedging for a short position in a European call option involves maintaining a long position of $N(d_1)$ for each option sold. Similarly, using delta hedging for a long position in a European call option involves maintaining a short position of $N(d_1)$ shares for each option purchased.

For a European put option on a non-dividend-paying stock, delta is given by

$$\Delta(\text{put}) = N(d_1) - 1$$

Delta is negative, which means that a long position in a put option should be hedged with a long position in the underlying stock, and a short position in a put option should be hedged with a short position in the underlying stock. Figure 6-3 shows the variation of the delta of a call option and a put option with the stock price. Figure 6-4 shows the variation of delta with the time to maturity for in-the-money, at-the-money, and out-of-the-money call options.

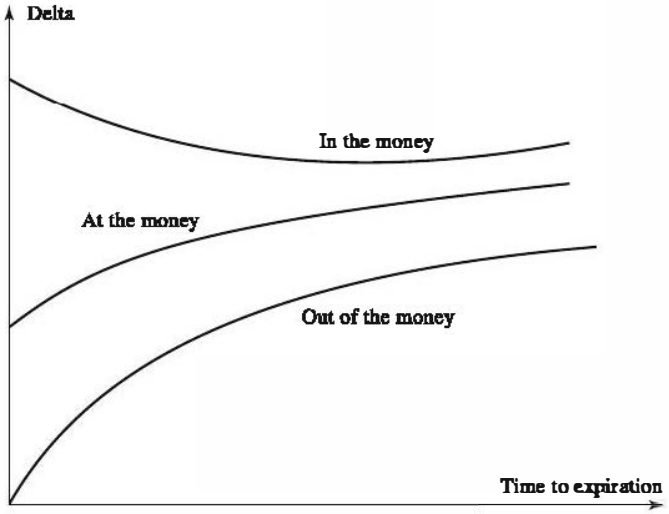

FIGURE 6-4 Typical patterns for variation of delta with time to maturity for a call option.

Example 6.1

Consider again the call option on a non-dividend-paying stock in the first section where the stock price is $49, the strike price is $50, the risk-free rate is 5%, the time to

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maturity is 20 weeks (= 0.3846 years), and the volatility is 20%. In this case,

$$d_1 = \frac{\ln(49/50) + (0.05 + 0.2^2/2) \times 0.3846}{0.2 \times \sqrt{0.3846}} = 0.0542$$

Delta is $N(d_1)$, or 0.522. When the stock price changes by ΔS, the option price changes by $0.522\Delta S$.

Dynamic Aspects of Delta Hedging

Tables 6-2 and 6-3 provide two examples of the operation of delta hedging for the example in the first section, where 100,000 call options are sold. The hedge is assumed to be adjusted or rebalanced weekly. The initial value of delta for a single option is calculated in Example 6.1 as 0.522. This means that the delta of the option position is initially −100,000 × 0.522, or −52,200. As soon as the option is written, $2,557,800 must be borrowed to buy 52,200 shares at a price of $49 to create a delta-neutral position. The rate of interest is 5%. An interest cost of approximately $2,500 is therefore incurred in the first week.

In Table 6-2, the stock price falls by the end of the first week to $48.12. The delta of the option declines to 0.458,

TABLE 6-2 Simulation of Delta Hedging (Option closes in the money and cost of hedging is $263,300)

Week	Stock Price	Delta	Shares Purchased	Cost of Shares Purchased ($000)	Cumulative Cost Including Interest ($000)	Interest Cost ($000)
0	49.00	0.522	52,200	2,557.8	2,557.8	2.5
1	48.12	0.458	(6,400)	(308.0)	2,252.3	2.2
2	47.37	0.400	(5,800)	(274.7)	1,979.8	1.9
3	50.25	0.596	19,600	984.9	2,966.6	2.9
4	51.75	0.693	9,700	502.0	3,471.5	3.3
5	53.12	0.774	8,100	430.3	3,905.1	3.8
6	53.00	0.771	(300)	(15.9)	3,893.0	3.7
7	51.87	0.706	(6,500)	(337.2)	3,559.5	3.4
8	51.38	0.674	(3,200)	(164.4)	3,398.5	3.3
9	53.00	0.787	11,300	598.9	4,000.7	3.8
10	49.88	0.550	(23,700)	(1,182.2)	2,822.3	2.7
11	48.50	0.413	(13,700)	(664.4)	2,160.6	2.1
12	49.88	0.542	12,900	643.5	2,806.2	2.7
13	50.37	0.591	4,900	246.8	3,055.7	2.9
14	52.13	0.768	17,700	922.7	3,981.3	3.8
15	51.88	0.759	(900)	(46.7)	3,938.4	3.8
16	52.87	0.865	10,600	560.4	4,502.6	4.3
17	54.87	0.978	11,300	620.0	5,126.9	4.9
18	54.62	0.990	1,200	65.5	5,197.3	5.0
19	55.87	1.000	1,000	55.9	5,258.2	5.1
20	57.25	1.000	0	0.0	5,263.3	

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TABLE 6-3 Simulation of Delta Hedging (Option closes out of the money and cost of hedging is $256,600)

Week	Stock Price	Delta	Shares Purchased	Cost of Shares Purchased ($000)	Cumulative Cost Including Interest ($000)	Interest Cost ($000)
0	49.00	0.522	52,200	2,557.8	2,557.8	2.5
1	49.75	0.568	4,600	228.9	2,789.2	2.7
2	52.00	0.705	13,700	712.4	3,504.3	3.4
3	50.00	0.579	(12,600)	(630.0)	2,877.7	2.8
4	48.38	0.459	(12,000)	(580.6)	2,299.9	2.2
5	48.25	0.443	(1,600)	(77.2)	2,224.9	2.1
6	48.75	0.475	3,200	156.0	2,383.0	2.3
7	49.63	0.540	6,500	322.6	2,707.9	2.6
8	48.25	0.420	(12,000)	(579.0)	2,131.5	2.1
9	48.25	0.410	(1,000)	(48.2)	2,085.4	2.0
10	51.12	0.658	24,800	1,267.8	3,355.2	3.2
11	51.50	0.692	3,400	175.1	3,533.5	3.4
12	49.88	0.542	(15,000)	(748.2)	2,788.7	2.7
13	49.88	0.538	(400)	(20.0)	2,771.4	2.7
14	48.75	0.400	(13,800)	(672.7)	2,101.4	2.0
15	47.50	0.236	(16,400)	(779.0)	1,324.4	1.3
16	48.00	0.261	2,500	120.0	1,445.7	1.4
17	46.25	0.062	(19,900)	(920.4)	526.7	0.5
18	48.13	0.183	12,100	582.4	1,109.6	1.1
19	46.63	0.007	(17,600)	(820.7)	290.0	0.3
20	48.12	0.000	(700)	(33.7)	256.6	

so that the new delta of the option position is −45,800. This means that 6,400 of the shares initially purchased are sold to maintain the delta-neutral hedge. The strategy realizes $308,000 in cash, and the cumulative borrowings at the end of Week 1 are reduced to $2,252,300. During the second week, the stock price reduces to $47.37, delta declines again, and so on. Toward the end of the life of the option, it becomes apparent that the option will be exercised and the delta of the option approaches 1.0. By Week 20, therefore, the hedger has a fully covered position. The hedger receives $5 million for the stock held, so that the total cost of writing the option and hedging it is $263,300.

Table 6-3 illustrates an alternative sequence of events such that the option closes out of the money. As it becomes clear that the option will not be exercised, delta approaches zero. By Week 20 the hedger has a naked position and has incurred costs totaling $256,600.

In Tables 6-2 and 6-3, the costs of hedging the option, when discounted to the beginning of the period, are close to but not exactly the same as the Black–Scholes–Merton

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TABLE 6-4 Performance of Delta Hedging (The performance measure is the ratio of the standard deviation of the cost of writing the option and hedging it to the theoretical price of the option)

Time between hedge rebalancing (weeks):	5	4	2	1	0.5	0.25
Performance measure:	0.42	0.38	0.28	0.21	0.16	0.13

price of $240,000. If the hedging worked perfectly, the cost of hedging would, after discounting, be exactly equal to the Black-Scholes-Merton price for every simulated stock price path. The reason for the variation in the cost of hedging is that the hedge is rebalanced only once a week. As rebalancing takes place more frequently, the variation in the cost of hedging is reduced. Of course, the examples in Tables 6-2 and 6-3 are idealized in that they assume that the volatility is constant and there are no transaction costs.

Table 6-4 shows statistics on the performance of delta hedging obtained from one million random stock price paths in our example. The performance measure is calculated similarly to Table 6-1 as the ratio of the standard deviation of the cost of hedging the option to the Black-Scholes-Merton price of the option. It is clear that delta hedging is a great improvement over a stop-loss strategy. Unlike a stop-loss strategy, the performance of a delta-hedging strategy gets steadily better as the hedge is monitored more frequently.

Delta hedging aims to keep the value of the financial institution's position as close to unchanged as possible. Initially, the value of the written option is $240,000. In the situation depicted in Table 6-2, the value of the option can be calculated as $414,500 in Week 9. Thus, the financial institution has lost $174,500 on its short option position. Its cash position, as measured by the cumulative cost, is $1,442,900 worse in Week 9 than in Week 0. The value of the shares held has increased from $2,557,800 to $4,171,100. The net effect of all this is that the value of the financial institution's position has changed by only $4,100 between Week 0 and Week 9.

Where the Cost Comes From

The delta-hedging procedure in Tables 6-2 and 6-3 creates the equivalent of a long position in the option. This neutralizes the short position the financial institution created by writing the option. As the tables illustrate, delta hedging a short position generally involves selling stock just after the price has gone down and buying stock just after the price has gone up. It might be termed a buy-high, sell-low trading strategy! The average cost of $240,000 comes from the present value of the difference between the price at which stock is purchased and the price at which it is sold.

Delta of a Portfolio

The delta of a portfolio of options or other derivatives dependent on a single asset whose price is S is

$$\frac{\partial \Pi}{\partial S}$$

where Π is the value of the portfolio.

The delta of the portfolio can be calculated from the deltas of the individual options in the portfolio. If a portfolio consists of a quantity w_i of option i ($1 \le i \le n$), the delta of the portfolio is given by

$$\Delta = \sum_{i=1}^{n} w_i \Delta_i$$

where Δ_i is the delta of the ith option. The formula can be used to calculate the position in the underlying asset necessary to make the delta of the portfolio zero. When this position has been taken, the portfolio is referred to as being *delta neutral*.

Suppose a financial institution has the following three positions in options on a stock:

1. A long position in 100,000 call options with strike price $55 and an expiration date in 3 months. The delta of each option is 0.533.
2. A short position in 200,000 call options with strike price $56 and an expiration date in 5 months. The delta of each option is 0.468.
3. A short position in 50,000 put options with strike price $56 and an expiration date in 2 months. The delta of each option is −0.508.

The delta of the whole portfolio is

$$100{,}000 \times 0.533 - 200{,}000 \times 0.468 - 50{,}000 \times (-0.508) = -14{,}900$$

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This means that the portfolio can be made delta neutral by buying 14,900 shares.

Transaction Costs

Derivatives dealers usually rebalance their positions once a day to maintain delta neutrality. When the dealer has a small number of options on a particular asset, this is liable to be prohibitively expensive because of the bid–offer spreads the dealer is subject to on trades. For a large portfolio of options, it is more feasible. Only one trade in the underlying asset is necessary to zero out delta for the whole portfolio. The bid–offer spread transaction costs are absorbed by the profits on many different trades.

THETA

The *theta* (Θ) of a portfolio of options is the rate of change of the value of the portfolio with respect to the passage of time with all else remaining the same. Theta is sometimes referred to as the *time decay* of the portfolio. For a European call option on a non-dividend-paying stock, it can be shown from the Black–Scholes–Merton formula that

$$\Theta(\text{call}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

where d_1 and d_2 are defined as in Equation (5.20) and

$$N'(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2} \qquad (6.2)$$

is the probability density function for a standard normal distribution.

For a European put option on the stock,

$$\Theta(\text{put}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

Because $N(-d_2) = 1 - N(d_2)$, the theta of a put exceeds the theta of the corresponding call by rKe^{-rT}.

In these formulas, time is measured in years. Usually, when theta is quoted, time is measured in days, so that theta is the change in the portfolio value when 1 day passes with all else remaining the same. We can measure theta either "per calendar day" or "per trading day". To obtain the theta per calendar day, the formula for theta must be divided by 365; to obtain theta per trading day, it must be divided by 252. (DerivaGem measures theta per calendar day.)

Example 6.2

As in Example 6.1, consider a call option on a non-dividend-paying stock where the stock price is \$49, the strike price is \$50, the risk-free rate is 5%, the time to maturity is 20 weeks (= 0.3846 years), and the volatility is 20%. In this case, $S_0 = 49$, $K = 50$, $r = 0.05$, $\sigma = 0.2$, and $T = 0.3846$.

The option's theta is

$$-\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2) = -4.31$$

The theta is $-4.31/365 = -0.0118$ per calendar day, or $-4.31/252 = -0.0171$ per trading day.

Theta is usually negative for an option.[7] This is because, as time passes with all else remaining the same, the option tends to become less valuable. The variation of Θ with stock price for a call option on a stock is shown in Figure 6-5. When the stock price is very low, theta is close to zero. For an at-the-money call option, theta is large and negative. As the stock price becomes larger, theta tends to $-rKe^{-rT}$. Figure 6-6 shows typical patterns for the variation of Θ with the time to maturity for in-the-money, at-the-money, and out-of-the-money call options.

Theta is not the same type of hedge parameter as delta. There is uncertainty about the future stock price, but there is no uncertainty about the passage of time. It makes sense to hedge against changes in the price of the

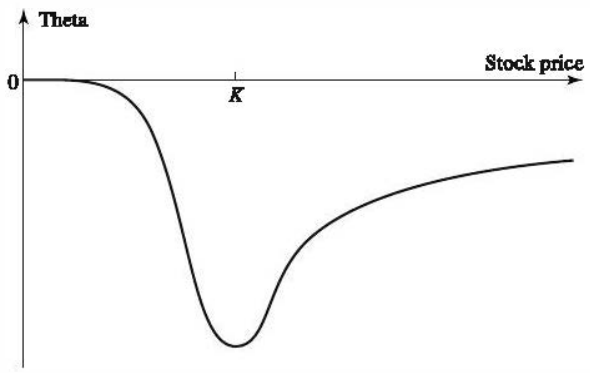

FIGURE 6-5 Variation of theta of a European call option with stock price.

[7] An exception to this could be an in-the-money European put option on a non-dividend-paying stock or an in-the-money European call option on a currency with a very high interest rate.

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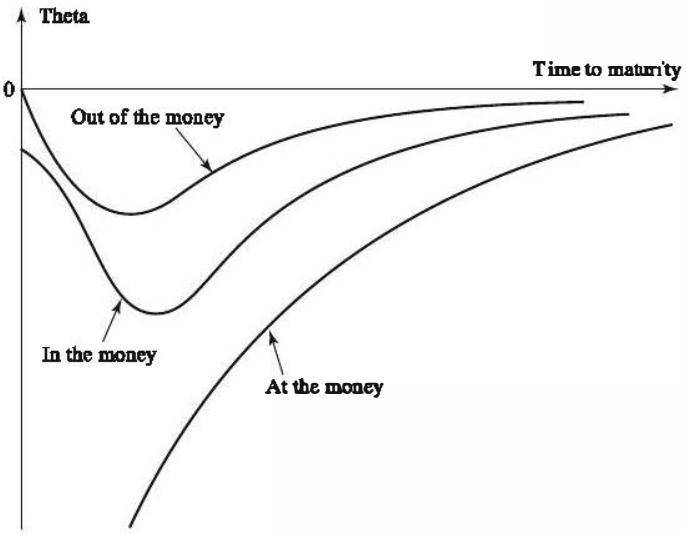

FIGURE 6-6 Typical patterns for variation of theta of a European call option with time to maturity.

underlying asset, but it does not make any sense to hedge against the passage of time. In spite of this, many traders regard theta as a useful descriptive statistic for a portfolio. This is because, as we shall see later, in a delta-neutral portfolio theta is a proxy for gamma.

GAMMA

The *gamma* (Γ) of a portfolio of options on an underlying asset is the rate of change of the portfolio's delta with respect to the price of the underlying asset. It is the second partial derivative of the portfolio with respect to asset price:

$$\Gamma = \frac{\partial^2 \Pi}{\partial S^2}$$

If gamma is small, delta changes slowly, and adjustments to keep a portfolio delta neutral need to be made only relatively infrequently. However, if gamma is highly negative or highly positive, delta is very sensitive to the price of the underlying asset. It is then quite risky to leave a delta-neutral portfolio unchanged for any length of time. Figure 6-7 illustrates this point. When the stock price moves from S to S', delta hedging assumes that the option price moves from C to C', when in fact it moves from C to C''. The difference between C' and C'' leads to a hedging error. The size of the error depends on the curvature of

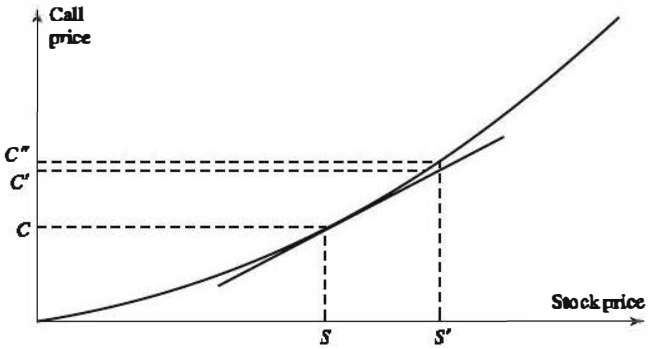

FIGURE 6-7 Hedging error introduced by nonlinearity.

the relationship between the option price and the stock price. Gamma measures this curvature.

Suppose that ΔS is the price change of an underlying asset during a small interval of time, Δt, and $\Delta\Pi$ is the corresponding price change in the portfolio. The Appendix at the end of this chapter shows that, if terms of order higher than Δt are ignored,

$$\Delta\Pi = \Theta\Delta t + \frac{1}{2}\Gamma\Delta S^2 \qquad (6.3)$$

for a delta-neutral portfolio, where Θ is the theta of the portfolio. Figure 6-8 shows the nature of this relationship between $\Delta\Pi$ and ΔS. When gamma is positive, theta tends to be negative. The portfolio declines in value if there is no change in S, but increases in value if there is a large positive or negative change in S. When gamma is negative, theta tends to be positive and the reverse is true: the portfolio increases in value if there is no change in S but decreases in value if there is a large positive or negative change in S. As the absolute value of gamma increases, the sensitivity of the value of the portfolio to S increases.

Example 6.3

Suppose that the gamma of a delta-neutral portfolio of options on an asset is −10,000. Equation (6.3) shows that, if a change of +2 or −2 in the price of the asset occurs over a short period of time, there is an unexpected decrease in the value of the portfolio of approximately $0.5 \times 10{,}000 \times 2^2 = \$20{,}000$.

Making a Portfolio Gamma Neutral

A position in the underlying asset has zero gamma and cannot be used to change the gamma of a portfolio. What

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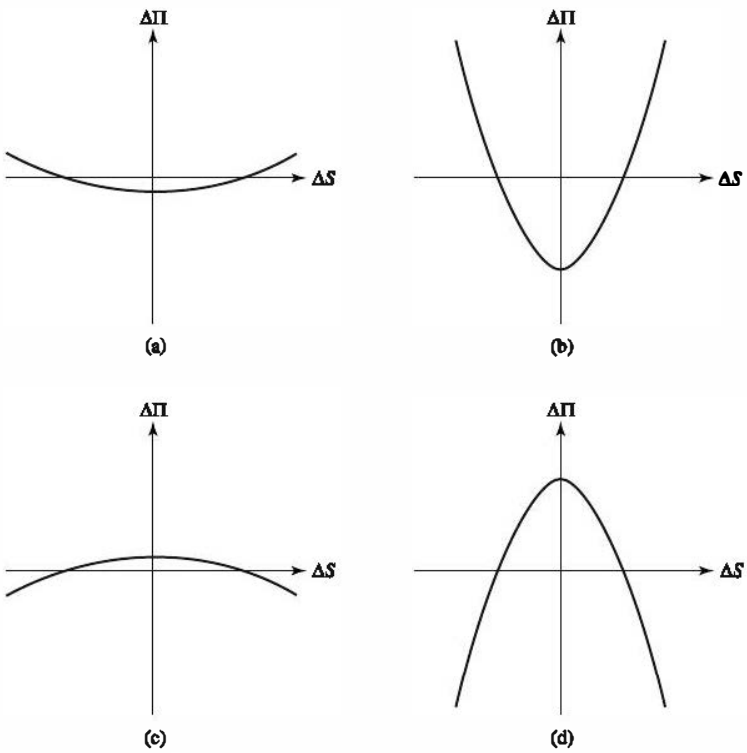

FIGURE 6-8 Relationship between $\Delta\Pi$ and ΔS in time Δt for a delta-neutral portfolio with (a) slightly positive gamma, (b) large positive gamma, (c) slightly negative gamma, and (d) large negative gamma.

is required is a position in an instrument such as an option that is not linearly dependent on the underlying asset.

Suppose that a delta-neutral portfolio has a gamma equal to Γ, and a traded option has a gamma equal to Γ_T. If the number of traded options added to the portfolio is w_T, the gamma of the portfolio is

$$w_T\Gamma_T + \Gamma$$

Hence, the position in the traded option necessary to make the portfolio gamma neutral is $-\Gamma/\Gamma_T$. Including the traded option is likely to change the delta of the portfolio, so the position in the underlying asset then has to be changed to maintain delta neutrality. Note that the portfolio is gamma neutral only for a short period of time. As time passes, gamma neutrality can be maintained only if the position in the traded option is adjusted so that it is always equal to $-\Gamma/\Gamma_T$.

Making a portfolio gamma neutral as well as delta-neutral can be regarded as a correction for the hedging error illustrated in Figure 6-7. Delta neutrality provides protection against relatively small stock price moves between rebalancing. Gamma neutrality provides protection against larger movements in this stock price between hedge rebalancing. Suppose that a portfolio is delta neutral and has a gamma of −3,000. The delta and gamma of a particular traded call option are 0.62 and 1.50, respectively. The portfolio can be made gamma neutral by including in the portfolio a long position of

$$\frac{3{,}000}{1.5} = 2{,}000$$

in the call option. However, the delta of the portfolio will then change from zero to 2,000 × 0.62 = 1,240. Therefore 1,240 units of the underlying asset must be sold from the portfolio to keep it delta neutral.

Calculation of Gamma

For a European call or put option on a non-dividend-paying stock, the gamma is given by

$$\Gamma = \frac{N'(d_1)}{S_0\sigma\sqrt{T}}$$

where d_1 is defined as in Equation (5.20) and $N'(x)$ is as given by Equation (6.2). The gamma of a long position is always positive and varies with S_0 in the way indicated in Figure 6-9. The variation of gamma with time to maturity for out-of-the-money, at-the-money, and in-the-money options is shown in Figure 6-10. For an at-the-money option, gamma increases as the time to maturity decreases. Short-life at-the-money options have very high gammas, which means

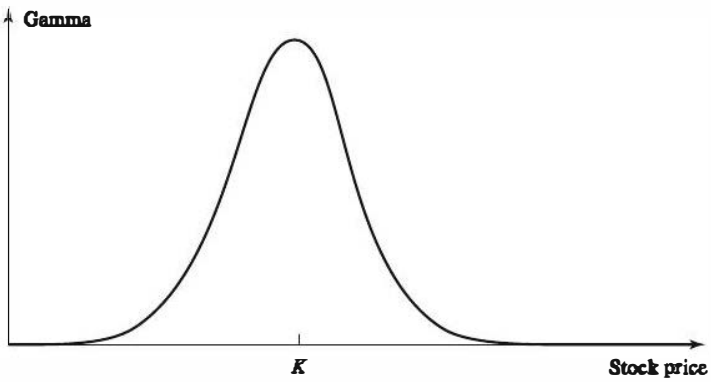

FIGURE 6-9 Variation of gamma with stock price for an option.

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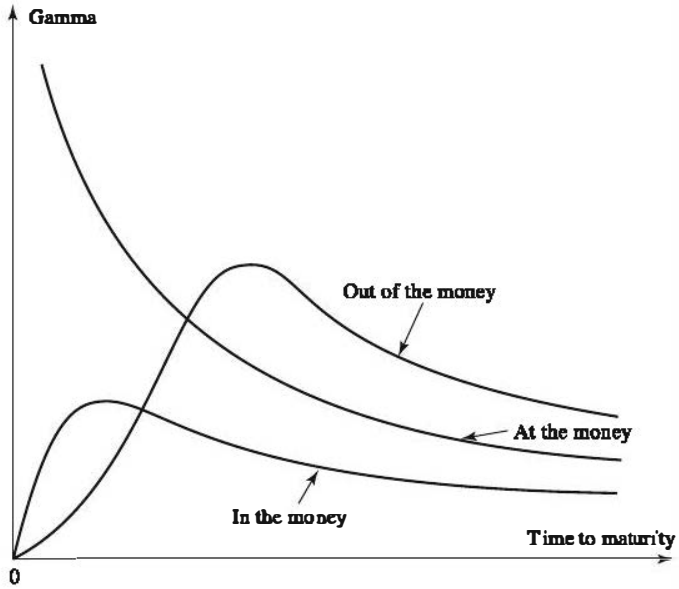

FIGURE 6-10 Variation of gamma with time to maturity for a stock option.

that the value of the option holder's position is highly sensitive to jumps in the stock price.

Example 6.4

As in Example 6.1, consider a call option on a non-dividend-paying stock where the stock price is \$49, the strike price is \$50, the risk-free rate is 5%, the time to maturity is 20 weeks (= 0.3846 years), and the volatility is 20%. In this case, $S_0 = 49$, $K = 50$, $r = 0.05$, $\sigma = 0.2$, and $T = 0.3846$.

The option's gamma is

$$\frac{N'(d_1)}{S_0\sigma\sqrt{T}} = 0.066$$

When the stock price changes by ΔS, the delta of the option changes by $0.066\Delta S$.

RELATIONSHIP BETWEEN DELTA, THETA, AND GAMMA

The price of a single derivative dependent on a non-dividend-paying stock must satisfy the differential Equation (5.16). It follows that the value of Π of a portfolio of such derivatives also satisfies the differential equation

$$\frac{\partial\Pi}{\partial t} + rS\frac{\partial\Pi}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2\Pi}{\partial S^2} = r\Pi$$

Since

$$\Theta = \frac{\partial\Pi}{\partial t}, \qquad \Delta = \frac{\partial\Pi}{\partial S}, \qquad \Gamma = \frac{\partial^2\Pi}{\partial S^2}$$

it follows that

$$\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2\Gamma = r\Pi \tag{6.4}$$

Similar results can be produced for other underlying assets.

For a delta-neutral portfolio, $\Delta = 0$ and

$$\Theta + \frac{1}{2}\sigma^2 S^2\Gamma = r\Pi$$

This shows that, when Θ is large and positive, gamma of a portfolio tends to be large and negative, and vice versa. This is consistent with the way in which Figure 6-8 has been drawn and explains why theta can to some extent be regarded as a proxy for gamma in a delta-neutral portfolio.

VEGA

Up to now we have implicitly assumed that the volatility of the asset underlying a derivative is constant. In practice, volatilities change over time. This means that the value of a derivative is liable to change because of movements in volatility as well as because of changes in the asset price and the passage of time.

The *vega* of a portfolio of derivatives, $\mathcal{V}$, is the rate of change of the value of the portfolio with respect to the volatility of the underlying asset.[8]

$$\mathcal{V} = \frac{\partial\Pi}{\partial\sigma}$$

If vega is highly positive or highly negative, the portfolio's value is very sensitive to small changes in volatility. If it is close to zero, volatility changes have relatively little impact on the value of the portfolio.

A position in the underlying asset has zero vega. However, the vega of a portfolio can be changed, similarly to the way gamma can be changed, by adding a position in a traded option. If $\mathcal{V}$ is the vega of the portfolio and $\mathcal{V}_T$ is the vega of a traded option, a position of $-\mathcal{V}/\mathcal{V}_T$ in the traded

[8] Vega is the name given to one of the "Greek letters" in option pricing, but it is not one of the letters in the Greek alphabet.

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option makes the portfolio instantaneously vega neutral. Unfortunately, a portfolio that is gamma neutral will not in general be vega neutral, and vice versa. If a hedger requires a portfolio to be both gamma and vega neutral, at least two traded derivatives dependent on the underlying asset must usually be used.

Example 6.5

Consider a portfolio that is delta neutral, with a gamma of −5,000 and a vega of −8,000. The options shown in the following table can be traded. The portfolio can be made vega neutral by including a long position in 4,000 of Option 1. This would increase delta to 2,400 and require that 2,400 units of the asset be sold to maintain delta neutrality. The gamma of the portfolio would change from −5,000 to −3,000.

	Delta	Gamma	Vega
Portfolio	0	−5000	−8000
Option 1	0.6	0.5	2.0
Option 2	0.5	0.8	1.2

To make the portfolio gamma and vega neutral, both Option 1 and Option 2 can be used. If w_1 and w_2 are the quantities of Option 1 and Option 2 that are added to the portfolio, we require that

$$-5{,}000 + 0.5w_1 + 0.8w_2 = 0$$

and

$$-8{,}000 + 2.0w_1 + 1.2w_2 = 0$$

The solution to these equations is $w_1 = 400$, $w_2 = 6{,}000$. The portfolio can therefore be made gamma and vega neutral by including 400 of Option 1 and 6,000 of Option 2. The delta of the portfolio, after the addition of the positions in the two traded options, is 400 × 0.6 + 6,000 × 0.5 = 3,240. Hence, 3,240 units of the asset would have to be sold to maintain delta neutrality.

For a European call or put option on a non-dividend-paying stock, vega is given by

$$\mathcal{V} = S_0\sqrt{T}\,N'(d_1)$$

where d_1 is defined as in Equation (5.20). The formula for $N'(x)$ is given in Equation (6.2). The vega of a long position in a European or American option is always positive. The general way in which vega varies with S_0 is shown in Figure 6-11.

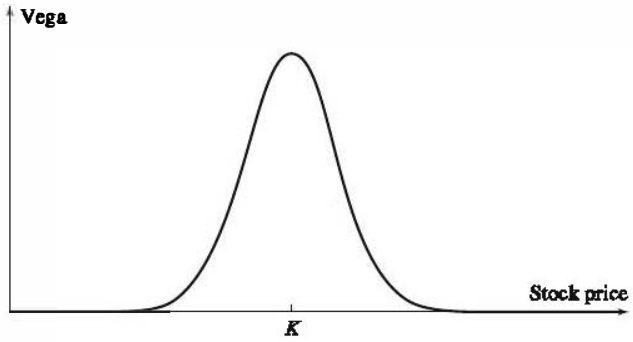

FIGURE 6-11 Variation of vega with stock price for an option.

Example 6.6

As in Example 6.1, consider a call option on a non-dividend-paying stock where the stock price is \$49, the strike price is \$50, the risk-free rate is 5%, the time to maturity is 20 weeks (= 0.3846 years), and the volatility is 20%. In this case, $S_0 = 49$, $K = 50$, $r = 0.05$, $\sigma = 0.2$, and $T = 0.3846$.

The option's vega is

$$S_0\sqrt{T}\,N'(d_1) = 12.1$$

Thus a 1% (0.01) increase in the volatility from (20% to 21%) increases the value of the option by approximately 0.01 × 12.1 = 0.121.

Calculating vega from the Black–Scholes–Merton model and its extensions may seem strange because one of the assumptions underlying the model is that volatility is constant. It would be theoretically more correct to calculate vega from a model in which volatility is assumed to be stochastic. However, it turns out that the vega calculated from a stochastic volatility model is very similar to the Black–Scholes–Merton vega, so the practice of calculating vega from a model in which volatility is constant works reasonably well.[9]

Gamma neutrality protects against large changes in the price of the underlying asset between hedge rebalancing. Vega neutrality protects against a variable σ. As might be expected, whether it is best to use an available traded

[9] See J. C. Hull and A. White, "The Pricing of Options on Assets with Stochastic Volatilities," *Journal of Finance* 42 (June 1987): 281–300; J. C. Hull and A. White, "An Analysis of the Bias in Option Pricing Caused by a Stochastic Volatility," *Advances in Futures and Options Research* 3 (1988): 27–61.

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option for vega or gamma hedging depends on the time between hedge rebalancing and the volatility of the volatility.[10]

When volatilities change, the implied volatilities of short-dated options tend to change by more than the implied volatilities of long-dated options. The vega of a portfolio is therefore often calculated by changing the volatilities of long-dated options by less than that of short-dated options.

RHO

The *rho* of a portfolio of options is the rate of change of the value of the portfolio with respect to the interest rate:

$$\frac{\partial \Pi}{\partial r}$$

It measures the sensitivity of the value of a portfolio to a change in the interest rate when all else remains the same. For a European call option on a non-dividend-paying stock,

$$\text{rho (call)} = KTe^{-rT}N(d_2)$$

where d_2 is defined as in Equation (5.20). For a European put option,

$$\text{rho (put)} = -KTe^{-rT}N(-d_2)$$

Example 6.7

As in Example 6.1, consider a call option on a non-dividend-paying stock where the stock price is \$49, the strike price is \$50, the risk-free rate is 5%, the time to maturity is 20 weeks (= 0.3846 years), and the volatility is 20%. In this case, $S_0 = 49$, $K = 50$, $r = 0.05$, $\sigma = 0.2$, and $T = 0.3846$.

The option's rho is

$$KTe^{-rT}N(d_2) = 8.91$$

This means that a 1% (0.01) increase in the risk-free rate (from 5% to 6%) increases the value of the option by approximately $0.01 \times 8.91 = 0.0891$.

THE REALITIES OF HEDGING

In an ideal world, traders working for financial institutions would be able to rebalance their portfolios very frequently in order to maintain all Greeks equal to zero. In practice, this is not possible. When managing a large portfolio dependent on a single underlying asset, traders usually make delta zero, or close to zero, at least once a day by trading the underlying asset. Unfortunately, a zero gamma and a zero vega are less easy to achieve because it is difficult to find options or other nonlinear derivatives that can be traded in the volume required at competitive prices. Box 6-1 provides a discussion of how dynamic hedging is organized at financial institutions.

As already mentioned, there are big economies of scale in trading derivatives. Maintaining delta neutrality for a small number of options on an asset by trading daily is usually not economically feasible because of trading costs.[11] But when a derivatives dealer maintains delta neutrality for a large portfolio of options on an asset, the trading costs per option hedged are likely to be much more reasonable.

SCENARIO ANALYSIS

In addition to monitoring risks such as delta, gamma, and vega, option traders often also carry out a scenario analysis. The analysis involves calculating the gain or loss on their portfolio over a specified period under a variety of different scenarios. The time period chosen is likely to depend on the liquidity of the instruments. The scenarios can be either chosen by management or generated by a model.

Consider a bank with a portfolio of options on a foreign currency. There are two main variables on which the value of the portfolio depends. These are the exchange rate and the exchange-rate volatility. Suppose that the exchange rate is currently 1.0000 and its volatility is 10% per annum. The bank could calculate a table such as Table 6-5 showing the profit or loss experienced during a 2-week period under different scenarios. This table considers seven different exchange rates and three different volatilities. Because a one-standard-deviation move in the exchange

[10] For a discussion of this issue, see J. C. Hull and A. White, "Hedging the Risks from Writing Foreign Currency Options," *Journal of International Money and Finance* 6 (June 1987): 131–52.

[11] The trading costs arise from the fact that each day the hedger buys some of the underlying asset at the offer price or sells some of the underlying asset at the bid price.

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BOX 6-1 Dynamic Hedging in Practice

In a typical arrangement at a financial institution, the responsibility for a portfolio of derivatives dependent on a particular underlying asset is assigned to one trader or to a group of traders working together. For example, one trader at Goldman Sachs might be assigned responsibility for all derivatives dependent on the value of the Australian dollar. A computer system calculates the value of the portfolio and Greek letters for the portfolio. Limits are defined for each Greek letter and special permission is required if a trader wants to exceed a limit at the end of a trading day.

The delta limit is often expressed as the equivalent maximum position in the underlying asset. For example, the delta limit of Goldman Sachs for a stock might be $1 million. If the stock price is $50, this means that the absolute value of delta as we have calculated it can be no more than 20,000. The vega limit is usually expressed as a maximum dollar exposure per 1% change in the volatility.

As a matter of course, options traders make themselves delta neutral—or close to delta neutral—at the end of each day. Gamma and vega are monitored, but are not usually managed on a daily basis. Financial institutions often find that their business with clients involves writing options and that as a result they accumulate negative gamma and vega. They are then always looking out for opportunities to manage their gamma and vega risks by buying options at competitive prices.

There is one aspect of an options portfolio that mitigates problems of managing gamma and vega somewhat. Options are often close to the money when they are first sold, so that they have relatively high gammas and vegas. But after some time has elapsed, the underlying asset price has often changed enough for them to become deep out of the money or deep in the money. Their gammas and vegas are then very small and of little consequence. A nightmare scenario for an options trader is where written options remain very close to the money as the maturity date is approached.

TABLE 6-5 Profit or Loss Realized in 2 Weeks under Different Scenarios ($ million)

	Exchange Rate						
Volatility	**0.94**	**0.96**	**0.98**	**1.00**	**1.02**	**1.04**	**1.06**
8%	+102	+55	+25	+6	−10	−34	−80
10%	+80	+40	+17	+2	−14	−38	−85
12%	+60	+25	+9	−2	−18	−42	−90

rate during a 2-week period is about 0.02, the exchange rate moves considered are approximately zero, one, two, and three standard deviations.

In Table 6-5, the greatest loss is in the lower right corner of the table. The loss corresponds to the volatility increasing to 12% and the exchange rate moving up to 1.06. Usually the greatest loss in a table such as Table 6-5 occurs at one of the corners, but this is not always so. Consider, for example, the situation where a bank's portfolio consists of a short position in a butterfly spread. The greatest loss will be experienced if the exchange rate stays where it is.

EXTENSION OF FORMULAS

The formulas produced so far for delta, theta, gamma, vega, and rho have been for a European option on a non-dividend-paying stock. Table 6-6 shows how they change when the stock pays a continuous dividend yield at rate q. By setting q equal to the dividend yield on an index, we obtain the Greek letters for European options on indices. By setting q equal to the foreign risk-free rate, we obtain the Greek letters for European options on a currency. By setting $q = r$, we obtain delta, gamma, theta, and vega for European options on a futures contract. The rho for a call futures option is $-cT$ and the rho for a European put futures option is $-pT$.

In the case of currency options, there are two rhos corresponding to the two interest rates. The rho corresponding to the domestic interest rate is given by the formula in Table 6-6. The rho corresponding to the foreign interest rate for a European call on a currency is

$$\text{rho(call, foreign rate)} = -Te^{-r_fT}S_0N(d_1)$$

For a European put, it is

$$\text{rho(put, foreign rate)} = Te^{-r_fT}S_0N(-d_1)$$

Delta of Forward Contracts

The concept of delta can be applied to financial instruments other than options. Consider a forward contract on a non-dividend-paying stock. The value of a forward contract is $S_0 - Ke^{-rT}$, where K is the delivery price and T is the forward contract's time to maturity. When the price of the stock changes by ΔS, with all else remaining the same, the value of a forward contract on the stock also changes by ΔS. The delta of a long forward contract

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TABLE 6-6 Greek Letters for European Options on an Asset That Provides a Yield at Rate q

Greek Letter	Call Option	Put Option
Delta	$e^{-qT}N(d_1)$	$e^{-qT}[N(d_1)-1]$
Gamma	$\frac{N'(d_1)e^{-qT}}{S_0\sigma\sqrt{T}}$	$\frac{N'(d_1)e^{-qT}}{S_0\sigma\sqrt{T}}$
Theta	$-S_0N'(d_1)\sigma e^{-qT}/(2\sqrt{T})$ $+qS_0N(d_1)e^{-qT}-rKe^{-rT}N(d_2)$	$-S_0N'(d_1)\sigma e^{-qT}/(2\sqrt{T})$ $-qS_0N(-d_1)e^{-qT}+rKe^{-rT}N(-d_2)$
Vega	$S_0\sqrt{T}\,N'(d_1)e^{-qT}$	$S_0\sqrt{T}\,N'(d_1)e^{-qT}$
Rho	$KTe^{-rT}N(d_2)$	$-KTe^{-rT}N(-d_2)$

on one share of the stock is therefore always 1.0. This means that a long forward contract on one share can be hedged by shorting one share; a short forward contract on one share can be hedged by purchasing one share.[12]

For an asset providing a dividend yield at rate q, the forward contract's delta is e^{-qT}. For the delta of a forward contract on a stock index, q is set equal to the dividend yield on the index in this expression. For the delta of a forward foreign exchange contract, it is set equal to the foreign risk-free rate, r_f.

Delta of a Futures Contract

The futures price for a contract on a non-dividend-paying stock is S_0e^{rT}, where T is the time to maturity of the futures contract. This shows that when the price of the stock changes by ΔS, with all else remaining the same, the futures price changes by ΔSe^{rT}. Since futures contracts are settled daily, the holder of a long futures position makes an almost immediate gain of this amount. The delta of a futures contract is therefore e^{rT}. For a futures position on an asset providing a dividend yield at rate q, delta is $e^{(r-q)T}$.

It is interesting that daily settlement makes the deltas of futures and forward contracts slightly different. This is true even when interest rates are constant and the forward price equals the futures price.

[12] These are hedge-and-forget schemes. Since delta is always 1.0, no changes need to be made to the position in the stock during the life of the contract.

Sometimes a futures contract is used to achieve a delta-neutral position. Define:

T: Maturity of futures contract

H_A: Required position in asset for delta hedging

H_F: Alternative required position in futures contracts for delta hedging

If the underlying asset is a non-dividend-paying stock, the analysis we have just given shows that

$$H_F = e^{-rT}H_A \tag{6.5}$$

When the underlying asset pays a dividend yield q,

$$H_F = e^{-(r-q)T}H_A \tag{6.6}$$

For a stock index, we set q equal to the dividend yield on the index; for a currency, we set it equal to the foreign risk-free rate, r_f, so that

$$H_F = e^{-(r-r_f)T}H_A \tag{6.7}$$

Example 6.8

Suppose that a portfolio of currency options held by a US bank can be made delta neutral with a short position of 458,000 pounds sterling. Risk-free rates are 4% in the US and 7% in the UK. From Equation (6.7), hedging using 9-month currency futures requires a short futures position

$$e^{-(0.04-0.07)\times 9/12} \times 458{,}000$$

or £468,442. Since each futures contract is for the purchase or sale of £62,500, seven contracts would be shorted. (Seven is the nearest whole number to 468,442/62,500.)

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PORTFOLIO INSURANCE

A portfolio manager is often interested in acquiring a put option on his or her portfolio. This provides protection against market declines while preserving the potential for a gain if the market does well. One approach is to buy put options on a market index such as the S&P 500. An alternative is to create the options synthetically.

Creating an option synthetically involves maintaining a position in the underlying asset (or futures on the underlying asset) so that the delta of the position is equal to the delta of the required option. The position necessary to create an option synthetically is the reverse of that necessary to hedge it. This is because the procedure for hedging an option involves the creation of an equal and opposite option synthetically.

There are two reasons why it may be more attractive for the portfolio manager to create the required put option synthetically than to buy it in the market. First, option markets do not always have the liquidity to absorb the trades required by managers of large funds. Second, fund managers often require strike prices and exercise dates that are different from those available in exchange-traded options markets.

The synthetic option can be created from trading the portfolio or from trading in index futures contracts. We first examine the creation of a put option by trading the portfolio. From Table 6-6, the delta of a European put on the portfolio is

$$\Delta = e^{-qT}[N(d_1) - 1] \tag{6.8}$$

where, with our usual notation,

$$d_1 = \frac{\ln(S_0 / K) + (r - q + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

The other variables are defined as usual: S_0 is the value of the portfolio, K is the strike price, r is the risk-free rate, q is the dividend yield on the portfolio, σ is the volatility of the portfolio, and T is the life of the option. The volatility of the portfolio can usually be assumed to be its beta times the volatility of a well-diversified market index.

To create the put option synthetically, the fund manager should ensure that at any given time a proportion

$$e^{-qT}[1 - N(d_1)]$$

of the stocks in the original portfolio has been sold and the proceeds invested in riskless assets. As the value of the original portfolio declines, the delta of the put given by Equation (6.8) becomes more negative and the proportion of the original portfolio sold must be increased. As the value of the original portfolio increases, the delta of the put becomes less negative and the proportion of the original portfolio sold must be decreased (i.e., some of the original portfolio must be repurchased).

Using this strategy to create portfolio insurance means that at any given time funds are divided between the stock portfolio on which insurance is required and riskless assets. As the value of the stock portfolio increases, riskless assets are sold and the position in the stock portfolio is increased. As the value of the stock portfolio declines, the position in the stock portfolio is decreased and riskless assets are purchased. The cost of the insurance arises from the fact that the portfolio manager is always selling after a decline in the market and buying after a rise in the market.

Example 6.9

A portfolio is worth \$90 million. To protect against market downturns the managers of the portfolio require a 6-month European put option on the portfolio with a strike price of \$87 million. The risk-free rate is 9% per annum, the dividend yield is 3% per annum, and the volatility of the portfolio is 25% per annum. The S&P 500 index stands at 900. As the portfolio is considered to mimic the S&P 500 fairly closely, one alternative is to buy 1,000 put option contracts on the S&P 500 with a strike price of 870. Another alternative is to create the required option synthetically. In this case, $S_0 = 90$ million, $K = 87$ million, $r = 0.09$, $q = 0.03$, $\sigma = 0.25$, and $T = 0.5$, so that

$$d_1 = \frac{\ln(90/87) + (0.09 - 0.03 + 0.25^2/2)0.5}{0.25\sqrt{0.5}} = 0.4499$$

and the delta of the required option is

$$e^{-qT}[N(d_1) - 1] = -0.3215$$

This shows that 32.15% of the portfolio should be sold initially and invested in risk-free assets to match the delta of the required option. The amount of the portfolio sold must be monitored frequently. For example, if the value of the portfolio reduces to \$88 million after 1 day, the delta of the required option changes to 0.3679 and a further

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4.64% of the original portfolio should be sold and invested in risk-free assets. If the value of the portfolio increases to $92 million, the delta of the required option changes to −0.2787 and 4.28% of the original portfolio should be repurchased.

Use of Index Futures

Using index futures to create options synthetically can be preferable to using the underlying stocks because the transaction costs associated with trades in index futures are generally lower than those associated with the corresponding trades in the underlying stocks. The dollar amount of the futures contracts shorted as a proportion of the value of the portfolio should from Equations (6.6) and (6.8) be

$$e^{-qT}e^{-(r-q)T^*}[1 - N(d_1)] = e^{qT^*-T}e^{-rT^*}[1 - N(d_1)]$$

where T^* is the maturity of the futures contract. If the portfolio is worth A_1 times the index and each index futures contract is on A_2 times the index, the number of futures contracts shorted at any given time should be

$$e^{qT^*-T}e^{-rT^*}[1 - N(d_1)]A_1/A_2$$

Example 6.10

Suppose that in the previous example futures contracts on the S&P 500 maturing in 9 months are used to create the option synthetically. In this case initially $T = 0.5$, $T^* = 0.75$, $A_1 = 100{,}000$, and $d_1 = 0.4499$. Each index futures contract is on 250 times the index, so that $A_2 = 250$. The number of futures contracts shorted should be

$$e^{qT^*-T}e^{-rT^*}[1 - N(d_1)]A_1/A_2 = 122.96$$

or 123, rounding to the nearest whole number. As time passes and the index changes, the position in futures contracts must be adjusted.

This analysis assumes that the portfolio mirrors the index. When this is not the case, it is necessary to (a) calculate the portfolio's beta, (b) find the position in options on the index that gives the required protection, and (c) choose a position in index futures to create the options synthetically. The strike price for the options should be the expected level of the market index when the portfolio reaches its insured value. The number of options required is beta times the number that would be required if the portfolio had a beta of 1.0.

STOCK MARKET VOLATILITY

We discussed in Chapter 5 the issue of whether volatility is caused solely by the arrival of new information or whether trading itself generates volatility. Portfolio insurance strategies such as those just described have the potential to increase volatility. When the market declines, they cause portfolio managers either to sell stock or to sell index futures contracts. Either action may accentuate the decline (see Box 6-2). The sale of stock is liable to drive down the market index further in a direct way. The sale of index futures contracts is liable to drive down futures prices. This creates selling pressure on stocks via

BOX 6-2 Was Portfolio Insurance to Blame for the Crash of 1987?

On Monday, October 19, 1987, the Dow Jones Industrial Average dropped by more than 20%. Many people feel that portfolio insurance played a major role in this crash. In October 1987 between $60 billion and $90 billion of equity assets were subject to portfolio insurance trading rules where put options were created synthetically. During the period Wednesday, October 14, 1987, to Friday, October 16, 1987, the market declined by about 10%, with much of this decline taking place on Friday afternoon. The portfolio trading rules should have generated at least $12 billion of equity or index futures sales as a result of this decline. In fact, portfolio insurers had time to sell only $4 billion and they approached the following week with huge amounts of selling already dictated by their models. It is estimated that on Monday, October 19, sell programs by three portfolio insurers accounted for almost 10% of the sales on the New York Stock Exchange, and that portfolio insurance sales amounted to 21.3% of all sales in index futures markets. It is likely that the decline in equity prices was exacerbated by investors other than portfolio insurers selling heavily because they anticipated the actions of portfolio insurers.

Because the market declined so fast and the stock exchange systems were overloaded, many portfolio insurers were unable to execute the trades generated by their models and failed to obtain the protection they required. Needless to say, the popularity of portfolio insurance schemes has declined significantly since 1987. One of the morals of this story is that it is dangerous to follow a particular trading strategy—even a hedging strategy—when many other market participants are doing the same thing.

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the mechanism of index arbitrage, so that the market index is liable to be driven down in this case as well. Similarly, when the market rises, the portfolio insurance strategies cause portfolio managers either to buy stock or to buy futures contracts. This may accentuate the rise.

In addition to formal portfolio trading strategies, we can speculate that many investors consciously or subconsciously follow portfolio insurance rules of their own. For example, an investor may choose to sell when the market is falling to limit the downside risk.

Whether portfolio insurance trading strategies (formal or informal) affect volatility depends on how easily the market can absorb the trades that are generated by portfolio insurance. If portfolio insurance trades are a very small fraction of all trades, there is likely to be no effect. But if portfolio insurance becomes very popular, it is liable to have a destabilizing effect on the market, as it did in 1987.

SUMMARY

Financial institutions offer a variety of option products to their clients. Often the options do not correspond to the standardized products traded by exchanges. The financial institutions are then faced with the problem of hedging their exposure. Naked and covered positions leave them subject to an unacceptable level of risk. One course of action that is sometimes proposed is a stop-loss strategy. This involves holding a naked position when an option is out of the money and converting it to a covered position as soon as the option moves into the money. Although superficially attractive, the strategy does not provide a good hedge.

The delta (Δ) of an option is the rate of change of its price with respect to the price of the underlying asset. Delta hedging involves creating a position with zero delta (sometimes referred to as a delta-neutral position). Because the delta of the underlying asset is 1.0, one way of hedging is to take a position of $-\Delta$ in the underlying asset for each long option being hedged. The delta of an option changes over time. This means that the position in the underlying asset has to be frequently adjusted.

Once an option position has been made delta neutral, the next stage is often to look at its gamma (Γ). The gamma of an option is the rate of change of its delta with respect to the price of the underlying asset. It is a measure of the curvature of the relationship between the option price and the asset price. The impact of this curvature on the performance of delta hedging can be reduced by making an option position gamma neutral. If Γ is the gamma of the position being hedged, this reduction is usually achieved by taking a position in a traded option that has a gamma of $-\Gamma$.

Delta and gamma hedging are both based on the assumption that the volatility of the underlying asset is constant. In practice, volatilities do change over time. The vega of an option or an option portfolio measures the rate of change of its value with respect to volatility. A trader who wishes to hedge an option position against volatility changes can make the position vega neutral. As with the procedure for creating gamma neutrality, this usually involves taking an offsetting position in a traded option. If the trader wishes to achieve both gamma and vega neutrality, two traded options are usually required.

Two other measures of the risk of an option position are theta and rho. Theta measures the rate of change of the value of the position with respect to the passage of time, with all else remaining constant. Rho measures the rate of change of the value of the position with respect to the interest rate, with all else remaining constant.

In practice, option traders usually rebalance their portfolios at least once a day to maintain delta neutrality. It is usually not feasible to maintain gamma and vega neutrality on a regular basis. Typically a trader monitors these measures. If they get too large, either corrective action is taken or trading is curtailed.

Portfolio managers are sometimes interested in creating put options synthetically for the purposes of insuring an equity portfolio. They can do so either by trading the portfolio or by trading index futures on the portfolio. Trading the portfolio involves splitting the portfolio between equities and risk-free securities. As the market declines, more is invested in risk-free securities. As the market increases, more is invested in equities. Trading index futures involves keeping the equity portfolio intact and selling index futures. As the market declines, more index futures are sold; as it rises, fewer are sold. This type of portfolio insurance works well in normal market conditions. On Monday, October 19, 1987, when the Dow Jones Industrial Average dropped very sharply, it worked badly. Portfolio insurers were unable to sell either stocks or index futures fast enough to protect their positions.

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Further Reading

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Taleb, N. N., *Dynamic Hedging: Managing Vanilla and Exotic Options*. New York: Wiley, 1996.

APPENDIX

Taylor Series Expansions and Hedge Parameters

A Taylor series expansion of the change in the portfolio value in a short period of time shows the role played by different Greek letters. If the volatility of the underlying asset is assumed to be constant, the value Π of the portfolio is a function of the asset price S, and time t. The Taylor series expansion gives

$$\Delta\Pi = \frac{\partial\Pi}{\partial S}\Delta S + \frac{\partial\Pi}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2\Pi}{\partial S^2}\Delta S^2 + \frac{1}{2}\frac{\partial^2\Pi}{\partial t^2}\Delta t^2 + \frac{\partial^2\Pi}{\partial S\partial t}\Delta S\Delta t + \cdots \quad (6.9)$$

where $\Delta\Pi$ and ΔS are the change in Π and S in a small time interval Δt. Delta hedging eliminates the first term on the right-hand side. The second term is nonstochastic. The third term (which is of order Δt) can be made zero by ensuring that the portfolio is gamma neutral as well as delta neutral. Other terms are of order higher than Δt.

For a delta-neutral portfolio, the first term on the right-hand side of Equation (6.9) is zero, so that

$$\Delta\Pi = \Theta\Delta t + \frac{1}{2}\Gamma\Delta S^2$$

when terms of order higher than Δt are ignored. This is Equation (6.3).

When the volatility of the underlying asset is uncertain, Π is a function of σ, S, and t. Equation (6.9) then becomes

$$\Delta\Pi = \frac{\partial\Pi}{\partial S}\Delta S + \frac{\partial\Pi}{\partial\sigma}\Delta\sigma + \frac{\partial\Pi}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2\Pi}{\partial S^2}\Delta S^2 + \frac{1}{2}\frac{\partial^2\Pi}{\partial\sigma^2}\Delta\sigma^2 + \cdots$$

where $\Delta\sigma$ is the change in σ in time Δt. In this case, delta hedging eliminates the first term on the right-hand side. The second term is eliminated by making the portfolio vega neutral. The third term is nonstochastic. The fourth term is eliminated by making the portfolio gamma neutral. Traders sometimes define other Greek letters to correspond to later terms in the expansion.

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Prices, Discount Factors, and Arbitrage

7

Learning Objectives

After completing this reading you should be able to:

- Define discount factor and use a discount function to compute present and future values.
- Define the "law of one price," explain it using an arbitrage argument, and describe how it can be applied to bond pricing.
- Identify the components of a U.S. Treasury coupon bond, and compare and contrast the structure to Treasury STRIPS, including the difference between P-STRIPS and C-STRIPS.
- Construct a replicating portfolio using multiple fixed income securities to match the cash flows of a given fixed income security.
- Identify arbitrage opportunities for fixed income securities with certain cash flows.
- Differentiate between "clean" and "dirty" bond pricing and explain the implications of accrued interest with respect to bond pricing.
- Describe the common day-count conventions used in bond pricing.

Excerpt is Chapter 1 of Fixed Income Securities, *Third Edition, by Bruce Tuckman.*

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This chapter begins by introducing the cash flows of fixed-rate, government coupon bonds. It shows that prices of these bonds can be used to extract *discount factors*, which are the market prices of one unit of currency to be received on various dates in the future.

Relying on a principle known as *the law of one price*, discount factors extracted from a particular set of bonds can be used to price other bonds, outside the original set. A more complex but more convincing relative pricing methodology, known as *arbitrage pricing*, turns out to be mathematically identical to pricing with discount factors. Hence, discounting can rightly be used and regarded as shorthand for arbitrage pricing.

The application of this chapter uses the U.S. Treasury coupon bond and Separate Trading of Registered Interest and Principal of Securities (STRIPS) markets to illustrate that bonds are not commodities, meaning that their prices reflect individual characteristics other than their scheduled cash flows. This idiosyncratic component of bond valuation implies that the predictions of the simplest relative pricing methodologies only approximate the complex reality of bond markets.

The chapter concludes with a discussion of day-counts and accrued interest, pricing conventions used throughout fixed income markets and, consequently, throughout this book.

THE CASH FLOWS FROM FIXED-RATE GOVERNMENT COUPON BONDS

The cash flows from fixed-rate, government coupon bonds are defined by *face amount, principal amount*, or *par value*; *coupon rate*; and *maturity date*. For example, in May 2010 the U.S. Treasury sold a bond with a coupon rate of 2⅛% and a maturity date of May 31, 2015. Purchasing $1 million face amount of these "2⅛s of May 31, 2015," entitles the buyer to the schedule of payments in Table 7-1. The Treasury promises to make a coupon payment every six months equal to half the note's annual coupon rate of 2⅛% times the face amount, i.e., ½ x 2⅛% × $1,000,000, or $10,625. Then, on the maturity date of May 31, 2015, in addition to the coupon payment on that date, the Treasury promises to pay the bond's face amount of $1,000,000. One fact worth mentioning, although too small a detail to receive much attention in this book, is that scheduled payments that do not fall on a business day are made on the following business day. For example, the payments of the 2⅛s scheduled for Sunday, May 31, 2015, would be made on Monday, June 1, 2015.

TABLE 7-1 Cash Flows of the U.S. 2⅛s of May 31, 2015

Date	Coupon Payment	Principal Payment
11/30/2010	$10,625	
5/31/2011	$10,625	
11/30/2011	$10,625	
5/31/2012	$10,625	
11/30/2012	$10,625	
5/31/2013	$10,625	
11/30/2013	$10,625	
5/31/2014	$10,625	
11/30/2014	$10,625	
5/31/2015	$10,625	$1,000,000

For concreteness and continuity of exposition this chapter restricts attention to U.S. Treasury bonds. But the analytics of the chapter apply easily to bonds issued by other countries because the cash flows of all fixed-rate government coupon bonds are qualitatively similar. The most significant difference across issues is the frequency of coupon payments, which can be semiannual or annual; government bond issues in France and Germany make annual coupon payments, while those in Italy, Japan, and the UK make semiannual payments.

Returning to the U.S. Treasury market, then, Table 7-2 reports the coupons and maturity dates of selected U.S. Treasury bonds, along with their prices as of the close of business on Friday, May 28, 2010. Almost all U.S. Treasury trades settle $T + 1$, which means that the exchange of bonds for cash happens one business day after the trade date. In this case, the next business day was Tuesday, June 1, 2010.

The prices given in Table 7-2 are *mid-market*, *full* (or *invoice*) prices per 100 face amount. A mid-market price is an average of a lower *bid* price, at which traders stand ready to buy a bond, and a higher *ask* price, at which

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TABLE 7-2 Selected U.S. Treasury Bond Prices as of May 28, 2010

Coupon	Maturity	Price
1¼%	11/30/2010	100.550
4⅞%	5/31/2011	104.513
4½%	11/30/2011	105.856
4¾%	5/31/2012	107.966
3⅜%	11/30/2012	105.869
3½%	5/31/2013	106.760
2%	11/30/2013	101.552
2¼%	5/31/2014	101.936
2⅛%	11/30/2014	100.834

traders stand ready to sell a bond. A *full* price is the total amount a buyer pays for a bond, which is the sum of the *flat* or *quoted* price of the bond and *accrued interest*. This division of full price will be explained later in this chapter. In any case, to take an example from Table 7-2, purchasing $100,000 face amount of the 3½s of May 31, 2013, costs a total of $106,760.

The bonds in Table 7-2 were selected from the broader list of U.S. Treasuries because they all mature and make payments on the same *cycle*, in this case at the end of May and November each year. This means, for example, that all of the bonds make a payment on November 30, 2010, and, therefore, that all their prices incorporate information about the value of a dollar to be received on that date. Similarly, all of the bonds apart from the 1¼s of November 30, 2010, which will have already matured, make a payment on May 31, 2011, and their prices incorporate information about the value of a dollar to be received on that date, etc. The next section describes how to extract information about the value of a dollar to be received on each of the payment dates in the May–November cycle from the prices in Table 7-2.

DISCOUNT FACTORS

The *discount factor* for a particular term gives the value today, or the *present value* of one unit of currency to be received at the end of that term. Denote the discount factor for t years by $d(t)$. Then, for example, if $d(.5)$ equals .99925, the present value of $1 to be received in six months is 99.925 cents. Another security, which pays $1,050,000 in six months, would have a present value of .99925 x $1,050,000 or $1,049,213.

Since Treasury bonds promise future cash flows, discount factors can be extracted from Treasury bond prices. In fact, each of the rows of Table 7-2 can be used to write one equation that relates prices to discount factors. The equation from the 1¼s of November 30, 2010, is

$$100.550 = \left(100 + \frac{1\frac{1}{4}}{2}\right)d(.5) \qquad (7.1)$$

In words, Equation (7.1) says that the price of the bond equals the present value of its future cash flows, namely its principal plus coupon payment, all times the discount factor for funds to be received in six months. Solving reveals that $d(.5)$ equals .99925.

By the same reasoning, the equations relating prices to discount factors can be written for the other bonds listed in Table 7-2. The next two of these equations are

$$104.513 = \frac{4\frac{7}{8}}{2} \times d(.5) + \left(100 + \frac{4\frac{7}{8}}{2}\right)d(1) \qquad (7.2)$$

$$105.856 = \frac{4\frac{1}{2}}{2} \times d(.5) + \frac{4\frac{1}{2}}{2} \times d(1) + \left(100 + \frac{4\frac{1}{2}}{2}\right)d(1.5) \qquad (7.3)$$

Given the solution for $d(.5)$ from Equation (7.1), Equation (7.2) can be solved for $d(1)$. Then, given the solutions for $d(.5)$ and $d(1)$, Equation (7.3) can be solved for $d(1.5)$. Continuing in this fashion through the rows of Table 7-2 generates the discount factors, in six-month intervals, out to four and one-half years, which are reported in Table 7-3. Note how these discount factors, falling with term, reflect the time value of money: the longer a payment of $1 is delayed, the less it is worth today.

THE LAW OF ONE PRICE

Another U.S. Treasury bond issue, one not included in the set of base bonds in Table 7-2, is the ¾s of November 30, 2011. How should this bond be priced? A natural answer is to apply the discount factors of Table 7-3 to this bond's cash flows. After all, the base bonds are all U.S. Treasury bonds and the value to investors of receiving $1 from a Treasury on some future date should not depend very

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TABLE 7-3 Discount Factors from U.S. Treasury Note and Bond Prices as of May 28, 2010

Term	Discount Factor
11/30/2010	.99925
5/31/2011	.99648
11/30/2011	.99135
5/31/2012	.98532
11/30/2012	.97520
5/31/2013	.96414
11/30/2013	.94693
5/31/2014	.93172
11/30/2014	.91584

much on which particular bond paid that $1. This reasoning is an application of the *law of one price*: absent confounding factors (e.g., liquidity, financing, taxes, credit risk), identical sets of cash flows should sell for the same price.

According to the law of one price, the price of the ¾s of November 30, 2011 should be

$$.375 \times .99925 + .375 \times .99648 + 100.375 \times .99135 = 100.255 \quad (7.4)$$

where each cash flow is multiplied by the discount factor from Table 7-3 that corresponds to that cash flow's payment date. As it turns out, the market price of this bond is 100.190, close to, but not equal to, the prediction of 100.255 in Equation (7.4).

Table 7-4 compares the market prices of three bonds as of May 28, 2010, to their present values (PVs), i.e., to their prices as predicted by the law of one price. The differences range from −2.8 cents to +6.5 cents per 100 face value, indicating that the law of one price describes the pricing of these bonds relatively well but not perfectly.

According to the last row of Table 7-4, the ⅞s of May 31, 2011, trade 2.8 cents *rich* to the base bonds, i.e., its market price is high relative to the discount factors in Table 7-3. In the same sense, the ¾s of November 30, 2011, and the ¾s of May 31, 2012, trade *cheap*. In fact, were these price discrepancies sufficiently large relative to transaction costs, an arbitrageur might consider trying to profit by selling

TABLE 7-4 Testing the Law of One Price for Three U.S. Treasury Notes as of May 28, 2010

Bond	⅞s 5/31/11	¾s 11/30/11	¾s 5/31/12
PV	100.521	100.255	100.022
Price	100.549	100.190	99.963
PV-Price	−.028	.065	.059

the rich ⅞s and simultaneously buying some combination of the base bonds; by buying either of the cheap bonds and simultaneously selling base bonds; or by selling the rich ⅞s and buying both of the cheap bonds in the table. Trades of this type, arising from deviations from the law of one price, are the subject of the next section.

ARBITRAGE AND THE LAW OF ONE PRICE

While the law of one price is intuitively reasonable, its justification rests on a stronger foundation. It turns out that a deviation from the law of one price implies the existence of an *arbitrage opportunity*, that is, a trade that generates profits without any chance of losing money.[1] But since arbitrageurs would rush *en masse* to do any such trade, market prices would quickly adjust to rule out any such opportunity. Hence, arbitrage activity can be expected to do away with significant deviations from the law of one price. And it is for this reason that the law of one price usually describes security prices quite well.

To make this argument more concrete, the discussion turns to an arbitrage trade based on the results of Table 7-4, which showed that the ¾s of November 30, 2011, are cheap relative to the discount factors in Table 7-3 or, equivalently, to the bonds listed in Table 7-2. The trade is to purchase the ¾s of November 30, 2011, and simultaneously sell or *short*[2] a portfolio of bonds from Table 7-2

[1] Market participants often use the term arbitrage more broadly to encompass trades that could conceivably lose money, but promise large profits relative to the risks borne.

[2] To short a security means to sell a security one does not own. For now, assume that a trader shorting a bond receives the price of the bond and is obliged to pay all its coupon and principal cash flows.

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TABLE 7-5 The Replicating Portfolio of the ¾s of November 30, 2011, with Prices as of May 28, 2010

	(1)	(2)	(3)	(4)	(5)	(6)
(i)	Coupon	1¼s	4⅞s	4½s		¾s
(ii)	Maturity	11/30/10	5/31/11	11/30/11	Portfolio	11/30/11
(iii)	Face Amount	−1.779	−1.790	98.166		100
	Date	**Cash Flows**				
(iv)	11/30/10	−1.790	−.044	2.209	.375	.375
(v)	5/31/11		−1.834	2.209	.375	.375
(vi)	11/30/11			100.375	100.375	100.375
(vii)	Price	100.550	104.513	105.856		100.190
(viii)	Cost	−1.789	−1.871	103.915	100.255	100.190
(ix)	Net Proceeds	.065				

that replicates the cash flows of the ¾s. Table 7-5 describes this *replicating portfolio* and the arbitrage trade.

Columns (2) to (4) of Table 7-5 correspond to the three bonds chosen from Table 7-2 to construct the replicating portfolio: the ¼s of November 30, 2010; the 4⅞s of May 31, 2011; and the 4½s of November 30, 2011. Row (iii) gives the face amount of each bond in the replicating portfolio, so that this portfolio is long 98.166 face amount of the 4½s, short 1.790 of the 4⅞s, and short 1.779 of the 1¼s. Rows (iv) through (vi) show the cash flows from those face amounts of each bond. For example, 98.166 face amount of the 4½s, which pay a coupon of 2.25% on May 31, 2011, generates a cash flow of 98.166 × 2.25% or 2.209 on that date. Similarly, −1.779 of the 1¼s, which pay coupon and principal totalling 100 + 1.25/2 or 100.625 per 100 face value on November 30, 2010, produces a cash flow of −1.779 × 100.625% or −1.790 on that date. Row (vii) gives the price of each bond per 100 face amount, simply copied from Table 7-2. Row (viii) gives the initial cost of purchasing the indicated face amount of each bond. So, for example, the "cost" of "purchasing" −1.790 face amount of the 4⅞s is −1.790 × 104.513% or −1.871. Said more naturally, the proceeds from selling 1.790 face amount of the 4⅞s are 1.871.

Column (5) of Table 7-5 sums columns (2) through (4) to obtain the cash flows and cost of the replicating portfolio. Rows (iv) through (vi) of column (5) confirm that the cash flows of the replicating portfolio do indeed match the cash flows of 100 face amount of the ¾s of November 30, 2011, given in the same rows of column (6). Note that most of the work of replicating the ¾s of November 30, 2011, is accomplished by the 4½s maturing on the same date. The other two bonds in the replicating portfolio are used for minor adjustments to the cash flows in six months and one year. Appendix A in this chapter shows how to derive the face amounts of the bonds in this or any such replicating portfolio.

With the construction of the replicating portfolio completed, the discussion returns to the arbitrage trade. According to row (viii) of Table 7-5, an arbitrageur can buy 100 face amount of the ¾s of November 30, 2011, for 100.190, sell the replicating portfolio for 100.255, pocket the difference or "net proceeds" of 6.5 cents, shown in row (ix), and not owe anything on any future date. And while a 6.5-cent profit may seem small, the trade can be scaled up: for $500 million face of the ¾s, which would not be an abnormally large position, the riskless profit increases to $500.000,000 × .065% or $325,000.

As stated at the start of this section, if a riskless and profitable trade like the one just described were really available, arbitrageurs would rush to do the trade and, in so doing, force prices to relative levels that admit no arbitrage opportunities. More specifically, arbitrageurs would drive the prices of the ¾s and of the replicating portfolio together until the two were equal.

The crucial link between arbitrage and the law of one price can now be explained. The total cost of the replicating portfolio, 100.255, given in column (5), row (viii) of Table 7-5, exactly equals the present value of the ¾s of November 30, 2011, computed in Table 7-4. In other words, the law of one price methodology of pricing the ¾s (i.e., discounting with factors derived from the 1¼s, 4⅞s, and 4½s) comes up with exactly the same value as does the arbitrage pricing methodology (i.e., calculating the value of portfolio of the 1¼s, 4⅞s, and 4½s that replicates the cash flows of the ¾s). This is not a coincidence. In fact, Appendix B in this chapter proves that these

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two pricing methodologies are mathematically identical. Hence, applying the law of one price, i.e., pricing with discount factors, is identical to relying on the activity of arbitrageurs to eliminate relative mispricings, i.e., pricing by arbitrage. Expressed another way, discounting can be justifiably regarded as shorthand for the more complex and persuasive arbitrage pricing methodology.

Despite this discussion, of course, the market price of the ¾s was quoted at a level somewhat below the level predicted by the law of one price. This can be attributed to one or a combination of the following reasons. First, there are transaction costs in doing arbitrage trades which could significantly lower or wipe out any arbitrage profit. In particular, the prices in Table 7-2 are mid-market whereas, in reality, an arbitrageur would have to buy securities at higher ask prices and sell at lower bid prices. Second, bid-ask spreads in the financing markets, incurred when shorting securities, might also overwhelm any arbitrage profit. Third, it is only in theory that U.S. Treasury bonds are commodities, i.e., fungible collections of cash flows. In reality, bonds have idiosyncratic differences that are recognized by the market and priced accordingly. And it is this last point that is the subject of the next section.

APPLICATION: STRIPS AND THE IDIOSYNCRATIC PRICING OF U.S. TREASURY NOTES AND BONDS

STRIPS

In contrast to coupon bonds that make payments every six months, *zero-coupon* bonds make no payments until maturity. Zero-coupon bonds issued by the U.S. Treasury are called STRIPS. For example, $1,000,000 face amount of STRIPS maturing on May 15, 2020, promises only one payment: $1,000,000 on that date. STRIPS are created when a particular coupon bond is delivered to the Treasury in exchange for its coupon and principal components. Table 7-6 illustrates the stripping of $1,000,000 face amount of the 3½s of May 15, 2020, which was issued in May 2010, to create coupon STRIPS maturing on the 20 coupon payment dates and principal STRIPS maturing on the maturity date. Coupon or interest STRIPS are called TINTs, INTs, or C-STRIPS while principal STRIPS are called TPs, Ps, or P-STRIPS. Note that the face amount of C-STRIPS on each date is 1/2 × 3.5% × $1,000,000 or $17,500.

TABLE 7-6 STRIPS Face Amounts from 1,000,000 Face Amount of the 3½s of May 15, 2020

Date	C-STRIP Face Amount	P-STRIP Face Amount
11/15/10	$17,500	0
5/15/11	$17,500	0
11/15/11	$17,500	0
⋮	⋮	⋮
5/15/19	$17,500	0
11/15/19	$17,500	0
5/15/20	$17,500	$1,000,000

The Treasury not only creates STRIPS but retires them as well. For example, upon delivery of the set of STRIPS in Table 7-6 the Treasury would *reconstitute* the $1,000,000 face amount of the 3½s of May 15, 2020. But in this context it is crucial to note that C-STRIPS are fungible while P-STRIPS are not. When reconstituting a bond, any C-STRIPS maturing on a particular date may be applied toward the coupon payment of that bond on that date. By contrast, only P-STRIPS that were stripped from a particular bond may be used to reconstitute the principal payment of that bond.[3] This feature of the STRIPS program implies that P-STRIPS, and not C-STRIPS, inherit the cheapness or richness of the bonds from which they came, an implication that will be demonstrated in the following subsection.

STRIPS prices are essentially discount factors. If the price of the C-STRIPS maturing on May 31, 2015, is 89.494 per 100 face amount, then the implied discount factor to that date is .89494. With this in mind, Figure 7-1 graphs the C-STRIPS prices per unit face amount as of May 28, 2010.

[3] Making P-STRIPS fungible would not affect either the total or the timing of cash flows owed by the Treasury, but could change the amounts outstanding of particular securities.

