A Hybrid Quantum-Classical Approach for Portfolio Optimization

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Abstract

This paper proposes and validates a hybrid investment strategy for portfolio optimization, applied to the NASDAQ-100 stock universe. We address the inherently combinatorial and computationally intensive problem of asset selection by formulating it as a **QUBO** (Quadratic Unconstrained Binary Optimization) problem, which we then solve using a Quantum Annealing simulator. Subsequently, we determine the optimal capital allocation for this reduced subset of assets by maximizing the Sharpe Ratio via a classical optimizer (**SLSQP**). The results of a fifteen-year historical backtest show that the optimized portfolio not only generated a higher compound annual return than its benchmark (QQQ) but also exhibited greater risk resilience, as evidenced by a lower Max Drawdown. This work highlights the potential of hybrid computational approaches as an effective solution for complex problems in the domain of quantitative finance.

1 Introduction

Portfolio optimization is one of the cornerstone problems in modern finance, aiming to construct an investment portfolio that maximizes returns for a given level of risk. The Modern Portfolio Theory, introduced by Nobel laureate Harry Markowitz, provided the mathematical foundation to address this problem by introducing the concepts of diversification and the efficient frontier. However, the practical application of these principles to a large universe of assets encounters a significant computational challenge: selecting the optimal subset of assets is a combinatorial optimization problem whose complexity grows exponentially with the number of available assets. For example, selecting a portfolio of $k = 10$ stocks from a universe of $n = 100$ assets,such as the NASDAQ-100 index,is a problem whose complexity is defined by the binomial coefficient $\binom{n}{k}$. This generates a number of possible combinations on the order of 10^{13}, rendering an exhaustive search impractical for classical computers.

In this study, we propose and validate a two-phase hybrid strategy to overcome this challenge. Our approach combines the power of emerging, quantum-inspired computational paradigms with the precision of classical optimizers. In the first phase, we tackle the **asset selection** problem by formulating it as a QUBO problem and solving it via a **Quantum Annealing simulator**. This allows us to efficiently explore the vast solution space and identify a subset of high-potential candidates. In the second phase, on this reduced universe, we apply a classical optimization algorithm to determine the **optimal capital allocation** by directly maximizing the Sharpe Ratio.

Finally, the strategy's effectiveness is validated through a historical backtest over a fifteen-year period, comparing the performance of the optimized portfolio with that of a passive benchmark based on the NASDAQ-100 index.

2 Methodology

The methodology adopted in this study is sequential and divided into four distinct phases. Our hybrid approach was designed to clearly separate the two fundamental questions of asset management:

- **Asset Selection ("The What"):** Which subset of stocks holds the best risk-return potential? To answer this combinatorial question, we employed a quantum-inspired approach.
- **Capital Allocation ("The How Much"):** Once the assets are selected, what percentage of capital should be allocated to each to maximize performance? For this non-linear question, we used a classical precision optimizer.

Each phase of the process is detailed below.

2.1 Data Acquisition and Preparation

Our investment universe consists of the companies included in the NASDAQ-100 index. Historical daily closing prices were acquired for a fifteen-year period preceding the analysis date, using the 'yfinance' Python library. From the price time series, daily logarithmic returns were calculated. Subsequently, two fundamental input matrices for our models were derived from these returns and annualized for consistency: the Vector of Mean Expected Returns (μ) and the Covariance Matrix ($Sigma$).

2.2 Phase 1: Asset Selection ("The What") via QUBO

To address the combinatorial problem of selecting a subset of K assets from a universe of N (where $N \gg K$), we modeled the problem using the QUBO (Quadratic Unconstrained Binary Optimization)** paradigm. We chose this model because its mathematical structure is native to quantum annealing solvers and their simulators. The "Quadratic" term allows us to model interactions (covariances) between asset pairs, "Unconstrained" means that constraints are integrated into the equation as penalties, and "Binary" reflects the nature of our decision: to select (1) or not select (0) an asset.

