Adaptive Gaussian Mixture Model for Background Subtraction

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I. Introduction

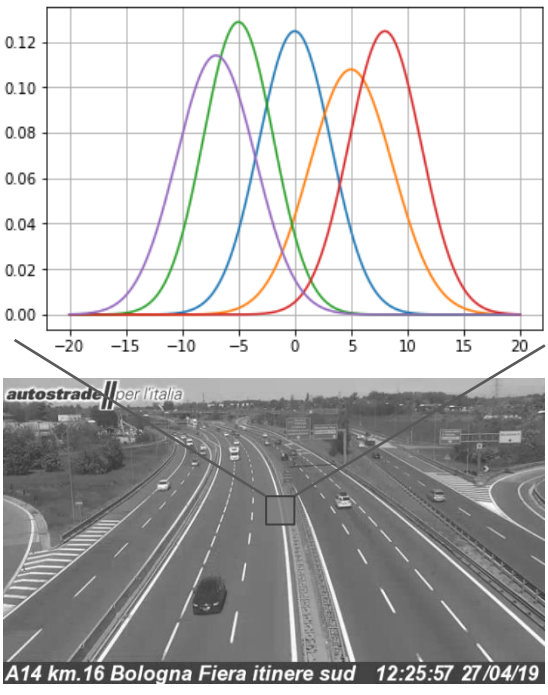
Background Subtraction is a *Computer Vision* problem of understanding and concretely detecting what is background in a scene, clean from any kind of foreground of that scene. It is applied to a video as input and it suggests frame by frame the detected background.

This problem was issued in different way, that's why there are in literature many kinds of solutions to perform the background subtraction. We tried to use the one based on gaussians model, called *Gaussian Mixture Model* (GMM), and make it adaptive to a 24h video.

Assumptions: we applied this model to gray-scale videos, with any condition of persistent traffic and a medium quality of videos, that let us understand the scene.

II. Model definition

Let's considering that GMM mixes the distribution of more than one gaussian, generating a mixture as in the figure below.



Let's considering a GMM for each pixel of each frame and let's see what happen to a pixel (x_0, y_0): at any frame *t*, you have *k* gaussian distributions (in our case we chose five gaussian distribution) where for each of them consider that they have:

- $w_{i,t} \rightarrow$ estimation of the weight for the i^{th} gaussian in the mixture, at frame *t*.
- $\mu_{i,t} \rightarrow$ mean value of i^{th} gaussian in the mixture, at frame *t*.
- $\Sigma_{i,t} \rightarrow$ covariance matrix of i^{th} gaussian in the mixture, at frame *t*.

So, the probability of observing the current pixel is:

$$p(X_t) = \sum_{i=1}^{k} w_{i,t} * N(X_i, \mu_{i,t}, \Sigma_{i,t})$$

where the function $N(X_i, \mu_{i,t}, \Sigma_{i,t})$ is the probability density function of the gaussian distribution and the *k* term is the summation is the number of gaussians.

Gaussian Mixture Model concerns of two steps: the first one, dealing with the update methods of mixture model of each pixel; the second one, dealing with estimation of the pixel value that belongs to the background.

III. Update the model

For each frame *t*, each pixel will update in the following way: given the pixel value X_t the algorithm checks if there is a match with one of the existing gaussians in that pixel model. A pixel matches a gaussian distribution when the Mahalanobis distance is satisfied. This distance is represented by the following inequation:

$$\left(\sqrt{(x-\mu)^{\gamma} * \Sigma(x-\mu)}\right) < T\sigma$$

where the T represent the parameter for the threshold value. Mahalanobis distance is used to decide if the new pixel value is well describing the background model. Now there are two possible cases, depending on the match or not of the inequation:

- ***There is a match with one gaussian***: considering the gaussian relative to the match, the parameters of this gaussian will be updated in this way:

  - $w_{i,t+1} = (1-\alpha)w_{i,t} + \alpha$
  - $\mu_{i,t+1} = (1-\rho)\mu_{i,t} + \rho X_{t+1}$
  - $\sigma^2{}_{i,t+1} = (1-\rho)\sigma^2_{i,t} + \rho\left(X_{t+1} - \mu_{i,t+1}\right)\left(X_{t+1} - \mu_{i,t+1}\right)^T$

  where $\rho = \alpha N(X_{t+1}, \mu_i, \Sigma_i)$ and α is the learning rate.