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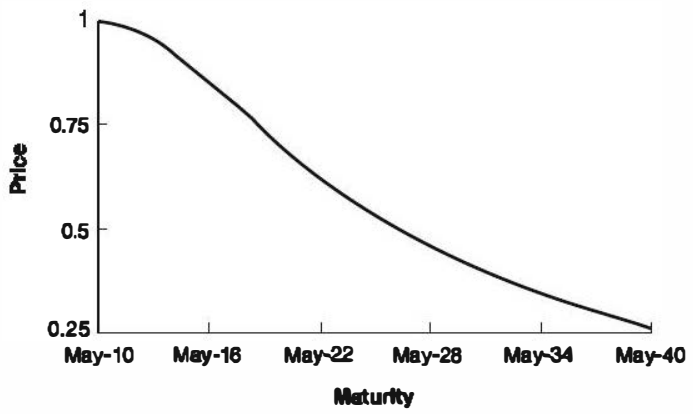

FIGURE 7-1 Discount factors from C-STRIPS prices as of May 28, 2010.

The Idiosyncratic Pricing of U.S. Treasury Notes and Bonds

If U.S. Treasury bonds were commodities, with each regarded solely as a particular collection of cash flows, then the price of each would be well approximated by discounting its cash flows with the C-STRIPS discount factors of Figure 7-1. If however individual bonds have unique characteristics that are reflected in pricing, the law of one price would not be as accurate an approximation. Furthermore, since C-STRIPS are fungible while P-STRIPS are not, any such pricing idiosyncrasies would manifest themselves as differences between the prices of P-STRIPS and C-STRIPS of the same maturity. To this end, Figure 7-2 graphs the differences between the prices of P-STRIPS and C-STRIPS that mature on the same date as of May 28, 2010. So, for example, with the price of P-STRIPS and C-STRIPS, both maturing on May 31, 2015, at 89.865 and 89.494, respectively, Figure 7-2 records the difference for May 31, 2015, as 89.865 − 89.494 or .371. Note that Figure 7-2 shows two sets of P-STRIPS prices, those P-STRIPS originating from Treasury bonds and those originating from Treasury notes.[4]

Inspection of Figure 7-2 shows that there are indeed significant pricing differences between P-STRIPS and C-STRIPS that mature on the same date. This does not necessarily imply the existence of arbitrage opportunities, as discussed at the end of the previous section. However, the results do suggest that bonds have idiosyncratic pricing differences and that these differences are inherited by their respective P-STRIPS. Of particular interest, for example, is the largest price difference in the figure, the 2.16 price difference between the P-STRIPS and C-STRIPS maturing on May 15, 2020. These P-STRIPS come from the most recently sold or *on-the-run* 10-year note, an issue which traditionally trades rich to other bonds because of its superior liquidity and financing characteristics. In any case, to determine whether idiosyncratic bond characteristics are indeed inherited by P-STRIPS, Table 7-7 analyzes the pricing of selected U.S. Treasury coupon securities in terms of STRIPS prices. The particular securities selected are those on the mid-month, May-November cycle with 10 or more years to maturity as of May 2010.

Columns (1) to (3) of Table 7-7 give the coupon, maturity, and market price of each bond. Column (4) computes a price for each bond by discounting all of its cash flows using the C-STRIPS prices in Figure 7-1, and column (5) gives the difference between the market price and that

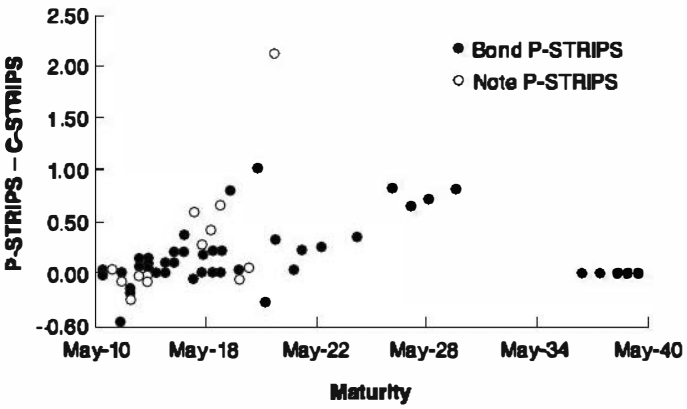

FIGURE 7-2 Differences between the prices of P-STRIPS and C-STRIPS maturing on the same date per 100 face amount as of May 28, 2010.

[4] The difference between notes and bonds is of historical interest only.

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TABLE 7-7 Market Prices Compared with Pricing Using C-STRIPS and with Pricing Using C-STRIPS for Coupon Payments and the Respective P-STRIPS for Principal Payments

(1) Coupon	(2) Maturity	(3) Market Price	(4) C-Pricing	(5) Error	(6) C-and P-Pricing	(7) Error
3½	5/15/20	101.896	99.820	2.076	101.982	−.086
8¾	5/15/20	146.076	145.738	.338	146.070	.006
6⅛	5/15/21	142.438	142.357	.080	142.407	.031
8	11/15/21	141.916	141.750	.167	141.980	−.063
7⅝	11/15/22	139.696	139.545	.151	139.805	−.109
7½	11/15/24	140.971	140.694	.277	141.059	−.087
6½	11/15/26	131.582	130.894	.687	131.716	−.134
6⅛	11/15/27	127.220	126.643	.578	127.291	−.070
5¼	11/15/28	116.118	115.456	.661	116.175	−.058
6¼	5/15/30	130.523	129.815	.708	130.639	−.116
5	5/15/37	113.840	112.916	.924	113.943	−.102
4½	5/15/38	105.114	104.625	.490	105.214	−.100
4¼	5/15/39	100.681	100.425	.256	100.764	−.083
4⅜	11/15/39	102.780	102.638	.143	102.905	−.124
4⅜	5/15/40	102.999	102.308	.691	102.969	.030

computed price. By the simplest application of the law of one price, these computed prices should be a good approximation of market prices. There are, however, some very significant discrepancies. The approximation misses the price of the 3½s of May 15, 2020, the 10-year on-the-run security, by a very large 2.076; the 5s of May 15, 2037, by .924; and the 6¼s of 5/15/30 by .708.

Column (6) of Table 7-7 computes the price of each bond by discounting its coupon payments with C-STRIPS prices and its principal payment with the P-STRIPS of that bond. Column (7) gives the difference between the market price and that computed price. To the extent that P-STRIPS prices inherit pricing idiosyncrasies of their respective bonds, these computed prices should be better approximations to market prices than the prices computed using C-STRIPS prices alone. And, in fact, this is the case. Comparing the absolute values of the two error columns reveals that the approximation in column (6) is better than the approximation in column (4) for every bond in the table.

In conclusion, then, individual Treasury bonds have idiosyncratic characteristics that are reflected in market prices. Furthermore, since P-STRIPS are not fungible across bonds, their prices inherit the idiosyncratic pricing of their respective bond issues.

ACCRUED INTEREST

This section describes the useful market practice of separating the full price of a bond, which is the price paid by a buyer to a seller, into two parts: a quoted or flat price, which is the price that appears on trading screens and is used when negotiating transactions; and *accrued interest*. The full and quoted prices are also known as the *dirty* and *clean* prices, respectively.

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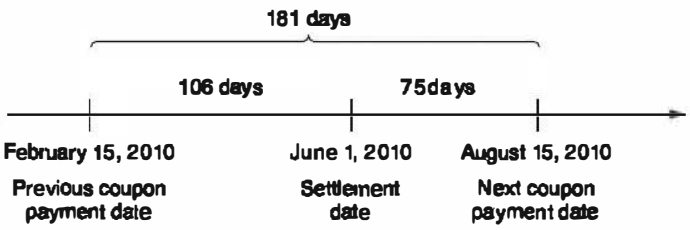

FIGURE 7-3 Example of accrued interest time line.

Definition

To make the concepts concrete, consider an investor who purchases $10,000 face amount of the U.S. Treasury 3⅝s of August 15, 2019, for settlement on June 1, 2010. The bond made a coupon payment of ½ × 3⅝% × $10,000 or $181.25 on February 15, 2010, and will make its next coupon payment of $181.25 on August 15, 2010. See the time line in Figure 7-3.

Assuming the purchaser holds the bond through the next coupon date, the purchaser will collect the coupon on that date. But it can be argued that the purchaser is not entitled to the full semiannual coupon payment on August 15 because, as of that time, the purchaser will have held the bond for only two and a half months of a six-month coupon period. More precisely, using what is known as the *actual/actual day-count convention*, which will be explained later in this section, and referring again to Figure 7-3, the purchaser should receive only 75 of 181 days of the coupon payment, that is, 75/181 × $181.25, or $75.10. The seller of the bond, whose cash was invested in the bond from February 15 to June 1, should collect the rest of the coupon, i.e., 106/181 × $181.25, or $106.15. A conceivable institutional arrangement is for the seller and purchaser to divide the coupon on the payment date, but this would undesirably require additional arrangements to ensure that this split of the coupon actually takes place. Consequently, market convention dictates instead that the purchaser pay the $106.15 of accrued interest to the seller on the settlement date and that the purchaser keep the entire coupon of $181.25 on the coupon payment date.

On May 28, 2010, for delivery on June 1, 2010, the flat or quoted price of the 3⅝s was 102-26, meaning 102 + 26/32 or 102.8125. The full or invoice price of the bond per 100 face amount is defined as the quoted price plus accrued interest, which, in this case, is 102.8125 + 1.0615 or 103.8740. For this particular trade, of $10,000 face amount, the invoice price is $10,387.40.

At this point, by the way, it becomes clear why discussion earlier in the chapter had to make reference to the fact that prices were full prices. When trading bonds that make coupon payments on May 31, 2010, for settlement on June 1, 2010, purchasers have to pay one day of accrued interest to sellers.

Pricing Implications

The present value of a bond's cash flows should be equated or compared with its full price, that is, with the amount a purchaser actually pays to purchase those cash flows. Conceptually, denoting the flat price by p, accrued interest by AI, the present value of the cash flows by PV, and the full price, as before, by P,

$$P = p + AI = PV \tag{7.5}$$

Equation (7.5) reveals an important point about accrued interest: the particular market convention used in calculating accrued interest does not really matter. Say, for example, that everyone recognizes that the convention in place is too generous to the seller because, instead of being made to wait for a share of the interest until the next coupon date, the seller receives that share at settlement. In that case, by Equation (7.5), the flat price would adjust downward to mitigate this advantage. Put another way, the only quantity that matters is the invoice price, which determines the amount of money that changes hands.

Having made this argument, why is the accrued interest convention useful in practice? The answer is told in Figure 7-4, which draws the full and flat prices of the 3⅝s of August 15, 2019, from February 15, 2010, to September 15, 2010, under several assumptions, with the most important being that 1) the discount function does not change, i.e., $d(t)$ does not change, where t is the number of days from settlement; and 2) the flat price of the bond for settlement on June 1 is 102.8125. In words, then, Figure 7-4 says that the full price changes dramatically over time even when the market is unchanged, including a discontinuous jump on coupon payment dates, while the flat price changes only gradually over time. Therefore, when trading bonds day to day, it is more intuitive to track flat prices and negotiate transactions in those terms.

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FIGURE 7-4 Full and flat prices for the 3⅝s of August 15, 2019, over time with a constant discount function.

The shapes of the price functions in Figure 7-4 can be understood as follows. Within a coupon period, the full price of the bond, which is just the present value of its cash flows, increases over time as the bond's payments draw near. But from an instant before the coupon payment date to an instant after, the full price falls by the coupon payment: the coupon is included in the present value of the remaining cash flows at the instant before the payment, but not at the instant after. Basically, however, the flat price of a bond like the 3⅝s, which sells for more than its face value, will trend down to its value at maturity, i.e., par.

Day-Count Conventions

Accrued interest equals the coupon times the fraction of the coupon period from the previous coupon payment date to the settlement date. For the 3⅝s, as for most government bonds, this fraction is calculated by dividing the actual number of days since the previous coupon date by the actual number of days in the coupon period. Hence the term "actual/actual" for this day-count convention.

Other day-count conventions, however, are applied in other markets. Two of the most common are *actual/360* and *30/360*. The actual/360 convention divides the actual number of days between two dates by 360, and is commonly used in money markets, i.e., for short-term, *discount* (i.e., zero-coupon) securities, and for the floating legs of interest rate swaps. The *30/360* convention assumes that there are 30 days in a month when calculating the difference between two dates and then divides by 360. Applying this convention, the number of days between June 1 and August 15 is 74 (29 days left in June, 30 days in July, and 15 days in August), as opposed to the 75 days using an actual day count. The 30/360 convention is used most commonly for corporate bonds and for the fixed leg of interest rate swaps.

APPENDIX A

Deriving Replicating Portfolios

To replicate the ¾s of November 30, 2011, Table 7-5 uses the 1¼s due November 30, 2010, the 4⅞s due May 31, 2011, and the 4½s due November 30, 2011. Number these bonds from 1 to 3 and let F^i be the face amount of bond i in the replicating portfolio. Then, the following equations express the requirement that the cash flows of the replicating portfolio equal those of the ¾s on each of the three cash flow dates.

For the cash flow on November 30, 2010:

$$\left(100\% + \frac{1\frac{1}{4}\%}{2}\right)F^1 + \left(\frac{4\frac{7}{8}\%}{2}\right)F^2 + \left(\frac{4\frac{1}{2}\%}{2}\right)F^3 = \frac{\frac{3}{4}\%}{2} \quad (7.6)$$

For the cash flow on May 31, 2011:

$$0 \times F^1 + \left(100\% + \frac{4\frac{7}{8}\%}{2}\right)F^2 + \left(\frac{4\frac{1}{2}\%}{2}\right)F^3 = \frac{\frac{3}{4}\%}{2} \quad (7.7)$$

And, for the cash flow on November 30, 2011:

$$0 \times F^1 + 0 \times F^2 + \left(100\% + \frac{4\frac{1}{2}\%}{2}\right)F^3 = 100\% + \frac{\frac{3}{4}\%}{2} \quad (7.8)$$

Solving Equations (7.6), (7.7), and (7.8) for F^1, F^2, and F^3 gives the replicating portfolio's face amounts in Table 7-5. Note that since one bond matures on each date, these equations can be solved one-at-a-time instead of simultaneously, i.e., solve (7.8) for F^3, then, using that result, solve (7.7) for F^2, and then, using both results, solve (7.6) for F^1. In any case, the results are as follows:

$$F^1 = -1.779\% \quad (7.9)$$

$$F^2 = -1.790\% \quad (7.10)$$

$$F^3 = 98.166\% \quad (7.11)$$

Replicating portfolios are easier to describe and manipulate using matrix algebra. To illustrate, Equations (7.6) through (7.8) are expressed in matrix form as follows:

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$$\begin{pmatrix} 1+\frac{1.25\%}{2} & \frac{4.875\%}{2} & \frac{4.5\%}{2} \\ 0 & 1+\frac{4.875\%}{2} & \frac{4.5\%}{2} \\ 0 & 0 & 1+\frac{4.5\%}{2} \end{pmatrix} \begin{pmatrix} F^1 \\ F^2 \\ F^3 \end{pmatrix} = \begin{pmatrix} \frac{.75\%}{2} \\ \frac{.75\%}{2} \\ 1+\frac{.75\%}{2} \end{pmatrix} \quad (7.12)$$

Note that each column of the leftmost matrix describes the cash flows of one of the bonds in the replicating portfolio; the elements of the vector to the right of this matrix are the face amounts of each bond for which Equation (7.12) has to be solved; and the rightmost vector contains the cash flows of the bond to be replicated. This equation can easily be solved by pre-multiplying each side by the inverse of the leftmost matrix.

In general then, suppose that the bond to be replicated makes payments on T dates. Let C be the $T \times T$ matrix of cash flows, principal plus interest, with the T columns representing the T bonds in the replicating portfolio and the T rows the dates on which those bonds make payments. Let $\vec{F}$ be the $T \times 1$ vector of face amounts in the replicating portfolio and let $\vec{c}$ be the vector of cash flows, principal plus interest, of the bond to be replicated. Then, the replication equation is

$$C\,\vec{F} = \vec{c} \quad (7.13)$$

with solution

$$\vec{F} = C^{-1}\,\vec{c} \quad (7.14)$$

The only complication is in ensuring that the matrix C does have an inverse. Essentially, any set of T bonds will do so long as there is at least one bond in the replicating portfolio making a payment on each of the T dates. In this case, the T bonds would be said to *span* the payment dates. So, for example, T bonds all maturing on the last date would work, but T bonds all maturing on the second-to-last date would not work: in the latter case there would be no bond in the replicating portfolio making a payment on date T.

APPENDIX B

The Equivalence of the Discounting and Arbitrage Pricing Approaches

Proposition: Pricing a bond according to either of the following methods gives the same price:

- Derive a set of discount factors from some set of spanning bonds and price the bond in question using those discount factors.
- Find the replicating portfolio of the bond in question using that same set of spanning bonds and calculate the price of the bond as the price of this portfolio.

Proof: Continue using the notation introduced at the end of Appendix A. Also, let $\vec{d}$ be the $T \times 1$ vector of discount factors for each date and let $\vec{P}$ be the vector of prices of each bond in the replicating portfolio, which is the same as the vector of prices of each bond used to compute the discount factors. Generalizing the "Discount Factors" section of this chapter, one can solve for discount factors using the following equation:

$$\vec{d} = (C')^{-1}\,\vec{P} \quad (7.15)$$

where the ′ denotes the transpose. Then, the price of the bond according to the first method is $c'\vec{d}$. The price according to the second method is $\vec{P}'\vec{F}$ where $\vec{F}$ is as derived in Equation (7.14).

Hence, the two methods give the same price if

$$\vec{c}'\vec{d} = \vec{P}'\vec{F} \quad (7.16)$$

Expanding the left-hand side of Equation (7.16) with (7.15) and the righthand side with (7.14),

$$\vec{c}'(C')^{-1}\vec{P} = \vec{P}'C^{-1}\vec{c} \quad (7.17)$$

And since both sides of this equation are just numbers, take the transpose of the left-hand side to show that Equation (7.17) is true.

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Spot, Forward, and Par Rates

8

Learning Objectives

After completing this reading you should be able to:

- Calculate and interpret the impact of different compounding frequencies on a bond's value.
- Calculate discount factors given interest rate swap rates.
- Compute spot rates given discount factors.
- Interpret the forward rate, and compute forward rates given spot rates.
- Define par rate and describe the equation for the par rate of a bond.
- Interpret the relationship between spot, forward, and par rates.
- Assess the impact of maturity on the price of a bond and the returns generated by bonds.
- Define the "flattening" and "steepening" of rate curves and describe a trade to reflect expectations that a curve will flatten or steepen.

Excerpt is Chapter 2 of Fixed Income Securities, *Third Edition, by Bruce Tuckman.*

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It is clear from Chapter 7 that price and cash flows completely describe any fixed-rate investment. Nevertheless, investors and traders almost always find it more intuitive to express the time value of money in terms of interest rates. This chapter, therefore, introduces the most commonly-used interest rates, which are spot rates, forward rates, and par rates. The relationships linking these rates to discount factors and to each other reveal why interest rates are so intuitively appealing.

Given the importance of interest rate swaps as a benchmark of market interest rates, the illustrative examples and the trading case study of this chapter are taken from global swap markets. The valuation of interest rates swaps, however, is not covered by this book. The reader is asked to accept the assertion, made here, that interest rates embedded in the swap market can be properly extracted by treating the fixed side of a swap as if it were a coupon bond and the floating side as if it were a floating rate bond worth par.

The trading case study of this chapter begins by highlighting the abnormally downward-sloping forward rates of the EUR swap curve in the second quarter of 2010. Then, in the context of macroeconomic factors and market technicals, a trade is constructed to take advantage of this abnormally-shaped curve.

SIMPLE INTEREST AND COMPOUNDING

Price and cash flows completely describe an investment: a bond might cost 101.980 today and pay 103 in six months; a 100,000,000 1.5-year loan, six months *forward* (i.e., a loan made in six months for 1.5 years) might pay 103,797,070 in two years. But investors and traders often prefer to quote and think in terms of interest rates, saying that the bond just described earns 2% and the forward loan 2.5%. Interest rates are more intuitive than prices because they automatically normalize for the amount invested and, expressed as annual rates, normalize for the investment horizon as well. So even though the bond costs 101.98 and matures in six months while the forward loan invests 100,000,000 for 1.5 years, the interest rates on the two investments can be sensibly and intuitively compared.

The purpose of this section is to describe the conventions through which interest rates are quoted given prices and cash flows. The most straightforward convention is *simple interest*, in which interest paid is the quoted, annualized rate times the term of the investment, in years. While the discussion of day-count conventions in Chapter 4 showed that there are many ways to define the term of an investment in years, in the context of this chapter semiannual periods are defined to have a term of half a year. Continuing then with the bond example of the previous paragraph, the six-month bond earns 2% because

$$101.98 + 101.98 \times \frac{2\%}{2} = 101.98 \times \left(1 + \frac{2\%}{2}\right) = 103 \qquad (8.1)$$

In words, a simple interest investment is conceptualized as making a single payment at maturity equal to the initial investment amount plus interest on that initial investment. In Equation (8.1), the initial investment is 101.980 and the interest earned is that 101.98 times 2%/2, where the latter is one-half the quoted, annual rate of 2%. The sum of these two is the bond's total payment of 103.

The forward loan example introduced at the start of this section has a term of 1.5 years or of three semiannual periods, requiring an outlay of 100 million in six months for a terminal payment of 103,797,070 in two years. Under the convention of *semiannual compounding*, an investment is conceptualized as follows. First, simple interest is earned within each six-month period. Second, each six-month period's total proceeds, that is, both principal and interest, are reinvested for the subsequent six-month period. So, in the case of the forward loan earning a rate of 2.5%, the proceeds from earning simple interest over the first six months are

$$100{,}000{,}000 \times \left(1 + \frac{2.5\%}{2}\right) = 101{,}250{,}000 \qquad (8.2)$$

Then, reinvesting this total amount over the subsequent six months at the same rate produces a total of

$$101{,}250{,}000 \times \left(1 + \frac{2.5\%}{2}\right) = 100{,}000{,}000 \times \left(1 + \frac{2.5\%}{2}\right)^2 = 102{,}515{,}625 \qquad (8.3)$$

To appreciate the impact of compounding, note that an investment of 100 million earning simple interest of 2.5% over a year would be worth 100,000,000 × (1 + 1 × 2.5%) or 102,500,000. The 15,625 difference between the semiannually compounded proceeds in (8.3) and this simple interest amount is exactly equal to the *interest on interest*, that is, the interest earned in the second six-month period on the interest earned over the first six-month

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period. More specifically, the interest over the first period, from (8.2), is 1,250,000 and the interest on that amount for six months is 1,250,000 × 2.5%/2 or 15,625.

Returning now to the forward loan, over the last of its three semiannual periods, the proceeds grow to

$$102{,}515{,}625 \times \left(1+\frac{2.5\%}{2}\right) = 100{,}000{,}000 \times \left(1+\frac{2.5\%}{2}\right)^3 \quad \textbf{(8.4)}$$
$$= 103{,}797{,}070$$

which is the terminal payoff set out in the example.

Generalizing this discussion, investing F at a rate of $\hat{r}$ compounded semiannually for T years generates

$$F \times \left(1+\frac{\hat{r}}{2}\right)^{2T} \quad \textbf{(8.5)}$$

at the end of those T years. (Note that the power in this expression is $2T$ since an investment for T years compounded semiannually is, in fact, an investment for $2T$ half-year periods.)

This discussion has been framed in terms of semiannual compounding because coupon bonds and the fixed side of interest rate swaps most commonly pay interest semiannually. Other compounding conventions, including *continuous compounding* (for which interest is assumed to be paid every instant), are useful in other contexts and are presented in Appendix A in this chapter.

EXTRACTING DISCOUNT FACTORS FROM INTEREST RATE SWAPS

As the examples of this chapter are drawn from global swap markets, this section digresses with a very brief introduction to interest rate swaps.

Two parties might agree, on May 28, 2010, to enter into an *interest rate swap* with the following terms. Starting in two business days, on June 2, 2010, party A agrees to pay a fixed rate of 1.235% on a notional amount of $100 million to party B for two years, who, in return, agrees to pay *three-month LIBOR (London Interbank Offered Rate)* on this same notional to Party A. See Figure 8-1. For the present, suffice it to say that three-month LIBOR is the rate at which the most creditworthy banks can borrow money from each other for three months and that a *fixing* of this rate is published once each trading day.

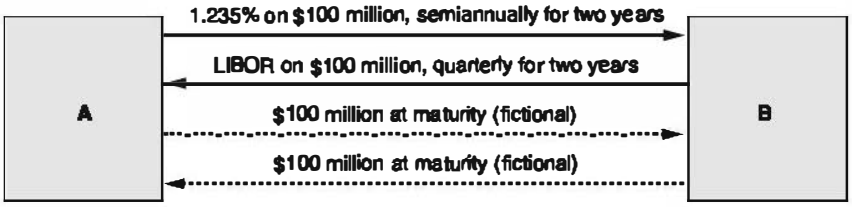

FIGURE 8-1 An example of an interest rate swap.

The $100 million in the example is called the notional amount of a swap, rather than the face, par, or principal amount, because it is used only to compute the fixed- and floating-rate payments: the $100 million itself is never paid or received by either party. In any case, party A, who *pays fixed* and *receives floating*, makes fixed payments of 1.235%/2 × $100,000,000, or $617,500 every six months. Party B, who *receives fixed* and *pays floating*, makes floating rate payments quarterly.

While swap contracts do not include any payment of the notional amount, it is convenient to assume that, at maturity, party A pays the notional amount to party B and that party B pays that same notional amount to party A. Once again, see Figure 8-1. There are three points to be made about these fictional payments. First, since they cancel each other, their inclusion has no effect on the value of the swap. Second, adding the fictional notional amount to the fixed side makes that leg of the swap look like a coupon bond, i.e., a security with semiannual, fixed coupon payments and a terminal principal payment. Third, adding the fictional notional amount to the floating side makes that leg look like a floating rate bond, i.e., a security with semiannual, floating coupon payments and a terminal principal payment.

The widely-used valuation methodology in which the floating leg of the swap, with its fictional notional amount, is worth par, or $100 million, on payment dates. Taking this as given for the purposes of this chapter, an interest rate swap can be viewed in a very simple way: party B, the fixed receiver, "buys" a 1.235% semiannually-paying coupon bond (i.e., the fixed leg) for $100 million (i.e., the value of the floating leg). Party A, the fixed payer, "sells" a 1.235% bond for $100 million. This interpretation of swaps is so useful and commonplace that the phrase, the "fixed leg of a swap," is almost always meant to include the fictional notional payment at maturity.

Invoking the interpretation of swaps in the previous paragraph, discount factors can be derived from swaps using

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TABLE 8-1 Discount Factors, Spot Rates, and Forward Rates Implied by Par USD Swap Rates as of May 28, 2010

Term in Years	Swap Rate	Discount Factor	Spot Rate	Forward Rate
0.5	.705%	.996489	.705%	.705%
1.0	.875%	.991306	.875%	1.046%
1.5	1.043%	.984494	1.045%	1.384%
2.0	1.235%	.975616	1.238%	1.820%
2.5	1.445%	.964519	1.450%	2.301%

the methodology of Chapter 7, developed in the context of coupon bonds. To illustrate this, along with the rate calculations of later sections, Table 8-1 presents some data on shorter-maturity, USD interest rate swaps as of May 28, 2010. The second column gives the rates that are quoted and observed in swap market trading. These indicate that counterparties are willing to exchange fixed payments of .875% against three-month LIBOR for one year, 1.043% against three-month LIBOR for 1.5 years, etc. The 2-year swap rate, depicted in Figure 8-1, is 1.235%. In any case, to derive the third column of Table 8-1, the discount factors implied by swap rates, proceed as in Chapter 7. Write an equation for each "bond" that equates the present value of its cash flows to its price of par, i.e.,

$$\left(100+\frac{.705}{2}\right)d(.5)=100 \tag{8.6}$$

$$\frac{.875}{2}d(.5)+\left(100+\frac{.875}{2}\right)d(1)=100 \tag{6.7}$$

etc. The set of five such equations, corresponding to the maturities .5 through 2.5, allows for the solution of the discount factors given in the third column of the table.

The derivation of spot and forward rates, the fourth and fifth columns of Table 8-1, along with the relationships across all of these rates, is the subject of the rest of the chapter.

DEFINITIONS OF SPOT, FORWARD, AND PAR RATES

Chapter 7 defined a curve of discount factors, $d(t)$, which gives the present values of one unit of currency to be received at various times t. This section expresses the information in the discount curve as a *term structure of interest rates* and, in particular, in terms of semiannually-compounded *spot*, *forward*, and *par* rates. Definitions of continuously compounded spot and forward rates can be found in Appendix B in this chapter.

Spot Rates

A spot rate is the rate on a *spot loan*, an agreement in which a lender gives money to the borrower at the time of the agreement to be repaid at some single, specified time in the future. Denote the semiannually compounded t-year spot rate by $\hat{r}(t)$. Then, following (8.5), investing 1 unit of currency from now to year t will generate proceeds at that time of

$$\left(1+\frac{\hat{r}(t)}{2}\right)^{2t} \tag{6.6}$$

To link spot rates and discount factors, note that if \$1 grows to the quantity (8.8) in t years, then the present value of that quantity is \$1. Using discount factors to compute that present value,

$$\left(1+\frac{\hat{r}(t)}{2}\right)^{2t}d(t)=1 \tag{6.9}$$

Then, solving for $d(t)$ gives

$$d(t)=\frac{1}{\left(1+\frac{\hat{r}(t)}{2}\right)^{2t}} \tag{8.10}$$

Table 8-1 gives the discount factors from the USD swap curve as of May 28, 2010. Taking the 2-year discount factor of .975616 from that table, Equation (8.10) can be used to derive the 2-year, semiannually-compounded spot rate of 1.238%:

$$d(2)=\frac{1}{\left(1+\frac{1.238\%}{2}\right)^{2\times 2}}=.975616 \tag{8.11}$$

From (8.8), this rate implies that, in two years, an investment of \$100 grows to

$$\$100\times\left(1+\frac{1.238\%}{2}\right)^{2\times 2}=\$102.499 \tag{8.12}$$

Forward Rates

A *forward* rate is the rate on a *forward loan*, which is an agreement to lend money at some time in the future to be repaid some time after that. There are many possible forward rates: the rate on a loan given in six months for a

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subsequent term of 1.5 years; the rate in five years for six months; etc. This subsection, however, focuses exclusively on forward rates over sequential, six-month periods. Let $f(t)$ denote the forward rate on a loan from year $t - .5$ to year t. Then, investing 1 unit of currency from year $t - .5$ for six months generates proceeds, at year t, of

$$\left(1+\frac{f(t)}{2}\right) \tag{8.13}$$

To link forward rates to spot rates, note that a spot loan for $t - .5$ years combined with a forward loan from year $t - .5$ to year t covers the same investment period as a spot loan to year t. To ensure that rates are quoted consistently, that is, to ensure that the proceeds from these identical investments are the same,

$$\begin{aligned}\left(1+\frac{\hat{r}(t)}{2}\right)^{2t} &= \left(1+\frac{\hat{r}(t-.5)}{2}\right)^{2(t-.5)}\left(1+\frac{f(t)}{2}\right) \\ &= \left(1+\frac{\hat{r}(t-.5)}{2}\right)^{2t-1}\left(1+\frac{f(t)}{2}\right)\end{aligned} \tag{8.14}$$

This logic can be extended further, to write the spot rate of term t as a function of all forward rates up to $f(t)$:

$$\left(1+\frac{\hat{r}(t)}{2}\right)^{2t} = \left(1+\frac{f(.5)}{2}\right)\left(1+\frac{f(1)}{2}\right)\cdots\left(1+\frac{f(t)}{2}\right) \tag{8.15}$$

Finally, to express forward rates in terms of discount factors, simply use Equation (8.10) to replace the spot rates in (8.14) with discount factors:

$$\left(1+\frac{f(t)}{2}\right) = \frac{d(t-.5)}{d(t)} \tag{8.16}$$

Continuing with the USD swap data in Table 8-1, use the 2- and 2.5-year spot rates or discount factors from the table, together with (8.14) or (8.16), to derive that $f(2.5) = 2.301\%$. This value implies that an investment of \$100 in 2 years will, in 2.5 years, be worth

$$\$100 \times \left(1+\frac{2.301\%}{2}\right) = \$101.151 \tag{8.17}$$

In passing, note that if the term structure of spot interest rates is *flat*, so that all spot rates are the same, i.e., $\hat{r}(t) = \hat{r}$ for all t, then, from (8.14), each forward rate must equal that same $\hat{r}$ and the term structure of forward interest rates is flat as well.

Par Rates

Consider 100 face or notional amount of a fixed-rate asset that makes regular semiannual coupon or fixed-rate payments of 100 × ½ and a terminal payment at year T of that 100. The T-year, semiannual *par* rate is the rate $C(T)$ such that the present value of this asset equals par or 100. But that is exactly the definition of swap rates given earlier in this chapter. Hence, swap rates in Table 8-1 are, in fact, par rates. For example, for the 2-year swap rate of 1.235%,

$$\frac{1.235}{2}[d(.5) + d(1) + d(1.5) + d(2)] + 100d(2) = 100 \tag{8.18}$$

This equality can be verified by substituting the discount factors from Table 8-1 into (8.18), but this comes as no surprise: the discount factors from that table are derived from a set of pricing equations that included (8.18).

In general, for an asset with a par amount of one unit that makes semiannual payments and matures in T years,

$$\frac{C(T)}{2}\sum_{t=1}^{2T} d\left(\frac{t}{2}\right) + d(T) = 1 \tag{8.19}$$

The sum in Equation (8.19), i.e., the value of one unit of currency to be received on every payment date until maturity in T years, is often called an *annuity factor* and denoted by $A(T)$. For semiannual payments,

$$A(T) = \sum_{t=1}^{2T} d\left(\frac{t}{2}\right) \tag{8.20}$$

Using the discount factors from Table 8-1, for example, $A(2)$ is about 3.948. In any case, substituting the annuity notation of (8.20) into (8.19), the par rate equation can also be written as

$$\frac{C(T)}{2}A(T) + d(T) = 1 \tag{8.21}$$

If the term structure of spot or forward rates is flat at some rate, then the term structure of par rates is flat at that same rate. This is proven in Appendix C in this chapter.

Before closing this subsection it is important to point out that a bond with a price of par, or the fixed leg of a swap worth par, may he valued at par only for a moment. As interest rates and discount factors change, the present values of these bonds or swaps change as well and the assets cease to be "par" bonds or swaps.

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Synopsis: Quoting Prices with Semiannual Spot, Forward, and Par Rates

Chapter 7 showed that prices of fixed-rate assets can be expressed in terms of discount factors and this section showed that spot, forward, and par rates can be expressed in terms of discount factors. Hence, prices of fixed-rate assets can be expressed in terms of either discount factors or rates. For review and easy reference, this subsection collects these relationships for a unit par amount of a fixed-rate asset with price P that makes semiannual payments at a rate c for T years and then returns par. Using discount and annuity factors,

$$P = \frac{c}{2}A(T) + d(T) \tag{8.22}$$

Using spot rates,

$$P = \frac{c}{2}\left[\frac{1}{\left(1+\frac{\hat{r}(.5)}{2}\right)} + \frac{1}{\left(1+\frac{\hat{r}(1)}{2}\right)^2} + \cdots + \frac{1}{\left(1+\frac{\hat{r}(T)}{2}\right)^{2T}}\right] + \frac{1}{\left(1+\frac{\hat{r}(T)}{2}\right)^{2T}} \tag{8.23}$$

Using forward rates,

$$P = \frac{c}{2}\left[\frac{1}{\left(1+\frac{f(.5)}{2}\right)} + \frac{1}{\left(1+\frac{f(.5)}{2}\right)\left(1+\frac{f(1)}{2}\right)} + \cdots + \frac{1}{\left(1+\frac{f(.5)}{2}\right)\left(1+\frac{f(1)}{2}\right)\cdots\left(1+\frac{f(T)}{2}\right)}\right] + \frac{1}{\left(1+\frac{f(.5)}{2}\right)\left(1+\frac{f(1)}{2}\right)\cdots\left(1+\frac{f(T)}{2}\right)} \tag{8.24}$$

And finally, using the par rate, $C(T)$, subtract (8.21) from (8.22) to obtain

$$P = 1 + \frac{c - C(T)}{2}A(T) \tag{8.25}$$

CHARACTERISTICS OF SPOT, FORWARD, AND PAR RATES

The six-month spot rate is identically equal to the corresponding forward rate: both are rates on a six-month loan starting on the settlement date. But an interesting first observation from Table 8-1 is that each of the other spot rates is nearly equal to the average of all the forward rates of equal and lower term. Taking the 2.5-year spot rate, for example,

$$1.450\% \approx \frac{.705\% + 1.046\% + 1.384\% + 1.820\% + 2.301\%}{5} \tag{8.26}$$

Intuitively this is not at all surprising: the interest rate on a 2.5-year loan is approximately equal to the average of the rates on a six-month loan and on six-month loans six months, one year, one and a half years, and two years forward. Mathematically, the proceeds from the 2.5-year spot loan must be the same as those from the five forward loans:

$$\left(1+\frac{\hat{r}(2.5)}{2}\right)^5 = \left(1+\frac{f(.5)}{2}\right)\left(1+\frac{f(1)}{2}\right)\left(1+\frac{f(1.5)}{2}\right) \times \left(1+\frac{f(2)}{2}\right)\left(1+\frac{f(2.5)}{2}\right) \tag{8.27}$$

So while the 2.5-year spot rate is, strictly speaking, a complex average of the first five six-month forward rates, the simple average is usually a very good approximation.[1]

A second observation from Table 8-1 is that spot rates are increasing with term while forward rates are greater than spot rates. This is not a coincidence. It has just been established that spot rates are an average of forward rates. Furthermore, adding a number to an average increases that average if and only if the added number is larger than the pre-existing average. Using the data in the table, adding the 2-year forward of 2.301% to the 2-year "average" or spot rate of 1.238%, gives a higher new "average" or 2.5-year spot rate of 1.450%. Appendix E in this chapter proves in general that, for any t, $\hat{r}(t) > \hat{r}(t - .5)$ if and only if $f(t) > \hat{r}(t - .5)$ and that $\hat{r}(t) < \hat{r}(t - .5)$ if and only if $f(t) < \hat{r}(t - .5)$. These are period-by-period statements and, as such, do not necessarily extend to entire spot and forward rate curves. In practice, however, spot rates increase or decrease over relatively wide maturity ranges and therefore forward rates are above or below spot rates over relatively wide maturity ranges. Figures 8-2 and 8-3, of the EUR and GBP swap curves as of May 28, 2010, illustrate typical relationships between spot and forward rate curves. In each

[1] Very precisely, one plus half the spot rate is a geometric average of one plus half of each of the forward rates. But a first-order Taylor series approximation to the geometric average is, in fact, the arithmetic average, and is relatively accurate since interest rates are usually small numbers.

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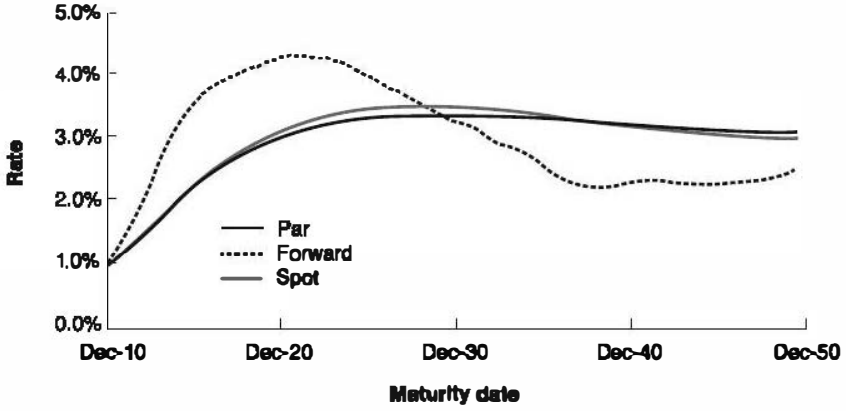

FIGURE 8-2 EUR swap curves as of May 28, 2010.

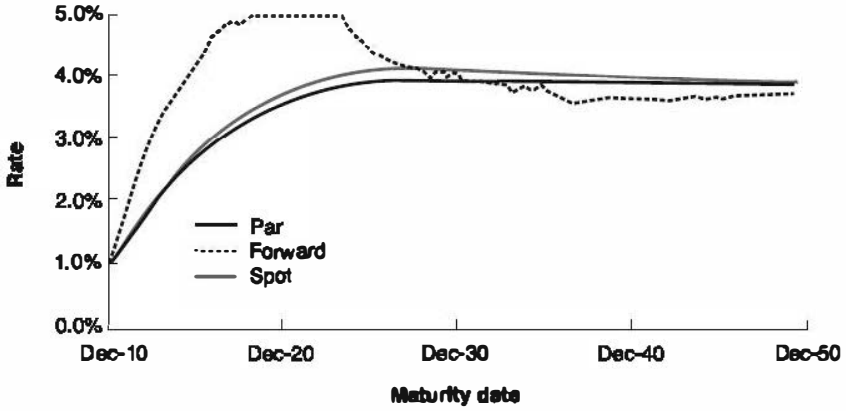

FIGURE 8-3 GBP swap curves as of May 28, 2010.

currency, the spot rate curve increases with term while forward rates are above spot rates, but, as forward rates cross from above to below the spot rates, the spot rate curve begins to decrease with term.

A third and final observation from Table 8-1 is that while spot rates are increasing with term, par rates are near, but below, spot rates. To understand the intuition here, consider the 2.5-year par and spot rates of 1.445% and 1.450%, respectively. From the discussion earlier in this chapter, were the spot rate curve flat at 1.450%, the par rate would be 1.450% as well. In other words, discounting fixed payments of 1.450% at a flat spot rate curve of 1.450% would give a price of par. But this means that discounting 1.450% payments at the spot rates in Table 8-1, which are all less than or equal to 1.450%, would give a price greater than par. Hence, discounting with the spot rates in the table, the par rate must be below 1.450%. More generally, Appendix F in this chapter proves that when the spot rate curve is strictly upward-sloping, par rates are below equal-maturity spot rates and that when spot rates are strictly downward-sloping, par rates are above equal-maturity spot rates. USD swap rate curves as of May 28, 2010, shown in Figure 8-4, illustrate how par rates are below spot rates as spot rates increase over most of the maturity range. By the end of the year 2041 the spot rate curve starts to decrease very gradually, but not nearly enough for par rates to exceed spot rates. By contrast, the EUR spot rate curve in Figure 8-2 does decrease rapidly enough at the longer maturities for the par rate curve to rise above the spot rate curve.

Maturity and Price or Present Value

If the term structure of rates remains completely unchanged over a six-month period, will the price of a bond or the present value of the fixed side of a swap increase or decrease over the period?

Table 8-2 explores this question by computing the present value of the fixed sides of swaps paying 1.445% to different maturities using the discount factors or rates from Table 8-1. Since 1.445% is the 2.5-year par rate, the present value of 100 face amount of the fixed side of the 2.5-year swap is 100. Six months later, should the term structure be exactly the same, the swap would be a two-year swap and this present value would rise to 100.41. Then, after another six months, the swap would be a 1.5-year swap and, with the term structure still unchanged, would have a present value of 100.60, etc. The third column of the table simply reproduces the forward rates of Table 8-1.

To understand why the present value behaves as it does, rising and then falling, begin by comparing the six-month and 1-year swaps. Both swaps pay 1.445% over the first six months. But then the 1-year swap pays 1.445% for an additional six months while the forward rate over that additional six-month period is only 1.046%. This paying of

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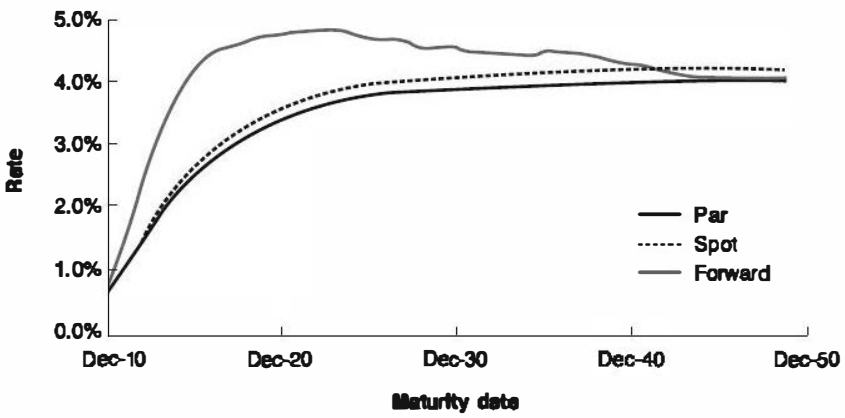

FIGURE 8-4 USD swap rates as of May 28, 2010.

TABLE 8-2 Present Values of 100 Face Amount of the Fixed Sides of 1.445% Swaps as of May 28, 2010

Maturity	Present Value	Forward Rate
.5	100.37	.705%
1	100.57	1.046%
1.5	100.60	1.384%
2	100.41	1.820%
2.5	100.00	2.301%

an above market rate makes the 1-year swap more valuable than the six-month swap and so its price is higher. And so with the 1.5-year swap: it pays 1.445% for the six months from 1-year to 1.5-years from now while the forward rate over that period is only 1.384%. And so, again, the present value increases as maturity increases from one to 1.5 years. But now consider the 2-year swap relative to the 1.5-year swap. The 2-year swap pays 1.445% for an additional six months while the forward rate for that six months is 1.820%. Hence the 2-year swap pays a below-market rate for the additional six months and has a present value less than that of the 1.5-year swap. Finally, the present value of the 2.5-year swap is less than that of the 2-year swap because the 2.5-year swap pays 1.445% for an additional six months while the forward rate is 2.301%.

Returning to the original question, then, if the term structure of rates remains unchanged over a six-month period, the present value will rise as the swap matures if its fixed rate is less than the forward rate corresponding to the expiring six-month period. The present value will fall as the swap matures if its fixed rate is greater than that forward rate. Appendix G in this chapter proves this general result.

TRADING CASE STUDY: TRADING AN ABNORMALLY DOWNWARD-SLOPING 10S-30S EUR FORWARD RATE CURVE IN Q2 2010

Figure 8-5 graphs six-month forward rate curves for USD, EUR, GBP, and JPY as of May 28, 2010. In EUR for example, the six-month rate, 10-years forward, or the 10y6m rate, is about 4.25% while the USD six-month rate, 30-years forward, or the 30y6m rate, is about 4%. By historical standards the EUR curve is remarkable in how the "10s-30s" forward curve, i.e., the curve from 10- to 30-year terms, slopes so steeply downward. The more usual historical shape is more like that of the other curves in the figure, sloping upward from short- to intermediate-maturities and then flattening out and falling gradually at the long end.

The macroeconomic context at the time was concerned about the fiscal difficulties and economic prospects of EUR countries triggered by fears that Greece and a number of other countries might default on their government debts. These fears were somewhat mitigated by a bailout fund proposed by EUR countries and the International Monetary Fund.

The technical context of these curves at this time was a particular theme of the Overview, namely, the need for European pension funds and insurance companies to invest in long-dated assets, or, in swap language, to receive fixed on the long end, so that their asset profiles better matched their long-term liabilities. This need was particularly acute after the approval of the Solvency II directive, which required additional capital to reflect any asset and liability mismatches. In any case, this institutional pressure to receive fixed on the long end, without

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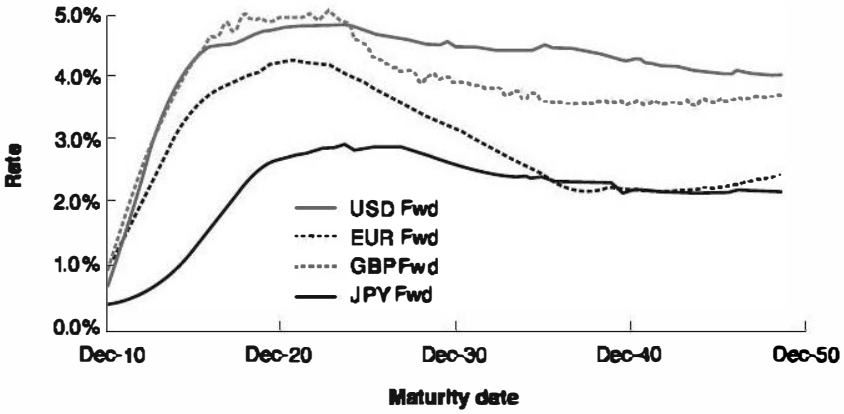

FIGURE 8-5 Forward swap rates in USD, EUR, GBP, and JPY as of May 28, 2010.

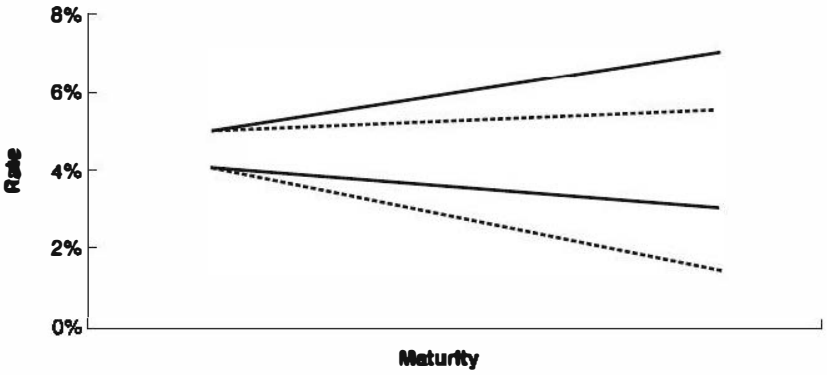

FIGURE 8-6 Shifting from either solid line to its dashed line is called a "flattening" of the term structure.

any commensurately sized payers on the long end, drove down long-term swap rates and was one factor responsible for the abnormally downward sloping EUR forward curve.

Before moving on to trade ideas, it will be useful to explain some market jargon. Consider the two pairs of abstract term structures of rates depicted in Figure 8-6. Market practitioners use the word *flattening* to describe shifts in which either 1) longer-term rates fall by more than shorter-term rates, or 2) shorter-term rates rise by more than longer-term rates. Therefore, by 1), a shift from either of the solid lines in the figure to its corresponding dashed line would be called a flattening even though everyday usage of the word "flatten" would not apply to the shift from the lower solid line to the lower dashed line. Similarly, market practitioners use the word *steepening* to describe shifts in which either 3) longer-term rates increase by more than shorter-term rates, or 4) shorter-term rates fall by more than longer term rates. Therefore, by 3), a shift from either of the dashed lines to its corresponding solid line would be called a steepening even though everyday normal usage of "steepen" would not apply to the shift from the lower dashed line to the lower solid line.

Returning now to the case, many market participants wanted to bet that the EUR forward curve in Figure 8-5 would revert to a more normal shape, i.e., that the 10s-30s forward curve would steepen. It was argued that the institutional demand to receive fixed would eventually be absorbed by the market so that a more normally sloped curve could be obtained. Furthermore, the technical factors holding down the long end would soon be overpowered by trading to follow in the wake of the resolution of macroeconomic uncertainty in Europe. More precisely, should the fiscal and economic situation in the EUR seriously deteriorate, the EUR forward curve would converge to the JPY forward curve and 10s-30s would steepen. On the other hand, should the fiscal and economic situation in the EUR improve, the EUR forward curve would converge to the USD and GBP curve and, once again, 10s-30s would steepen.

It might be the case, of course, that 10s-30s does not steepen. First, the institutional demand to receive fixed in the long end might so overwhelm the supply of payers that no amount of trading driven by macroeconomic considerations would drive 10s-30s EUR forwards back to historical norms. In fact, should incremental institutional demand to receive fixed continue to exceed incremental supply, 10s-30s might flatten even more. Also, global macroeconomic forces might flatten 10s-30s across the globe, which may very well have nothing to do with EUR technicals but which would still result in the EUR curve's flattening.

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A trader who comes to the conclusion that the risk-return characteristics of the steepening bet are appealing can implement the bet through the following trade: receive fixed in the relatively high EUR 10y6m rate and pay fixed in the abnormally low long end.[2] Put another way, lock in a rate to receive 10y6m and lock in a rate to pay in the long end as a bet that the 10y6m forward is going to fall relative to the longer-dated forwards. In addition, construct the trade so that if the 10y6m and longer-dated forward rates both increase by one basis point (i.e., .01%), the loss from the 10-year leg is offset by the gain from the longer-dated leg and the trade neither makes nor loses money. (Part Two shows how this type of hedge is constructed.)

TABLE 8-3 Selected EUR and JPY Forward Rates as of May 28, 2010

	10y6m	9y6m	25y6m	24y6m	30y6m	29y6m
EUR	4.254%	4.127%	2.550%	2.724%	2.293%	2.237%
JPY	2.712%	2.594%	2.433%	2.452%	2.219%	2.339%

TABLE 8-4 One-Year Roll-Down from Receiving 10y6m EUR and Paying 30y6m EUR as of May 28, 2010, Assuming an Unchanged Term Structure

	Today		One Year Later		Gain/Loss
	Forward	Rate	Forward	Rate	(bps)
Receive	10y6m	4.254	9y6m	4.127	+12.7
Pay	30y6m	2.293	29y6m	2.237	−5.6
Total					+7.1

A final aspect of the trade to consider is *roll-down*,[3] i.e., how the trade fares if rates do not change much at all, which would be the case, for example, if the forward rate curve remains the same. For if the trade does lose money over time as nothing happens, then the trader may not be able to stay in the trade long enough to realize the anticipated profits. This implied impatience can arise from internal risk management controls that force the closure of trades hitting *stop-losses* (i.e., loss threshholds). Impatience can also arise from the inability or reluctance, as trades lose money, to post more and more collateral to counterparties to ensure performance of increasingly under-water contracts. In any case, to analyze the roll-down of the trade discussed thus far, Table 8-3 gives six-month forward rates of various terms in EUR and, for later use, in JPY as of May 28, 2010.

Say that a trader decides to implement the suggested trade by receiving in EUR 10y6m and paying in EUR 30y6m. How does this trade roll-down over a year in which the term structure does not change? Table 8-4, using the forward rates in Table 8-3, outlines the answer. After one year the trader will have a position receiving 4.254% in 9y6m and paying 2.293% in 29y6m, but the market rates for those forwards will have fallen to 4.127% and 2.237%, respectively. As the table shows, this means a gain of 12.7 basis points (i.e., 4.254% − 4.127%) on the receiving leg of the trade and a loss of 5.6 basis points (i.e., −2.293% + 2.237%) on the paying leg of the trade. Furthermore, since the trade is constructed so that each leg has the same exposure to a change in interest rates, the net result would simply be the sum of the individual results or +7.1 basis points. So, for example, a trade scaled to have an interest rate exposure of €10,000 per basis point would gain €71,000.

But what if, instead of selling the 30y6m forward, the trader pays fixed in the 25y6m forward? This may be harder to transact, as the 30-year maturity is more liquidly traded, but it is a choice to be considered. Table 8-5 computes the roll-down in this case, again using the forward rates in Table 8-3. The receiving leg is unchanged and still gains 12.7 basis points. But the paying leg, since the 24y6m rate is greater than the 25y6m rate, gains as well, in the amount of 17.4 basis points. Hence the total roll-down, the sum of the roll-down of the two legs, is 30.1 basis points. This revised trade, then,

[2] It is possible that the trade would be implemented in exactly this way, but as six-month forwards at long maturities are not liquid, a much more likely implementation would use portfolios of par swaps. For clarity of exposition, however, the text assumes direct trading in short-term forwards.

[3] Some practitioners would call this *carry* or *carry-roll-down*. See the discussion in Chapter 9.

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TABLE 8-5 One-Year Roll-Down from Receiving 10y6m EUR and Paying 25y6m EUR as of May 28, 2010, Assuming an Unchanged Term Structure

	Today		One Year Later		Gain/Loss
	Forward	Rate	Forward	Rate	(bps)
Receive	10y6m	4.254	9y6m	4.127	+12.7
Pay	25y6m	2.550	24y6m	2.724	+17.4
Total					+30.1

has much better roll-down properties than the originally conceived trade.

It was noted above that the proposed trade would lose money if 10s-30s around the globe flattened due to shared macroeconomic shocks. A possible hedge to this losing scenario is to put on the opposite trade in another currency, e.g., to pay fixed in 10y6m and to receive fixed in 25y6m in JPY. It makes sense to put on this hedge only if two conditions are met. One, 10s-30s in that currency is not likely to experience any idiosyncratic moves over the time horizon of the trade; if such idiosyncratic moves were likely, the hedge might very well increase rather than decrease the volatility of the trade's results. Two, the roll-down of the hedge is not so negative as to spoil the appealing risk-return profile of the original trade.

As it turns out, the JPY curve seems very suitable for this hedge, i.e., paying in 10y6m and receiving in 25y6m. First, resolution of Japan's fiscal and economic situation and, therefore, a reshaping of its swap curve, is expected to happen much more slowly than a resolution of the situation in the EUR countries. Second, using the data in Table 8-3, the incremental roll down of this trade is a negative 2.712% −2.594% or −11.8 basis points from the 10-year leg and a negative 2.433% − 2.453% or −2 basis points from the 25-year leg for a total of − 13.8 basis points. Noting that the overall roll-down of the trade, the original 30.1 basis points minus the 13.8 basis points of the macroeconomic hedge, is a reasonable +16.3 basis points, a trader might very well choose to purchase this insurance by adding the hedge to the original trade.

It is possible, of course, that 10s-30s in EUR becomes more steeply downward sloping at the same time that JPY 10s-30s becomes less steeply downward sloping, in which case both the original trade and the hedge lose money. But nothing in the analysis of the macroeconomic and technical foundations of the trade suggests this eventuality. And, after all, a trade is always a bet on something!

APPENDIX A

Compounding Conventions

The text discussed semiannual compounding, which assumes that interest is paid twice a year, and showed that one unit of currency invested at the rate $\hat{r}^{sa}$ for T years would grow to

$$\left(1+\frac{\hat{r}^{sa}}{2}\right)^{2T} \tag{8.28}$$

Similarly, it is easy to see that one unit of currency invested at an annual rate $\hat{r}^{a}$, a monthly rate $\hat{r}^{m}$, or a daily rate $\hat{r}^{d}$, would grow after T years to the following quantities, respectively,

$$(1+\hat{r}^{a})^{T} \tag{8.29}$$

$$\left(1+\frac{\hat{r}^{m}}{12}\right)^{12T} \tag{8.30}$$

$$\left(1+\frac{\hat{r}^{d}}{365}\right)^{365T} \tag{8.31}$$

More generally, if interest at a rate $\hat{r}$ is paid n times per year, the proceeds after T years will be

$$\left(1+\frac{\hat{r}}{n}\right)^{nT} \tag{8.32}$$

One would expect that, holding all other characteristics of investment constant, the market would offer a single terminal amount for having invested one unit of currency for T years. Given the quantities in Equations (8.28) through (8.32), this means that the market could offer many different rates of interest for that investment, each associated with a different compounding convention. So, for example, if the market offers 2% annually compounded for a one-year investment, so that a unit investment grows to 1.02 at the end of a year, rates of other compounding conventions would be determined by the equations

$$\left(1+\frac{\hat{r}^{sa}}{2}\right)^{2}=\left(1+\frac{\hat{r}^{m}}{12}\right)^{12}=\left(1+\frac{\hat{r}^{d}}{365}\right)^{365}=1.02 \tag{8.33}$$

Solving Equation (8.33) for each rate, $\hat{r}^{sa}$ = 1.9901%; $\hat{r}^{m}$ = 1.9819; and $\hat{r}^{d}$ = 1.9803%. Note that the more often interest is paid, the more interest can earn interest on interest, and

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the lower the rate required to earn the fixed amount 1.02 over the year.

Under continuous compounding, interest is paid every instant, resulting in a terminal value equal to the limit of the quantity (8.32) as n approaches infinity. Taking the natural logarithm of both sides of that equation and rearranging terms,

$$nT\ln\left(1+\frac{r}{n}\right)=\frac{T\ln\left(1+\frac{r}{n}\right)}{\frac{1}{n}} \tag{8.34}$$

Using l'Hôpital's rule, the limit of the right-hand side of (8.34) as n becomes large is rT. Hence, the limit of (8.32) must be e^{rT}, where e = 2.71828 . . . is the base of the natural logarithm. Therefore, if interest is paid at a rate r^c every instant, an investment of one unit of currency will grow after T years to

$$e^{r^cT} \tag{8.35}$$

Equivalently, the value of one unit of currency to be received in T years is

$$e^{-r^cT} \tag{8.36}$$

APPENDIX B

Continuously Compounded Spot and Forward Rates

Let $\hat{r}^c(t)$ be the continuously compounded spot rate from time 0 to t, let $f^c(t)$ be the continuously compounded forward rate at time t, and let $f^c(t-\Delta, t)$ be the forward rate from time $t-\Delta t$ to time t, which approaches $f^c(t)$ as Δt approaches 0. From the discussion on spot rates in the text and the discussion on continuous compounding in Appendix A and Equation (8.36) in particular, the continuously compounded spot rate is defined such that

$$d(t) = e^{-\hat{r}^c(t)t} \tag{8.37}$$

With respect to forward rates, the continuously compounded analogue of Equation (8.14) of the text is

$$e^{\hat{r}^c(t-\Delta)\times(t-\Delta)}e^{f^c(t-\Delta,t)\Delta} = e^{\hat{r}^c(t)t} \tag{8.38}$$

Substituting for each of the two spot rates using Equation (8.37) and rearranging terms,

$$e^{f^c(t-\Delta,t)\Delta} = \frac{d(t-\Delta)}{d(t)} \tag{8.39}$$

Next, taking the natural logarithm of both sides and rearranging terms,

$$f^c(t-\Delta,t) = -\frac{\ln[d(t)]-\ln[d(t-\Delta)]}{\Delta} \tag{8.40}$$

Finally, taking the limit of both sides, recognizing the limit of the right-hand side of (8.40) as the derivative of $\ln[d(t)]$,

$$f^c(t) = \frac{d'(t)}{d(t)} \tag{8.41}$$

where $d'(t)$ is the derivative of the discount function.

APPENDIX C

Flat Spot Rates Imply Flat Par Rates

Proposition: If spot rates are flat at the rate $\hat{r}$, then par rates are flat at that same rate.

Proof: Write Equation (8.19) in terms of the single spot rate, $\hat{r}$:

$$\frac{C(T)}{2}\sum_{t=1}^{2T}\frac{1}{\left(1+\frac{\hat{r}}{2}\right)^t}+\frac{1}{\left(1+\frac{\hat{r}}{2}\right)^{2T}}=1 \tag{8.42}$$

Using (8.49) in Appendix D, rewrite this equation as

$$\frac{C(T)}{\hat{r}}\left[1-\frac{1}{\left(1+\frac{\hat{r}}{2}\right)^{2T}}\right]+\frac{1}{\left(1+\frac{\hat{r}}{2}\right)^{2T}}=1 \tag{8.43}$$

But solving (8.43) for $C(T)$ shows that $C(T) = \hat{r}$ for all T.

APPENDIX D

A Useful Summation Formula

Proposition:

$$\sum_{t=a}^{b} z^t = \frac{z^a - z^{b+1}}{1-z} \tag{8.44}$$

Proof: Define S such that

$$S = \sum_{t=a}^{b} z^t \tag{8.45}$$

Then,

$$zS = \sum_{t=a+1}^{b+1} z^t \tag{8.46}$$

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And, subtracting (8.46) from (8.45),

$$S(1-z) = z^{b} - z^{b+1} \tag{8.47}$$

Finally, dividing both sides of (8.47) by $1 - z$ gives Equation (8.44), as desired.

This proposition is quite useful in fixed income where expressions like the one in Equation (8.42) of Appendix C are common:

$$\sum_{t=1}^{2T} \frac{1}{\left(1+\frac{f}{2}\right)^{t}} \tag{8.48}$$

Setting $z = 1/(1+f/2)$ and applying the proposition of this appendix gives the result

$$\sum_{t=1}^{2T} \frac{1}{\left(1+\frac{f}{2}\right)^{t}} = \frac{\frac{1}{\left(1+\frac{f}{2}\right)} - \frac{1}{\left(1+\frac{f}{2}\right)^{2T+1}}}{1-\frac{1}{\left(1+\frac{f}{2}\right)}} = \frac{1-\frac{1}{\left(1+\frac{f}{2}\right)^{2T}}}{\frac{f}{2}} = \frac{2}{f}\left[1-\frac{1}{\left(1+\frac{f}{2}\right)^{2T}}\right] \tag{8.49}$$

APPENDIX E

The Relationship Between Spot and Forward Rates and the Slope of the Term Structure

Proposition: For semiannually compounded rates, $f(t) > \hat{r}(t-.5)$ if and only if $\hat{r}(t) > \hat{r}(t-.5)$.

Proof: $f(t) > \hat{r}(t-.5)$ is equivalent to

$$\left(1+\frac{\hat{r}(t-.5)}{2}\right)^{2t-1}\left(1+\frac{f(t)}{2}\right) > \left(1+\frac{\hat{r}(t-.5)}{2}\right)^{2t-1}\left(1+\frac{\hat{r}(t-.5)}{2}\right) > \left(1+\frac{\hat{r}(t-.5)}{2}\right)^{2t} \tag{8.50}$$

But the left-hand side of this equation can be written in terms of $\hat{r}(t)$ using Equation (8.14).

$$\left(1+\frac{\hat{r}(t)}{2}\right)^{2t} > \left(1+\frac{\hat{r}(t-.5)}{2}\right)^{2t} \tag{8.51}$$

And this is equivalent to $\hat{r}(t) > \hat{r}(t-.5)$.

Proposition: For semiannually compounded rates, $f(t) < \hat{r}(t-.5)$ if and only if $\hat{r}(t) < \hat{r}(t-.5)$.

Proof: Reverse the inequalities in the previous proof.

Proposition: For continuously compounded rates, $f^c(t) \geq \hat{r}^c(t)$ if and only if $d\hat{r}^c(t)/dt \geq 0$ and $f^c(t) \leq \hat{r}^c(t)$ if and only if $d\hat{r}^c(t)/dt \leq 0$.

Proof: Taking the derivative of Equation (8.37),

$$d'(t) = -\left[\hat{r}^c(t) + t\frac{d\hat{r}^c(t)}{dt}\right]d(t) \tag{8.52}$$

Dividing both sides by $-d(t)$ and then substituting for the left-hand side using (8.41),

$$f^c(t) = \hat{r}^c(t) + t\frac{d\hat{r}^c(t)}{dt} \tag{8.53}$$

Rearranging terms,

$$\frac{d\hat{r}^c(t)}{dt} = \frac{f^c(t) - \hat{r}^c(t)}{t} \tag{8.54}$$

By inspection, then, $d\hat{r}^c(t)/dt$ has the same sign as $f^c(t) - \hat{r}^c(t)$.

APPENDIX F

The Relationship Between Spot and Par Rates and the Slope of the Term Structure

Proposition: If $\hat{r}(.5) < \hat{r}(1) < \cdots < \hat{r}(T)$ then $C(T) < \hat{r}(T)$.

Proof: By the definition of the par rate, $C(T)$,

$$\frac{C(T)}{2}\left[\frac{1}{\left(1+\frac{\hat{r}(.5)}{2}\right)} + \cdots + \frac{1}{\left(1+\frac{\hat{r}(T)}{2}\right)^{2T}}\right] + \frac{1}{\left(1+\frac{\hat{r}(T)}{2}\right)^{2T}} = 1 \tag{8.55}$$

Also, setting all spot rates in (8.55) equal to $C(T)$, it follows from (8.49) of Appendix D that

$$\frac{C(T)}{2}\left[\frac{1}{\left(1+\frac{C(T)}{2}\right)} + \cdots + \frac{1}{\left(1+\frac{C(T)}{2}\right)^{2T}}\right] + \frac{1}{\left(1+\frac{C(T)}{2}\right)^{2T}} = 1 \tag{8.56}$$

Furthermore, since $\hat{r}(.5) < \hat{r}(1) < \cdots < \hat{r}(T)$, the expression

$$\frac{C(T)}{2}\left[\frac{1}{\left(1+\frac{\hat{r}(T)}{2}\right)} + \cdots + \frac{1}{\left(1+\frac{\hat{r}(T)}{2}\right)^{2T}}\right] + \frac{1}{\left(1+\frac{\hat{r}(T)}{2}\right)^{2T}} \tag{8.57}$$

which sets all of the discounting rates to $\hat{r}(T)$, is less than the left-hand side of Equation (8.55). But since the left-hand sides of both (8.55) and (8.56) equal 1, this implies that (8.57) is also less than the left-hand side of (8.56). And this, in turn, implies that $C(T) < \hat{r}(T)$, as was to be proved.

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Proposition: If $\hat{r}(.5) > \hat{r}(1) > \cdots > \hat{r}(T)$ then $C(T) > \hat{r}(T)$.

Proof: In this case, (8.57) is greater than the left-hand side of Equation (8.55) and, therefore, of (8.56). But this implies that $C(T) > \hat{r}(T)$, as was to be proved.

APPENDIX G

Maturity, Present Value, and Forward Rates

Proposition: The sign of $P(T) - P(T - .5)$ equals the sign of $c - f(T)$.

Proof: Using Equation (8.24) for $P(T)$ and for $P(T - .5)$ it can be shown that

$$P(T) - P(T - .5) = \frac{1 + \frac{c}{2}}{\left(1 + \frac{f(.5)}{2}\right)\left(1 + \frac{f(1)}{2}\right)\cdots\left(1 + \frac{f(T)}{2}\right)} - \frac{1}{\left(1 + \frac{f(.5)}{2}\right)\cdots\left(1 + \frac{f(T-.5)}{2}\right)} \quad \textbf{(8.58)}$$

Or,

$$P(T) - P(T - .5) = \frac{1 + \frac{c}{2} - \left(1 + \frac{f(T)}{2}\right)}{\left(1 + \frac{f(.5)}{2}\right)\left(1 + \frac{f(1)}{2}\right)\cdots\left(1 + \frac{f(T)}{2}\right)} \quad \textbf{(8.59)}$$

Or, again,

$$P(T) - P(T - .5) = \frac{\frac{1}{2}(c - f(T))}{\left(1 + \frac{f(.5)}{2}\right)\left(1 + \frac{f(1)}{2}\right)\cdots\left(1 + \frac{f(T)}{2}\right)} \quad \textbf{(8.60)}$$

Therefore the sign of $P(T) - P(T - .5)$ equals the sign of $c - f(T)$.

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Returns, Spreads, and Yields

9

Learning Objectives

After completing this reading you should be able to:

- Distinguish between gross and net realized returns, and calculate the realized return for a bond over a holding period including reinvestments.
- Define and interpret the spread of a bond, and explain how a spread is derived from a bond price and a term structure of rates.
- Define, interpret, and apply a bond's yield-to-maturity (YTM) to bond pricing.
- Compute a bond's YTM given a bond structure and price.
- Calculate the price of an annuity and a perpetuity.
- Explain the relationship between spot rates and YTM.
- Define the coupon effect and explain the relationship between coupon rate, YTM, and bond prices.
- Explain the decomposition of P&L for a bond into separate factors including carry roll-down, rate change and spread change effects.
- Identify the most common assumptions in carry roll-down scenarios, including realized forwards, unchanged term structure, and unchanged yields.

Excerpt is Chapter 3 of Fixed Income Securities, *Third Edition, by Bruce Tuckman.*

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Spot, forward, and par rates, presented in Chapter 8, intuitively describe the time value of money embedded in market prices. To analyze the *ex-post* performance and the *ex-ante* relative attractiveness of individual securities, however, market participants rely on returns, spreads, and yields.

The first section of this chapter defines these terms. Horizon returns in the fixed income context have to account for intermediate cash flows and are often computed both on a gross basis and net of financing, but are otherwise similar to the returns calculated for any asset. Spreads measure the pricing of an individual fixed income security relative to a benchmark curve, usually of swaps or government bonds. Yield is a practical and intuitive way to quote price and is used extensively for quick insight and analysis. It cannot be used, however, as a precise measure of relative value. This first section concludes with a brief news excerpt about the sale of Greek government bonds that illustrates the convenience of speaking in terms of spreads and yields.

The second section of the chapter shows how the profit-and-loss (P&L) or return of a fixed income security can be decomposed into component parts. Such decompositions are defined differently by different market participants, but this book will define terms as follows. Cash-carry is a security's coupon income minus its financing cost, a quantity that will be particularly useful in the context of forwards and futures. Carry-roll-down is the change in the (flat) price of a security if rates move "as expected," where one common interpretation of "as expected" is the scenario of realized forwards and another is the scenario of an unchanged term structure, both of which are described in this chapter.

The third and final section of the chapter presents several carry-roll-down scenarios, partly to complete the discussion of return decompositions, but partly for the insights these scenarios provide with respect to bond returns. Two such insights are the following: 1) if realized forward rates exceed the forward rates embedded in bond prices, a strategy of rolling over short-term bonds outperforms an investment in long-term bonds; 2) a bond's return equals its yield only if its yield stays constant and if all coupons are reinvested at that same yield.

DEFINITIONS

Realized Returns

This section begins the chapter by defining *gross* and *net* realized returns over a single period and over several periods. Very simply, net returns are gross returns minus financing costs. For concreteness and ease of exposition this chapter focuses exclusively on bonds, but the principles and definitions presented can easily be extended to other securities. For the same reasons, this chapter calculates returns only over holding periods equal to the length of time between cash flows, so that, for example, the returns of semiannual coupon bonds are calculated only over six-month holding periods.

Since Chapters 7 and 8 have dealt extensively with the details of semiannual cash flows, this chapter simplifies notation by not explicitly recording the length of each period. Denote the price of a particular bond at time t by P_t per unit face value and the price of that same bond, after one period of unspecified length, as P_{t+1}. Also, denote the bond's periodic coupon payment per unit face value by c. Numerical examples, however, will explicitly incorporate semiannual cash flow conventions and will assume face values of 100.

An investor purchasing a bond at time t pays P_t and then, at time $t + 1$, receives a coupon c and has a bond worth P_{t+1}. The gross realized return on that bond from t to $t + 1$, $R_{t,t+1}$, is defined as the total value at the end of the period minus the starting value all divided by the starting value. Mathematically,

$$R_{t,t+1} = \frac{P_{t+1} + c - P_t}{P_t} \tag{9.1}$$

Continuing with the U.S. Treasury example of Chapter 8, say that an investor bought the U.S. Treasury 4½s of November 30, 2011, for 105.856 for settlement on June 1, 2010. Then suppose that the price of the bond one coupon-period later, on November 30, 2010, turned out to be 105. The six-month return on that investment would have been

$$\frac{105 + 2.25 - 105.856}{105.856} = 1.317\% \tag{9.2}$$

where the 2.25 in the numerator is the bond's semiannual coupon payment.

