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Dynamic Programming with tree decomposition

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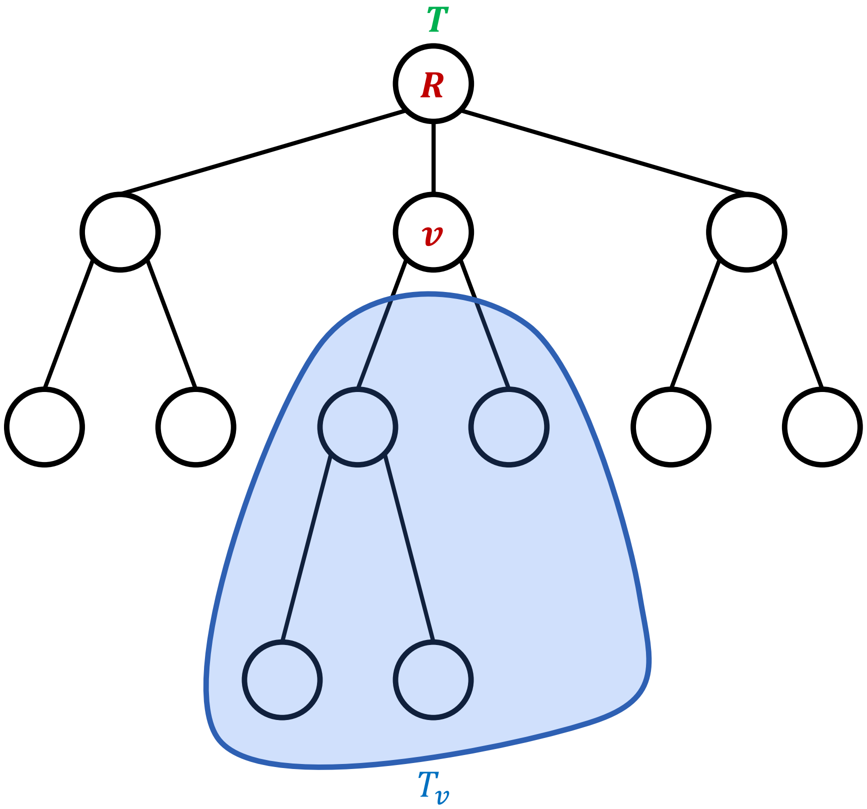
Recap

Maximum Independent Set on Trees



For each vertex v we compute:

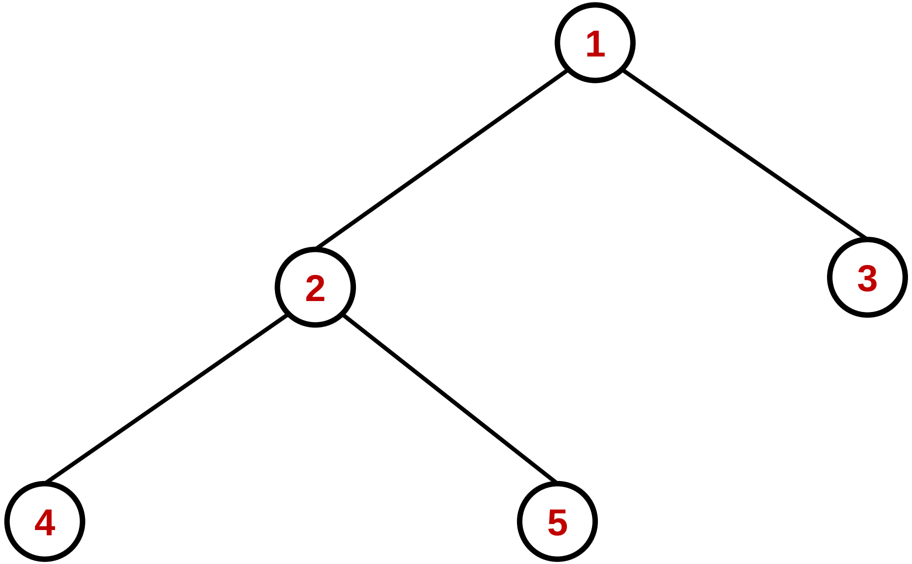
- $M^+[v] = |MIS(T_v) \cup \{v\}|$
- $M^-[v] = |MIS(T_v)|$

For a vertex v with children $w_1, \dots, w_d$

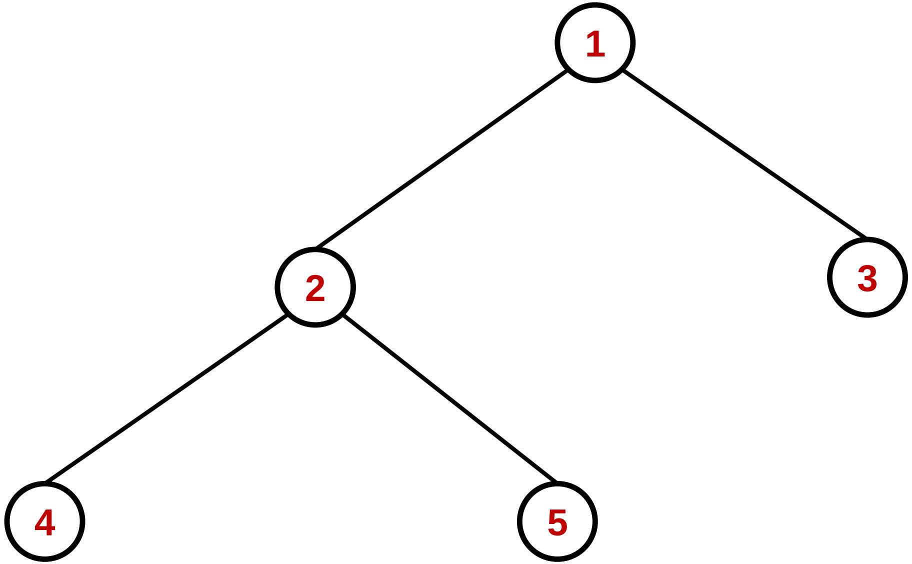
- $M^+[v] = 1 + \sum_{i=1}^{d} M^-[w_i]$
- $M^-[v] = \sum_{i=1}^{d} \max\{M^+[w_i], M^-[w_i]\}$