  For the unmatched gaussians the weight is the only one parameter to be updated:

  - $w_{j,t+1} = (1-\alpha)w_{j,t}$

  The "match-update" means that any new pixel value which match the Mahalanobis distance, will update the weight, mean and variance in such a way the older weight will become less relevant then the new one. The learning rate parameter is describing how much faster the model is updating; in other words, it represents how faster the model forgive the previous pixel value.

- ***There is no match with any gaussians***: the algorithm evaluates the probability of each gaussian of the mixture and the least probable distribution goes out from the mixture. A new distribution is created, with the current value as its mean value, an initially high variance and a lowest priority of weight. This new distribution is introduced in the gaussian mixture of the considered pixel.

IV. BACKGROUND MODEL ESTIMATION

The gaussian mixture model of each pixel is ordered by w/σ value. The first D distribution will be chosen to represent the background model, where in particular

$$D = argmin_k\left(\sum_{i=1}^{k} w_{i,t} > B\right).$$

B is the background ratio, i.e. is a measure of the minimum portion of the data, in terms of weight representation of the pixel value, that should be accounted for by the background.

What about the adaptive ability of the model?

Up to now the model performs with slow adaptation, in other words it is possible to see that during the transition times, i.e. sunset and sunrise, it takes too much time to update the background. The consequence is that if the current frame has just overcome the transition time and is going to the night-time or the day-time, it is possible to see that the model detects the background very delayed with respect to the current frame. It happens because the model has its own update time, i.e. called *Learning Rate*, that remain the same for all time.

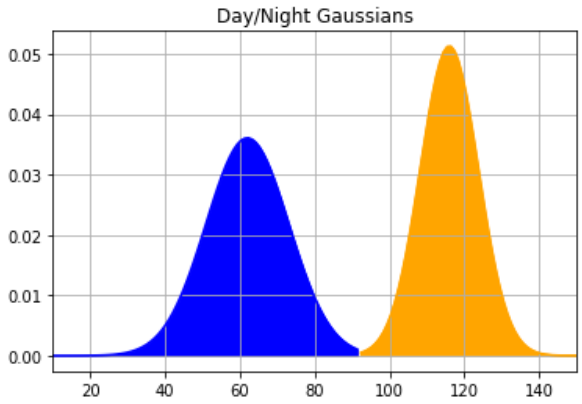
How can we make the model more adaptive?

The idea is to change the learning rate during the transition time, in order to make the model adaptive to the transition times, where the light goes up or down faster. To do this it is necessary to understand automatically from the pixel values, when the current frame belongs to night-time, transition time or day-time.

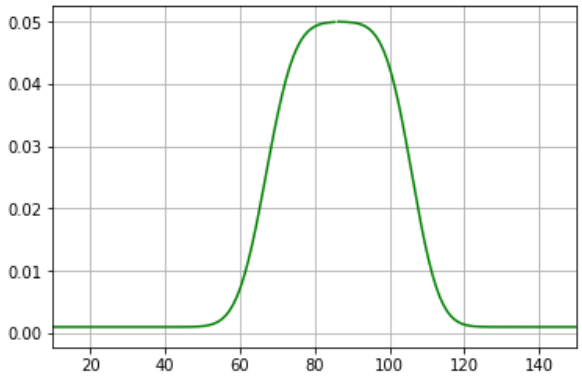
One possible way to overcome this problem is to consider the pixel values for some random pixel of a frame. In particular, we considered five 80x80 pixels square, taking 100 pixels values randomly for each one of these square, as showed in the following figure.

The white squares are the five squares in which 100 pixels values are taken. The 500 pixels coordinates taken are represented with white points inside the white squares. The five squares are not placed in a random way, but they are placed in those positions in order to consider the five important parts of the frame: the upper ones take into consideration the eventually presence of the sky (fundamental to understand the time of a frame), the center one and the lower ones take into consideration different point of view of the street, such that if in a square there are stationary vehicles, it is less probable that the same happens in the opposite square. Studying the trend of these 500 pixels values in a 24h video, we are able to set a correct value of learning rate in each moment.