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Computing a realized return over a longer holding period requires keeping track of the rate at which coupons are reinvested over the holding period. Consider an investment in the same bond for one year, that is, to May 31, 2011. The total proceeds at the end of the year consist not only of the value of the bond and the coupon payment on May 31, 2011, but also of the reinvested proceeds of the coupon paid on November 30, 2010. Assuming that this November coupon is invested at a semiannually compounded rate of .60% and that the price of the bond on May 31, 2011, is 105, the realized gross holding period return over the year would be

$$\frac{105 + 2.25 + 2.25 \times \left(1 + \frac{.60\%}{2}\right) - 105.856}{105.856} = 3.449\% \qquad \textbf{(9.3)}$$

Now consider an investor in the 4½s of November 30, 2011, who *financed* the purchase of the bond, that is, who borrowed cash to make the investment. While not usually the case, assume for the purposes of this chapter that the investor could borrow the entire purchase price of the bond. Assume a rate of .2% for .5 years on the amount borrowed so that paying off the loan costs 105.856 × (1 + .2%/2) or 105.962. Also assume, as before, that the price of the bond is 105 on November 30, 2010. Then, this investment over a six-month horizon is described as in Table 9-1.

One obvious problem in calculating a return on this investment is that it requires no initial cash and the final value cannot be divided by zero. But even if the investor did have to put up some amount of initial cash, so that borrowing was 90% rather than 100% of the purchase price, it would still not be sensible to divide the final value by the amount invested when trying to describe the return on the 4½s of November 30, 2011. After all, another investor might have borrowed 95% of the purchase price and a third investor only 85%. Hence it would be sensible to divide by the investor's outlay only to calculate a return on capital for that investor. But that is not the exercise here. Therefore, when calculating realized returns on securities, even when those securities are financed, it is conventional to divide that final value by the initial price of the security.

TABLE 9-1 A Financed Purchase of the 4½s of November 30, 2011

Settlement Date	Transaction	Proceeds	Total Proceeds
June 1, 2010	buy bond	−105.856	0
	borrow price	105.856	
November 30, 2010	collect coupon	2.250	
	sell bond	105.000	1.288
	pay off loan	−105.962	

With this choice of a denominator, the net realized return on the security looks almost, but not exactly, like the gross return in (9.2):

$$\frac{105 + 2.25 - 105.962}{105.856} = 1.217\% \qquad \textbf{(9.4)}$$

In fact, the net return is simply the previously calculated gross return of 1.317% minus the 0.1% cost of six-month financing. To make this a bit more explicit,

$$\frac{105 + 2.25 - 105.856 \times \left(1 + \frac{.2\%}{2}\right)}{105.856} = \frac{105 + 2.25 - 105.856}{105.856} - \frac{.2\%}{2} = 1.317\% - .1\% \qquad \textbf{(9.5)}$$

Without going into further detail here, calculating a multiperiod net return requires not only the reinvestment rates of the coupons but the future financing costs as well.

Spreads

As mentioned in the introduction to the chapter, spreads are important measures of relative value and their convergence or divergence is an important component of return.

The market price of any security can be thought of as its value computed using some term structure of interest rates, denoted generically by $\mathbb{R}$, plus a premium or discount, ϵ, relative to that term structure:

$$P = P(\mathbb{R}) + \epsilon \qquad \textbf{(9.6)}$$

Furthermore, the premium or discount ϵ is often expressed in terms of a spread to interest rates, s, rather than in terms of price. Mathematically, first

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write $P(\mathbb{R})$ using forward rates (as in Equation (8.24) but with periods of unspecified length):

$$P = \frac{c}{(1+f(1))} + \frac{c}{(1+f(1))(1+f(2))} + \cdots + \frac{1+c}{(1+f(1))(1+f(2))\cdots(1+f(T))} + \epsilon \tag{9.7}$$

Then, instead of defining the deviation of the market from $P(\mathbb{R})$ through ϵ, define it through a spread. In other words, find s such that the following equation is identically true:

$$P = \frac{c}{(1+f(1)+s)} + \frac{c}{(1+f(1)+s)(1+f(2)+s)} + \cdots + \frac{1+c}{(1+f(1)+s)(1+f(2)+s)\cdots(1+f(T)+s)} \tag{9.8}$$

In words, the market price is recovered by discounting a bond's cash flows using an appropriate term structure plus a spread.

Spreads defined as in Equation (9.8) are usually intended to be either bond- or sector-specific. As an example of the former, recall the testing of the law of one price in Table 9-4. The ¾s of November 30, 2011, when priced using the discount curve derived in Chapter 7, gave a present value of 100.255 compared with a market price of 100.190. To express this price deviation or ϵ in terms of spread, express the discount factors in Table 7-3 as forward rates and solve the following equation for s:

$$100.190 = \frac{.375}{\left(1+\frac{.149\%}{2}+\frac{s}{2}\right)} + \frac{.375}{\left(1+\frac{.149\%}{2}+\frac{s}{2}\right)\left(1+\frac{.556\%}{2}+\frac{s}{2}\right)} + \frac{100.375}{\left(1+\frac{.149\%}{2}+\frac{s}{2}\right)\left(1+\frac{.556\%}{2}+\frac{s}{2}\right)\left(1+\frac{1.016\%}{2}+\frac{s}{2}\right)} \tag{9.9}$$

The result is s = .044% or 4.4 basis points. With this spread result, instead of saying that the ¾s of November 30, 2011, trade 6.5 cents cheap relative to the reference bonds, one could say that they trade 4.4 basis points cheap. Sometimes speaking in terms of price is more useful, as when saying that buying the ¾s and selling its replicating portfolio will produce a P&L of 6.5 cents per $100. But sometimes speaking in terms of spread is more intuitive, as when saying that the ¾s trade at 4.4 basis points above the Treasury curve. There is also an interpretation of that 4.4 basis points in terms of the bond's return, which will be presented in the third section of this chapter.

Equation (9.8) and the U.S. Treasury note example illustrate bond-specific spreads. A common example of sector-specific spreads would be corporate bonds. In that context bonds issued by a particular corporation are thought of as trading at a spread curve to government bonds or swaps, where a spread curve means that the forward spread at each term is different. The pricing equation for a bond in that case might take the following form:

$$P = \frac{c}{(1+f(1)+s(1))} + \frac{c}{(1+f(1)+s(1))(1+f(2)+s(2))} + \cdots + \frac{1+c}{(1+f(1)+s(1))\cdots(1+f(T)+s(T))} \tag{9.10}$$

Yield-to-Maturity

While par, spot, and forward rates are in many contexts more intuitive than prices, their appeal suffers from needing so many rates to describe the pricing of a single bond. As a result, *yield-to-maturity* is often quoted when describing a security in terms of rates rather than price.

Yield-to-maturity is the single rate such that discounting a security's cash flows at that rate gives that security's market price. For example, Table 7-2 reported that, with 1.5 years to maturity, the price of the 4½s of November 30, 2011, was 105.856. The yield-to-maturity, y, of this bond is therefore defined such that[1]

$$105.856 = \frac{2.25}{\left(1+\frac{y}{2}\right)} + \frac{2.25}{\left(1+\frac{y}{2}\right)^2} + \frac{102.25}{\left(1+\frac{y}{2}\right)^3} \tag{9.11}$$

Juxtaposing Equation (9.11) with Equations (8.23), (8.24), and (9.8) or (9.10) reveals that yield summarizes both the term structure of interest rates as well as any spread or spread curve for this bond relative to that term structure. In any case, solving (9.11) for y by trial-and-error or some numerical method shows that the yield of the 4½s is about .574%. While it is much easier to solve for price given yield than for yield given price, many calculators and computer applications are readily available to move from price to yield or *vice versa*. Yield is often used as an alternate way to quote price: a trader could bid to buy the 4½s of November 30, 2011, at a price of 105.856 or at a yield of .574%. Needless to say, market practice is not such that a trader can bid to buy the bond with three spot or forward rates instead of a price.

[1] This is not perfectly correct since the prices in Table 7-2 were for settlement on June 1, 2010, rather than May 31, 2010. See Appendix A in this chapter for a more precise definition.

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The definition of yield for a coupon bond for settlement on a coupon payment date is[2]

$$P = \frac{\frac{1}{2}c}{\left(1+\frac{y}{2}\right)} + \frac{\frac{1}{2}c}{\left(1+\frac{y}{2}\right)^2} + \cdots + \frac{1+\frac{1}{2}c}{\left(1+\frac{y}{2}\right)^{2T}} \tag{9.12}$$

Or, more compactly,

$$P = \frac{c}{2}\sum_{t=1}^{2T}\frac{1}{\left(1+\frac{y}{2}\right)^t} + \frac{1}{\left(1+\frac{y}{2}\right)^{2T}} \tag{9.13}$$

And simplifying using the summation formula given in Appendix D in Chapter 8,

$$P = \frac{c}{y}\left(1-\frac{1}{\left(1+\frac{y}{2}\right)^{2T}}\right) + \frac{1}{\left(1+\frac{y}{2}\right)^{2T}} \tag{9.14}$$

Equation (9.14) provides several immediate facts about the price-yield relationship. First, when $c = y$, $P(T) = 1$. In words, when the yield is equal to the coupon rate, the bond sells for its face value. Second, when $c > y$, $P(T) > 1$: when the coupon rate exceeds the yield, the bond sells at a premium to its face value. Third, when $c < y$, $P(T) < 1$: when the yield exceeds the coupon rate, the bond sells at a discount to its face value.

Figure 9-1 illustrates these first three implications of Equation (9.14). Fixing all yields at 3%, each curve gives the price of a bond with a particular coupon rate as a function of years remaining to maturity. The bond with a coupon of 3% has a price of 100 at all terms. With 30 years to maturity, the 4% and 5.5% bonds sell at substantial premiums to par. As these bonds mature, however, the value of an above-market coupon falls: receiving a coupon 1% or 2.5% above market for 20 years is not so valuable as receiving those above-market coupons for 30 years. Hence, the prices of these premium bonds fall over time until they are worth par at maturity. Conversely, the .5% and 2% bonds sell at substantial discounts to par with 30 years to maturity and rise in price as they mature. The time trend of bond prices depicted in the figure is known as the *pull to par*. Of course, the realized price paths of these bonds will differ dramatically from those in Figure 9-1 (which fixes all yields at 3%) according to the actual realization of yields.

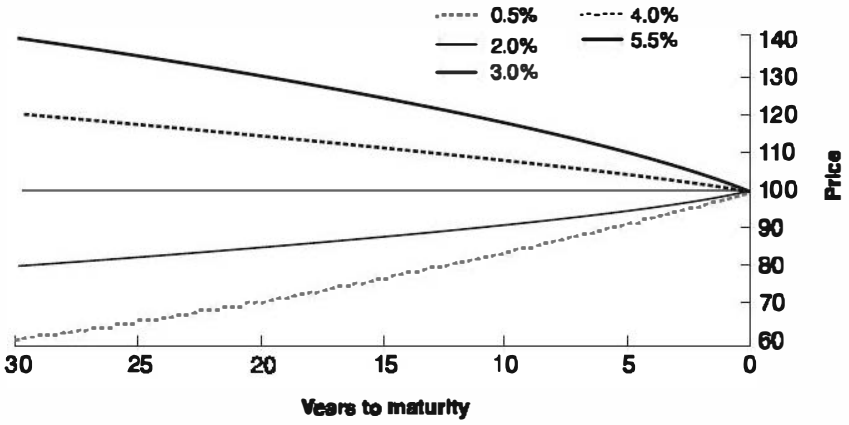

FIGURE 9-1 Prices of bonds with varying coupons over time with yields fixed at 3%.

The fourth lesson from the price-yield relationship of Equation (9.14) is the *annuity* formula. An annuity makes annual payments of 1 until date T with no final principal payment. In this case, the second term of (9.14) vanishes and, with $c = 1$, the value of the annuity, $A(T)$, becomes

$$A(T) = \frac{1}{y}\left(1-\frac{1}{\left(1+\frac{y}{2}\right)^{2T}}\right) \tag{9.15}$$

The annuity formula appears frequently in fixed income as the present value factor for a bond's coupons, a swap's fixed-rate cash flows, or a mortgage's payments, which are most often structured as a series of equal payments.

A fifth implication of Equation (9.14) is that the value of a *perpetuity*, a security that makes the fixed payment c forever, can be found by letting T approach infinity in (9.14) and multiplying by c, which gives c/y.

A sixth and final implication of the definition of yield is that if the term structure is flat, so that all spot rates and all forward rates equal some single rate, then the yield-to-maturity of all bonds equals that rate as well. This is easily seen by observing that, in the case of a flat spot rate curve, the pricing equation for each bond would take exactly the same form as Equation (9.12) with the yield equal to the single spot rate.

[2] The formula for other settlement dates is given in Appendix A in this chapter.

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Yield Curves and the Coupon Effect

The phrase "yield curve" is used often, but its meaning is not very precise because the concept of yield is intertwined with the cash flows of a particular bond. Spot, forward, and par rate curves can, as shown in Chapter 8, be used to price any similar security. By contrast, the yield of a particular security derived from (9.14) can be used to price only that security. To illustrate this point, Figure 9-2, using C-STRIPS prices as of May 28, 2010, graphs the yields on hypothetical but fairly priced zero-coupon bonds, par bonds, and 9% coupon bonds of various maturities on the mid-month, May-November cycle. In other words, using discount factors derived from C-STRIPS prices, the prices of these hypothetical bonds are computed along the lines of Chapter 7. Then the yields of these bonds are calculated. Figure 9-2 also shows the yields of actual U.S. Treasury notes and bonds on the same payment cycle and as of the same pricing date. Figure 9-3 shows the same data as Figure 9-2, but zooms in on a narrower yield range by focusing on the longer maturities.

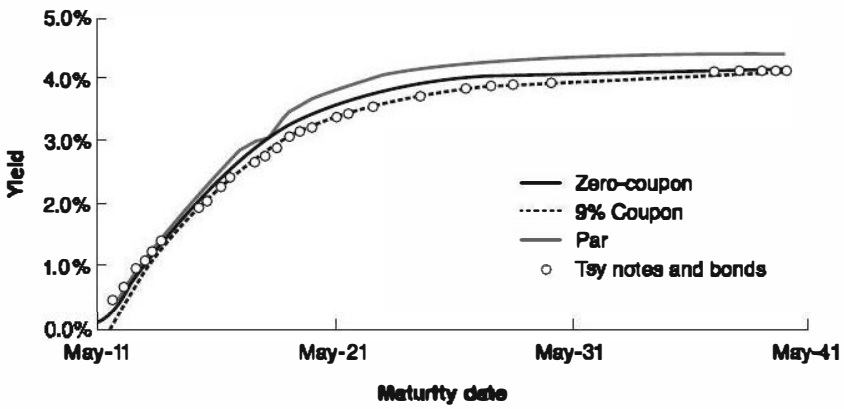

FIGURE 9-2 Yields of hypothetical securities priced with C-STRIPS as of May 28, 2010.

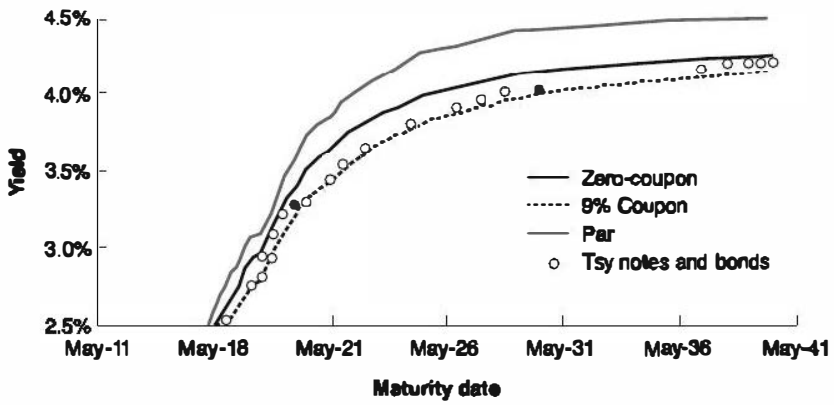

FIGURE 9-3 Yields of long-term hypothetical securities priced with C-STRIPS as of May 28, 2010.

These figures show that the "zero-coupon yield curve," the "par yield curve," and the "9% coupon yield curve," are indeed all different. In other words, a yield curve is not well defined until particular cash flows have been defined. And securities with a structure different from that of a coupon bond, like an amortizing bond or a fixed-rate mortgage, which spread principal payments out over time, would generate more dramatically different "yield curves."

In Figures 9-2 and 9-3, for any given maturity, zero-coupon yields exceed par yields, which, in turn, exceed the 9% coupon yields. This can be explained by the fact that yield is the one rate that describes how a security's cash flows are being discounted. Since a zero-coupon bond has only one cash flow at maturity, its yield is simply the spot rate corresponding to that maturity. A 9% coupon bond, on the other hand, makes cash flows every six months. Its yield, therefore, is a complex average of all of the spot rates from terms of six months to the bond's maturity, although the greatest weight is on the spot rate corresponding to the bond's largest present value, namely, that of the final payment of coupon plus principal. Furthermore, since the term structure of interest rates in the figures slopes upward, any weight this complex average places on the shorter-term spot rates lowers that average below the spot rate at maturity. Hence the yield on the 9% bond has to be lower than the yield on the 0% bond. The par bonds, with coupons between 0% and 9%, discount a lot of their present value at the shorter-term spot rates relative to zero-coupon bonds, but discount little of their present value at those shorter-term rates relative to the

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9% bonds. Hence, the yield of a par bond of a given maturity will be between the yield of the 0% and 9% bonds of that maturity. While not illustrated here, if the term structure slopes downward, then the argument just made would be reversed and the zero-coupon yield curve would be below the 9%-coupon yield curve.

The fact that fairly priced bonds of the same maturity but different coupons have different yields-to-maturity is called the *coupon effect.* The implication of this effect is that yield is not a reliable measure of relative value. Just because one fixed income security has a higher yield than another does not necessarily mean that it is a better investment. Any such difference may very well be due to the relationship between the time pattern of the security's cash flows and the term structure of spot rates, as discussed in the previous paragraph.

The yields on the actual notes and bonds are seen most easily in Figure 9-3. Many of the bonds, particularly those of longer term, are closest to the 9% coupon yield curve because those bonds, having been issued relatively long ago when rates were much higher, do indeed have very high coupons. The 6½s of November 15, 2026, the 6⅛s of November 15, 2027, the 5¼s of November 15, 2028, and the 6¼s of May 15, 2030, are all easily seen in the figure to fall into this category. Other bonds, however, were issued more recently at lower coupons and trade closer to the par yield curve. The three bonds in the figure with longest maturities, which were issued relatively recently, fall into this category: the 4¼s of May 15, 2039, the 4⅜s of November 15, 2039, and the 4⅜s of May 15, 2040.

Japanese Simple Yield

Before concluding the discussion of yield, it is noted here that Japanese government bonds are quoted on a *simple yield* basis. With a flat price p per unit face amount, a coupon rate c, and a maturity in years, T, this simple yield, y, is given by $y = c/p + (1/T) \times (1 - p)/p$. So, for example, if $p = 101.45\%$, $c = 2\%$, and $T = 20$, then $y = 1.90\%$.

News Excerpt: Sale of Greek Government Bonds in March, 2010

At the end of March, 2010, investors around the world were concerned that Greece might not be able to meet all its debt obligations. At that time, Bloomberg reported the following[3] about the Greek government's sale of new, seven-year bonds:

> *Greece priced the 5 billion euros ($6.7 billion) of seven-year bonds to yield 310 basis points more than the benchmark mid-swap rate, according to a banker involved in the transaction . . .*
>
> *The bonds' 6 percent yield equates to 334 basis points more than seven-year German bunds, Europe's benchmark government securities. That compares with a yield premium, or spread, of 61 basis points for similar maturity Spanish debt and 114 basis points on Portugal's government bonds due 2017, according to composite prices on Bloomberg. Italy's seven-year bonds yield 45 basis points more than bunds, the prices show.*
>
> *"Greece's borrowing costs exceed those of Spain and Portugal as it still needs to convince the market that it can roll over existing debt . . ."*

COMPONENTS OF P&L AND RETURN

As stated in the introduction to this chapter, breaking down P&L or return into component parts is extremely useful for understanding how money is being made or lost in a trading book or investment portfolio. In addition, many sorts of errors can often be caught by a thorough analysis of *ex-post* profitability or loss.

For expositional ease, this section makes the following choices. First, it decomposes P&L; a return decomposition can then be found by dividing each P&L component by the initial price. Second, the P&L considered is that of a single bond trading at a single spread, but the analysis can be extended to more general portfolios and term structures of spreads. Third, the holding period is assumed to be equal to a coupon payment period. Appendix B of this chapter gives the P&L decomposition for holding periods both within and across coupon payment periods.

P&L is generated by price appreciation plus *cash-carry,* which consists of explicit cash flows like coupon payments and financing costs. This section decomposes price

[3] "Greece Pays Bond Investors 5 Times Spain Yield Spread (Update1)," Bloomberg *BusinessWeek,* Thursday May 27, 2010.

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appreciation into three components and then presents a sample return decomposition. The next section focuses on one component of return, namely, carry-roll-down, in more detail.

Set the following notation:

- $P_t(\mathbb{R}_t, s_t)$: the price of a bond at time t, under term structure $\mathbb{R}_t$, and bond-specific spread s_t.
- c: periodic coupon payment of the bond.
- $P_{t+1}(\mathbb{R}_{t+1}, s_{t+1})$: the price of the bond at $t + 1$, with the term structure and bond-specific spreads changing as indicated.
- $\mathbb{R}^e_{t+1}$: some term structure of rates that is not necessarily the term structure at time t or $t + 1$. The choice of this term structure will be discussed shortly.

The total price appreciation and a breakdown of that appreciation into its component parts can be defined as follows.[4] Note that the sum of the component parts is, by design, identically equal to total price appreciation.

- *Total Price Appreciation:* $P_{t+1}(\mathbb{R}_{t+1}, s_{t+1}) - P_t(\mathbb{R}_t, s_t)$
- *Carry-Roll-Down:* $P_{t+1}(\mathbb{R}^e_{t+1}, s_t) - P_t(\mathbb{R}_t, s_t)$
- *Rate Changes:* $P_{t+1}(\mathbb{R}_{t+1}, s_t) - P_{t+1}(\mathbb{R}^e_{t+1}, s_t)$
- *Spread Change:* $P_{t+1}(\mathbb{R}_{t+1}, s_{t+1}) - P_{t+1}(\mathbb{R}_{t+1}, s_t)$

The first component of the decomposition, called carry-roll-down, is the price change due to the passage of time with rates moving "as expected," from $\mathbb{R}_t$ to $\mathbb{R}^e_{t+1}$, and with no change in spread. Before proceeding further, however, it is worthwhile to explain the name carry-roll-down by discussing the generic concepts of carry and roll-down, which are invoked often in practice, but tend to generate some confusion.

Most generally, P&L due to carry is meant to convey how much a position earns due to the passage of time, holding everything else constant. A clean example is a par bond when the term structure is flat and unchanging: since the bond's price is always par, its carry is clearly its coupon income minus its cost of financing. Another clean example is a premium bond when the term structure is, again, flat and unchanging. Since this bond's price is pulled to par over time (see Figure 9-1), its carry is easily defined as its coupon income minus the decline in its price minus its cost of financing. Note, by the way, that the concept of carry just described, by including pull-to-par P&L, is broader than cash-carry, defined earlier as coupon income minus financing costs. Cash-carry plays an important role in describing bond forward and futures prices.

P&L due to roll-down is meant to convey how much a position earns due to the fact that, as a security matures, its cash flows are priced at earlier points on the term structure. A clean example of this is the 10y6m forward highlighted in the case study of Chapter 8. At the time of that case, an investor might agree to lend EUR for six months, 10-years forward, at a rate of 4.254%. That trade has no carry in the sense of the previous paragraph: it pays no coupon, it costs nothing to finance, and, if the market rate of the forward trade remains at 4.254%, then its P&L is zero. But if at the time of the trade the 9y6m rate was 4.127%, then the trade would be said to have roll-down P&L in the following sense. If the term structure does not change, then, after a year, the 10y6m forward trade at 4.254% matures into a 9y6m forward with an appropriate market rate of 4.127%. Hence, the investor would gain the difference between 4.254% and 4.127%, or 12.7 basis points, because the forward trade had "rolled-down" the curve.

The examples in the previous two paragraphs cleanly illustrate the concepts of carry and roll-down, but the division of P&L between the two often requires further calculation. Consider a premium bond when the term structure is upward-sloping and unchanging. The resulting P&L over time would be a combination of carry, i.e., pull-to-par plus coupon minus financing costs, and roll-down, as the bond's cash flows are discounted at lower rates. While an investor could define some separation of this P&L into distinct carry and roll-down components, the separation would not be as clean as in the earlier examples and, more importantly, would probably not be worth the effort. From the perspective of understanding P&L over time, the more important objective is to separate out what happens to a position when rates move "as expected" from what happens as rates and spread change.

Taking all of these considerations into account, this book preserves a separate accounting for cash-carry, i.e., coupon income minus financing costs, so as to be consistent with concepts in forward and futures markets. The

[4] Defining the breakdown in a different order can change the allocation of the total price appreciation, but the magnitude of this change is usually very small except for securities with values that are very nonlinear in rates or spreads.

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remaining P&L due to the passage of time, i.e., the P&L due to the passage of time excluding cash-carry, is called carry-roll-down. This name reflects the fact that carry-roll-down is a mix of P&L that might otherwise be classified as either carry or roll-down.

Returning then to the P&L decomposition given previously, carry-roll-down P&L is the price appreciation due to the bond's maturing over the period and rates moving from the original term structure $\mathbb{R}_t$ to some hypothetical, "expected," or intermediate term structure, $\mathbb{R}^e_{t+1}$. There are many possible choices for $\mathbb{R}^e_{t+1}$ and some common ones are discussed in the next section, but no choice clearly dominates another. In any case, note that carry-roll-down price appreciation assumes that the bond's individual spread has not changed over the period. Also note that practitioners often calculate carry-roll-down in advance, that is, at time t they are interested in knowing the carry-roll-down from time t to time $t + 1$.

The price appreciation due to rate changes is the price effect of rates changing from the intermediate term structure, $\mathbb{R}^e_{t+1}$, to the term structure that actually prevails at time $t + 1$, namely $\mathbb{R}_{t+1}$. Note that spread is assumed unchanged here as well. Note also that price appreciation due to changes in rates might be calculated in advance as part of a scenario analysis, but is usually reserved for calculations done *ex-post* as part of realized return.

Finally, the price appreciation due to a spread change is the price effect due to the bond's individual spread changing from s_t to s_{t+1}. The spread is, in fact, the focus or bet of many trades. Is this U.S. Treasury too cheap relative to others? Is that corporate bond too expensive relative to swaps? Price appreciation due to a spread change, like that due to rate changes, may be calculated in advance as part of a scenario analysis or *ex-post* in the process of computing realized returns.

Note that dividing each of the components of price appreciation and then cash-carry by the initial price, $P_t(\mathbb{R}_t, s_t)$, gives the respective components of bond return.

A Sample P&L Decomposition

This subsection works through an example of decomposing the return of the ¾s of November 30, 2011, over the six months from May 28, 2010, to November 30, 2010. The example assumes that:

- The initial term structure and spreads are as in Equation (9.9);
- The carry-roll-down scenario is *realized forwards*, which will be explained shortly;
- The term structure falls in parallel by 10 basis points over the six-month holding period;
- The bond's spread converges from its initial 4.4 basis points to 0 over the holding period.

Table 9-2 shows how forward rates and prices change from their initial values to the values in each step of the decomposition. The initial forwards used to price the ¾s on May 28, 2010, given in row (i) of the table, are the sums of the initial base forwards on that date, row (ii), and the computed spread of the ¾s on that date, row (iii). The price of the bond using these forwards and this spread is 100.190, given in the rightmost column of row (i). See Equation (9.9). Rows (iv) through (xii) of the table describe the pricing of the ¾s at the end of the holding period, on November 30, 2010.

The first price change, due to carry-roll-down, is presented in rows (iv) through (vi) of Table 9-2. The assumption of realized forwards means the following. As of the initial date, May 28, 2010, the forward rate curve in row (ii) "anticipated" a rate of .556% from November 30, 2010, to May 31, 2011, and a rate of 1.036% from May 31, 2011, to November 31, 2011. Then, six months later, these anticipated rates were realized: on November 30, 2010, the forward rate curve in row (v) is taken to be .556% in the first period and 1.036% in the second. The justification for the assumption of realized forwards will be described in the next section. Under these forwards in row (v), however, along with an unchanged spread of .044%, row (vi), the price of the now one-year bond is 99.911, given in the rightmost column of row (iv). Hence, the price appreciation due to carry-roll-down in this example is 99.911 − 100.190 or −.279. (Of course, the bond paid a coupon on November 30, 2010, but that will be handled in the cash-carry part of the calculations.)

The next price change, due to rate changes, is presented in rows (vii) through (ix). For this example it is assumed that all forward rates fell by 10 basis points. Therefore, the term structure of forwards falls from .556% and 1.036% in row (v) to .456% and .936% in row (viii). The spreads in

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TABLE 9-2 A Decomposition of the Price Appreciation for the ¾s of November 30, 2011, over a Six-Month Holding Period

	Start Period End Period	5/30/10 11/30/10	11/30/10 5/31/11	5/31/11 11/30/11	Price
	Pricing Date 5/28/10				
(i)	Initial Forwards	.193%	.600%	1.080%	100.190
(ii)	Term Structure	.149%	.556%	1.036%	
(iii)	Spreads	.044%	.044%	.044%	
	Pricing Date 11/30/10				
(iv)	Carry-Roll-Down Forwards		.600%	1.080%	99.911
(v)	Term Structure		.556%	1.036%	
(vi)	Spreads		.044%	.044%	
(vii)	Rate-Change Forwards		.500%	.980%	100.011
(viii)	Term Structure		.456%	.936%	
(ix)	Spreads		.044%	.044%	
(x)	Spread-Change Forwards		.456%	.936%	100.054
(xi)	Term Structure		.456%	.936%	
(xii)	Spreads		.000	.000	

row (ix) remain again at 4.4 basis points, so the new forwards for pricing the ¾s in row (vii) are .500% and .980%. These new forwards give a bond price of 100.011 in the rightmost column of row (vii) and a price appreciation due to rate changes of 100.011 − 99.911 or .1.

The final price change, due to the change of the spread from .044% to 0%, is presented in rows (x) through (xii). Keeping the new term structure in row (xi) the same as in row (viii) and using a zero spread in row (xii), the new forwards in row (x) are .456% and .936%, which gives a final bond price of 100.054 in the rightmost column of row (x). Hence, the price appreciation due to spread change is 100.054 − 100.011 or .043.

Table 9-3 summarizes the components of price appreciation and adds the coupon payment to complete the decomposition of gross dollar return. Were the position financed, the financing cost would be included in the carry so as to compute net dollar returns. Finally, these dollar returns can be divided by the initial price to obtain percentage returns, although, since the initial price is very near 100, percentage returns do not add much insight in this particular example.

CARRY-ROLL-DOWN SCENARIOS

When considering potential trades or investments, many practitioners want to calculate the dollar return of the trade or investment under the expectation or scenario of "no change" in rates. So the question with respect to carry-roll-down is, "What are good choices for no change scenarios?"

One common choice is to assume that forward rates equal expectations of future rates and that, as time passes, these forward rates are realized. So, for example, today's six-month rate two years forward is the realized six-month rate two years from today. This realized forward assumption was used in the sample P&L decomposition of the previous section. A second common choice

TABLE 9-3 Decomposition of P&L of the ¾s of November 30, 2011, over a Six-Month Holding Period

	$
Initial Price	100.190
Price Appreciation	**−.136**
Carry-Roll-Down	−.279
Rates	+.100
Spread	+.043
Cash-Carry	**.375**
Coupon	.375
Financing	0.000
P&L	**+.239**

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assumes that the entire term structure of interest rates remains unchanged over time. So, for example, today's six-month rate two years forward will be the six-month rate two years forward a week from now, a month from now, a year from now, etc.

This section derives some implications of the realized forward and unchanged term structure assumptions, in addition to the related assumption of unchanged yields. To conclude, the section considers one alternative assumption which, while conceptually attractive, is hardly used in practice.

Realized Forwards

Given the example of realized forwards in the previous section, this subsection proceeds directly to the mathematics. Recall the pricing equation of a bond in terms of forwards, omitting any spreads to the base curve:

$$P_0(\mathbb{R}_0) = \frac{c}{(1+f(1))} + \frac{c}{(1+f(1))(1+f(2))} + \cdots + \frac{1+c}{(1+f(1))(1+f(2))\cdots(1+f(T))} \quad \textbf{(9.16)}$$

Under the assumption of realized forwards, the price of the bond after one period becomes

$$P_1(\mathbb{R}_1) = \frac{c}{(1+f(2))} + \frac{c}{(1+f(2))(1+f(3))} + \cdots + \frac{1+c}{(1+f(2))(1+f(3))\cdots(1+f(T))} \quad \textbf{(9.17)}$$

Combining Equations (9.16) and (9.17) it is easy to see that

$$\frac{P_1(\mathbb{R}_1) + c - P_0(\mathbb{R}_0)}{P_0(\mathbb{R}_0)} = f(1) \quad \textbf{(9.18)}$$

In words, Equation (9.18) says that the gross, single-period return of any security is the prevailing one-period rate. A two-year bond and a 10-year bond, over the next period, both earn the short-term rate. This result and the underlying assumption of realized forwards is not particularly satisfying. It is more common to assume that, since the 10-year bond has more interest rate risk than the two-year bond, investors demand a higher return for the 10-year bond. In any case, under the reasonable assumption that the one-period financing rate is $f(1)$, subtracting this rate from the gross return in (9.18) shows that the single-period, net return of any security is 0.

In a similar manner it is easy to show that in the presence of a term structure of spreads, i.e., with price given by (9.10), the one-period gross return of a bond in the case of realized forward rates and spreads is $f(1) + s(1)$, i.e., the short-term rate plus the short-term spread.

The gross return under the realized forward assumption can be calculated over many periods as well. In general, it can be shown that the return to maturity under realized forwards is

$$\frac{c(1+f(2))(1+f(3))\cdots(1+f(T)) + \cdots}{P_0(\mathbb{R}_0)} + \frac{P_T(\mathbb{R}_T) - P_0(\mathbb{R}_0)}{P_0(\mathbb{R}_0)} = (1+f(1))(1+f(2))\cdots(1+f(T)) - 1 \quad \textbf{(9.19)}$$

In words, the return to a bond held to maturity under the assumption of realized forward rates is the same as rolling a $1 investment one period at a time at those forward rates.

The discussion of this subsection has interesting implications in the answer to the following question. Which of the following two strategies is more profitable, rolling over one-period bonds or investing in a long term bond and reinvesting coupons at prevailing short-term rates? As just demonstrated, if forward rates are realized, the two strategies are equally profitable. But if realized forwards are greater than the forwards implicit in the initial bond price, rolling over one-period bonds is more profitable. And if realized forwards are less than those implicit in the initial bond price, investing in the long-term bond is more profitable. Hence, the decision to roll short-term investments or to purchase long-term bonds depends on how the decision maker's forecast of rates compares with market forward rates. Note, however, that while this reasoning provides a good deal of intuition about the returns of short- *versus* long-term bonds, it says nothing about the more realistic case of some forwards being realized above the initial forwards and some being realized below.

Unchanged Term Structure

A very common carry-roll-down assumption is that the term structure stays unchanged. If the six-month rate two years forward is 1.25% today, then, six months from now, the six-month rate two-years forward will still be 1.25%. Under this assumption, the prices of a bond today and after one period are

$$P_0(\mathbb{R}_0) = \frac{c}{(1+f(1))} + \frac{c}{(1+f(1))(1+f(2))} + \cdots + \frac{1+c}{(1+f(1))(1+f(2))\cdots(1+f(T))} \quad \textbf{(9.20)}$$

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$$P_1(\mathbb{R}_1) = \frac{c}{(1+f(1))} + \frac{c}{(1+f(1))(1+f(2))} + \cdots + \frac{1+c}{(1)+f(1))(1+f(2))\cdots(1+f(T-1))} \quad (9.21)$$

Combining the two Equations (9.20) and (9.21) reveals the one-period gross return under the assumption of an unchanged term structure:

$$\frac{P_1(\mathbb{R}_1)+c-P_0(\mathbb{R}_0)}{P_0(\mathbb{R}_0)} = \left[\frac{f(T)-c}{(1+f(1))\cdots(1+f(T))}+c\right]\frac{1}{P_0(\mathbb{R}_0)} \quad (9.22)$$

While Equation (9.22) does not have as neat an interpretation as the analogous equation for realized forwards, it does make the following point. The gross return under the assumption of an unchanged term structure depends most crucially on the last relevant forward rate, that is the forward rate from one-period before maturity to maturity, *versus* the bond's coupon rate. The intuition for this result parallels the discussion in the "Maturity and Price or Present Value" subsection of Chapter 8. Finally, it is easy to show that in the presence of a term structure of spreads, the relevant quantity for determining the return becomes $f(T) + s(T) - c$.

The realized forward assumption implicitly assumes that there is no risk premium built into forward rates. The unchanged term structure implicitly assumes the opposite extreme. If the term structure slopes upward on average and yet remains unchanged on average, it must be that the upward-sloping shape is completely explained by investors' requiring a risk premium that increases with term.

Unchanged Yields

Yet another carry-roll-down assumption is that a bond's yield remains unchanged. This assumption is useful not so much for explicit carry-roll-down calculations but for interpreting yield-to-maturity as a measure of return. The bond pricing equation in terms of yield, Equation (9.12) without the explicit semiannual payment convention, is

$$P_0(\mathbb{R}_0) = \frac{c}{(1+y)} + \frac{c}{(1+y)^2} + \cdots + \frac{1+c}{(1+y)^T} \quad (9.23)$$

Under the assumption of unchanged yields,

$$P_1(\mathbb{R}_1) = \frac{c}{(1+y)} + \frac{c}{(1+y)^2} + \cdots + \frac{1+c}{(1+y)^{T-1}} \quad (9.24)$$

And, along the lines of the previous subsections, combine Equations (9.23) and (9.24) to see that

$$\frac{P_1(\mathbb{R}_1)+c-P_0(\mathbb{R}_0)}{P_0(\mathbb{R}_0)} = y \quad (9.25)$$

In words, the one-period gross return, assuming that yield remains unchanged, is the yield. It is in this sense that an investor in a bond earns its yield-to-maturity.

Extending this analysis to many periods, it can be shown that, under the assumption of constant yields,

$$\frac{c(1+y)^{T-1}+c(1+y)^{T-2}+\cdots}{P_0(\mathbb{R}_0)} + \frac{P_T(\mathbb{R}_T)-P_0(\mathbb{R}_0)}{P_0(\mathbb{R}_0)} = (1+y)^T - 1 \quad (9.26)$$

In words, an investor to maturity earns the bond's yield in the sense that, if the yield does not change and if all coupons are reinvested at that yield, then the return of the bond to maturity equals the return of rolling over a $1 investment period-by-period at that yield. Now while this sounds similar to the statement made in the context of realized forwards, the unchanged yield scenario is even less satisfying. The assumptions that yield stays unchanged over the life of a bond and that all coupons can be reinvested at that same yield are particularly flawed: the fact that there is a term structure of interest rates implies that a bond's yield will change with maturity and that single-period reinvestment rates should not equal bond yield. The unchanged yield assumption is less problematic for these reasons if the term structure is always flat, but that condition is quite unrealistic as well.

Expectations of Short-Term Rates Are Realized

A more conceptually appealing scenario for computing carry-roll-down is that expectations of short-term rates are realized. This is much more difficult to implement than the other scenarios presented in this section because an investor has to specify expectations of rates in the future and then describe how forwards rates are formed relative to those expectations. The outcome, arguably more sensible than others in this section, is that the expected return of a bond, which is not the same as the roll-down return,[5] is equal to the short-term rate plus a risk premium that depends on the riskiness of the bond.

[5] The expected return of a bond is not the same as the return of the bond should rates evolve according to expectation. Mathematically, a price at expected rates is not equal to the expected price.

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APPENDIX A

Yield on Settlement Dates Other than Coupon Payment Dates

To keep the presentation of ideas simple, the "Yield-to-Maturity" subsection earlier in this chapter considered only settlement dates that fall on coupon payment dates. This appendix gives the formula for yield-to-maturity when the settlement date does not fall on a coupon payment date. The definition of yield is expressed in the text as Equation (9.13) or (9.14):

$$P = \frac{c}{2}\sum_{t=1}^{2T}\frac{1}{\left(1+\frac{y}{2}\right)^{t}} + \frac{1}{\left(1+\frac{y}{2}\right)^{2T}} \tag{9.27}$$

$$P = \frac{c}{y}\left(1-\frac{1}{\left(1+\frac{y}{2}\right)^{2T}}\right) + \frac{1}{\left(1+\frac{y}{2}\right)^{2T}} \tag{9.28}$$

Equation (9.27) has to change in two ways to take account of a settlement date between coupon dates. First, price has to be interpreted to be the full price of the bond. See the "Accrued Interest" section of Chapter 7. Second, the exponents of Equation (9.27) have to be adjusted to reflect the timing of the cash flows. When the coupon payments arrive in semiannual intervals, then, following the semiannual compounding convention, the first payment is discounted by dividing by $1 + y/2$, the second by dividing by $(1 + y/2)^2$, etc. But what if the first payment is paid in a fraction τ of a semiannual period? (If the next coupon were paid in five months, for example, then $\tau = 5/6$.)[6]

Market convention for the purpose of calculating yield (which cannot really be justified in terms of the logic of semiannual compounding) is to discount the next coupon payment by

$$\frac{1}{\left(1+\frac{y}{2}\right)^{\tau}} \tag{9.29}$$

and a subsequent payment i semiannual periods later by

$$\frac{1}{\left(1+\frac{y}{2}\right)^{\tau+i}} \tag{9.30}$$

Under this convention, the price-yield equation for a bond making its next payment in a fraction τ of a semiannual period and then making $2T - 1$ subsequent semiannual payments is

$$P = \frac{c}{2}\sum_{t=0}^{2T-1}\frac{1}{\left(1+\frac{y}{2}\right)^{\tau+t}} + \frac{1}{\left(1+\frac{y}{2}\right)^{\tau+2T-1}} \tag{9.31}$$

Finally, applying the summation formula in Appendix D of Chapter 8 to (9.31) in order to derive the generalization of Equation (9.28) gives the relatively simple

$$P = \left(1+\frac{y}{2}\right)^{1-\tau}\left[\frac{c}{y}\left(1-\frac{1}{\left(1+\frac{y}{2}\right)^{2T}}\right) + \frac{1}{\left(1+\frac{y}{2}\right)^{2T}}\right] \tag{9.32}$$

[6] More accurately, τ would be calculated with the day-count convention appropriate for the security in question.

APPENDIX B

P&L Decomposition on Dates Other than Coupon Payment Dates

For ease of exposition, the text assumed that dates t and $t + 1$ are both coupon payment dates. To generalize the P&L decomposition, this appendix allows these dates to fall between coupon payment dates. The notation of the text continues here, with the following qualifications and additions. Let P_t denote the full price of a bond, p_t denote its quoted price, and $AI(t)$ denote its accrued interest, so that $P_t = p_t + AI(t)$. The coupon rate is c, as in the text, and let the financing rate be r. Finally, let there be d days between dates t and $t + 1$.

Begin with the case in which there is no coupon paid between dates t and $t + 1$. Then the total P&L of a bond, including the cost of financing the full price of the bond for d days, is

$$P_{t+1}(\mathbb{R}_{t+1}, s_{t+1}) - P_t(\mathbb{R}_t, s_t)\left(1-\frac{rd}{360}\right) \tag{9.33}$$

Using the breakdown of full price into quoted price plus accrued interest and rearranging terms, the P&L becomes

$$p_{t+1}(\mathbb{R}_{t+1}, s_{t+1}) - p_t(\mathbb{R}_t, s_t) + AI(t+1) - AI(t) - P_t(\mathbb{R}_t, s_t)\frac{rd}{360} \tag{9.34}$$

Applying the breakdown in the text to the quoted price appreciation in (9.34) gives

$$\begin{aligned}&[p_{t+1}(\mathbb{R}^e_{t+1}, s_t) - p_t(\mathbb{R}_t, s_t)] + [p_{t+1}(\mathbb{R}_{t+1}, s_t) - p_{t+1}(\mathbb{R}^e_{t+1}, s_t)]\\ &\quad + [p_{t+1}(\mathbb{R}_{t+1}, s_{t+1}) - p_{t+1}(\mathbb{R}_{t+1}, s_t)]\\ &\quad + \left[AI(t+1) - AI(t) - P_t(\mathbb{R}_t, s_t)\frac{rd}{360}\right]\end{aligned} \tag{9.35}$$

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The P&L terms of (9.35) are, in order, the contributions due to carry-roll-down, rates, spread, and cash-carry.

In the case that there is a coupon payment between dates t and $t + 1$, then, ignoring the second order amount of interest on the coupon payment from its payment date to $t + 1$, the P&L expression (per unit face amount) changes only with the cash-carry term in (9.35) changing to

$$\frac{c}{2} + AI(t+1) - AI(t) - P_t(R_t, s_t)\frac{rd}{360} \qquad \textbf{(9.36)}$$

Note, however, that in (9.35), $AI(t + 1) > AI(t)$ because there is no coupon paid between t and $t + 1$. By contrast, in (9.36), $AI(t + 1)$ may be greater or less than $AI(t)$ depending on where the two dates fall in the coupon cycle.

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One-Factor Risk Metrics and Hedges

10

Learning Objectives

After completing this reading you should be able to:

- Describe an interest rate factor and identify common examples of interest rate factors.
- Define and compute the DV01 of a fixed income security given a change in yield and the resulting change in price.
- Calculate the face amount of bonds required to hedge an option position given the DV01 of each.
- Define, compute, and interpret the effective duration of a fixed income security given a change in yield and the resulting change in price.
- Compare and contrast DV01 and effective duration as measures of price sensitivity.
- Define, compute, and interpret the convexity of a fixed income security given a change in yield and the resulting change in price.
- Explain the process of calculating the effective duration and convexity of a portfolio of fixed income securities.
- Explain the impact of negative convexity on the hedging of fixed income securities.
- Construct a barbell portfolio to match the cost and duration of a given bullet investment, and explain the advantages and disadvantages of bullet versus barbell portfolios.

Excerpt is Chapter 4 of Fixed Income Securities, *Third Edition, by Bruce Tuckman.*

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This chapter presents some of the most important concepts used to measure and hedge risk in fixed income markets, namely, *DV*01, duration, and convexity. These concepts are first presented in a very general, one-factor framework, meaning that the only significant assumption made about how the term structure changes is that all rate changes are driven by one factor. As an application used to illustrate concepts, the chapter focuses on a market maker who shorts futures options and hedges with futures, although the reader need not know anything about futures at this point.

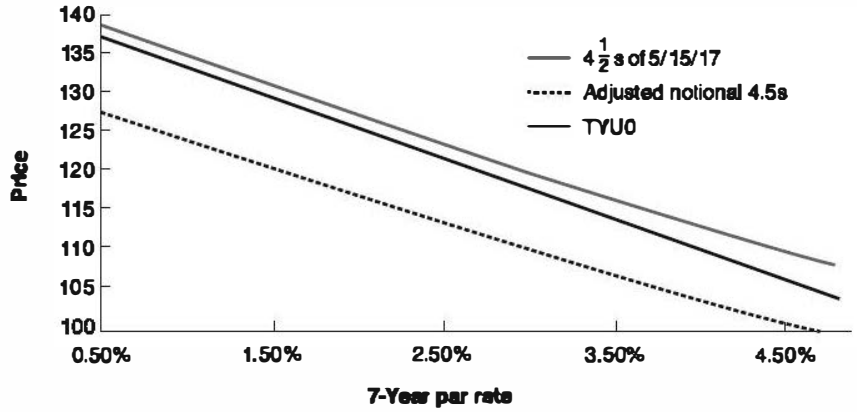

FIGURE 10-1 4½s of 5/15/2017 and TYU0 price-rate curves as of May 28, 2010.

The chapter then presents the yield-based equivalents of these more general concepts, i.e., yield-based *DV*01, duration, and convexity. Because these can be expressed through relatively simple formulas, they are very useful for building intuition about the interest rate risk of bonds and are widely used in practice. They cannot, however, be applied to securities with interest-rate contingent payoffs, like options.

The chapter concludes with an application in which a portfolio manager is deciding whether to purchase duration in the form of a bullet or barbell portfolio. As it turns out, the choice depends on the manager's view on future interest rate volatility.

*DV*01

Denote the price-rate function of a fixed income security by $P(y)$, where y is an interest rate factor. Despite the usual use of y to denote a yield, this factor might be a yield, a spot rate, a forward rate, or a factor in one of the models. In any case, since this chapter describes one-factor measures of price sensitivity, the single number y completely describes the term structure of interest rates.

This chapter uses three securities, with prices as of May 28, 2010, to illustrate concepts: the U.S. Treasury 4½s of May 15, 2017; the 10-year U.S. note futures contract maturing in September 2010, whose ticker is TYU0; and a call option on TYU0 with a strike of 120 and a maturity of August 27, 2010, whose ticker is TYU0C 120. For the purposes of this chapter, the reader need not know anything about futures and futures options. Understanding the interest rate risk of a security from its price-rate function can be separated from the creation of that price-rate function. For completeness, however, it is noted here that the price-rate curves of the three illustrative securities were created using a particular calibration of the Vasicek model.

Figure 10-1 graphs three price-rate curves as a function of a (hypothetical) seven-year U.S. Treasury par rate, which, on the pricing date, was 2.77%. The three curves are for TYU0, for 100 notional amount of the 4½s, and for an adjusted notional amount of the 4½s which, because of the technicalities of the futures contract, is more comparable to TYU0.[1] This adjusted notional position is included in Figure 10-1 to highlight the difference between the shape of a bond's price-rate curve and that of a futures contract. The price-rate curve of the 4½s is typical of all coupon bonds; it decreases with rates and is very slightly convex,[2] though that is hard to see from this figure. The price-rate curve of TYU0 is typical of futures, decreasing with rates but with both convex and concave[3] regions. The convex region is to the left of the graph, for low values of rates, while the concave region is to the right of the graph, most easily recognized in contrast with the convexity of the two bond curves over that same region.

[1] The notional amount is 100 divided by the conversion factor of the bond for delivery into TYU0.

[2] A line connecting any two points of a convex curve lies above the curve over that region.

[3] The line connecting any two points of a concave curve lies below the curve over that region.

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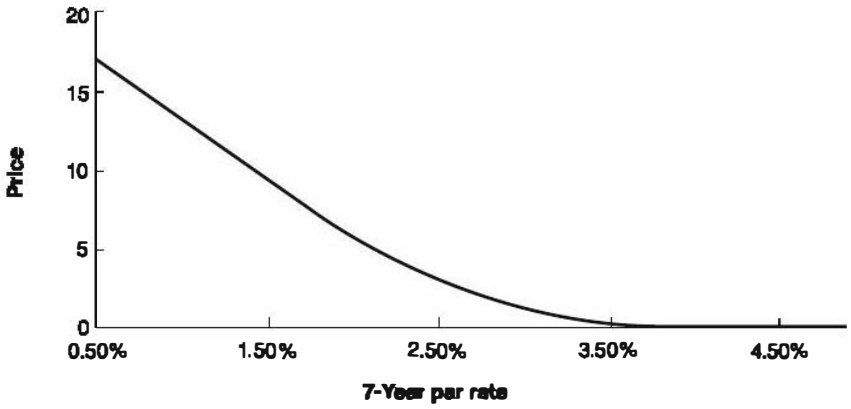

FIGURE 10-2 TYUOC 120 price-rate curve as of May 28, 2010.

Figure 10-2 graphs the price-rate curve of TYUOC 120. Its shape is typical for a call option on a fixed income security, decreasing to zero as rates increase and highly convex between a decreasing linear segment on the left and a flat, zero-valued segment on the right.[4]

The price-rate curves in Figures 10-1 and 10-2 can be used to compute the price sensitivities of the three securities with respect to interest rates. From Figure 10-2, for example, if rates rise 10 basis points from .95% to 1.05%, the price of the option falls from 13.550 to 12.755, for a slope of $\frac{13.550-12.755}{1.05\%-.95\%}$, which is −795 or −7.95 cents per basis point. If rates rise from 2.45% to 2.55% the same option falls in price from 3.096 to 2.622, for a slope of −474 or −4.74 cents per basis point. And finally, if rates rise from 3.45% to 3.55% the option falls from .310 to .225, for a slope of −85 or −.85 cents per basis point. The fact that price sensitivity changes as rates change will be explored in later sections.

To define a measure of interest rates more generally, let ΔP and Δy denote the changes in price and rate, respectively, and note that the change in rate measured in basis points is $10{,}000 \times \Delta y$. Then, consider the following measure of price sensitivity:

$$DV01 = -\frac{\Delta P}{10{,}000 \times \Delta y} \tag{10.1}$$

[4] The typical shape of an option price-price curve is a hockey stick increasing to the right. Figure 10-2, however, is a price-rate curve.

*DV*01 is an acronym for *dollar value of an 01* (i.e., of .01%) and gives the change in the value of a fixed income security for a one-basis point decline in rates. The negative sign defines *DV*01 to be positive if price increases when rates decline and negative if price decreases when rates decline. This convention has been adopted so that *DV*01 is positive most of the time: all fixed coupon bonds and most other fixed income securities do rise in price when rates decline.

In the discussion of Figure 10-2, the slope of the call is estimated using pairs of option prices valued at rates which are 10 basis points apart: the points (.95%, 13.550) and (1.05%, 12.755) are used to provide an estimate of the slope at a rate of 1%, the points (2.45%, 3.096) and (2.55%, 2.622) are used to provide an estimate at a rate of 2.5%, etc. Since the slope of the call does change with rates, using points closer together, e.g., at 2.49% and 2.51% for an estimate of the slope at 2.50%, would—so long as the price of the call can be computed accurately enough—give a more precise estimate of the slope at a single point on the curve. In the limit of moving these points together, the estimation gives the slope of the line *tangent* to the price-rate curve at the chosen rate level. Figure 10-3 graphs two such tangent lines to TYUOC 120, one tangent at 2.50% and one at 3.50%. That the former is steeper than the latter shows that the option is more sensitive to rates at 2.50% than it is at 3.50%.

In the calculus, the slope of the tangent line at a particular rate level is called the *derivative* of the price-rate function at that rate and is denoted dP/dy. In some special cases, e.g., the yield-based metrics discussed later in this chapter or certain model-based metrics, the derivative of the price-rate function can be written in closed form, i.e., as a relatively simple mathematical formula. In other cases it has to be calculated numerically as in the calculations for TYUOC 120 shown previously. In either case, in terms of the derivative, Equation (10.1) for *DV*01 becomes

$$DV01 = -\frac{1}{10{,}000}\frac{dP}{dy} \tag{10.2}$$

Before closing this section, a note on terminology is in order. Most market participants use *DV*01 to mean yield-based *DV*01, which is discussed later in this chapter. Yield-based *DV*01 assumes that the yield-to-maturity of a particular security changes by one basis point while, in

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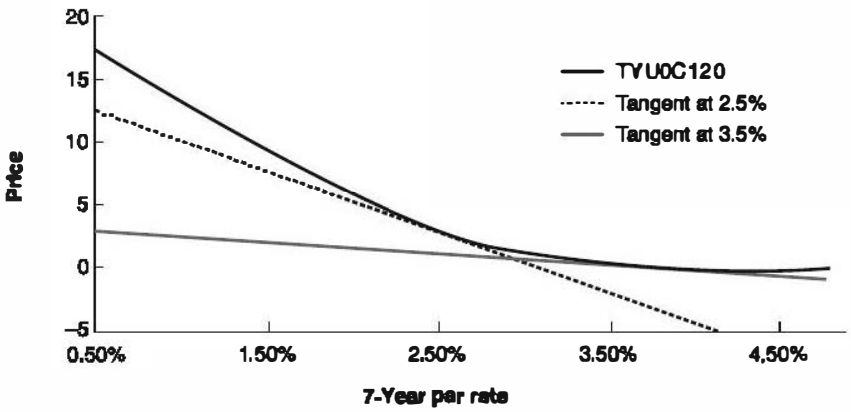

FIGURE 10-3 Tangent lines at 2.50% and 3.50% to the TYUOC 120 price-rate curve as of May 28, 2010.

the general definition of *DV*01 in this section, some factor changes by one basis point, which then propagates in some way across the rest of the term structure. To avoid confusion, some market participants have different names for *DV*01 measures according to the assumed change in rates. For example, the change in price after a parallel shift in forward rates might be called *DVDF* or *DPDF* while the change in price after a parallel shift in spot or zero-coupon rates might be called *DVDZ* or *DPDZ*.[5]

A HEDGING APPLICATION, PART 1: HEDGING A FUTURES OPTION

Say that in the course of business on May 28, 2010, a market maker sells $100 million face amount of the option, TYUOC 120, when the seven-year par rate used in the figures of the previous section is 2.77%. How might the market maker hedge the resulting interest rate exposure by trading in the underlying futures contract, TYUO?[6] Since the market maker has sold the option and stands to lose money if rates fall, purchasing futures can hedge the resulting exposure.

[5] The term *PV*01 will be discussed in the next chapter.

[6] For expositional reasons this application is somewhat contrived. Since futures options are traded on exchanges, a broker-dealer would, in reality, act as an agent to purchase TYUOC for a customer's account rather than act as a principal to sell the option to a customer from its own account. Over-the-counter derivatives, on the other hand, would be more strictly consistent with the spirit of the application.

The *DV*01 of the two securities can be used to figure out exactly how many futures should be bought against the short option position.

Table 10-1 gives selected price-rate pairs for TYUO and for TYUOC 120 along with a calculated *DV*01. Note that, along the lines of the previous section, the calculated *DV*01 at 2.77% uses the prices at rates of 2.72% and 2.82%, but not the price at 2.77% itself. In any case, let *F* be the face amount of futures the market maker needs to hedge the $100 million short option position. Then, set *F* such that, after a one basis-point decline in rates, the change in the price of the hedge position plus the change in the price of the option position equals zero. Mathematically,

$$F\frac{.07442}{100} - 100{,}000{,}000 \times \frac{.03505}{100} = 0 \qquad \textbf{(10.3)}$$

There is a negative sign in front of the second term on the left-hand side because the option position is short $100 million. Also, since *DV*01 values quoted in the text and shown in the figures are for 100 face amount, they have to be divided by 100 before being multiplied by face amounts. Rearranging terms of (10.3) shows that

$$F = 100{,}000{,}000 \times \frac{.03505}{.07442} \qquad \textbf{(10.4)}$$

Solving (10.4) for *F*, the market maker should purchase $47.098 million face amount of TYUO.

To summarize this hedging strategy, the change in value of the short option position for each basis point decline in rates is

$$-\$100{,}000{,}000 \times \frac{.03505}{100} = -\$35{,}050 \qquad \textbf{(10.5)}$$

TABLE 10-1 Selected Model Prices and *DV*01s for TYUO and TYUOC 120 as of May 28, 2010

7-Year Par Rate	TYUO	*DV*01	TYUOC 120	*DV*01
2.72%	120.0780		1.9194	
2.77%	119.7061	.07442	1.7383	.03505
2.82%	119.3338		1.5689	

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The change in the value of the hedge, the $47 million face amount of TYU0, offsets this loss:

$$\$47{,}098{,}000 \times \frac{.07442}{100} = \$35{,}050 \qquad (10.6)$$

Generally, if *DV*01 is expressed in terms of a fixed face amount, hedging a position of F^A face amount of security *A* requires a position of F^B of security *B* where

$$F^B = -\frac{F^A \times DV01^A}{DV01^B} \qquad (10.7)$$

To avoid careless trading mistakes, it is worth emphasizing the simple implications of Equation (10.7), assuming for the moment that, as usually is the case, each *DV*01 is positive. First, hedging a long position in security *A* requires a short position in security *B* and *vice versa*. In the example, the market maker sells futures options and buys futures. Second, the security with the higher *DV*01 is traded in smaller quantity than the security with the lower *DV*01. In the example, the market maker buys only $47.098 million futures against the sale of $100 million options.

There are securities for which *DV*01 is negative, most notably in mortgage derivatives. Hedging such a security with a positive-*DV*01 security would, by (10.7), require both sides of the trade to be long or short.

Return to the market maker who sells $100 million of TYUOC 120 and buys $47.098 million TYU0 when rates are 2.77%. Using the prices in Table 10-1, the value of the hedged position immediately after the trades is

$$-\$100{,}000{,}000 \times \frac{1.7383}{100} + \$47{,}098{,}000 \times \frac{119.7061}{100} = \$54{,}640{,}879 \qquad (10.8)$$

Now say that rates fall by 5 basis points to 2.72%. Using the prices in Table 10-1 at the new rate level, the value of the position becomes

$$-\$100{,}000{,}000 \times \frac{1.9194}{100} + \$47{,}098{,}000 \times \frac{120.0780}{100} = \$54{,}634{,}936 \qquad (10.9)$$

The hedge has succeeded in that the value of the position has hardly changed even though rates have changed.

To avoid misconceptions about market making, note that the market maker in this example makes no money. In reality, the market maker would sell the options at some premium to their fair value. Taking half a tick, for example, the market maker would take an immediate value gain of half of 1/32 or .015625 on the $100 million options for a total of $15,625. This spread compensates the market maker for executing the original trade and for managing the hedge of the position over the time. Some of the challenges of hedging the option after the initial trade are discussed in the continuation of this application later in this chapter.

DURATION

*DV*01 measures the dollar change in the value of a security for a basis point change in interest rates. Another measure of interest rate sensitivity, *duration*, measures the percentage change in the value of a security for a unit change in rates. Mathematically, letting *D* denote duration,

$$D = -\frac{1}{P}\frac{\Delta P}{\Delta y} \qquad (10.10)$$

As in the case of *DV*01, when an explicit formula for the price-rate function is available, the derivative of the price-rate function may be used for the change in price divided by the change in rate:

$$D = -\frac{1}{P}\frac{dP}{dy} \qquad (10.11)$$

Otherwise, prices at various rates must be substituted into (10.10) to estimate duration.

Table 10-2 gives the same rate levels and prices as Table 10-1 but computes duration instead of *DV*01. Once again, rates above and below the rate level in question are used to compute changes. The duration of TYU0 at 2.77% is given by

$$D = -\frac{1}{119.7061}\frac{(119.3338 - 120.0780)}{2.82\% - 2.72\%} = 6.217 \qquad (10.12)$$

TABLE 10-2 Selected Model Prices and Durations for TYU0 and TYUOC 120 as of May 28, 2010

7-Year Par Rate	TYU0	Duration	TYUOC 120	Duration
2.72%	120.0780		1.9194	
2.77%	119.7061	6.217	1.7383	201.6
2.82%	119.3338		1.5689	

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One way to interpret the duration number of 6.217 is to multiply both sides of definition (10.10) by Δy:

$$\frac{\Delta P}{P} = -D\Delta y \tag{10.13}$$

In the case of TYU0, Equation (10.13) says that the percentage change in price equals −6.217 times the change in rate. Therefore, a one-basis point decrease in rate will result in a percentage price change of 6.217 × .0001 or .06217%. Since the price of TYU0 at 2.77% is 119.7061, this percentage change translates into a dollar change of .06217% × 119.7061 or .07442 per basis point, which is, of course, the *DV*01 of the futures at that rate level.

When speaking about duration, it is conventional to normalize for a 100 basis-point change in rates. In the present case, for example, practitioners would say that TYU0's price changes by 6.217% for a 100 basis-point change in rates. This is a convention of language, not of practice, because duration, like *DV*01, changes with the level of rates so that the actual price change for a move as large as 100 basis points will not be particularly well approximated by 6.217%.

Duration tends to be more convenient than *DV*01 in the investing context, as opposed to the trading context. If an institutional investor has funds to invest when rates are 2.77%, the fact that the duration of TYU0C 120 vastly exceeds that of TYU0 alerts the investor to the far greater risk of investing money in options. With a duration of 6.215, the funds invested in TYU0 will change in value by about .62% for a 10-basis point change in rates. However, with a duration of 201.381, the same funds invested in the option will gain or lose about 20.1% for the same 10-basis point change in rates!

By contrast, in a trading or hedging problem percentage changes are not particularly useful because the dollar amounts of the two sides of the trade are usually not the same. In the example of the previous section, the market maker sells options worth about $1.74 million and buys futures with a bond-equivalent value of $56.38 million. Hence it is much more useful to compute the dollar sensitivity of each position, as in Equations (10.5) and (10.6).

Another difference between *DV*01 and duration is their behavior as rates change. Figure 10-3 showed that the *DV*01 of TYU0C 120 decreases as rates increase. As it turns out, however, the duration of the option increases as rates increase because the value of the option, which appears in the denominator of the definition of duration, decreases rapidly with rates. For example, at a rate of 2.77%, the option's *DV*01 is .0351 (Table 10-1) and its duration is 201.6 (Table 10-2). At a rate of 3.50%, however, the *DV*01 (calculated earlier in this chapter) is lower, at .0085, while its duration is higher, at .0085 × 10,000/.265 or about 321, where .265 is the option price at 3.50%.

Like the section on *DV*01, this section closes with a note on terminology. As defined in this chapter, duration may be computed for any assumed change in the term structure of interest rates. This very general definition is sometimes also called *effective duration*. In any case, note that when using the term duration many market participants mean yield-based duration, which is discussed later in this chapter.

CONVEXITY

As first mentioned in the discussion of Figure 10-3, interest rate sensitivity changes with the level of rates. *Convexity* measures this sensitivity. To start the discussion, Figure 10-4 graphs the *DV*01 of the adjusted notional amount of the 4½s of May 15, 2017, TYU0, and TYU0C 120, all as a function of the level of rates. The *DV*01 of the bond declines relatively gently as rates rise. The *DV*01 of the futures changes gently as well, although it first declines with rates, then increases, and then declines again. (This shape is usual for futures contracts.) Finally, the *DV*01 of the futures option declines gradually or steeply, depending on the level of rates.

Mathematically, convexity is defined as

$$C = \frac{1}{P}\frac{d^2P}{dy^2} \tag{10.14}$$

where the second multiplicand is the *second derivative* of the price-rate function. While the first derivative measures how price changes with rates, the second derivative measures how the first derivative changes with rates. As with *DV*01 and duration, if there is an explicit formula for the price-rate function then (10.14) may be used to compute convexity. Without such a formula, convexity must be estimated numerically.

Tables 10-3, 10-4, and 10-5 show how to estimate the convexity of the adjusted notional of the 4½s, TYU0, and TYU0C 120, respectively, at three rate levels, namely, 1.77%, 2.77%, and 3.77%. Prices have been recorded to three decimal places, but calculations have been performed using

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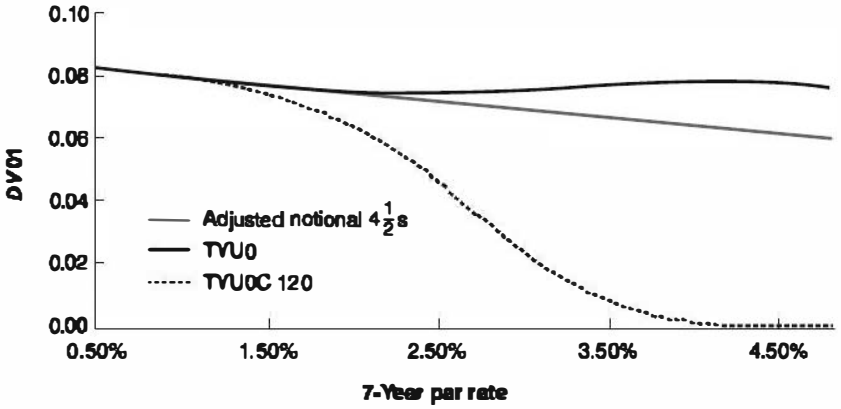

FIGURE 10-4 *DV*01-rate curves for the adjusted notional of the 4½s of 5/15/2017, TYU0, and TYU0C 120 as of May 28, 2010.

greater accuracy. (This does make a difference in the calculations of second derivatives which divide twice by a small number, namely, by the .05% difference between rates.)

The convexity of the futures contract at 1.77%, as reported in Table 10-4, is estimated as follows. Start by estimating the first derivative between 1.72% and 1.77%, i.e., at 1.745%, by dividing the change in price by the change in rate:

$$\frac{127.172545 - 127.552549}{1.77\% - 1.72\%} = -760.008 \qquad (10.15)$$

Then estimate the derivative between 1.77% and 1.82%, i.e., at 1.795%, in the same way to get −757.956. Next, estimate the second

TABLE 10-3 Model Convexity Calculations for the Adjusted Notional Amount of the 4½s of May 15, 2017, as of May 28, 2010

Rate	Price	1st Derivative	Convexity
1.72%	129.043		
1.745%		−755.304	
1.77%	128.665		41.5
1.795%		−752.637	
1.82%	128.289		
2.72%	121.737		
2.745%		−703.902	
2.77%	121.385		40.6
2.795%		−701.436	
2.82%	121.035		
3.72%	114.927		
3.745%		−656.370	
3.77%	114.599		39.8
3.795%		−654.090	
3.82%	114.272		

TABLE 10-4 Model Convexity Calculations for TYU0 as of May 28, 2010

Rate	Price	1st Derivative	Convexity
1.72%	127.553		
1.745%		−760.008	
1.77%	127.173		32.3
1.795%		−757.956	
1.82%	126.794		
2.72%	120.078		
2.745%		−743.792	
2.77%	119.706		−14.3
2.795%		−744.648	
2.82%	119.334		
3.72%	112.505		
3.745%		−773.593	
3.77%	112.119		−19.2
3.795%		−774.669	
3.82%	111.731		

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TABLE 10-5 Model Convexity Calculations for TYUOC 120 as of May 28, 2010

Rate	Price	1st Derivative	Convexity
1.72%	7.657		
1.745%		−715.275	
1.77%	7.299		2,575.0
1.795%		−705.878	
1.82%	6.946		
2.72%	1.919		
2.745%		−362.117	
2.77%	1.738		26,860.0
2.795%		−338.771	
2.82%	1.569		
3.72%	.126		
3.745%		−41.434	
3.77%	.105		113,382.0
3.795%		−35.480	
3.82%	.087		

derivative at 1.77% by dividing the change in the first derivative by the change in rates:

$$\frac{-757.956 + 760.008}{1.795\% - 1.745\%} = 4{,}104 \qquad \textbf{(10.16)}$$

Finally, to estimate the convexity, divide the estimate of the second derivative by the price of the futures contract at 1.77%:

$$\frac{1}{127.172545} \times 4{,}104 = 32.3 \qquad \textbf{(10.17)}$$

In Tables 10-3 and 10-5 the second derivatives of the bond and option are always positive so that convexity is always positive. These securities would be said to exhibit *positive convexity*. Graphically this means that their price-rate curves are convex and that, as shown in Figure 10-4, their *DV*01s fall as rates increase.

The futures contract, by contrast, is convex over part but not all of its range: in Table 10-4 TYUO exhibits positive convexity at 1.77% but negative convexity at 2.77% and at 3.77%. In terms of Figure 10-4, the *DV*01 of the futures contract is falling at 1.77% but is rising at 2.77% and also at 3.77%.

The convexity values for the option calculated in Table 10-5 are relatively large. At intermediate rate levels this is certainly due in part to the rapid fall in *DV*01 as seen in Figure 10-4. At low and high levels of rates, however, the relatively large convexity values are mostly due to the relatively low price of the option. At 3.77%, for example, the change in the first derivative is about 2.3 for the bond and 6.0 for the option. But because the option price at 3.77% is .105, compared with 114.599 for the bond, the convexity of the option is thousands of times bigger. In short, a price factor distinguishes convexity from the second derivative just as a price factor distinguishes duration from *DV*01.

A HEDGING APPLICATION, PART II: A SHORT CONVEXITY POSITION

In the first part of this hedging application the market maker buys $47.098 million face amount of TYUO against a short position of $100 million TYUOC 120. Figure 10-5 shows the profit and loss, or P&L, of a long position of $47.098 million futures and of a *long* position of $100 million options as rates change. Since the market maker is actually short the options, the P&L of the position at any rate level is the P&L of the long futures position minus the P&L of the long option position.

By construction, the *DV*01 of the long futures and option positions are the same at a rate of 2.77%. In other words, for small rate changes, the change in the value of one position equals the change in the value of the other. Graphically, the P&L curves are tangent at 2.77%.

The first part of this hedging application showed that the hedge performs well in that the market maker neither makes nor loses money after a five-basis point change in rates. At first glance it may appear from Figure 10-5 that the hedge works well after moves of 25 or even 50 basis points. The values on the vertical axis, however, are measured in millions of dollars. After a move of only 25 basis points the hedge is off by about $150,000, which is a very large number in light of the approximately $15,625 the market maker collected in spread. Worse yet, since the

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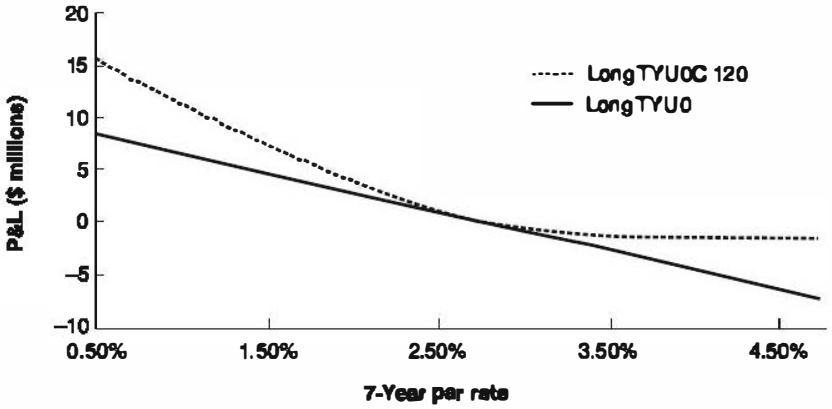

FIGURE 10-5 P&L-rate curve for a $100 million long in TYUOC 120 and a *DV*01-equivalent long in TYUO as of May 28, 2010.