$$MIS(T) = \max\{M^+[R], M^-[R]\}$$

Maximum Independent Set on Trees

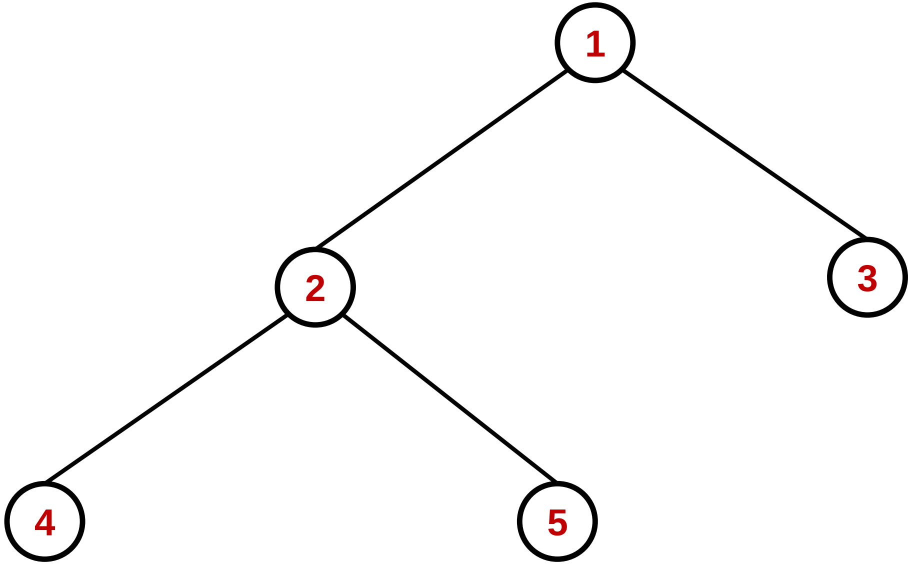


Maximum Independent Set on Trees



$M^+[4] = 1$
$M^-[4] = 0$

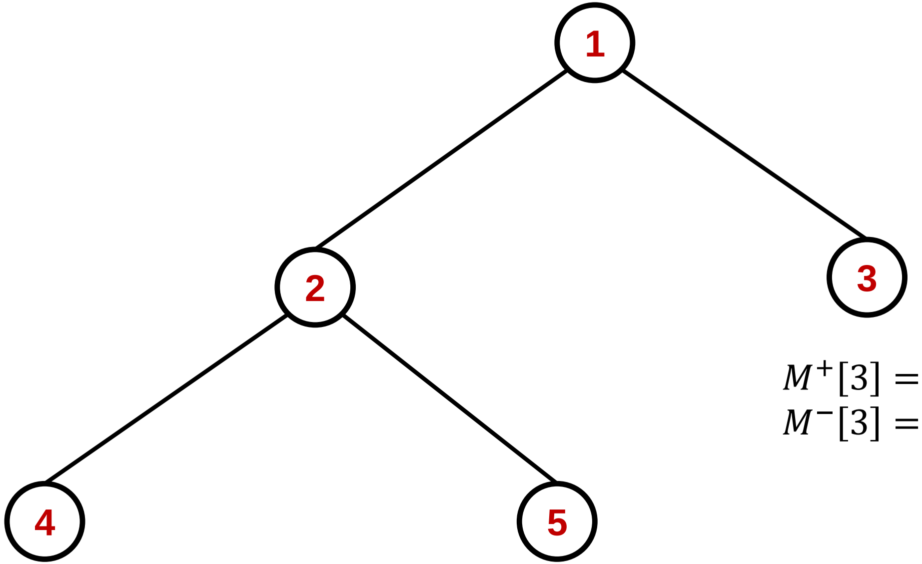
Maximum Independent Set on Trees



$M^+[4] = 1$
$M^-[4] = 0$

$M^+[5] = 1$