The figure below shows the trend of the average of the pixel values.



The x-axis represents the pixel intensity, the blue gaussian represent the trend of the average of pixel intensity during the night-time, the orange one represents the same trend but during the day-time. So, the pixel intensity values between the two gaussians represent the transition time (sunset or sunrise). Depending on transition time, we have to modulate the learning rate and we do that as showed in the following figure:

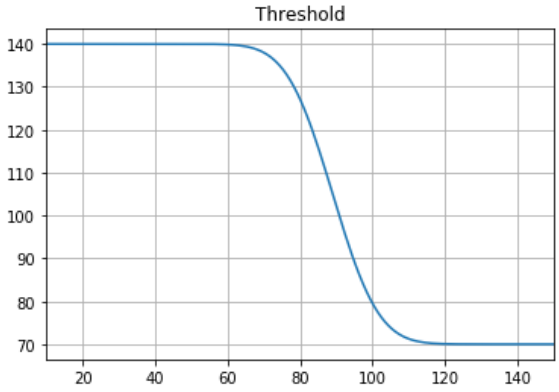


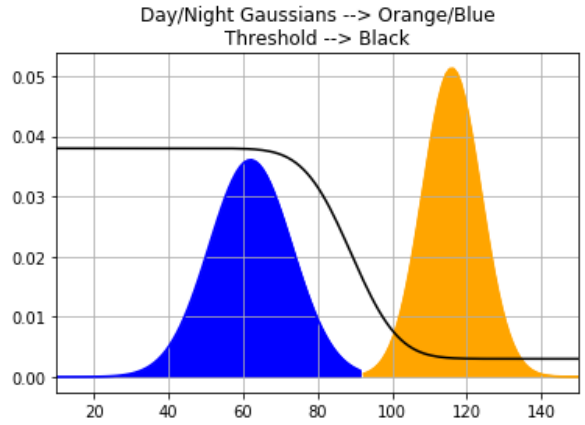
This green line is the learning rate function, computed over the two gaussians above. The variance and mean of this function are the hyper-parameters of the model, that we set in such a way to produce this result. Our setting is based on our experience after looking the average of pixel values for each frame of different videos and what was the best situation with some value of learning rate.

Another attempt to improve the performance is given by the threshold function. The threshold value is fundamental for the foreground evaluation for each frame, so we noticed that during the night-time, the car headlights affect the pixel intensity values. Increasing the value of threshold during the night-time, we reduce the car headlights area, making the average of pixel intensity representing better the time then in the previous case.

The threshold function is approximated with a sigmoid function with a lower bound and an upper bound.

The figure depicted below shows the threshold function, with its upper bound and lower bound, that are hyper-parameters.





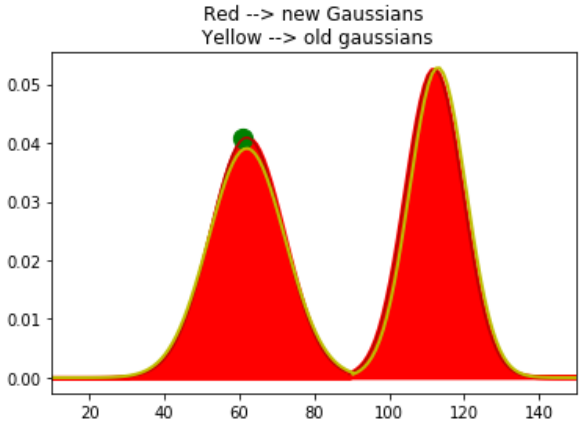
Instead in the above image, there is a comparison of the threshold with the two gaussians, plotted with respect to the gaussians reference, showing when the threshold value is high and when it is low.

V. Update the adaptive model

Gaussians and learning rate functions are the main characteristics for our adaptation to the 24h videos, for this reason they had to be updated as much as possible, in order to produce a background as much real-time as possible. The threshold function behaves always in the same way, so it does not affect the adaptive ability of the model, so we can express this function only one time.