P&L of the long option is always above that of the long futures position, the market maker loses this $150,000 whether rates rise or fall by 25 basis points.

The hedged position loses whether rates rise or fall because the option is more convex than the bond. In market jargon, the hedged position is *short convexity*. For small rate changes away from 2.77% the values of the futures and option positions change by the same amount. Due to its greater convexity, however, the sensitivity of the option changes by more than the sensitivity of the bond. When rates increase, the *DV*01 of the option falls by more. Hence, after further rate increases, the option falls in value less than the futures, and the P&L of the option position stays above that of the futures position. Similarly, when rates decline below 2.77%, the *DV*01 of both the futures and option rise, but the *DV*01 of the option rises by more. Hence, after further rate declines the option rises in value more than the futures and the P&L of the option position again stays above that of the futures position.

This discussion reveals that *DV*01 hedging is local, that is, valid in a particular neighborhood of rates. As rates move, the quality of the hedge deteriorates. Consequently, the market maker will need to re-hedge the position. If rates rise above 2.77% so that the *DV*01 of the option position falls by more than the *DV*01 of the futures position, the market maker will have to sell futures to re-equate *DV*01s at the higher level of rates. If, on the other hand, rates fall below 2.77% so that the *DV*01 of the option position rises by more than the *DV*01 of the futures position, the market maker will have to buy futures to re-equate *DV*01s at the lower level of rates.

An erroneous conclusion might be drawn at this point. Figure 10-5 shows that the value of the option position exceeds the value of the futures position at any rate level. Nevertheless, it is not correct to conclude that the option position is a superior holding to the futures position. The market price of an option will be set high enough relative to the price of the futures to reflect its convexity advantages. In particular, if rates do not change by very much, then as time passes the futures will perform better than the option, a disadvantage of the long option position that is not captured in Figure 10-5. In summary, the long option position will outperform the long futures position if rates move a lot while the long futures position will outperform if rates stay about the same. It is in this sense, by the way, that a long convexity position is long volatility while a short convexity position is short volatility.

ESTIMATING PRICE CHANGES AND RETURNS WITH *DV*01, DURATION, AND CONVEXITY

Price changes and returns as a result of changes in rates can be estimated with the measures of price sensitivity used in previous sections. Despite the abundance of calculating machines that, strictly speaking, makes these approximations unnecessary, an understanding of these estimation techniques builds intuition about the behavior of fixed income securities and, with practice, allows for some rapid mental calculations.

A *second-order Taylor approximation* of the price-rate function with respect to rates gives the following approximation for the price of a security after a small change in rate:

$$P(y+\Delta y) \approx P(y) + \frac{dP}{dy}\Delta y + \frac{1}{2}\frac{d^2P}{dy^2}\Delta y^2 \qquad (10.18)$$

Equation (10.18) can be rewritten in several useful ways. First, subtracting P from both sides gives an approximation for the change in price:

$$\Delta P \approx \frac{dP}{dy}\Delta y + \frac{1}{2}\frac{d^2P}{dy^2}\Delta y^2 \qquad (10.19)$$

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Second, dividing (10.19) by P gives an approximation for the percentage change in price:

$$\frac{\Delta P}{P} \approx \frac{1}{P}\frac{dP}{dy}\Delta y + \frac{1}{2}\frac{1}{P}\frac{d^2P}{dy^2}\Delta y^2 \qquad \textbf{(10.20)}$$

Third, using the definitions of duration and convexity in Equations (10.11) and (10.14), (10.20) can be rewritten as

$$\frac{\Delta P}{P} \approx -D\Delta y + \frac{1}{2}C\Delta y^2 \qquad \textbf{(10.21)}$$

As an example, given data on the price and interest rate sensitivity of TYUOC 120 at 2.77% from previous sections, what is an estimate of the price at 2.50%? Any of Equations (10.18) through (10.21) could be applied, but choose (10.18) for now. Table 10-1 reports that at 2.77% the price of the option is 1.738 and its *DV*01 is .03505, which, multiplying by −10,000, implies a first derivative of −350.5. Table 10-5 reports that at 2.77% the convexity of the option is 26,860.0, which, multiplied by its price of 1.738, implies a second derivative of 46,682.7. Substituting all these quantities into (10.18) gives the following price estimate at 2.50%:

$$P(2.50\%) \approx P(2.77\%) + \frac{dP}{dy}(2.50\% - 2.77\%) + \frac{1}{2}\frac{d^2P}{dy^2}(2.50\% - 2.77\%)^2 \qquad \textbf{(10.22)}$$

$$\approx 1.738 - 350.5 \times (-.27\%) + \frac{1}{2} \times 46{,}682.7 \times (-.27\%)^2$$

$$\approx 1.738 + .946 + .170 = 2.854$$

To three decimals the price of TYUOC 120 at 2.50% is 2.854, so the approximation given by (10.22) is quite accurate.

Note that the first derivative or *DV*01-like term of (10.22), .946, is much larger than the second derivative term, .170. Or, were the approximation (10.21) used instead, the duration term is much larger than the convexity term. This is generally true for individual securities because, while convexity is usually a larger number than duration, the change in rate is so much larger than the change in rate squared that the duration effect dominates.[7]

[7] This need not be true, of course, for manufactured securities or positions, e.g., hedged positions constructed to have zero duration.

This fact suggests that it may sometimes be safe to drop the convexity term completely and to use the *first-order approximation* for the change in price or the percentage change in price, which follow from (10.19) and (10.21), respectively:

$$\Delta P \approx \frac{dP}{dy}\Delta y \qquad \textbf{(10.23)}$$

$$\frac{\Delta P}{P} \approx -D\Delta y \qquad \textbf{(10.24)}$$

Figure 10-6 graphs the option price along with the first-order and second-order approximations at a starting rate of 2.77%. Both approximations work very well for very small changes in rate. For larger changes the second-order approximation still works well, but for very large changes it eventually fails. The figure makes clear that approximating price changes with *DV*01 or duration alone ignores the curvature or convexity of the price-rate function while adding the convexity term captures a good deal of this curvature.

In the case of a bond or futures price, with price-rate curves that exhibit much less convexity than that of the option—compare Figure 10-1 with Figure 10-2—both first- and second-order approximations work so well that they would be difficult to distinguish graphically over a relevant range of interest rates.

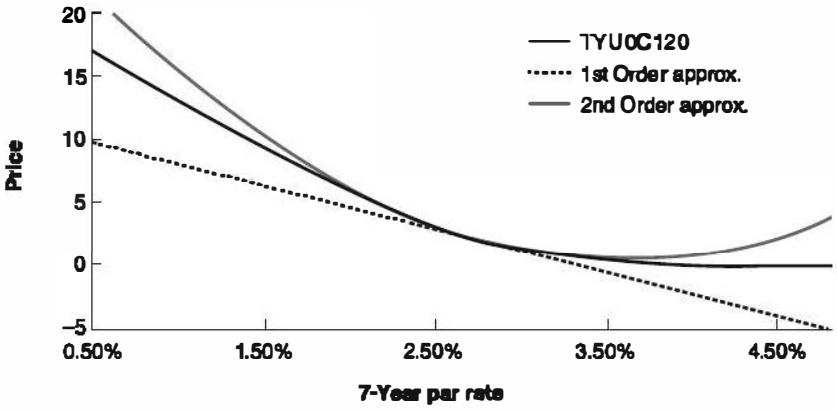

FIGURE 10-6 Price-rate curve for TYUOC 120 and its first- and second-order approximations as of May 28, 2010.

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CONVEXITY IN THE INVESTMENT AND ASSET-LIABILITY MANAGEMENT CONTEXTS

It was mentioned earlier in this chapter, in the discussion of Figure 10-5, that the option, as the more positively convex security, outperforms a *DV*01-matched position in futures if rates move a lot. This effect, that convexity is an exposure to volatility, can be seen directly from the approximation (10.21). Since Δy^2 is always positive, positive convexity increases return so long as interest rates move. The bigger the move in either direction, the greater the gains from positive convexity. Negative convexity works in the reverse. If *C* is negative, then rate moves in either direction reduce returns. In the investment context, choosing among securities with the same duration expresses a view on interest rate volatility. Choosing a very positively convex security would essentially be choosing to be long volatility, while choosing a negatively convex security would essentially be choosing to be short volatility.

Figure 10-6 suggests that asset-liability managers (or hedgers, more generally) can achieve greater protection against interest rate changes by hedging duration and convexity instead of duration alone. Consider an asset-liability manager who sets both the duration and convexity of assets equal to those of liabilities. Since both the first- and second-derivative terms of the asset and liability price-rate functions match, changes in the value of assets will more closely resemble changes in the value of liabilities than had their durations alone been matched. Furthermore, since matching convexity also sets the initial change in interest rate sensitivity of the assets equal to that of the liabilities, the sensitivity of the assets will be very close to the sensitivity of the liabilities even after a small change in rate. Put another way, the asset-liability manager need not rebalance so often as in the case of matching duration alone.

MEASURING THE PRICE SENSITIVITY OF PORTFOLIOS

This section shows how measures of a portfolio's price sensitivity are related to the measures of its component securities. Computing price sensitivities can be a time-consuming process. Since a typical investor or trader focuses on a particular set of securities at one time and constantly searches for desirable portfolios from that set, it is often inefficient to compute the sensitivity of every portfolio from scratch. A better solution is to compute sensitivity measures for all the individual securities and then to use the rules of this section to compute portfolio sensitivity measures.

A price or a measure of sensitivity for security *i* is indicated by the superscript *i*, while quantities without superscripts denote portfolio quantities. By definition, the value of a portfolio equals the sum of the value of the individual securities in the portfolio:

$$P = \sum P^i \tag{10.25}$$

Recall that in this chapter *y* has been a single rate or factor sufficient to determine the prices of all securities. Therefore, one can compute the derivative of price with respect to this rate or factor for all securities in the portfolio and, from (10.25),

$$\frac{dP}{dy} = \sum \frac{dP^i}{dy} \tag{10.26}$$

Then, dividing both sides by 10,000 and using the definition of *DV*01 in (10.1) shows that the *DV*01 of a portfolio equals the sum of the individual security *DV*01s:

$$DV01 = \sum DV01^i \tag{10.27}$$

The rule for duration is only a bit more complex. Starting from Equation (10.26), divide both sides by $-P$:

$$-\frac{1}{P}\frac{dP}{dy} = \sum -\frac{1}{P}\frac{dP^i}{dy} \tag{10.26}$$

Now multiply each term in the summation by one in the form of P^i/P^i.

$$-\frac{1}{P}\frac{dP}{dy} = \sum -\frac{P^i}{P}\frac{1}{P^i}\frac{dP^i}{dy} \tag{10.29}$$

Finally, using the definition of duration in (10.11),

$$D = \sum \frac{P^i}{P} D^i \tag{10.30}$$

In words, the duration of a portfolio equals a weighted sum of individual durations, where each security's weight is its value as a percentage of portfolio value.

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Since the formula for the convexity of a portfolio can be derived along the same lines as the duration of a portfolio, it is given here without proof:

$$C = \sum \frac{P^i}{P} C^i \tag{10.31}$$

YIELD-BASED RISK METRICS

As a special case of the metrics defined so far in this chapter, this section defines yield-based measures of price sensitivity. These measures have two important weaknesses. First, they are defined only for securities with fixed cash flows. Second, as will be seen shortly, their use implicitly assumes parallel shifts in yield, which is not a particularly good assumption. Despite these weaknesses, however, there are several reasons fixed income professionals must understand these measures. First, these measures of price sensitivity are simple to compute, easy to understand, and, in many situations, perfectly reasonable to use. Second, these measures are widely used in the financial industry. Third, much of the intuition gained from a full understanding of these measures carries over to more general measures of price sensitivity.

Yield-Based *DV*01 and Duration

Yield-based *DV*01 and duration are special cases of the metrics introduced earlier in this chapter. In particular, these yield-based measures assume that the yield of a security is the interest rate factor and that the price-rate relationship is the price-yield function introduced in Equations (9.13) and (9.14). For convenience, these equations are reproduced here for a face value of 100 and with price written explicitly as a function of that security's yield, y:

$$P(y) = \frac{100c}{2}\sum_{t=1}^{2T}\frac{1}{(1+\frac{y}{2})^t} + \frac{100}{(1+\frac{y}{2})^{2T}} \tag{10.32}$$

$$P(y) = \frac{100c}{y}\left(1-\frac{1}{(1+\frac{y}{2})^{2T}}\right) + \frac{100}{(1+\frac{y}{2})^{2T}} \tag{10.33}$$

Taking the negative of the derivative of the two pricing expressions, (10.32) and (10.33), dividing by 10,000, and applying the definition of *DV*01 in (10.2), gives two expressions for yield-based *DV*01. Note that, to avoid clutter, this section will use the simple notations *DV*01 and *D* even when referring to the special cases of yield-based *DV*01 and duration.

$$DV01 = \frac{1}{10{,}000}\frac{1}{1+\frac{y}{2}}\left[\frac{100c}{2}\sum_{t=1}^{2T}\frac{t}{2}\frac{1}{(1+\frac{y}{2})^t} + T\frac{100}{(1+\frac{y}{2})^{2T}}\right] \tag{10.34}$$

$$DV01 = \frac{1}{10{,}000}\left[\frac{100c}{y^2}\left(1-\frac{1}{(1+\frac{y}{2})^{2T}}\right) + T\left(1-\frac{c}{y}\right)\frac{100}{(1+\frac{y}{2})^{2T+1}}\right] \tag{10.35}$$

Similarly, applying the definition of duration in (10.11) to the pricing Equations (10.32) and (10.33) gives the special cases of yield-based duration:

$$D = \frac{1}{P}\frac{1}{1+\frac{y}{2}}\left[\frac{100c}{2}\sum_{t=1}^{2T}\frac{t}{2}\frac{1}{(1+\frac{y}{2})^t} + T\frac{100}{(1+\frac{y}{2})^{2T}}\right] \tag{10.36}$$

$$D = \frac{1}{P}\left[\frac{100c}{y^2}\left(1-\frac{1}{(1+\frac{y}{2})^{2T}}\right) + T\left(1-\frac{c}{y}\right)\frac{100}{(1+\frac{y}{2})^{2T+1}}\right] \tag{10.37}$$

These special cases are also known in the industry as *modified* or *adjusted duration*.[8]

There is a certain structure to Equations (10.34) and (10.36). Each term in the brackets is the present value of a bond payment multiplied by the time to receipt of that payment, ½. The contribution of a payment to the interest rate risk of a bond varies directly with its present value and with its time to receipt. In addition, duration can be viewed as a weighted-sum of times to receipt, with each weight equal to the corresponding present value divided by the total of the present values, i.e., the price. Viewed this way, duration is a weighted-sum of times to receipt of payments and can be said to be measured in years. Hence, practitioners often refer to a duration of six as *six years*.

Table 10-6 calculates the *DV*01 and duration of the U.S. Treasury 2⅛s due May 31, 2015, as of May 28, 2010, using Equations (10.34) and (10.36) and the market yield of the bond on that date, namely 2.092%.[9] The present value of each payment is computed using the market yield. For

[8] This terminology is used because the first metric of this sort was *Macaulay Duration*. But the definition of the text, which divided Macaulay Duration by 1 + ½, became the industry standard.

[9] The use of these equations in this case is actually an approximation since the settlement date is June 1, 2010, and not May 31. See Appendix A in Chapter 9.

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TABLE 10-6 *DV*01 and Duration Calculations for the 2½s of May 31, 2015, as of May 28, 2010, at a Yield of 2.092 Percent

Date	Term	Cash Flow	Present Value	Time-Wtd. PV	% of Wtd. Sum
11/30/10	0.5	1.0625	1.0515	.5258	.1%
5/31/11	1.0	1.0625	1.0406	1.0406	.2%
11/30/11	1.5	1.0625	1.0298	1.5448	.3%
5/31/12	2.0	1.0625	1.0192	2.0384	.4%
11/30/12	2.5	1.0625	1.0086	2.5216	.5%
5/31/13	3.0	1.0625	.9982	2.9946	.6%
11/30/13	3.5	1.0625	.9879	3.4575	.7%
5/31/14	4.0	1.0625	.9776	3.9105	.8%
11/30/14	4.5	1.0625	.9675	4.3538	.9%
5/31/15	5.0	101.0625	91.0749	455.3746	95.3%
Total			100.1559	477.7621	
***DV*01**		**.04728**			
Duration		**4.7208**			

example, the present value of the coupon payment due on May 31, 2014, is

$$\frac{1.0625}{(1+\frac{2.092\%}{2})^8} = .97763 \qquad \textbf{(10.38)}$$

The time-weighted present value of each cash flow is its present value times its term. For the cash flow on May 31, 2014, the time-weighted present value is .97763 × 4.0 or 3.9105.

From Equation (10.34), the *DV*01 of the bond is the sum of the time-weighted present values divided by one plus half the yield and divided by 10,000. Using the total from the table, this bond's *DV*01 is

$$\frac{1}{10{,}000} \times \frac{1}{(1+\frac{2.092\%}{2})} \times 477.7621 = .04728 \qquad \textbf{(10.39)}$$

From Equation (10.36), the duration of the bond is the sum of the time weighted present values divided by one plus half the yield and divided by the price, the price just being the sum of the present values:

$$\frac{1}{100.1559} \times \frac{1}{(1+\frac{2.092\%}{2})} \times 477.7621 = 4.7208 \qquad \textbf{(10.40)}$$

The rightmost column of Table 10-6 gives the time-weighted present value of each cash flow as a percent of the total of these weighted values.

Given the definitions of *DV*01 and duration in Equations (10.34) and (10.36), these percentages are also the contribution of each cash flow to the interest rate risk of the bond. Far and away the largest contributor is the large cash flow at maturity. But considering the coupon flows alone, the contribution increases with term. Even though the present values of the longer-term coupon payments decline with term, their contributions to interest rate risk increase with term. Longer-dated cash flows are more sensitive to interest rate changes because they are discounted over longer periods of time.

Having defined and illustrated yield-based measures of interest rate sensitivity, an important limitation of their use becomes clear. Constructing a hedge so that the yield-based *DV*01 of a bond bought equals the yield-based *DV*01 of a bond sold will work as intended only if the two bond yields change by the same amount, i.e., only if their yields move in parallel. Of course, the efficacy of any hedge depends on the validity of its assumptions. In the examples of the previous sections, an underlying pricing model was used to relate the prices of the various securities to the seven-year par rate, and the quality of those hedges depends on that relationship being valid. Nevertheless, a well-thought-out model, or well-researched empirical relationships, are more likely to produce valid pricing relationships and hedges than the assumption of parallel yield shifts.

Yield-Based *DV*01 and Duration for Zero-Coupon Bonds, Par Bonds, and Perpetuities

Yield-based measures are particularly useful because of the intuition furnished by their easy-to-derive formulas.

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This and the next several subsections exploit this usefulness to compare and contrast the risk profiles of bonds with different cash-flow characteristics.

The yield-based *DV*01 and duration of a zero-coupon bond can be found by setting the coupon rate *c* equal to zero in Equations (10.35) and (10.37) and noting for the latter that, for a *T*-year zero-coupon bond with 100 face amount,

$$P = \frac{100}{(1+\frac{y}{2})^{2T}} \tag{10.41}$$

Hence,

$$DV01_{c=0} = \frac{T}{100(1+\frac{y}{2})^{2T+1}} = \frac{TP}{10{,}000(1+\frac{y}{2})} \tag{10.42}$$

$$D_{c=0} = \frac{T}{(1+\frac{y}{2})} \tag{10.43}$$

From (10.43), the duration of a zero-coupon bond is its years to maturity divided by a factor only slightly greater than one. Also, the duration of a zero, for a fixed yield, always increases with maturity. From (10.42), however, for long maturity zero-coupon bonds, the *DV*01 may not increase with maturity because a falling price may outweigh the increase in maturity. This last point will be illustrated in the next subsection.

The yield-based *DV*01 and duration of par bonds are useful formulae as relatively simple approximations for bonds with prices close to par. For a par bond (see Chapter 9), $P = 100$ and $c = y$. Substituting these values into Equations (10.35) and (10.37) shows that

$$DV01_{c=y} = \frac{1}{100y}\left(1-\frac{1}{(1+\frac{y}{2})^{2T}}\right) \tag{10.44}$$

$$D_{c=y} = \frac{1}{y}\left(1-\frac{1}{(1+\frac{y}{2})^{2T}}\right) \tag{10.45}$$

The last cases to be considered here are the *DV*01 and duration of perpetuities, which are sometimes useful as rough approximations for the risk of extremely long-term fixed income securities. Letting *T* approach infinity in Equations (10.35) and (10.37) and recalling from Chapter 9 that the price of a perpetuity with 100 face amount is $^{100c}/_{y}$,

$$DV01_{T=\infty} = \frac{1}{100}\frac{c}{y^2} \tag{10.46}$$

$$D_{T=\infty} = \frac{1}{y} \tag{10.47}$$

Duration, *DV*01, Maturity, and Coupon: A Graphical Analysis

Figure 10-7 uses the equations in this section to show how duration varies across bonds. For the purposes of this figure, all yields are fixed at 3.50%. At this yield, the duration of a perpetuity is 28.6. Since a perpetuity has no maturity, this duration is shown in Figure 10-7 as a horizontal line. Also, since by Equation (10.47) the duration of a perpetuity does not depend on coupon, this line is a benchmark for the duration of any coupon bond with a sufficiently long maturity.

From Equation (10.43), and as evident from Figure 10-7, the duration of zero-coupon bonds is linear in maturity.

The duration of the par bond in Figure 10-7 increases with maturity. Inspection of Equation (10.45) makes it clear that this is always the case and that the duration of a par bond rises from zero at a maturity of zero and steadily approaches the duration of a perpetuity.

Considering all of the curves of Figure 10-7 together reveals that for any given maturity duration falls as coupon increases. (Recognize that the par bond in the figure has a coupon equal to the yield of 3.50%.) The intuition behind this fact is that higher-coupon bonds have a greater fraction of their value paid earlier. The higher the coupon, the larger the weights on the duration terms of early years relative to those of later years. Hence, higher-coupon bonds are effectively shorter-term bonds and therefore have lower durations.

A little-known fact about duration can be extracted from Figure 10-7. The duration of a bond with a very low, near zero, coupon would be just below the zero-coupon line of the figure. Furthermore, the coupon could be set low enough such that the bond's duration is still just below the zero-coupon line but above the duration of a perpetuity.[10] Eventually, however, as maturity increases, the low coupon bond must approach the duration of a perpetuity, i.e., its duration must fall with maturity. This fact is somewhat of a mathematical curiosity if—as at the time of this writing—yields are low relative to the coupons of outstanding

[10] In the example of the text, a bond with a coupon of .5% would have a duration that peaked above the duration of a perpetuity.

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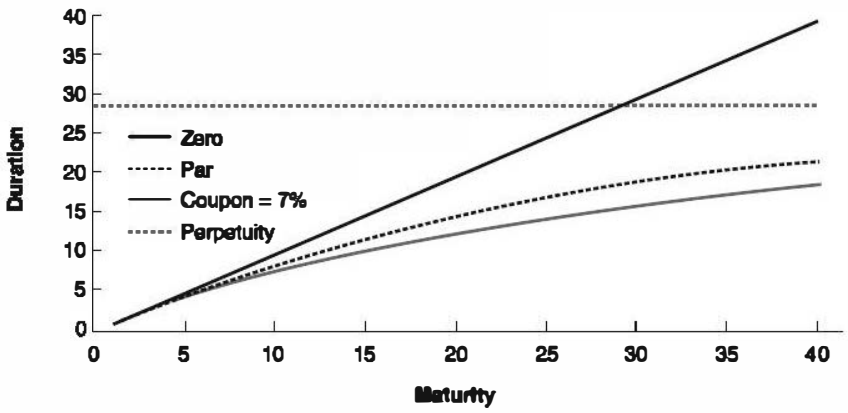

FIGURE 10-7 Duration across bonds yielding 3.50%.

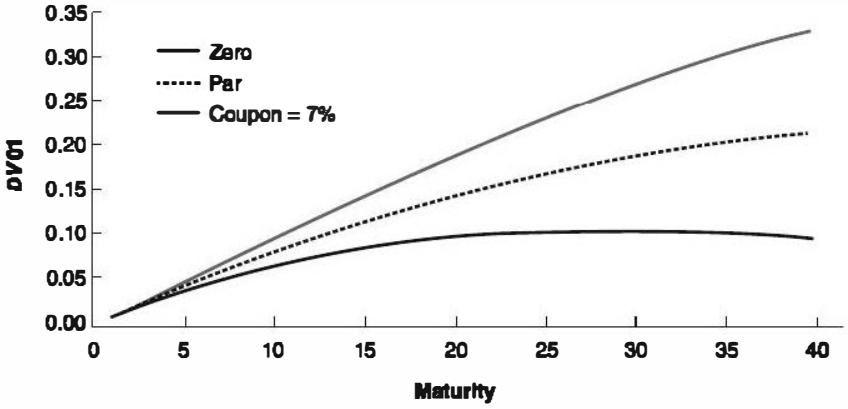

FIGURE 10-8 *DV*01 across bonds yielding 3.50%.

bonds so that few if any bonds exist with the prerequisite long maturities and deep discounts.

The next figure will show how *DV*01 varies across bonds. For this discussion it is useful to combine explicitly the definitions of *DV*01 and duration from (10.2) and (10.11) to write that

$$DV01 = \frac{P \times D}{10{,}000} \tag{10.48}$$

As discussed in the context of Figure 10-7, duration almost always increases with maturity. According to Equation (10.48), however, the effect of maturity on *DV*01 is more complex since it depends not only on how duration changes with maturity but also on how price changes with maturity. What will be called the *duration effect* tends to increase *DV*01 with maturity while what will be called the *price effect* can either increase or decrease *DV*01 with maturity.

Figure 10-8 graphs *DV*01 as a function of maturity under the same assumptions used in Figure 10-7. Since the *DV*01 of a perpetuity, unlike its duration, depends on the coupon rate, the perpetuity line is removed.

Inspection of Equation (10.44) reveals that the *DV*01 of par bonds always increases with maturity. Since the price of par bonds is always 100, the price effect does not come into play, and, as in the case of duration, longer par bonds have greater price sensitivity. The curve approaches .286, the *DV*01 of a par perpetuity at a yield of 3.50%.

As discussed in Chapter 9, extending the maturity of a premium bond increases its price. As a result, the price and duration effects combine so that the *DV*01 of a premium bond increases with maturity faster than the *DV*01 of a par bond. Of course, at some maturity beyond the range of the graph, the price of the bond increases very slowly and the price effect becomes less important. The *DV*01 of the 7% bond eventually approaches that of a perpetuity with a coupon of 7% (i.e., .571).

The *DV*01 of a zero behaves initially like that of a coupon bond, but it eventually falls to zero. With no coupon payments the present value of a zero with a longer and longer maturity approaches zero, and so does its *DV*01.

Figure 10-8 also shows that, unlike duration, *DV*01 rises with coupon. This fact is immediately evident from Equation (10.34).

Duration, *DV*01, and Yield

Inspection of Equation (10.34) reveals that increasing yield lowers *DV*01. This fact was already introduced when

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showing that coupon bonds display positive convexity, that is, that their *DV*01s fall as interest rates increase. As it turns out, increasing yield also lowers duration. The intuition behind this fact is that increasing yield lowers the present value of all payments but lowers the present value of the longer payments the most. Therefore, the value of the longer payments falls relative to the value of the whole bond. But since the duration of these longer payments is greatest, lowering their corresponding weights in the duration calculation must lower the duration of the whole bond.

To illustrate the effect of yield on duration, return to the example in Table 10-6. At a yield of 2.092%, the duration of the 2⅛s of May 31, 2015, is 4.7208. Also, the time-weighted present value of the payment at maturity, as a percentage of the sum of those values, is 95.3%. Reworking the calculations at a yield of 6%, the percentage of the sum attributable to the payment at maturity falls to 95% which, along with the increased relative importance of the shorter coupon payments, drives the duration down to 3.8375.

Yield-Based Convexity

Following the general definition of convexity in (10.14), yield-based convexity can be derived by taking the second derivative of (10.32) and dividing by price. The resulting formula is

$$C = \frac{1}{P}\frac{1}{(1+\frac{y}{2})^2}\left[\frac{100c}{2}\sum_{t=1}^{2T}\frac{t}{2}\frac{t+1}{2}\frac{1}{(1+\frac{y}{2})^t} + T(T+.5)\frac{100}{(1+\frac{y}{2})^{2T}}\right] \quad \textbf{(10.49)}$$

The structure of this equation is similar to those of the expressions for yield-based *DV*01 and duration, but the time weights are $\frac{1}{2}\times\frac{t+1}{2}$ instead of ½, or, loosely speaking, more like t^2 than like t. With this in mind, the convexity of the 2⅛s due May 31, 2015, can be calculated using the first four columns of Table 10-6 but then substituting the weighted present value terms from (10.49) for those appropriate for the duration calculation. Doing this, the sum of the weighted present values, corresponding to the bracketed term in (10.49), is about 2,586 and, therefore, the bond's convexity is

$$\frac{1}{100.1559}\times\frac{1}{(1+\frac{2.092\%}{2})^2}\times 2{,}586 = 25.29 \quad \textbf{(10.50)}$$

For intuition, a useful special case of (10.49) is that of a zero-coupon bond. Setting c = 0 and $P = 100(1 + \frac{y}{2})^{-2T}$,

$$C_{c=0} = \frac{T(T+.5)}{(1+\frac{y}{2})^2} \quad \textbf{(10.51)}$$

Applying (10.51), a five-year zero-coupon bond yielding 2.092% would have a convexity of $5 \times 5.5 \times (1 + \frac{2.092\%}{2})^{-2}$ or 26.93.

This exceeds the convexity of the five-year 2⅛s yielding 2.092%: since a coupon bond has some of its present value in earlier payments, and since the convexity contributions of those payments are less than that of the final payment at maturity, a coupon bond will have a lower convexity than a maturity- and yield-equivalent zero.

From (10.51) it is clear that longer-maturity zeros have greater convexity. In fact, the convexity of a zero increases with the square of maturity. Furthermore, thinking of a coupon bond as a portfolio of zeros, longer-maturity coupon bonds usually have greater convexity than shorter-maturity coupon bonds.

For easy reference, another useful special case of convexity is presented here, namely, the convexity of a par bond. This is obtained by differentiating Equation (10.33) twice with respect to yield, evaluating the result at $y = c$, and dividing by the price, which, for par bonds is 100:

$$C_{c=y} = \frac{2}{y^2}\left[1-\frac{1}{(1+\frac{y}{2})^{2T}}\right] - \frac{2T}{y(1+\frac{y}{2})^{2T+1}} \quad \textbf{(10.52)}$$

APPLICATION: THE BARBELL VERSUS THE BULLET

On May 28, 2010, a portfolio manager is considering the purchase of $100 million face amount of the U.S. Treasury 3⅜s due November 15, 2019, at a cost of $100,859,000. After an analysis of the interest rate environment, the manager is comfortable with the pricing of the bond at a yield of 3.288% and with its duration of 8.033. But, after considering the data on two other Treasury bonds in Table 10-7, the manager wishes to consider an alternate investment.

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TABLE 10-7 Data on Three U.S. Treasury Bonds as of May 28, 2010

Coupon	Maturity	Price	Yield	Duration	Convexity
2½	3/31/15	102.5954	2.025%	4.520	23.4
3⅜	11/15/19	100.8590	3.288%	8.033	74.8
4⅜	11/15/39	102.7802	4.221%	16.611	389.7

The three bonds in the table have maturities of approximately five years, 10 years, and 30 years, respectively. Thus, an alternative to purchasing a *bullet* investment in the 10-year 3⅜s is to purchase a *barbell* portfolio of the shorter maturity, 5-year 2½s, and the longer maturity, 30-year 4⅜s. In particular, the barbell portfolio would be constructed to cost the same and have the same duration as the bullet investment. The advantages and disadvantages of this *barbell* relative to this *bullet* will be discussed after deriving the composition of the barbell portfolio.

Let V^5 and V^{30} be the value in the barbell portfolio of the 5-year and 30-year bonds, respectively. Then, for the barbell to have the same value as the bullet,

$$V^5 + V^{30} = 100{,}859{,}000 \qquad \textbf{(10.53)}$$

Furthermore, using the data in Table 10-7 and Equation (10.30), which describes how to compute the duration of a portfolio, the duration of the barbell equals the duration of the bullet if

$$\frac{V^5}{100{,}859{,}000} \times 4.520 + \frac{V^{30}}{100{,}859{,}000} \times 16.611 = 8.033 \qquad \textbf{(10.54)}$$

Solving Equations (10.53) and (10.54) shows that V^5 is \$71.555 million or 70.95% of the portfolio and that V^{30} is \$29.304 million or 29.05% of the portfolio. Finally, the convexity of the portfolio, using the data in Table 10-7 and Equation (10.31), which describes how to compute the convexity of a portfolio, is

$$70.95\% \times 23.4 + 29.05\% \times 389.7 = 129.8 \qquad \textbf{(10.55)}$$

The barbell has greater convexity than the bullet because duration increases linearly with maturity while convexity increases with the square of maturity. If a combination of short and long durations, essentially maturities, equals the duration of the bullet, that same combination of the two convexities, essentially maturities squared, must be greater than the convexity of the bullet. In the current context, the particularly high convexity of the 4⅜ more than compensates for the lower convexity of the 2½. As a result, the convexity of the portfolio exceeds the convexity of the 3⅜. The general lesson is that spreading out the cash flows of a portfolio, without changing duration, raises convexity.

Return now to the decision of the portfolio manager. For the same amount of duration risk, the barbell portfolio has greater convexity, which means that its value will increase more than the value of the bullet when rates rise or fall. This is completely analogous to the price-rate profile of the option TYUOC 120 relative to the *DV*01-equivalent position in the futures TYUO depicted in Figure 10-5: the barbell portfolio benefits more from interest rate volatility than does the bullet portfolio. What then is the disadvantage of the barbell portfolio? The weighted yield of the barbell portfolio is

$$70.95\% \times 2.025\% + 29.05\% \times 4.221\% = 2.663\% \qquad \textbf{(10.58)}$$

compared with the yield of the bullet of 3.288%. Hence, the barbell will not do as well as the bullet portfolio if yields remain at current levels while, as just argued, the barbell will outperform if rates move sufficiently higher or lower.

In short, the manager's work in choosing to bear a level of interest rate risk consistent with a portfolio duration of about eight is not sufficient to complete the investment decision. A manager believing that rates will be particularly volatile will prefer the barbell portfolio while a manager believing that rates will not be particularly volatile will prefer the bullet portfolio. Of course, further calculations can establish exactly how volatile rates have to be for the barbell portfolio to outperform.

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Multi-Factor Risk Metrics and Hedges

11

Learning Objectives

After completing this reading you should be able to:

- Describe and assess the major weakness attributable to single-factor approaches when hedging portfolios or implementing asset liability techniques.
- Define key rate exposures and know the characteristics of key rate exposure factors including partial '01s and forward-bucket '01s.
- Describe key-rate shift analysis.
- Define, calculate, and interpret key rate '01 and key rate duration.
- Describe the key rate exposure technique in multi-factor hedging applications; summarize its advantages and disadvantages.
- Calculate the key rate exposures for a given security, and compute the appropriate hedging positions given a specific key rate exposure profile.
- Relate key rates, partial '01s and forward-bucket '01s, and calculate the forward-bucket '01 for a shift in rates in one or more buckets.
- Construct an appropriate hedge for a position across its entire range of forward-bucket exposures.
- Apply key rate and multi-factor analysis to estimating portfolio volatility.

Excerpt is Chapter 5 of Fixed Income Securities, *Third Edition, by Bruce Tuckman.*

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A major weakness of the approach in Chapter 10, is the assumption that movements in the entire term structure can be described by one interest rate factor. To make the case in the extreme, because the six-month rate is unrealistically assumed to predict perfectly the change in the 30-year rate, a (naive) *DV*01 analysis leads to hedging a 30-year bond with a six-month bill. In reality, of course, it is widely recognized that rates in different regions of the term structure are far from perfectly correlated. Put another way, predicted changes in the 30-year rate relative to changes in the six-month rate can be wildly off target, whether these predicted changes come from a model, like the one implicitly used in the first part of Chapter 10, or from the implicit assumption when using yield-based *DV*01 that the two rates move by the same amount. The risk that rates along the term structure move by different amounts is known as *curve risk*.

This chapter discusses how to measure and hedge the risks of a security or portfolio in terms of several other securities, where each hedging security is most sensitive to a different part of the term structure. The more securities used in the hedge, the less important are any assumptions linking the behavior of one rate with another. At the extreme discussed in the previous paragraph, hedging with one security requires extremely strong assumptions about how rates move together. At the other extreme, a hedge that uses one security for every cash flow being hedged requires no assumptions about how rates move together because risk will have been *immunized* against any and all interest rate scenarios. Such a hedge, however, is almost certainly to be excessively costly. The methods presented in this chapter have been found to strike a sensible balance between hedging effectiveness and cost or practicality.

Key rate exposures are used for measuring and hedging the risk of bond portfolios in terms of a relatively small number of the most liquid bonds available, usually the most recently issued, near-par, government bonds. *Partial '01s* are used for measuring and hedging the risk of portfolios of swaps or portfolios that contain both bonds and swaps in terms of the most liquid money market and swap instruments. As these instruments are almost always those whose prices are used to build a swap curve, the number of securities used in this methodology is usually greater than the number used in a key rate framework. Finally, *forward-bucket '01s*, mostly used in the swap or combined bond and swap contexts as well, measure the risk of a portfolio not in terms of other securities but in terms of direct changes in the shape of the term structure. As a result, forward-bucket '01s are often the most intuitive way to understand the curve risks of a portfolio, but not the quickest way to see which hedges are required to neutralize such risks. This chapter concludes with a comment on the use of these methods to measure the volatility of a portfolio.

KEY RATE '01s AND DURATIONS

Key rate exposures are designed to describe how the risk of a bond portfolio is distributed along the term structure and how to implement any desired hedge, all in terms of some set of benchmark bonds, usually the more liquid government securities.[1] Table 11-1, as an example, shows a key rate exposure report for the U.S. Lehman Aggregate Bond Index,[2] a benchmark portfolio of U.S. governments, agencies, mortgages, and corporates. The duration of the portfolio with respect to U.S. government rates is 4.339, as reported in the last row of the table. While this one number certainly quantifies interest rate risk, along the lines explained in Chapter 10, the rest of the table adds

TABLE 11-1 Key Rate Duration Profile of the U.S. Lehman Aggregate Bond Index as of December 31, 2004

Key Rate	Duration
6-Month	0.145
2-Year	0.655
5-Year	1.151
10-Year	1.239
20-Year	0.800
30-Year	0.349
Total	4.339

Source: The Lehman Brothers Global Risk Model: A Portfolio Manager's Guide, April 2005.

[1] The idea was proposed in Thomas Ho, "Key Rate Duration: A Measure of Interest Rate Risk," *Journal of Fixed Income*, September, 1992.

[2] This set of indexes is now run by Barclays Capital.

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information about the distribution of this risk across the curve. For example, more than half of the portfolio's duration risk is closely related to—and could be hedged with—5- and 10-year bonds.

Continuing with this example for a moment, consider a portfolio manager whose performance is judged against the performance of this index. And say in addition that the manager's portfolio has the same duration as the index but is concentrated in 30-year bonds. If rates move up or down in parallel, the manager's performance will match that of the index. But if the government bond curve steepens the manager's portfolio will underperform, while if it flattens the manager's portfolio will outperform.[3]

The next three subsections discuss defining key rate shifts, computing key rate exposures, and then hedging with these exposures.

Key Rate Shifts

The crucial assumption of the key rate approach is that all rates can be determined as a function of a relatively small number of key rates. Therefore, the following decisions have to be made in order to implement the methodology: the number of key rates, the type of the key rates (e.g., spot rates, par yields), the terms of the key rates, and the rule for computing all other rates given the key rates.

In order to cover risk across the term structure, to keep the number of key rates as few as reasonable, and to rely only on the most liquid government securities, one popular choice of key rates for the U.S. Treasury and related markets are the 2-, 5-, 10-, and 30-year par yields. Then, motivated mostly by simplicity, the change in the term structure of par yields given a one-basis point change in each of the key rates is assumed to be as in Figure 11-1. Each of the four shapes is called a *key rate shift*. Each key rate affects par yields from the term of the previous key rate (or zero) to the term of the next key rate (or the last term). For example, the 10-year key rate affects par yields of terms 5 to 30 years only. Furthermore, the impact of each key rate is normalized to be one basis point at its own maturity and then assumed to decline linearly, reaching zero at the terms of the adjacent key rates. For the two-year shift at terms of less than 2 years and for the 30-year shift at terms greater than 30 years, however, the assumed change is constant at one basis point.

By construction, the four key rate shifts sum to a constant shift of one basis point. This allows for the interpretation of key rate exposures as a decomposition of the total *DV*01 or duration of a security or a portfolio into exposures to four different regions of the term structure.

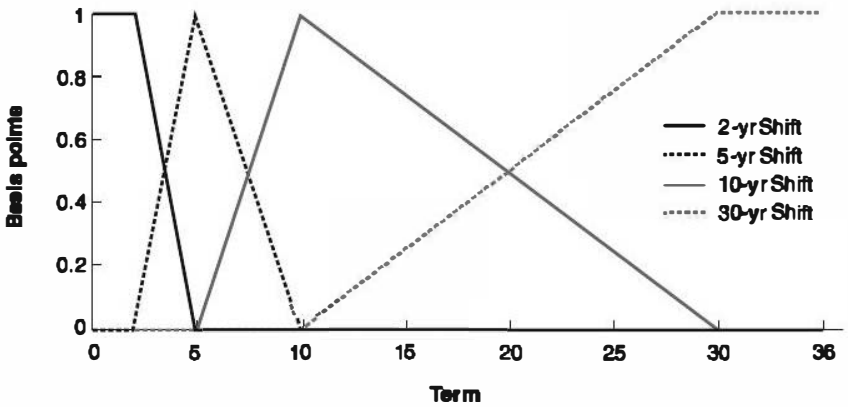

FIGURE 11-1 A specification of key rate shifts.

While the key rate shifts in Figure 11-1 turn out to be very tractable and useful, they implicitly make quite strong assumptions about the behavior of the term structure. Consider the assumption that the rate of a given term is affected only by its neighboring key rates. The 7-year rate, for example, is assumed to be a function of changes in the 5- and 10-year rates only. Empirically, however, were the 2-year rate to change while the 5- and 10-year rates stayed the same, the 7-year rate would probably change as well so as to maintain reasonable curvature across the term structure. The linearity of the shifts is also not likely to be an empirically valid assumption. All in all, however, the great tractability of working with the shifts in Figure 11-1 has been found to compensate for these theoretical and empirical shortcomings.

[3] For a definition of steepening and flattening, see Figure 8.6 and the surrounding discussion.

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TABLE 11-2 Key Rate *DV*01s and Durations of the May 15, 2040, C-STRIP as of May 28, 2010

	(1) Value	(2) Key Rate '01	(3) Key Rate Duration
Initial Curve	26.22311		
2-year Shift	26.22411	−.0010	−.38
5-year Shift	26.22664	−.0035	−1.35
10-year Shift	26.25763	−.0345	−13.16
30-year Shift	26.10121	.1219	46.49
Total		.0829	31.60

Calculating Key Rate '01s and Durations

As a simple introduction to the calculation of key rate '01s and duration, this subsection takes the example of a 30-year zero-coupon bond. While the exposure of a 30-year zero to spot rates is very simple, its exposure to par yields and, therefore, to key rates (as defined in the previous subsection), is somewhat complicated. Basically, the risk along the curve of a 30-year zero is not equivalent to that of a 30-year par bond because of the latter's coupon payments.

Table 11-2 illustrates the calculations of key rate *DV*01s and durations for 100 face amount of the C-STRIP due May 15, 2040, as of May 28, 2010. The C-STRIP curve on that day was taken as the base pricing curve, with the key rate shifts of Figure 11-1 superimposed as appropriate.

Column (1) of Table 11-2 gives the initial price of the C-STRIP and its present value after applying the key rate one-basis point shifts of Figure 11-1. Column (2) gives the key rate '01s. Denoting the key rate '01 with respect to key rate y^k as $DV01^k$, these are defined analogously to *DV*01 as

$$DV01^k = -\frac{1}{10{,}000}\frac{\partial P}{\partial y^k} \tag{11.1}$$

where $\partial P/\partial y^k$ denotes the partial derivative of price with respect to that one key rate. Applying this definition to the C-STRIP described in Table 11-2, the key rate '01 with respect to the 5-year shift is

$$-\frac{1}{10{,}000}\frac{26.22664 - 26.22311}{.01\%} = -.0035 \tag{11.2}$$

Or, in words, the C-STRIP *increases* in price by .0035 per 100 face amount for a *positive* one-basis-point five-year shift. The intuition behind the sign of this '01 will be explained in a moment.

The key rate durations, denoted here as D^k, are also defined analogously to duration so that,

$$D^k = -\frac{1}{P}\frac{\partial P}{\partial y^k} \tag{11.3}$$

Continuing with the five-year shift in Table 11-2, the key rate duration is

$$-\frac{1}{26.22311}\frac{26.22664 - 26.22311}{.01\%} = -1.35 \tag{11.4}$$

Turning now to interpreting the results, the key rate profile in Table 11-2 shows that the interest rate exposure of the 30-year C-STRIPS is equivalent to that of a long position in a 30-year par bond, a smaller, short position in a 10-year par bond, and even smaller short positions in five- and two-year par bonds. This accords with the intuition stated at the beginning of this subsection, that the 30-year par bond's coupons create exposures at shorter terms that have to be offset by shorts of short-term par bonds.

In addition to this replicating portfolio intuition, it is useful to understand the precise mechanics by which the shorter-term key rate '01s and durations in Table 11-2 turn out to be negative. To this end, Figure 11-2 graphs the 10-year key rate (i.e., par yield) shift along with the resulting, implied shift of spot rates. (An analogous figure could be constructed for the five- and two-year key rate shifts.)

From the implied spot rate shift in Figure 11-2 it is immediately apparent why the 10-year, key rate sensitivities of the 30-year C-STRIPS in Table 11-2 are negative. By definition, the 30-year par yield is unchanged by the 10-year key rate shift. But, according to the figure, the 30-year spot rate declines by about .45 basis points, meaning the 30-year C-STRIPS increases in value. Hence, the *DV*01 or duration of the 30-year STRIPS with respect to the 10-year par yield is negative. Since this spot rate declines by only .45 basis points per basis-point increase in the 10-year par rate, however, the absolute magnitude of this sensitivity is relatively small.

As for the intuition behind the shape of the implied spot-rate shift in Figure 11-2, the interested reader can note that with par yields with from zero- to five-year terms remaining unchanged, spot rates of those terms have to remain unchanged as well. Therefore, any increases in par rates of

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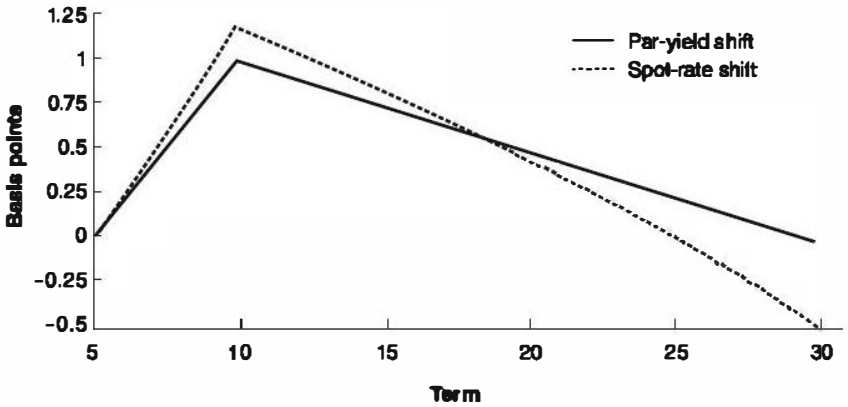

FIGURE 11-2 The assumed 10-year key rate shift of par yields and its implied shift of spot rates.

terms between 5 and 10 years cannot be spread out across the spot rate curve but have to be concentrated in spot rates with terms greater than 5 years. But this implies that spot rates of terms between 5 and 10 years have to increase by more than par rates. Similarly, as par rates with terms greater than 10 years decrease, all spot rates with terms up to 10 years have already been fixed, implying that all of the decrease in par rates with terms greater than 10 years has to be concentrated in spot rates with terms beyond 10 years. Thus, the decline in spot rates has to be steeper than the decline in par rates. Finally, note that it would be impossible for the change in the 30-year par yield to be zero if all of the spot rates with terms from 5 to 30 years have increased. Hence, the longest-term spot rates have to decline as part of this key rate shift of par yields.

A final technical point should be made about the last row of Table 11-2, namely, the sum of the key rate '01s and durations. Since the sum of the key rate shifts is a parallel shift of par yields, the sums of the key rate '01s and durations are, as mentioned earlier, conceptually comparable to the one-factor, yield-based *DV*01 and duration metrics, respectively. But key rate exposures, which shift par yields, will not exactly match yield-based metrics, which shift security-specific yields.[4]

[4] For example, it turns out that the sum of the changes in the 30-year spot rate across all the key rate shifts is 1.08 basis points. Therefore, the sum of the key rate exposures of a 30-year zero is about 1.08 times its exposure to the 30-year spot rate, which is the same as 1.08 times its yield-based exposure.

Hedging with Key Rate Exposures

This subsection illustrates how to hedge with key rate exposures using a stylized example of a trader making markets in U.S. Treasury bonds. On May 28, 2010, the trader executed two large trades:

1. The trader shorted $100 million face amount of a 30-year STRIPS to a customer, buying about $47 million face of the 30-year bond to hedge the resulting interest rate risk.
2. The trader facilitated a customer 5s–10s curve trade by shorting $40 million face of the 10-year note and buying about $72 million of the 5-year note.

Table 11-3 lists these trades in column (2), with two hedges, to be discussed presently, in the other columns. The coupon bonds featured in the rows of the table are the on-the-run 2-, 5-, 10-, and 30-year U.S. Treasuries, which, consistent with the motivation of key rates, are used by the trader to hedge risk. The other bond in the table is the STRIPS due May 15, 2040, discussed in the previous subsection. Table 11-4 gives the key rate '01 profiles for 100 face amount of these bonds in rows (i) through (v) and the '01 profiles for particular portfolios, again, to be discussed presently, in rows (vi) through (ix).

If the maturity of a coupon bond were exactly equal to the term of a key rate and if the price of that bond were

TABLE 11-3 Stylized Market Maker Positions and Hedges as of May 28, 2010

(1)	(2)	(3)	(4)
	Face Amount ($ millions)		
Bond	Position	Hedge	Alternate Hedge
.75s of 5/31/12		−5.190	
2.125s of 5/31/15	72.446	−80.006	−80.008
3.5s of 5/15/20	−40	−.487	
0s of 5/15/40	−100		
4.375s of 5/15/40	47.077	22.633	21.806

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TABLE 11-4 Key Rate '01 Profile of a Stylized Market Maker's Position and Hedges as of May 28, 2010

		Key Rate '01 (per 100 face amount)				
	Bond	2-Year	5-Year	10-Year	30-Year	Sum
(i)	.75s of 5/31/12	.0199	.0000	.0000	.0000	.0199
(ii)	2.125s of 5/31/15	.0000	.0480	.0000	.0000	.0480
(iii)	3.5s of 5/15/20	.0000	−.0001	.0870	.0000	.0869
(iv)	0s of 5/15/40	−.0010	− .0035	−.0345	.1219	.0829
(v)	4.375s of 5/15/40	.0000	.0001	.0010	.1749	.1760
(vi)	Total Position ($)	1,000	38,377	198	−39,578	0
(vii)	Hedge($)	−1,000	−38,377	−198	39,578	0
(viii)	Alternate Hedge ($)	31	−38,379	217	38,131	0
(ix)	Total+Alt. Hedge($)	1,031	−2	415	−1,447	0

exactly par, then that bond's yield would be identical to that key rate. By definition, then, that bond's key rate '01 with respect to that key rate would equal its yield-based *DV*01 while its key rate '01 with respect to all other key rates would be zero. Since the on-the-run bonds profiled in Table 11-4 are close to 2-, 5-, 10-, and 30-year maturities, and since they do sell for about par, their key rate exposures in rows (i), (ii), (iii), and (v) are heavily concentrated in the respective buckets. In row (iii), for example, the 10-year, key rate '01 of the 3.5s of May 15, 2020, is .0870, while the rest of its key rate '01s are near zero. Note that the key rate profile of the 30-year STRIPS in row (iv) is as presented in Table 11-2.

The sums of the key rate '01s for each of the bonds in rows (i) through (v) are given in the rightmost column of Table 11-4. The trader uses these sums for initial hedging, which, as discussed previously, is very much like single-factor, *DV*01 hedging. So, the trader bought $72.4 million of the five-year against the $40mm short of the 10-year because

$$\frac{.0869}{.0480} \times \$40\text{mm} = \$72.4\text{mm} \tag{11.5}$$

Similarly, the trader bought $47.1 million of 30-year bonds against the $100 million short of 30-year STRIPS because

$$\frac{.0829}{.1760} \times \$100\text{mm} = \$47.1\text{mm} \tag{11.6}$$

Row (vi) of Table 11-4 gives the key rate '01 profile, in dollars, of the trader's position recorded in column (2) of Table 11-3. The five-year key rate '01 in millions of dollars, for example, is calculated as

$$\begin{aligned} &72.446 \times \frac{.048}{100} - 40 \times \left(\frac{-.0001}{100}\right) \\ &- 100 \times \left(\frac{-.0035}{100}\right) \\ &+ 47.077 \times \frac{.0001}{100} \\ &= .03836\text{l} \end{aligned} \tag{11.7}$$

which is $38,361. (The small difference between this number and the $38,377 in Table 11-4 is due to the rounding of the '01s and the position amounts.)

Because the trader's initial hedges were constructed to be *DV*01-neutral, the trader has no net *DV*01-type exposure, i.e., the sum of the '01s across row (vi) of Table 11-4 is zero. As can be seen from the rest of that row, however, the key rate profile of the trader's book is not flat. In fact, the trader essentially has on a substantial 5s-30s steepener, meaning a position that will make money if 30-year yields rise relative to 5-year yields but lose money if the opposite occurs. But this accumulated steepener is a by-product of market making and not the result of deliberate risk taking. The trader, therefore, will construct a hedge to flatten out the key rate profile in row (vi).

The hedging problem is to find the face amount of each of the key rate securities such that the net key rate '01s of the overall position are all zero. Let the face amount of each of the hedging securities be F^2, F^5, F^{10}, and F^{30} for the 2-, 5-, 10-, and 30-year bonds, respectively. Then the equations for setting the overall 2-, 5-, 10-, and 30-key rate '01s to zero are, respectively,

$$\frac{.0199}{100}F^2 + 0 \times F^5 + 0 \times F^{10} + 0 \times F^{30} + \$1{,}000 = 0 \tag{11.8}$$

$$\frac{.048}{100} \times F^5 - \frac{.0001}{100} \times F^{10} + \frac{.0001}{100} \times F^{30} + \$38{,}377 = 0 \tag{11.9}$$

$$\frac{.0870}{100} \times F^{10} + \frac{.001}{100} \times F^{30} + \$198 = 0 \tag{11.10}$$

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$$\frac{.1749}{100} \times F^{30} - \$39{,}578 = 0 \qquad (11.11)$$

Solving Equations (11.8) through (11.11) gives the face amounts in column (3) of Table 11-3. By construction, then, the key rate profile of the hedging portfolio, shown in row (vii) of Table 11-4, is the negative of the profile of row (vi) so that these two rows sum to zero.

This precisely constructed hedge, with its four equations and four unknowns, may look somewhat daunting. But this should not obscure the essentials of the hedge. The five-year key rate '01 to be hedged is $38,377 and the five-year key rate '01 of the five-year on-the-run bond is .048, so the approximate face amount of the five-year bond that has to be sold is $\$38{,}377/.048\%$ or about $79.95 million. Similarly, the 30-year '01 to be hedged is −39,578 and the 30-year '01 of the 30-year on-the-run is .1749, so the face amount of the 30-year bond that has to be bought is about $\$39{,}578/.1749\%$ or about $22.63 million. These results are very close to the precise results reported in column (3) of Table 11-3.

In practice, a trader might very well recognize that the biggest risk of the position, from row (vi) of Table 11-4, is the 5s–30s steepener. The trader might then sell the $80 million of the five-year on-the-run, as computed in the previous paragraph. Then, to keep a flat overall *DV*01, the trader might purchase an amount F^{30} such that

$$F^{30}\frac{.1760}{100} = \$80\text{mm} \times \frac{.0480}{100} \qquad (11.12)$$

And solving, F^{30} is $21.8 million. This quicker, alternate hedge is recorded in column (4) of Table 11-3. Its key rate profile is given in row (viii) of Table 11-4 and the net key rate profile of the original position and this alternate hedge is given in row (ix). This net profile is very close to flat, although the residual is a very small 2s–30s steepener!

PARTIAL '01s AND *PV*01

Swaps have become the most popular interest rare benchmark. Interest rate risk is measured in terms of swap curves not only by swaps traders, but also by asset managers that run portfolios that combine bonds and swaps.[5] Examples of such managers include these: a life insurance company or pension fund that selects attractive corporate credits or mortgage exposures and hedges interest rate risk with swaps; a manager who supervises several traders or portfolio managers, some of whom trade bonds and some of whom trade swaps; or a relative value government bond investor who hedges curve risk with swaps. In any case, when swaps are taken as the benchmark for interest rates, risk along the curve is usually measured with *Partial '01s* or *Partial PV01s* rather than with key rate '01s. This section discusses these swap-based methodologies without introducing additional numerical examples since the underlying concepts are very similar to those of key rate exposures.

Swap market participants fit a par swap rate curve every day, if not more frequently, from a set of traded or observable par swap rates and shorter-term money market and futures rates. Leveraging this curve-fitting machinery, sensitivities of a portfolio or trading book are measured in terms of changes in the rates of the fitting securities. More specifically, the partial '01 with respect to a particular fitted rate is defined as the change in the value of the portfolio after a one-basis-point decline in that fitted rate and a refitting of the curve. All other fitted rates are unchanged. So, for example, if a curve fitting algorithm fits the three-month *London Interbank Offered Rate (LIBOR)* rate and par rates at 2-, 5-, 10-, and 30-year maturities, then the two-year partial '01 would be the change in the value of a portfolio for a one-basis point decline in the two-year par rate and a refitting of the curve, where the three-month LIBOR and the par 5-, 10-, and 30-year rates are kept the same. Note how the details of calculating partial '01s are intertwined with the details of constructing the swap curve.

Given the partial '01 profile of a portfolio, hedges to zero-out this profile are particularly easy to calculate. As pointed out in the previous section, with key rate shifts defined in terms of par yields, the key rate profile of the 10-year bond, for example, would be its *DV*01 for the 10-year shift and zero for all other shifts only if the 10-year bond matured in exactly 10 years and were priced at exactly par. By contrast, in the case of partial '01s, the shifts are defined precisely in terms of the fitting securities. Therefore, by construction, all of the '01 of a fitting security is concentrated in the partial '01 calculated by shifting its rate, making calculating hedges particularly easy. Nevertheless, since there are typically many fitting

[5] In addition to managing interest rate risk, these managers must also manage *spread risk*, i.e., the risk that spreads between bond and swap rates change.

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securities, market practice is to trade enough of the fitting securities so as to achieve an acceptable profile of partial '01s rather than trading every single fitting security so as to zero-out all partial '01s.

The *PV*01 of a security is defined as the change in the value of the security if the rates of all fitting securities decline by one basis point. Hence *PV*01 is conceptually equivalent to *DV*01, where the underlying curve fitting methodology defines rates at all terms given the changes in the rates of the fitting securities. Furthermore, since the sum of all the partial '01 shifts is the *PV*01 shift—with one caveat to be raised presently—the partial '01s may be thought of as a decomposition of the *PV*01 into risks along the curve. The technical caveat is that money market rates and swap rates are quoted under different day-count conventions, namely, actual/360 for LIBOR-related rates and 30/360 for the fixed side of swaps. So, if money market rates and swap rates are mixed when fitting swap curves, as they usually are, changing each market rate by a basis point is not the same as changing all actual/360 rates by a basis point or all 30/360 rates by a basis point. To ensure that the sum of the partial '01s does equal the *PV*01, all rates could be converted into a single day-count convention. This normalization, however, sacrifices the desirable property that the '01 of each fitting security equals its '01 with respect to its own quoted rate.

In passing, it is worth noting that the *CV*01 of a swap is the change in value of a swap for a one basis-point decrease in its coupon rate. A moment's reflection reveals that this quantity is proportional to the annuity factor to the swap's maturity. See Equation (8.21). The two metrics, *CV*01 and *PV*01, are sometimes used interchangeably, and sometimes confused, because the two are essentially equal for par swaps. To see this, note that the expression for the annuity factor in Equation (9.15) is 100 times the expression for the *DV*01 of a par swap in Equation (10.44).

FORWARD-BUCKET '01s

While key rates and partial '01s conveniently express the exposures of a position in terms of hedging securities, *forward-bucket '01s* convey the exposures of a position to different parts of the curve in a much more direct and intuitive way. Basically, forward-bucket '01s are computed by shifting the forward rate over each of several defined regions of the term structure, one region at a time.

The starting point of the methodology is the division of the term structure into buckets. For the illustration of this section, the term structure is divided into five buckets: 0–2 years, 2–5 years, 5–10 years, 10–15 years, and 20–30 years. The best choice of buckets depends, of course, on the application at hand. A financing desk that does most of its trading in very short-term securities would define many, narrow buckets in the short end and relatively few, wide buckets in the long end. A swaps market-making desk, with business across the curve, might use the buckets defined for this section, although it would likely prefer a greater number of narrower buckets and, particularly in Europe, might need buckets to cover maturities beyond 30 years.

Forward-Bucket Shifts and '01 Calculations

Each forward-bucket '01 is computed by shifting the forward rates in that bucket by one basis point. Depending on how rate curves are stored, this may mean shifting all of a bucket's semiannual forward rates, quarterly forward rates, or rates of even shorter term. This section shifts semiannual rates.

As a first example, consider a 2.12% five-year swap as of May 28, 2010. Table 11-5 lists the cash flows of the fixed side of 100 notional amount of the swap, the "Current" forward rates as of the pricing date, and the three shifted forward curves. For the "0–2 Shift," forward rates of term .5 to 2.0 years are shifted up by one basis point while all other forward rates stay the same. For the "2–5 Shift," forward rates in that bucket, and that bucket only, are shifted. Lastly, for "Shift All," the entire forward curve is shifted.

The row of Table 11-5 labeled "Present Value" gives the present value of the cash flows under the initial forward rate curve and under each of the shifted curves. The forward-bucket '01 for each shift is then the negative of the difference between the shifted and initial present values. For the 2–5-year shift, for example, the '01 is −(99.9679 − 99.9955), or .0276.

The '01 of the "Shift All" scenario is analogous to a *DV*01. The forward bucket analysis decomposes this total '01 into .0196 due to the 0–2-year part of the curve and .0276 due to the 2–5-year part of the curve. The factors that determine the exact distribution of a total '01 across buckets are described in the next section.

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TABLE 11-5 Computation of the Forward-Bucket '01s of a Five-Year 2.12 Percent EUR Swap as of May 28, 2010

Term	Cash Flow	Forward Rates (%)			
		Current	0–2 Shift	2–5 Shift	Shift All
.5	1.06	1.012	1.022	1.012	1.022
1.0	1.06	1.248	1.258	1.248	1.258
1.5	1.06	1.412	1.422	1.412	1.422
2.0	1.06	1.652	1.662	1.652	1.662
2.5	1.06	1.945	1.945	1.955	1.955
3.0	1.06	2.288	2.288	2.298	2.298
3.5	1.06	2.614	2.614	2.624	2.624
4.0	1.06	2.846	2.846	2.856	2.856
4.5	1.06	3.121	3.121	3.131	3.131
5.0	101.06	3.321	3.321	3.331	3.331
Present Value		99.9955	99.9760	99.9679	99.9483
'01			.0196	.0276	.0472

Understanding Forward-Bucket '01s: A Payer Swaption

This subsection analyzes the forward-bucket '01s of a payer swaption. A payer swaption gives the purchaser the right to pay a fixed rate on a swap at some time in the future. More specifically, consider an EUR 5×10 *payer swaption* struck at 4.044% as of May 28, 2010, which gives the purchaser the right to pay a fixed rate of 4.044% on a 10-year EUR swap in five years, that is, at the end of May 2015. The underlying security of this option is a 10-year swap for settlement in five years, otherwise known as a "5×10" swap. See Figure 11-3.[6] As of May 28, 2010, the rate on the EUR 5×10 swap was 4.044%, so the swaption of this application was at-the-money.

Table 11-6 gives the forward-bucket '01s of the EUR 5×10 payer swaption, along with the forward-bucket '01s of an EUR 5-year swap, 10-year swap, 15-year swap, and 5×10 swap. The column labeled "All" gives the '01 from shifting all forward rates.

Computing the '01s of the swaption requires a pricing model, which is not covered here. The intuition behind the results, however, is straightforward. The overall '01 of the payer swaption is negative: as rates increase, the value of the option to pay a fixed rate of 4.044% in exchange for a floating side worth par increases. Furthermore, since the swaption gives the right to pay fixed on a 5×10 swap, the '01 of the swaption will be most concentrated in the buckets that determine the value of that 5×10 swap, i.e., the 5–10 and 10–15 buckets. The swaption has some positive '01 in the 0–2 and 2–5 buckets, as well, because the forward rates in that part of the curve affect the present value of the option's payoff at its expiration in five years' time.

The bucket '01 profiles of the 5-, 10-, and 15-year swaps are determined by several effects. First, and most obvious, each swap is exposed to all parts of the curve up to, but not past, its maturity. Second, the wider buckets, which shift the forward curve over a wider range, tend to generate larger '01s. For example, the 10-year swap's 5–10 bucket '01, which shifts forward rates over five years, is greater than its 2–5 bucket '01, which shifts rates over

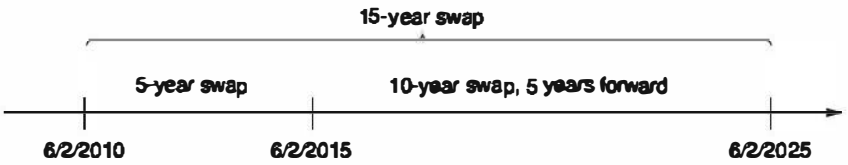

FIGURE 11-3 An example of spot-starting and forward-starting swaps.

[6] This forward swap is a contract to enter into a 10-year swap in five years. Note from the figure that, since swaps settle $T + 2$, the spot-starting swaps begin on June 2, 2010, and the forward starting swap begins on June 2, 2015.

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TABLE 11-6 Forward-Bucket Exposures of Selected EUR-Denominated Securities as of May 28, 2010

		Forward-Bucket Exposures					
Security	**Rate**	**0–2**	**2–5**	**5–10**	**10–15**	**20–30**	**All**
5×10 Payer Swaption	4.044%	.0010	.0016	−.0218	−.0188	.0000	−.0380
5-Year Swap	2.120%	.0196	.0276	.0000	.0000	.0000	.0472
10-Year Swap	2.943%	.0194	.0269	.0394	.0000	.0000	.0857
15-Year Swap	3.290%	.0194	.0265	.0383	.0323	.0000	.1164
5×10 Swap	4.044%	.0000	.0000	.0449	.0366	.0000	.0815

three years. Third, the further a shift is along the curve, the fewer of a swap's coupon payments are affected. This tends to lower the '01s of the longer-term buckets relative to the shorter-term buckets. Fourth, the larger the forward rate in a bucket, the lower the '01, for the same reason that *DV*01 falls with rate, as shown in Chapter 10. In Table 11-6 the term structure of forward rates is, in fact, upward-sloping,[7] so this effect, combined with the third, lowers the 15-year swap's 10–15 bucket '01 relative to its 5–10 bucket '01.

TABLE 11-7 Forward-Bucket Exposures of Three Hedges of a Payer Swaption as of May 28, 2010

		Forward-Bucket Exposures				
	Security or Portfolio	**0–2**	**2–5**	**5–10**	**10–15**	**All**
(I)	5×10 Payer Swaption	.0010	.0016	−.0218	−.0188	−.0380
	Hedge #1:					
(II)	Long 44.34% 10-Year Swaps	.0086	.0119	.0175	.0000	.0380
(III)	Net Position	.0096	.0135	−.0043	−.0188	.0000
	Hedge #2:					
(IV)	Long 46.66% 5×10 Swaps			.0209	.0171	.0380
(V)	Net Position	.0010	.0016	−.0009	−.0017	.0000
	Hedge #3:					
(VI)	Long 57.55% 15-Year Swaps	.0112	.0153	.0220	.0186	.0670
(VII)	Short 61.55% 5-Year Swaps	−.0120	−.0170			−.0290
(VIII)	Net Position	.0002	−.0001	.0002	−.0002	.0000

The 5×10 swap has no exposure to forward rates with a term less then 5 years or greater than 15 years, which is easily apparent from Figure 11-3. Its total '01 of .0815 is divided between the 5–10 and 10–15-year buckets, according to the third and fourth effects described in the previous paragraph.

The Appendix in this chapter presents a very simple demonstration of the third and fourth effects just invoked.

[7] This follows from the upward-sloping par rates in the table, or, more directly, from the graph of the EUR forward rates in Figure 8-2.

Hedging with Forward-Bucket '01s: A Payer Swaption

Table 11-7 shows the forward-bucket exposure of the payer swaption hedged in three different ways: with a 10-year swap, with a 5×10 swap, and with a combination of 5- and 15-year swaps.

The full '01 of the payer swaption and the 10-year swap are, from Table 11-6, −.0380 and .0857, respectively. Therefore, hedging the payer swaption requires a long position of $\frac{.0380}{.0857}$ or approximately 44.34% of the 10-year.

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Multiplying each of the forward-bucket exposures of the 10-year swap in Table 11-6 by this face amount gives row (ii) of Table 11-7. Then, adding the '01s of this hedge to those of the payer swaption gives the net bucket exposures in row (iii). So, while buying 10-year swaps in a *DV*01-neutral way may be a good first pass at a hedge, that is, a quick way to neutralize the rate risk of the payer swaption with the most liquid security available, the net bucket exposures show that the resulting position is at risk of a flattening.

Hedging the payer swaption by receiving in a *DV*01-weighted 5×10 swap, depicted in rows (iv) and (v) of Table 11-7, is a better hedge than receiving in the 10-year swap. This is not particularly surprising since the swaption is the right to pay fixed on that very swap. In any case, the resulting hedged position has a very slight exposure to flattening, but, for the most part, is neutral to rates and the term structure.

Since forward swaps are, in practice, not as easy to execute as par swaps, the final hedge of Table 11-7 considers hedging the swaption with 5- and 15-year par swaps. This hedge, depicted in rows (vi) through (viii) of the table, chooses a long face amount of the 15-year swap to neutralize the 5–10 and 10–15 bucket exposures of the payer swaption and a short face amount of the five-year swap to neutralize the 0–2 and 2–5 bucket exposures arising in small part from the original payer position but in large part from the 15-year swap bought as a hedge. The result, given in row (viii), shows that this hedge neutralizes the risk of each bucket quite closely.

MULTI-FACTOR EXPOSURES AND MEASURING PORTFOLIO VOLATILITY

The facts that a portfolio has a *DV*01 of $10,000 and that interest rates have a volatility of 100 basis points per year leads to the conclusion that the portfolio has an annual volatility of $10,000 × 100 or $1 million. But this measure has the same drawback as one-factor measures of price sensitivity: the volatility of the entire term structure cannot be adequately summarized with just one number. Just as there is a term structure of interest rates, there is a term structure of volatility. The 10-year par rate, for example, is usually more volatile than the 30-year par rate.

In general, portfolios are exposed to interest rates all along the curve and changes in these rates are not perfectly correlated. The frameworks of this chapter, therefore, can be used to estimate volatility more precisely. The presentation here will be in terms of key rates; the discussion would be similar in terms of partial '01s or forward bucket '01s.

First, estimate a volatility for each of the key rates and estimate a correlation for each pair of key rates. Second, compute the key rate '01s of the portfolio. Third, compute the variance and volatility of the portfolio. This computation is quite straightforward given the required inputs. Say that there are only two key rates, C_1 and C_2, that the key rates of the portfolio are $KR01_1$ and $KR01_2$, that the value of the portfolio is P, and that changes are denoted by Δ. Then, by the definition of key rates,

$$\Delta P = KR01_1 \times \Delta C_1 + KR01_2 \times \Delta C_2 \tag{11.13}$$

Furthermore, letting σ_P^2, σ_1^2, and σ_2^2 denote the variances of the portfolio and of the key rates and letting ρ denote the correlation of the key rates, Equation (11.13) implies that

$$\sigma_P^2 = KR01_1^2\sigma_1^2 + KR01_2^2\sigma_2^2 + 2KR01_1KR01_2\rho\sigma_1\sigma_2 \tag{11.14}$$

The standard deviation of the portfolio, of course, is just σ_P. While, as mentioned, this reasoning can be applied equally well to partial '01s or forward-bucket '01s, those two frameworks tend to have more reference rates than a typical key rate framework and, therefore, would require the estimation of a greater number of volatilities and a much greater number of correlation pairs.

APPENDIX

Selected Determinants of Forward-Bucket '01s

Write the price of a two-year bond or fixed leg of a swap, with its fictional notional, in terms of forward rates, as

$$P = \frac{c}{1+f_1} + \frac{1+c}{(1+f_1)(1+f_2)} \tag{11.15}$$

Differentiating with respect to each of the forward rates and multiplying by −1,

$$-\frac{\partial P}{\partial f_1} = \frac{c}{(1+f_1)^2} + \frac{1+c}{(1+f_1)^2(1+f_2)} \tag{11.16}$$

$$-\frac{\partial P}{\partial f_2} = \frac{1+c}{(1+f_1)(1+f_2)^2} \tag{11.17}$$

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To consider the effects of the term of the bucket alone, let $f_1 = f_2$. Then,

$$-\frac{\partial P}{\partial f_1} > -\frac{\partial P}{\partial f_2} \tag{11.18}$$

showing that the '01 of the first bucket, from date 0 to date 1, is greater than the '01 of the second bucket, from date 1 to date 2, precisely because f_1 is used to discount more cash flows than is f_2.

