Teaching Statement

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Main teaching goals. A Mathematics course is an opportunity to:

1. Learn to think in a logical and ordered way.
2. Improve their problem-solving skills.
3. Learn to appreciate the power of abstraction.
4. Learn to express their thoughts precisely and understandably.
5. Understand some deep theoretical concepts.
6. Learn some techniques and tools.

I organize my courses intending to teach all these different aspects, which are essential for all students, and not only for Mathematics majors. For example, learning logical reasoning and problem-solving skills is crucial for students of every discipline because it gives them the habit of thinking creatively, precisely, and quantitatively. This is what gives Mathematics most of its educational value.

Teaching style. Teaching Mathematics goes beyond providing a sequence of computation rules or, for the proof-based courses, a sequence of definitions, theorems, and proofs. During a lecture, the instructor should give meaning, purpose and life to the mathematical theories.

Every mathematical notion requires an explanation: during a Calculus course, I do not just introduce the derivative as "the limit of the ratio of the increments," but I take a function from real life, and I explain that the derivative measures the rate of change of its values. I also draw several pictures with tangent lines and secant lines to explain the same formulae graphically.

Similarly, during a proof-based course, when I introduce a theorem, besides giving the statement, I also emphasize its meaning, its importance, possible applications, and its relationships with other theorems. After introducing a theorem properly, I discuss why it can be expected to hold and how to find ideas to prove it. Only after this preliminary discussion I present the actual proof.

This approach stimulates critical thinking and shows that Mathematics is not a dry and purely formal discipline but requires intuition and vision.

Promoting active learning. Understanding Mathematics requires a lot of active work from students: they must think, compute, and play with the course material. Passively listening to a lecture is simply not enough.

For this reason, my lectures are always accompanied by homework exercises. They force the students to actively review the lecture's content, giving them precise questions to think about by themselves.

During a Calculus course, I solve in class exercises that are similar to the homework ones. When the students review the lecture, they must first try to solve the same exercises by themselves, and then compare their work with my solution. After that, they can solve the homework exercises.

During a proof-based course for students majoring in Mathematics, when presenting a proof I often leave small parts unexplained, and I encourage the students to complete them. In this way,

the students take an active role in developing the theory they are learning. I always balance the need for clear explanations with the need to leave small challenges open.

An effective learning strategy for students is to discuss the course material with their peers. I encourage them to work at the homework in little groups. In this way, they can ask questions and teach one another. Moreover, this makes the homework more fun, and students will be motivated to spend more time on it. Even if they work in a group, I require the students to write their solutions by themselves, in their own words.

Inclusive teaching. Different students have different needs, motivations, interests, and skills; hence I adapt my teaching style and material to my class. For example, when teaching students whose major is not Mathematics, I change the ratio between theory and exercises, and I emphasize the motivations coming from real-life situations, usually drawing ideas from physics, finance, economics, biology, and games.

This becomes harder when students with different backgrounds and interests attend the same lecture. This always happens at Columbia University, where the student population is rather diverse. I am used to having students interested in different majors and from different ethnic groups, geographical origins, ages, gender, and socio-economic backgrounds. I have plenty of experience in dealing with such a diverse audience.

I usually spend the first two weeks of a course reviewing the prerequisites and covering key introductory material. This ensures that students have the opportunity to identify and fill critical gaps in their knowledge.

During the entire course, I always emphasize the basic material because, in my experience, most of the difficulty the students have when following a course is due to a poor grasp of the basics. The more advanced topics come only when the students are ready for them. This approach is especially helpful for students who start with a weaker preparation or have a low level of self-confidence.

I always strive to know the students attending the course. I ask them questions during the lectures to gauge their background knowledge, and if I notice some gap, I give additional explanations. During office hours, I ask students what their major is and where they studied before coming to Columbia. I make this look like an innocent ice-breaker, but I use this information to understand my audience and adapt my lectures. Moreover, this gives them an opportunity to talk about themselves and tell me if they have particular needs.

At the end of the course, I know almost all the student's names. The students seem to trust me, and every semester several students belonging to under-represented groups ask me for letters of recommendation for applying to summer programs and grad schools.

Interaction with students. During my lectures, I create an interactive atmosphere. I regularly encourage questions and answer thoroughly. Questions are helpful for the entire class: they can stimulate further discussion, and the students feel like the entire group is following and trying their best.

I often ask questions to the students; for example, I ask them how they would approach a specific exercise to stimulate them to think independently. I often ask them if they already know a concept to decide how long I have to spend on it. Asking questions to students contributes to the class atmosphere and makes them feel part of a group.

The office hours give me a chance to give students personalized feedback and support. This opportunity is invaluable for all students, but especially for the more introverted and less confident ones. Offering different communication channels allows me to reach every student and bring

everyone up to speed; this is especially important with heterogeneous classes.

Seminars and projects I organized several student seminars, one semester here at Columbia and six semesters in Europe. I met once or twice with every student, helping them to prepare their talk. This allowed me to work in close contact with the students and give them personalized advice. I answer their mathematical questions and give them advice on how to give a talk. During the seminar, I encourage the other students to ask questions and to have a group discussion at the end.

When I did this in Germany, after giving the talk, the student also needed to submit a text with the talk's content. This is comparable to a small writing project, and correspondingly the students needed advice and guidance.

I also supervised a student's Master thesis and a student's Bachelor thesis, where I worked with the students for some months while they were learning some advanced research-level material, and they wrote it in an understandable text in the form of a dissertation.

Both student seminars, projects, and theses are valuable activities for the students. On the one hand, they have a way to learn mathematics with individual guidance; on the other hand, they have an opportunity to learn transferable skills, such as giving a presentation and writing a technical text.

Supervised homework When I was in Italy and in Switzerland, my teaching duty included weekly Exercise Sessions: hours when the students were solving their homework in class in small groups, while I patrolled the class answering their questions and giving them hints. I observed the beneficial effect of these Exercise Sessions for the students.

In the US, courses don't usually include Exercise Sessions, but I saw something similar in courses taught under the so called "flipped class model": an experimental teaching model where the students learn the theory at home, usually from a video lecture, and the class time is dedicated only to answering student's questions and to supervising the students while they solve the homework.

I never managed to implement this teaching model, because my Calculus classes were always too crowded. But I am looking forward to an opportunity to teach a small Calculus class to try this model. The experience I obtained from teaching Exercise Sessions will help me to get the most out of this method.

Online Teaching I have taught online for 3 semesters in 2020 and 2021, and I learnt how to give online courses that are as similar as possible to in-person courses. To my experience, the key ingredient is to convince the students to turn their video on. I don't force them to do so, it would violate their privacy. But by explaining them, at the beginning of every lecture, that this helps to recreate the class atmosphere and to enhance the communication, a majority of students will accept to turn their video on. These students will engage with my online lectures as much as they would do during the usual in-person lectures. In this way, the entire class can have a good learning experience, including the few students who chose to keep their video off. I also encourage the students to create a chat group for the entire class, that I don't join, to ensure peer-to-peer communication and allowing the formation of study groups.

Conclusion. In conclusion, during a course I strive to teach something more than just the course material. Mathematics is also a way of thinking, it is learning how to solve problems, a way to model the real world and a source of abstract beauty. Moreover, Mathematics is fun! When I teach, my goal is to communicate all this to my students.