$M^-[5] = 0$

Maximum Independent Set on Trees

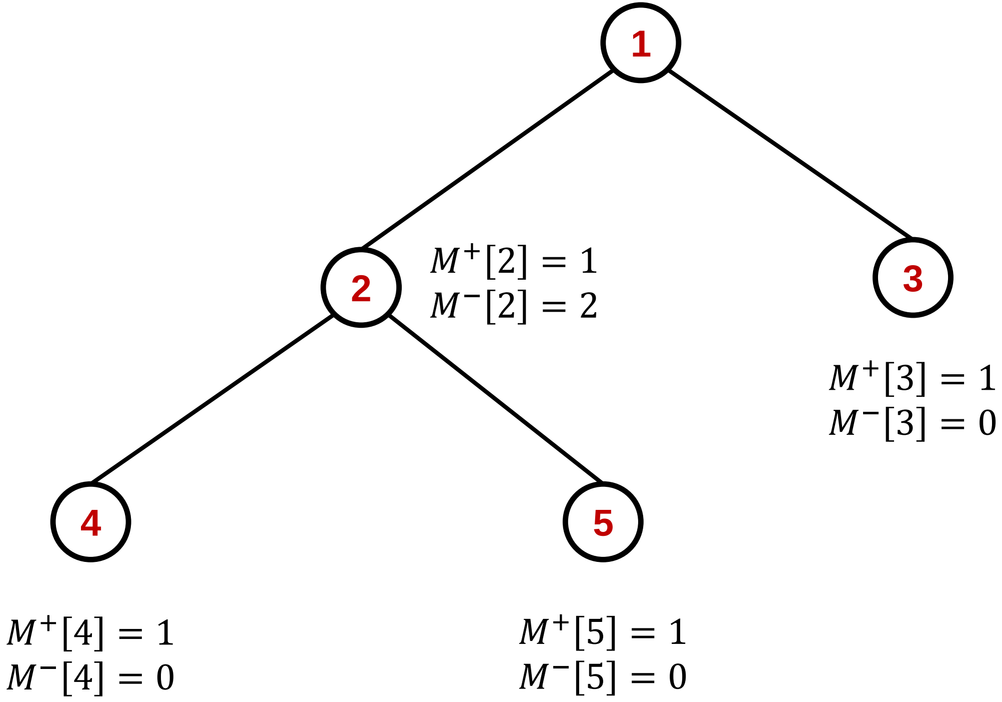


$M^+[3] = 1$
$M^-[3] = 0$

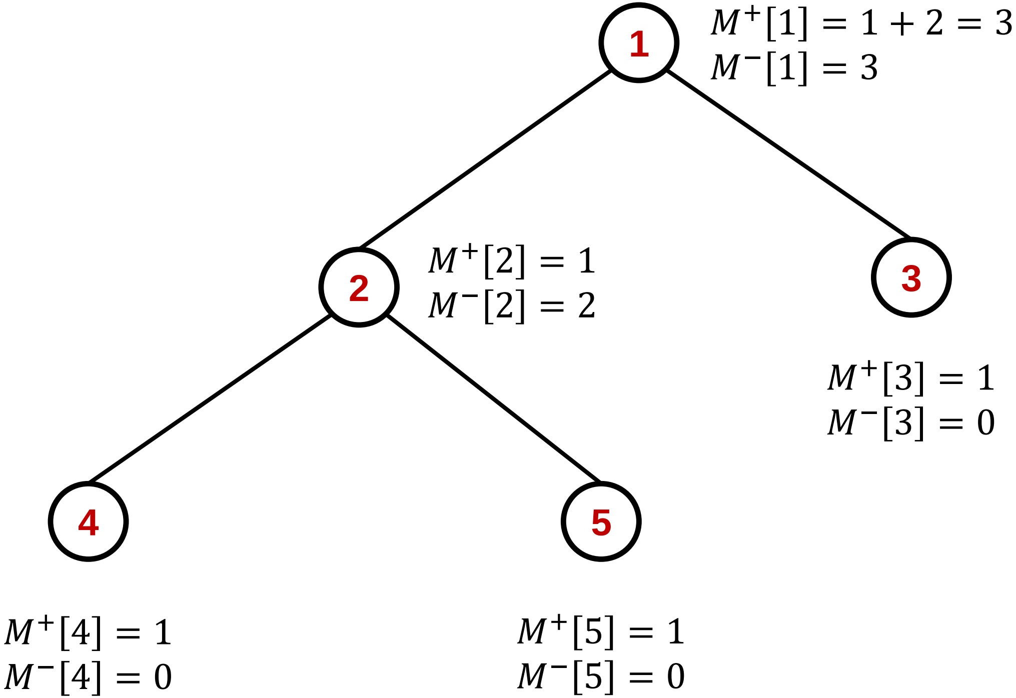
$M^+[4] = 1$
$M^-[4] = 0$

$M^+[5] = 1$
$M^-[5] = 0$

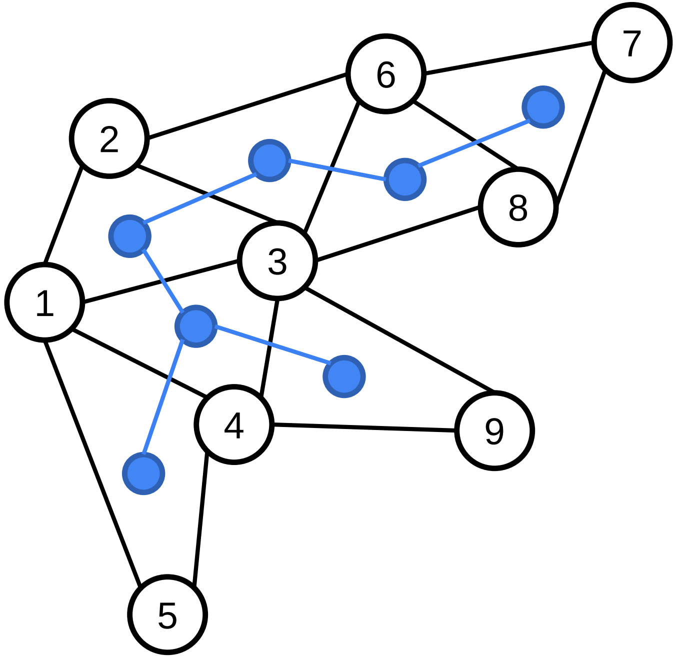
Maximum Independent Set on Trees