To consider the effects of the term structure alone, let c = 0. Then, the second bucket risk is less than the first if

$$-\frac{\partial P}{\partial f_2} = \frac{1}{(1+f_1)(1+f_2)^2} < -\frac{\partial P}{\partial f_1} = \frac{1}{(1+f_1)^2(1+f_2)} \tag{11.19}$$

which simplifies to

$$f_1 < f_2 \tag{11.20}$$

Hence, an upward-sloping term structure, because of discounting, lowers the second bucket risk relative to the first.

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Country Risk: Determinants, Measures and Implications

12

Learning Objectives

After completing this reading you should be able to:

- Identify sources of country risk.
- Explain how a country's position in the economic growth life cycle, political risk, legal risk, and economic structure affect its risk exposure.
- Evaluate composite measures of risk that incorporate all types of country risk and explain limitations of the risk services.
- Compare instances of sovereign default in both foreign currency debt and local currency debt, and explain common causes of sovereign defaults.
- Describe the consequences of sovereign default.
- Describe factors that influence the level of sovereign default risk; explain and assess how rating agencies measure sovereign default risks.
- Describe the advantages and disadvantages of using the sovereign default spread as a predictor of defaults.

Excerpt is Country Risk: Determinants, Measures and Implications—The 2015 Edition, *by Aswath Damodaran. New York University—Stern School of Business.*

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As companies and investors globalize, we are increasingly faced with estimation questions about the risk associated with this globalization. When investors invest in China Mobile, Infosys or Vale, they may be rewarded with higher returns but they are also exposed to additional risk. When Siemens and Apple push for growth in Asia and Latin America, they clearly are exposed to the political and economic turmoil that often characterize these markets. In practical terms, how, if at all, should we adjust for this additional risk? We will begin the paper with an overview of overall country risk, its sources and measures. We will continue with a discussion of sovereign default risk and examine sovereign ratings and credit default swaps (CDS) as measures of that risk. We will extend that discussion to look at country risk from the perspective of equity investors, by looking at equity risk premiums for different countries and consequences for valuation. In the final section, we will argue that a company's exposure to country risk should not be determined by where it is incorporated and traded. By that measure, neither Coca Cola nor Nestle are exposed to country risk. Exposure to country risk should come from a company's operations, making country risk a critical component of the valuation of almost every large multinational corporation. We will also look at how to move across currencies in valuation and capital budgeting, and how to avoid mismatching errors.

Globalization has been the dominant theme for investors and businesses over the last two decades. As we shift from the comfort of local markets to foreign ones, we face questions about whether investments in different countries are exposed to different amounts of risk, whether this risk is diversifiable in global portfolios and whether we should be demanding higher returns in some countries, for the same investments, than in others. In this paper, we propose to answer all three questions.

In the first part, we begin by taking a big picture view of country risk, its sources and its consequences for investors, companies and governments. We then move on to assess the history of government defaults over time as well as sovereign ratings and credit default swaps (CDS) as measures of sovereign default risk. In the third part, we extend the analysis to look at investing in equities in different countries by looking at whether equity risk premiums should vary across countries, and if they do, how best to estimate these premiums. In the final part, we look at the implications of differences in equity risk premiums across countries for the valuation of companies.

COUNTRY RISK

Are you exposed to more risk when you invest in some countries than others? The answer is obviously affirmative but analyzing this risk requires a closer look at why risk varies across countries. In this section, we begin by looking at why we care about risk differences across countries and break down country risk into constituent (though interrelated) parts. We also look at services that try to measure country risk and whether these country risk measures can be used by investors and businesses.

Why We Care!

The reasons we pay attention to country risk are pragmatic. In an environment where growth often is global and the economic fates of countries are linked together, we are all exposed to variations in country risk in small and big ways.

Let's start with investors in financial markets. Heeding the advice of experts, investors in many developed markets have expanded their portfolios to include nondomestic companies. They have been aided in the process by an explosion of investment options ranging from listings of foreign companies on their markets (ADRs in the US markets, GDRs in European markets) to mutual funds that specialize in emerging or foreign markets (both active and passive) and exchange-traded funds (ETFs). While this diversification has provided some protection against some risks, it has also exposed investors to political and economic risks that they are unfamiliar with, including nationalization and government overthrows. Even those investors who have chosen to stay invested in domestic companies have been exposed to emerging market risk indirectly because of investments made by these companies.

Building on the last point, the need to understand, analyze and incorporate country risk has also become a priority at most large corporations, as they have globalized and become more dependent upon growth in foreign markets for their success. Thus, a chemical company based in the United States now has to decide whether the hurdle rate (or cost of capital) that it uses for a new investment should be different for a new plant that it is considering building in Brazil, as opposed to the United States, and if so, how best to estimate these country-specific hurdle rates.

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Finally, governments are not bystanders in this process, since their actions often have a direct effect on country risk, with increased country risk often translating into less foreign investment in the country, leading to lower economic growth and potentially political turmoil, which feeds back into more country risk.

Sources of Country Risk

If we accept the common sense proposition that your exposure to risk can vary across countries, the next step is looking at the sources that cause this variation. Some of the variation can be attributed to where a country is in the economic growth life cycle, with countries in early growth being more exposed to risk than mature companies. Some of it can be explained by differences in political risk, a category that includes everything from whether the country is a democracy or dictatorship to how smoothly political power is transferred in the country. Some variation can be traced to the legal system in a country, in terms of both structure (the protection of property rights) and efficiency (the speed with which legal disputes are resolved). Finally, country risk can also come from an economy's disproportionate dependence on a particular product or service. Thus, countries that derive the bulk of their economic output from one commodity (such as oil) or one service (insurance) can be devastated when the price of that commodity or the demand for that service plummets.

Life Cycle

In company valuation, where a company is in its life cycle can affect its exposure to risk. Young, growth companies are more exposed to risk partly because they have limited resources to overcome setbacks and partly because they are far more dependent on the macro environment staying stable to succeed. The same can be said about countries in the life cycle, with countries that are in early growth, with few established business and small markets, being more exposed to risk than larger, more mature countries.

We see this phenomenon in both economic and market reactions to shocks. A global recession generally takes a far greater toll of small, emerging markets than it does in mature markets, with biggest swings in economic growth and employment. Thus, a typical recession in mature markets like the United States or Germany may translate into only a 1–2% drop in the gross domestic products of these countries and a good economic year will often result in growth of 3–4% in the overall economy. In an emerging market, a recession or recovery can easily translate into double-digit growth, in positive or negative terms. In markets, a shock to global markets will travel across the world, but emerging market equities will often show much greater reactions, both positive and negative to the same news. For instance, the banking crisis of 2008, which caused equity markets in the United States and Western Europe to drop by about 25%–30%, resulted in drops of 50% or greater in many emerging markets.

The link between life cycle and economic risk is worth emphasizing because it illustrates the limitations on the powers that countries have over their exposure to risk. A country that is still in the early stages of economic growth will generally have more risk exposure than a mature country, even it is well governed and has a solid legal system.

Political Risk

While a country's risk exposure is a function of where it is in the growth cycle, that risk exposure can be affected by the political system in place in that country, with some systems clearly augmenting risk far more than others.

1. *Continuous versus discontinuous risk*: Let's start with the first and perhaps trickiest question on whether democratic countries are less or more risky than their authoritarian counterparts. Investors and companies that value government stability (and fixed policies) sometimes choose the latter, because a strong government can essentially lock in policies for the long term and push through changes that a democracy may never be able to do or do only in steps. The cautionary note that should be added is that while the chaos of democracy does create more continuous risk (policies that change as governments shift), dictatorships create more discontinuous risk. While change may happen infrequently in an authoritarian system, it is also likely to be wrenching and difficult to protect against. It is also worth noting that the nature of authoritarian systems is such that the more stable policies that they offer can be accompanied by other costs (political corruption and ineffective legal systems) that overwhelm the benefits of policy stability.

   The trade-off between the stability (artificial though it might be) of dictatorships and the volatility of

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democracy makes it difficult to draw a strong conclusion about which system is more conducive to higher economic growth. Przeworski and Limongi (1993) provide a summary of the studies through 1993 on the link between economic growth and democracy and report mixed results.[1] Of the 19 studies that they quote, seven find that dictatorships grow faster, seven conclude that democracies grow at a higher rate and five find no difference. In an interesting twist, Glaeser, La Porta, Lopez-de-Silane and Shleifer (2004) argue that it is not political institutions that create growth but that economic growth that allows countries to become more democratic.[2]

2. *Corruption and side costs*: Investors and businesses have to make decisions based upon rules or laws, which are then enforced by a bureaucracy. If those who enforce the rules are capricious, inefficient or corrupt in their judgments, there is a cost imposed on all who operate under the system. Transparency International tracks perceptions of corruption across the globe, using surveys of experts living and working in different countries, and ranks countries from most to least corrupt. Based on the scores from these surveys,[3] *Transparency International* also provides a listing of the ten least and most corrupt countries in the world in Table 12-1 (with higher scores indicating less corruption) for 2014.

   In business terms, it can be argued that corruption is an implicit tax on income (that does not show up in conventional income statements as such) that reduces the profitability and returns on investments for businesses in that country directly and for investors in these businesses indirectly. Since the tax is not specifically stated, it is also likely to be more uncertain than an explicit tax, especially if there are legal sanctions that can be faced as a consequence, and thus add to total risk.

3. *Physical violence*: Countries that are in the midst of physical conflicts, either internal or external, will expose investors/businesses to the risks of these conflicts. Those costs are not only economic (taking the form of higher costs for buying insurance or protecting business interests) but are also physical (with employees and managers of businesses facing harm). Figure 12-1 provides a measure of violence around the world in the form of a *Global Peace Index* map generated and updated every year by the Institute for Economics and Peace.[4]

4. *Nationalization/expropriation risk*: If you invest in a business and it does well, the pay off comes in the form of higher profits (if you are a business) or higher value (if you are an investor). If your profits can be expropriated by the business (with arbitrary and specific taxes imposed just upon you) or your business can be nationalized (with you receiving well below the fair value as compensation), you will be less likely to invest and more likely to perceive risk in the investment. Some businesses seem to be more exposed to nationalization risk than others, with natural resource companies at the top of the target list. An Ernst and Young assessment of risks facing mining companies in 2012, lists nationalization at the very top of the list of risk in 2012, a stark contrast with the list in 2008, where nationalization was ranked eighth of the top ten risks.[5]

TABLE 12-1 Most and Least Corrupt Countries—2014

Least Corrupt		Most Corrupt	
Country	*Score*	*Country*	*Score*
Denmark	92	Korea (North)	8
New Zealand	91	Somalia	8
Finland	89	Sudan	11
Sweden	87	Afghanistan	12
Norway	86	South Sudan	15
Switzerland	86	Iraq	16
Singapore	84	Turkmenistan	17
Netherlands	83	Eritrea	18
Luxembourg	82	Libya	18
Canada	81	Uzbekistan	18

Source: Transparency International.

[1] Przeworski, A. and F. Limongi, 1993, Political Regimes and Economic Growth, *The Journal of Economic Perspectives*, v7, 51–69.

[2] Glaeser, E.L., R. La Porta, F. Lopez-de-Silane, A. Shleifer, 2004, Do Institutions Cause Growth?, NBER Working Paper # 10568.

[3] See Transperancy.org for specifics on how they come up with corruption scores and update them.

[4] See http://www.visionofhumanity.org.

[5] Business Risks facing mining and metals, 2012–2013, Ernst & Young, www.ey.com.

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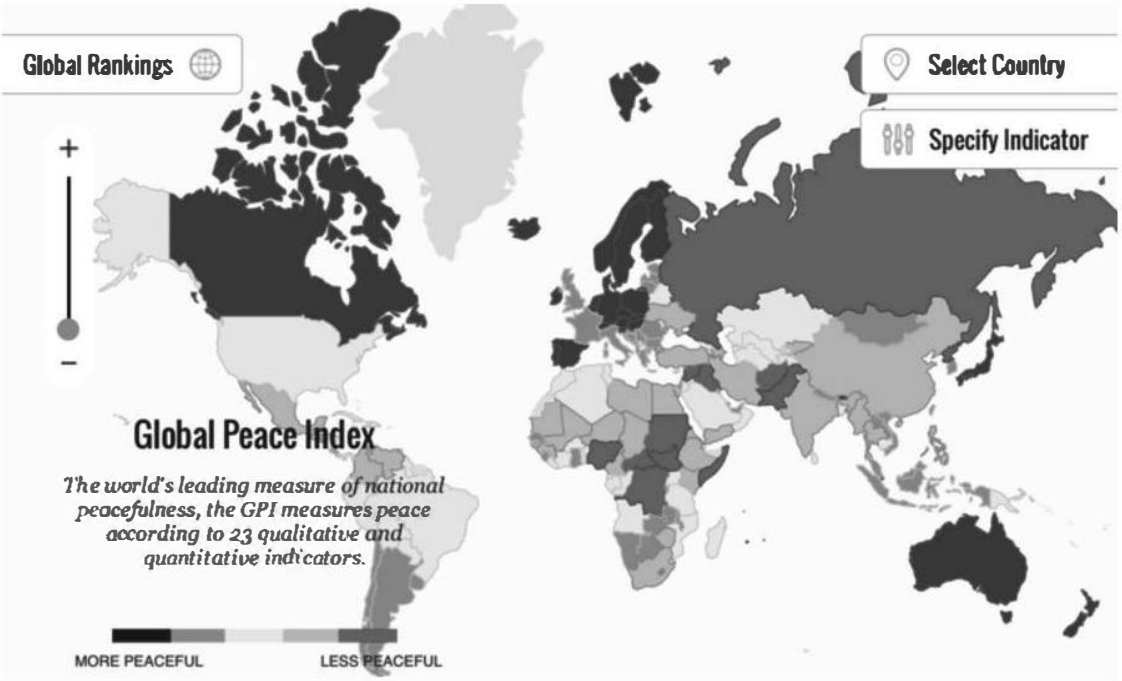

FIGURE 12-1 Global peace index in 2014.

Source: Institute for Peace and Economics.

Legal Risk

Investors and businesses are dependent upon legal systems that respect their property rights and enforce those rights in a timely manner. To the extent that a legal system fails on one or both counts, the consequences are negative not only for those who are immediately affected by the failing but for potential investors who have to build in this behavior into their expectations. Thus, if a country allows insiders in companies to issue additional shares to themselves at well below the market price without paying heed to the remaining shareholders, potential investors in these companies will pay less (or even nothing) for shares. Similarly, companies considering starting new ventures in that country may determine that they are exposed to the risk of expropriation and either demand extremely high returns or not invest at all.

It is worth emphasizing, though, that legal risk is a function not only of whether it pays heed to property and contract rights, but also how efficiently the system operates. If enforcing a contract or property rights takes years or even decades, it is essentially the equivalent of a system that does not protect these rights in the first place, since neither investors nor businesses can wait in legal limbo for that long. A group of nongovernment organizations has created an international property rights index, measuring the protection provided for property rights in different countries.[6] The summary results, by region, are provided in Table 12-2, with the ranking from best protection (highest scores) to worst in 2014.

Based on these measures, property right protections are strongest in North America and weakest in Latin America and Africa. In an interesting illustration of differences within geographic regions, within Latin America, Chile ranks 24th in the world in property protection rights but Argentina and Venezuela fall towards the bottom of the rankings.

Economic Structure

Some countries are dependent upon a specific commodity, product or service for their economic success. That dependence can create additional risk for investors and businesses, since a drop in the commodity's price or demand for the product/service can create severe economic pain that spreads well beyond the companies

[6] See the International Property Rights Index, http://www.internationalpropertyrightsindex.org/ranking

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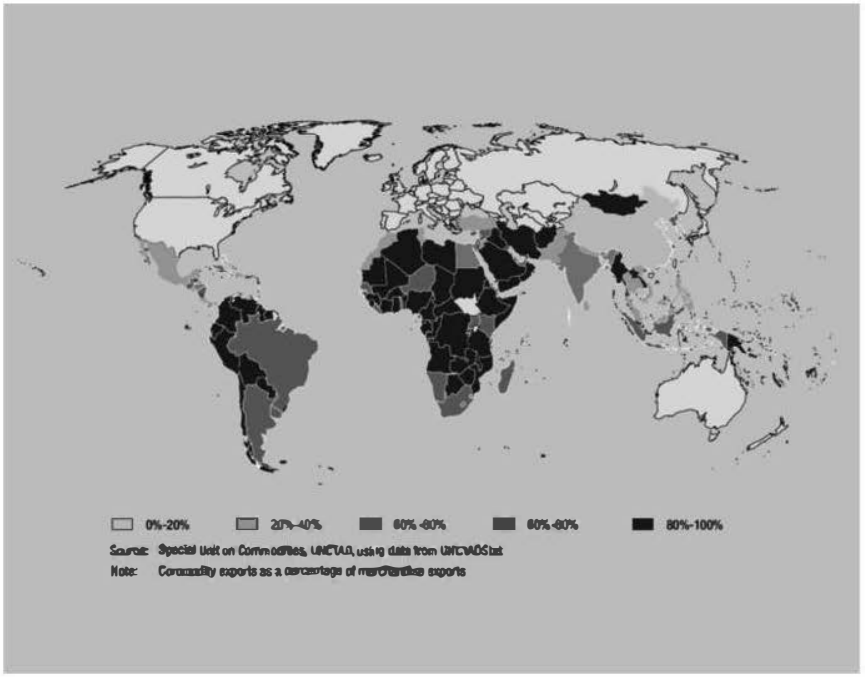
TABLE 12-2 Property Right Protection by Region

Region	Overall Property Rights	Legal Property Rights	Physical Property Rights	Intellectual Property Rights
North America	5.23	4.95	5.76	4.98
Western Europe	5.19	4.91	5.73	4.92
Central/Eastern Europe	4.78	4.65	5.47	4.22
Asia & Oceania	4.77	4.42	5.44	4.44
Middle East & North Africa	4.76	4.61	5.42	4.26
Latin America	4.57	4.23	5.23	4.25
Africa	4.53	4.26	5.17	4.16

Source: International Property Rights Index.



DEVELOPING COUNTRY DEPENDENCY ON COMMODITIES, 2012–2013 (map)

FIGURE 12-2 Commodity dependence of countries.

immediately affected. Thus, if a country derives 50% of its economic output from iron ore, a drop in the price of iron ore will cause pain not only for mining companies but also for retailers, restaurants and consumer product companies in the country.

In a comprehensive study of commodity dependent countries, the United National Conference on Trade and Development (UNCTAD) measures the degree of dependence upon commodities across emerging markets and Figure 12-2 reports the statistics.[7] Note the disproportional dependence on commodity exports that countries in Africa and Latin America have, making their economies and markets very sensitive to changes in commodity prices.

Why don't countries that derive a disproportionate amount of their economy from a single source diversify their economies? That is easier said than done, for two reasons. First, while it is feasible for larger countries like Brazil, India and China to try to broaden their economic bases, it is much more difficult for small countries like Peru or Angola to do the same. Like small companies, these small countries have to find a niche where there can specialize, and by definition, niches will lead to over dependence upon one or a few sources. Second, and this is especially the case with natural resource dependent countries, the wealth that can be created by exploiting the natural resource will usually be far greater than using the resources elsewhere in the economy. Put differently, if a country with ample oil reserves decides to diversify its economic base by directing its resources into manufacturing or service businesses, it may have to give up a significant portion of near term growth for a long-term objective of having a more diverse economy.

[7] The State of Commodity Dependence 2014, United Nations Conference on Trade and Development (UNCTAD), http://unctad.org/en/PublicationsLibrary/suc2014d7_en.pdf

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Measuring Country Risk

As the discussion in the last section should make clear, country risk can come from many different sources. While we have provided risk measures on each dimension, it would be useful to have composite measures of risk that incorporate all types of country risk. These composite measures should incorporate all of the dimensions of risk and allow for easy comparisons across countries.

Risk Services

There are several services that attempt to measure country risk, though not always from the same perspective or for the same audience. For instance, Political Risk Services (PRS) provides numerical measures of country risk for more than a hundred countries.[8] The service is commercial and the scores are made available only to paying members, but *PRS uses twenty two variables to measure risk* in countries on three dimensions: political, financial and economic. It provides country risk scores on each dimension separately, as well as a composite score for the country. The scores range from zero to one hundred, with high scores (80–100) indicating low risk and low scores indicating high risk. In the July 2015 update, the 15 countries that emerged as safest and riskiest are listed in Table 12-3.

In addition to providing current assessments, PRS provides forecasts of country risk scores for the countries that it follows.

There are other services that attempt to do what PRS does, with difference in both how the scores are developed and what they measure. Euromoney has country risk scores, based on surveys of 400 economists that

[8] See http://www.prsgroup.com/ICRG_Methodology.aspx#RiskForecasts for a discussion of the factors that PRS considers in assessing country risk scores.

TABLE 12-3 Highest and Lowest Risk Countries: PRS Scores (July 2015)

Riskiest Countries		Safest Countries	
Country	**Composite PRS Score**	**Country**	**Composite PRS Score**
Syria	35.3	Switzerland	88.5
Somalia	41.8	Norway	88.3
Sudan	46.8	Singapore	85.8
Liberia	49.8	Luxembourg	84.8
Libya	50.3	Brunei	84.5
Guinea	50.8	Sweden	84.5
Venezuela	52.0	Germany	83.5
Yemen, Republic	53.8	Taiwan	83.3
Ukraine	54.0	Canada	83.0
Niger	54.3	Qatar	82.3
Zimbabwe	55.3	United Kingdom	81.8
Korea, D.P.R.	55.8	Denmark	81.3
Mozambique	55.8	Korea, Republic	81.0
Congo, Dem. Republic	56.0	New Zealand	81.0
Belarus	57.5	Hong Kong	80.8

Source: Political Risk Services (PRS).

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range from zero to one hundred.[9] It updates these scores, by country and region, at regular intervals. The Economist developed its own variant on country risk scores that are developed internally, based upon currency risk, sovereign debt risk and banking risk. The World Bank provides a collected resource base that draws together risk measures from different services into one database of governance indicators.[10] There are six indicators provided for 215 countries, measuring corruption, government effectiveness, political stability, regulatory quality, rule of law and voice/accountability, with a scaling around zero, with negative numbers indicating more risk and positive numbers less risk.

Limitations

The services that measure country risk with scores provide some valuable information about risk variations across countries, but it is unclear how useful these measures are for investors and businesses interested in investing in emerging markets for many reasons:

- *Measurement models/methods:* Many of the entities that develop the methodology and convert them into scores are not business entities and consider risks that may have little relevance for businesses. In fact, the scores in some of these services are more directed at policy makers and macroeconomists than businesses.
- *No standardization:* The scores are not standardized and each service uses it own protocol. Thus, higher scores go with lower risk with PRS and Euromoney risk measures but with higher risk in the Economist risk measure. The World Bank's measures of risk are scaled around zero, with more negative numbers indicating higher risk.
- *More rankings than scores:* Even if you stay with the numbers from one service, the country risk scores are more useful for ranking the countries than for measuring relative risk. Thus, a country with a risk score of 80, in the PRS scoring mechanism, is safer than a country with a risk score of 40, but it would be dangerous to read the scores to imply that it is twice as safe.

In summary, as data gets richer and easier to access, there will be more services trying to measure country risk and even more divergences in approaches and measurement mechanisms.

SOVEREIGN DEFAULT RISK

The most direct measure of country risk is a measure of default risk when lending to the government of that country. This risk, termed sovereign default risk, has a long history of measurement attempts stretching back to the nineteenth century. In this section, we begin by looking at the history of sovereign defaults, both in foreign currency and local currency terms, and follow up by looking at measures of sovereign default risk, ranging from sovereign ratings to market-based measures.

A History of Sovereign Defaults

In this section, we will examine the history of sovereign default, by first looking at governments that default on foreign currency debt (which is understandable) and then looking at governments that default on local currency debt (which is more difficult to explain).

Foreign Currency Defaults

Through time, many governments have been dependent on debt borrowed from other countries (or banks in those countries), usually denominated in a foreign currency. A large proportion of sovereign defaults have occurred with this type of sovereign borrowing, as the borrowing country finds its short of the foreign currency to meet its obligations, without the recourse of being able to print money in that currency. Starting with the most recent history from 2000–2014, sovereign defaults have mostly been on foreign currency debt, starting with a relatively small default by Ukraine in January 2000, followed by the largest sovereign default of the last decade with Argentina in November 2001. Table 12-4 lists the sovereign defaults, with details of each.

Going back further in time, sovereign defaults have occurred frequently over the last two centuries, though the defaults have been bunched up in eight periods. A

[9] http://www.euromoney.com/Poll/10683/PollsAndAwards/Country-Risk.html

[10] http://data.worldbank.org/data-catalog/worldwide-governance-indicators

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TABLE 12-4 Sovereign Defaults: 2000–2014

Default Date	Country	$ Value of Defaulted Debt	Details
January 2000	Ukraine	$1,064 m	Defaulted on DM and US dollar denominated bonds. Offered exchange for longer term, lower coupon bonds to lenders.
September 2000	Peru	$4,870 m	Missed payment on Brady bonds.
November 2001	Argentina	$82,268 m	Missed payment on foreign currency debt in November 2001. Debt was restructured.
January 2002	Moldova	$145 m	Missed payment on bond but bought back 50% of bonds, before defaulting.
May 2003	Uruguay	$5,744 m	Contagion effect from Argentina led to currency crisis and default.
July 2003	Nicaragua	$320 m	Debt exchange, replacing higher interest rate debt with lower interest rate debt.
April 2005	Dominican Republic	$1,622 m	Defaulted on debt and exchanged for new bonds with longer maturity.
December 2006	Belize	$242 m	Defaulted on bonds and exchanged for new bonds with step-up coupons.
December 2008	Ecuador	$510 m	Failed to make interest payment of $30.6 million on the bonds.
February 2010	Jamaica	$7.9 billion	Completed a debt exchange resulting in a loss of between 11% and 17% of principal.
January 2011	Ivory Coast	$2.3 billion	Defaulted on Eurobonds.
July 2014	Argentina	$13 billion	US Judge ruled that Argentina could not pay current bondholders unless old debt holders also got paid.

survey article on sovereign default, Hatchondo, Martinez and Sapriza (2007) summarizes defaults over time for most countries in Europe and Latin America and their findings are captured in Table 12-5:[11]

While Table 12-5 does not list defaults in Asia and Africa, there have been defaults in those regions over the last 50 years as well.

In a study of sovereign defaults between 1975 and 2004, Standard and Poor's notes the following facts about the phenomenon:[12]

1. Countries have been more likely to default on bank debt owed than on sovereign bonds issued. Figure 12-3 summarizes default rates on each.

[11] J.C. Hatchondo, L. Martinez, and H. Sapriza, 2007, *The Economics of Sovereign Default, Economic Quarterly*, v93, pg 163-187.

[12] S&P Ratings Report, "Sovereign Defaults set to fall again in 2005," September 28, 2004.

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TABLE 12-5 Defaults Over Time: 1820–2003

	1824–34	1867–82	1890–1900	1911–1921	1931–40	1976–89	1998–2003
Europe							
Austria		1868		1914	1932		
Bulgaria				1915	1932		
Germany					1932		
Greece	1824		1893				
Hungary					1931		
Italy					1940		
Moldova							2002
Poland					1936	1981	
Portugal	1834		1892				
Romania				1915	1933	1981	
Russia				1917			1998
Serbia-Yugoslavia			1895		1933	1983	
Spain	1831	1867					
Turkey		1976		1915	1940	1978	
Ukraine							1998
Latin America							
Argentina	1830		1890	1915	1930	1982	2001
Bolivia		1874			1931	1980	
Brazil	1826		1898	1914	1931	1983	
Chile	1826	1880			1931	1983	
Colombia	1826	1879	1900		1932		
Costa Rica	1827	1874	1895		1937	1983	
Cuba					1933	1982	
Dominica							2003
Dominican Republic		1869	1899		1931	1982	
Ecuador	1832	1868		1911, '14	1931	1982	1999
El Salvador	1827			1921	1931		
Guatemala	1828	1876	1894	1933			

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	1824–34	1867–82	1890–1900	1911–1921	1931–40	1976–89	1998–2003
Honduras	1827	1873		1914		1981	
Mexico	1827	1867		1914		1982	
Nicaragua	1828		1894	1911	1932	1980	
Panama					1932	1982	
Paraguay	1827	1874	1892	1920	1932	1986	
Peru	1826	1876			1931	1983	
Uruguay		1876	1892			1983	2003
Venezuela	1832	1878	1892			1982	

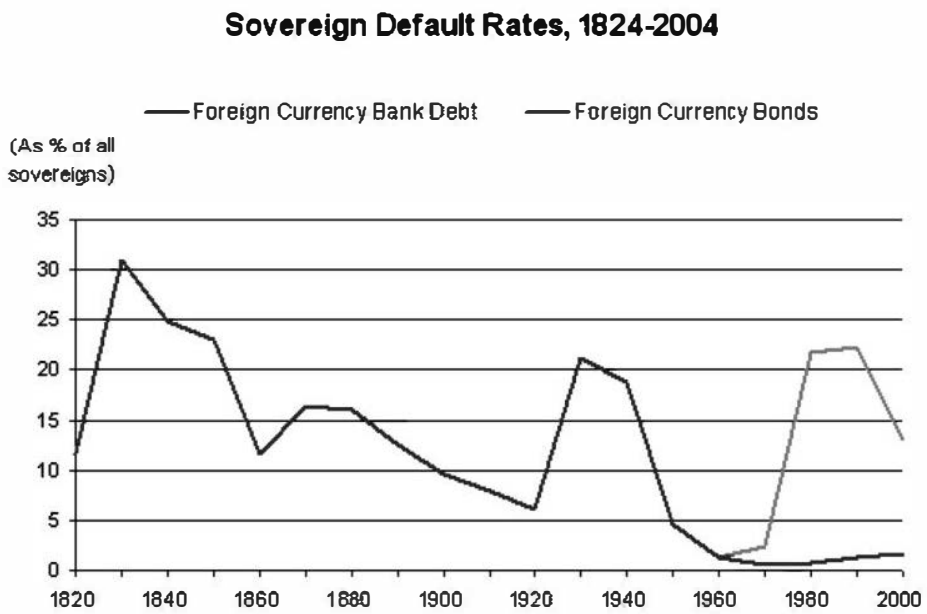

FIGURE 12-3 Percent of sovereign debt in default.

Note that while bank loans were the only recourse available to governments that wanted to borrow prior to the 1960s, sovereign bond markets have expanded access in the last few decades. Defaults since then have been more likely on foreign currency debt than on foreign currency bonds.

2. In dollar value terms, Latin American countries have accounted for much of sovereign defaulted debt in the last 50 years. Figure 12-4 summarizes the statistics:

   In fact, the 1990s represent the only decade in the last 5 decades, where Latin American countries did not account for 60% or more of defaulted sovereign debt.

Since Latin America has been at the epicenter of sovereign default for most of the last two centuries, we may be able to learn more about why default occurs by looking at its history, especially in the nineteenth century, when the region was a prime destination for British, French and Spanish capital. Lacking significant domestic savings and possessing the allure of natural resources, the newly

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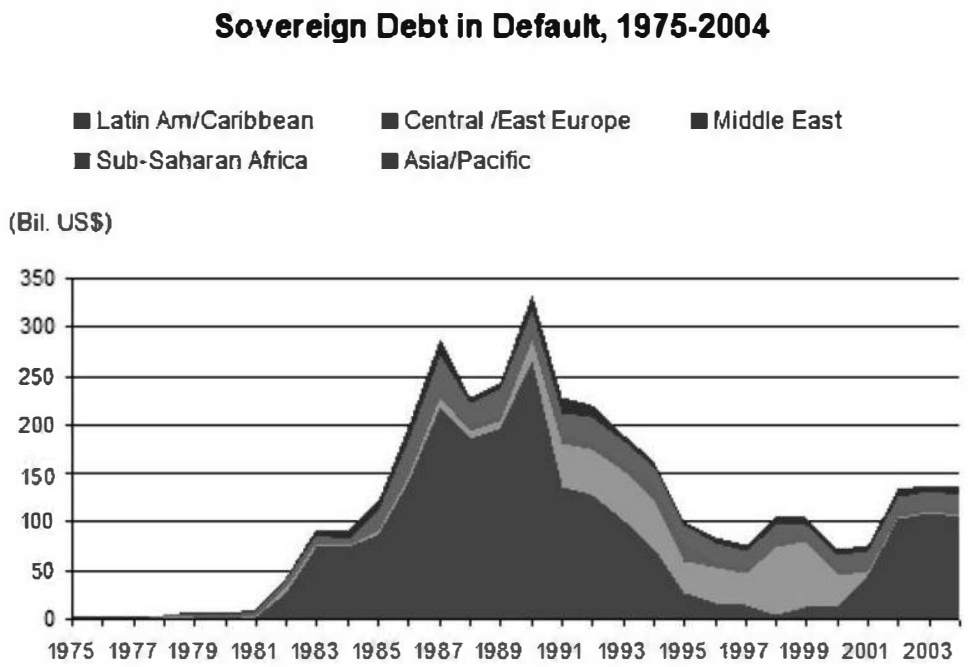

FIGURE 12-4 Sovereign default by region.

independent Latin American countries borrowed heavily, usually in foreign currency or gold and for very long maturities (exceeding 20 years). Brazil and Argentina also issued domestic debt, with gold clauses, where the lender could choose to be paid in gold. The primary trigger for default was military conflicts between countries or coups within, with weak institutional structures exacerbating the problems. Of the 77 government defaults between 1820 and 1914, 58 were in Latin America and as Figure 12-5 indicates, these countries collectively spent 38% of the period between 1820 and 1940 in default.

The percentage of years that each country spent in default during the entire period is in parentheses next to the country; for instance, Honduras spent 79% of the 115 years in default.

Local Currency Defaults

While defaulting on foreign currency debt draws more headlines, some of the countries listed in Tables 12-2 and 12-3 also defaulted contemporaneously on domestic currency debt.[13] A survey of defaults by S&P since 1975 notes that 23 issuers have defaulted on local currency debt, including Argentina (2002–2004), Madagascar (2002), Dominica (2003–2004), Mongolia (1997–2000), Ukraine (1998–2000), and Russia (1998–1999). Russia's default on $39 billion worth of ruble debt stands out as the largest local currency default since Brazil defaulted on $62 billion of local currency debt in 1990. Figure 12-6 summarizes the percentage of countries that defaulted in local currency debt between 1975 and 2004 and compares it to sovereign defaults in foreign currency.[14]

Moody's broke down sovereign defaults in local currency and foreign currency debt and uncovered an interesting feature: countries are increasingly defaulting on both local and foreign currency debt at the same time, as evidenced in Figure 12-7.

[13] In 1992, Kuwait defaulted on its local currency debt, while meeting its foreign currency obligations.

[14] S&P Ratings Report, "Sovereign Defaults set to fall again in 2005," September 28, 2004.

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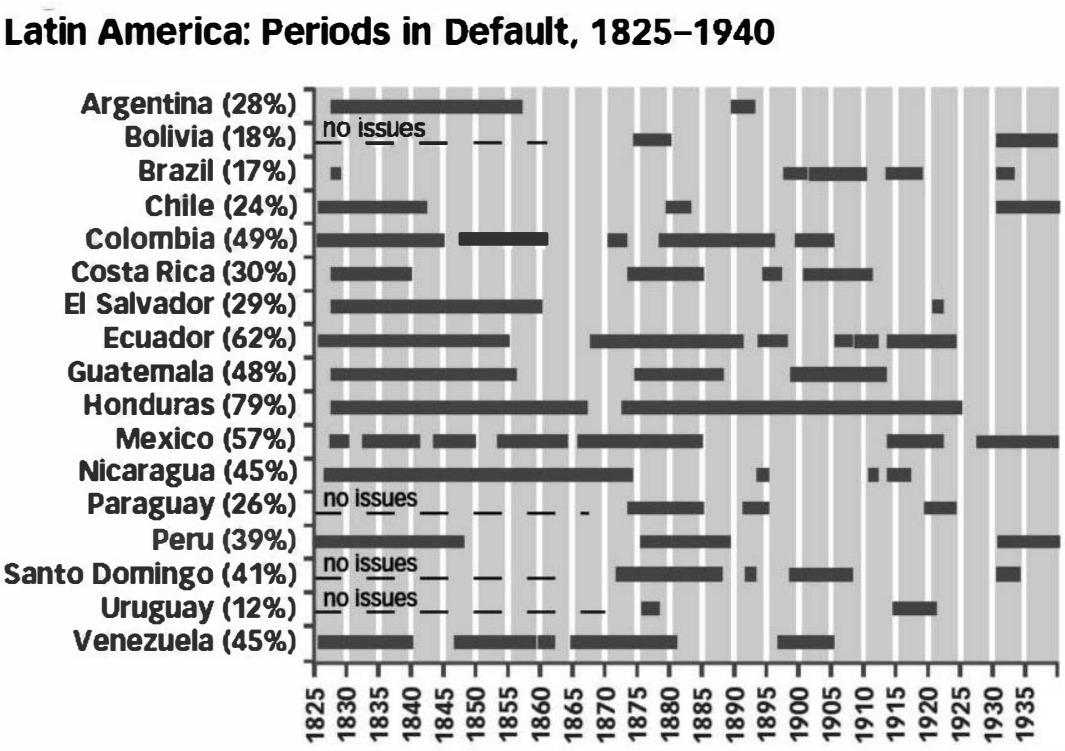

Sources: Taylor (2003); default data from Tomz (2001); issue dates from Marichal (1989).

FIGURE 12-5 Latin America—The sovereign default epicenter.

Sources: Taylor (2003); default data from Tomz (2001); issue dates from Marichal (1989).

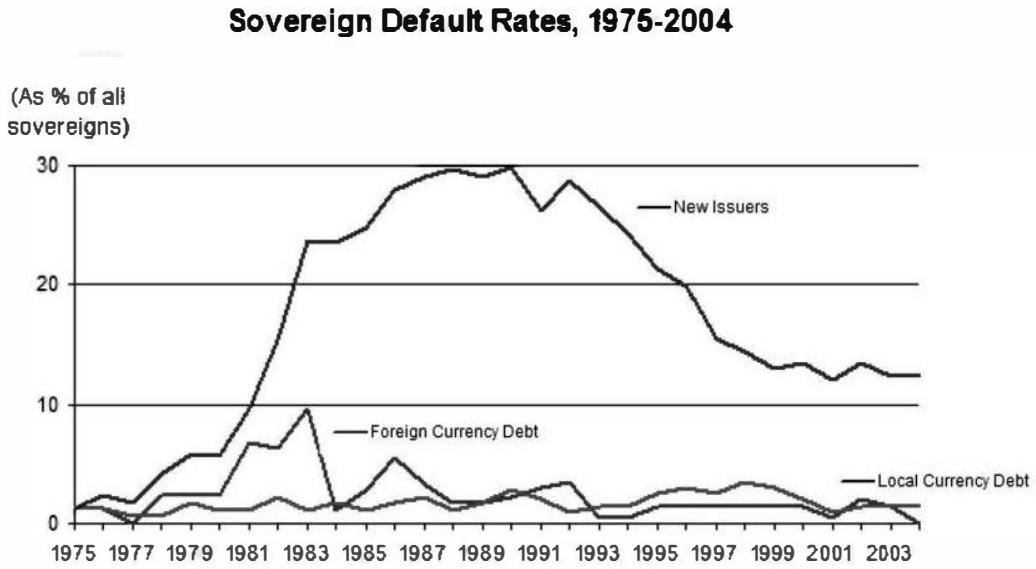

FIGURE 12-6 Defaults on foreign and local currency debt.

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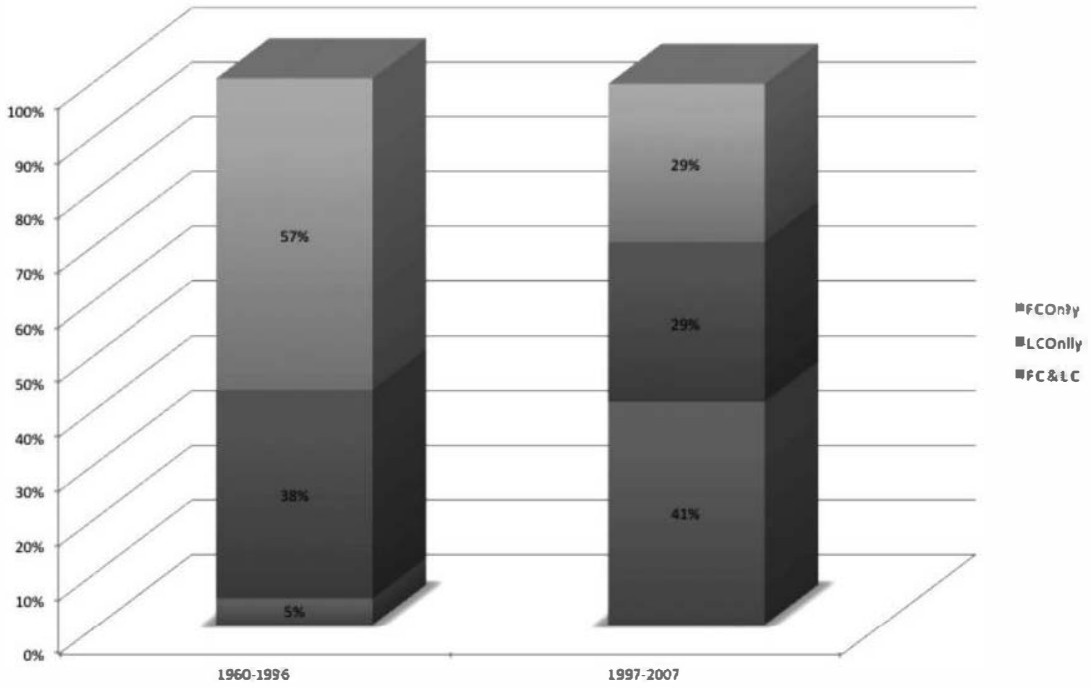

FIGURE 12-7 Foreign currency & local currency sovereign debt default.

While it is easy to see how countries can default on foreign currency debt, it is more difficult to explain why they default on local currency debt. As some have argued, countries should be able to print more of the local currency to meet their obligations and thus should never default. There are three reasons why local currency default occurs and will continue to do so.

The first two reasons for default in the local currency can be traced to a loss of power in printing currency.

1. *Gold standard:* In the decades prior to 1971, when some countries followed the gold standard, currency had to be backed up with gold reserves. As a consequence, the extent of these reserves put a limit on how much currency could be printed.
2. *Shared currency:* The crisis in Greece has brought home one of the costs of a shared currency. When the Euro was adopted as the common currency for the Euro zone, the countries involved accepted a trade off. In return for a common market and the convenience of a common currency, they gave up the power to control how much of the currency they could print. Thus, in July 2015, the Greek government cannot print more Euros to pay off outstanding debt.

The third reason for local currency default is more intriguing. In the next section, we will argue that default has negative consequences: reputation loss, economic recessions and political instability. The alternative of printing more currency to pay debt obligations also has costs. It debases and devalues the currency and causes inflation to increase exponentially, which in turn can cause the real economy to shrink. Investors abandon financial assets (and markets) and move to real assets (real estate, gold) and firms shift from real investments to financial speculation. Countries therefore have to trade off between which action—default or currency debasement—has lower long-term costs and pick one; many choose default as the less costly option.

An intriguing explanation for why some countries choose to default in local currency debt whereas other prefer to print money (and debase their currencies) is based on whether companies in the country have foreign currency debt funding local currency assets. If they do, the cost of printing more local currency, pushing up inflation and devaluing the local currency, can be catastrophic for corporations, as the local currency devaluation lays waste to their assets while liabilities remain relatively unchanged.

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Consequences of Default

What happens when a government defaults? In the eighteenth century, government defaults were followed often by shows of military force. When Turkey defaulted in the 1880s, the British and the French governments intervened and appointed commissioners to oversee the Ottoman Empire to ensure discipline. When Egypt defaulted around the same point in time, the British used military force to take over the government. A default by Venezuela in the early part of the 20th century led to a European blockade of that country and a reaction from President Theodore Roosevelt and the United States government, who viewed the blockade as a threat to the US power in the hemisphere.

In the twentieth century, the consequences of sovereign default have been both economic and political. Besides the obvious implication that lenders to that government lose some or a great deal of what is owed to them, there are other consequences as well:

1. *Reputation loss:* A government that defaults is tagged with the "deadbeat" label for years after the event, making it more difficult for it to raise financing in future rounds.
2. *Capital market turmoil:* Defaulting on sovereign debt has repercussions for all capital markets. Investors withdraw from equity and bond markets, making it more difficult for private enterprises in the defaulting country to raise funds for projects.
3. *Real output:* The uncertainty created by sovereign default also has ripple effects on real investment and consumption. In general, sovereign defaults are followed by economic recessions, as consumers hold back on spending and firms are reluctant to commit resources to long-term investments.
4. *Political instability:* Default can also strike a blow to the national psyche, which in turn can put the leadership class at risk. The wave of defaults that swept through Europe in the 1930s, with Germany, Austria, Hungary and Italy all falling victims, allowed for the rise of the Nazis and set the stage for the Second World War. In Latin America, defaults and coups have gone hand in hand for much of the last two centuries.

In short, sovereign default has serious and painful effects on the defaulting entity that may last for long periods.

It is also worth emphasizing that default has seldom involved total repudiation of the debt. Most defaults are followed by negotiations for either a debt exchange or restructuring, where the defaulting government is given more time, lower principal and/or lower interest payments. Credit agencies usually define the duration of a default episode as lasting from when the default occurs to when the debt is restructured. Defaulting governments can mitigate the reputation loss and return to markets sooner, if they can minimize losses to lenders.

Researchers who have examined the aftermath of default have come to the following conclusions about the short and long term effects of defaulting on debt:

1. *Default has a negative impact on real GDP growth of between 0.5% and 2%,* but the bulk of the decline is in the first year after the default and seems to be short lived.
2. *Default does affect a country's long term sovereign rating and borrowing costs.* One study of credit ratings in 1995 found that the ratings for countries that had defaulted at least once since 1970 were one to two notches lower than otherwise similar countries that had not defaulted. In the same vein, defaulting countries have borrowing costs that are about 0.5 to 1% higher than countries that have not defaulted. Here again, though, the effects of default dissipate over time.
3. *Sovereign default can cause trade retaliation.* One study indicates a drop of 8% in bilateral trade after default, with the effects lasting for up to 15 years, and another one that uses industry level data finds that export oriented industries are particularly hurt by sovereign default.
4. *Sovereign default can make banking systems more fragile.* A study of 149 countries between 1975 and 2000 indicates that the probability of a banking crisis is 14% in countries that have defaulted, an eleven percentage-point increase over non-defaulting countries.
5. *Sovereign default also increases the likelihood of political change.* While none of the studies focus on defaults per se, there are several that have examined the aftereffects of sharp devaluations, which often accompany default. A study of devaluations between 1971 and 2003 finds a 45% increase in the probability of change in the top leader (prime minister or president) in the country and a 64% increase in the probability of change in the finance executive (minister of finance or head of central bank).

In summary, default is costly and countries do not (and should not) take the possibility of default lightly. Default is particularly expensive when it leads to banking crises and

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currency devaluations; the former have a longstanding impact on the capacity of firms to fund their investments whereas the latter create political and institutional instability that lasts for long periods.

Measuring Sovereign Default Risk

If governments can default, we need measures of sovereign default risk not only to set interest rates on sovereign bonds and loans but to price all other assets. In this section, we will first look at why governments default and then at how ratings agencies, markets and services measure this default risk.

Factors Determining Sovereign Default Risk

Governments default for the same reason that individuals and firms default. In good times, they borrow far more than they can afford, given their assets and earning power, and then find themselves unable to meet their debt obligations during downturns. To determine a country's default risk, we would look at the following variables:

1. *Degree of indebtedness:* The most logical place to start assessing default risk is by looking at how much a sovereign entity owes not only to foreign banks/investors but also to its own citizens. Since larger countries can borrow more money, in absolute terms, the debt owed is usually scaled to the GDP of the country. Table 12-6 lists the 20 countries that owe the most, relative to GDP, in 2014.[15]

   The list suggests that this statistic (government debt as percent of GDP) is an incomplete measure of default risk. The list includes some countries with high default risk (Zimbabwe, Lebanon) but is also includes some countries that were viewed as among the most credit worthy by ratings agencies and markets (Japan, France and Canada). However, the list did also include Portugal, Greece and Italy, countries that had high credit ratings prior to the 2008 banking crisis, but have gone through repeated bouts of debt worries since. As a final note, it is worth looking at how this statistic (debt as a percent of GDP) has changed in the United States over its last few decades. Figure 12-8 shows public debt as a percent of GDP for the US from 1966 to 2014:[16]

[15] The World Factbook, 2015, Central Intelligence Agency*

[16] The statistic varies depending upon the data source you use, with some reporting higher numbers and others lower. This data was obtained from usgovernmentspending.com.

TABLE 12-6 Debt as % of Gross Domestic Product

Country	Government Debt as % of GDP
Japan	227.70%
Zimbabwe	181.00%
Greece	174.50%
Lebanon	142.40%
Italy	134.10%
Jamaica	132.00%
Portugal	131.00%
Cyprus	119.40%
Ireland	118.90%
Grenada	110.00%
Singapore	106.70%
Belgium	101.90%
Eritrea	101.30%
Barbados	101.20%
Spain	97.60%
France	95.50%
Iceland	94.00%
Egypt	93.80%
Puerto Rico	93.60%
Canada	92.60%

Source: The CIA World Factbook.

At 102% of GDP, federal debt in the United States is approaching levels not seen since the Second World War, with much of the surge coming after 2008. If there is a link between debt levels and default risk, it is not surprising that questions about default risk in the US government have risen to the surface.

2. *Pensions/social service commitments:* In addition to traditional debt obligations, governments also make commitments to their citizens to pay pensions and cover health care. Since these obligations also compete for the limited revenues that the government has, countries that have larger commitments on these

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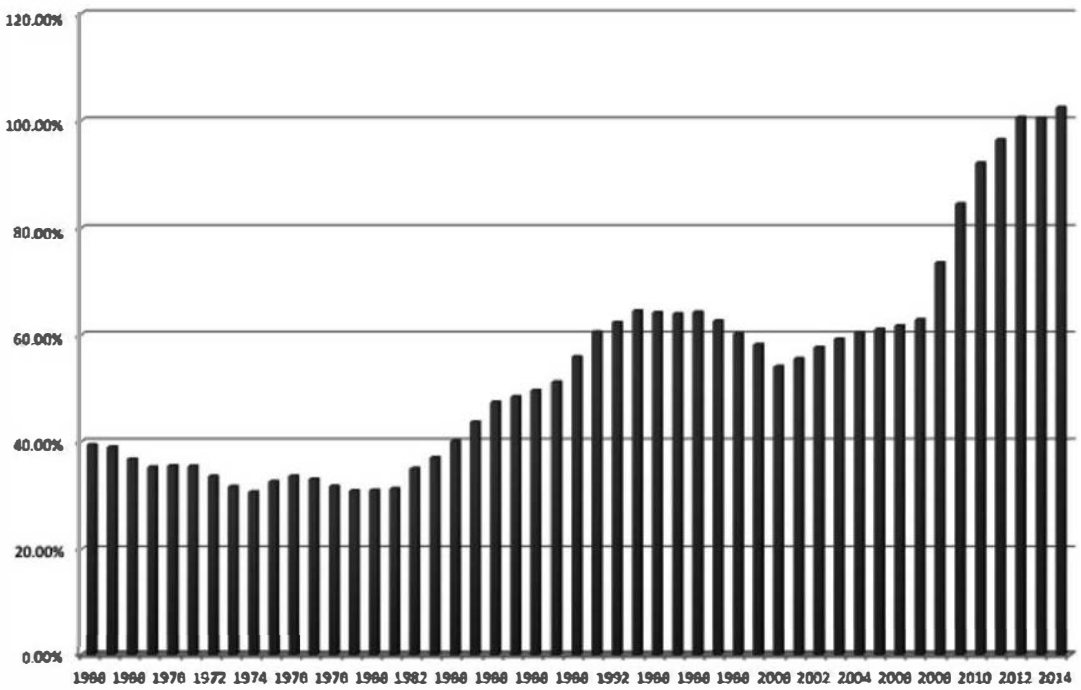

FIGURE 12-8 Debt as % of GDP—United States.

Source: FRED, Federal Reserve Bank of St. Louis.

counts should have higher default risk than countries that do not.[17]

3. *Revenues/inflows to government:* Government revenues usually come from tax receipts, which in turn are a function of both the tax code and the tax base. Holding all else constant, access to a larger tax base should increase potential tax revenues, which, in turn, can be used to meet debt obligations.
4. *Stability of revenues:* The essence of debt is that it gives rise to fixed obligations that have to be covered in both good and bad times. Countries with more stable revenue streams should therefore face less default risk, other things remaining equal, than countries with volatile revenues. But what is it that drives revenue stability? Since revenues come from taxing income and consumption in the nation's economy, countries with more diversified economies should have more stable tax revenues than countries that are dependent on one or a few sectors for their prosperity. To illustrate, Peru, with its reliance on copper and silver production and Jamaica, an economy dependent upon tourism, face more default risk than Brazil or India, which are larger, more diversified economies. The other factor that determines revenue stability is type of tax system used by the country. Generally, income tax based systems generate more volatile revenues than sales tax (or value added tax systems).
5. *Political risk:* Ultimately, the decision to default is as much a political decision as it is an economic decision. Given that sovereign default often exposes the political leadership to pressure, it is entirely possible that autocracies (where there is less worry about political backlash) are more likely to default than democracies. Since the alternative to default is printing more

[17] Since pension and health care costs increase as people age, countries with aging populations (and fewer working age people) face more default risk.

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money, the independence and power of the central bank will also affect assessments of default risk.

6. *Implicit backing from other entities:* When Greece, Portugal and Spain entered the European Union, investors, analysts and ratings agencies reduced their assessments of default risk in these countries. Implicitly, they were assuming that the stronger European Union countries—Germany, France and the Scandinavian countries—would step in to protect the weaker countries from defaulting. The danger, of course, is that the backing is implicit and not explicit, and lenders may very well find themselves disappointed by lack of backing, and no legal recourse. In summary, a full assessment of default risk in a sovereign entity requires the assessor to go beyond the numbers and understand how the country's economy works, the strength of its tax system and the trustworthiness of its governing institutions.

Sovereign Ratings

Since few of us have the resources or the time to dedicate to understanding small and unfamiliar countries, it is no surprise that third parties have stepped into the breach, with their assessments of sovereign default risk. Of these third party assessors, bond ratings agencies came in with the biggest advantages:

1. They have been assessing default risk in corporations for a hundred years or more and presumably can transfer some of their skills to assessing sovereign risk.
2. Bond investors who are familiar with the ratings measures, from investing in corporate bonds, find it easy to extend their use to assessing sovereign bonds. Thus, a AAA rated country is viewed as close to riskless whereas a C rated country is very risky.

In spite of these advantages, there are critiques that have been leveled at ratings agencies by both the sovereigns they rate and the investors that use these ratings. In this section, we will begin by looking at how ratings agencies come up with sovereign ratings (and change them) and then evaluate how well sovereign ratings measure default risk.

The Evolution of Sovereign Ratings

Moody's, Standard and Poor's and Fitch's have been rating corporate bond offerings since the early part of the twentieth century. Moody's has been rating corporate bonds since 1919 and started rating government bonds in the 1920s, when that market was an active one. By 1929, Moody's provided ratings for almost fifty central governments. With the great depression and the Second World War, investments in government bonds abated and with it, the interest in government bond ratings. In the 1970s, the business picked up again slowly. As recently as the early 1980s, only about fifteen, more mature governments had ratings, with most of them commanding the highest level (Aaa). The decade from 1985 to 1994 added 35 companies to the sovereign rating list, with many of them having speculative or lower ratings. Table 12-7 summarizes the growth of sovereign ratings from 1975 to 1994.

Since 1994, the number of countries with sovereign ratings has surged, just as the market for sovereign bonds has expanded. In 2015, Moody's, S&P and Fitch had ratings available for more than a hundred countries apiece.

In addition to more countries being rated, the ratings themselves have become richer. Moody's and S&P now provide two ratings for each country—a local currency rating (for domestic currency debt/ bonds) and a foreign currency rating (for government borrowings in a foreign currency). As an illustration, Table 12-8 summarizes the local and foreign currency ratings, from Moody's, for Latin American countries in July 2015.

For Ecuador and Panama, there is only a foreign currency rating, and the outlook on each country provides Moody's views on potential ratings changes, with negative (NEG) reflecting at least the possibility of a ratings downgrade. For the most part, local currency ratings are at least as high or higher than the foreign currency rating, for the obvious reason that governments have more power to print more of their own currency. There are, however, notable exceptions, where the local currency rating is lower than the foreign

TABLE 12-7 Sovereign Ratings—1975–1994

Year	Number of Newly Rated Sovereigns	Median Rating
Pre-1975	3	AAA/Aaa
1975–79	9	AAA/Aaa
1980–84	3	AAA/Aaa
1985–1989	19	A/A2
1990–94	15	BBB-/Baa3

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TABLE 12-8 Local and Foreign Currency Ratings—Latin America in July 2015

Sovereigns	Foreign Currency		Local Currency	
	Rating	Outlook	Rating	Outlook
Argentina	Caa1	NEG	Caa1	NEG
Belize	Caa2	STA	Caa2	STA
Bolivia	Ba3	STA	Ba3	STA
Brazil	Baa2	NEG	Baa2	NEG
Colombia	Baa2	STA	Baa2	STA
Costa Rica	Ba1	STA	Ba1	STA
Ecuador	B3	STA	—	—
El Salvador	Ba3	STA	—	—
Guatemala	Ba1	NEG	Ba1	NEG
Honduras	B3	POS	B3	POS
Mexico	A3	STA	A3	STA
Nicaragua	B3	STA	B3	STA
Panama	Baa2	STA	—	—
Paraguay	Ba1	STA	Ba1	STA
Peru	A3	STA	A3	STA
Uruguay	Baa2	STA	Baa2	STA
Venezuela	Caa3	STA	Caa3	STA

Source: Moody's.

currency rating. In March 2010, for instance, India was assigned a local currency rating of Ba2 and a foreign currency rating of Baa3.

Do the ratings agencies agree on sovereign risk? For the most part, there is consensus in the ratings, but there can be significant differences on individual countries. These differences can come from very different assessments of political and economic risk in these countries by the ratings teams at the different agencies.

Do sovereign ratings change over time? Yes, but far less than corporate ratings do. The best measure of sovereign ratings changes is a ratings transition matrix, which captures the changes that occur across ratings classes. Using Fitch ratings to illustrate our point, Table 12-9 summarizes the annual probability of ratings transitions, by rating, from 1995 to 2008.

This table provides evidence on how little sovereign ratings change on an annual basis, especially for higher rated countries. A AAA rated sovereign has a 99.42% chance of remaining AAA rated the next year; a BBB rated sovereign has an 8.11% chance of being upgraded, an 87.84% chance of remaining unchanged and a 4.06% chance of being downgraded. The ratings transition tables at Moody's and S&P tell the same story of ratings stickiness. As we will see later in this paper, one of the critiques of sovereign ratings is that they do not change quickly enough to alert investors to imminent danger. There is some evidence in S&P's latest update on transition probabilities that sovereign

TABLE 12-9 Annual Ratings Transitions—1995 to 2008

Fitch Sovereign Transition Rates across the Major Rating Categories: 1995–2008

(%, Average Annual)

	AAA	AA	A	BBB	BB	B	CCC to C	D	Total
AAA	99.42	0.58	0.00	0.00	0.00	0.00	0.00	0.00	100.00
AA	4.12	94.12	1.18	0.00	0.00	0.59	0.00	0.00	100.00
A	0.00	3.55	92.91	3.55	0.00	0.00	0.00	0.00	100.00
BBB	0.00	0.00	8.11	87.84	3.38	0.68	0.00	0.00	100.00
BB	0.00	0.00	0.00	9.04	83.51	5.85	0.00	1.60	100.00
B	0.00	0.00	0.00	0.00	12.12	84.09	3.03	0.76	100.00
CCC to C	0.00	0.00	0.00	0.00	0.00	23.08	53.85	23.08	100.00

Source: Fitch.

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ratings have become more volatile, with BBB rated countries showing only a 57.1% likelihood of staying with the same rating from 2010–2012.[18]

As the number of rated countries around the globe increases, we are opening a window on how ratings agencies assess risk at the broader regional level. One of the criticisms that rated countries have mounted against the ratings agencies is that they have regional biases, leading them to underrate entire regions of the world (Latin America and Africa). The defense that ratings agencies would offer is that past default history is a good predictor of future default and that Latin America has a great deal of bad history to overcome.

What Goes Into a Sovereign Rating?

The ratings agencies started with a template that they developed and fine tuned with corporations and have modified it to estimate sovereign ratings. While each agency has its own system for estimating sovereign ratings, the processes share a great deal in common.

- *Ratings measure:* A sovereign rating is focused on the credit worthiness of the sovereign to private creditors (bondholders and private banks) and not to official creditors (which may include the World Bank, the IMF and other entities). Ratings agencies also vary on whether their rating captures only the probability of default or also incorporates the expected severity, if it does occur. S&P's ratings are designed to capture the probability that default will occur and not necessarily the severity of the default, whereas Moody's focus on both the probability of default and severity (captured in the expected recovery rate). Default at all of the agencies is defined as either a failure to pay interest or principal on a debt instrument on the due date (outright default) or a rescheduling, exchange or other restructuring of the debt (restructuring default).
- *Determinants of ratings:* In a publication that explains its process for sovereign ratings, Standard and Poor's lists out the variables that it considers when rating a country. These variables encompass both political, economic and institutional variables and are summarized in Table 12-10.

[18] Standard & Poor's, 2013, Default Study: Sovereign Defaults And Rating Transition Data, 2012 Update.

While Moody's and Fitch have their own set of variables that they use to estimate sovereign ratings, they parallel S&P in their focus on economic, political and institutional detail.

- *Rating process:* The analyst with primary responsibility for the sovereign rating prepares a ratings recommendation with a draft report, which is then assessed by a ratings committee composed of 5–10 analysts, who debate each analytical category and vote on a score. Following closing arguments, the ratings are decided by a vote of the committee.
- *Local versus foreign currency ratings:* As we noted earlier, the ratings agencies usually assign two ratings for each sovereign—a local currency rating and a foreign currency rating. There are two approaches used by ratings agencies to differentiate between these ratings. In the first, called the notch-up approach, the foreign currency rating is viewed as the primary measure of sovereign credit risk and the local currency rating is notched up, based upon domestic debt market factors. In the notch down approach, it is the local currency rating that is the anchor, with the foreign currency rating notched down, reflecting foreign exchange constraints. The differential between foreign and local currency ratings is primarily a function of monetary policy independence. Countries that maintain floating rate exchange regimes and fund borrowing from deep domestic markets will have the largest differences between local and foreign currency ratings, whereas countries that have given up monetary policy independence, either through dollarization or joining a monetary union, will see local currency ratings converge on foreign currency ratings.
- *Ratings review and updates:* Sovereign ratings are reviewed and updated by the ratings agencies and these reviews can be both at regular periods and also triggered by news items. Thus, news of a political coup or an economic disaster can lead to a ratings review not just for the country in question but for surrounding countries (that may face a contagion effect).

Do Sovereign Ratings Measure Default Risk?

The sales pitch from ratings agencies for sovereign ratings is that they are effective measures of default risk in bonds (or loans) issued by that sovereign. But do they work as advertised? Each of the ratings agencies goes to great pains to argue that notwithstanding errors on some countries, there is a high correlation between sovereign ratings

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TABLE 12-10 Factors Considered While Assigning Sovereign Ratings

Sovereign Ratings Methodology Profile
Political risk • Stability and legitimacy of political institutions • Popular participation in political processes • Orderliness of leadership succession • Transparency in economic policy decisions and objectives • Public security • Geopolitical risk
Economic structure • Prosperity, diversity, and degree to which economy is market oriented • Income disparities • Effectiveness of financial sector in intermediating funds; availability of credit • Competitiveness and profitability of nonfinancial private sector • Efficiency of public sector • Protectionism and other nonmarket influences • Labor flexibility
Economic growth prospects • Size and composition of savings and investment • Rate and pattern of economic growth
Fiscal flexibility • General government revenue, expenditure, and surplus/deficit trends • Compatibility of fiscal stance with monetary and external factors • Revenue-raising flexibility and efficiency • Expenditure effectiveness and pressures • Timeliness, coverage, and transparency in reporting • Pension obligations
General Government Debt Burden • General government gross and net (of liquid assets) debt • Share of revenue devoted to interest • Currency composition and maturity profile • Depth and breadth of local capital markets
Offshore and contingent liabilities • Size and health of NFPEs • Robustness of financial sector
Monetary flexibility • Price behavior in economic cycles • Money and credit expansion • Compatibility of exchange-rate regime and monetary goals • Institutional factors, such as central bank independence • Range and efficiency of monetary policy tools, particularly in light of the fiscal stance and capital market characteristics • Indexation and dollarization
External liquidity • Impact of fiscal and monetary policies on external accounts • Structure of the current account • Composition of capital flows • Reserve adequacy
External debt burden • Gross and net external debt, including nonresident deposits and structured debt • Maturity profile, currency composition, and sensitivity to interest rate changes • Access to concessional funding • Debt service burden
NFPEs—Nonfinancial public sector enterprises.

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and sovereign defaults. In Table 12-11, we summarize S&P's estimates of cumulative default rates for bonds in each ratings class from 1975 to 2012.

Fitch and Moody's also report default rates by ratings classes and in summary, all of the ratings agencies seem to have, on average, delivered the goods. Sovereign bonds with investment grade ratings have defaulted far less frequently than sovereign bonds with speculative ratings.

Notwithstanding this overall track record of success, ratings agencies have been criticized for failing investors on the following counts:

1. *Ratings are upward biased:* Ratings agencies have been accused of being far too optimistic in their assessments of both corporate and sovereign ratings. While the conflict of interest of having issuers pay for the rating is offered as the rationale for the upward bias in corporate ratings, that argument does not hold up when it comes to sovereign ratings, since the issuing government does not pay ratings agencies.
2. *There is herd behavior:* When one ratings agency lowers or raises a sovereign rating, other ratings agencies seem to follow suit. This herd behavior reduces the value of having three separate ratings agencies, since their assessments of sovereign risk are no longer independent.
3. *Too little, too late:* To price sovereign bonds (or set interest rates on sovereign loans), investors (banks) need assessments of default risk that are updated and timely. It has long been argued that ratings agencies

TABLE 12-11 S&P Sovereign Foreign Currency Ratings and Default Probabilities—1975 to 2012

	Time Horizon														
Rating	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
AAA	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AA+	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AA	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AA-	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
A+	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.9%	3.7%	3.7%	3.7%	3.7%	3.7%	3.7%	3.7%
A	0.0%	0.0%	0.0%	0.8%	1.8%	3.0%	4.3%	4.6%	5.2%	6.9%	8.6%	8.6%	8.6%	8.6%	8.6%
A-	0.0%	0.0%	0.9%	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	1.3%	5.1%	6.2%	6.2%	6.2%
BBB+	0.0%	0.3%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%
BBB	0.0%	0.7%	2.0%	3.4%	3.4%	3.4%	3.4%	3.4%	3.4%	3.4%	3.4%	3.4%	6.3%	7.4%	7.4%
BBB-	0.0%	0.8%	1.7%	2.8%	5.0%	7.2%	7.9%	7.9%	7.9%	7.9%	7.9%	7.9%	7.9%	9.6%	12.6%
BB+	0.1%	1.3%	1.3%	1.3%	1.3%	1.4%	2.9%	4.6%	6.4%	6.9%	6.9%	6.9%	6.9%	6.9%	6.9%
BB	0.0%	0.9%	1.9%	2.9%	3.6%	4.6%	5.0%	5.0%	5.0%	5.0%	5.5%	8.3%	11.7%	13.6%	13.6%
BB-	1.7%	4.0%	6.1%	6.6%	9.8%	13.0%	16.5%	19.3%	19.9%	19.9%	21.0%	21.0%	21.0%	21.0%	21.0%
B+	0.6%	1.7%	3.4%	6.6%	8.0%	10.9%	15.4%	20.6%	22.4%	25.3%	26.9%	26.9%	26.9%	30.8%	39.8%
B	2.4%	6.1%	9.8%	14.3%	19.4%	23.1%	25.6%	28.2%	31.6%	35.1%	35.8%	35.8%	35.8%	35.8%	35.8%
B-	7.4%	11.7%	14.6%	17.5%	19.7%	21.3%	23.7%	24.8%	25.9%	25.9%	25.9%	25.9%	25.9%	25.9%	NA
CCC+	19.6%	24.7%	33.1%	38.5%	50.7%	68.7%	82.1%	91.0%	91.0%	91.0%	NA	NA	NA	NA	NA
CCC	39.6%	66.0%	66.0%	66.0%	66.0%	66.0%	66.0%	66.0%	NA	NA	NA	NA	NA	NA	NA
CCC-	77.8%	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
CC	100.0%	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
Investment grade	0.0%	0.1%	0.4%	0.6%	0.9%	1.2%	1.4%	1.5%	1.6%	1.7%	1.9%	2.0%	2.2%	2.4%	2.5%
Speculative grade	2.7%	5.1%	7.1%	9.1%	11.3%	13.6%	16.1%	18.4%	19.7%	20.6%	21.2%	21.8%	22.6%	23.5%	24.8%
All rated	0.9%	1.8%	2.5%	3.3%	4.2%	5.0%	5.9%	6.5%	6.9%	7.3%	7.5%	7.7%	8.0%	8.3%	8.6%

Source: Standard and Poor's.

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take too long to change ratings, and that these changes happen too late to protect investors from a crisis.

4. *Vicious cycle:* Once a market is in crisis, there is the perception that ratings agencies sometimes overreact and lower ratings too much, thus creating a feedback effect that makes the crisis worse.
5. *Ratings failures:* At the other end of the spectrum, it can be argued that when a ratings agency changes the rating for a sovereign multiple times in a short time period, it is admitting to failure in its initial rating assessment. In a paper on the topic, Bhatia (2004) looks at sovereigns where S&P and Moody changed ratings multiple times during the course of a year between 1997 and 2002. His findings are reproduced in Table 12-12.

Why do ratings agencies sometimes fail? Bhatia provides some possible answers:

1. *Information problems:* The data that the agencies use to rate sovereigns generally come from the governments. Not only are there wide variations in the quantity and quality of information across governments, but there is also the potential for governments holding back bad news and revealing only good news. This, in turn, may explain the upward bias in sovereign ratings.

TABLE 12-12 Ratings Failures

Failure	Failed Rating (& date)	Corrected Rating (& date)	Notches Adjusted	Key Factor
S&P				
1997: Thailand	A (Sept. 3, 1997)	BBB− (Jan. 8, 1998)	4 ↓	Evaporation of reserves
1997: Indonesia	BBB (Oct. 10, 1997)	B− (Mar. 11, 1998)	7 ↓	Collapse of asset quality
1997: Korea	AA− (Oct. 24, 1997)	B+ (Dec. 22, 1997)	10 ↓	Evaporation of reserves
1997: Malaysia	A+ (Dec. 23, 1997)	BBB− (Sept. 15, 1998)	5 ↓	Collapse of asset quality
1998: Korea	B+ (Feb. 18, 1998)	BBB− (Jan. 25, 1999)	4 ↑	Reserves replenishment
1998: Romania	BB− (May 20, 1998)	B− (Oct. 19, 1998)	3 ↓	Evaporation of reserves
1998: Russia	BB− (June 9, 1998)	B− (Aug. 13, 1998)	3 ↓	Evaporation of reserves
2000: Argentina	BB (Nov. 14, 2000)	B− (July 12, 2001)	4 ↓	Fiscal slippage
2002: Uruguay	BBB− (Feb. 14, 2002)	B (July 26, 2002)	5 ↓	Evaporation of reserves
Moody's				
1997: Thailand	A2 (Apr. 8, 1997)	Ba1 (Dec. 21, 1997)	5 ↓	Evaporation of reserves
1997: Korea	A1 (Nov. 27, 1997)	Ba1 (Dec. 21, 1997)	6 ↓	Evaporation of reserves
1997: Indonesia	Baa3 (Dec. 21, 1997)	B3 (Mar. 20, 1998)	6 ↓	Collapse of asset quality
1997: Malaysia	A1 (Dec. 21, 1997)	Baa2 (Sept. 14, 1998)	4 ↓	Collapse of asset quality
1998: Russia	Ba2 (Mar. 11, 1998)	B3 (Aug. 21, 1998)	4 ↓	Evaporation of reserves
1998: Moldova	Ba2 (July 14, 1998)	B2 (July 14, 1998)	3 ↓	Evaporation of reserves
1998: Romania	Ba3 (Sept. 14, 1998)	B3 (Nov. 6, 1998)	3 ↓	Evaporation of reserves
2002: Uruguay	Baa3 (May 3, 2002)	B3 (July 31, 2002)	6 ↓	Evaporation of reserves

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2. *Limited resources:* To the extent that the sovereign rating business generates only limited revenues for the agencies and it is required to at least break even in terms of costs, the agencies cannot afford to hire too many analysts. These analysts are then spread thin globally, being asked to assess the ratings of dozens of low-profile countries. In 2003, it was estimated that each analyst at the agencies was called up to rate between four and five sovereign governments. It has been argued by some that it is this overload that leads analysts to use common information (rather than do their own research) and to herd behavior.
3. *Revenue bias:* Since ratings agencies offer sovereign ratings gratis to most users, the revenues from ratings either have to come from the issuers or from other business that stems from the sovereign ratings business. When it comes from the issuing sovereigns or sub-sovereigns, it can be argued that agencies will hold back on assigning harsh ratings. In particular, ratings agencies generate significant revenues from rating sub-sovereign issuers. Thus, a sovereign ratings downgrade will be followed by a series of sub-sovereign ratings downgrades. Indirectly, therefore, these sub-sovereign entities will fight a sovereign downgrade, again explaining the upward bias in ratings.
4. *Other incentive problems:* While it is possible that some of the analysts who work for S&P and Moody's may seek work with the governments that they rate, it is uncommon and thus should not pose a problem with conflict of interest. However, the ratings agencies have created other businesses, including market indices, ratings evaluation services and risk management services, which may be lucrative enough to influence sovereign ratings.

Market Interest Rates

The growth of the sovereign ratings business reflected the growth in sovereign bonds in the 1980s and 1990s. As more countries have shifted from bank loans to bonds, the market prices commanded by these bonds (and the resulting interest rates) have yielded an alternate measure of sovereign default risk, continuously updated in real time. In this section, we will examine the information in sovereign bond markets that can be used to estimate sovereign default risk.