Gaussian distribution of day/night and the learning rate are updated approximately every 30 seconds: for each frame the model computes the average of pixel intensity of those 500 pixel; it adds this average to a vector that contains all the previous averages, with a weight factor

of three, because we want that the model updates considering more the new averages then the previous. From this updated array we evaluate again its trend, so the gaussians change and consequently the learning rate changes.



This is an example of the new gaussians with respect to the previous ones. The yellow line represents the old gaussians, while the red one represents the updated gaussian. The same thing happens for the learning rate. In this way we have an adaptive model.

What do we update if we do not have anything to update at the first frame of a video?

We overcome this problem giving to the model an initialization array that holds the averages of pixel intensity: it comes from the accumulation of the averages of pixel intensity taken from five videos. Then computing the gaussian trend of the five videos we obtain a mean gaussian trend from the trends of the five gaussians model. Starting from this initialization, the model knows which part of day is expecting from each single average of pixel value, but not in a precise way initially. After some frames, it will go to update the gaussian model considering more the new average values than the initialization ones.

VI. DATA

In total we analyzed twenty webcams taken from the website *https://www.autostrade.it/autostrade-gis/webcam.do*.

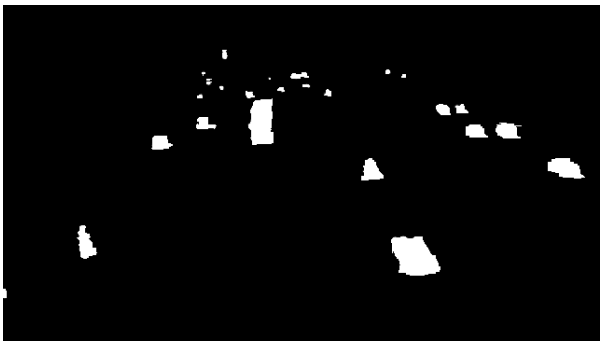
Only five webcams have been deeply analyzed, the ones that have the cleanest video.

The analyzed videos have a 24 hours duration and includes all lightning changes and weather conditions: sunny, cloudy and raining.

To evaluate the performance of our method, we have created a ground-truth images for each webcam video analyzed. A ground-truth image has all objects that belong to foreground correctly marked.

These images are created manually and due to the lot of works required only two frames for each video are taken. One at midday and one at midnight. For a total of 10 ground-truth images.

An example of ground-truth image is the following:



Moreover, for a further analysis, for each video have been created manually four images that represent the background without any elements that belongs to the foreground in it.

One image for each lightning condition in the 24 hours: day, sunset, night and sunrise.

An examples are reported below:

VII. DATA ANALYSIS

Two kinds of analysis have been performed, the first works with ground-truth images and the second one with the background images.

In the first analysis the detected model using our algorithm is compared to ground truth image. The comparison is done pixel per pixel and is used the following criteria:

- If a pixel is classified as foreground in both model and ground-truth is labelled **True Positive**.
- If a pixel is classified as background in both model and ground-truth is labelled **True Negative**.
- If a pixel is classified as foreground in model and as background in ground-truth is labelled **False Positive**.
- If a pixel is classified as background in model and as foreground in ground-truth is labelled **False Negative**.

From these acquired data, we are able to extract four important evaluation metrics: accuracy, precision, recall and F-measure.

- **Accuracy** is simply the ration of correctly predicted observation to the total observations.

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

- **Precision** is the ratio of correctly predicted positive observation to the total predicted positive observations.

$$Precision = \frac{TP}{TP + FP}$$

- **Recall** is the ratio of correctly predicted positive observations to the all observations in the positive class.

$$Precision = \frac{TP}{TP + FN}$$

- **F-measure** or **F-score** is the harmonic average of the precision and recall.

$$Fscore = 2 \times \frac{Recall \times Precision}{Recall + Precision}$$

The second type of analysis is a graphical way to analyze the background detected and the background images manually created.

This analysis is made up by a pixel by pixel value comparison between the two images. Due to lightning variance we consider a little threshold.

