Allocating Firefighting Vehicles in New York City: A Linear Optimization Model for Balancing Proximity and Coverage

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***Abstract*—This paper introduces a linear optimization model designed to enhance the allocation and distribution of firefighting vehicles across designated stations in New York City. The model, based on available data, addresses the dual objectives of maximizing population and emergency coverage while minimizing distances to neighborhoods. To achieve this, the model leverages standardized factors such as neighborhood population density, historical usage of specific vehicle types in emergencies, and distances to the different neighborhoods.**

Several constraints are incorporated to ensure equitable vehicle distribution across the city and between shifts, in adherence to technical, ethical, and administrative considerations. Additionally, the model defines maximum operating distances for various vehicle types and establishes neighborhood serviceability requirements.

Results demonstrate the model's effectiveness in achieving a fair and balanced distribution of vehicle types, with a focus on strategically deploying critical vehicles in high-relevance areas. Notably, the model optimizes and restricts vehicle transfers between shifts, mitigating operator fatigue. Experimentation reveals the model's flexibility, as administrators can easily adjust weights associated with different objective function factors to tailor it to specific needs.

***Index Terms*—minimizing distance, maximizing coverage, vehicles allocation, linear optimization**

I. Introduction

In the heart of urban management, the optimal allocation of resources for emergency response plays a pivotal role in ensuring the safety and security of a city's residents. With its vast and diverse landscape, New York City is no exception to this challenge.

The crux of this paper revolves around allocating five different types of fire department vehicles to stations across the city. The primary goal is to devise an optimization model that maximizes the coverage of neighborhoods, minimizes vehicle distance from these neighborhoods, and leverages incident data from the previous year for enhanced incident coverage.

This problem holds significant societal importance. The effectiveness of an emergency response system is central to the well-being and security of urban residents. It directly impacts the outcomes of emergencies, saving lives and minimizing property damage. An efficient allocation system ensures public safety, and translates into cost savings and an improved economic environment for the city.

To address this challenge, we employ linear optimization technique, a powerful mathematical method for resource allocation. Linear optimization provides a systematic approach to allocating fire department vehicles while adhering to various constraints. Doing so helps balance coverage, proximity, and historical data, resulting in an efficient and effective distribution strategy.

This paper is organized through the following sections:

- II - Related Work: we reviewed existing literature related to optimization models and real-world applications of similar methodologies.
- III - Proposed model: we delve into the core of our approach. We detail the mathematical and methodology used to formulate the linear optimization model.
- IV - Implementation: we describe the practical implementation of our model in the context of the city's specific data and infrastructure.[1]
- V - Experiments: we present the results of experiments and simulations based on real data.
- VI - Conclusions: the final section summarizes the findings, discusses the practical implications, and outlines potential future directions.

II. Related work

Badri et al. (1998) in [5] determined that traditional models proposed up to that point did not consider a sufficient number of criteria. For this reason, the article proposes a multi-objective model of integer goal programming capable of taking into account a wide range of criteria for positioning fire stations. Among the criteria considered, particular emphasis is placed on maximizing the coverage of areas that need it the most based on past incidents, and minimizing travel times to emergencies. Conversely as we did in our model, population density is not considered.

The article also considers the development of a set-covering problem to identify stations capable of covering specific neighborhoods within a certain time limit. The article demonstrates how different time limits lead to the need to build a different number of stations. A different solution from our approach, which has an already limited number of vehicles.

[1] https://gitlab.com/drvicsana/cop-proyecto-2023/

The model has proven effective in station placement, especially in real-world application contexts. However, the importance of a long-term perspective and close contact with local administrators has been highlighted. This enables the adaptation of these models to the specific situation and better decision-making regarding the prioritization of each criterion.

Huang et al. (2008) in [2] analyzed the optimal management of emergency resources in settings characterized by large geographic areas. The main focus of this study is the efficient allocation of limited vehicles, including fire trucks and ambulances, in a predetermined set of candidate stations. A linear mixed integer programming model aims to maximize service coverage for critical transportation infrastructure (CTI) by considering the impacts of demand at CTI nodes and transportation network performance on optimal CTI coverage.

The experiment focuses on high-density metropolitan areas, using the city of Singapore as a case study. In this way, the experiment makes a significant contribution to the understanding and optimal management of emergency resources in complex urban environments.

In contrast to our problem, this model focuses on the optimal distribution of investment among different types of emergency service vehicles within a set budget. In addition, the uncertainty associated with crucial parameters such as, for example, the traffic network's performance is considered. Formulations based on different risk factors are proposed. Interestingly, the final section introduces the concept of regret, which assesses the robustness of proposed resource allocation strategies, offering more flexible management under unpredictable scenarios.