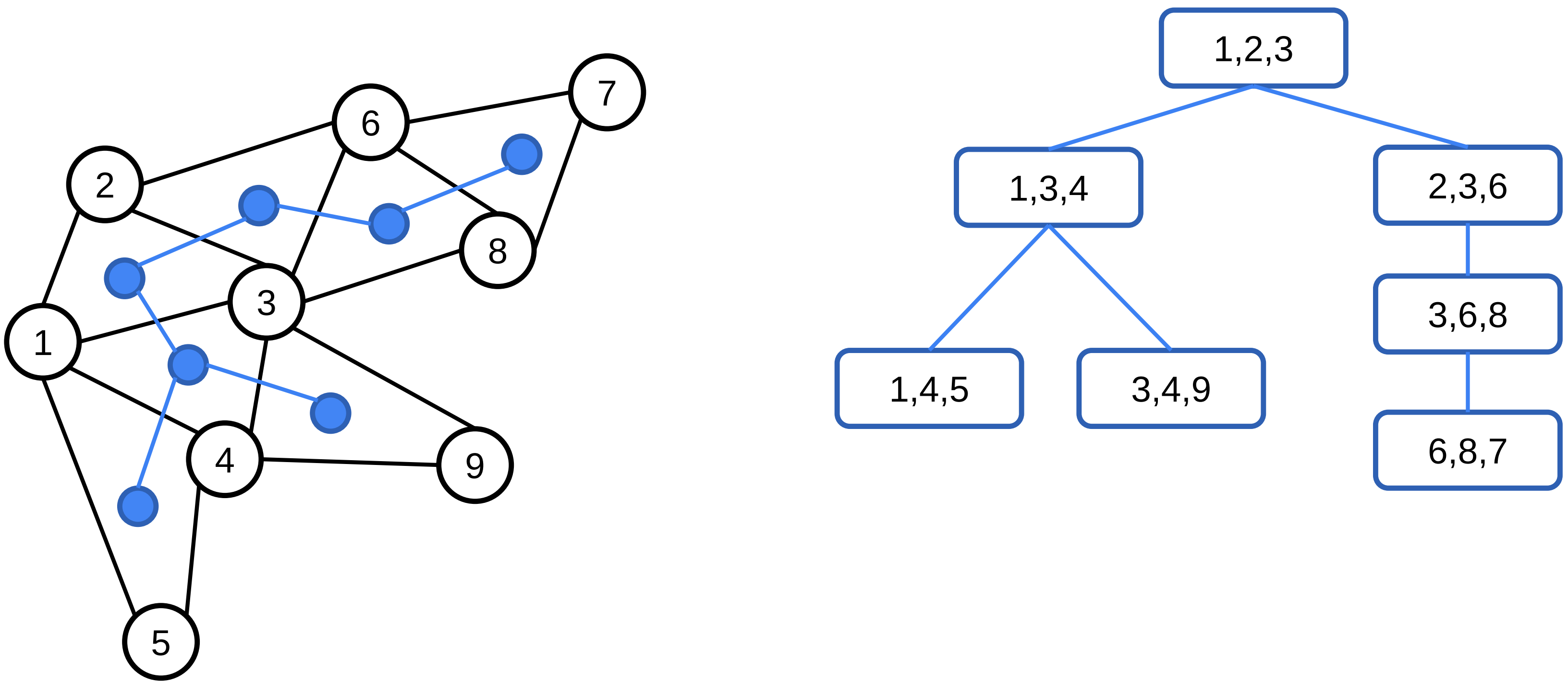
Maximum Independent Set on Trees



Tree decomposition: intuition



Tree decomposition



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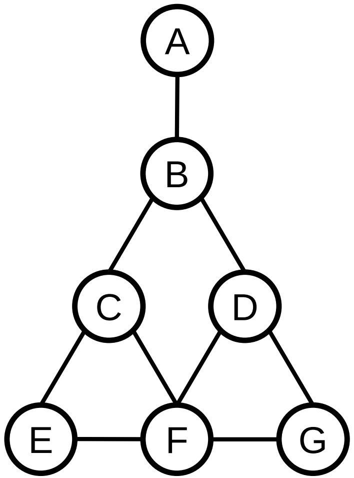
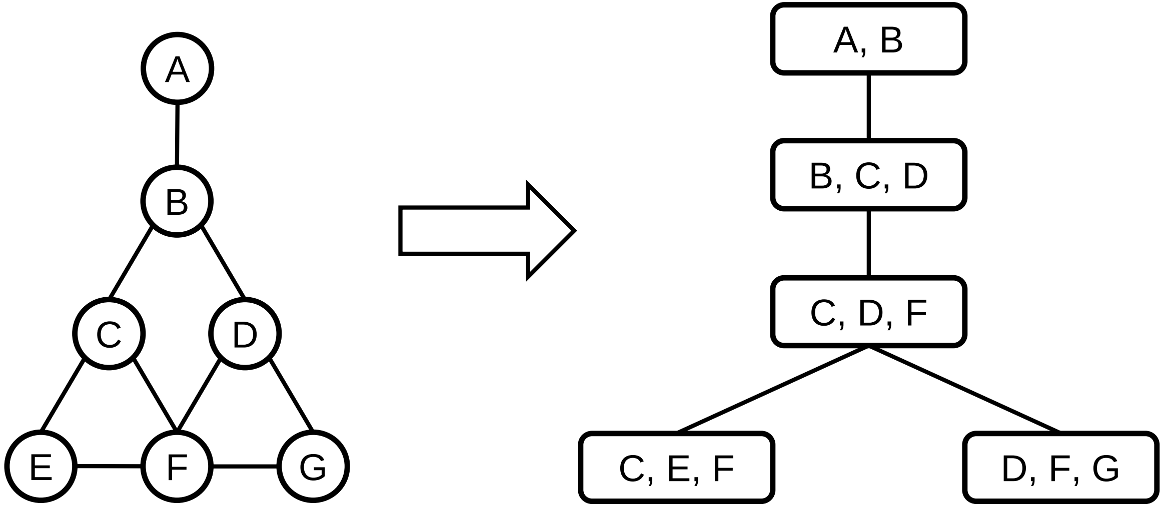
Dynamic Programming with Tree Decompositions