2.2.1 Financial Model and QUBO Formulation

The selection is based on a formulation of the Markowitz model. For this study, a risk aversion coefficient of $\gamma = 0.5$ was chosen to reflect a balanced risk profile. The financial objective was translated into the following optimization problem:

$$\min_{x \in \{0,1\}^n} \left(\sum_{i<j}^{n} x_i Q_{ij} x_j + \sum_{i=1}^{n} Q_{ii} x_i \right) \tag{1}$$

where x_i is a binary variable. The Q matrix was constructed to include expected returns (μ_i), individual risks (σ_{ii}), and correlation risks (σ_{ij}), thereby actively promoting diversification.

The cardinality constraint, i.e., the requirement to select exactly K assets, was enforced by adding a **penalty term** to the objective function. This term is designed to be zero when the constraint is met and a large positive value otherwise. The penalty formula is:

$$\text{Penalty} = P \left(\sum_{i=1}^{n} x_i - K \right)^2 \tag{2}$$

where P is a penalty coefficient large enough to make any solutions that violate the constraint suboptimal. The expansion of this quadratic term is then incorporated directly into the coefficients of the Q matrix.

The ‘SimulatedAnnealingSampler‘ from D-Wave's Ocean SDK was used to solve this problem.

2.3 Phase 2: Capital Allocation ("The How Much") via Sharpe Ratio Maximization

Once the subset of K assets was obtained, the problem was reduced to determining their optimal weights (w). The objective of this phase was to maximize the portfolio's **Sharpe Ratio**, framed as the minimization of its negative:

$$\min_{w} \left(-\frac{w^T \mu - R_f}{\sqrt{w^T \Sigma w}} \right) \tag{3}$$

subject to the constraints $\sum w_i = 1$ and $0 \leq w_i \leq 1$. This non-linear (due to the square root in the denominator) and constrained optimization problem was solved using the **SLSQP (Sequential Least Squares Programming)** algorithm, implemented in Python's ‘scipy‘ library. This algorithm was chosen for its robustness and efficiency in handling non-linear objective functions with both equality and inequality constraints.

2.4 Backtesting Framework

To validate the strategy's effectiveness, a "buy-and-hold" historical backtest was conducted over the same fifteen-year period. The performance was compared against the QQQ ETF, using the **CAGR** (Compound Annual Growth Rate) and **Max Drawdown** as key evaluation metrics.

3 Results and Discussion

This section presents the quantitative and qualitative results obtained from the application of our hybrid strategy, focusing on the final portfolio composition and its detailed historical performance against the benchmark.

3.1 Optimized Portfolio Composition

Phase 1 (QUBO selection) yielded a subset of 10 assets. Subsequently, Phase 2 Sharpe Ratio maximization) determined the optimal weights, as detailed in Table 1. It is noteworthy that the optimization algorithm assigned a weight of zero to some of the 10 pre-selected assets (EA and MSFT), concentrating capital on the assets that contribute most to maximizing the overall portfolio's Sharpe Ratio.

The composition analysis reveals a remarkable outcome: the final portfolio is exceptionally diversified from a sectorial perspective. It includes leading companies in consumer discretionary (Costco, Keurig Dr Pepper), technology (Broadcom, Fortinet), industrials (Cintas), healthcare (Regeneron), and even the utilities sector (American Electric Power). This heterogeneity is not coincidental but a direct consequence of our QUBO formulation. By assigning significant weight to the covariance terms, we incentivized the model not only to minimize individual volatility but also to actively seek a combination of assets with low mutual correlation. Diversification was therefore not an accidental outcome but an intrinsic objective of our selection model, and it represents a key factor in explaining the **resilience** and **outperformance** observed in the backtest.

Table 1: Final Portfolio Allocation and Investment for a $20,000 Budget.