Dibene et al. (2017) in [4] conducted a study on optimizing the location of ambulances for the Red Cross in Tijuana (RCT) in Tijuana, Mexico, which is the largest provider of ambulance services in the region. To solve this optimisation problem, integer linear programming is used. In the optimisation problem, constraints are used to ensure that the distribution of emergency vehicles provides both fast response and fair coverage. Furthermore, there are constraints on the number of available emergency vehicles that can be distributed to 1000 possible base locations, as well as a minimum distance requirement. Similar to the problem presented in this report, fast response time and ensuring that all parts of the city are covered efficiently are emphasised. Although the two optimisation problems share some common objectives and constraints, they differ in terms of specific approaches and constraints, reflecting the different characteristics of the ambulances and the firefighting vehicles.

Sebastian et al. (2021) in [3] present a groundbreaking simulation-optimization approach aimed at addressing the facility location and vehicle assignment problem in firefighting. Their iterative methodology dynamically refines the optimal positioning of both vehicles and fire stations based on precomputed utilization parameters. Using a novel spatio-temporal sampling method in their simulation model, the authors generate emergency events, offering a more accurate depiction of the emergency arrival process compared to previous methodologies.

This innovative strategy not only showcases superior effectiveness but also establishes a comprehensive decision-making framework for strategically locating and assigning resources in emergency response scenarios. The document marks a significant advancement in the field, providing valuable insights for optimizing emergency resource allocation.

On the contrary, we do not take into account other parameters like the type of incident, of the type of districts, the implementation of new fire stations, relocation variables or the auxiliary variable representing the minimum expected demand coverage.

H. Liu et al. (2022) in [1] presented a Joint location and assignment optimization (JLAO) problem, where fire vehicles from different fire stations in Harbin City (China) were considered. The objective was to create a model that helped make the best decisions about where to place fire stations, how many of each type of fire vehicle should be at each station, and how to assign these vehicles to different fire incidents. This mixed integer non-linear model aimed to minimize the total system cost, including facility construction, operations cost, and fire damage losses. An efficient Stingy and Interchange (SI) algorithm was used to solve JLAO problems much more efficiently than other commercial solvers like Gurobi. Then, a sensitivity analysis was performed to determine how the parameter changes affected the optimal design of the JLAO problem.

One of the similarities with the present study is the consideration of some parameters like having different types of fire vehicles to locate, the capacity of stations, the location of fire stations, or the response time for a fire vehicle located at the station to serve an incident. In this case, we would contemplate the response time as the distance between the station and the centroid of the neighborhood where the incident occurs.

On the contrary, we do not take into account other parameters like the travel cost of fire vehicles, the cost of locating one type of vehicle in one station, the fire damage of the incident, the type of incident, or the weight level of the incident.

III. Proposed model

In this section, we explore key elements of the proposed linear programming model, with a focused examination of the decision variables, the objective function, and the corresponding constraints.

All the outlined aspects are grounded in a specific set of parameters, comprehensively detailed in the table I.

TABLE I
MODEL PARAMETERS

Parameter	Domain	Description
J	$\{1, ..., 219\}$	Stations
I	$\{1, ..., 5\}$	Vehicle types
N	$\{1, ..., 195\}$	Neighborhoods
K	$\{1, 2\}$	Shifts

A. Decision variables

Firstly, four decision variables are defined:

1) $X_{i,j,k}$ - number of vehicles of type i deployed in station j during shift k
$$X_{i,j,k} \geq 0,\, X_{i,j,k} \in \mathbb{Z}, i \in I, j \in J, k \in K$$
2) $Y_{i,a,b}$ - number of vehicles of type i moved from station a to station b at shift change
$$Y_{i,a,b} \geq 0,\, Y_{i,a,b} \in \mathbb{Z}, i \in I, \{a, b\} \in J$$
3) $T_{i,j,k}$ - 1 if a vehicle of type i is deployed in station j during shift k, 0 otherwise
$$T_{i,j,k} \geq 0,\, T_{i,j,k} \in \{0, 1\}, i \in I, j \in J, k \in K$$
4) $Q_{j,k}$ - 1 if there are more than two vehicles in station j during shift k, 0 otherwise
$$Q_{j,k} \geq 0,\, Q_{j,k} \in \{0, 1\}, j \in J, k \in K$$

B. Constants

Secondly, several constants for the model are defined:

1) $s_{i,j,n}$ - 1 if from station j we can serve the neighborhood n with a vehicle of type i, 0 otherwise[2]
2) l_i - distance (in seconds) limit for transfer of vehicle type i at the end of a shift[3]
3) c_j - maximum vehicles capacity of station j
4) $r_{a,b}$ - distance (in seconds) between station a and b
5) v_i - maximum number of available vehicles of type i
6) p_n - population density of neighborhood n
7) $e_{i,k,n}$ - number of vehicles of type i needed during emergencies in neighborhood n during shift k
8) $d_{j,n}$ - distance between station j and the centroid of neighborhood n
9) w_f - importance of a factor $f \in \{p, e, d\}$ for calculating the overall importance of the deployment of a vehicle in a station for a neighborhood during a specific shift.