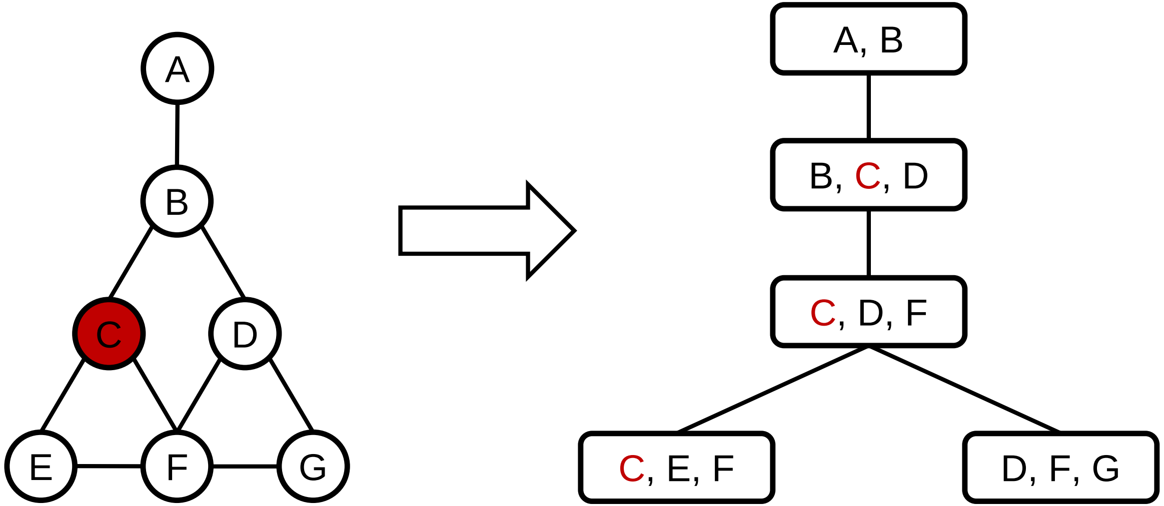
MIS on Graphs of bounded treewidth



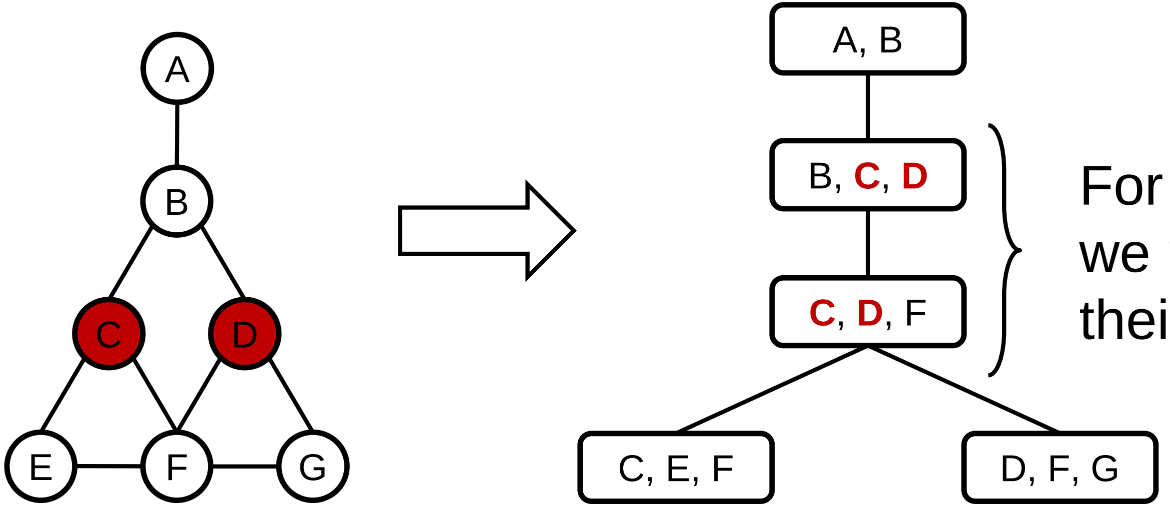
MIS on Graphs of bounded treewidth



MIS on Graphs of bounded treewidth

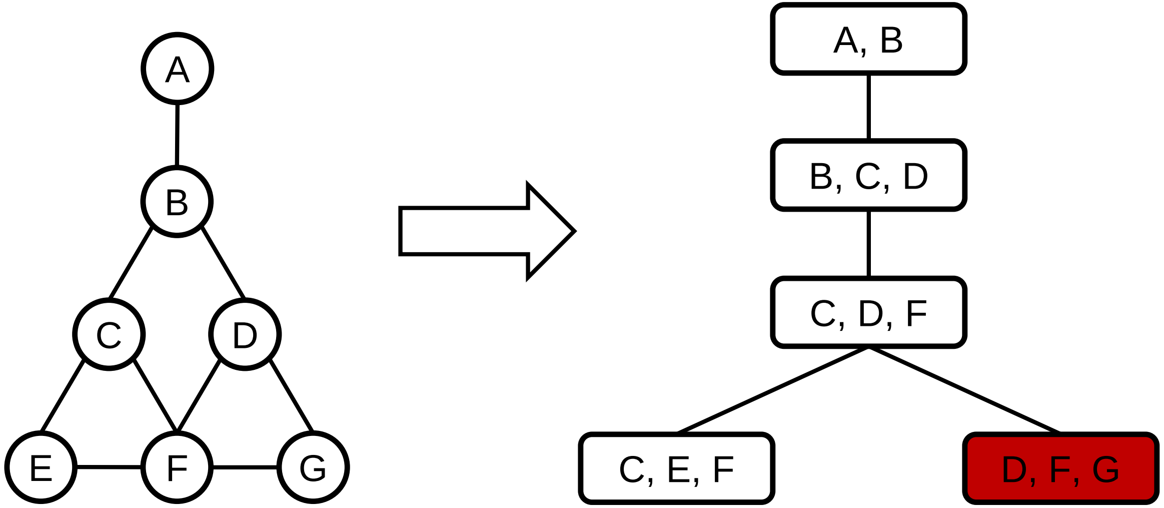


MIS on Graphs of bounded treewidth



For all parent-child bags, we need to take care of their intersection

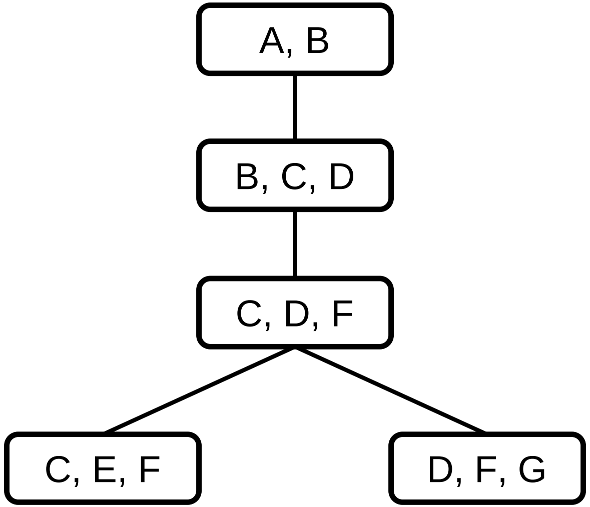
MIS on Graphs of bounded treewidth



Solve the subproblem for all subsets:

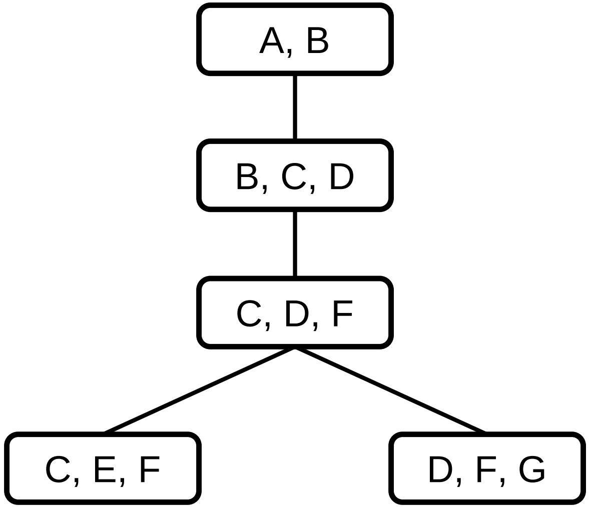
$$\{D\}, \{F\}, \{G\}, \{D, F\}, \{D, G\}, \{F, G\}, \{D, F, G\}$$

