

Chapter 1

Formule varie

$$\begin{cases} \partial_t u = \epsilon^2 \Delta u + \alpha(x) u^2 v - u + (\tau \gamma)^{-1} v \\ \partial_t v = \frac{D}{\tau} \Delta v - \frac{1}{\tau} v + \frac{1}{\tau} - \gamma (\alpha(x) u^2 v - u) - \frac{\beta \gamma}{\tau} u \end{cases} \quad (1.1)$$

Sintetic Operators without coefficients

$$\mathcal{L} = \begin{bmatrix} \Delta_s + \mathbb{1} & \mathbb{1} \\ \mathbb{1} & \Delta_s + \mathbb{1} \end{bmatrix}$$

$$\mathcal{N}(U) = \begin{bmatrix} u^2 v \\ u^2 v \end{bmatrix}$$

1st option: Implicit Euler + Newton's method (complete)

$$U^{n+1} = U^n + \Delta t \mathcal{L} U^{n+1} + \Delta t \mathcal{N}(U^{n+1}) \Leftrightarrow G(U) = (\mathbf{1} - \mathcal{L}) U - \Delta t \mathcal{N}(U) - U^n = 0 \quad (1.2)$$

Newton method... full matrix

2nd option: Implicit Euler + semi-implicit (these morning hyp.)

$$U = (u, v)$$

$$(\mathbb{1} - \Delta t \mathcal{L}) U^{n+1} - \Delta t \mathcal{N}(U^{n+1}, U^n) - U^n = 0$$

with

$$\mathcal{N}(U^{n+1}, U^n) = ((U^n)^2 V^{n+1}, (U^n)^2 V^{n+1})'$$

Linear but still not sparse

$$\begin{cases} (1 - \Delta t \epsilon^2 \Delta_s) u^{n+1} + \Delta t u^{n+1} - \Delta t \alpha(x) (u^n)^2 v^{n+1} - \Delta t (\tau \gamma)^{-1} v^{n+1} = u^n \\ (1 - \Delta t \frac{D}{\tau} \Delta_s) v^{n+1} + \frac{\Delta t}{\tau} v^{n+1} + \Delta t \gamma \alpha(x) (u^n)^2 v^{n+1} - \Delta t \gamma u^{n+1} + \Delta t \frac{\beta \gamma}{\tau} u^{n+1} = \frac{\Delta t}{\tau} + v^n \end{cases} \quad (1.3)$$

$$\begin{cases} (\frac{1}{\Delta t} - \epsilon^2 \Delta_s + 1) u^{n+1} - (\tau \gamma)^{-1} v^{n+1} - \alpha(x) (u^n)^2 v^{n+1} = \frac{u^n}{\Delta t} \\ (\frac{1}{\Delta t} - \frac{D}{\tau} \Delta_s + \frac{1}{\tau}) v^{n+1} + (-\gamma + \frac{\beta \gamma}{\tau}) u^{n+1} + \gamma \alpha(x) (u^n)^2 v^{n+1} = \frac{1}{\tau} + \frac{v^n}{\Delta t} \end{cases} \quad (1.4)$$

$$\begin{bmatrix} A_u & B_u \\ B_v & A_v \end{bmatrix} * \begin{bmatrix} \mathbf{U}^{n+1} \\ \mathbf{V}^{n+1} \end{bmatrix} + \begin{bmatrix} C_u(\mathbf{U}^n) \mathbf{V}^{n+1} \\ C_v(\mathbf{U}^n) \mathbf{V}^{n+1} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (1.5)$$