C. Objective function

As anticipated, the model is designed to enhance the efficiency of vehicle deployment by optimizing conflicting factors such as distance, people coverage, and the number of vehicles of a specific types used in past emergencies. To achieve this, we have introduced an importance index ($\text{IMPORTANCE}_{j,i,k,n}$) as detailed in Equation 1. This index helps assess the importance of placing a vehicle type i at a station j that can cover a *specific* neighborhood n during a shift k.

$$\text{IMPORTANCE}_{j,i,k,n} = w_p p_n^{\varsigma} + w_e e_{i,k,n}^{\varsigma} - w_d d_{j,n}^{\varsigma} - v_i^{\varsigma} \quad (1)$$

The index considers three key aspects: the population density (p_n) of the neighborhood, the usage of a vehicle type during past emergencies ($e_{i,k,n}$), and the distance ($d_{j,n}$) in seconds from a station to a neighborhood. Population density emerges as a more reliable parameter than simple population count, given the possible variability in the surface area of neighborhoods (see figure 1): a seemingly sizable population may, therefore, be distributed across a vast area. Each factor f was also given a weight (w_f), providing the flexibility to adjust the importance of each factor within the index.

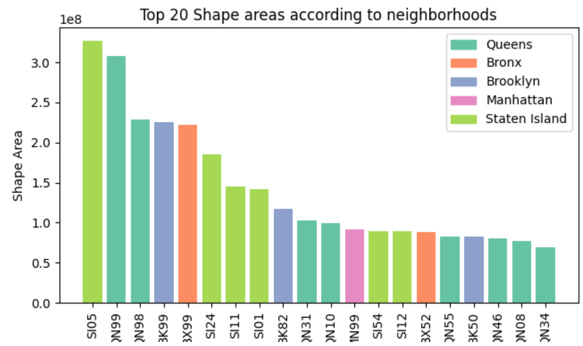

Fig. 1. Differences between the top 20 neighborhood shape areas

Additionally, the model assesses the importance of deploying a vehicle based on the availability of a specific type (v_i). This ensures that vehicles with limited availability are not placed in areas with low population density, prioritizing their deployment in strategic locations over more abundant vehicle types.

To account for the different nature of these factors, we standardized (ς) the values to a common range of 0 to 1. Standardizing allows us to combine these factors into a single index.[4]

To establish overall relevance, it is necessary to sum the importance criterion of each neighborhood that can be served by a specific station. The summation $\sum_{n \in N} s_{i,j,n}\text{IMPORTANCE}_{j,i,k,n}$ allows us to calculate the actual importance of placing a vehicle of type i within a station j during shift k.

Since our goal is to achieve the highest possible overall importance by the deployment of the vehicles, we decided to structure the problem as a maximization problem.

$$max Z : \sum_{k \in K} \sum_{i \in I} \sum_{j \in J} X_{i,j,k} \left(\sum_{n \in N} s_{i,j,n} \text{IMPORTANCE}_{j,i,k,n} \right) \quad (2)$$

The Objective Function (equation 2) establishes the number $X_{j,i,k}$ of vehicles of type i to be deployed in a station j during the shift k. The optimization is performed for all stations J, for all vehicle types I and for both shifts K.

D. Constraints

The problem of allocating firefighting vehicles within a city is complex, as it faces both *technical* and *ethical* requirements. For these reasons, a rich group of constraints are defined:

[2]For more details on how we established the serviceability of a neighborhood refer to section IV

[3]For more details on how we calculated the constant l_i refer to section IV

[4]Standardized constants between 0 and 1 are denoted by the letter ς in superscript

1) LINKING CONSTRAINTS: To ensure the correct functioning of the binary variables $T_{i,j,k}$ and $Q_{j,k}$, it is necessary to define *linking constraints* (or *system constraints).*

Equation 3 ensures that variable $T_{i,j,k}$ will be equal to 1 if a vehicle of type i is deployed in station j during shift k.

$$T_{i,j,k} \leq X_{i,j,k} \leq c_j T_{i,j,k}, \forall i \in I, j \in J, k \in K \tag{3}$$

At the same time, equation 4 ensures that variable $Q_{j,k}$ will be equal to 1 if there are more than two vehicles in the same station j during shift k.

$$\sum_{i \in I} X_{i,j,k} - 1 \leq c_j Q_{j,k}, \forall j \in J, k \in K \tag{4}$$