MIS on Graphs of bounded treewidth



R is the root
X_i is a bag
X_j is a child of X_i
$X_{k,i}$ is the parent of X_i

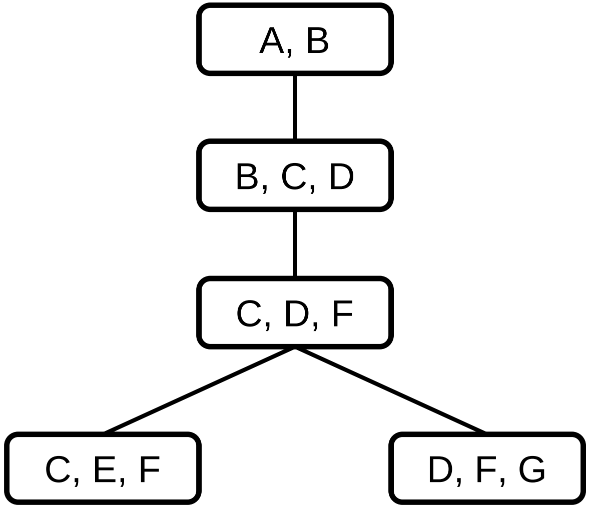
MIS on Graphs of bounded treewidth



R is the root
X_i is a bag
X_j is a child of X_i
$X_{k,i}$ is the parent of X_i

$$A[S,i] = MIS(S)\forall S \in X_i + \sum_j MIS(X_j)$$

MIS on Graphs of bounded treewidth

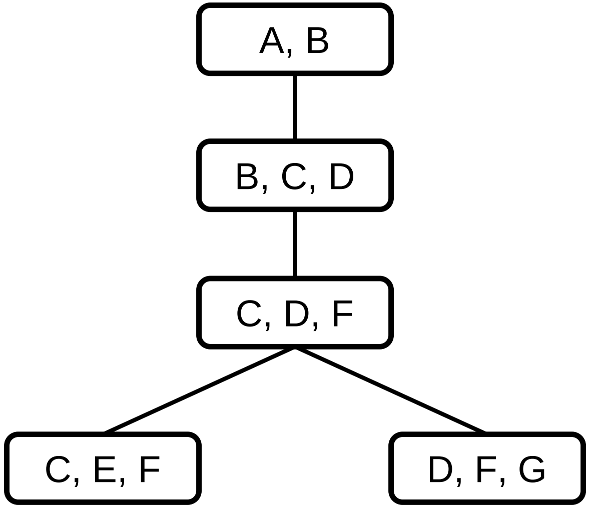


R is the root
X_i is a bag
X_j is a child of X_i
$X_{k,i}$ is the parent of X_i

$$A[S, i] = MIS(S) \forall S \in X_i + \sum_j MIS(X_j)$$

$$B[S, i] = \max_{S' \subset X_i} A[S', i], \qquad S = S' \cap X_{k,i}$$

MIS on Graphs of bounded treewidth



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X_i is a bag
X_j is a child of X_i
$X_{k,i}$ is the parent of X_i

$$A[S,i] = MIS(S) \forall S \in X_i + \sum_j MIS(X_j)$$

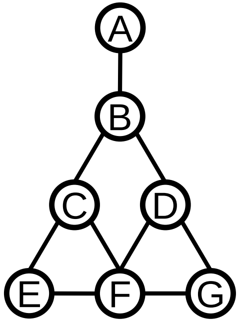
$$B[S,i] = \max_{S' \subset X_i} A[S',i], \qquad S = S' \cap X_{k,i}$$