where

$$\begin{aligned} [A_u]_{i,j} &= \int_{\Omega} \left(\frac{1}{\Delta t} + 1 \right) \phi_j \phi_i + \int_{\Omega} \epsilon^2 \nabla_s \phi_j \cdot \nabla_s \phi_j \\ [A_v]_{i,j} &= \int_{\Omega} \left(\frac{1}{\Delta t} + \frac{1}{\tau} \right) \phi_j \phi_i + \int_{\Omega} \frac{D}{\tau} \nabla_s \phi_j \cdot \nabla_s \phi_j \\ [B_u]_{i,j} &= \int_{\Omega} \left(\frac{-1}{\tau \gamma} \right) \phi_j \phi_i \\ [B_v]_{i,j} &= \int_{\Omega} \left(\frac{\beta \gamma}{\tau} - \gamma \right) \phi_j \phi_i \\ [C_u(u^n)]_{i,j} &= \int_{\Omega} -\alpha(x) (u^n)^2 \phi_j \phi_i \\ [C_v(u^n)]_{i,j} &= \int_{\Omega} \gamma \alpha(x) (u^n)^2 \phi_j \phi_i \\ [F_1]_i &= \int_{\Omega} \frac{u^n}{\Delta t} \phi_i \\ [F_2]_i &= \int_{\Omega} \frac{v^n}{\Delta t} \phi_i + \frac{1}{\tau} \phi_i \end{aligned} \quad (1.6)$$

Initialization of the variables: in paper of year 2015 at page 5 is stated that:

”As initial conditions for our time-dependent computations, we take a small random perturbation to:

$$U_0 \equiv \frac{1}{\gamma \beta} \quad V_0 \equiv \frac{\tau \beta \gamma}{\tau + \beta^2 \gamma}, \quad (1.7)$$

”

Since you told me to initialize the system as in Fig 2(a) in 1.1, I decided upto now to set::

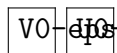
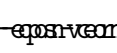
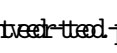
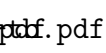
   

Figure 1.1: Hyp. initialization from Fig.2(a), 2015 paper

$$\begin{aligned}
\mathbf{U}^0 &= (U_0 - (x - 0.05)) \mathbb{1}_{x < 0.05} \text{ active ROPs solution at instant } t_k = 0 \\
\mathbf{V}^0 &= (V_0 - (x - 0.05)) \mathbb{1}_{x < 0.05} \text{ inactive ROPs solution at instant } t_k = 0
\end{aligned} \tag{1.8}$$

(sort of step function, but continuous, that near the left border assume the values U_0, V_0). This option results in **FreeFem++** as in Fig 1.2

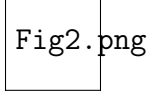


Figure 1.2: Fig.2 take from 2015 paper

3rd option NOT TO CONSIDER: Innmplicit Euler + "semi-implicit" (decoupled eqs)
Non linear problem

$$\begin{aligned}
&(\mathbb{1} - \Delta t \mathcal{L}) U^{n+1} - \Delta t \mathcal{N}(U^{n+1}) - U^n = 0 \\
&\begin{bmatrix} 1 - \Delta t & \Delta_s \\ 1 - \Delta t & \Delta_s \end{bmatrix} * \begin{bmatrix} u^{n+1} \\ v^{n+1} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} * \begin{bmatrix} v^{n+1} \\ u^{n+1} \end{bmatrix} - \Delta t \mathcal{N}(U^{n+1}) - U^n = 0
\end{aligned} \tag{1.9}$$

Take explicit terms in the Non linear part and in the second contribute:

$$\begin{aligned}
&\begin{bmatrix} 1 - \Delta t & \Delta_s \\ 1 - \Delta t & \Delta_s \end{bmatrix} * \begin{bmatrix} u^{n+1} \\ v^{n+1} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} * \begin{bmatrix} v^n \\ u^n \end{bmatrix} - \Delta t \hat{\mathcal{N}}(U^{n+1}, U^n) - U^n = 0 \\
&\text{where} \\
&\hat{\mathcal{N}}(U^{n+1}, U^n) = \begin{bmatrix} u^n v^n u^{n+1} \\ (u^n)^2 v^{n+1} \end{bmatrix}
\end{aligned} \tag{1.10}$$

Explicitly with all the coefficients

$$\begin{cases} u^{n+1} - \Delta t \epsilon^2 \Delta u^{n+1} + \Delta t u^{n+1} - \Delta t \alpha(x) u^n v^n u^{n+1} = +\Delta t (\tau \gamma)^{-1} v^n + u^n \\ v^{n+1} - \Delta t \frac{D}{\tau} \Delta v^{n+1} + \frac{\Delta t}{\tau} v^{n+1} + \Delta t \gamma \alpha(x) (u^n)^2 v^{n+1} = \frac{\Delta t}{\tau} + \Delta t \gamma u^n - \Delta t \beta \gamma u^n + v^n \end{cases} \tag{1.11}$$