The used criteria is the following:

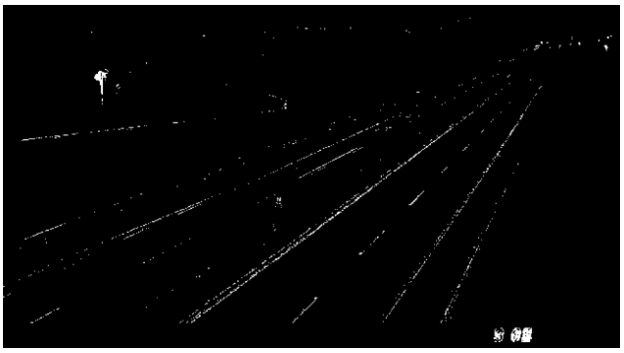
$$\begin{cases} v \in [u - t, u + t] \rightarrow positive\ check \\ otherwise\ negative\ check \end{cases}$$

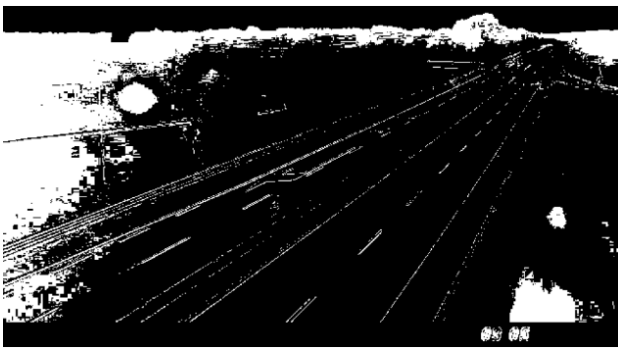
Where:

$$v\ is\ the\ value\ of\ background\ detected\ pixel$$
$$u\ is\ the\ value\ of\ background\ image\ pixel$$

The accuracy is then computed by the ratio of number of positive comparisons to the total comparisons. Starting from this analysis the error between the detected background and the background image is plotted.

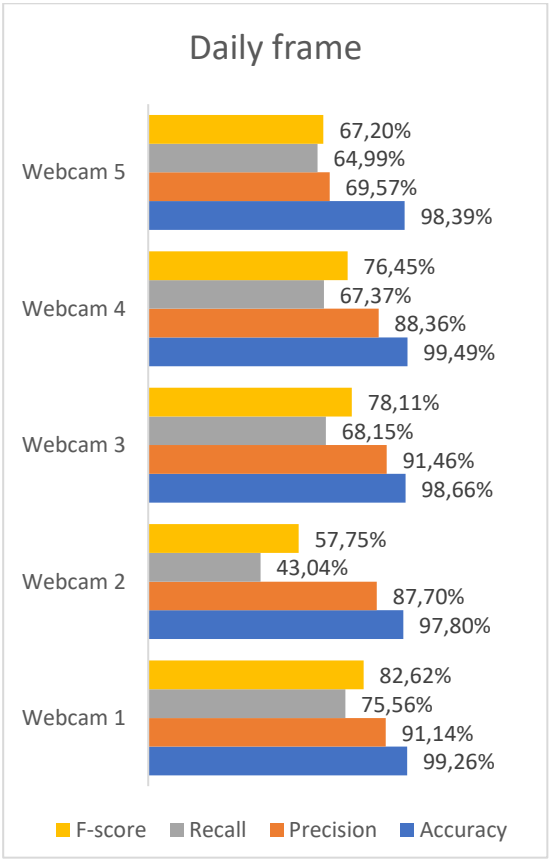
Two examples are reported below:

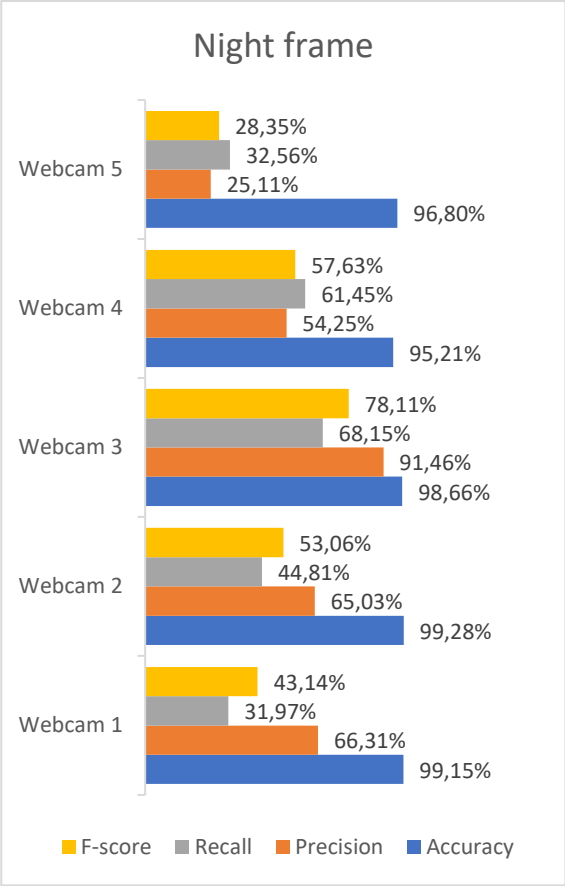




VIII. RESULTS ANALYSIS

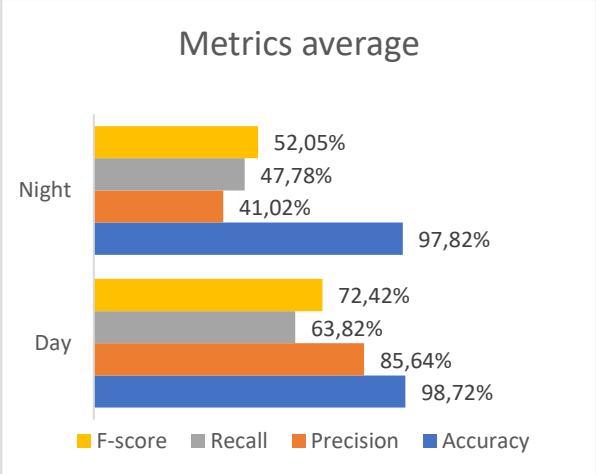
Concerning the ground truth analysis, the following charts include all results:





As is possible to see, the results obtained in daily frames are better than those obtained in night frames.

Below are reported the average value of all metrics to make easier the comparison between day and night.



The only metric that stay approximately at the same value is the accuracy.

All the other metrics in the night scenario have a lower value than daily scenario. This happens because in the night scenario there are two important factors that influence the results: vehicle headlights and the higher threshold used in Gaussian Mixture Model.

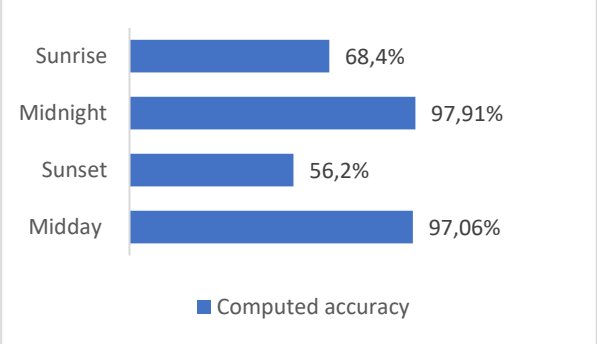
These two factors introduce an error that lead to less realistic foreground detection.

Furthermore, it is needed to remember that all these metrics depend on the ground-truth images and these are realized manually. It is probable that in the creation process are introduced some imperfections.

The second analysis has fluctuating results. Daily and night scenarios have a good accuracy, always greater than 97%. Instead, the transition scenarios have uncertain results. Them can vary from 10% to 90%.

This depends on lightning condition and mostly on the background updating speed. Even if our adaptive model improves the performance of standard OpenCV background subtraction implementation, there is still a little delay in updating the background in the scene with a large lightning change.

As the previous analysis, also in this case are reported the results:



And it is possible to see what has been explained above: transition scenes have lower accuracy than stable scenes.