The Sovereign Default Spread

When a government issues bonds, denominated in a foreign currency, the interest rate on the bond can be compared to a rate on a riskless investment in that currency to get a market measure of the default spread for that country. To illustrate, the Brazilian government had a 10-year dollar denominated bond outstanding in July 2015, with a market interest rate of 4.5%. At the same time, the 10-year US treasury bond rate was 2.47%. If we assume that the US treasury is default free, the difference between the two rates can be attributed (2.03%) can be viewed as the market's assessment of the default spread for Brazil. Table 12-13 summarizes interest rates and default spreads for Latin American countries in July 2015, using dollar denominated bonds issued by these countries, as well as the sovereign foreign currency ratings (from Moody's) at the time.

While there is a strong correlation between sovereign ratings and market default spreads, there are advantages to using the default spreads. The first is that the market differentiation for risk is more granular than the ratings agencies; thus, Peru and Brazil have the same Moody's rating (Baa2) but the market sees more default risk in Brazil than in Peru. The second is that the market-based spreads are more dynamic than ratings, with changes occurring in real time. In Figure 12-9, we graph the shifts in the default spreads for Brazil and Venezuela between 2006 and the end of 2009.

In December 2005, the default spreads for Brazil and Venezuela were similar; the Brazilian default spread was 3.18%

TABLE 12-13 Default Spreads on Dollar Denominated Bonds—Latin America

Country	Moody's Rating	Interest Rate on $ Denominated Bond (10 Year)	10-year US Treasury Bond Rate	Default Spread
Mexico	Baa1	3.92%	2.47%	1.45%
Brazil	Baa2	4.50%	2.47%	2.03%
Colombia	Baa3	4.05%	2.47%	1.58%
Peru	Baa2	3.93%	2.47%	1.46%

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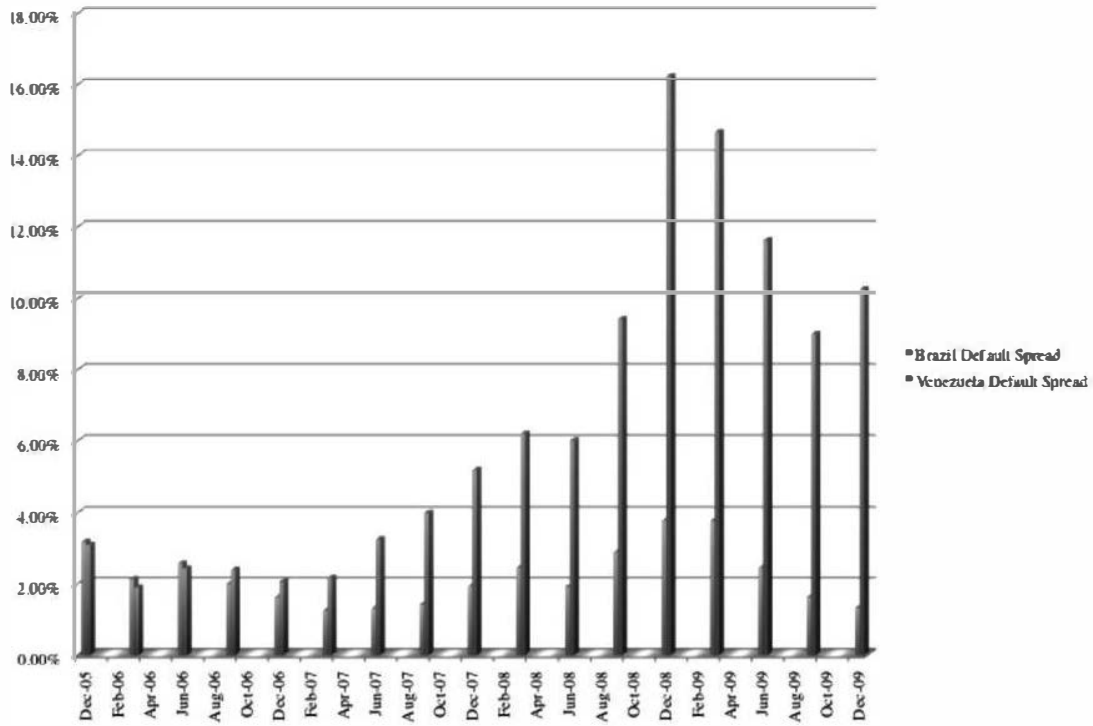

FIGURE 12-9 Default spreads for $ denominated bonds: Brazil vs Venezuela.

and the Venezuelan default spread was 3.09%. Between 2006 and 2009, the spreads diverged, with Brazilian default spreads dropping to 1.32% by December 2009 and Venezuelan default spreads widening to 10.26%.

To use market-based default spreads as a measure of country default risk, there has to be a default free security in the currency in which the bonds are issued. Local currency bonds issued by governments cannot be compared to each other, since the differences in rates can be due to differences in expected inflation. Even with dollar-denominated bonds, it is only the assumption that the US Treasury bond rate is default free that allows us to back out default spreads from the interest rates.

The Spread as a Predictor of Default

Are market default spreads better predictors of default risk than ratings? One advantage that market spreads have over ratings is that they can adjust quickly to information. As a consequence, they provide earlier signals of imminent danger (and default) than ratings agencies do. However, market-based default measures carry their own costs. They tend to be far more volatile than ratings and can be affected by variables that have nothing to do with default. Liquidity and investor demand can sometimes cause shifts in spreads that have little or nothing to do with default risk.

Studies of the efficacy of default spreads as measures of country default risk reveal some consensus. First, default spreads are for the most part correlated with both sovereign ratings and ultimate default risk. In other words, sovereign bonds with low ratings tend trade at much higher interest rates and also are more likely to default. Second, the sovereign bond market leads ratings agencies, with default spreads usually climbing ahead of a rating downgrade and dropping before an upgrade. Third, notwithstanding the lead-lag relationship, a change in sovereign ratings is still an informational event that creates a price impact at the time that it occurs. In summary, it would be a mistake to conclude that sovereign ratings are useless, since sovereign bond markets seems to draw on ratings

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(and changes in these ratings) when pricing bonds, just as ratings agencies draw on market data to make changes in ratings.

Credit Default Swaps

The last decade has seen the evolution of the Credit Default Swap (CDS) market, where investors try to put a price on the default risk in an entity and trade at that price. In conjunction with CDS contracts on companies, we have seen the development of a market for sovereign CDS contracts. The prices of these contracts represent market assessments of default risk in countries, updated constantly.

How Does a CDS Work?

The CDS market allows investors to buy protection against default in a security. The buyer of a CDS on a specific bond makes payments of the "spread" each period to the seller of the CDS; the payment is specified as a percentage (spread) of the notional or face value of the bond being insured. In return, the seller agrees to make the buyer whole if the issuer of the bond (reference entity) fails to pay, restructures or goes bankrupt (credit event), by doing one of the following:

1. *Physical settlement:* The buyer of the CDS can deliver the "defaulted" bond to the seller and get par value for the bond.
2. *Cash settlement:* The seller of the CDS can pay the buyer the difference between par value of the defaulted bond and the market price, which will reflects the expected recovery from the issuer.

In effect, the buyer of the CDS is protected from losses arising from credit events over the life of the CDS.

Assume, for instance, that you own 5-year Colombian government bonds, with a par value of $10 million, and that you are worried about default over the life of the bond. Assume also that the price of a 5-year CDS on the Colombian government is 250 basis points (2.5%). If you buy the CDS, you will be obligated to pay $250,000 each year for the next 5 years and the seller of the CDS would receive this payment. If the Colombian government fails to fulfill its obligations on the bond or restructures the bond any time over the next 5 years, the seller of the CDS can fulfill his obligations by either buying the bonds from you for $10 million or by paying you the difference between $10 million and the market price of the bond after the credit event happens.

There are two points worth emphasizing about a CDS that may undercut the protection against default that it is designed to offer. The first is that the protection against failure is triggered by a credit event; if there is no credit event, and the market price of the bond collapses, you as the buyer will not be compensated. The second is that the guarantee is only as good as the credit standing of the seller of the CDS. If the seller defaults, the insurance guarantee will fail. On the other side of the transaction, the buyer may default on the spread payments that he has contractually agreed to make.

Market Background

J.P. Morgan is credited with creating the first CDS, when it extended a $4.8 billion credit line to Exxon and then sold the credit risk in the transaction to investors. Over the last decade and a half, the CDS market has surged in size. By the end of 2007, the notional value of the securities on which CDS had been sold amounted to more than $60 trillion, though the market crisis caused a pullback to about $39 trillion by December 2008.

You can categorize the CDS market based upon the reference entity, i.e., the issuer of the bond underlying the CDS. While our focus is on sovereign CDS, they represent a small proportion of the overall market. Corporate CDS represent the bulk of the market, followed by bank CDS and then sovereign CDS. While the notional value of the securities underlying the CDS market is huge, the market itself is a fair narrow one, insofar that a few investors account for the bulk of the trading in the market. While the market was initially dominated by banks buying protection against default risk, the market has attracted investors, portfolio managers and speculators, but the number of players in the market remains small, especially given the size of the market. The narrowness of the market does make it vulnerable, since the failure of one or more of the big players can throw the market into tumult and cause spreads to shift dramatically. The failure of Lehman Brothers in 2008, during the banking crisis, threw the CDS market into turmoil for several weeks.

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CDS and Default Risk

If we assume away counterparty risk and liquidity, the prices that investors set for credit default swaps should provide us with updated measures of default risk in the reference entity. In contrast to ratings, that get updated infrequently, CDS prices should reflect adjust to reflect current information on default risk.

To illustrate this point, let us consider the evolution of sovereign risk in Greece during 2009 and 2010. In Figure 12-10, we graph out the CDS spreads for Greece on a month-by-month basis from 2006 to 2010 and ratings actions taken by one agency (Fitch) during that period.

While ratings stayed stagnant for the bulk of the period, before moving late in 2009 and 2010, when Greece was downgraded, the CDS spread and default spreads for Greece changed each month. The changes in both market-based measures reflect market reassessments of default risk in Greece, using updated information.

While it is easy to show that CDS spreads are more timely and dynamic than sovereign ratings and that they reflect fundamental changes in the issuing entities, the fundamental question remains: Are changes in CDS spreads better predictors of future default risk than sovereign ratings or default spreads? The findings are significant. First, changes in CDS spreads lead changes in the sovereign bond yields and in sovereign ratings.[19] Second, it is not clear that the CDS market is quicker or better at assessing default risks than the government bond market, from which we can extract default spreads. Third, there seems to be clustering in the CDS market, where CDS prices across groups of companies move together in the same

[19] Ismailescu, I., 2007, The Reaction of Emerging Markets Credit Default Swap Spreads to Sovereign Credit Rating Changes and Country Fundamentals, Working Paper, Pace University. This study finds that CDS prices provide more advance warning of ratings downgrades.

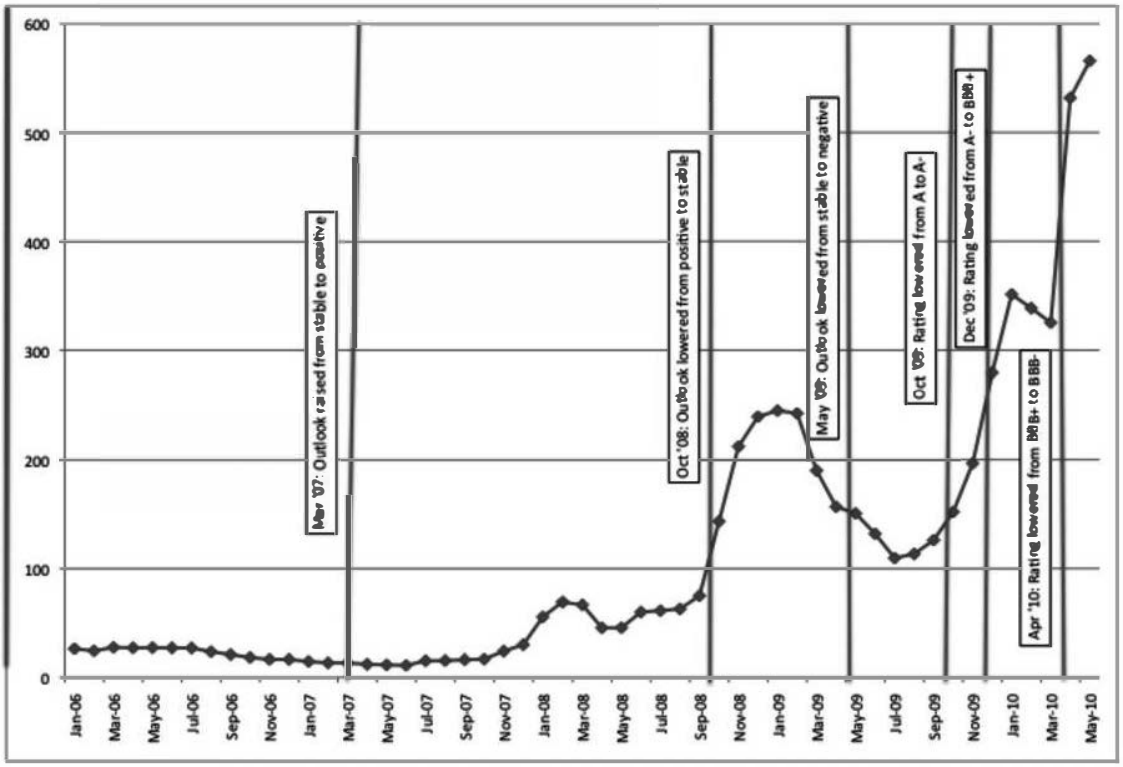

FIGURE 12-10 Greece CDS prices and ratings.

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TABLE 12-14 Clusters of Emerging Markets: CDS Market

	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Cluster 6
Countries in Cluster	Brazil Bulgaria Mexico Poland Russia Slovak Turkey	Chile China Korea Thailand Venezuela	Croatia Hungary Malaysia Romania S. Africa	Colombia Panama Peru	Pakistan Philippines Ukraine	Israel Qatar
Ave. Corr. Internal	0.516	0.596	0.402	0.588	0.517	0.466
Ave. Corr. External	0.210	0.220	0.278	0.245	0.218	0.102
Ave. CDS Spread	287.30	114.83	96.10	243.63	262.37	30.12

direction. A study suggests six clusters of emerging market countries, captured in Table 12-14:

The correlation within the cluster and outside the cluster, are provided towards the bottom. Thus, the correlation between countries in cluster 1 is 0.516, whereas the correlation between countries in cluster 1 and the rest of the market is only 0.210.

There are inherent limitations with using CDS prices as predictors of country default risk. The first is that the exposure to counterparty and liquidity risk, endemic to the CDS market, can cause changes in CDS prices that have little to do with default risk. Thus, a significant portion of the surge in CDS prices in the last quarter of 2008 can be traced to the failure of Lehman and the subsequent surge in concerns about counterparty risk. The second and related problem is that the narrowness of the CDS market can make individual CDS susceptible to illiquidity problems, with a concurrent effect on prices. Notwithstanding these limitations, it is undeniable that changes in CDS prices supply important information about shifts in default risk in entities. In summary, the evidence, at least as of now, is that changes in CDS prices provide information, albeit noisy, of changes in default risk. However, there is little to indicate that it is superior to market default spreads (obtained from government bonds) in assessing this risk.

Sovereign Risk in the CDS Market

Notwithstanding both the limitations of the market and the criticism that has been directed at it, the CDS market continues to grow. In July 2015, there were 61 countries with sovereign CDS trading on them. Figure 12-11 captures the differences in CDS spreads across the globe (for the countries for which it is available) in July 2015.

Not surprisingly, much of Africa remains uncovered, there are large swaths in Latin America with high default risk, Asia has seen a fairly dramatic drop-off in risk largely because of the rise of China and Southern Europe is becoming a hotbed for default risk, at least according to the CDS market.

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FIGURE 12-11 CDS spreads by country—July 2015.

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External and Internal Ratings

13

Learning Objectives

After completing this reading you should be able to:

- Describe external rating scales, the rating process, and the link between ratings and default.
- Describe the impact of time horizon, economic cycle, industry, and geography on external ratings.
- Explain the potential impact of ratings changes on bond and stock prices.
- Compare external and internal ratings approaches.
- Explain and compare the through-the-cycle and at-the-point-in-time internal ratings approaches.
- Describe a ratings transition matrix and explain its uses.
- Describe the process for and issues with building, calibrating, and backtesting an internal rating system.
- Identify and describe the biases that may affect a rating system.

Excerpt is Chapter 2 of Measuring and Managing Credit Risk, *by Arnaud de Servigny and Olivier Renault.*

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In order to assess default risk, it is customary to oppose qualitative tools and quantitative approaches. Ratings are among the best-known forms of qualitative measurements. In this chapter we review rating methodologies and assess their strengths and weaknesses.

Rating agencies fulfill a mission of delegated monitoring for the benefit of investors active in bond markets. The objective of rating agencies is to provide an independent credit opinion based on a set of precise criteria. Their contribution is reflected through rating grades that convey information about the credit quality of a borrower. Rating agencies strive to make their grades consistent across regions, industries, and time. Over the past 20 years, rating agencies have played an increasingly important role in financial markets, and their ratings have had a greater impact on corporate security prices.

It is important to stress that delegated monitoring is also a mission of the banking firm. A large part of the competitive advantage of banks lies in their ability to assess risks in a timely manner and accurately, based on relevant information. Ideally banks would like to assign analysts to the monitoring of each of their counterparts. Indeed, who better than a senior industry analyst is able to capture the dynamics of a company's creditworthiness, based on a mix of criteria: financial ratios, business factors, strategic performance, industrial market cyclicality, changes in competitiveness, products innovation, etc.?

Assigning an analyst to every counterpart is, of course, not realistic for cost reasons. The cost of the time spent by an analyst gathering and processing the data may not be recouped (in terms of reduced default losses) for smaller loans. A bank will therefore have to rely on quantitative techniques for small and midsize enterprises (SMEs).

In this chapter we focus exclusively on borrower ratings and not on facility ratings. We first present the most significant elements regarding the rating methodology and criteria that external agencies use. Then we consider comments and criticisms about ratings and finally turn our attention to internal rating systems.

RATINGS AND EXTERNAL AGENCIES

The Role of Rating Agencies in the Financial Markets

A rating agency is an organization that provides analytical services. These services are based on independent, objective, credible, and transparent assessments. The agency's recognition depends on the investor's willingness to accept its judgment.

Credit Ratings

Rating Scales A credit rating represents the agency's opinion about the creditworthiness of an obligor with respect to a particular debt security or other financial obligation (*issue-specific credit ratings*). It also applies to an issuer's general creditworthiness (*issuer credit ratings*). There are generally two types of assessment corresponding to different financial instruments: long-term and short-term ones. We should stress that ratings from various agencies do not convey the same information. Standard & Poor's perceives its ratings primarily as an opinion on the likelihood of default of an issuer, whereas Moody's ratings tend to reflect the agency's opinion on the expected loss (probability of default times loss severity) on a facility.

Long-term issue-specific credit ratings and issuer ratings are divided into several categories, e.g., from AAA to D for Standard & Poor's. Short-term issue-specific ratings can use a different scale (e.g., from A-1 to D). Figure 13-1 shows Moody's and S&P's rating scales. Although these grades are not directly comparable as mentioned earlier, it is common to put them in parallel. The rated universe is broken down into two very broad categories: investment grade (IG) and non-investment-grade (NIG), or speculative, issuers. IG firms are relatively stable issuers with moderate default risk, while bonds issued in the NIG category, often called junk bonds, are much more likely to default.

The credit quality of firms is best for Aaa/AAA ratings and deteriorates as ratings go down the alphabet. The coarse grid AAA, AA, A, . . . CCC can be supplemented with pluses and minuses in order to provide a finer indication of risk.

The Rating Process A rating agency supplies a rating only if there is adequate information available to provide a credible credit opinion. This opinion relies on various analyses based on a defined analytical framework. The criteria according to which any assessment is provided are very strictly defined and constitute the intangible assets of rating agencies, accumulated over years of experience. Any change in criteria is typically discussed at a worldwide level.

For industrial companies, the analysis is commonly split between business reviews (firm competitiveness, quality

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of the management and of its policies, business fundamentals, regulatory actions, markets, operations, cost control, etc.) and quantitative analyses (financial ratios, etc.). The impact of these factors depends highly on the industry.

Figure 13-2 shows how various factors may impact differently on various industries. It also reports various business factors that impact on different sectors.

Following meetings with the management of the firm that is asking for a rating, the rating agency reviews qualitative as well as quantitative factors and compares the company's performance with that of its peers. (See the ratio medians per rating in Table 13-1.) After this review, a rating committee meeting is convened. The committee discusses the lead analyst's recommendation before voting on it.

The issuer is subsequently notified of the rating and the major considerations supporting it. A rating can be appealed prior to its publication if meaningful new or additional information is to be presented by the issuer. But there is no guarantee that a revision will be granted. When a rating is assigned, it is disseminated to the public through the news media.

All ratings are monitored on an ongoing basis. Any new qualitative or quantitative piece of information is under surveillance. Regular meetings with the issuer's management are organized. As a result of the surveillance process, the rating agency may decide to initiate a review (i.e., put the firm on credit watch) and change the current rating. When a rating is put on a credit watch list, a comprehensive analysis is undertaken. After the process, the rating change or affirmation is announced.

More recently the "outlook" concept has been introduced. It provides information about the rating trend. If, for instance, the outlook is positive, it means that there is some potential upside conditional to the realization of current assumptions regarding the company. On the flip side, a negative outlook suggests that the creditworthiness of the company follows a negative trend.

Description	Moody's	S&P	
Investment grade			
	Aaa	AAA	Maximum safety
	Aa	AA	
	A	A	
	Baa	BBB	
Speculative grade			
	Ba	BB	
	B	B	
	Caa	CCC	
			Worst credit quality

FIGURE 13-1 Moody's and S&P's rating scales.

Indicative Averages	Retail	Airlines	Property	Pharmaceuticals
Investment and speculative grade(%)	Investment grade: 82% Speculative grade: 18%	Investment grade: 24% Speculative grade: 76%	Investment grade: 90% Speculative grade: 10%	Investment grade: 78% Speculative grade: 22%
Business risk weight	*High*	*Low*	*High*	*High*
Financial risk weight	*Low*	*High*	*Low*	*Low*
Business qualitative factors	• Discretionary vs. nondiscretionary • Scale and geographic profile • Position on price, value, and service • Regulatory environment	• Market position (share capacity) • Utilization of capacity • Aircraft fleet (type, age) • Cost control (labor, fuel)	• Quality and location of the assets • Quality of the tenants • Lease structure • Country-specific criteria (laws, taxation, and market liquidity)	• R&D programs • Product portfolio • Patent expirations

FIGURE 13-2 Examples of various possible determinants of ratings.

A very important fact that the agencies persistently emphasize is that their ratings are mere opinions. They do not constitute any recommendation to purchase, sell, or hold any type of security. A rating in itself indeed says nothing about the price or relative value of specific securities. A CCC bond may well be underpriced while an AA security may be trading at an overvalued price, although the risk may be appropriately reflected by their respective ratings.

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TABLE 13-1 Financial Ratios per Rating (3-Year Medians for 1998–2000), U.S. Firms

	AAA	AA	A	BBB	BB	B	CCC
EBIT int. cov. (x)	21.4	10.1	6.1	3.7	2.1	0.8	0.1
EBITDA int. cov. (x)	26.5	12.9	9.1	5.3	3.4	1.8	1.3
Free oper. cash flow/ total debt (%)	84.2	25.2	15.0	8.5	2.6	(3.2)	(12.9)
Funds from oper./total debt (%)	128.8	55.4	43.2	30.8	18.8	7.8	1.6
Return on capital (%)	34.9	21.7	19.4	13.6	11.6	6.6	1.0
Operating income/sales (%)	27.0	22.1	18.6	15.4	15.9	11.9	11.9
Long-term debt/capital (%)	13.3	28.2	33.9	42.5	57.2	69.7	68.8
Total debt/capital (%)	22.9	37.7	42.5	48.2	62.6	74.8	87.7
Number of Companies	8	29	136	218	273	281	22

The Link between Ratings and Probabilities of Default Although a rating is meant to be forward-looking, it is not devised to pinpoint a precise probability of default, but rather to point to a broad risk bucket. Rating agencies publish on a regular basis tables reporting observed default rates per rating category, per year, per industry, and per region. These tables reflect the empirical average defaulting frequencies of firms per rating category within the rated universe. The primary goal of these statistics is to verify that better (worse) ratings are indeed associated with lower (higher) default rates. They show that ratings tend to have homogeneous default rates across industries, as illustrated in Table 13-2.

Figure 13-3 displays cumulative default rates in S&P's universe per rating category. There is a striking difference in default patterns between investment-grade and speculative-grade categories. The clear link between observed default rates and rating categories is the best support for claims by agencies that their grades are appropriate measures of creditworthiness.

TABLE 13-2 Average 1-Year Default Rates per Industry (in Percent)*

	Trans.	Util.	Tele.	Media	Insur.	High Tech	Chem.	Build.	Fin.	Ener.	Cons.	Auto.
AAA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A	0.00	0.11	0.00	0.00	0.09	0.00	0.00	0.42	0.00	0.00	0.00	0.00
BBB	0.00	0.14	0.00	0.27	0.67	0.73	0.19	0.64	0.32	0.22	0.17	0.29
BB	1.46	0.25	0.00	1.24	1.59	0.75	1.12	0.89	0.86	0.98	1.77	1.47
B	6.50	6.31	5.86	4.97	2.38	4.35	5.29	5.41	8.97	9.57	6.77	5.19
CCC	19.4	71.4	35.9	29.3	10.5	9.52	21.6	21.9	24.7	14.4	26.0	33.3

*Default rates for CCC bonds are based on a very small sample and may not be statistically robust.
Source: S&P CreditPro, 1981–2001.

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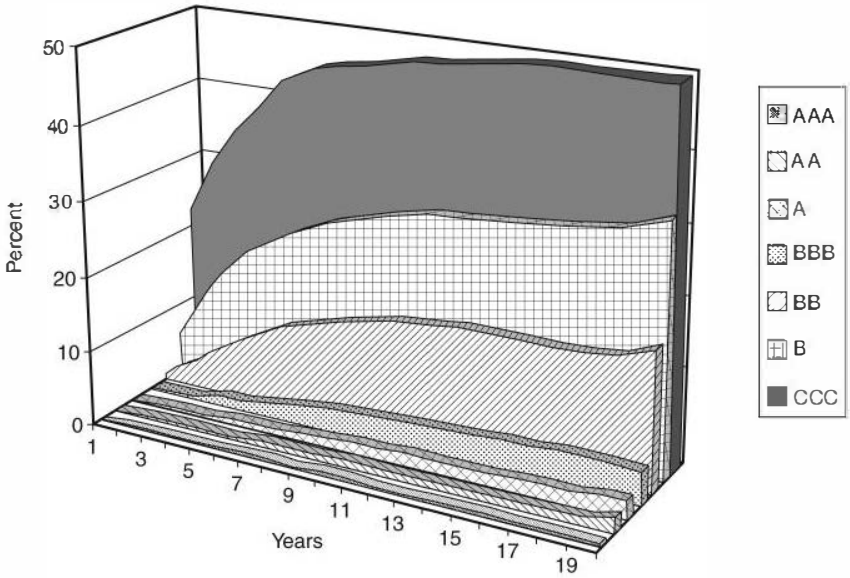

FIGURE 13-3 Cumulative default rates per rating category.

Source: S&P CreditPro, 1981–2001.

Rating agencies also calculate transition matrices, which are tables reporting probabilities of migrations from one rating category to another. They serve as indicators of the likely path of a given credit at a given horizon. Ex post information such as that provided in default tables or transition matrices does not guarantee to provide ex ante insights regarding future probabilities of default or migration. Both the stability over time of default probability in a given rating class and the stability of rating criteria used by agencies also contribute to making ratings forward-looking predictors of default.

COMMENTS AND CRITICISMS ABOUT EXTERNAL RATINGS

We have discussed above the general process that agencies use to determine their ratings, and we have described how these assessments give an appropriate broad ranking of firms in terms of creditworthiness. We now focus on three specific issues related to agency ratings. The first issue deals with the horizon of ratings and their dependence on the business cycle. The second is the consistency of transition matrices across time and regions with particular emphasis on the academic literature on the topic. Finally we consider the documented impact of rating changes on corporate security prices and firm value.

Ratings, Related Time Horizon, and Economic Cycles

Time Horizon for External Ratings

Rating agencies are very clear about the fact that issuer credit ratings or long-term issue ratings should not correspond to a mere snapshot of the present situation, but should focus on the long term. The agencies try to factor in the effect of cycles, though they recognize it is not always easy to anticipate them and though cycles are not fully repetitive in terms of duration, magnitude, and dynamics. The confluence of different types of cycles (macroeconomic and industrial, for example) is not unusual and contributes to making the task of rating analysts harder.

A careful assessment of business-risk sensitivity to cycles, for given industry categories, is an important part of the due diligence performed by analysts. Once this has been assessed, analysts try to mitigate the effect of cycles on ratings by incorporating the effect of an "average cycle" in their scenarios. This helps to make the final rating less volatile and less sensitive to expected changes in the business cycle. Rating agencies are therefore associated with "through-the-cycle" ratings.

Figure 13-4 shows how a through-the-cycle rating can filter out cycle effects: A through-the-cycle rating does not fluctuate much with temporary changes in microeconomic conditions (e.g., expected or likely changes in quarter-on-quarter sales) since they are already factored in the rating. However, once the analyst is convinced that a worsening of economic conditions both at the firm level and at the macro level is persistent, then the rating is downgraded on several occasions.

We stated earlier that ratings were broad indicators of probabilities of default (PD) and do not pinpoint a specific PD at a given horizon. This is illustrated in Figure 13-5. The figure shows how a persistent downturn in the economy, such as those observed in the early parts of the 1980s, 1990s and 2000s, significantly raises 1-year default rates within a given rating class. This emphasizes the fact that

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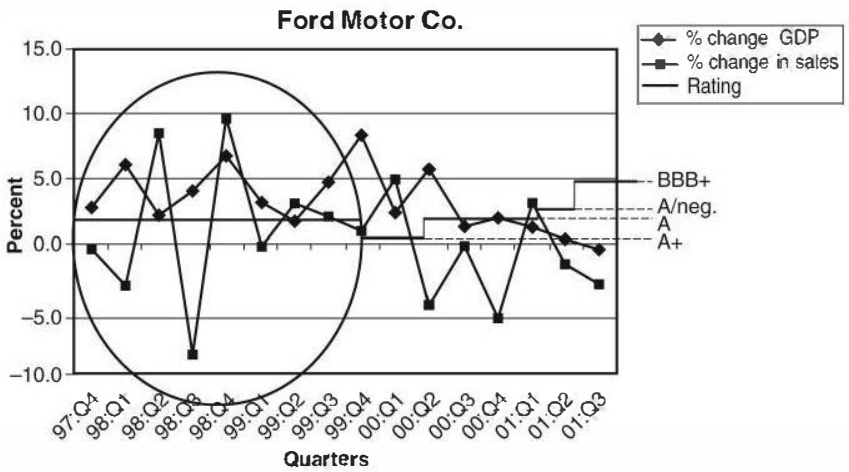

FIGURE 13-4 Ratings as through-the-cycle indicators.

Source: S&P Risk Solutions.

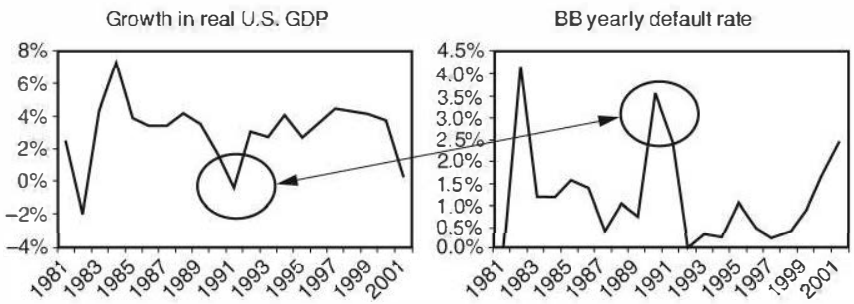

FIGURE 13-5 Impact of macroeconomic shock on default rates.

Source: CreditPro and Federal Reserve.

although the ranking of firms (AAA, AA, etc.) tends to work well on average, the absolute level of riskiness within a rating category fluctuates: Ratings incorporate an average cycle but may overshoot or undershoot when economic conditions deviate too strongly from "average."

To be fair, Figure 13-5 overstates the real variability of default rates within rating categories for at least two reasons: First, volatility has to be expected at the bottom end of the rating scale (for speculative-grade issuers). If we had considered the AAA category, we would have observed a perfectly consistent zero default rate throughout the period. Furthermore, the small number of firms rated BB also contributes to the volatility and explains, for example, why there was no default at all in this category in 1992.

Quality of Transition Matrices over Time and Regions

In this section, we address the following questions: Are migration probabilities, based on past data, useful to predict future migrations? Are transition matrices stable through time?

Nickell, Perraudin, and Varotto (2000) test the stability of transition matrices, based on several drivers: time, the type of borrower, and the position in the economic cycle. Their study is based on a sample of 6534 issuers over the period from December 1970 to December 1997. The authors first calculate a transition matrix on the whole period unconditional on economic cycles and show that migration volatility is higher for low ratings. The calculated transition matrix is also different from those calculated in a previous study using the same data source but for a different time period (Carty and Fons, 1993). This may come as a surprise, as it means that a single transition matrix, independent from the economic cycle, is not really time stationary even when the averaging is performed on a very long time period.

As a second step, Nickell, Perraudin, and Varotto (2000) carry out an analysis by type of borrower and by geographic area. Their conclusions indicate that transition matrices tend to be stable *within* broad homogeneous economic sectors and by geographic areas. However, differences are noticeable *across* sectors, especially for investment-grade issuers: It can be observed that components from transition matrices by sector tend to differ by more than 5 percent from the global multisectors transition matrix. Major discrepancies tend to occur for best ratings. For example, migration volatility is higher for banks than for corporates (they are more likely to change ratings), but conversely large movements are more frequent in industrial sectors than in the banking industry.

As for regional homogeneity, North American matrices per activity are close to the global one. This is natural given the large share of the region in the global

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sample. This is not the case for the Japanese sample, which may not be of sufficient size to draw statistically robust inference.

Transition matrices also appear to be dependent on the economic cycle as downgrades and default probabilities increase significantly during recessions. Nickell, Perraudin, and Varotto (2000) classify the years between 1970 and 1997 into three categories (growth, stability, and recession) according to GDP growth for the G7 countries. One of their observations is that for investment-grade counterparts, migration volatility is much lower during growth periods than during recessions. Therefore, their conclusion is that transition matrices unconditional on the economic cycle cannot be considered as Markovian.

In another study based on S&P data, Bangia, Diebold, Kronimus, Schagen, and Schuermann (2002) observe that the more the time horizon of an independent transition matrix increases, the less monotonic the matrix becomes. This point illustrates nonstationarity. Regarding its Markovian property, the authors tend to be less affirmative than Nickell, Perraudin, and Varotto (2000); i.e., their tests show that the Markovian hypothesis is not strongly rejected. The authors, however, acknowledge that one can observe path dependency in transition probabilities. For example, a past history of downgrades has an impact on future migrations. Such path dependency is significant since future PDs can increase up to five times for recently downgraded companies.

The authors then focus on the impact of economic cycles on transition matrices. They select two types of periods (expansion and recession) according to NBER indicators. The major difference between the two matrices corresponds mainly to a higher frequency of downgrades during recession periods. Splitting transition matrices in two periods is helpful; i.e., out-of-diagonal terms are much more stable. Their conclusion is that choosing two transition matrices conditional on the economic cycle gives much better results in terms of Markovian stability than considering only one matrix unconditional on the economic cycle.

In order to investigate further the impact of cycles on transition matrices and Credit VaR (value-at-risk), Bangia, et. al. (2002) use a version of CreditMetrics on a portfolio of 148 bonds. They show that the necessary economic capital increases substantially during recessions compared with growth periods (by 30 percent for a 99-percent confidence level of Credit VaR, or 25 percent for a 99.9-percent confidence level). Note that the authors ignore the increase in correlation during recessions. This latter factor alone contributes substantially to the increase in portfolio losses during recessions, particularly at higher confidence levels.

Industry and Geography Homogeneity

External rating agencies as well as internal credit departments within banks aim at using the same rating grades to characterize default risk for all countries and for the various asset classes they cover, such as large corporates, financial institutions, municipalities, sovereigns, etc.

Two initial remarks often appear regarding homogeneity and external rating agencies:

- First, because rating agencies have originally developed their methodologies in the United States, there could be differences in performance between U.S. firms and non-U.S. firms. If such a bias existed, it could come from the fact that the rating history outside the United States is much shorter.
- Second, Morgan (1997) shows that the level of consensus among rating agencies is much lower for financial institutions than it is for corporates. The rationale for such differences is often linked with the opacity of financial institutions. As a result, different levels of transparency between sectors could lead to rating heterogeneity.

Nickell, Perraudin, and Varotto (2000), as well as Ammer and Packer (2000), review these two issues. The empirical study of the latter is based on Moody's database between 1983 and 1998. Their conclusion is twofold:

- Geographic homogeneity is not questionable.
- For a given rating category, banks tend to show higher default rates than corporates.

External rating agencies have recently put a lot of emphasis on ratings homogeneity (Standard & Poor's, 1999). In the light of the Basel II reform, it is also important that rating agencies provide broadly similar assessments of risk, at least on average. In their "standardized approach," the Basel proposals enable banks to rely on external agency ratings to calculate the risk weights used

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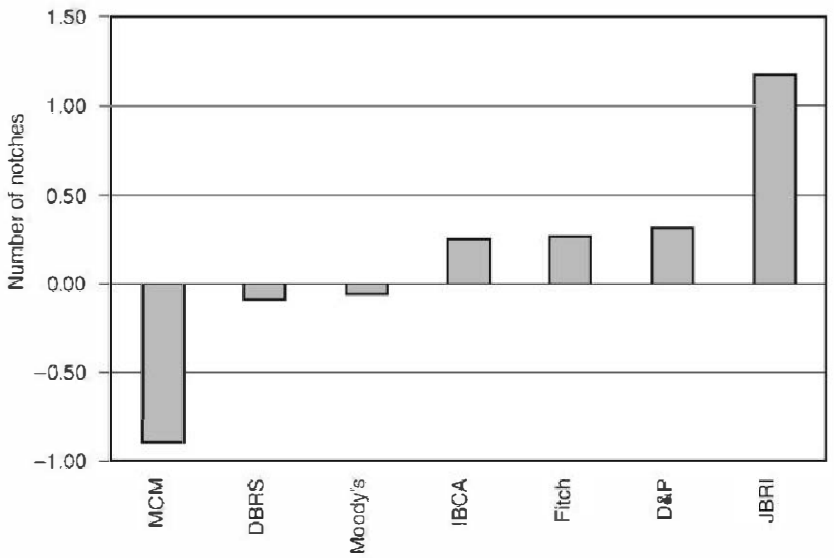

FIGURE 13-6 Average rating difference compared with S&P's.*

*Notches below zero = more conservative than S&P; notches above zero = more lenient.
Source: Beattie and Searle (1992).

in calculating capital requirements. Wide discrepancies across agencies would induce banks to select the most lenient rating provider. In order to preclude "agency arbitrage," i.e., to choose the rating agency providing the most favorable rating, the regulators have to ensure that there is no obvious systematic underestimation of risk by authorized agencies.

There have been relatively few empirical studies on comparing agencies' output, probably due to the difficulty of gathering data from all providers. Beattie and Searle (1992) provide a comprehensive analysis of the assessment of eight rating agencies (Figure 13-6). Their results show that larger players (Moody's and S&P) exhibit very similar average assessments. Neither of them exhibits significantly more conservative behavior than the other. However, there are some large differences with more specialized or regional agencies. Unfortunately, Beattie and Searle's (1992) paper is now quite old, and we are not aware of any more recent studies on a similar scale.

Impact of Rating Changes on Corporate Security Prices

If ratings bring information about the credit quality of firms, a change in rating should lead to a reassessment of the firm's risk by market participants and therefore to changes in the prices of corporate securities such as bonds and equity issued by the firm. The impacts of upgrades and downgrades have attracted a lot of academic attention, which we now briefly summarize.

Effect of Rating Changes on Bond Prices

We have seen earlier that rating categories were associated with different default probabilities. The expected sign of the impact of a rating change on bond prices should be and is actually unambiguous. Given that ratings act as a proxy for default probability or expected loss, a downgrade (upgrade) is likely to have a negative (positive) impact on bond prices.

This intuition is supported by most studies on the topic, such as that of Hand, Holthausen, and Leftwich (1992) among many others. Most articles rely on event study methodologies and report a statistically significant underperformance of recently downgraded bonds. Recently upgraded bonds tend to exhibit overperforming returns, but this result is generally less statistically significant. The findings are very sensitive to the frequency of observation (monthly bond return versus daily) and the possible "contamination" of rating changes by other events impacting on bond prices. For example, if a firm is downgraded at the beginning of a month and announces a substantial restructuring during the same month, the negative price impact of the rating may be compensated for by a positive change linked to the restructuring. Overall the price may rise during the observation month although the actual event of interest (downgrade) had the expected negative effect. This may explain the results of early studies, such as that of Weinstein (1977), that do not find a price reaction at the time of rating changes.

The well-documented link between default probability and rating (see, e.g., Figure 13-3) is in itself insufficient for rating changes to have some bearing on prices. It is also important for investors that the information content of ratings not be fully anticipated and previously incorporated in asset prices. Alternatively ratings may influence the supply of and demand for securities and therefore trigger price changes irrespective of informational issues.

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A lot of debate has recently focused on whether rating analysts should incorporate more timely market information in their assessment. Ratings have indeed been shown in some cases to lag equity prices in capturing deterioration in credit quality. We will not enter this debate here but want to point out that the value of ratings resides to a large extent in the fact that agency analysts bring new information to the market. If ratings were to mimic market fluctuations, their usefulness would be severely jeopardized. The argument that ratings do not bring information and that the signal brought by ratings is fully anticipated by the market is contradicted by the fact that rating changes do affect corporate security prices.

Supply and demand effects also partly explain why rating changes translate into price shocks. Some market participants such as asset managers often have self-imposed restrictions on the credit quality of the assets they can invest in. In particular, many funds have a policy to invest only in investment grade bonds. A downgrade to speculative grade therefore leads to significant sales by asset managers and contributes to depressing the prices of bonds issued by the downgraded company. Banking regulation also leads to the segmentation of bond markets. Under the current Basel guidelines (whereby all corporate bonds bear a 100 percent risk weight irrespective of the credit quality of their issuer), banks are at a competitive disadvantage compared with funds and insurance companies on the investment-grade market. Banks indeed have to put capital aside to cover potential losses, while other investment houses are not subject to the same constraints. This explains why banks tend not to be the dominant players in the investment-grade market where spreads are too narrow to compensate them for the cost of capital. By making explicit the relationship between regulatory risk weights and ratings in the standardized approach, the purchases and sales of corporate bonds by banks (and their induced price effects) will arguably be more dependent on rating changes and should reinforce the effects of rating changes on bond prices.

Rating triggers, i.e., bond covenants based on the rating of a bond issue, are also instrumental in explaining the underperformance of downgraded bonds in some cases. The most common type of securities with rating triggers is step-up bonds whose coupons increase when the issuer is downgraded below a predefined threshold. While these features may at first seem attractive for bondholders, they are double-edged swords: When a company starts entering into difficulties and gets downgraded, it is further penalized by the rating triggers (higher interest payments). Therefore, not only does the downgraded firm find new funding to be more expensive (because the rating change leads to higher spreads), but its current source of funds becomes more costly as well. This has been shown to lead to vicious-circle effects, with recently downgraded firms more likely to be downgraded again. Rating triggers were particularly popular with telecom issuers, who found them a convenient way to raise reasonably cheap capital in good times.

More recently, credit derivatives have led to price volatility in the corporate bond market. One of the main novelties introduced by credit derivatives has been to allow market participants to sell credit short. The ability of traders to "short" corporate bonds leads to more ample price fluctuations than those that were previously observed. Some of this volatility is generated at times of rating changes as some credit products are based on the rating of an underlying firm or security. The rebalancing of hedging portfolios leads to large purchases and sales of corporate bonds around times of rating changes, which increases price volatility.

The Impact of Rating Changes on Stock Prices

We have seen that the link between the probability of default and rating brings an intuitive connection between rating changes and bond returns. The impact of these events on stock prices is less obvious. If rating changes leave the value of the firm unchanged, equity prices should, of course, jump in the opposite direction to bond prices.

A downgrade due to an increase in firm risk (volatility of assets) can indeed be beneficial to equity holders who own a call on the value of the firm. Kliger and Sarig (2000) find such an overall neutral effect in their experiment. They analyze the impact of Moody's shift from a coarse rating grid to a finer one, which occurred in 1982. This was not accompanied by any fundamental change in issuers' risks but brought a more precise assessment of the default probability. The authors report that the incremental rating information did not affect firm value although individual claims (debt and equity) were affected.

Goh and Ederington (1993) make a distinction between downgrades associated with increases in leverage and

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those linked to deteriorating financial prospects. While the latter type of downgrades is bad news for bondholders and shareholders alike, the former case corresponds to a wealth transfer from bondholders to shareholders and should be associated with an increase in the price of equity. They find on a sample of Moody's ratings, that downgrades related to falling expectations of the firm's future earnings or sales are associated with stock price falls, whereas downgrades linked to increased leverage do not have any impact. They interpret the latter result as a sign that changes in leverage are generally anticipated by the market.

On the whole there is no reason to believe that rating revisions should not affect the value of the firm. Many articles, (Dichev and Piotroski, 2001; Holthausen and Leftwich, 1986; and Pinches and Singleton, 1978) indeed report falls in the value of equity. Bankruptcy costs, for example, can lead to a drop in the value of the firm as the probability of default increases and some of the value is transferred to third parties (lawyers, etc.). A segmentation of the bond market, particularly between investment-grade and non-investment-grade categories, can also lead to a downgrade being associated with a drop in the overall asset value.

A persistent finding in almost all papers is that downgrades affect stock prices significantly but upgrades do not. Authors disagree on the explanation for this fact. One possibility could be that firms' managers tend to divulge good news and retain bad news so that an upgrade is more likely to be expected than a downgrade. Another alternative would be asymmetric utility functions with downside risk priced more dearly than upside potential.

Given the overwhelming share of the rated universe accounted for by the United States, very few authors have carried out similar studies outside the United States. Two noticeable exceptions are Barron, Clare, and Thomas (1997) and Matolcsy and Lianto (1995), who report results for U.K. and Australian stock returns, respectively, that are broadly similar to the U.S. experience. Both studies are limited to a very small sample (less than 100 observations), and so tests on subsamples such as downgrades should therefore be interpreted with caution.

APPROACHING CREDIT RISK THROUGH INTERNAL RATINGS OR SCORE-BASED RATINGS

Over the past few years, banks have attempted to mirror the rating behavior of external rating agencies. Given that the core business of a bank is not to provide assessments of the creditworthiness of companies but to lend money, it is a natural incentive for a bank's credit analysts to set up processes similar to those that have been thoroughly tested and validated over time by agencies.

Not long ago, the initial question asked by many bank credit committees was whether the creditworthiness of a company was good or bad, leading directly to a yes or no lending decision. To some extent this policy persists in the personal loan business where customers either satisfy a list of criteria and are granted the loan or fail to satisfy one criterion and their application is rejected. The problem with this black-and-white assessment is that there are no distinctions among "good" customers, and so all of them are assigned the same average interest rate based on an average probability of default and recovery rate.

This approach has evolved with time notably because of two major drivers: First, external rating agencies' scales are being used extensively as a common language in financial markets and banks. Second, regulators, in the context of the Basel II new rules, have strongly recommended the use of a relatively refined rating scale to assess credit quality. Such scales make sense from a statistical point of view. Indeed, empirical tests performed on a historical basis show that in a vast majority of cases a default is the consequence of several rating downgrades. Sudden defaults without preliminary downgrades are much rarer (11 percent on average according to a study by Moody's in 1997).

Any internal rating approach, however, raises a lot of questions: the objectivity of qualitative judgments, the validity of the rating allocation process, the quality of forecast information embedded in ratings, the time horizon, the consistency with external ratings, etc.

We will now raise the very important issue of the time horizon associated with internal ratings before turning to the process of building an internal rating system.

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Internal Ratings, Scores, and Time Horizons

Internal ratings generally refer to a time-consuming qualitative assessment process devised to identify the credit quality of a firm. They generally use either letter-labeled classes similar to those of rating agencies (e.g., BBB or Baa) or numbers (1, 2, . . .).

Scores tend to use quantitative methodologies based on financial and sometimes nonfinancial information. One of the best-known initial approaches was the Z-score proposed by Altman (1968). It assumes that past accounting variables provide predictive information on the default probability of any firm. Default probability information corresponds to a percentage extracted from the [0 percent, 100 percent] continuous scale.

The link between continuous scales and discrete ones is often built through an internal "mapping" process. Most of the time the continuous scale is split either in buckets reflecting scores or directly in internal rating categories. We should stress that such a mapping between probabilities of default and internal ratings only makes sense if the time horizons corresponding to the two approaches are comparable.

Two Ways to Rate or Score a Company

One way to rate a company is to use an "at-the-point-in-time" approach. This kind of approach assesses the credit quality of a firm over the coming months (generally 1 year). This approach is widely used by banks that use quantitative scoring systems, for example, based on discriminant analysis or logit models (linear, quadratic, etc.). All tools based on arbitrage between equity and debt markets, through to structural models, like KMV Credit Monitor EDFs (expected default frequencies), also fall into the at-the-point-in-time category of default estimates.

A second way to rate a company is to use a through-the-cycle approach. As explained earlier, a through-the-cycle approach tries to capture the creditworthiness of a firm over a longer time horizon, including the impact of normal cycles of the economy. A through-the-cycle assessment therefore embeds scenarios about the economy as well as business and financial factors. Because the economic cycle is factored in, such ratings are supposed to be much more stable than at-the-point-in-time estimates.

The Incompatibility of the Two Approaches

In many banks it is common to follow a qualitative process for large corporates, based on a comparison with ratings from rating agencies, and at the same time use a scoring approach for the middle market or SMEs, with a very basic mapping process to revert to the bank's master rating scale. Such a mix may not be optimal, as the same internal rating scale is used to convey at the same time through-the-cycle and point-in-time information. This homogeneity issue corresponds to a real stake for banks' internal rating systems and may lead banks to significant biases regarding their economic capital allocation process. Indeed asset classes rated with through-the-cycle tools would be penalized during growth periods compared with asset classes rated with at-the-point-in-time tools, and vice versa in recessions.

At-the-point-in-time score volatility is much higher than through-the-cycle score volatility. But this volatility is not comparable across the rating scale: Median at-the-point-in-time scores tend to display significant volatility, whereas high and low at-the-point-in-time scores often exhibit a more moderate level of volatility more akin to through-the-cycle ratings.

For these reasons the two approaches are not comparable and should not be mixed. Banks try to build a consistent view of the creditworthiness of their counterparts for all their asset classes. As a result, they should exercise great care if they use, for example, at-the-point-in-time scores for their SMEs or private corporates and through-the-cycle ratings for their public corporates at the same time.

A practical way to observe the differences is to calculate 1-year transition matrices for a typical scoring system and compare them to those of an external rating agency. A transition matrix is devised to display average 1-year migrations for all scores or ratings, i.e., probabilities to move from one rating category to another. Considering both Standard & Poor's rating universe and a common scored universe (see Figure 13-7), we observe that an AA trajectory is very different from a "2" trajectory although their mean 1-year PD may be similar: The probability of an AA firm to remain an AA a year after is between 80 and

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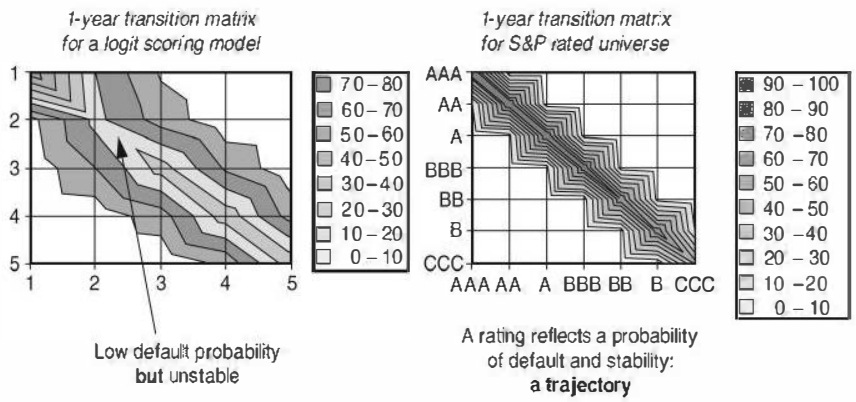

FIGURE 13-7 Scoring versus rating transition matrix.

90 percent, whereas the probability of a 2 to remain a 2 one year after is only between 30 and 40 percent. Therefore these two creditworthiness indicators are not comparable.

Two results are found persistently when analyzing transition matrices derived from scores: The weight on the diagonal (the probability of remaining in the same rating) is (1) fairly low and (2) nonmonotonic as a function of score level. In contrast, rating transition matrices are heavily concentrated on the diagonal and exhibit lower volatility as one reaches higher grades.

Attempts to Extract Through-the-Cycle Information from At-the-Point-in-Time Scores

From a risk-mitigation standpoint, it is not only default risk for today or tomorrow that has to be forecast. For buy-and-hold strategies (typical of banks' lending books) what matters is default risk at any time until the horizon of the underlying credit instruments. As a result an appropriate credit assessment should in theory not just be limited to a probability of default at a given horizon but also reflect its variability through time and its sensitivity to changes in the major factors affecting a given company. One needs to consider not only a short-term PD, but also the estimated *trajectory* of this PD over a longer horizon.

Most quantitative analysts trying to build a scoring system tend to face a difficult dilemma: Either target the highest level of predictive power at a given horizon and fail to obtain a stable through-the-cycle system, or reduce the level of predictive power in order to increase stability. The obtained trade-off is in general not fully satisfactory.

Chassang and de Servigny (2002) propose a way to extract through-the-cycle predictive default information from financial input. They show that with a sufficiently large history of past short-term PDs, it is possible to obtain through-the-cycle equivalent ratings, thanks to a mapping exercise based on the estimation of appropriate rating buckets defined on a mean, variance, skewness (of PDs) space. The main underlying idea is that a through-the-cycle rating is the combination of at-the-point-in-time PD information and a range of different trajectories along time, which are representative of a given rating category.

Löffler (2002) tries another interesting way to capture the through-the-cycle information, using a Kalman filter. His approach relies on the underlying assumption that a Merton-type distance to default is the single driver for creditworthiness.

How to Build an Internal Rating System

Using Rating Templates to Mirror the Behavior of External Agencies Ratings

As mentioned above, one way for banks to obtain an internal rating system is to try and mirror the behavior of rating agencies' analysts. This is particularly necessary for asset classes where default observations are very scarce, for example for financial institutions, insurance, or project finance. Such methodology is very straightforward, as it consists of identifying the most meaningful ratios and risk factors (financial or nonfinancial ones) and assigning weights to each of them in order to derive a rating estimate close to what an analyst from a rating agency would calculate. Of course, the agency analyst does not use a model to rate a company, but a model can integrate the most meaningful factors considered by this analyst. The weights on each of the factors can either be defined qualitatively, based on discussions with the analysts, or be extracted quantitatively through various statistical methodologies.

Rating templates allow banks to calibrate their internal rating process. They also enable them to use, in a consistent manner, rating agencies' transition matrices for portfolio management matters. Figure 13-8 is an illustrative

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example of a summarized template. The analyst would enter his opinion on all relevant variables in the form of a score. All scores are then weighted to obtain a global score that is mapped to a rating category. Obviously, the choice of weight is crucial, and weights need to be calibrated on a fairly large sample and back-tested regularly. The usual way to check the appropriateness of weights is to compare external ratings with internal ones on a sample of firms. If a systematic difference (overestimation or underestimation of risk) is observed, the weights should be amended.

	Weighting	Scoring	
Corporate Credit Score	A	C	(B × C)
	Weight (%)	Score (0 – 100)	Weighted Score
1. Industry characteristics			
2. Market position			
3. Management			
Total score* for business profile			
4. Financial policy			
5. Profitability			
6. Cash flow			
7. Capital structure			
8. Financial flexibility			
Total score* for financial profile			
Total score			

FIGURE 13-8 Example of an internal rating template.

Calibrating and Back-Testing a Rating System Requires a Long Time Horizon

When banks build their internal rating system, their objective is twofold. First they want to assess the creditworthiness of companies during the loan application process. Second they want to use rating information to feed their portfolio management tools designed to produce regulatory capital or economic capital measures. As a result, banks have to devise links between their internal rating scale and tables displaying cumulative probabilities of default at horizons ranging from 1 year to the longest maturity of the debt instruments they hold.

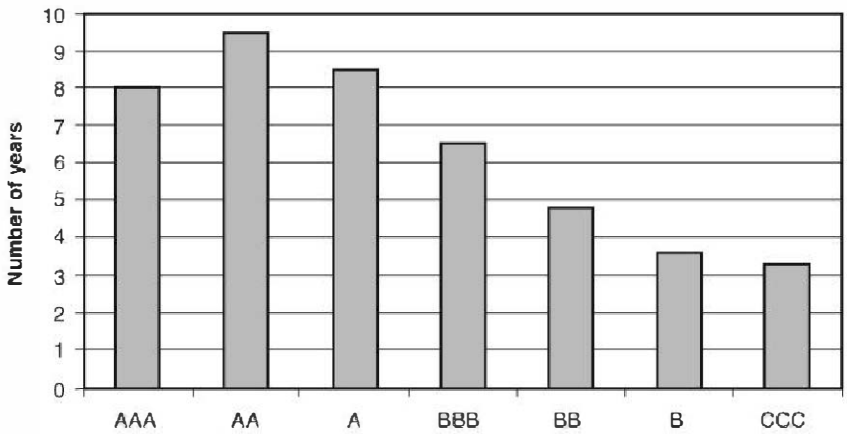

FIGURE 13-9 Time to default per rating category.*

*Data period: 1981–2001.
Source: S&P Credit Pro.

When banks define their internal scale, they have no track record of default rates per rating category, per industry, or per region. They may also have a rated universe that is by construction too small to provide strong statistics about empirical default rates.

The second step for banks, just as for rating agencies, is to test the stability of their internal transition matrices. If this assumption is found to be acceptable, then and only then should banks be entitled to devise a link between internal ratings and probabilities of default. Some banks might find that they need to build different transition matrices that are specific to their different asset classes or to the economic cycle. The question then is how many years should be required to perform such a complete analysis?

Based on Moody's database, Carey and Hrycay (2001) estimate that a historical sample between 11 and 18 years should be necessary in order to test the validity of internal ratings. Based on Standard & Poor's universe, we think that a time period of 10 years should be considered as a minimum for investment-grade securities, while 5 years should be enough to back-test non-investment-grade issues. Figure 13-9 is calculated from a sample of defaulted firms and reports the average time it took for firms in a given rating grade to drift down to default.

In many banks we are still far from ex post statistical testing, because the rating history is in general too short. In

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the future many banks will probably discover that their internal rating system is weaker than they expected.

Banks using at-the-point-in-time tools as the backbone of their internal rating system have two options, each associated with a specific risk. One option is to stick to probabilities of default, without using any internal rating scale. Such an approach will convey an accurate measure of the creditworthiness of the bank's counterparts. The associated risk is procyclicality since changes in the credit quality of the portfolio can evolve very quickly. If, on the contrary, banks using at-the-point-in-time methodologies revert to an internal rating scale, the main risk is providing highly unstable transition matrices and no guidance for the long term.

Impact of Internal Models at the Macro Level

So far we have only considered the impact of PD measures on banks but have ignored their systemic or macroeconomic effects.

Procyclicality is a topic that is becoming a central issue with Basel II regulation. It is the fact that linking capital requirements to PDs may induce all banks to overlend in good times and underlend in bad times, thereby reinforcing credit and economic cycles. Many academics and practitioners have recently considered this issue. One of the major dangers with the procyclical effects of the new techniques that banks use to evaluate their economic or regulatory capital requirement lies in the risk of a sudden liquidity crisis affecting the whole economy.

Procyclicality could affect even more those banks that have chosen to set their internal credit limits in terms of expected loss rather than exposure. Expected loss will be very volatile due to the high volatility of PDs calculated using at-the-point-in-time methods. Consequently, during a recession period, short-term PDs will increase sharply and the bank will have to reduce significantly its loan exposures in order to maintain stable expected losses. If such types of PD measures are used by a majority of banks, then firms will face liquidity shortages because of unexpected credit rationing (all their lending banks may simultaneously refuse to grant them further credit). As a consequence, real economic cycles may be amplified.

Another type of cyclical effect comes from the use of at-the-point-in-time measures of risk in economic capital calculations and in the lending process. These models tend to underevaluate risk during growth periods (and overestimate it in recessions). Because defaults take some time to materialize, banks lack incentives to adjust their credit policy before entering into recession: The last years have shown few defaults in their portfolio, and their model (calibrated on previous years' data) still reports low probabilities of default for most firms. After 1 or 2 years (when the economy is in a trough), the number of cases of financial distress increases significantly, and lending conditions are tightened by banks. As a result, the credit cycle tends to lag the economic cycle. Credit rationing may result as a consequence of the contraction of the lending activity by banks. This will in turn exacerbate economic downturn. Credit rationing will impact first and foremost asset classes that are highly dependent on banks because they are too small or have not yet established sufficient reputation to tap financial markets. In particular, the SME sector is very sensitive to banks' lending policies.

Finally, at-the-point-in-time measures of risk present another risk for the aggregate economy. Short-term PD measures tend to bias loan procedures in favor of short-term projects. The selection of short-term projects can lead to suboptimal investment decisions.

Granularity of Rating Scales

There has recently been intense discussion comparing the output of external rating agencies with the output of structural models, such as the KMV Credit Monitor. The core topics discussed focused on the reactivity of structural models versus the stability of ratings. The acquisition of KMV by Moody's has in fact given practical evidence of the complementarity of the two approaches. But the question of the appropriate rating scale to reflect such reactivity is still an open one within banks.

In this respect, a bank and an external rating agency may not share the same objectives. For the latter, communication to investors is dominant. A downgrade or an upgrade is an important event, with various consequences. Having a discrete scale with a limited level of granularity reinforces the informational impact of any migration, sending a strong signal reflecting substantial changes in firms. Banks do not use their internal ratings for external communication and, provided they have enough data and are sufficiently confident in their own systems, they could choose a more granular approach.

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Consequences

This approach of evaluating default risk through ratings migration is quite attractive because of its simplicity. Its robustness is undoubtedly very good for investment grade (IG) counterparts. Regarding non-investment-grade (NIG) counterparts, banks and rating agencies are usually very cautious because migration volatility is strongly related to the economic cycle.

Recent history has shown on many occasions how high-yield markets can be volatile and unpredictable. The split between IG and NIG universes may seem unfair to companies crossing the fence, given its impact in terms of bond spreads. But from a global standpoint, it really seems to correspond to different firms' behavior, different credibility, and different risk profiles. It is also meaningful in terms of segmentation of the demand for such products: Investors in investment-grade and non-investment-grade bonds exhibit very different risk-aversion profiles.

SUMMARY

Rating agencies have developed very precise methodologies to assess the creditworthiness of companies. The stability over time of their approach and of their criteria is key to their success. The main challenge for rating agencies is to convey through-the-cycle information (i.e., about the trajectory of an issuer's creditworthiness) while maintaining a sufficient level of reactivity in order to incorporate early warning predictive power.

The task related to internal ratings assigned to banks in the Basel II accord is very challenging. Banks have to rate a very large universe corresponding to most of the asset classes they are dealing with. For most banks it is a new task that they have to perform. They suffer from a lack of data history, and it will take years before they have sufficient results to back-test their methodologies. Many are at the stage of choosing their approaches for the various asset classes: A qualitative approach (internal rating) is generally adopted for larger positions, and a scoring model deals with smaller exposures. The next step will be to integrate the two approaches consistently in a portfolio model.

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Capital Structure in Banks

14

Learning Objectives

After completing this reading you should be able to:

- Evaluate a bank's economic capital relative to its level of credit risk.
- Identify and describe important factors used to calculate economic capital for credit risk: probability of default, exposure, and loss rate.
- Define and calculate expected loss (EL).
- Define and calculate unexpected loss (UL).
- Estimate the variance of default probability assuming a binomial distribution.
- Calculate UL for a portfolio and the risk contribution of each asset.
- Describe how economic capital is derived.
- Explain how the credit loss distribution is modeled.
- Describe challenges to quantifying credit risk.

Excerpt is from Chapter 5 of Risk Management and Value Creation in Financial Institutions, *by Gerhard Schroeck.*

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In this section we will first define what credit risk is. We will then discuss the steps to derive economic capital for credit risk and the problems related to this approach.

Definition of Credit Risk

Credit risk is the risk that arises from any nonpayment or rescheduling of any promised payments (i.e., default-related events) or from (unexpected) credit migrations (i.e., events that are related to changes in the credit quality of a borrower) of a loan[1] and that gives rise to an economic loss to the bank.[2] This includes events resulting from changes in the counterparty as well as the country[3] characteristics. Since credit losses are a predictable element of the lending business, it is useful to distinguish between so-called expected losses and unexpected losses[4] when attempting to quantify the risk of a credit portfolio and, eventually, the required amount of economic capital, introduced in Box 14-1.

Steps to Derive Economic Capital for Credit Risk

In this section, we will discuss the steps for deriving economic capital for credit risk. These are the quantification of Expected Losses (*EL*), Unexpected Losses (*UL*–Standalone), Unexpected Loss Contribution (*ULC*), and Economic Capital for Credit Risk.

[1] This includes all credit exposures of the bank, such as bonds, customer credits, credit cards, derivatives, and so on.

[2] See Ong (1999), p. 56. Rolfes (1999), p. 332, also distinguishes between default risk and migration risk.

[3] Country risk is also often labeled transfer risk and is defined as the risk to the bank that solvent foreign borrowers will be unable to meet their obligations due to the fact that they are unable to obtain the convertible currency needed because of transfer restrictions. Note that the economic health of the customer is not by definition affected in this case. However, any changes in the macroeconomic environment that lead to changes in the credit quality of the counterparty should be captured in the counterparty rating.

[4] See, for example, Ong (1999), pp. 56, 94+, and 109+, Kealhofer (1995), pp. 52+, Asarnow and Edwards (1995), pp. 11+.

BOX 14-1 Introduction to Economic Capital

Economic capital is an estimate of the overall capital reserve needed to guarantee the solvency of a bank for a given confidence level. A bank will typically set the confidence level to be consistent with its target credit rating.

For credit risk, the amount of economic capital needed is derived from the expected loss and unexpected loss measures discussed in this chapter. For a portfolio of credit assets, expected loss is the amount a bank can expect to lose, on average, over a predetermined period of time when extending credits to its customers. Unexpected loss is the volatility of credit losses around its expected loss. To survive in the event that a greater-than-expected loss is realized, the bank must hold enough capital to cover unexpected losses, subject to a predetermined confidence level—this is the economic capital amount.

Economic capital is dependent upon two parameters, the confidence level used and the riskiness of the bank's assets. As the confidence level increases, so does the economic capital needed. Consider a bank that wants to target a very high credit rating, which implies that the bank must be able to remain solvent even during a very high loss event. This bank must choose a very high confidence level (e.g., 99.97%), which corresponds to a higher capital multiplier (CM) being applied to unexpected losses, increasing the amount of the loss distribution that is covered (as seen in Figure 14-2). Alternatively, a more aggressive bank would target a lower credit rating, which corresponds to a lower CM being applied to unexpected losses, decreasing the amount of the loss distribution that is covered.

Similarly, as the riskiness of the bank's assets increases, so does the economic capital needed. Relative to a bank with low-risk credit assets, a bank with riskier credit assets will have a higher unexpected loss. Therefore, to meet the same confidence level, the bank with riskier credit assets will need greater economic capital.

Holding less capital allows a bank the opportunity to achieve higher returns as it can use that capital to generate returns elsewhere. Therefore, economic capital is an important feature of effective bank management for achieving the desired balance between risk and return.

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Expected Losses (*EL*)

A bank can expect to lose, on average, a certain amount of money over a predetermined period of time[5] when extending credits to its customers. These losses should, therefore, not come as a surprise to the bank, and a prudent bank should set aside a certain amount of money (often called loan loss reserves or [standard] risk costs[6]) to cover these losses that occur during the normal course of their credit business.[7]

Even though these credit loss levels will fluctuate from year to year, there is an anticipated average (annual) level of losses over time that can be statistically determined. This actuarial-type average credit loss is called expected loss (*EL*), can therefore be viewed as payments to an insurance pool,[8] and is typically calculated from the bottom up, that is, transaction by transaction. *EL* must be treated as the foreseeable cost of doing business in lending markets. It, therefore, needs to be reflected in differentiated risk costs and reimbursed through adequate loan pricing. It is important to recognize that *EL* is not the level of losses predicted for the following year based on the economic cycle, but rather the long-run average loss level across a range of typical economic conditions.[9]

There are three components that determine *EL*:

- The probability of default (*PD*),[10] which is the probability that a borrower will default before the end of a predetermined period of time (the estimation horizon typically chosen is one year) or at any time before the maturity of the loan
- The exposure amount (*EA*) of the loan at the time of default

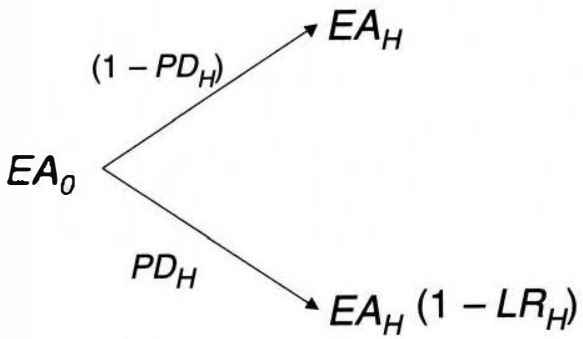

FIGURE 14-1 Deriving expected losses.

Source: Adapted from Ong (1999), p. 101.

- The loss rate (*LR*), that is, the fraction of the exposure amount that is lost in the event of default,[11] meaning the amount that is not recovered after the sale of the collateral

Since the default event *D* is a Bernoulli variable,[12] that is, *D* equals 1 in the event of default and 0 otherwise, we can define the expected amount lost (*EL*) in the event of a default as above (see Figure 14-1):

Hence,

$$\begin{aligned} EL_H &= EA_H - E(EA_H) \\ &= EA_H - [(1 - PD_H) \cdot EA_H + PD_H \cdot (EA_H \cdot (1 - LR_H))] \\ &= PD_H \cdot EA_H \cdot LR_H \end{aligned} \tag{14.1}$$

where PD_H = Probability of default up to time H (horizon)

EA_H = Exposure amount at time H

LR_H = Loss rate experienced at time H

$E(\cdot)$ = Expected Value of $(\cdot)$

The expected loss experienced at time H (EL_H), that is, at the end of the predetermined estimation period, is the difference between the promised exposure amount (EA_H) at that time (including all promised interest payments) and the amount that the bank can expect to receive at that time—given that, with a certain probability of default

[5] Following the annual (balance sheet) review cycle in banks, this period of time is most often set to be one year.

[6] See for example, Rolfes (1999), p. 14, and the list of references to the literature presented there.

[7] See Ong (1999), p. 56.

[8] See, for example, the ACRA (Actuarial Credit Risk Accounting) approach used by Union Bank of Switzerland as described in Garside et. al. (1999), p. 206.

[9] Note that Expected Losses are the unconditional estimate of losses for a given (customer) credit rating. However, for a portfolio, the grade distribution is conditional on the recent economic cycle. Thus, losses from a portfolio as predicted by a rating model will have some cyclical elements.

[10] Often also labeled expected default frequency (EDF); see, for example, Kealhofer (1995), p. 53, Ong (1999), pp. 101-102.

[11] Therefore also called severity, loss given default (LGD), or loss in the event of default (LIED); see, for example, Asarnow and Edwards (1995), p. 12. The loss rate equals (1 − recovery rate), see, for example, Mark (1995), pp. 113+.

[12] See Bamberg and Baur (1991), pp. 100-101, that is, a binomial $B(1; p)$ random variable, where $p = PD$.

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(PD_H) between time 0 and H, a loss ($EA_H \cdot LR_H$) will be experienced.[13]

Therefore, *EL* is the product of its three determining components, which we will briefly describe in turn below:

1. **Probability of default** (*PD*): This probability determines whether a counterparty or client goes into default[14] over a predetermined period of time. *PD* is a borrower-specific estimate[15] that is typically linked to the borrower's risk rating, that is, estimated independently[16] of the specifics of the credit facility such as collateral and/or exposure structure.[17] Although the probability of default can be calculated for any period of time, probabilities are generally estimated at an annual horizon. However, *PD* can and does change over time. A counterparty's *PD* in the second year of a loan is typically higher than its *PD* in the first year.[18] This behavior can be modeled by using so-called migration or transition matrices.[19] Since these matrices are based on the Markov property,[20] they can be used to derive multiperiod *PD*s—both cumulative[21] and marginal[22] default probabilities.[23]

The remaining two components reflect and model the product specifics of a borrower's liability.

2. **Exposure amount** (*EA*): The exposure amount *EA*, for the purposes of the *EL* calculation, is the expected amount of the bank's credit exposure to a customer or counterparty at the time of default. As described above, this amount includes all outstanding payments (including interest) at that time.[24] These overall outstandings can often be very different from the outstandings at the initiation of the credit. This is especially true for the credit risk of derivative transactions (such as swaps), where the quantification of *EA* can be difficult and subject to Monte Carlo simulation.[25]
3. **Loss rate** (*LR*): When a borrower defaults, the bank does not necessarily lose the full amount of the loan. *LR* represents the ratio of actual losses incurred at the time of default (including all costs associated with the collection and sale of collateral) to *EA*. *LR* is, therefore, largely a function of collateral. Uncollateralized, unsecured loans typically have much higher ultimate losses than do collateralized or secured loans.

EL due to transfer or country risk can be modeled similarly to this approach and has basically the same three components (*PD* of the country,[26] *EA*, and *LR* due to country risk[27]). However, there are some more specific aspects

[13] This assumes—for the sake of both simplicity and practicability—that all default events occurring between time 0 and the predetermined period of time ending at H will be considered in this framework. However, the exposure amount and the loss experienced after recoveries will be considered/calculated only at time H and not exactly at the time when the actual default occurs.