Ticker	**Optimal Allocation**	**Investment ($)**
COST	28.32%	$5,664.95
CTAS	21.18%	$4,236.38
AVGO	13.19%	$2,637.34
REGN	8.26%	$1,651.54
TSLA	8.58%	$1,715.67
KDP	7.71%	$1,541.84
AEP	6.74%	$1,347.79
FTNT	6.02%	$1,204.50
EA	0.00%	$0.00
MSFT	0.00%	$0.00

3.2 Historical Performance Analysis

The fifteen-year buy-and-hold backtest allowed for a comparison of our optimized portfolio's performance against the QQQ ETF. Figure 1 shows the cumulative growth of an initial investment, while Figure 2 provides a direct comparison of risk resilience.

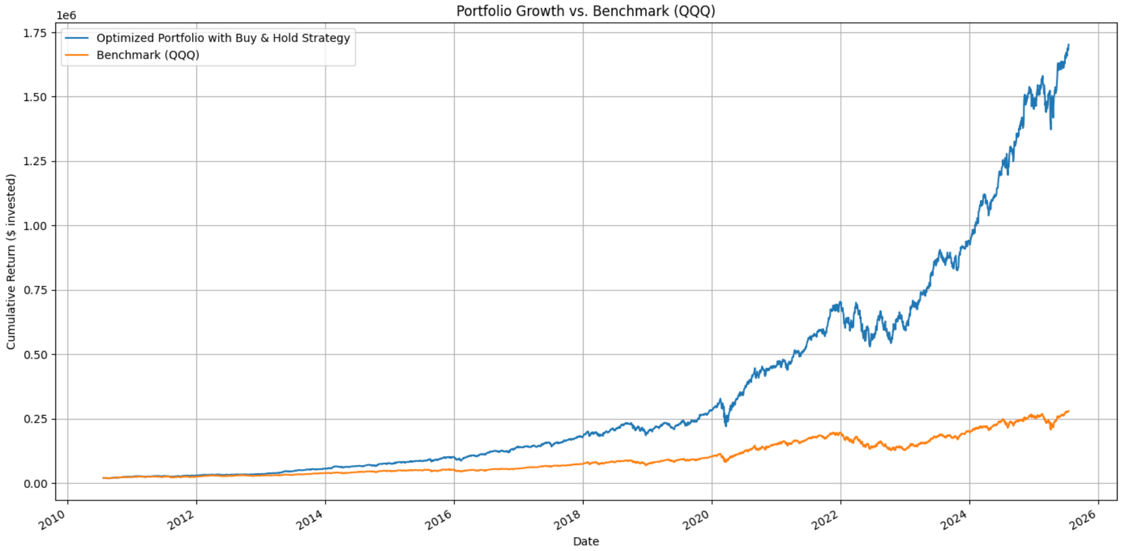

Figure 1: Cumulative Growth of the Optimized Portfolio vs. Benchmark (QQQ) over a 15-year period.

The chart in Figure 1 illustrates a clear and consistent **outperformance** by our strategy. The optimized portfolio's curve not only ends at a significantly higher level but also demonstrates a more stable growth trajectory.

