

## 1 Ideal gas mixer

### 1.1 Utility functions

For emptying the input pipe  $I$  into the furnace

```
fillFurnace:
yield
s Furnace SettingInput 100
l x IAnalyzer TotalMoles
brgtz -3
s Furnace SettingInput 0
j ra
```

And for emptying the furnace into the filtration system, which filters all  $\text{CO}_2$  back into the input pipe.

```
emptyFurnace:
yield
s Furnace SettingOutput 100
l x Furnace TotalMoles
brgtz -3
s Furnace SettingOutput 0
j ra
```

### 1.2 Mixing algorithm

Let  $n_R = 0$  be the moles to remove, and calculate constant terms.

$$e_T = t_T n_T$$
$$n_{T-F} = n_T - n_F$$
$$t_{H-C} = t_H - t_C$$

```
move nR 0
mul eT tT nT
sub dn nT nF
sub dt tH tC
```

### 1.3 Calculate input moles $n_I = n_H + n_C$ given $n_R$

Calculate total moles input

$$n_I = n_{T-F} + n_R$$

```
calculateInputMoles:
add nI dn nR
```

Calculate temperature input:

$$t_I = \frac{e_T - t_F(n_F - n_R)}{n_I}$$

```
sub x nF nR
mul x tF x
sub tI eT x
div tI tI nI
```

Calculate moles from hot/cold source:

$$n_H = \frac{n_I(t_I - t_C)}{t_{H-C}}, \quad n_C = n_I - n_H$$

```
sub nH tI tC
mul nH nI nH
div nH nH dt
sub nC nI nH
j ra
```

## 1.4 Searching for $n_R$

For  $n_R$  to be valid,  $n_H$  and  $n_C$  must both be positive.

```
checkInputMoles:
min x nH nC
sgtz x x # x = nH > 0 and nC > 0
j ra
```

For  $n_R = 0$ , if this is the case we are done.

```
move nR 0
jal calculateInputMoles
jal checkInputMoles
beq x 1 compose # nR=0, nH>0, nC>0: skip
```

Else, we begin a binary search of the domain  $[0, n_F]$  for a satisfactory value of  $n_R$ , each time recalculating and rechecking the input moles via `??`. Beginning with  $n_R = n_F/2$ , we bisect  $i$  times.

```
define BisectionIterations 20
# ...
move i BisectionIterations
div nR nF 2
div step nF 4
jal calculateInputMoles
jal checkInputMoles
select x x 1 -1
mul x x step
add nR nR x # nR += (nH>0 && nC>0) ? step : -step
div step step 2
sub i i 1
brgtz i -7
```

## 1.5 Removing $n_R$ and composing $I$

Having found satisfactory  $n_R$ ,  $n_H$  and  $n_C$ , we first remove  $n_R$  moles from the furnace (or remove none when  $n_R$  is zero). The furnace, containing only  $\text{CO}_2$ , is emptied into the input pipe  $I$ . Moles are removed, via a volume pump, until the total moles in the pipe  $n_I$  is less-than or equal to the initial number of moles  $n_F$  minus the number of moles to remove  $n_R$ . Then this potentially reduced amount is reinserted into the furnace.

```
jal emptyFurnace
sub nF nF nR
yield
s InputDumpPump On 1
l x IAnalyzer TotalMoles
brgt x nF -3
s InputDumpPump On 0
jal fillFurnace
```

We then compose the  $I$  mixture by first adding  $H$  moles via a volume pump while the moles in  $I$  is less-than to  $n_H$ , then doing the same for  $C$  until moles in  $I$  is less-than to  $n_I$ .

```
alias HPump d0
alias CPump d1
alias HAnalyzer d2
alias CAnalyzer d3
```

```
alias nH r0
alias hC r1
alias i r2

move i 0

fillUntil:
yield
s dr2 0n 1
add i i 2
l x dr3 TotalMoles
sub i i 2
brlt x rr2 - 3
s dr0 0n 0
add i i 1
```

$$\begin{aligned}
n_T &= n_F - n_R + n_T \\
\Rightarrow n_I &= n_T - n_F + n_R
\end{aligned}$$

$$\begin{aligned}
t_T n_T &= t_F(n_F - n_R) - t_I n_I \\
&= t_F(n_F - n_R) - t_I(n_T - n_F + n_R) \\
\Rightarrow f(t_I) &= n_R = n_F - \frac{n_T(t_T + t_I)}{t_F + t_I}
\end{aligned}$$