[14] Default is typically defined as a failure to make a payment of either principal or interest, or a restructuring of obligations to avoid a payment failure. This is the definition also used by most external rating agencies, such as Standard & Poor's and Moody's. Independently of what default definition has been chosen, a bank should ensure an application of this definition of default as consistent as possible across the credit portfolio.

[15] This assumes that either all credit obligations of one borrower are in default or none of them.

[16] This is not true for some facility types such as project finance or commercial real estate lending where the probability of default (*PD*) is not necessarily linked to a specific borrower but rather to the underlying business. Additionally, *PD* is not independent from the loss rate (*LR* – as discussed later), that is, the recovery rates change with the credit quality of the underlying business. This requires obviously a different modeling approach (usually a Monte Carlo simulation).

[17] Amortization schedules and credit lines (i.e., limit vs. utilization) can have a significant impact on the exposure amount outstanding at the time of default. The same is true for the credit exposure of derivatives.

[18] This statement is only true (on average) for credits with initially low *PD*s.

[19] See, for example, Standard & Poor's (1997) and Moody's Investor Services (1997).

[20] See, for example, Bhat (1984), pp. 38+.

[21] That is the overall probability to default between time 0 and the estimation horizon n.

[22] That is the probability of not defaulting until period i, but defaulting between period i and $i + 1$. These are also often derived as forward *PD*s (similar to forward interest rates).

[23] However, this can—by definition—only reflect the average behavior of a cohort of similarly rated counterparties and not the customer-specific development path.

[24] Obviously, there are differing opinions as to when the measurement actually should take place. See Ong (1999), pp. 94+.

[25] See, for example, Dowd (1998), p. 174.

[26] Typically estimated using the input from the Economics/Research Department of the bank and/or using the information from the spreads of sovereign Eurobonds, see Meybom and Reinhart (1999).

[27] The calculation of *LR* due to country risk is broken into (the product of) two parts: (1) loss rate given a country risk event, which is a function of the characteristics of the country of risk (i.e., where *EA* is located) and (2) the country risk type, which is a function of the facility type (e.g., recognizing the differences between short-term export finance and long-term project finance that can be subject to nationalization, and so on).

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to consider. For instance, since a borrower can default due to counterparty and country risk at the same time, one would need to adjust for the "overlap" because the bank can only lose its money once.

Likewise, we will not deal with the parameterization[28] of this model in this book, but there are many pitfalls when correctly determining the components in practice.

By definition, *EL* does not itself constitute risk. If losses always equaled their expected levels, there would be no uncertainty, and there would be no economic rationale to hold capital against credit risk. Risk arises from the variation in loss levels—which for credit risk is due to unexpected losses (*UL*). As we will see shortly, unexpected loss is the standard deviation of credit losses, and can be calculated at the transaction and portfolio level. Unexpected loss is the primary driver of the amount of economic capital required for credit risk.

Unexpected loss is translated into economic capital for credit risk in three steps, which are—as already indicated—discussed in turn: first, the standalone unexpected loss is calculated (see the "Unexpected Losses" section which follows). Then, the contribution of the standalone *UL* to the *UL* of the bank portfolio is determined (see the "Unexpected Loss Contribution" section later in this chapter). Finally, this unexpected loss contribution (*ULC*) is translated into economic capital by determining the distance between *EL* and the confidence level to which the portfolio is intended to be backed by economic capital (see the "Economic Capital for Credit Risk" section later in this chapter).

Unexpected Losses (*UL*-Standalone)

As we have defined previously, risk arises from (unexpected) variations in credit loss levels. These unexpected losses (*UL*)[29] are—like *EL*—an integral part of the business of lending and stem from the (unexpected) occurrence of defaults and (unexpected) credit migration.[30] However, these *UL*s cannot be anticipated and hence cannot be adequately priced for in a loan's interest rate. They require a cushion of economic capital, which needs to be differentiated by the risk characteristics of a specific loan.[31]

UL, in statistical terms, is the standard deviation of credit losses, that is, the standard deviation of actual credit losses around the expected loss average (*EL*). The *UL* of a specific loan on a standalone basis (i.e., ignoring diversification effects) can be derived from the components of *EL*. Just as *EL* is calculated as the mean of a distribution, *UL* is calculated as the standard deviation of the same distribution.

Recall that *EL* is the product of three factors: *PD*, *EA*, and *LR*. For an individual loan, *PD* is (by definition) independent of the *EA* and the *LR*, because default is a binary event. Moreover, in most situations, *EA* and the *LR* can be viewed as being independent.[32] Thus, we can apply standard statistics to derive the standard deviation of the product of three independent factors and arrive at:[33]

$$UL = EA \cdot \sqrt{PD \cdot \sigma^2_{LR} + LR^2 \cdot \sigma^2_{PD}} \qquad (14.2)$$

where σ_{LR} = Standard deviation of the loss rate *LR*

σ_{PD} = Standard deviation of the default probability *PD*

Since the expected exposure amount *EA* can vary, but is (typically) not subject to changes in the credit characteristics itself, *UL* is dependent on the default probability *PD*, the loss rate *LR*, and their corresponding variances, σ^2_{LR} and σ^2_{PD}. If there were no uncertainty in the default event and no uncertainty about the recovery rate, both variances would be equal to zero, and hence *UL* would also be equal to zero, indicating that there would be no credit risk. For simplicity, we have ignored the time index in this derivation. But all parameters are estimated, as was done previously, at time *H*.

Note that, since default is a Bernoulli variable with a binomial *B*(1;*PD*)-distribution:[34]

$$\sigma^2_{PD} = PD \cdot (1 - PD) \qquad (14.3)$$

[28] We will not deal with the estimation and determination of the various input factors for specific customer and product segments. See, for a discussion, Ong (1999), pp. 104–108.

[29] For a detailed discussion of *UL* see, for example, Ong (1999), Chapter 14, pp. 109–118.

[30] See Ong (1999), p. 111.

[31] To be more precise and as we will see shortly below, the amount of economic capital depends on the risk contribution of a specific loan to the overall riskiness of a loan portfolio.

[32] However, in practice it is not clear as to whether the assumption of statistical independence is well justified. See Ong (1999), p. 114. If they were not independent, a covariance cross-term needs to be introduced, but would have only a small overall impact on the absolute amount of *UL* in practice.

[33] See Ong (1999), pp. 116–118, for a detailed derivation.

[34] See Bamberg and Baur (1991), p. 123.

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Since it is typically difficult in practice to measure the variance of the loss rate σ^2_{LR} due to the lack of sufficient historical data, we will have to assume in most cases a reasonable distribution for the variations in the loss rate. Unfortunately, unlike the distribution for *PD*, the loss rate distribution can take a number of shapes, which result in different equations for the variance of *LR*. Possible candidates are the binomial, the uniform, or the normal distribution. Whereas the binomial distribution overstates the variance of *LR* (when a customer defaults, either all of the exposure amount is lost or nothing), the uniform distribution assumes that all defaulted borrowers would have the same probability of losing anywhere between 0% and 100%. Therefore, the most reasonable assumption is the normal distribution, because of the lack of better knowledge in most cases.[35] The shape of this assumed normal distribution should take into account the empirical fact that some customers lose almost nothing, that is, almost fully recover, and it is very unlikely that all of the money is lost during the work-out process.[36]

Like *EL*, *UL* can also be calculated for various time periods and for rolling time windows across time. By convention, almost always one-year intervals are used.[37] Hence, all measures of volatility need to be annualized to allow comparisons among different products and business units.[38] Again, the same methodology can be applied to derive the *UL* resulting from country risk using the three components of country *EL*.

Unexpected Loss Contribution (*ULC*)

Credit risk cannot be completely eliminated by hedging it through the securities markets like market risk.[39] Even credit derivatives and asset securitizations can only shift credit risk to other market players. These actions will not eliminate the downside risk associated with lending. However, they can transfer credit risk to the market participant best suited to bear it, because the only way to reduce credit risk is by holding it in a well-diversified portfolio (of other credit risks).[40] Therefore, we need to change our perspective of looking at credit risk from the single, standalone credit to credit risk in a portfolio context.

The expected loss of a portfolio of credits is straightforward to calculate because *EL* is linear and additive.[41] Therefore:

$$EL_p = \sum_{i=1}^{n} EL_i = \sum_{i=1}^{n} EA_i \cdot PD_i \cdot LR_i \qquad (14.4)$$

where EL_p = Expected loss of a portfolio of *n* credits.

However, when measuring unexpected loss at the portfolio level, we need to consider the effects of diversification because—as always in portfolio theory—only the contribution of an asset to the overall portfolio risk matters in a portfolio context. In its most general form, we can define the unexpected loss of a portfolio UL_p as:

$$UL_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i \omega_j \rho_{ij} UL_i UL_j} \qquad (14.5)$$

where

$$\sum_{i=1}^{n} \omega_i = 1 \text{ and } \omega_i = \frac{EA_i}{\sum_{i=1}^{n} EA_i} \qquad (14.8)$$

ω_i = Portfolio weight of the *i*-th credit asset

ρ_{ij} = Correlation that default or a credit migration (in the same direction) of asset *i* and asset *j* will occur over the same predetermined period of time (usually, again, between time 0 and *H* [one] year)

UL_i = Unexpected Loss of the *i*-th credit asset as defined above in Equation (14.2).

Therefore, considering a loan at the portfolio level, the contribution of a single UL_i to the overall portfolio risk is a function of:

- The loan's expected loss (*EL*), because default probability (*PD*), loss rate (*LR*), and exposure amount (*EA*) all enter the *UL*-equation

[35] Also see Ong (1999), p. 132.

[36] As mentioned above, even unsecured loans almost always recover some amount in the bankruptcy court, see, for example, Eales and Bosworth (1998), p. 62, or Carty and Lieberman (1996), p. 5.

[37] See Ong (199), p. 121.

[38] For convenience and again due to lack of data, the volatility of *LR* is assumed to be constant over time (intervals).

[39] Credit risk only has a downside potential (i.e., to lose money), but no upside potential (the maximum return on a credit is limited because the best possible outcome is that all promised payments will be made according to schedule).

[40] See Mason (1995), pp. 14–24, and Ong (1999), p. 119. As Mason shows, the same argument can be applied to the management of insurance risk.

[41] See Ong (1999), p. 123.

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- The loan's exposure amount (i.e., the weight of the loan in the portfolio)
- The correlation of the exposure to the rest of the portfolio

To calculate the unexpected loss contribution[42] ULC_i of a single loan i analytically, we first need to determine the marginal impact of the inclusion of this loan on the overall credit portfolio risk. This is done by taking the first partial derivative of the portfolio UL with respect to UL_i (for loan i):

$$ULMC_i = \frac{\partial UL_p}{\partial UL_i} = \frac{\partial\left(UL_p^2\right)^{1/2}}{\partial UL_i} = \left(\frac{1}{2}\right)\cdot\left(UL_p^2\right)^{-1/2}\cdot\frac{\partial\left(UL_p^2\right)}{\partial UL_i}$$

$$= \left(\frac{1}{2UL_p}\right)\cdot\frac{\partial\left(\sum_{j=1}^{n}\sum_{k=1}^{n}UL_j\cdot UL_k\rho_{jk}\right)}{\partial UL_i} = \frac{\sum_{j=1}^{n}UL_j\rho_{ij}}{UL_p} \quad (14.7)$$

where $ULMC_i$ is the marginal contribution of loan i to the overall portfolio unexpected loss.

Note that in the above formula, the marginal contribution only depends on the (UL-) weights of the different loans in the portfolio, not on the size of the portfolio itself. In order to calculate the portfolio volatility attributable to loan i, we use the following property for a marginal change in portfolio volatility:

$$dUL_{Port} = \sum_{i=1}^{n}\frac{\partial UL_p}{\partial UL_i}\cdot dUL_i = \sum_{i=1}^{n}ULMC_i\cdot dUL_i \quad (14.8)$$

The marginal contribution of each loan is constant if the weights of each loan in the portfolio are held constant. Hence, integrating the above equation, holding the weight of each loan constant (i.e., UL_i/UL_p is constant, which is true for practical purposes on average), we obtain:

$$UL_{Port} = \sum_{i=1}^{n}ULMC_i\cdot UL_i \quad (14.9)$$

Therefore, the portfolio UL can be viewed to split into n components, each of which corresponds to the marginal loss volatility contribution of each loan multiplied by its standalone loss volatility. Hence, we define the total contribution to the portfolio's UL as:[43]

$$ULC_i = ULMC_i\cdot UL_i = \frac{\sum_{j=1}^{n}UL_j\rho_{ij}}{UL_p}\cdot UL_i \quad (14.10)$$

It is easy to see from the above formula that ULC has the important property that the sum of the ULCs of all loans will equal the portfolio-level UL (i.e., the sum of the parts equals the whole, which is exactly the intended result):[44]

$$\sum_{i=1}^{n}ULC_i = \sum_{i=1}^{n}UL_i\frac{\sum_{j=1}^{n}UL_j\rho_{ij}}{UL_p} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}UL_i\cdot UL_j\cdot\rho_{ij}}{UL_p} = \frac{UL_p^2}{UL_p} = UL_p \quad (14.11)$$

Assuming now that the portfolio consists of n loans that have approximately the same characteristics and size ($1/n$), we can set $\rho_{ij} = \rho =$ constant (for all $i \neq j$). Rewriting Equation (14.5) according to standard portfolio theory:

$$UL_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\text{cov}_{i,j}} = \sqrt{\sum_{i=1}^{n}\text{var}_i + \sum_{i,j\neq j}^{n}\text{cov}_{i,j}} \quad (14.12)$$

where cov_{ij} is defined as the covariance and var_i as the variance of losses; one could further derive:

$$UL_p = \sqrt{\sum_{i}^{n}UL_i^2 + \sum_{i,j<i}^{n}\text{cov}_{i,j}} = \sqrt{\sum_{i}^{n}UL_i^2 + 2\sum_{i,j<i}^{n}\rho UL_iUL_j}$$

$$\approx \sqrt{n\cdot UL_i^2 + 2\frac{n(n-1)}{2}\cdot\rho\cdot UL_i^2} = \sqrt{[n+\rho n(n-1)]\cdot UL_i^2} \quad (14.13)$$

and hence:

$$UL_p = UL_i\sqrt{n+\rho(n^2-n)} \quad (14.14)$$

Using the assumption of similar credits within the portfolio previously described, we can now rewrite:

$$ULC_i = \frac{UL_p}{n} = \frac{1}{n}UL_i\sqrt{n+\rho(n^2-n)} = UL_i\sqrt{\frac{1}{n}+\rho\left(1-\frac{1}{n}\right)} \quad (14.15)$$

which reduces for large n to:

$$ULC_i = UL_i\sqrt{\rho} \quad (14.16)$$

Combining Equation (14.10) with (14.16) and rearranging the terms, we can arrive at:

$$\sqrt{\rho} = \frac{\sum_{j=1}^{n}UL_j\rho_{ij}}{UL_p} \quad (14.17)$$

which clearly shows that ρ is the (weighted) average correlation between loans in the portfolio (as was assumed above).

[42] Note that we follow the argument made by Ong (1999), p. 133, in this discussion and ignore the weights w_i in the derivation of ULC. We can do so if we assume that UL_i is measured in dollar terms rather than as a percentage of the overall portfolio.

[43] See Ong (1999), p. 126, for more details on his derivation of this equation pp. 132–134.

[44] See Ong (1999), p. 127.

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This derivation provides some important insights:

- If one tried to estimate the portfolio *UL* by using Equation (14.5), one would need to estimate $[n(n-1)]/2$ pairwise default correlations.[45] Given that typical loan portfolios contain many thousand credits, this is impossible to do. Additionally, one needs to consider the fact that default correlations are very difficult, if not impossible, to observe.[46]
- Equation (14.16) is a practicable way to calculate *ULC*. However, it basically ignores the fact that loans are of different sizes and show different correlations (e.g., by industry, geography, etc.). Therefore, using Equation (14.16) does not reveal potential concentrations in the credit portfolio. But banks try to avoid exactly these concentrations. It is easy to show[47] that Equation (14.16) can be decomposed for various segments of the portfolio so that, for example, default correlations between various industries or even of a single credit can be included. Using this approach (instead of the impractical "full-blown" approach, as indicated by Equation (14.5), allows banks to quantify exactly what they have done by intuition, prudent lending policies, and guidelines for a very long time.[48]
- Default correlations are small, but positive. Therefore, and as indicated previously, there are considerable benefits to diversification in credit portfolios.
- Overall, the analytical approach is very cumbersome and prone to estimation errors and problems. To avoid these difficulties, banks now use numerical procedures[49] to derive more exact and reliable results.

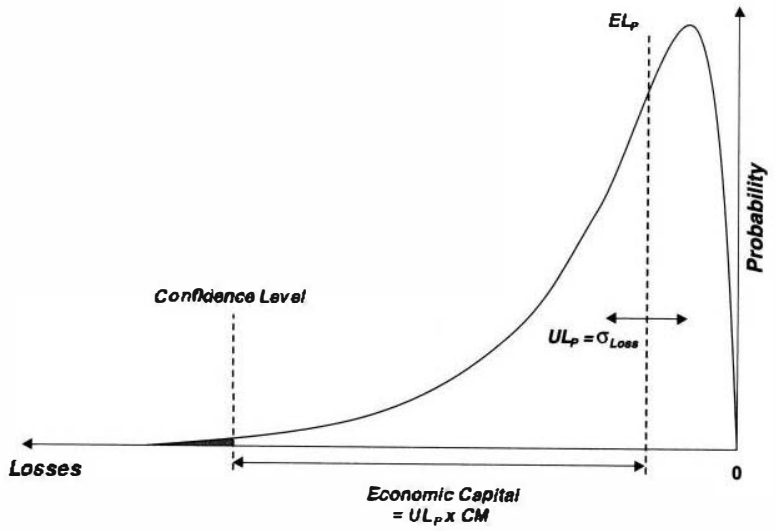

FIGURE 14-2 Economic capital for credit risk.

Source: Adapted from Ong (1999), p. 169.

Viewing the *UL* of a single credit in the context of a credit portfolio[50] reduces the standalone risk considerably in terms of its risk contribution (*ULC*).[51]

Economic Capital for Credit Risk

As defined previously, the amount of economic capital needed is the distance between the expected outcome and the unexpected (negative) outcome at a certain confidence level. As we saw in the last section, the unexpected outcomes at the portfolio level are driven by UL_P, the estimated volatility around the expected loss. Knowing the shape of the loss distribution, EL_P, and UL_P, one can estimate the distance between the expected outcome and the chosen confidence level as a multiple (often labeled as capital multiplier, or *CM*[52]) of UL_P, as shown in Figure 14-2.

Since the sum of ULC_is equals UL_P, we can attribute the necessary economic capital at the single transaction level as follows:

$$Economic\ Capital_P = UL_P \cdot CM \qquad (14.18)$$

[45] As indicated above, one would also need to estimate the correlation of a joint movement in credit quality.

[46] However, they can be estimated from observable asset correlations. See e.g., Gupton et. al. (1997), Ong (1999), pp. 143–145, Pfingsten and Schrock (2000), pp. 14–15.

[47] See Ong (1999), pp. 133–134.

[48] These guidelines often state that a bank should not lend too much money to a single counterparty (i.e., the size effect ignored in Equation [14.16]), the same industry or geography (i .e., the correlation effect ignored in Equation [14.16]).

[49] Such as Monte Carlo simulations; see, for example, Wilson (1997a) and (1997b).

[50] An alternative for determining this marginal risk contribution would be to calculate the *UL* of the portfolio once without and once with the transaction and to build the difference between the two results.

[51] The same approach is applicable to country risk. However, instead of borrower default correlations, country default correlations are applied.

[52] See Ong (1999), p. 163.

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Therefore:

$$Economic\ Capital_i = ULC_i \cdot CM \tag{14.19}$$

that is, the required economic capital at the single credit transaction level is directly proportional to its contribution to the overall portfolio credit risk.

The crucial task in estimating economic capital is, therefore, the choice of the probability distribution, because we are only interested in the tail of this distribution. Credit risks are not normally distributed but highly skewed because, as mentioned previously, the upward potential is limited to receiving at maximum the promised payments and only in very rare events to losing a lot of money.

One distribution often recommended[53] and suitable for this practical purpose is the beta distribution. This kind of distribution is especially useful in modeling a random variable that varies between 0 and c (> 0). And, in modeling credit events,[54] losses can vary between 0 and 100%, so that $c = 1$.[55] The beta distribution is extremely flexible in the shapes of the distribution it can accommodate. When defined between 0 and 1, the beta distribution has the following probability density function:[56]

$$f(x;\alpha,\beta) = \begin{cases} \dfrac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} x^{\alpha-1}(1-x)^{\beta-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases} \tag{14.20}$$

where

$$\Gamma(z) = \int_0^\infty t^z e^{-t} dt$$

By specifying the parameters α and β, we completely determine the shape of the beta distribution. It can be shown[57] that if $\alpha = \beta$, the beta distribution is symmetric and that in our case ($0 < c < 1$) the mean of the beta distribution equals:

$$\mu = EL_p = \int_0^1 x f(x;\alpha,\beta)\, dx = \frac{\alpha}{\alpha+\beta} \tag{14.21}$$

and that the variance equals:

$$\sigma^2 = UL_p^{\ 2} = \int_0^1 x^2 f(x;\alpha,\beta)\, dx - \mu^2 = \left[\frac{\alpha\beta}{(\alpha+\beta)^2 \cdot (\alpha+\beta+1)}\right] \tag{14.22}$$

Therefore, the form of the beta distribution is fully characterized by two parameters: EL_p and UL_p. However, the difficulty is fitting the beta distribution exactly to the tail of the risk profile of the credit portfolio.[58] This tail-fitting exercise is best accomplished by combining the analytical (beta distribution) solution with a numerical procedure such as a Monte Carlo simulation.[59]

Since we try to determine the distance between EL_p and the confidence level, we try to estimate:

$$p\left(\frac{X_H - EL_p}{UL_p} \le CM\right) \le \alpha \tag{14.23}$$

the probability p that the negative deviation of the random variable X exceeds the confidence level only in α% of the cases[60] (as indicated by the gray shaded area in Figure 14-2) in the end of the predetermined measurement period, that is, at time horizon H. Taking the inverse of the beta function at the chosen confidence level, we can determine CM, the capital multiplier, to determine the required amount of economic capital. Obviously, CM is dependent on the overall credit quality of the portfolio and the confidence level. At the typically chosen 99.97% confidence level, CM is between 7.0 and 7.5,[61] which is—given the skewness of the loss distribution—far higher than the capital multiples for the normally distributed events in market risk.

[53] See Ong (1999), p. 164. Other recommended distributions for finding an analytic solution to economic capital are the inverse normal distribution (see Ong (1999), p. 184) or distributions that are also used in extreme value theory (*EVT*) such as Cauchy, Gumbel, or Pareto distributions. For a detailed discussion of *EVT*, see Reiss and Thomas (1997), Embrechts et. al. (1997 and 1998), McNeil and Saladin (1997), and McNeil (1998).

[54] It can be shown that the beta distribution is a continuous approximation of a binomial distribution (the sum of independent two-point distributions).

[55] In Figure 14-2, a credit loss is depicted as a negative deviation, so that $c = -1$ in that case.

[56] See Greene (1993), p. 61.

[57] See Greene (1993), p. 61, and Ong (1999), pp. 165–166.

[58] The tail of a fitted beta distribution depends on the ratio of EL_p/UL_p. For high-quality portfolios ($EL_p > UL_p$) the beta distribution has too fat a tail. Here, the beta distribution usually overestimates economic capital. In contrast, for lower-quality portfolios ($EL_p < UL_p$) it has too thin a tail. See Ong (1999), pp. 184–185.

[59] See Ong (1999), pp. 164 and 170–177, as well as, for a detailed description of the workings of such a model, pp. 179–196.

[60] Mathematically, this implies that the bank needs to hold an economic capital cushion ($CM \times UL_p$) sufficient to make the area under its loss probability distribution equal to 99.97%, if it targets a AA target solvency.

[61] See Ong (1999), pp. 173–177.

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Note that the derivation of the economic capital cushion for country risk is identical to the previously described derivation. However, country risk is more "lumpy," that is, the correlations between single transfer events are higher and there are fewer benefits to diversification because there are only a limited number of countries in the world. Additionally, one needs to consider the correlation between country and counterparty events in deriving the overall economic capital amount.

Problems with the Quantification of Credit Risk

Despite the beauty[62] and simplicity of the bottom-up (total) risk measurement approach just described, there are a number of caveats that need to be addressed:

- This approach assumes that credits are illiquid assets. Therefore, it measures only the risk contribution (i.e., the internal "betas") to the losses of the existing credit portfolio and not the correlation with risk factors as priced in liquid markets. Since the credit risk of bank loans becomes more and more liquid and is traded in the capital markets, a value approach would be more suitable. Such an approach would estimate the expected return and value of the promised payments and would try to model the probability distribution of changes in the value of the loan portfolio to derive the necessary economic capital.
- This, however, would require modeling the multi-period nature of credits and, hence, the expected and unexpected changes in the credit quality of the borrowers (and their correlations). Even though this can be easily included in the analytical approach, the more precise numerical solutions get very complex and cumbersome. Therefore, almost all of the internal credit risk models used in practice[63] use only a one-year estimation horizon.[64]
- Although this approach considers correlations at a practicable level, that is, within the same risk type, it assumes, when measuring, that all other risk components (such as market and operational risk) are separated and are measured and managed in different departments within the bank.

. . .

[62] Contrary to the regulatory approach that assigns roughly 8% equity capital to credits on a standalone basis, this approach reflects the economic perspective with respect to both a differentiated capital attribution by borrower quality as well as in a portfolio context reflecting the benefits to diversification.

[63] For instance CreditMetrics™/CreditManager™ as described by Gupton et. al. (1997), CreditPortfolioView as described by Wilson (1997a and 1997b), and CreditRisk+ as described by CSFP (1997).

[64] See Ong (1999), p. 122.

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Operational Risk

15

Learning Objectives

After completing this reading you should be able to:

- Compare three approaches for calculating regulatory capital.
- Describe the Basel Committee's seven categories of operational risk.
- Derive a loss distribution from the loss frequency distribution and loss severity distribution using Monte Carlo simulations.
- Describe the common data issues that can introduce inaccuracies and biases in the estimation of loss frequency and severity distributions.
- Describe how to use scenario analysis in instances when data is scarce.
- Describe how to identify causal relationships and how to use risk and control self-assessment (RCSA) and key risk indicators (KRIs) to measure and manage operational risks.
- Describe the allocation of operational risk capital to business units.
- Explain how to use the power law to measure operational risk.
- Explain the risks of moral hazard and adverse selection when using insurance to mitigate operational risks.

Excerpt is Chapter 23 of Risk Management and Financial Institutions, *Fourth Edition, by John Hull.*

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In 1999, bank supervisors announced plans to assign capital for operational risk in the new Basel II regulations. This met with some opposition from banks. The chairman and CEO of one major international bank described it as "the dopiest thing I have ever seen." However, as the implementation date for Basel II was approached, bank supervisors did not back down. They listed more than 100 operational risk losses by banks, each exceeding $100 million. Here are some of those losses:

Internal fraud: Allied Irish Bank, Barings, and Daiwa lost $700 million, $1 billion, and $1.4 billion, respectively, from fraudulent trading.

External fraud: Republic New York Corp. lost $611 million because of fraud committed by a custodial client.

Employment practices and workplace safety: Merrill Lynch lost $250 million in a legal settlement regarding gender discrimination.

Clients, products, and business practices: Household International lost $484 million from improper lending practices; Providian Financial Corporation lost $405 million from improper sales and billing practices.

Damage to physical assets: Bank of New York lost $140 million because of damage to its facilities related to the September 11, 2001, terrorist attack.

Business disruption and system failures: Salomon Brothers lost $303 million from a change in computing technology.

Execution, delivery, and process management: Bank of America and Wells Fargo Bank lost $225 million and $150 million, respectively, from systems integration failures and transaction processing failures.

More recently, there has been no shortage of other examples of big operational risk losses. A big rogue trader loss occurred at Société Générale in 2008 and there was another similar loss at UBS in 2011. The London Whale loss occurred at JPMorgan Chase in 2012. In 2014, it was announced that the French bank BNP Paribas would pay $9 billion (roughly one year's profit) to the U.S. government for violating economic sanctions by moving dollar-denominated transactions through the American banking system on behalf of Sudanese, Iranian, and Cuban parties. The bank was also banned from conducting certain U.S. transactions for a year.

An increasingly important type of operational risk for banks is cyber risk. Banks have sophisticated systems in place to protect themselves from cyber attacks, but the attacks themselves are becoming more sophisticated. Also, banks are making increasing use of computer systems and the Internet, providing more opportunities for cyber fraud. Customers and employees have to be continually educated so that the bank's data remains secure. Cyber attacks on banks are attractive to criminals because, to quote the bank robber Willie Surron, "that's where the money is." They are also attractive to terrorists because of their potential to damage a nation's economic security and way of life.

Some regulators now regard operational risk as the most important risk facing banks. To quote Thomas J. Curry, head of the Office of the Comptroller of the Currency (OCC) in the United States, in 2012: "Given the complexity of today's banking markets and the sophistication of the technology that underpins it, it is no surprise that the OCC deems operational risk to be high and increasing. Indeed, it is currently at the top of the list of safety and soundness issues for the institutions we supervise." He goes on to argue that operational risk is more important than credit risk.[1] Most banks have always had some framework in place for managing operational risk. However, the prospect of new capital requirements led them to greatly increase the resources they devote to measuring and monitoring operational risk.

It is much more difficult to quantify operational risk than credit or market risk. Operational risk is also more difficult to manage. Financial institutions make a conscious decision to take a certain amount of credit and market risk, and there are many traded instruments that can be used to reduce these risks. Operational risk, by contrast, is a necessary part of doing business. An important part of operational risk management is identifying the types of risk that are being taken and which should be insured against. There is always a danger that a huge loss will be incurred from taking an operational risk that ex ante was not even recognized as a risk.

It might be thought that a loss such as that at Société Générale was a result of market risk because it was movements in market variables that led to it. However, it should be classified as an operational risk loss because it involved fraud. (Jérôme Kerviel created fictitious trades

[1] Speech to the Exchequer Club, May 16, 2012.

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to hide the big bets he was taking.) Suppose there was no fraud. If it was part of the bank's policy to let traders take huge risks, then the loss would be classified as market risk. But, if this was not part of the bank's policy and there was a breakdown in its controls, it would be classified as operational risk. The SocGen example illustrates that operational risk losses are often contingent on market movements. If the market had moved in Kerviel's favor, there would have been no loss. The fraud and the breakdown in SocGen's control systems might then never have come to light.

There are some parallels between the operational risk losses of banks and the losses of insurance companies. Insurance companies face a small probability of a large loss from a hurricane, earthquake, or other natural disaster. Similarly, banks face a small probability of a large operational risk loss. But there is one important difference. When insurance companies lose a large amount of money because of a natural disaster, all companies in the industry tend to be affected and often premiums rise the next year to cover losses. Operational risk losses tend to affect only one bank. Because it operates in a competitive environment, the bank does not have the luxury of increasing prices for the services it offers during the following year.

DEFINING OPERATIONAL RISK

There are many different ways in which operational risk can be defined. It is tempting to consider operational risk as a residual risk and define it as any risk faced by a financial institution that is not market risk or credit risk. To produce an estimate of operational risk, we could then look at the financial institution's financial statements and remove from the income statement (a) the impact of credit losses and (b) the profits or losses from market risk exposure. The variation in the resulting income would then be attributed to operational risk.

Most people agree that this definition of operational risk is too broad. It includes the risks associated with entering new markets, developing new products, economic factors, and so on. Another possible definition is that operational risk, as its name implies, is the risk arising from operations. This includes the risk of mistakes in processing transactions, making payments, and so on. This definition of risk is too narrow. It does not include major risks such as the risk of a rogue trader such as Jérôme Kerviel.

We can distinguish between internal risks and external risks. Internal risks are those over which the company has control. The company chooses whom it employs, what computer systems it develops, what controls are in place, and so on. Some people define operational risks as all internal risks. Operational risk then includes more than just the risk arising from operations. It includes risks arising from inadequate controls such as the rogue trader risk and the risks of other sorts of employee fraud.

Bank regulators favor including more than just internal risks in their definition of operational risk. They include the impact of external events such as natural disasters (for example, a fire or an earthquake that affects the bank's operations), political and regulatory risk (for example, being prevented from operating in a foreign country by that country's government), security breaches, and so on. All of this is reflected in the following definition of operational risk produced by the Basel Committee on Banking Supervision in 2001:

> *The risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events.*

Note that this definition includes legal risk but does not include reputation risk and the risk resulting from strategic decisions.

Operational risks result in increases in the bank's costs or decreases in its revenue. Some operational risks interact with credit and market risk. For example, when mistakes are made in a loan's documentation, it is usually the case that losses result if and only if the counterparty defaults. When a trader exceeds limits and misreports positions, losses result if and only if the market moves against the trader.

DETERMINATION OF REGULATORY CAPITAL

Banks have three alternatives for determining operational risk regulatory capital. The simplest approach is the *basic indicator approach*. Under this approach, operational risk capital is set equal to 15% of annual gross income over the previous three years. Gross income is defined as net interest income plus noninterest income.[2] A slightly more

[2] Net interest income is the excess of income earned on loans over interest paid on deposits and other instruments that are used to fund the loans.

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TABLE 15-1 Beta Factors in Standardized Approach

Business Line	Beta Factor
Corporate finance	18%
Trading and sales	18%
Retail banking	12%
Commercial banking	15%
Payment and settlement	18%
Agency services	15%
Asset management	12%
Retail brokerage	12%

complicated approach is the *standardized approach*. In this, a bank's activities are divided into eight business lines: corporate finance, trading and sales, retail banking, commercial banking, payment and settlement, agency services, asset management, and retail brokerage. The average gross income over the last three years for each business line is multiplied by a "beta factor" for that business line and the result summed to determine the total capital. The beta factors are shown in Table 15-1. The third alternative is the *advanced measurement approach* (AMA). In this, the operational risk regulatory capital requirement is calculated by the bank internally using qualitative and quantitative criteria. Similarly to credit capital, it is based on a one-year 99.9% VaR.

The Basel Committee has listed conditions that a bank must satisfy in order to use the standardized approach or the AMA approach. It expects large internationally active banks to move toward adopting the AMA approach through time. To use the standardized approach a bank must satisfy the following conditions:

1. The bank must have an operational risk management function that is responsible for identifying, assessing, monitoring, and controlling operational risk.
2. The bank must keep track of relevant losses by business line and must create incentives for the improvement of operational risk.
3. There must be regular reporting of operational risk losses throughout the hank.
4. The bank's operational risk management system must be well documented.
5. The bank's operational risk management processes and assessment system must be subject to regular independent reviews by internal auditors. It must also be subject to regular review by external auditors or supervisors or both.

To use the AMA approach, the bank must satisfy additional requirements. It must be able to estimate unexpected losses based on an analysis of relevant internal and external data, and scenario analyses. The bank's system must be capable of allocating economic capital for operational risk across business lines in a way that creates incentives for the business lines to improve operational risk management.

The objective of banks using the AMA approach for operational risk is analogous to their objectives when they attempt to quantify credit risk. They would like to produce a probability distribution of losses such as that shown in Figure 15-1. Assuming that they can convince regulators that their expected operational risk cost is incorporated into their pricing of products, capital is assigned to cover unexpected costs. The unexpected loss is the difference between the one-year 99.9% VaR and the expected one-year loss.

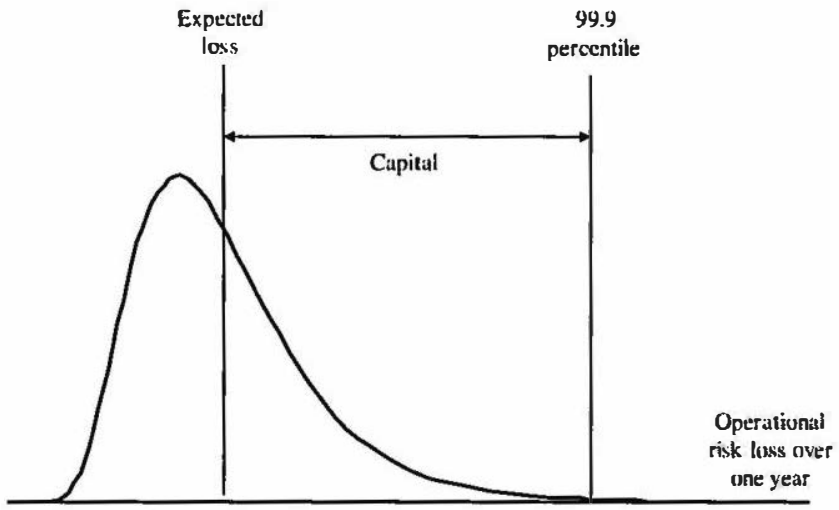

FIGURE 15-1 Calculation of capital for operational risk.

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CATEGORIZATION OF OPERATIONAL RISKS

The Basel Committee on Banking Supervision has identified seven categories of operational risk.[3] These are:

1. *Internal fraud:* Acts of a type intended to defraud, misappropriate property, or circumvent regulations, the law, or company policy (excluding those concerned with diversity or discrimination) involving at least one internal party. Examples include intentional misreporting of positions, employee theft, and insider trading on an employee's own account.
2. *External fraud:* Acts by a third party of a type intended to defraud, misappropriate property, or circumvent the law. Examples include robbery, forgery, check kiting, and damage from computer hacking.
3. *Employment practices and workplace safety:* Acts inconsistent with employment, health or safety laws or agreements, or which result in payment of personal injury claims, or claims relating to diversity or discrimination issues. Examples include workers compensation claims, violation of employee health and safety rules, organized labor activities, discrimination claims, and general liability (for example, a customer slipping and falling at a branch office).
4. *Clients, products, and business practices:* Unintentional or negligent failure to meet a professional obligation to clients and the use of inappropriate products or business practices. Examples are fiduciary breaches, misuse of confidential customer information, improper trading activities on the bank's account, money laundering, and the sale of unauthorized products.
5. *Damage to physical assets:* Loss or damage to physical assets from natural disasters or other events. Examples include terrorism, vandalism, earthquakes, fires, and floods.
6. *Business disruption and system failures:* Disruption of business or system failures. Examples include hardware and software failures, telecommunication problems, and utility outages.
7. *Execution, delivery, and process management:* Failed transaction processing or process management, and disputes with trade counterparties and vendors. Examples include data entry errors, collateral management failures, incomplete legal documentation, unapproved access given to clients accounts, nonclient counterparty misperformance, and vendor disputes.

There are 7 × 8 = 56 combinations of these seven risk types with the eight business lines in Table 15-1. Banks must estimate one-year 99.9% VaRs for each combination and then aggregate them, to determine a single one-year 99.9% operational risk VaR measure.

LOSS SEVERITY AND LOSS FREQUENCY

There are two distributions that are important in estimating potential operational risk losses for a risk type/business line combination. One is the *loss frequency distribution* and the other is the *loss severity distribution*. The loss frequency distribution is the distribution of the number of losses observed during one year. The loss severity distribution is the distribution of the size of a loss, given that a loss occurs. It is usually assumed that loss severity and loss frequency are independent.

For loss frequency, the natural probability distribution to use is a Poisson distribution. This distribution assumes that losses happen randomly through time so that in any short period of time Δt there is a probability $\lambda\Delta t$ of a loss occurring. The probability of n losses in T years is

$$e^{-\lambda T}\frac{(\lambda T)^n}{n!}$$

The parameter λ can be estimated as the average number of losses per year. For example, if during a 10-year period there were a total of 12 losses, λ would be estimated as 1.2 per year. A Poisson distribution has the property that the mean frequency of losses equals the variance of the frequency of losses.[4]

[3] See Basel Committee on Bank Supervision, "Sound Practices for the Management and Supervision of Operational Risk," Bank for International Settlements, July 2002.

[4] If the mean frequency is greater than the variance of the frequency, a binomial distribution may be more appropriate. If the mean frequency is less than the variance, a negative binomial distribution (mixed Poisson distribution) may be more appropriate.

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For the loss-severity probability distribution, a lognormal probability distribution is often used. The parameters of this probability distribution are the mean and standard deviation of the logarithm of the loss.

The loss-frequency distribution must be combined with the loss severity distribution for each risk type/business line combination to determine a loss distribution. Monte Carlo simulation can be used for this purpose.[5] As mentioned earlier, the usual assumption is that loss severity is independent of loss frequency. On each simulation trial, we proceed as follows:

1. We sample from the frequency distribution to determine the number of loss events ($= n$) in one year.
2. We sample n times from the loss severity distribution to determine the loss experienced for each loss event ($L_1, L_2, \ldots, L_n$).
3. We determine the total loss experienced ($= L_1 + L_2 + \ldots + L_n$).

When many simulation trials are used, we obtain a total loss distribution for losses of the type being considered. The 99.9 percentile of the distribution can then be determined.

Figure 15-2 illustrates the procedure. In this example, the expected loss frequency is 3 per year and the loss severity is drawn from a lognormal distribution. The logarithm of each loss ($ millions) is assumed to have a mean of zero and a standard deviation of 0.4. The Excel worksheet used to produce Figure 15-2 is on the author's website: www-2.rotman.utoronto.ca/hull/riskman.

IMPLEMENTATION OF AMA

We now discuss how the advanced measurement approach is implemented in practice. The Basel Committee requires the implementation to involve four elements: internal data, external data, scenario analysis, and business environment and internal control factors.[6] We will consider each of these in turn.

Internal Data

Unfortunately, there is usually relatively little historical data available within a bank to estimate loss severity and loss frequency distributions for particular types of losses. Many banks have not in the past kept records of operational risk losses. They are doing so now, but it may be some time before a reasonable amount of historical data is available. It is interesting to compare operational risk losses with credit risk losses in this respect. Traditionally, banks have done a much better job at documenting their credit risk losses than their operational risk losses. Also, in the case of credit risks, a bank can rely on a wealth of

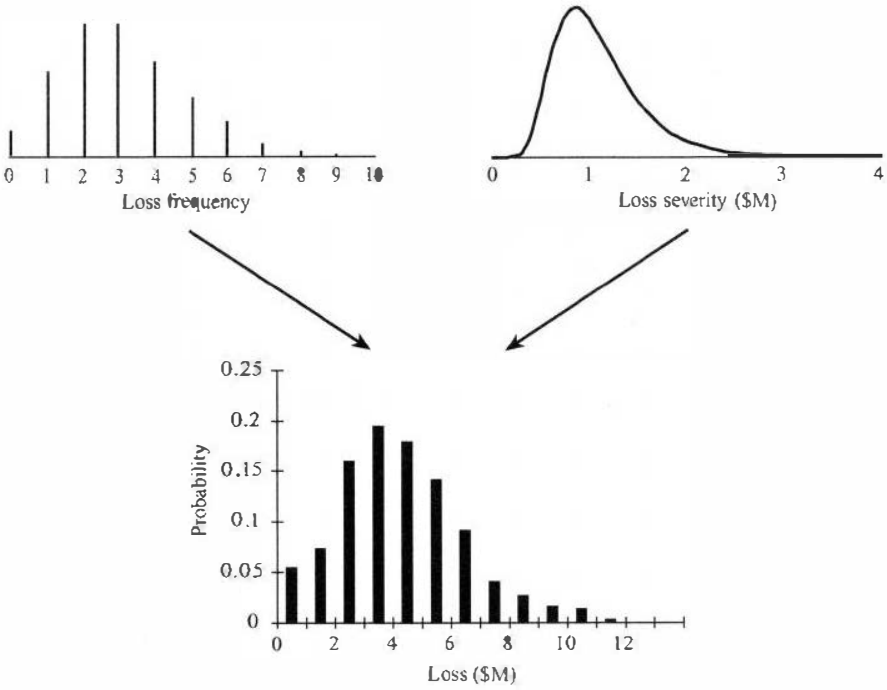

FIGURE 15-2 Calculation of loss distribution from loss frequency and loss severity.

[5] Combining the loss severity and loss frequency distribution is a very common problem in insurance. Apart from Monte Carlo simulation, two approaches that are used are Panjer's algorithm and fast Fourier transforms. See H. H. Panjer, "Recursive Evaluation of a Family of Compound Distributions," *ASTIN Bulletin* 12 (1981): 22–29.

[6] See Basel Committee on Banking Supervision, "Operational Risk: Supervisory Guidelines for the Advanced Measurement Approach," June 2011.

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information published by credit rating agencies to assess probabilities of default and expected losses given default. Similar data on operational risk have not been collected in a systematic way.

There are two types of operational risk losses: high-frequency low-severity losses (HFLSLs) and low-frequency high-severity losses (LFHSLs). An example of the first is credit card fraud losses. An example of the second is rogue trader losses. A bank should focus its attention on LFHSLs. These are what create the tail of the loss distribution. A particular percentile of the total loss distribution can be estimated as the corresponding percentile of the total LFHSL distribution plus the average of the total HFLSL. Another reason for focusing on LFHSLs is that HFLSLs are often taken into account in the pricing of products.

By definition, LFHSLs occur infrequently. Even if good records have been kept, internal data are liable to be inadequate, and must be supplemented with external data and scenario analysis. As we will describe, external data can be used for the loss severity distribution. The loss frequency distribution must be specific to the bank and based on internal data and scenario analysis estimates.

External Data

There are two sources of external data. The first is data consortia, which are companies that facilitate the sharing of data between banks. (The insurance industry has had mechanisms for sharing loss data for many years and banks are now doing this as well.) The second is data vendors, who are in the business of collecting publicly available data in a systematic way. External data increases the amount of data available to a bank for estimating potential losses. It also has the advantage that it can lead to a consideration of types of losses that have never been incurred by the bank, but which have been incurred by other banks.

Both internal and external historical data must be adjusted for inflation. In addition, a scale adjustment should be made to external data. If a bank with a revenue of $10 billion reports a loss of $8 million, how should the loss be scaled for a bank with a revenue of $5 billion? A natural assumption is that a similar loss for a bank with a revenue of $5 billion would be $4 million. But this estimate is probably too small. For example, research by Shih et. al. (2000) suggests that the effect of firm size on the size of a loss experienced is non-linear.[7] Their estimate is

$$\text{Estimated Loss for Bank A} = \text{Observed Loss for Bank B} \times \left(\frac{\text{Bank A Revenue}}{\text{Bank B Revenue}}\right)^{\alpha}$$

where $\alpha = 0.23$. This means that in our example the bank with a revenue of $5 billion would experience a loss of $8 \times 0.5^{0.23} = \$6.82$ million. After the appropriate scale adjustment, data obtained through sharing arrangements with other banks can be merged with the bank's own data to obtain a larger sample for determining the loss severity distribution.

The loss data available from data vendors is data taken from public sources such as newspapers and trade journals. Data from vendors cannot be used in the same way as internal data or data obtained through sharing arrangements because they are subject to biases. For example, only large losses are publicly reported, and the larger the loss, the more likely it is to be reported.

Data from vendors are most useful for determining *relative* loss severity. Suppose that a bank has good information on the mean and standard deviation of its loss severity distribution for internal fraud in corporate finance, but not for external fraud in corporate finance or for internal fraud in trading and sales. Suppose that the bank estimates the mean and standard deviation of its loss severity distribution for internal fraud in corporate finance as $50 million and $30 million. Suppose further that external data indicates that, for external fraud in corporate finance, the mean severity is twice that for internal fraud in corporate finance and the standard deviation of the severity is 1.5 times as great. In the absence of a better alternative, the bank might assume that its own severity for external fraud in corporate finance has a mean of $2 \times 50 = \$100$ million and a standard deviation of severity equal to $1.5 \times 30 = \$45$ million. Similarly, if the external data indicates that the mean severity for internal fraud in trading and sales is 2.5 times that for internal fraud in corporate

[7] See J. Shih, A. Samad-Khan, and P. Medapa, "Is the Size of an Operational Loss Related to Firm Size?" *Operational Risk Magazine* 2, no. 1 (January 2000). Whether Shih et. al.'s results apply to legal risks is debatable. The size of a settlement in a large lawsuit against a bank can be governed by how much the bank can afford.

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finance and the standard deviation is twice as great, the bank might assume that its own severity for internal fraud in trading and sales has a mean of 2.5 × 50 = $125 million and a standard deviation of 2 × 30 = $60 million.

Scenario Analysis

Scenario analysis has become a key tool for the assessment of operational risk under the AMA. The aim of scenario analysis is to generate scenarios covering the full range of possible LFHSLs. Some of these scenarios might come from the bank's own experience, some might be based on the experience of other banks, some might come from the work of consultants, and some might be generated by the risk management group in conjunction with senior management or business unit managers. The Basel Committee estimates that at many banks the number of scenarios considered that give rise to a loss of greater than 10 million euros is approximately 20 times larger than the number of internal losses of this amount.

An operational risk committee consisting of members of the risk management group and senior management should be asked to estimate loss severity and loss frequency parameters for the scenarios. As explained previously, a lognormal distribution is often used for loss severity and a Poisson distribution is often used for loss frequency. Data from other banks may be useful for estimating the loss severity parameters. The loss frequency parameters should reflect the controls in place at the bank and the type of business it is doing. They should reflect the views of the members of the operational risk committee. A number of categories of loss frequency can be defined such as:

1. Scenario happens once every 1,000 years on average ($\lambda = 0.001$)
2. Scenario happens once every 100 years on average ($\lambda = 0.01$)
3. Scenario happens once every 50 years on average ($\lambda = 0.02$)
4. Scenario happens once every 10 years on average ($\lambda = 0.1$)
5. Scenario happens once every 5 years on average ($\lambda = 0.2$)

The committee can be asked to assign each scenario that is developed to one of the categories.

One difference between this scenario analysis and others is that there is no model for determining losses and, if data is nor available, the parameters of the loss severity distribution have to be estimated by the committee. One approach is to ask the committee to estimate an average loss and a high loss that the committee is 99% certain will not be exceeded. A lognormal distribution can then be fitted to the estimates.

Fortunately, the operational risk environment does not usually change as fast as the market and credit risk environment so that the amount of work involved in developing scenarios and keeping them up to date should not be as onerous. Nevertheless, the approach we have described does require a great deal of senior management time. The relevant scenarios for one bank are often similar to those for other banks and, to lessen the burden on the operational risk committee, there is the potential for standard scenarios to be developed by consultants or by bank industry associations. However, the loss frequency estimates should always be specific to a bank and reflect the controls in place in the bank and the type of business it is currently doing.

As in the case of market and credit risk stress testing, the advantage of generating scenarios using managerial judgment is that they include losses that the financial institution has never experienced, but could incur. The scenario analysis approach leads to management thinking actively and creatively about potential adverse events. This can have a number of benefits. In some cases, strategies for responding to an event so as to minimize its severity are likely to be developed. In other cases, proposals may be made for reducing the probability of the event occurring at all.

Whether scenario analysis or internal/external data approaches are used, distributions for particular loss types have to be combined to produce a total operational risk loss distribution. The correlations assumed for the losses from different operational risk categories can make a big difference to the one-year 99.9% VaR that is calculated, and therefore to the AMA capital. Correlations can be used to aggregate economic capital requirements across market risk, credit risk, and operational risk. The same approach can be used to aggregate different operational risk capital requirements. It is often argued that operational risk losses are largely uncorrelated with each other and there is some empirical support for this view. If the zero-correlation assumption is made, Monte Carlo simulation can be used in a straightforward way to sample from the distribution of losses for each scenario to obtain a total distribution of risk losses.

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Business Environment and Internal Control Factors

Business environment and internal control factors (BEICFs) should be taken into account when loss severity and loss frequency are estimated. These include the complexity of the business line, the technology used, the pace of change, the level of supervision, staff turnover rates, and so on. For example, factors influencing the estimates made for the rogue trader scenario might be the level of supervision of traders, the level of trade surveillance, and the strengths or weaknesses of the systems used by the middle and back office.

PROACTIVE APPROACHES

Risk managers should try to be proactive in preventing losses from occurring. One approach is to monitor what is happening at other banks and try and learn from their mistakes. When a $700 million rogue trader loss happened at a Baltimore subsidiary of Allied Irish Bank in 2002, risk managers throughout the world studied the situation carefully and asked: "Could this happen to us?" Business Snapshot 15-1 describes a situation concerning a British local authority in the late 1980s. It immediately led to all banks instituting procedures for checking that counterparties had the authority to enter into derivatives transactions.

Causal Relationships

Operational risk managers should try to establish causal relations between decisions taken and operational risk losses. Does increasing the average educational qualifications of employees reduce losses arising from mistakes in the way transactions are processed? Will a new computer system reduce the probabilities of losses from system failures? Are operational risk losses correlated with the employee turnover rate? If so, can they be reduced by measures taken to improve employee retention? Can the risk of a rogue trader be reduced by the way responsibilities are divided between different individuals and by the way traders are motivated?

One approach to establishing causal relationships is statistical. If we look at 12 different locations where a bank operates and find a high negative correlation between the education of back-office employees and the cost of mistakes in processing transactions, it might well make sense to do a cost-benefit analysis of changing the educational requirements for a back-office job in some of the locations. In some cases, a detailed analysis of the cause of losses may provide insights. For example, if 40% of computer failures can be attributed to the fact that the current hardware is several years old and less reliable than newer versions, a cost-benefit analysis of upgrading is likely to be useful.

BUSINESS SNAPSHOT 15-1

The Hammersmith and Fulham Story

Between 1987 and 1989, the London Borough of Hammersmith and Fulham in Great Britain entered into about 600 interest rate swaps and related transactions with a total notional principal of about six billion pounds. The transactions appear to have been entered into for speculative rather than hedging purposes. The two employees of Hammersmith and Fulham that were responsible for the trades had only a sketchy understanding of the risks they were taking and how the products they were trading worked.

By 1989, because of movements in sterling interest rates, Hammersmith and Fulham had lost several hundred million pounds on the swaps. To the banks on the other side of the transactions, the swaps were worth several hundred million pounds. The banks were concerned about credit risk. They had entered into offsetting swaps to hedge their interest rate risks. If Hammersmith and Fulham defaulted, they would still have to honor their obligations on the offsetting swaps and would take a huge loss.

What actually happened was not a default. Hammersmith and Fulham's auditor asked to have the transactions declared void because Hammersmith and Fulham did not have the authority to enter into the transactions. The British courts agreed. The case was appealed and went all the way to the House of Lords, Britain's highest court. The final decision was that Hammersmith and Fulham did not have the authority to enter into the swaps, but that they ought to have the authority to do so in the future for risk management purposes. Needless to say, banks were furious that their contracts were overturned in this way by the courts.

RCSA and KRIs

Risk control and self-assessment (RCSA) is an important way in which banks try to achieve a better understanding of their operational risk exposures. It involves asking the managers of business units to identify their operational risks. Sometimes questionnaires and scorecards designed by senior management or consultants are used.

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A by-product of any program to measure and understand operational risk is likely to be the development of key risk indicators (KRIs).[8] Risk indicators are key tools in the management of operational risk. The most important indicators are prospective. They provide an early warning system to track the level of operational risk in the organization. Examples of key risk indicators that could be appropriate in particular situations are

1. Staff turnover
2. Number of failed transactions
3. Number of positions filled by temps
4. Ratio of supervisors to staff
5. Number of open positions
6. Percentage of staff that did not take 10 days consecutive leave in the last 12 months

The hope is that key risk indicators can identify potential problems and allow remedial action to be taken before losses are incurred. It is important for a bank to quantify operational risks, hut it is even more important to take actions that control and manage those risks.

E-Mails and Phone Calls

One way in which operational risk costs can be mitigated is by educating employees to be very careful about what they write in e-mails and, when they work on the trading floor, what they say in phone calls. Lawsuits or regulatory investigations contending that a financial institution has behaved inappropriately or illegally are a major source of operational risk. One of the first things that happens when a case is filed is that the financial institution is required to provide all relevant internal communications. These have often proved to be very embarrassing and have made it difficult for financial institutions to defend themselves. Before sending an e-mail or making a recorded phone call, an employee should consider the question "How could it hurt my employer if this became public knowledge?"

ALLOCATION OF OPERATIONAL RISK CAPITAL

Operational risk capital should be allocated to business units in a way that encourages them to improve their operational risk management. If a business unit can show that it has taken steps to reduce the frequency or severity of a particular risk, it should be allocated less capital. This will have the effect of improving the business unit's return on capital (and possibly lead to the business unit manager receiving an increased bonus).

Note that it is not always optimal for a manager to reduce a particular operational risk. Sometimes the costs of reducing the risk outweigh the benefits of reduced capital so that return on allocated capital decreases. A business unit should he encouraged to make appropriate calculations and determine the amount of operational risk that maximizes return on capital.

The overall result of operational risk assessment and operational risk capital allocation should be that business units become more sensitive to the need for managing operational risk. Hopefully, operational risk management will be seen to be an important part of every manager's job. A key ingredient for the success of any operational risk program is the support of senior management. The Basel Committee on Banking Supervision is very much aware of this. It recommends that a bank's board of directors be involved in the approval of a risk management program and that it reviews the program on a regular basis.

USE OF POWER LAW

The power law states that, for a wide range of variables

$$\text{Prob}(v > x) = Kx^{-\alpha}$$

where v is the value of the variable, x is a relatively large value of v, and K and α are constants.

De Fontnouvelle et. al. (2003), using data on losses from vendors, find that the power law holds well for the large losses experienced by banks.[9] This makes the calculation of VaR with high degrees of confidence, such as 99.9%, easier. Loss data (internal or external) and scenario analysis are employed to estimate the power law parameters using the maximum likelihood approach. The 99.9 percentile of the loss distribution can then be estimated.

When loss distributions are aggregated, the distribution with the heaviest tails tends to dominate. This means that

[8] These are sometimes referred to as Business Environment and Internal Control Factors (BEICFs).

[9] See P. De Fontnouvelle, V. DeJesus-Rueff, J. Jordan, and E. Rosengren, "Capital and Risk: New Evidence on Implications of Large Operational Risk Losses," Federal Reserve Board of Boston, Working Paper, September 2003.

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the loss with the lowest α defines the extreme tails of the total loss distribution.[10] Therefore, if all we are interested in is calculating the extreme tail of the total operational risk loss distribution, it may only be necessary to consider one of two business line/risk type combinations.

INSURANCE

An important decision for operational risk managers is the extent to which operational risks should be insured against. Insurance policies are available on many different kinds of risk ranging from fire losses to rogue trader losses. Provided that the insurance company's balance sheet satisfies certain criteria, a bank using AMA can reduce the capital it is required to hold by entering into insurance contracts. We now review the moral hazard and adverse selection risks faced by insurance companies in the context of operational risk.

Moral Hazard

One of the risks facing a company that insures a bank against operational risk losses is moral hazard. This is the risk that the existence of the insurance contract will cause the bank to behave differently than it otherwise would. This changed behavior increases the risks to the insurance company. Consider, for example, a bank that insures itself against robberies. As a result of the insurance policy, it may be tempted to be lax in its implementation of security measures—making a robbery more likely than it would otherwise have been.

Insurance companies have traditionally dealt with moral hazard in a number of ways. Typically there is a *deductible* in any insurance policy. This means that the bank is responsible for bearing the first part of any loss. Sometimes there is a *coinsurance provision* in a policy. In this case, the insurance company pays a predetermined percentage (less than 100%) of losses in excess of the deductible. In addition, there is nearly always a *policy limit*. This is a limit on the total liability of the insurer. Consider again a bank that has insured itself against robberies. The existence of deductibles, coinsurance provisions, and policy limits are likely to provide an incentive for a bank not to relax security measures in its branches.

[10] The parameter ξ in extreme value theory equals $1/\alpha$, so it is the loss distribution with the largest ξ that defines the extreme tails.

BUSINESS SNAPSHOT 15-2

Rogue Trader Insurance

A rogue trader insurance policy presents particularly tricky moral hazard problems. An unscrupulous bank could enter into an insurance contract to protect itself against losses from rogue trader risk and then choose to be lax in its implementation of trading limits. If a trader exceeds the trading limit and makes a large profit, the bank is better off than it would be otherwise. If a large loss results, a claim can be made under the rogue trader insurance policy. Deductibles, coinsurance provisions, and policy limits may mean that the amount recovered is less than the loss incurred by the trader. However, potential net losses to the bank are likely to be far less than potential profits making the lax trading limits strategy a good bet for the bank.

Given this problem, it is perhaps surprising that some insurance companies do offer rogue trader insurance policies. These companies tend to specify carefully how trading limits are implemented within the bank. They may require that the existence of the insurance policy not be revealed to anyone on the trading floor. They are also likely to want to retain the right to investigate the circumstances underlying any loss.

From the bank's point of view, the lax trading limits strategy we have outlined may be very short-sighted. The bank might well find that the costs of all types of insurance rise significantly as a result of a rogue trader claim. Also, a large rogue trader loss (even if insured) would cause its reputation to suffer.

The moral hazard problem in rogue trader insurance is discussed in Business Snapshot 15-2.

Adverse Selection

The other major problem facing insurance companies is adverse selection. This is where an insurance company cannot distinguish between good and bad risks. It offers the same price to everyone and inadvertently attracts more of the bad risks. For example, banks without good internal controls are more likely to enter into rogue trader insurance contracts; banks without good internal controls are more likely to buy insurance policies to protect themselves against external fraud.

To overcome the adverse selection problem, an insurance company must try to understand the controls that exist within banks and the losses that have been experienced. As a result of its initial assessment of risks, it may not charge the same premium for the same contract to all

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banks. As time goes by, it gains more information about the bank's operational risk losses and may increase or reduce the premium charged. This is much the same as the approach adopted by insurance companies when they sell automobile insurance to a driver. At the outset, the insurance company obtains as much information on the driver as possible. As time goes by, it collects more information on the driver's risk (number of accidents, number of speeding tickets, etc.) and modifies the premium charged accordingly.

SARBANES-OXLEY

Largely as a result of the Enron bankruptcy, the Sarbanes-Oxley Act was passed in the United States in 2002. This provides another dimension to operational risk management for both financial and nonfinancial institutions in the United States. The act requires boards of directors to become much more involved with day-to-day operations. They must monitor internal controls to ensure risks are being assessed and handled well.

The act specifies rules concerning the composition of the board of directors of public companies and lists the responsibilities of the board. It gives the SEC the power to censure the board or give it additional responsibilities. A company's auditors are not allowed to carry out any significant non-auditing services for the company.[11] Audit partners must be rotated. The audit committee of the board must be made aware of alternative accounting treatments. The CEO and CFO must prepare a statement to accompany the audit report to the effect that the financial statements are accurate. The CEO and CFO are required to return bonuses in the event that financial statements are restated. Other rules concern insider trading, disclosure, personal loans to executives, reporting of transactions by directors, and the monitoring of internal controls by directors.

SUMMARY

In 1999, bank supervisors indicated their intention to charge capital for operational risk. This has led banks to carefully consider how they should measure and manage operational risk. Bank supervisors have identified seven different types of operational risks and eight different business lines. The most sophisticated banks quantify risks for each of the 56 risk type/business line combinations.

Operational risk losses of a particular type can be treated in much the same way as actuaries treat losses from insurance policies. A frequency of loss distribution and a severity of loss distribution can be estimated and then combined to form a total loss distribution. When they use the advanced measurement approach (AMA), banks are required to use internal data, external data, scenario analysis, and business environment and risk control factors. The external data comes from other banks via data sharing arrangements or from data vendors. The most important tool is scenario analysis. Loss scenarios covering the full spectrum of large operational risk losses are identified. Loss severity can sometimes be estimated from internal and external data. Loss frequency is usually estimated subjectively by the risk management group in conjunction with senior management and business unit managers and should reflect the business environment and risk control factors at the bank.

Risk managers should try to be forward-looking in their approach to operational risk. They should try to understand what determines operational risk losses and try to develop key risk indicators to track the level of operational risk in different parts of the organization.

Once operational risk capital has been estimated, it is important to develop procedures for allocating it to business units. This should be done in a way that encourages business units to reduce operational risk when this can be done without incurring excessive costs.

The power law seems to apply to operational risk losses. This makes it possible to use extreme value theory to estimate the tails of a loss distribution from empirical data. When several loss distributions are aggregated, it is the loss distribution with the heaviest tail that dominates. In principle, this makes the calculation of VaR for total operational risk easier.

Many operational risks can be insured against. However, most policies include deductibles, coinsurance provisions, and policy limits. As a result, a bank is always left bearing part of any risk itself. Moreover, the way insurance premiums change as time passes is likely to depend on the claims made and other indicators that the insurance company has of how well operational risks are being managed.

[11] Enron's auditor, Arthur Andersen, provided a wide range of services in addition to auditing. It did not survive the litigation that followed the downfall of Enron.

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The whole process of measuring, managing, and allocating operational risk is still in its infancy. As time goes by and data are accumulated, more precise procedures than those we have mentioned in this chapter are likely to emerge. One of the key problems is that there are two sorts of operational risk: high-frequency low-severity risks and low-frequency high-severity risks. The former are relatively easy to quantify, but the one-year 99.9% operational risk VaR required by the AMA is largely driven by the latter.

Bank supervisors seem to be succeeding in their objective of making banks more sensitive to the importance of operational risk. In many ways, the key benefit of an operational risk management program is not the numbers that are produced, but the process that banks go through in producing the numbers. If handled well, the process makes managers more aware of the importance of operational risk and perhaps leads to them thinking about it differently.

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Governance over Stress Testing

16

Learning Objectives

After completing this reading you should be able to:

- Describe the key elements of effective governance over stress testing.
- Describe the responsibilities of the board of directors and senior management in stress-testing activities.
- Identify elements of clear and comprehensive policies, procedures and documentations on stress testing.
- Identify areas of validation and independent review for stress tests that require attention from a governance perspective.
- Describe the important role of the internal audit in stress-testing governance and control.
- Identify key aspects of stress-testing governance, including stress-testing coverage, stress-testing types and approaches, and capital and liquidity stress testing.

Excerpt is from Chapter 1 of Stress Testing: Approaches, Methods, and Applications, *by Akhtar Siddique and Iftekhar Hasan.*

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Governance and controls are a very important aspect of stress testing, yet are sometimes overlooked or given insufficient attention by institutions.[1] Proper governance and controls over stress-testing not only confirm that stress tests are conducted in a rigorous manner, but also help ensure that stress tests and their outcomes are subject to an appropriately critical eye. Governance and controls are particularly needed in the area of stress testing given the highly technical nature of many stress-testing activities, the generally large number of assumptions in stress-testing exercises and the inherent uncertainty in estimating the nature, likelihood and impact of stressful events and conditions.

While the exact form of governance and controls over stress-testing activities can and should vary across countries and financial institutions, there are some general principles, expectations and recommendations that financial institutions can follow. The manner in which the principles, expectations and recommendations outlined in this chapter are applied at any given financial institution should involve a "tailored" approach that is specifically tied to the size, complexity, risk profile, culture and individual characteristics of that institution.

This chapter discusses key elements of effective governance over stress testing, including: governance structure; policies, procedures and documentation; validation and independent review; and internal audit. It also discusses other aspects of stress-testing activities that should be considered and reviewed as part of the stress-testing governance process.

GOVERNANCE STRUCTURE

Governance structure is one of the primary elements for sound governance over stress testing. While institutions may have different structures based on the legal, regulatory or cultural norms in their countries, it is generally expected that every institution has separation of duties between a board of directors and senior management. This separation of duties is equally important for stress-testing activities, as it helps ensure there is proper oversight and action taken on an ongoing basis. The board and senior management should share some responsibilities—albeit to varying degrees of detail—but also have distinct responsibilities in other cases. Together, an institution's board and senior management should establish comprehensive, integrated and effective stress testing that fits into the broader risk management of the institution.

Board of Directors

In general, the board of directors has ultimate oversight responsibility and accountability for the entire organisation. It should be responsible for key strategies and decisions, define the culture of the organisation and set the "tone at the top". This applies to stress testing as well, as the board is ultimately responsible for the institution's stress-testing activities, even if the board is not intimately involved in the details. Board members should be sufficiently knowledgeable about stress-testing activities to ask informed questions, even if they are not experts in the technical details. The board should actively evaluate and discuss information received from senior management about stress testing, ensuring that the stress-testing activities are in line with the institution's risk appetite, overall strategies and business plans, and contingency plans—directing changes where appropriate.[2] Board members should also ensure they review that information with an appropriately critical eye, challenging key assumptions, ensuring that there is sufficient information with appropriate detail and supplementing the information with their own views and perspectives.

Stress-testing results should be used, along with other information, to inform the board about alignment of the institution's risk profile with the board's chosen risk appetite, as well as inform operating and strategic decisions. Stress-testing results should be considered directly for decisions relating to capital and liquidity adequacy, including capital contingency plans and contingency funding plans. While stress-testing exercises can be very helpful in providing a forward-looking assessment of the potential impact of adverse outcomes, board members should ensure they use the results of the stress tests with an appropriate degree of skepticism, given the assumptions, limitations and uncertainties inherent in any type of stress testing. In general, the board should not rely on just one stress-test exercise in making key decisions,

[1] For the purposes of this chapter, the term "stress testing" is defined as exercises used to conduct a forward-looking assessment of the potential impact of various adverse events and circumstances on a banking institution.

[2] Risk appetite is defined as the level and type of risk an institution is able and willing to assume in its exposures and business activities, given its business objectives and obligations to stakeholders.

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but should aim to have it supplemented with other tests and other quantitative and qualitative information. The board should be able to take action based on its review of stress-test results and accompanying information, which could include changing capital levels, bolstering liquidity, reducing risk, adjusting exposures, altering strategies or withdrawing from certain activities. In many cases, stress-testing activities can serve as a useful "early-warning" mechanism for the board, especially during benign times (i.e., non-stress periods), and thus can be useful in guiding the overall direction and strategy for the institution.

Senior Management

Senior management has the responsibility of ensuring that stress-testing activities authorised by the board are implemented in a satisfactory manner, and is accountable to the board for the effectiveness of those activities. That is, senior management should execute on the overall stress-testing strategy determined by the board. Senior management duties should include establishing adequate policies and procedures and ensuring compliance with them, allocating appropriate resources and assigning competent staff, overseeing stress-test development and implementation, evaluating stress-test results, reviewing any findings related to the functioning of stress-test processes and taking prompt remedial action where necessary.

In addition, whether directly or through relevant committees, senior management should be responsible for regularly reporting to the board on stress-testing developments (including the process to design tests and develop scenarios) and on stress-testing results (including those from individual tests, where material), as well as on compliance with stress-testing policies. Senior management should ensure there is appropriate buy-in at different levels of the institution, and that stress-testing activities are appropriately coordinated. Such coordination does not have to mean that all stress-testing exercises are built on the same assumptions or use the same information. Indeed, it can be very useful to conduct different types of stress tests to achieve a wide perspective. But senior management should be mindful of potential inconsistencies, contradictions or gaps among its stress tests and assess what actions should be taken as a result. At a minimum, this means that assumptions are transparent and that results are not used in a contradictory manner.

Senior management, in consultation with the board, should ensure that stress-testing activities include a sufficient range of stress-testing activities applied at the appropriate levels of the institution (i.e., not just one single stress test). Another key task is to ensure that stress-test results are appropriately aggregated, particularly for enterprise-wide tests. Senior management should maintain an internal summary of test results to document at a high level the range of its stress-testing activities and outcomes, as well as proposed follow-up actions. Sound governance at this level also includes using stress testing to consider the effectiveness of an institution's risk-mitigation techniques for various risk types over their respective time horizons, such as to explore what could occur if expected mitigation techniques break down during stressful periods.

Stress-test results should inform management's analysis and decision making related to business strategies, limits, capital and liquidity, risk profile and other aspects of risk management, consistent with the institution's established risk appetite. Wherever possible, benchmarking or other comparative analysis should be used to evaluate the stress-testing results relative to other tools and measures—both internal and external to the institution—to provide proper context and a check on results. Just as at the board level, senior management should challenge the results and workings of stress-testing exercises. In fact, senior management should be much more well versed in the details of stress testing and be able to drill down in many cases to discuss technical issues and challenge results on a granular level.

Senior management can and should use stress testing to supplement other information it develops and provides to the board, such as other risk metrics or measures of capital and liquidity adequacy. When reporting stress-testing information to the board, senior management should be able to explain the key elements of stress-testing activities, including assumptions, limitations and uncertainties. Reports from senior management to the board should be clear, comprehensive and current, providing a good balance of succinctness and detail. Those reports should include information about the extent to which stress test models are appropriately governed, including the extent to which they have been subject to validation or other type of independent review (see later in chapter). Senior management, as part of its overall efforts to ensure proper governance and controls, is also responsible for ensuring that staff involved in stress testing operate under the proper incentives. Finally, senior management should ensure that there is a regular assessment of stress-testing

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activities across the institution by an independent, unbiased party (such as internal audit—see later in chapter).

Senior management should ensure that stress-testing activities are updated in light of new risks, better understanding of the institution's exposures and activities, new stress-testing techniques, updated data sources and any changes in its operating structure and its internal and external environment. An institution's stress-testing development should be iterative, with ongoing adjustments and refinements to better calibrate the tests to provide current and relevant information. In addition, management should review stress-testing activities on a regular basis to confirm the general appropriateness of, among other things, the validity of the assumptions, the severity of tests, the robustness of the estimates, the performance of any underlying models and the stability and reasonableness of the results. In addition to conducting formal, routine stress tests, management should ensure the institution has the flexibility to conduct new or ad hoc stress tests in a timely manner to address rapidly emerging risks and vulnerabilities.

POLICIES, PROCEDURES, AND DOCUMENTATION

Having clear and comprehensive policies, procedures and documentation is integral to sound stress-testing governance. These areas provide the important codification of an institution's practices and allow the institution as a whole to follow the same general principles and standards for its stress-testing activities. Thus, in order to promote a sound control environment and allow for consistency and repeatability in stress-testing activities across the entity, the institution should have written policies that direct and govern the implementation of stress-testing activities in a clear and comprehensive manner. It is generally expected that these policies would be approved and annually reassessed by the board. Stress-testing policies, along with procedures to implement them, should:

- describe the overall purpose of stress-testing activities;
- articulate consistent and sufficiently rigorous stress-testing practices across the entire institution;
- indicate stress-testing roles and responsibilities, including controls over external resources used for any part of stress testing (such as vendors and data providers);
- describe the frequency and priority with which stress-testing activities should be conducted;
- outline the process for choosing appropriately stressful conditions for tests, including the manner in which scenarios are designed and selected;
- include information about validation and independent review of stress tests;
- provide transparency to third parties for their understanding of an institution's stress-testing activities;
- indicate how stress-test results are used and by whom, and outline instances in which remedial actions should be taken; and
- be reviewed and updated as necessary to ensure that stress-testing practices remain appropriate and keep up to date with changes in market conditions, the institution's products and strategies, its risks, exposures and activities, its established risk appetite and industry stress-testing practices.