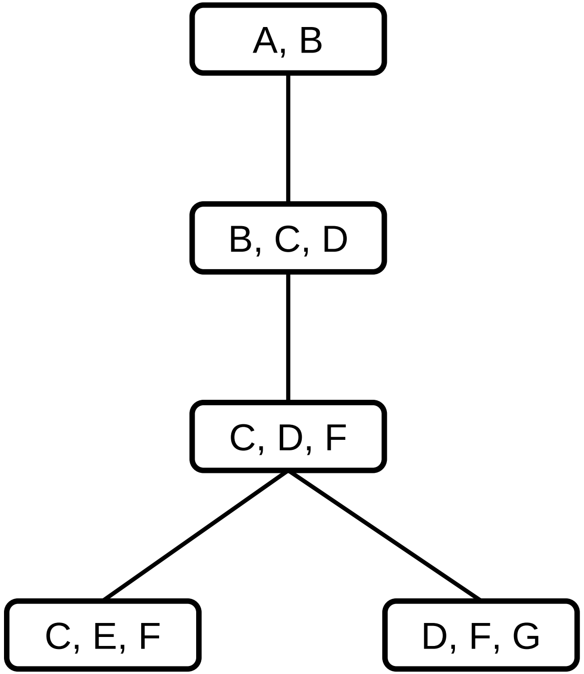
$$MIS(G) = \max_{S \subset R} B[S,R]$$

MIS on Graphs of bounded treewidth

$$A[S,i] = |S| + \sum_j B[S \cap X_j, j] - |S \cap X_j|$$

$$B[S,i] = \max_{S' \subset X_i} A[S', i], \qquad S = S' \cap X_{k,i}$$

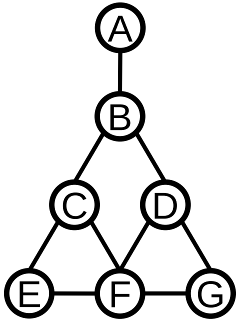


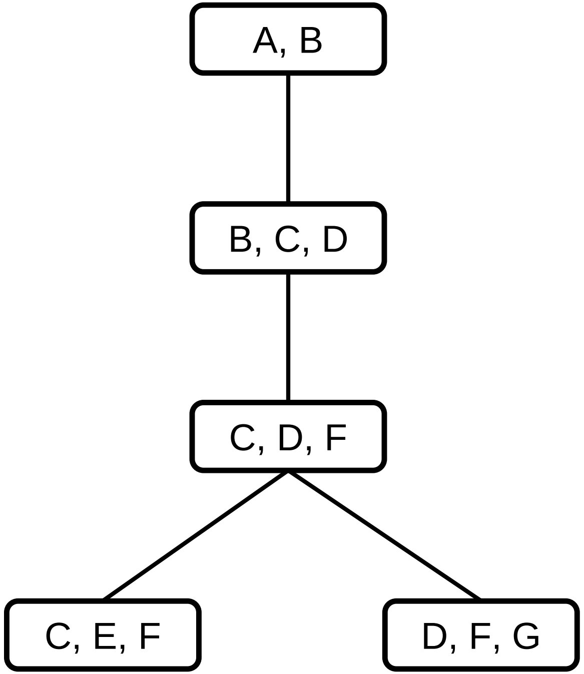


MIS on Graphs of bounded treewidth

$$A[S,i] = |S| + \sum_j B\left[S \cap X_j, j\right] - \left|S \cap X_j\right|$$

$$B[S,i] = \max_{S' \subset X_i} A[S', i], \qquad S = S' \cap X_{k,i}$$





$A[\emptyset] = 0 \quad A[E] = 1$

$A[C] = 1 \quad A[F] = 1$

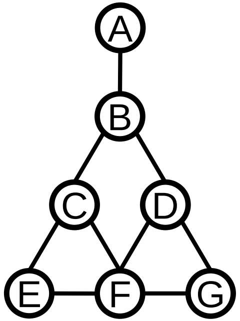
$B[\emptyset] = 0 \quad B[F] = 1$

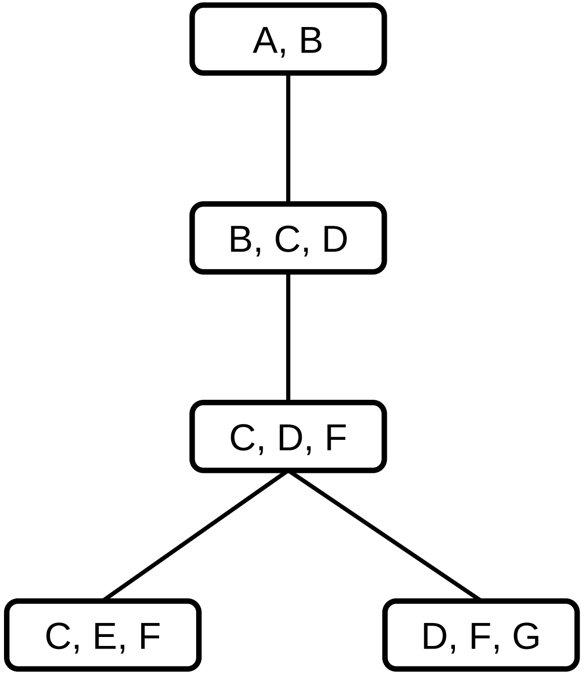
$B[C] = 1$

MIS on Graphs of bounded treewidth

$$A[S,i] = |S| + \sum_j B[S \cap X_j, j] - |S \cap X_j|$$

$$B[S,i] = \max_{S' \subset X_i} A[S',i], \qquad S = S' \cap X_{k,i}$$





$A[\emptyset] = 0 \quad A[E] = 1$
$A[C] = 1 \quad A[F] = 1$

$B[\emptyset] = 0 \quad B[F] = 1$
$B[C] = 1$

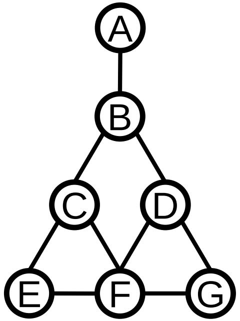
$A[\emptyset] = 0 \quad A[F] = 1$
$A[D] = 1 \quad A[G] = 1$

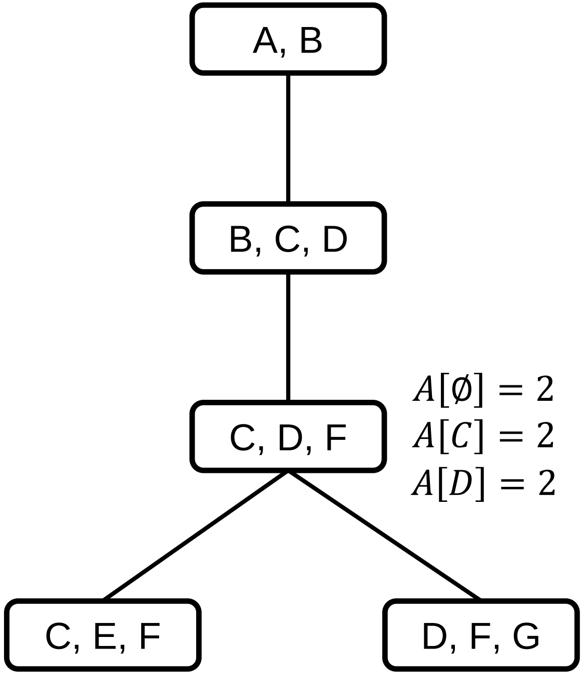
$B[\emptyset] = 0 \quad B[F] = 1$
$B[D] = 1$