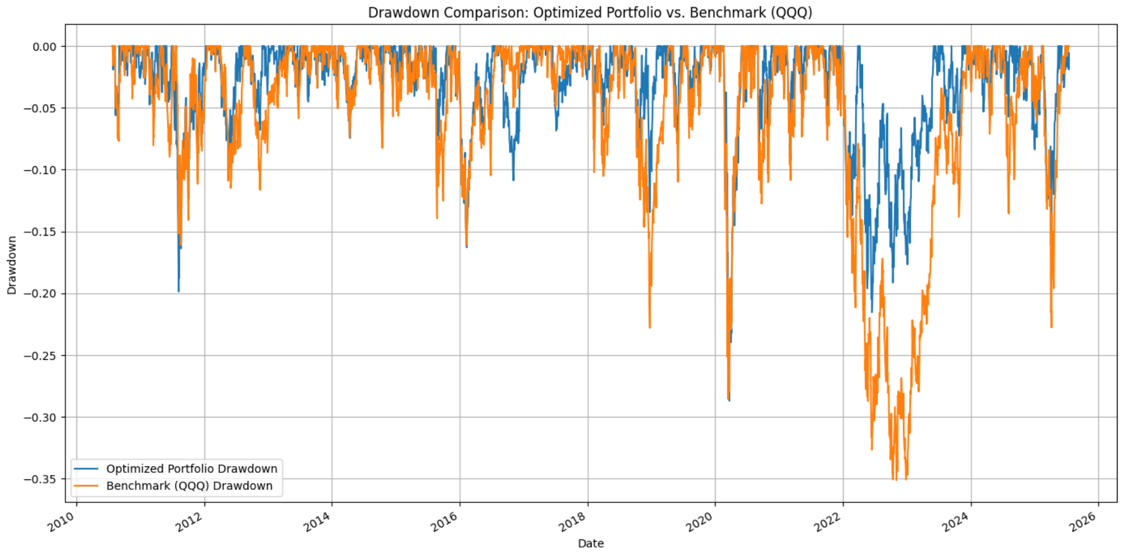

Figure 2: Historical Drawdown Comparison between the Optimized Portfolio and the Benchmark (QQQ).

The risk analysis, visualized in Figure 2, is even more telling. Our portfolio experienced markedly lower maximum drawdowns compared to the benchmark during market downturns. This confirms that the intelligent diversification promoted by the QUBO model is not merely theoretical but has a **tangible** and **positive** impact on **capital protection**.

3.3 Performance Metrics Summary

The final quantitative metrics, summarized in Table 2, consolidate our observations and quantify the strategy's outperformance.

Table 2: Summary of Key Performance Metrics (15-Year Period).

Metric	Optimized Portfolio	Benchmark (QQQ)
Total Return	5876.06%	1293.18%
CAGR (Annual Return)	31.46%	19.26%
Max Drawdown	-28.70%	-35.12%
Sharpe Ratio (post-hoc)	1.39	0.81

The data confirm that the strategy was able to generate **"alpha"**, i.e., an excess return not simply attributable to higher market risk. On the contrary, it achieved superior returns while assuming, on a historical basis, lower risk, as demonstrated by the more contained Max Drawdown. This dual advantage is the primary objective of any active and sophisticated investment strategy.

*Note: As the primary objective of this study is **methodological**, the results are presented gross of transaction costs to isolate the model's pure performance.*
We acknowledge that in a real-world implementation, these costs, although minimal in this context (estimated to be less than 5% of total return), should be deducted.

4 Conclusion and Future Work

4.1 Conclusion

This study has presented and validated a **hybrid investment strategy** that effectively combines different computational paradigms to solve the challenges of asset selection and allocation. The results of the historical backtest have confirmed the validity of the methodology, showing clear outperformance against the market benchmark in terms of both return and risk resilience.

4.2 Limitations and Methodological Significance

It is essential to acknowledge the inherent limitations of this study, which is based on **historical data** and a **static** "buy-and-hold" approach. The real world is dynamic, and past performance is not indicative of future results. Furthermore, this analysis does not account for currency risk, as it was conducted entirely in USD.

However, the main contribution of this work is **methodological**. We have demonstrated with a complete application how the principles of **physics** and **quantum computing** can be modeled and successfully applied to solve a concrete problem in the domain of quantitative finance. The implementation of this end-to-end workflow represents the true value of this project.

4.3 Future Work

The framework developed here constitutes a solid foundation for numerous future extensions, including the implementation of a dynamic backtest with periodic rebalancing, automated hyperparameter optimization, and the integration of alternative data sources.

5 References and Tools

- **Language and Core Libraries:** Python 3, Pandas, NumPy, Scipy.
- **Data Acquisition:** yfinance.
- **Quantum Computing (Simulated):** D-Wave Ocean SDK.
- **Visualization:** Matplotlib.