In addition to having clear and comprehensive policies and procedures, an institution should ensure that its stress tests are documented appropriately, including a description of the types of stress tests and methodologies used, test results, key assumptions, limitations and uncertainties, and suggested actions. Among other things, documentation:

- allows management to track results and analyse differences over time, including changes due to methodologies and assumptions as well as changes due to market conditions or other external factors;
- is a vital aspect of stress-testing governance as it allows third parties to evaluate stress tests and their components, including for validation and internal audit review;
- provides for continuity of operations, makes compliance with policy transparent and helps track recommendations, responses and exceptions; and
- is a useful tool for stress-test developers, as it forces them to think clearly about their stress tests, categorise the components of the tests and describe choices made and assumptions used.

Documenting stress tests takes time and effort, so institutions should therefore provide incentives to produce effective and complete documentation. Developers of stress tests should be given explicit responsibility during

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development for thorough documentation, which should be kept up to date as stress testing and application environment change. In addition, the institution should ensure that other participants document their work related to stress-testing activities, including validators, reviewers and senior management. For cases in which a bank uses stress tests from a vendor or other third party, it should ensure that appropriate documentation of the third-party approach is available so that the stress test can be appropriately understood, validated, reviewed, approved and used.

VALIDATION AND INDEPENDENT REVIEW

Another key element of governance over stress testing is validation and independent review. Stress-testing governance should incorporate validation or other type of independent review to ensure the integrity of stress-testing processes and results. Such unbiased, critical review of stress-testing activities gives additional assurance that stress tests are functioning as intended. In general, validation and independent review of stress-testing activities should be conducted on an ongoing basis, not just as a single event. In addition, validation and review work for stress testing should be integrated with an institution's general approach to validation and independent review of its quantitative estimation tools—although stress tests may need to be validated and reviewed in a particular manner. Specifically, because stress tests by definition aim to estimate the potential impact of rare events and circumstances, conducting more traditional outcomes analysis used in a more data-rich environment may not be possible. For instance, statistical backtesting of stress-test estimates against realised outcomes may not be feasible.

To address challenges associated with validating stress tests, some institutions may try to test their models using data from non-stress periods, i.e., during "good times" or in a "baseline" setting. Such testing can be beneficial to determine whether the stress test generally functions as a predictive model under those conditions. If the stress test does not perform well in a more data-rich environment, that would certainly raise questions about its usefulness. However, while "baseline" outcomes showing good test performance can provide some additional confidence in the stress test, those outcomes should not be interpreted as sufficient for the designated task of estimating stress outcomes. For instance, markets and market actors can behave quite differently in stress environments, and assumed interactions among variables can change markedly (such as higher incidence of nonlinearities). Thus, the model used in a baseline situation may actually require a different specification to properly estimate stress outcomes (or an entirely new model may be needed for stress periods). There can be additional challenges when upgrades or enhancements are made to stress tests, because it may not be immediately clear that the upgraded or enhanced model actually performs better. Here, too, assessing the baseline outcomes can provide some assurance about such changes, but cannot offer full confirmation. In sum, even with rigorous quantitative analytics, there can remain very real limitations in the extent to which stress tests can be formally validated or otherwise fully assessed in terms of quantitative performance.

As an additional response to these validation issues, given the limitations of relying on outcomes analysis, an institution may need to rely on other aspects of validation and independent review of stress tests—such as a greater emphasis on conceptual soundness of the stress test, additional sensitivity testing, and simulation techniques. Or an institution may choose to create holdout sample portfolios and run them through its stress-test model. Benchmarking to internal or external models, tools or results can also be beneficial but institutions should be careful that the benchmarks appropriately fit the institution's risks, exposures and activities. Finally, expert-based judgement should be applied to ensure that test results are intuitive and logical, and to add additional perspective on stress-test performance.

Despite these additional efforts, institutions may continue to be challenged in trying to fully validate their stress tests to the same extent as other models, given the limitations in conducting performance testing. Such limitations do not mean that those stress tests cannot be used, but there should be transparency about validation status, and information about the lack of full validation should be communicated to users of stress-test results. For cases in which validation and independent review have identified material deficiencies or limitations in a stress test, there should be a remediation plan to explain how the stress test will be enhanced or its use limited, or both. Identified deficiencies in stress tests should be communicated to all stress-test users.

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Additional areas of validation and independent review for stress tests that require attention from a governance perspective include:

- ensuring that there is appropriate independence and effective challenge in the validation and review process;
- including validation and independent review of the qualitative or judgemental aspects of a stress test—such aspects can be an integral component of a stress test and thus should be reviewed in some manner, even if they cannot be tested in a quantitative/statistical sense;
- ensuring that stress tests are subject to appropriate development standards, including a clear statement of purpose, proper theory and design, sound methodologies and processing components, and developmental testing (including testing of assumptions);
- acknowledging limitations in stress-testing methodologies, even if they represent best practices;
- recognising any data limitations or weaknesses in data quality;
- ensuring that stress tests are implemented in a rigorous manner that is appropriate for the stated use, and accounting for any changes to the developed stress test that occur during implementation;
- monitoring performance on an ongoing basis and assessing any degradation in performance (where possible);
- expressing stress-test uncertainty and inaccuracy, including in the form of confidence bands around estimates and/or factors not observable or not fully incorporated; and
- ensuring that vendor or other third-party models are sufficiently validated, including their implementation, to ensure they function as intended and are appropriate for the institution's use.

INTERNAL AUDIT

An additional aspect of governance and controls is the role of internal audit. An institution's internal audit function evaluates practices in a range of risk-management areas, and stress-testing activities should be among them. Internal audit should provide independent evaluation of the ongoing performance, integrity and reliability of stress-testing activities. It is not expected that internal audit will have full knowledge of all stress-test details, or will have to independently assess each stress test used. Rather, internal audit should look across the firm's stress-testing activities and ensure that, as a whole, they are being conducted in a sound manner, are appropriate for the intended purpose and remain current. There should also be an assessment of the staff involved in stress-testing activities regarding their expertise and roles and responsibilities.

Internal audit should also check that the manner in which all material changes to stress tests and their components are appropriately documented, reviewed and approved. In addition, it should evaluate the validation and independent review conducted for stress tests, including all the items listed above relating to validation. In order to conduct such evaluations, internal audit staff should possess sufficient technical expertise to understand the stress tests and challenge their processes and results. It is also important to review the manner in which stress-testing deficiencies are identified, tracked and remediated. On the whole, internal audit serves the valuable task of assessing the full suite of stress-testing activities across the institution on a regular basis to evaluate whether, as a whole, such activities are functioning as intended, in adherence with policies and procedures and serving the institution properly.

OTHER KEY ASPECTS OF STRESS-TESTING GOVERNANCE

This final section outlines some key aspects of stress-testing governance that should also receive attention, and areas in which governance should be exercised. These include stress-testing coverage, stress-testing types and approaches and capital/liquidity stress testing. The manner in which these areas are addressed can and should vary across institutions. But, at a minimum, these are areas of stress testing that should be addressed in some way by senior management, evaluated as part of the internal control framework summarised for review by the board.

Stress-testing Coverage

- Appropriate coverage in stress-testing activities is important, as stress-testing results could give a false sense of comfort if certain portfolios, exposures, liabilities or business-line activities are not included; this underscores the need to document clearly what is included in each stress test and what is not being covered.

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- Effective stress testing should be applied at various levels in the institution, such as business line, portfolio and risk type, as well as on an enterprise-wide basis; in some cases, stress testing can also be applied to individual exposures or instruments (e.g., structured products).
- Stress testing should capture the interplay among different exposures, activities and risks and their combined effects; while stress testing several types of risks or business lines simultaneously may prove operationally challenging, an institution should aim to identify concentrations and common risk drivers across risk types and business lines that can adversely affect its financial condition—including those not readily apparent during more benign periods.
- Stress testing should be conducted over various relevant time horizons to adequately capture both conditions that may materialize in the near term and adverse situations that take longer to develop.

Stress-testing Types and Approaches

- For any scenario analysis conducted, the scenarios used should be relevant to the direction and strategy set by its board of directors, as well as sufficiently severe to be credible to internal and external stakeholders; at least some scenarios should be of sufficient severity to challenge the viability of the institution.
- Scenarios should consider the impact of both firm-specific and systemic stress events and circumstances that are based on historical experience as well as on hypothetical occurrences that could have an adverse impact on an institution's operations and financial condition.
- An institution should carefully consider the incremental and cumulative effects of stress conditions, particularly with respect to potential interactions among exposures, activities, and risks and possible second-order or "knock-on" effects.
- For an enterprise-wide stress test, institutions should take care in aggregating results across the firm, and business lines and risk areas should use the same assumptions for the chosen scenario, since the objective is to see how the institution as a whole will be affected by a common scenario.
- Consideration should be given to reverse stress tests that "break the bank" to help an institution consider scenarios beyond its normal business expectations and see what kinds of events could threaten its viability (even if it is difficult to estimate their likelihood).

Capital and Liquidity Stress Testing

- Stress testing for capital and liquidity adequacy should be conducted in coordination with an institution's overall strategy and annual planning cycles; results should be refreshed in the event of major strategic decisions, or other decisions that can materially impact capital or liquidity.
- An institution's capital and liquidity stress testing should consider how losses, earnings, cashflows, capital and liquidity would be affected in an environment in which multiple risks manifest themselves at the same time—for example, an increase in credit losses during an adverse interest-rate environment.
- Stress testing can aid contingency planning by helping management identify exposures or risks in advance that would need to be reduced and actions that could be taken to bolster capital and liquidity positions or otherwise maintain capital and liquidity adequacy, as well as actions that in times of stress might not be possible—such as raising capital or accessing debt markets.
- Capital and liquidity stress testing should assess the potential impact of an institution's material subsidiaries suffering capital and liquidity problems on their own, even if the consolidated institution is not encountering problems.
- Effective stress testing should explore the potential for capital and liquidity problems to arise at the same time or exacerbate one another; for example, an institution in a stressed liquidity position is often required to take actions that have a negative direct or indirect capital impact (e.g., selling assets at a loss or incurring funding costs at above market rates to meet funding needs), which can then further exacerbate liquidity problems.
- For capital and liquidity stress tests, it is beneficial for an institution to articulate clearly its objectives for a post-stress outcome, for instance to remain a viable financial market participant that is able to meet its existing and prospective obligations and commitments.

CONCLUSION

Similar to other aspects of risk management, an institution's stress testing will be effective only if it is subject

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to strong governance and effective internal controls to ensure the stress testing activities are functioning as intended. Strong governance and effective internal controls help ensure that stress-testing activities contain core elements, from clearly defined stress-testing objectives to recommended actions. There are many elements that contribute to effective stress-testing governance, foremost being the role of the board and senior management. Stress testing can be a very powerful risk-management tool, but the board and senior management should challenge stress-testing processes and results, demonstrating a solid understanding of their assumptions, limitations and uncertainties. Additionally, strong governance helps ensure that stress testing is not isolated within its risk-management function, but is firmly integrated into business lines, capital and asset-liability committees and other decision-making bodies. Finally, strong governance can help institutions continue to recognise the difficulty in estimating the impact of stressful events and circumstances, thereby acknowledging that stress-test results should be used only with sound judgement and a healthy degree of scepticism.

The views expressed in this chapter do not necessarily represent the views of the Federal Reserve Board or the Federal Reserve System.

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Stress Testing and Other Risk-Management Tools

17

Learning Objectives

After completing this reading you should be able to:

- Describe the relationship between stress testing and other risk measures, particularly in enterprise-wide stress testing.
- Describe the various approaches to using VaR models in stress tests.
- Explain the importance of stressed inputs and their importance in stressed VaR.
- Identify the advantages and disadvantages of stressed risk metrics.

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Excerpt is from Chapter 2 of Stress Testing: Approaches, Methods, and Applications, *by Akhtar Siddique and Iftekhar Hasan.*

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Stress tests have gained in prominence since the financial crisis of 2007–9. However, stress testing existed in the arsenal of risk managers well before the financial crisis. But it has not existed in isolation: along with stress tests, risk managers have always used other tools.

In our experience, quite sophisticated stress testing existed in many banks' management of market risk before the 2007–9 crisis, and it often focused on the trading book. This included both transaction and portfolio-level stress testing. In contrast, stress testing of credit risk was more likely to be at a transaction level. Portfolio-level stress testing was often rudimentary, if it existed at all. Enterprise-wide stress tests tended to be rudimentary (with one or two notable exceptions), as well, especially for institutions that had large banking books.

Risk management in financial institutions has always relied on a panoply of tools and measures. Textbooks on risk management at financial institutions describe various other tools such as position limits and exposure limits, as well as limits on the Greeks, such as on delta or vega.[1]

In this chapter, we discuss the relationship between those other tools and stress testing. We first focus on similarities, differences consistencies between them. We then discuss the ongoing evolution whereby stress testing has affected other risk-management tools. We also discuss how other risk-management tools are affecting stress testing.

Of the other risk measures, we focus on the value-at-risk (VaR) measures. These include the economic capital (EC) measures. This choice is motivated by the fact that such metrics are designed to capture risk across different types (such as market, credit, interest rate, etc.) in a manner similar to stress testing. Additionally, regulatory capital models as used in Basel II/III can also be viewed as akin to EC models. Enterprise-wide risk limits have often been based on value at risk or its variants. More concretely, many institutions have expressed their risk appetite in terms of a very high percentile such as 99.97% EC.

[1] See, for example, Hull (2012).

ENTERPRISE-WIDE STRESS TESTING

As is well known, an important use of stress testing has been to acquire enterprise-wide views of risk, especially in the supervisory stress tests run by regulators around the world. These are the enterprise-wide stress tests.

At a basic level, different risk-management tools can produce different results because of differences in the inputs. For both VaR measures and stress tests, the inputs are data and scenarios.

A stress test may be viewed as translation of a scenario into a loss estimate. In a similar vein, EC or VaR methods also involve translation of scenarios into loss estimates. The distribution of the loss estimates are then used to derive the VaR at a high percentile such as 99% or 99.9%. In practice, stress tests usually focus on a few scenarios, whereas VaR measures commonly utilise a very large number of scenarios.

Hence, as long as identical inputs and similar definitions of loss estimates are used between stress tests and EC/VaR methods, there can be consistency between stress tests and EC/VaR methods, at least when identical scenarios are used.

However, in practice, the loss estimates are often defined quite differently between stress tests and EC methods. In particular, a significant difference is that losses in stress tests have more often than not taken an accounting view rather than a "market" view commonly attempted in EC methods.

The second significant difference has been the horizon. Enterprise-wide stress tests have often examined a long period such as losses over nine quarters in the Dodd-Frank stress tests in the U.S. In contrast, EC models have focused on losses at a point in time, such as the loss in value at the end of a year.

The final significant difference is the role of probabilities. Scenarios for stress tests can sometimes be generated using distributions of the macroeconomic variables. Therefore, the results of a scenario in a stress test can be assigned a probability, i.e., the probability of that scenario. However, probabilities have not played a prominent role in stress tests. For many stress tests conducted around the world, ordinal rank assignments such as "base", "adverse" and "severely adverse" have been done, but with little

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discussion of the cardinal probabilities attached to them. In contrast, cardinal probabilities generally play a large role in the VaR-type models. For the VaR/EC models using Monte Carlo simulation, there exist complex statistical models underneath. For the VaR models using historical simulation, the history has been viewed as the distribution to draw from. More importantly, in the interpretation and use of the VaR/EC model results, probabilities have played a very large role. A 99.9% VaR loss has often been viewed as a 1-in-1,000 event, albeit with uncertainty (or standard errors) around it.

The last difference has been the approach to scenarios. Stress-test scenarios are often *ad hoc* and conditional, rather than the unconditional scenarios typically generated in VaR-type metrics. Especially, for the regulatory stress tests, the scenario-generation process has looked at the present period as the starting point and then generated two or three hypothetical scenarios from that starting point.

A Simple Example: Stress Test

A concrete example can be given for a wholesale portfolio. It is a very simplified example designed to get the idea across rather than provide a guideline to follow. Let us assume the bank is using a two-year scenario that consists of GDP growth and unemployment (see Table 17-1). For wholesale exposures, let us assume that the bank has chosen to model at a portfolio (top-down) level rather than a loan level. At a basic level, the bank needs to estimate the sensitivities of losses in this portfolio to the changes in the two macro variables: GDP growth and unemployment.

Let us assume the following information on the bank's wholesale portfolio (see Table 17-2).

TABLE 17-1 Hypothetical Scenarios for Two Macro Variables

Macro Variable	1st-year Change from Base	2nd-year Change from Base
GDP	-1%	-0.5%
Unemployment	+1%	0%

TABLE 17-2 Portfolio Composition for the Hypothetical Bank

Rating Bucket	Balance	1-year Default Rate (%)	2-year Default Rate (%)
1	200	0.00	0.00
2	350	0.01	0.02
3	400	0.02	0.10
4	500	0.18	0.53
5	100	1.23	3.31
6	10	5.65	12.35
7	0	21.12	33.53

Let us assume that the bank chooses to use a PD LGD approach. Therefore, the bank needs to compute what the PD is in the stress scenario for each of the two years. Additionally, the bank needs to model which of the exposures transition to a lower rating. Finally, the bank needs to understand what new wholesale loans the bank will generate in the two years and what rating buckets (and PD) the new loans will be in.[2] Based on historical experience, the bank establishes the following first-year and second-year stressed PDs. This may be based on the bank's own historical experience or on industry data. The exposures (EAD) are not expected to change. However, the LGD does change. Using the experience of 2008, the bank finds that, according to Moody's URD data, the LGD for senior unsecured increases from 53% to 63%. The bank chooses to increase its LGD by 10% for all rating buckets. Table 17-3 presents the balances and the stressed parameters for the bank's wholesale portfolio. We are assuming no new business and are not taking into account migration between the two years.

The two-year cumulative loss rate comes out to be 10.61% with these assumptions.

[2] For the purposes of this simplified example, we are aware that we are making very strong assumptions and simplifications in this example and are ignoring many elements that banks take into account. For example, banks can find that the underwriting of new loans can actually be stricter in a recession, resulting in a lower PD for new business compared with the existing book.

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TABLE 17-3 Projected Stressed Parameters for the Bank's Portfolio

Rating Bucket	Balance	Stressed LGD	1st-year Stressed PD (%)	2nd-year Stressed PD (%)
1	100	60%	0.02	0.00
2	200	60%	0.03	0.02
3	400	70%	0.04	0.03
4	500	70%	0.25	0.20
5	100	80%	1.85	1.50
6	200	80%	8.00	8.50
7	500	90%	25.00	20.00

A Simple Example, Continued: EC/VaR

In the implementation of EC models, banks commonly use a Merton model framework to simulate the defaults and credit quality. In this framework, asset returns are simulated using a factor model framework, and default occurs when the simulated asset value is below a threshold (generally tied to the leverage of the borrower) at the one-year horizon.

In a multifactor setup, for a borrower i with default probability PD

$$Z_i < N^{-1}(PD_i) \text{ where}$$

$$Z_i = \beta_{i1} GDP + \beta_{i2} Unemployment + \left(\sqrt{1-\beta_{i1}^2-\beta_{i2}^2}\right)\eta_i$$

where Z_i is a unit normal variable and GDP and unemployment are simulated values for the two macroeconomic factors. For credit quality, the simulated asset value (and by extension the simulated leverage) is used to impute a spread. It is common for the shock to the spreads to be modelled as a function of Z_i as well. Banks generally generate the asset values using a correlation matrix using correlations between industries and countries.

Banks run these simulations a number of times, sort the losses from the draws, and arrive at the 99th or 99.9th percentile of the loss distribution as the 99th or 99.9th percentile VaR.

The loss for the stress test may correspond to one of the losses and can allow the user to roughly gauge the severity of the stress test.

USE OF VaR MODELS IN STRESS TESTS

Since the VaR models provide a mechanism for computing loss via

$$Loss = PD \times LGD \times EAD$$

One approach that some institutions have taken is to assess where the losses based on stress tests lie in the loss distribution used in the VaR/EC estimation. This process has been one mechanism to associate probability with a given hypothetical or historical stress scenario. Going one step further, some institutions have also used such mechanisms to tie together scenarios across disparate lines of business.

As an example, if a scenario's loss magnitude translates into a 90th percentile loss on the loss distribution for VaR, the bank may take the 90th percentile loss in the EC model as an approximation to the stressed loss for market risk.

No financial institution can be run with zero risk tolerance, nor can all sources of risk be eliminated. However, clearly, some losses are unacceptable because of their magnitudes, irrespective of the scenarios. For such losses, the likelihood (or probability) of the scenario is not that material. However, for most scenarios, the output tends to be used as the loss in that scenario and the likelihood of that scenario.

The assignment of probability via "matching" the stressed loss to a point on the loss distribution serves the useful purpose of coming up with the probability of that scenario. Since, for the practical implementation of stress tests in risk management, assignment of probabilities to the outcomes is important, the probability arrived via the loss distribution can help make the stress tests more actionable.[3]

STRESSED CALIBRATION OF VALUE AT RISK MEASURES

Another approach to incorporating stress into risk measurement methodologies has been the use of stressed inputs. There have been quite a few variants. This has been particularly useful in the market risk area. The

[3] Action triggers (when actions need to be taken) for stress tests can be tied to either the output—for example, if the losses exceed a certain level. Alternatively, they can be tied to the input, i.e., if the realised input into a test is below/above a threshold. As an example, a stress test can involve a scenario for GDP growth. If GDP growth in a quarter is below a trigger such as -2%, actions can be taken.

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incorporation of stress into the risk measurement as well as capital metrics has occurred in both the supervisory approaches and the many banks' internal approaches.

On the supervisory approaches, the 2011 Basel market risk rule requires banks to use stressed inputs, i.e., the revisions to the market risk capital framework (BCBS 2011a) states,

> In addition, a bank must calculate a "stressed value-at-risk" measure. This measure is intended to replicate a value-at-risk calculation that would be generated on the bank's current portfolio if the relevant market factors were experiencing a period of stress; and should therefore be based on the 10-day, 99th percentile, one-tailed confidence interval value-at-risk measure of the current portfolio, with model inputs calibrated to historical data from a continuous 12-month period of significant financial stress relevant to the bank's portfolio.

The revisions to the market risk capital framework also explicitly require the use of stress tests: "Banks that use the internal models approach for meeting market risk capital requirements must have in place a rigorous and comprehensive stress-testing program."

Similarly, in the revisions to Basel III (BCBS 2011b), stressed parameters are required: "To determine the default risk capital charge for counterparty credit risk as defined in paragraph 105, banks must use the greater of the portfolio-level capital charge (not including the CVA charge in paragraphs 97–104) based on Effective EPE using current market data and the portfolio-level capital charge based on Effective EPE using a stress calibration. The stress calibration should be a single consistent stress calibration for the whole portfolio of counterparties."

As an illustration, we present some results from Siddique (2010), with six risk factors to simulate the exposures. These are: (1) three month LIBOR (LIBOR3M); (2) the yield on BAA-rated bonds (BAA); (3) the spread between yields on BAA- and AAA-rated bonds (BAA–AAA); (4) the return on the S&P 500 index (SPX); (5) the change in the volatility option index (VIX); and (6) contract interest rates on commitments for fixed-rate first mortgages (from the Freddie Mac survey) (MORTG). MORTG is in a weekly frequency that is converted to daily data through imputation using a Markov chain Monte Carlo. There are a total of 2,103 daily observations over the period January 2, 2002, through May 10, 2010.

With Monte Carlo, the stressed VaR as 99.9th percentile of a distribution of P&Ls generated using stressed parameters can be constructed. Two separate sets of moments, (1) using the previous 180 days or 750 days' history of the risk factors and (2) the stress period (180 days or 750 days ending in 30.06.09), are used to simulate the risk factors. The 99.9th percentile of the portfolio value is then the 99.9th regular VaR or stressed VaR based on which sets of moments are used. Figure 17-1 illustrates VaR and stressed VaR with a balanced portfolio.

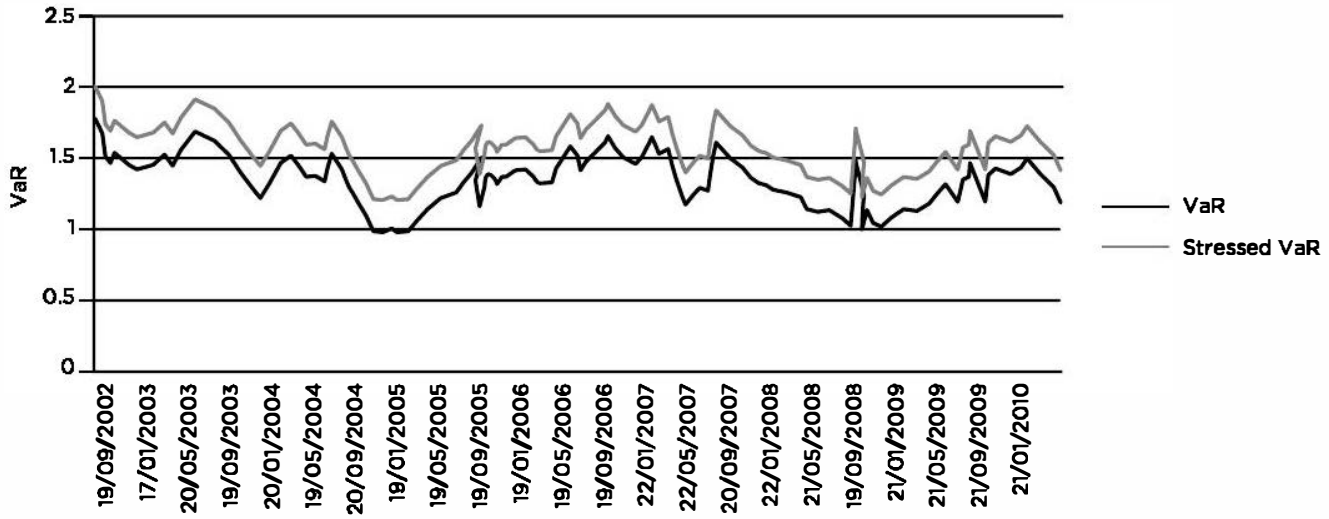

FIGURE 17-1 VaR and stressed VaR with a balanced portfolio.

Source: Siddique (2010).

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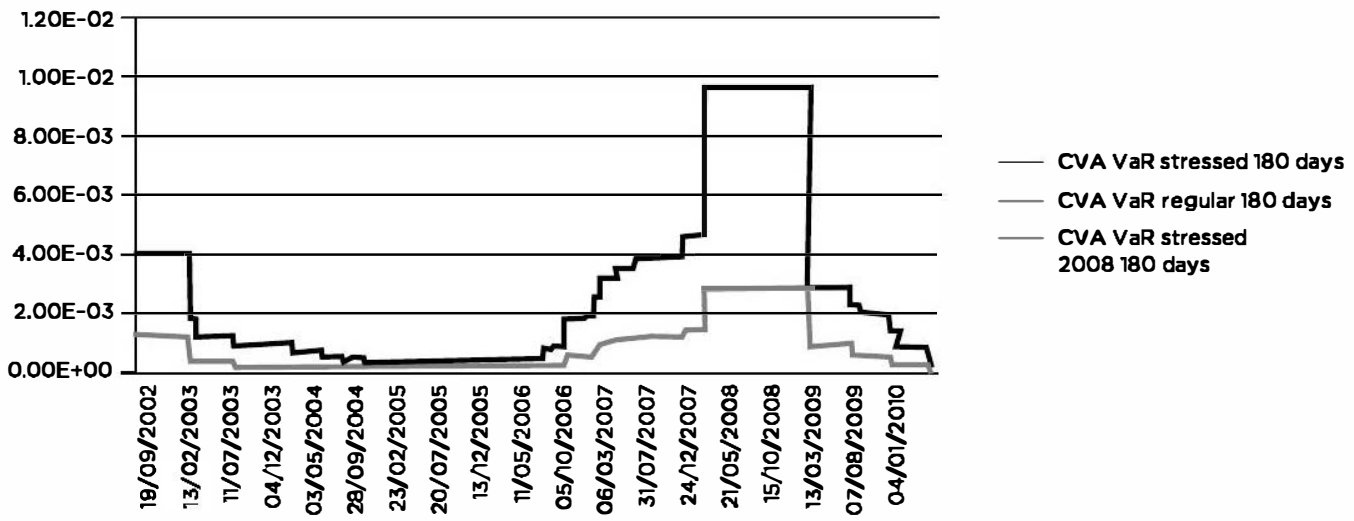

FIGURE 17-2 Comparison of VaR and stressed VaR of CVA: different stressed periods.

Source: Siddique (2010).

Stressed inputs are also used in the capital charge for credit valuation adjustment (CVA) as mentioned above. To assess the impact of the use of stressed inputs for those metrics, Siddique (2010) carries out some other simulations whose results are presented in Figure 17-2.

Two separate periods are used to compute the stressed calibration: (1) 180 days ending 30.09.08; and (2) 180 days ending 30.06.09. The impact of a stressed calibration appears in the early period in the data, where the CVA VaR is substantially higher than the unstressed (regular) CVA VaR. However, in the latter period the unstressed and stressed VaR are identical. It is important to note that an incorrect stress period (i.e., ending 30.09.08) can actually produce VaR lower than an unstressed CVA VaR.

There are both advantages and disadvantages of such stressed risk metrics. An obvious advantage is that, with capital for unexpected losses taking into account stressed environments, capital should be adequate when the next stress or shock occurs. That is, a risk metric with a stressed input is usually going to be more conservative.

However, given that the inputs are always stressed, the risk metric will no longer be responsive to the current market conditions, but primarily depend on the portfolio composition.

Only time will tell what the final impact of the incorporation of stress-testing elements into risk management and capital adequacy will be.

CONCLUSION

Stress testing has played a very large role in the assessment of capital adequacy. It has always played a role in risk management as well, which has become much larger as a result of the 2007–9 financial crisis. However, banks have continued to use other risk-management tools such as VaR as well. Nevertheless, stress testing has influenced those tools and those tools have also been used in stress testing.

The views expressed in this chapter are those of the authors alone and do not necessarily represent those of the Comptroller of the Currency or the Bank of Finland.

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Principles for Sound Stress Testing Practices and Supervision

18

Learning Objectives

After completing this reading you should be able to:

- Describe the rationale for the use of stress testing as a risk management tool.
- Describe weaknesses identified and recommendations for improvement in:
  - The use of stress testing and integration in risk governance
  - Stress testing methodologies
  - Stress testing scenarios
  - Stress testing handling of specific risks and products
- Describe stress testing principles for banks regarding the use of stress testing and integration in risk governance, stress testing methodology and scenario selection, and principles for supervisors.

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Excerpt is from Principles for Sound Stress Testing Practices and Supervision, *by Bank for International Settlements, Basel Committee on Banking Supervision Publication.*

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INTRODUCTION

The depth and duration of the financial crisis has led many banks and supervisory authorities to question whether stress testing practices were sufficient prior to the crisis and whether they were adequate to cope with rapidly changing circumstances. In particular, not only was the crisis far more severe in many respects than was indicated by banks' stress testing results, but it was possibly compounded by weaknesses in stress testing practices in reaction to the unfolding events. Even as the crisis is not over yet there are already lessons for banks and supervisors emerging from this episode.

Stress testing is an important risk management tool that is used by banks as part of their internal risk management and, through the Basel II capital adequacy framework, is promoted by supervisors. Stress testing alerts bank management to adverse unexpected outcomes related to a variety of risks and provides an indication of how much capital might be needed to absorb losses should large shocks occur. While stress tests provide an indication of the appropriate level of capital necessary to endure deteriorating economic conditions, a bank alternatively may employ other actions in order to help mitigate increasing levels of risk. Stress testing is a tool that supplements other risk management approaches and measures. It plays a particularly important role in:

- providing forward-looking assessments of risk;
- overcoming limitations of models and historical data;
- supporting internal and external communication;
- feeding into capital and liquidity planning procedures;
- informing the setting of a bank's risk tolerance; and
- facilitating the development of risk mitigation or contingency plans across a range of stressed conditions.

Stress testing is especially important after long periods of benign economic and financial conditions, when fading memory of negative conditions can lead to complacency and the underpricing of risk. It is also a key risk management tool during periods of expansion, when innovation leads to new products that grow rapidly and for which limited or no loss data is available.

Pillar 1 (minimum capital requirements) of the Basel II framework requires banks using the Internal Models Approach to determine market risk capital to have in place a rigorous programme of stress testing. Similarly, banks using the advanced and foundation internal ratings-based (IRB) approaches for credit risk are required to conduct credit risk stress tests to assess the robustness of their internal capital assessments and the capital cushions above the regulatory minimum. Basel II also requires that, at a minimum, banks subject their credit portfolios in the banking book to stress tests. Recent analysis has concluded that banks' stress tests did not produce large loss numbers in relation to their capital buffers going into the crisis or their actual loss experience. Further, banks' firm-wide stress tests should have included more severe scenarios than the ones used in order to produce results more in line with the actual stresses that were observed.

The Basel Committee has engaged with the industry in examining stress testing practices over this period and this paper is the result of that examination. Notwithstanding the ongoing evolution of the crisis and future lessons that may emerge, this paper assesses stress testing practices during the crisis. Based on that assessment and in an effort to improve practices, it develops sound principles for banks and supervisors. The principles cover the overall objectives, governance, design and implementation of stress testing programmes as well as issues related to stress testing of individual risks and products.

The recommendations are aimed at deepening and strengthening banks' stress testing practices and supervisory assessment of these practices. By itself, stress testing cannot address all risk management weaknesses, but as part of a comprehensive approach, it has a leading role to play in strengthening bank corporate governance and the resilience of individual banks and the financial system.

A stress test is commonly described as the evaluation of a bank's financial position under a severe but plausible scenario to assist in decision making within the bank. The term "stress testing" is also used to refer not only to the mechanics of applying specific individual tests, but also to the wider environment within which the tests are developed, evaluated and used within the decision-making process. In this paper, we use the term "stress testing" in this wider sense.

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PERFORMANCE OF STRESS TESTING DURING THE CRISIS[1]

The financial crisis has highlighted weaknesses in stress testing practices employed prior to the start of the crisis in four broad areas: (i) use of stress testing and integration in risk governance; (ii) stress testing methodologies; (iii) scenario selection; and (iv) stress testing of specific risks and products.

Use of Stress Testing and Integration in Risk Governance

Board and senior management involvement is critical in ensuring the appropriate use of stress testing in banks' risk governance and capital planning. This includes setting stress testing objectives, defining scenarios, discussing the results of stress tests, assessing potential actions and decision making. At banks that were highly exposed to the financial crisis and fared comparatively well, senior management as a whole took an active interest in the development and operation of stress testing, with the results of stress tests serving as an input into strategic decision making which benefited the bank. Stress testing practices at most banks, however, did not foster internal debate nor challenge prior assumptions such as the cost, risk and speed with which new capital could be raised or that positions could be hedged or sold.

The financial crisis has also revealed weaknesses in organisational aspects of stress testing programmes. Prior to the crisis, stress testing at some banks was performed mainly as an isolated exercise by the risk function with little interaction with business areas. This meant that, amongst other things, business areas often believed that the analysis was not credible. Moreover, at some banks, the stress testing programme was a mechanical exercise. While there is room for routinely operated stress tests within a comprehensive stress testing programme (e.g., for background monitoring), they do not provide a complete picture because mechanical approaches can neither fully take account of changing business conditions nor incorporate qualitative judgements from across the different areas of a bank. Furthermore, in many banks, stress tests were carried out by separate units focusing on particular business lines or risk types. This led to organisational barriers when aiming to integrate quantitative and qualitative stress testing results across a bank.

Prior to the crisis, many banks did not have an overarching stress testing programme in place but ran separate stress tests for particular risks or portfolios with limited firm-level integration. Risk-specific stress testing was usually conducted within business lines. While stress testing for market and interest rate risk had been practiced for several years, stress testing for credit risk in the banking book has only emerged more recently. Other types of stress tests are still in their infancy. As a result, there was insufficient ability to identify correlated tail exposures and risk concentrations across the bank.

Stress testing frameworks were usually not flexible enough to respond quickly as the crisis evolved (e.g., inability to aggregate exposures quickly, apply new scenarios or modify models). Further investments in IT infrastructure may be necessary to enhance the availability and granularity of risk information that will enable timely analysis and assessment of the impact of new stress scenarios designed to address a rapidly changing environment. For example, investing in liquidity risk management information systems would enhance a bank's ability to automate end-of-day information, obtain more granularity as to unencumbered assets and forecast balance sheet needs of business units.

Stress Testing Methodologies

Stress tests cover a range of methodologies. Complexity can vary, ranging from simple sensitivity tests to complex stress tests, which aim to assess the impact of a severe macroeconomic stress event on measures like earnings and economic capital.[2] Stress tests may be performed at

[1] The discussion of the market crisis is based on information provided to the Basel Committee through discussions with industry representatives; the work of the Senior Supervisors Group (SSG); industry reports such as that produced by the Institute of International Finance (IIF); questionnaires and industry workshops; and from the knowledge obtained by individual agencies through their own supervisory and information gathering activities.

[2] For an overview of different stress testing objectives and how these relate to modelling see Drehmann (2008), "Stress Tests: Objectives, Challenges and Modelling Choices", *Riksbank Economic Review*, June. For a discussion of economic capital, see *Range of practices and issues in economic capital frameworks*, Basel Committee on Banking Supervision, March 2009.

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varying degrees of aggregation, from the level of an individual instrument up to the institutional level. Stress tests are performed for different risk types including market, credit, operational and liquidity risk. Notwithstanding this wide range of methodologies, the crisis has highlighted several methodological weaknesses.

At the most fundamental level, weaknesses in infrastructure limited the ability of banks to identify and aggregate exposures across the bank. This weakness limits the effectiveness of risk management tools—including stress testing.

Most risk management models, including stress tests, use historical statistical relationships to assess risk. They assume that risk is driven by a known and constant statistical process, i.e., they assume that historical relationships constitute a good basis for forecasting the development of future risks. The crisis has revealed serious flaws with relying solely on such an approach.

First, given a long period of stability, backward-looking historical information indicated benign conditions so that these models did not pick up the possibility of severe shocks nor the build up of vulnerabilities within the system. Historical statistical relationships, such as correlations, proved to be unreliable once actual events started to unfold.

Second, the financial crisis has again shown that, especially in stressed conditions, risk characteristics can change rapidly as reactions by market participants within the system can induce feedback effects and lead to system-wide interactions. These effects can dramatically amplify initial shocks as recent events have illustrated.[3]

[3] At the outset of the crisis, mortgage default shocks played a part in the deterioration of market prices of collateralised debt obligations (CDOs). Simultaneously, these shocks revealed deficiencies in the models used to manage and price these products. The complexity and resulting lack of transparency led to uncertainty about the value of the underlying investment. Market participants then drastically scaled down their activity in the origination and distribution markets and liquidity disappeared. The standstill in the securitisation markets forced banks to warehouse loans that were intended to be sold in the secondary markets. Given a lack of transparency of the ultimate ownership of troubled investments, funding liquidity concerns were triggered within the banking sector as banks refused to provide sufficient funds to each other. This in turn led to the hoarding of liquidity, exacerbating further the funding pressures within the banking sector. The initial difficulties in subprime mortgages also fed through to a broader range of market instruments since the drying up of market and funding liquidity forced market participants to liquidate those positions which they could trade in order to scale back risk. An increase in risk aversion also led to a general flight to quality, an example of which was the high withdrawals by households from money market funds.

Extreme reactions (by definition) occur rarely and may carry little weight in models that rely on historical data. It also means that they are hard to model quantitatively. The management of most banks did not sufficiently question these limitations of more traditional risk management models used to derive stress testing outcomes nor did they sufficiently take account of qualitative expert judgement to develop innovative ad-hoc stress scenarios. Therefore, banks generally underestimated the strong interlinkages between, for example, the lack of market liquidity and funding liquidity pressures. The reliance on historical relationships and ignoring reactions within the system implied that firms underestimated the interaction between risks and the firm-wide impact of severe stress scenarios.

Prior to the crisis, most banks did not perform stress tests that took a comprehensive firm-wide perspective across risks and different books. Even if they did, the stress tests were insufficient in identifying and aggregating risks. As a result, banks did not have a comprehensive view across credit, market and liquidity risks of their various businesses. An appropriately conducted firm-wide stress test would have beneficially drawn together experts from across the organisation. For example, the expertise of retail lenders, who in some cases were reducing exposure to US subprime mortgages, should have counteracted the overly optimistic outlook of traders in securities backed by the same subprime loans.

Scenario Selection

Most bank stress tests were not designed to capture the extreme market events that were experienced. Most firms discovered that one or several aspects of their stress tests did not even broadly match actual developments. In particular, scenarios tended to reflect mild shocks, assume shorter durations and underestimate the correlations between different positions, risk types and markets due to system-wide interactions and feedback effects. Prior to the crisis, "severe" stress scenarios typically resulted in estimates of losses that were no more than a quarter's worth of earnings (and typically much less).

A range of techniques has been used to develop scenarios. Sensitivity tests, which are at the most basic level, generally shock individual parameters or inputs without relating those shocks to an underlying event or real-world outcomes. Given that these scenarios ignore multiple risk factors or feedback effects, their main benefit is that

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they can provide a fast initial assessment of portfolio sensitivity to a given risk factor and identify certain risk concentrations.

More sophisticated approaches apply shocks to many parameters simultaneously. Approaches are typically either historically based or hypothetical.

Historical scenarios were frequently implemented based on a significant market event experienced in the past. Such stress tests were not able to capture risks in new products that have been at the centre of the crisis. Furthermore, the severity levels and duration of stress indicated by previous episodes proved to be inadequate. The length of the stress period was viewed as unprecedented and so historically based stress tests underestimated the level of risk and interaction between risks.

Banks also implemented hypothetical stress tests, aiming to capture events that had not yet been experienced. Prior to the crisis, however, banks generally applied only moderate scenarios, either in terms of severity or the degree of interaction across portfolios or risk types. At many banks, it was difficult for risk managers to obtain senior management buy-in for more severe scenarios. Scenarios that were considered extreme or innovative were often regarded as implausible by the board and senior management.

Stress Testing of Specific Risks and Products

Particular risks that were not covered in sufficient detail in most stress tests include:

- the behaviour of complex structured products under stressed liquidity conditions;
- pipeline or securitisation risk;
- basis risk in relation to hedging strategies;
- counterparty credit risk;
- contingent risks; and
- funding liquidity risk.

Scenarios were not sufficiently severe when stress testing structured products and leveraged lending prior to the crisis. This may, to some degree, be attributed to reliance on historical data. In general, stress tests of structured products suffered from the same problems as other risk management models in this area in that they failed to recognise that risk dynamics for structured instruments are different from those of similarly-rated cash instruments such as bonds. These differences were particularly pronounced during the crisis, further degrading the performance of the stress tests. In particular, stress tests should specifically consider the credit quality of the underlying exposures, as well as the unique characteristics of structured products. Furthermore, stress tests also assumed that markets in structured products would remain liquid or, if market liquidity would be impaired, that this would not be the case for a prolonged period. This also meant that banks underestimated the pipeline risk related to issuing new structured products.

In many cases stress tests dealt only with directional risk and did not capture basis risk, thereby reducing the effectiveness of hedges. Another feature of the crisis was wrong-way risk, for example related to the credit protection purchased from monoline insurers.[4]

In addition, stress tests for counterparty credit risk typically only stressed a single risk factor for a counterparty, were insufficiently severe and usually omitted the interaction between credit risk and market risk (specific wrong way risk). Stress testing for counterparty credit risk should be improved by utilising stresses applied across counterparties and to multiple risk factors, as well as those that incorporate current valuation adjustments.

Another weakness of the models was that they did not adequately capture contingent risks that arose either from legally binding credit and liquidity lines or from reputational concerns related, for example, to off-balance sheet vehicles. Had stress tests adequately captured contractual and reputational risk associated with off-balance sheet exposures, concentrations in such exposures may have been avoided.

With regard to funding liquidity, stress tests did not capture the systemic nature of the crisis or the magnitude and duration of the disruption to interbank markets. For a more in-depth discussion of the shortcomings of liquidity stress tests, see the Basel Committee's *Principles for Sound Liquidity Risk Management and Supervision* (September 2008).

[4] Some credits on which banks and dealers had purchased protection from monolines to help manage risk on their structured credit origination activities became impaired at the same time that the creditworthiness of the monolines deteriorated.

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Changes in Stress Testing Practices Since the Outbreak of the Crisis

Given the unexpected severity of events, stress testing has gained greater prominence and credibility within banks as a complementary risk management and capital planning tool to provide a different risk perspective. It is important that this process continues so that stress testing programmes become embedded in banks' governance structures. Moreover, this process needs to be led by the board and senior management.

Banks recognise that current stress testing frameworks must be enhanced both in terms of granularity of risk representation and the range of risks considered. Some banks have started to address these issues and other weaknesses of stress tests for the specific risks identified above. More general areas in which banks are considering future improvement include:

- constantly reviewing scenarios and looking for new ones;
- examining new products to identify potential risks;
- improving the identification and aggregation of correlated risks across books as well as the interactions between market, credit and liquidity risk; and
- evaluating appropriate time horizons and feedback effects.

Generally, firm-wide stress testing is an area that many banks recognise they will need to improve to ensure appropriate risk capture and to aggregate risk more effectively across business lines. The principles set forth in this paper are intended to support and reinforce efforts made by banks to improve their practices, but banks should not restrict themselves to a checklist approach to improvement.

After the onset of the crisis, ad hoc "hot-spot" stress testing has been used by some banks as an important tool to inform senior management's crisis management decisions. The ability to conduct stress tests at very short notice has proven to be valuable during a period of rapidly changing market conditions.

The need for improvement in stress testing has also been recognised by the financial industry. In July 2008 the Institute of International Finance published its *Final Report of the IIF Committee on Market Best Practices: Principles of Conduct and Best Practice Recommendations*. The report among other things reviewed stress testing practices and set out two principles and five specific recommendations in this area. The principles include the need for stress testing to be carried out comprehensively and integrated with the overall risk management infrastructure. They also identified the need for stress testing to have a meaningful impact on business decisions, with the board and senior management having an important role in evaluating stress test results and impact on a bank's risk profile. Recommendations by the Counterparty Risk Management Policy Group (CRMPG III) in its August 2008 report (*Containing Systemic Risk: The Road to Reform—The Report of the CRMPG III*) include the need for firms to think creatively about how the value of stress tests can be maximised, including a so-called reverse stress test to explore the events that could cause a significant impact on the firm.

The following recommendations are formulated with a view towards application to large, complex banks. The extent of application should be commensurate with the size and complexity of a bank's business and the overall level of risk that it accepts. These recommendations should therefore be applied to banks on a proportionate basis.

PRINCIPLES FOR BANKS

Use of Stress Testing and Integration in Risk Governance

1. Stress testing should form an integral part of the overall governance and risk management culture of the bank. Stress testing should be actionable, with the results from stress testing analyses impacting decision making at the appropriate management level, including strategic business decisions of the board and senior management. Board and senior management involvement in the stress testing programme is essential for its effective operation.

The board has ultimate responsibility for the overall stress testing programme, whereas senior management is accountable for the programme's implementation, management and oversight. Recognising that many practical aspects of a stress testing programme will be delegated, the involvement of the board in the overall stress testing programme and of senior management in the programme's design are essential. This will help ensure the

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board's and senior management's buy-in to the process. It will also help maximise the effective use of stress tests, especially with respect to firm-wide stress testing. The rationale for particular choices, as well as their principal implications, should be explained and documented so that the board and senior management are aware of the limitations of the stress tests performed (e.g., key underlying assumptions, the extent of judgement in evaluating the impact of the stress test or the likelihood of the event occurring). Stress testing should promote candid discussion on modelling assumptions between the board and risk managers.

Senior management should be able to identify and clearly articulate the bank's risk appetite and understand the impact of stress events on the risk profile of the bank. Senior management must participate in the review and identification of potential stress scenarios, as well as contribute to risk mitigation strategies. In addition, senior management should consider an appropriate number of well-understood, documented, utilised and sufficiently severe scenarios that are relevant to their bank. Senior management's endorsement of stress testing as a guide in decision-making is particularly valuable when the tests reveal vulnerabilities that the bank finds costly to address.

A stress testing programme as a whole should be actionable and feed into the decision making process at the appropriate management level, including strategic business decisions of the board or senior management. Stress tests should be used to support a range of decisions. In particular but not exclusively, stress tests should be used as an input for setting the risk appetite of the firm or setting exposure limits. Stress tests should also be used to support the evaluation of strategic choices when undertaking and discussing longer term business planning. Importantly, stress tests should feed into the capital and liquidity planning process.

2. A bank should operate a stress testing programme that promotes risk identification and control; provides a complementary risk perspective to other risk management tools; improves capital and liquidity management; and enhances internal and external communication.

A stress testing programme is an integrated strategy for meeting a range of purposes (described below) by means of the origination, development, execution and application of a suitable range of stress tests. The range of purposes requires the use of a range of techniques since stress testing is not a one-size-fits-all approach.

To promote risk identification and control, stress testing should be included in risk management activities at various levels. This includes the use of stress testing for the risk management of individual or groups of borrowers and transactions, for portfolio risk management, as well as for adjusting a bank's business strategy. In particular, it should be used to address existing or potential firm-wide risk concentrations.

Stress testing should provide a complementary and independent risk perspective to other risk management tools such as value-at-risk (VaR) and economic capital. Stress tests should complement risk management approaches that are based on complex, quantitative models using backward looking data and estimated statistical relationships. In particular, stress testing outcomes for a particular portfolio can provide insights about the validity of statistical models at high confidence intervals, for example those used to determine VaR.

Importantly, as stress testing allows for the simulation of shocks which have not previously occurred, it should be used to assess the robustness of models to possible changes in the economic and financial environment. In particular, appropriate stress tests should challenge the projected risk characteristics of new products where limited historical data are available and which have not been subject to periods of stress. Users should also simulate stress scenarios in which the model-embedded statistical relationships break down as has been observed during the recent market crisis. Use of these various stress tests should help to detect vulnerabilities such as unidentified risk concentrations or potential interactions between types of risk that could threaten the viability of the bank, but may be concealed when relying purely on statistical risk management tools based on historical data.

Stress testing should form an integral part of the internal capital adequacy assessment process (ICAAP), which requires banks to undertake rigorous, forward-looking stress testing that identifies severe events or changes in market conditions that could adversely impact the bank. Stress testing should also be a central tool in identifying, measuring and controlling funding liquidity risks, in particular for assessing the bank's liquidity profile and the

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adequacy of liquidity buffers in case of both bank-specific and market-wide stress events.[5]

Stress tests should play an important role in the communication of risk within the bank. In contrast to purely statistical models, plausible forward-looking scenarios are more easily grasped and thereby assist in the assessment of vulnerabilities and evaluation of the feasibility and effectiveness of potential counter actions. Stress tests should also play an important role in external communication with supervisors to provide support for internal and regulatory capital adequacy assessments. A bank may also want to voluntarily disclose its stress test results more broadly to enable the market to better understand its risk profile and management. If a bank does voluntarily disclose its stress test results, it may also wish to provide relevant supporting information in order to ensure that informed judgements of the results can be made by third parties. This supporting information could include any major stress test limitations, underlying assumptions, the methodologies used and an evaluation of the impact of the stress test.

3. Stress testing programmes should take account of views from across the organisation and should cover a range of perspectives and techniques.

The identification of relevant stress events, the application of sound modelling approaches and the appropriate use of stress testing results each require the collaboration of different senior experts within a bank such as risk controllers, economists, business managers and traders. A stress testing programme should ensure that opinions of all relevant experts are taken into account, in particular for firm-wide stress tests. The unit with responsibility for implementing the stress testing programme should organise appropriate dialogue among these experts, challenge their opinions, check them for consistency (e.g., with other relevant stress tests) and decide on the design and the implementation of the stress tests, ensuring an adequate balance between usefulness, accuracy, comprehensiveness and tractability.

Banks should use multiple perspectives and a range of techniques in order to achieve comprehensive coverage in their stress testing programme. These include quantitative and qualitative techniques to support and complement the use of models and to extend stress testing to areas where effective risk management requires greater use of judgement. Stress tests should range from simple sensitivity analysis based on changes in a particular risk factor to more complex stress tests that revalue portfolios taking account of the interactions among risk drivers conditional on the stress event. Some stress tests should be run at regular intervals whilst the stress testing programme should also allow for the possibility of ad hoc stress testing.

Sensitivity analysis is generally intended to assess the output from quantitative approaches when certain inputs and parameters are stressed or shocked.[6] In most cases, sensitivity analysis involves changing inputs or parameters without relating those changes to an underlying event or real-world outcomes. For example, a sensitivity test might explore the impact of varying declines in equity prices (such as by 10%, 20%, 30%) or a range of increases in interest rates (such as by 100, 200, 300 basis points). While it is helpful to draw on extreme values from historical periods of stress, sensitivity analysis should also include hypothetical extreme values to ensure that a wide range of possibilities is included. In some cases, it can be helpful to conduct a scenario analysis of several factors at the same time because simply testing factors individually may not reveal their potential interaction (particularly if that interaction is complex and not intuitively clear). For example, scenarios can evaluate the combined impact on credit risk capital needs from sudden spikes in probabilities of default and concurrent changes in the dependence parameters of a credit capital model.

Sensitivity and scenario analysis has additional benefits in helping to reveal whether quantitative approaches are working as originally intended.[7] For example, one can check the assumption that a relationship continues to be linear when extreme inputs are used. If the analysis results show that a certain model is unstable or does not work as originally intended with extreme inputs, then management should consider rethinking the model, modifying certain parameters, or at least putting less weight on the

[5] See also *Principles for Sound Liquidity Risk Management and Supervision*, Basel Committee on Banking Supervision, September 2008.

[6] Note that using less extreme values of parameters and inputs can also be useful in sensitivity analysis.

[7] In this manner, sensitivity analysis can also play an important role in validation.

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accuracy of model output. Finally, sensitivity and scenario analysis should be conducted regularly (not just during model development), since models can deteriorate and relationships among variables can change over time.

4. A bank should have written policies and procedures governing the stress testing programme. The operation of the programme should be appropriately documented.

The stress testing programme should be governed by internal policies and procedures. These should be appropriately documented.

The programme should be documented particularly in relation to firm-wide stress tests. The following aspects should be detailed: (i) the type of stress testing and the main purpose of each component of the programme; (ii) frequency of stress testing exercises which is likely to vary depending on type and purpose; (iii) the methodological details of each component, including the methodologies for the definition of relevant scenarios and the role of expert judgement; and (iv) the range of remedial actions envisaged, based on the purpose, type and result of the stress testing, including an assessment of the feasibility of corrective actions in stress situations. Documentation requirements should not, however, impede the bank from being able to perform flexible ad-hoc stress testing, which by their nature need to be completed quickly and often to respond to emerging risk issues.

A bank should document the assumptions and fundamental elements for each stress testing exercise. These include the reasoning and judgements underlying the chosen scenarios and the sensitivity of stress testing results to the range and severity of the scenarios. An evaluation of such fundamental assumptions should be performed regularly or in light of changing external conditions. Furthermore, a bank should document the outcome of such assessments.

5. A bank should have a suitably robust infrastructure in place, which is sufficiently flexible to accommodate different and possibly changing stress tests at an appropriate level of granularity.

Commensurate with the principle of proportionality, a bank should have suitably flexible infrastructure as well as data of appropriate quality and granularity. The infrastructure should enable the bank on a timely basis to aggregate its exposures to a given risk factor, product or counterparty, and modify methodologies to apply new scenarios as needed.

The infrastructure should also be sufficiently flexible to allow for targeted or ad-hoc stress tests at the business line or firm-wide level to assess specific risks in times of stress. System flexibility is crucial to handle customised and changing stress tests and to aggregate comparable risks and exposures across a bank.

6. A bank should regularly maintain and update its stress testing framework. The effectiveness of the stress testing programme, as well as the robustness of major individual components, should be assessed regularly and independently.

The effectiveness and robustness of stress tests should be assessed qualitatively as well as quantitatively, given the importance of judgements and the severity of shocks considered. Areas for assessment should include:

- the effectiveness of the programme in meeting its intended purposes;
- documentation;
- development work;
- system implementation;
- management oversight;
- data quality; and
- assumptions used.

The quantitative processes should include benchmarking with other stress tests within and outside the bank.

Since the stress test development and maintenance processes often imply judgemental and expert decisions (e.g., assumptions to be tested, calibration of the stress, etc.), the independent control functions such as risk management and internal audit should also play a key role in the process.

Stress Testing Methodology and Scenario Selection

7. Stress tests should cover a range of risks and business areas, including at the firm-wide level. A bank should be able to integrate effectively, in a meaningful fashion, across the range of its stress

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testing activities to deliver a complete picture of firm-wide risk.

A stress testing programme should consistently and comprehensively cover product-, business- and entity-specific views. Using a level of granularity appropriate to the purpose of the stress test, stress testing programmes should examine the effect of shocks across all relevant risk factors, taking into account interrelations among them.

A bank should also use stress tests to identify, monitor and control risk concentrations.[8] In order to adequately address risk concentrations, the scenario should be firm-wide and comprehensive, covering balance sheet and off-balance sheet assets, contingent and non-contingent risks, independent of their contractual nature. Further, stress tests should identify and address potential changes in market conditions that could adversely impact a bank's exposure to risk concentrations.

The impact of stress tests is usually evaluated against one or more measures. The particular measures used will depend on the specific purpose of the stress test, the risks and portfolios being analysed and the particular issue under examination. A range of measures may need to be considered to convey an adequate impression of the impact. Typical measures used are:

- asset values;
- accounting profit and loss;
- economic profit and loss;
- regulatory capital or risk weighted assets;
- economic capital requirements; and
- liquidity and funding gaps.

Developing coherent stress testing scenarios on a firm-wide basis is a difficult task as risk factors for different portfolios differ widely and horizons vary.[9] For example, deriving a coherent scenario for market and credit risk is not straightforward as market risk materialises quickly whereas credit risk will need a longer time horizon to feed through the system. However, in order to effectively challenge the business model and support the decision-making process, the scenarios have to assess the nature of linked risks across portfolios and across time. A relevant aspect in this regard is the role played by liquidity conditions for determining the ultimate impact of a stress test.

8. Stress testing programmes should cover a range of scenarios, including forward-looking scenarios, and aim to take into account system-wide interactions and feedback effects.

An effective stress testing programme should comprise scenarios along a spectrum of events and severity levels. Doing so will help deepen management's understanding of vulnerabilities and the effect of non-linear loss profiles. Stress testing should be conducted flexibly and imaginatively, in order to better identify hidden vulnerabilities. A "failure of imagination" could lead to an underestimation of the likelihood and severity of extreme events and to a false sense of security about a bank's resilience.

The stress testing programme should cover forward-looking scenarios to incorporate changes in portfolio composition, new information and emerging risk possibilities which are not covered by relying on historical risk management or replicating previous stress episodes. The compilation of forward-looking scenarios requires combining the knowledge and judgement of experts across the organisation. The scenarios should be based on senior management dialogue and judgements. The challenge is to stimulate discussion and to use the information at different levels of the bank in a productive way.

An appropriate stress testing framework should comprise a broad range of scenarios covering risks at different levels of granularity, including firm-wide stress tests, as well as product-, business- and entity-specific stress tests. Some stress scenarios should provide insight into the firm-wide impact of severe stress events on a bank's financial strength and allow for an assessment of the bank's ability to react to events. In general, stress scenarios should reflect the materiality of particular business areas and their vulnerability to changes in economic and financial conditions.

The financial crisis has shown that estimating *ex ante* the probabilities of stress events is problematic. The statistical

[8] These may arise along different dimensions: single name concentrations; concentrations in regions or industries; concentrations in single risk factors; concentrations that are based on correlated risk factors that reflect subtler or more situation-specific factors, such as previously undetected correlations between market and credit risks, as well as between those risks and liquidity risk; concentrations in indirect exposures via posted collateral or hedge positions; concentrations in off-balance sheet exposure, contingent exposure, non-contractual obligations due to reputational reasons.

[9] As suggested in principle 21, supervisors should engage in cross-border efforts with other public bodies, as well as with the industry, to discuss stress testing practices.

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relationships used to derive the probability tend to break down in stressed conditions. In this respect, the crisis has underscored the importance of giving appropriate weight to expert judgement in defining relevant scenarios with a forward-looking perspective.

Stress testing should include various time horizons depending on the risk characteristics of the analysed exposures and whether the particular test is intended for tactical or strategic use. A natural starting point for stress tests conducted for risk management purposes is the relevant risk management horizon of the target portfolio and the liquidity of the underlying exposures. However, there is need to cover substantially longer periods than this as liquidity conditions can change rapidly in stressed conditions. The bank should also assess the impact of recession-type scenarios, including its ability to react over a medium to long time horizon. The bank should note the increased importance of assumptions as the time horizon of a stress test is lengthened. A bank should also consider incorporating feed-back effects and firm-specific and market-wide reactions into such stress tests.

When analysing the potential impact of a set of macroeconomic and financial shocks, a bank should aim to take into account system-wide interactions and feedback effects. Recent events have demonstrated that these effects have the capacity to transform isolated stress events into global crisis threatening even large, well capitalised banks, as well as systemic stability. As they occur rarely, they are generally not contained in historical data series used for daily risk management. A stress test supplemented with expert judgement can help to address these deficiencies in an iterative process and thereby improve risk identification.

9. Stress tests should feature a range of severities, including events capable of generating the most damage whether through size of loss or through loss of reputation. A stress testing programme should also determine what scenarios could challenge the viability of the bank (reverse stress tests) and thereby uncover hidden risks and interactions among risks.

Commensurate with the principle of proportionality, stress tests should feature the most material business areas and events that might be particularly damaging for the firm. This could include not only events that inflict large losses but which subsequently cause damage to the bank's reputation.

Reverse stress tests start from a known stress test outcome (such as breaching regulatory capital ratios, illiquidity or insolvency) and then asking what events could lead to such an outcome for the bank. As part of the overall stress testing programme, it is important to include some extreme scenarios which would cause the firm to be insolvent (i.e., stress events which threaten the viability of the whole firm). For a large complex firm, this is a challenging exercise requiring involvement of senior management and all material risk areas across the firm.[10]

A reverse stress test induces firms to consider scenarios beyond normal business settings and leads to events with contagion and systemic implications. Hence, reverse stress testing has important quantitative and qualitative uses, such as informing senior management's assessment of vulnerabilities. For example, a bank with a large exposure to complex structured credit products could have asked what kind of scenario would have led to widespread losses such as those observed in the financial crisis. Given this scenario, the bank would have then analysed its hedging strategy and assessed whether this strategy would be robust in the stressed market environment characterised by a lack of market liquidity and increased counterparty credit risk. Given the appropriate judgements, this type of stress test can reveal hidden vulnerabilities and inconsistencies in hedging strategies or other behavioural reactions.

Before the financial market crisis, such an analysis was considered of little value by most senior management since the event had only a remote chance of happening. However, banks now express the need for examining tail events and assessing the actions to deal with them. Some banks have expressed successes in using this kind of stress test to identify risk concentrations and vulnerabilities. A good reverse stress test also includes enough diagnostic support to investigate the reasons for potential failure.

Areas which benefit in particular from the use of reverse stress testing are business lines where traditional risk management models indicate an exceptionally good risk/return trade-off; new products and new markets which have not experienced severe strains; and exposures where there are no liquid two-way markets.

[10] See also *The Report of the CRMPG III* (August 2008).

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10. As part of an overall stress testing programme, a bank should aim to take account of simultaneous pressures in funding and asset markets, and the impact of a reduction in market liquidity on exposure valuation.

Funding and asset markets may be strongly interrelated, particularly during periods of stress. The recent crisis has demonstrated this fact in several circumstances, impacting severely on the financial condition of individual banks and affecting systemic stability. Banks did not address in their risk management approaches significant linkages between asset and funding liquidity.

A bank should enhance its stress testing practices by considering important interrelations between various factors, including:

- price shocks for specific asset categories;
- the drying-up of corresponding asset liquidity;
- the possibility of significant losses damaging the bank's financial strength;
- growth of liquidity needs as a consequence of liquidity commitments;
- taking on board affected assets; and
- diminished access to secured or unsecured funding markets.[11]

Specific Areas of Focus

The following recommendations to banks focus on the specific areas of risk mitigation and risk transfer that have been highlighted by the financial crisis.

11. The effectiveness of risk mitigation techniques should be systematically challenged.

Stress testing should facilitate the development of risk mitigation or contingency plans across a range of stressed conditions. The performance of risk mitigating techniques, like hedging, netting and the use of collateral, should be challenged and assessed systematically under stressed conditions when markets may not be fully functioning and multiple institutions simultaneously could be pursuing similar risk mitigating strategies.

[11] See also *Principles for Sound Liquidity Risk Management and Supervision*, Basel Committee on Banking Supervision (September 2008).

12. The stress testing programme should explicitly cover complex and bespoke products such as securitised exposures. Stress tests for securitised assets should consider the underlying assets, their exposure to systematic market factors, relevant contractual arrangements and embedded triggers, and the impact of leverage, particularly as it relates to the subordination level in the issue structure.

Banks have mistakenly assessed the risk of some products (e.g., CDOs of ABS) by relying on external credit ratings or historically observed credit spreads related to (seemingly) similar products like corporate bonds with the same external rating. Such approaches cannot capture relevant risk characteristics of complex, structured products under severely stressed conditions. A bank, therefore, should include in its stress tests all relevant information related to the underlying asset pools, their dependence on market conditions, complicated contractual arrangements as well as effects related to the subordination level of the specific tranches.

13. The stress testing programme should cover pipeline and warehousing risks. A bank should include such exposures in its stress tests regardless of their probability of being securitised.

Stress testing is particularly important in the management of warehouse and pipeline risk associated with underwriting and securitisation activities. Many of the risks associated with pipeline and warehoused exposures emerge when a bank is unable to access the securitisation market due to either bank specific or market stresses. A bank should therefore include such exposures in its regular stress tests regardless of the probability of the pipeline exposures being securitised.

14. A bank should enhance its stress testing methodologies to capture the effect of reputational risk. The bank should integrate risks arising from off-balance sheet vehicles and other related entities in its stress testing programme.

To mitigate reputational spill-over effects and maintain market confidence, a bank should develop methodologies to measure the effect of reputational risk on other risk types, with a particular focus on credit, liquidity and market risks. For instance, a bank should include non-contractual off-balance sheet exposures in its stress tests

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to determine the effect on its credit, liquidity and market risk profiles.

A bank should carefully assess the risks associated with commitments to off-balance sheet vehicles related to structured credit securities and the possibility that assets will need to be taken on balance sheet for reputational reasons. Therefore, in its stress testing programme, a bank should include scenarios assessing the size and soundness of such vehicles relative to its own financial, liquidity and regulatory capital positions. This analysis should include structural, solvency, liquidity and other risk issues, including the effects of covenants and triggers.

15. A bank should enhance its stress testing approaches for highly leveraged counterparties in considering its vulnerability to specific asset categories or market movements and in assessing potential wrong-way risk related to risk mitigating techniques.

A bank may have large gross exposures to leveraged counterparties including hedge funds, financial guarantors, investment banks and derivatives counterparties that may be particularly exposed to specific asset types and market movements. Under normal conditions, these exposures are typically completely secured by posted collateral and continuous re-margining agreements yielding zero or very small net exposures. In case of severe market shocks, however, these exposures may increase abruptly and potential cross-correlation of the creditworthiness of such counterparties with the risks of assets being hedged may emerge (i.e., wrong-way risk). A bank should enhance its stress testing approaches related to these counterparties in order to capture adequately such correlated tail risks.

PRINCIPLES FOR SUPERVISORS

16. Supervisors should make regular and comprehensive assessments of a bank's stress testing programme.

Supervisors should assess banks' compliance with sound stress testing practices, including the aspects listed under *Principles for banks*.

Supervisors should verify the active involvement of senior management in the stress testing programme and require a bank to submit at regular intervals the results of its firm-wide stress testing programme. Supervisors should evaluate how the stress testing analysis impacts the bank's decision making at different management levels, including strategic business decisions of the board and senior management.

Supervisors should verify that stress testing forms an integral part of the ICAAP and of the bank's liquidity risk management framework. Supervisors should also verify that a bank devotes sufficient resources and develops explicit procedures to undertake rigorous, forward-looking stress testing in order to identify possible adverse events that could significantly impact the bank and threaten its viability. Supervisors should engage senior management in regular communication to discuss its view on major macroeconomic and financial market vulnerabilities as well as threats specific to the bank's operations and business model.

17. Supervisors should require management to take corrective action if material deficiencies in the stress testing programme are identified or if the results of stress tests are not adequately taken into consideration in the decision-making process.

In making their assessments of a bank's stress testing programme, supervisors should assess the effectiveness of the programme in identifying relevant vulnerabilities. Supervisors should review the key assumptions driving stress testing results and challenge their continuing relevance in view of existing and potentially changing market conditions. Supervisors should challenge banks on how stress testing is used and the way it impacts upon decision-making. Where this assessment reveals material shortcomings, supervisors should require the bank to detail a plan of corrective action.

The range of remedial action should take into consideration the magnitude and likelihood of potential stress events and be proportionate to the severity of the impact of the stress test, the overall risk management framework and to other limiting or risk mitigating policies. The measures undertaken by supervisors may involve:

- the review of limits;
- the recourse to risk mitigation techniques;
- the reduction of exposures to specific sectors, countries, regions or portfolios;
- the revision of bank policies, such as those that relate to funding or capital adequacy; and
- the implementation of contingency plans.

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18. Supervisors should assess and if necessary challenge the scope and severity of firm-wide scenarios. Supervisors may ask banks to perform sensitivity analysis with respect to specific portfolios or parameters, use specific scenarios or to evaluate scenarios under which their viability is threatened (reverse stress testing scenarios).

Supervisors should question a bank's methodology when the impact of stress tests seems unrealistically low or when mitigating actions are unrealistic.

Supervisors should ensure that a bank conducts appropriate sensitivity analysis at multiple levels of the organisation. They should ensure that a bank's sensitivity analysis is rigorous, includes different types of tests, and incorporates a range of extreme values (from mild to extreme) for inputs and parameters. In their evaluations, supervisors should review whether the bank uses output from sensitivity tests appropriately, shares sensitivity analysis results within the organisation (such as with risk managers and senior management) and properly acts upon the results (e.g., by taking remedial actions if sensitivity tests show large adverse outcomes or reveal model weaknesses).

Supervisors should evaluate whether the scenarios are consistent with the risk appetite the bank has set for itself. Supervisors should ensure that the scenarios chosen by the bank are appropriate to its risk profile and business mix and that they include a severe and sustained downturn. The scenarios chosen should also include, where relevant, an episode of financial market turbulence or a shock to market liquidity.

Supervisors may ask a bank to evaluate scenarios under which the bank's viability is compromised and may ask the bank to test scenarios for specific lines of business, or to assess the plausibility of events that could lead to significant strategic or reputational risk, in particular for significant business lines.

19. Under Pillar 2 (supervisory review process) of the Basel II framework, supervisors should examine a bank's stress testing results as part of a supervisory review of both the bank's internal capital assessment and its liquidity risk management. In particular, supervisors should consider the results of forward-looking stress testing for assessing the adequacy of capital and liquidity.

Supervisors should examine the future capital resources and capital needs of a bank under adverse scenarios. In particular, supervisors should examine the results of forward-looking stress testing as part of a supervisory evaluation of the adequacy of capital buffers. Supervisors should assess capital adequacy under stressed conditions against a variety of capital ratios, including regulatory ratios, as well as ratios based on a bank's internal definition of capital resources.

Supervisors should take account of the extent to which capital might not be freely transferable within banking groups in periods of severe downturn or extended market disruption. Supervisors should also consider the possibility that a crisis impairs the ability of even very healthy banks to raise funds at reasonable cost.

Supervisors should review the range of remedial actions envisaged by a bank in response to the results of the stress testing programme and be able to understand the rationale for management decisions to take or not to take remedial actions. Supervisors should challenge whether such actions will be feasible in a period of stress and whether the institution will realistically be willing to carry them out.

Supervisors may wish to take actions in the light of this review. These actions might entail requiring a bank to raise the level of capital above the minimum Pillar 1 requirement to ensure that it continues to meet its minimum capital requirements over the capital planning horizon during a stress period.

Supervisors should also examine the liquidity needs of a bank under adverse scenarios and consider the adequacy of liquidity buffers under conditions of severe stress. Supervisors should review the use of stress test results to ensure that the potential impact on a bank's liquidity is fully considered and discussed at senior management level. Where deficiencies are noted, supervisors should ensure that management takes appropriate action, such as increasing the liquidity buffer of the bank, decreasing its liquidity risk, and strengthening its contingency funding plans. More detailed information on stress testing

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for liquidity risk is outlined in the Basel Committee's *Principles for Sound Liquidity Risk Management and Supervision.*

20. Supervisors should consider implementing stress test exercises based on common scenarios.

Supervisors should consider complementary supervisory stress test exercises, based on common scenarios for banks in their jurisdiction. They should ensure that banks have a common understanding as to the scope of such tests and the manner in which the tests complement individual bank stress testing programmes. These may be used to assess risk across banks at a range of levels (from the portfolio level to aggregate firm-wide exposures).

Supervisory determined stress scenarios can enhance the ability of supervisors and banks to assess the impact of specific stress events. Such stress tests could complement a bank's own stress testing programme, and should not be problematic to execute for a bank that has an adequate stress testing programme in place. However, supervisory stress tests should on their own not be considered as sufficient by a bank. In considering such stress test exercises, supervisors should make clear that these are not a substitute for stress tests designed by bank management, given that a common supervisory scenario is not tailored to the unique characteristics of individual banks.

21. Supervisors should engage in a constructive dialogue with other public authorities and the industry to identify systemic vulnerabilities. Supervisors should also ensure that they have the capacity and skills to assess a bank's stress testing programme.

Supervisors should engage with other public bodies and the industry to discuss stress testing practices. Discussion could include ways in which scenarios could unfold and systemic interactions could crystallise. A constructive, systematic dialogue with the industry should help the financial community to understand how the behaviour of banks and other market participants may contribute to the build up of financial imbalances and the crystallisation of systemic vulnerabilities.

Supervisors should possess expertise in quantitative modelling that is sufficient to be able to meaningfully review a bank's internal stress testing programmes. Supervisors should have adequate skill and ability to assess the scope and severity of stress scenarios and to form judgements on behavioural reactions, systemic interactions and feedback effects.

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