MIS on Graphs of bounded treewidth

$$A[S,i] = |S| + \sum_j B[S \cap X_j, j] - |S \cap X_j|$$

$$B[S,i] = \max_{S' \subset X_i} A[S',i], \qquad S = S' \cap X_{k,i}$$





$A[\emptyset] = 2 \quad A[F] = 1 \quad B[\emptyset] = 2 \quad B[D] = 2$

$A[C] = 2 \quad A[C,D] = 2 \quad B[C] = 2 \quad B[C,D] = 2$

$A[D] = 2$

$A[\emptyset] = 0 \quad A[E] = 1$

$A[C] = 1 \quad A[F] = 1$

$B[\emptyset] = 0 \quad B[F] = 1$

$B[C] = 1$

$A[\emptyset] = 0 \quad A[F] = 1$

$A[D] = 1 \quad A[G] = 1$

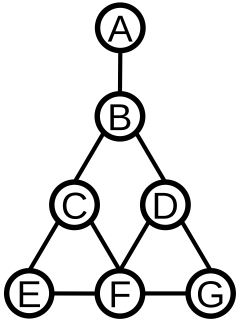
$B[\emptyset] = 0 \quad B[F] = 1$

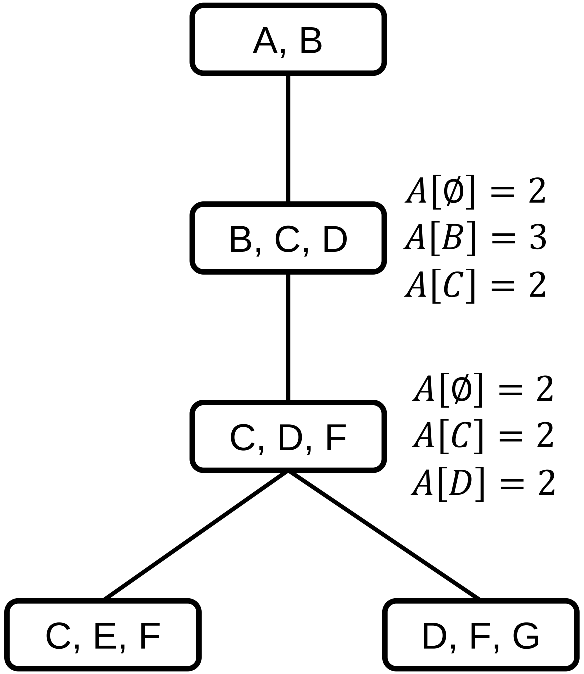
$B[D] = 1$

MIS on Graphs of bounded treewidth

$$A[S,i] = |S| + \sum_j B\left[S \cap X_j, j\right] - \left|S \cap X_j\right|$$

$$B[S,i] = \max_{S' \subset X_i} A[S',i], \qquad S = S' \cap X_{k,i}$$





Node B, C, D:

$A[\emptyset] = 2$ $\quad A[D] = 2$ $\quad B[\emptyset] = 2$ $\quad B[B] = 3$

$A[B] = 3$ $\quad A[C,D] = 2$

$A[C] = 2$

Node C, D, F:

$A[\emptyset] = 2$ $\quad A[F] = 1$ $\quad B[\emptyset] = 2$ $\quad B[D] = 2$

$A[C] = 2$ $\quad A[C,D] = 2$ $\quad B[C] = 2$ $\quad B[C,D] = 2$

$A[D] = 2$

Node C, E, F:

$A[\emptyset] = 0$ $\quad A[E] = 1$

$A[C] = 1$ $\quad A[F] = 1$

$B[\emptyset] = 0$ $\quad B[F] = 1$

$B[C] = 1$

Node D, F, G:

$A[\emptyset] = 0$ $\quad A[F] = 1$

$A[D] = 1$ $\quad A[G] = 1$

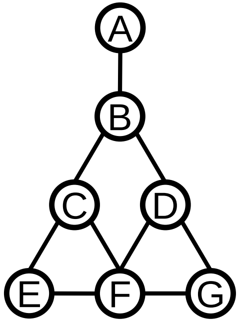
$B[\emptyset] = 0$ $\quad B[F] = 1$

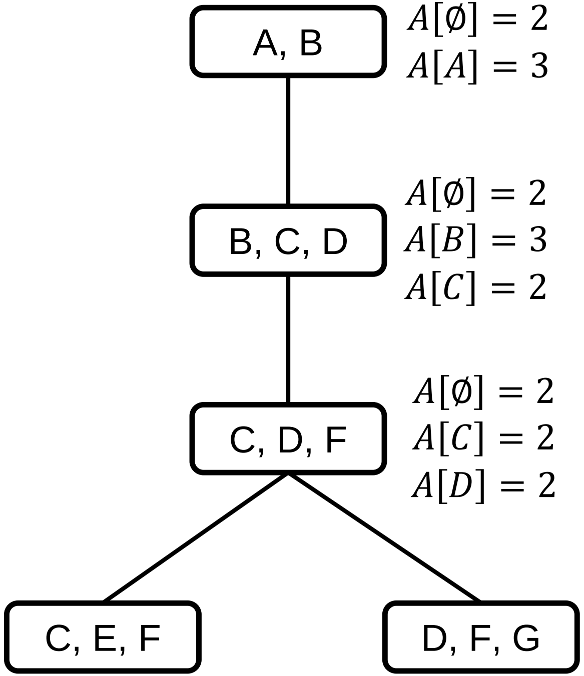
$B[D] = 1$

MIS on Graphs of bounded treewidth

$$A[S,i] = |S| + \sum_j B[S \cap X_j, j] - |S \cap X_j|$$

$$B[S,i] = \max_{S' \subset X_i} A[S',i], \qquad S = S' \cap X_{k,i}$$





A, B:

$A[\emptyset] = 2 \quad A[B] = 3$

$A[A] = 3$

$B[\emptyset] = 2 \quad B[A] = 3$

$B[B] = 3$

B, C, D:

$A[\emptyset] = 2 \quad A[D] = 2$

$A[B] = 3 \quad A[C,D] = 2$

$A[C] = 2$

$B[\emptyset] = 2 \quad B[B] = 3$

C, D, F:

$A[\emptyset] = 2 \quad A[F] = 1$

$A[C] = 2 \quad A[C,D] = 2$

$A[D] = 2$

$B[\emptyset] = 2 \quad B[D] = 2$

$B[C] = 2 \quad B[C,D] = 2$

C, E, F:

$A[\emptyset] = 0 \quad A[E] = 1$

$A[C] = 1 \quad A[F] = 1$

$B[\emptyset] = 0 \quad B[F] = 1$

$B[C] = 1$

D, F, G:

$A[\emptyset] = 0 \quad A[F] = 1$

$A[D] = 1 \quad A[G] = 1$

$B[\emptyset] = 0 \quad B[F] = 1$

$B[D] = 1$