2) STATION CAPACITY: Each station has a limited vehicle holding capacity. Equation 5 ensures that the number of vehicles $X_{i,j,k}$ deployed at a station j does not exceed its maximum capacity c_j.

$$\sum_{i \in I} X_{i,j,k} \leq c_j, \forall j \in J, k \in K \tag{5}$$

3) VEHICLE AVAILABILITY: Each vehicle type has a limited number of available vehicles, and unused resources should not be left. Equation 6 ensures that all available vehicles for a particular type i are deployed.

$$\sum_{j \in J} X_{i,j,k} = v_i, \forall i \in I, k \in K \tag{6}$$

4) AT LEAST ONE VEHICLE: There should not be unused stations. Then, equation 7 enforces the deployment of at least one vehicle per station j, so as to ensure more uniformity in the distribution of vehicles in the city.

$$\sum_{i \in I} X_{i,j,k} \geq 1, \forall j \in J, k \in K \tag{7}$$

5) SCARCE RESOURCES COVERAGE: Rescue and Squad vehicle types are available in very limited numbers. For this reason, it is important that each neighborhood be served by at least one vehicle of each of these two types.

Equation 8 ensures that each neighborhood has at least one Rescue ($i = 3$) and one Squad ($i = 4$) vehicle type in the set of stations that serve it.

$$\sum_{j \in J} s_{i,j,n} X_{i,j,k} \geq 1, \forall n \in N, i \in \{3, 4\}, k \in K, \{a, b\} \in J \tag{8}$$

6) AREA RESOURCES CONCENTRATION: Engine and Ladder vehicles are available in large numbers. However, to ensure a more even distribution of these vehicle types in the city, it is necessary to limit the concentration of these vehicles within an area θ (equation 9).[5]

[5]In this case, an area θ is defined as the number of stations that are able to serve the same neighborhood

$$\theta = \sum_{j \in J} s_{i,j,n} \tag{9}$$

Equation 10 allows for the placement of no more Engine ($i = 1$) or Ladder ($i = 2$) vehicles than the number of stations within an area. At the same moment, the constraint ensures that the number of vehicles available for an area is sufficiently large (half the number of stations in the area).

$$\frac{\theta}{2} \leq \sum_{j \in J} s_{i,j,n} X_{i,j,k} \leq \theta, \forall n \in N, k \in K, i \in \{1, 2\} \tag{10}$$

Differently, Rescue and Squad vehicles are available in much smaller quantities, and thus, their operating range is considerably wider.[6] For this reason, it is perfectly normal that there may be neighborhoods covered by more than one Rescue or Squad vehicle. A constraint such as that defined by equation 10 would not be suitable.

To avoid the concentration of these types of vehicles in the same area, equation 11 ensures that stations containing a vehicle of Rescue ($i = 3$) or Squad ($i = 4$) type are distant at least half the operational limit of that type of vehicle. Division by two is necessary to make the constraint less restrictive, ensuring a feasible solution.

$$r_{a,b} \geq \frac{l_i}{2} \left(T_{i,a,k} + T_{i,b,k} - 1 \right), \forall k \in K, i \in \{3, 4\}, \{a, b\} \in J \tag{11}$$

7) STATION RESOURCES CONCENTRATION: To avoid excessive concentration of vehicles, it is important to limit the quantity of the same vehicle types in a station. Since Engine and Ladder type vehicles are available in greater quantities in comparison to Rescue and Squad type, we defined two constraints with two different quantity limits.

Equation 12 allows no more than *two* vehicles of Engine ($i = 1$) or Ladder ($i = 2$) type to be deployed within each station j.

$$X_{i,j,k} \leq 2, \forall j \in J, k \in K, i \in \{1, 2\} \tag{12}$$

Similarly, equation 13 allows no more than *one* vehicle of Rescue ($i = 3$) or Squad ($i = 4$) type to be deployed within each station j.

$$X_{i,j,k} \leq 1, \forall j \in J, k \in K, i \in \{3, 4\} \tag{13}$$

8) AT LEAST TWO VEHICLE TYPES: In the eyes of local administrators, it might seem strange that a station would have vehicles all of the same type. Equation 14 ensures that, if there are two or more vehicles in a station j, they cannot be all of the same type.

$$\sum_{i \in I} T_{i,j,k} \geq 2Q_{j,k}, \forall j \in J, k \in K \tag{14}$$

[6]In this case, the operational limit of certain type i of a vehicle is represented by constant l_i

9) VEHICLES TRANSFER: Vehicles can be transferred from one station to another at the end of a shift to optimize deployment in each shift individually. Equation 15 ensures that the difference between the number of vehicles present at the first shift ($X_{i,a,1}$) and the second shift ($X_{i,a,2}$) in a station a has to be equal to the difference between the number of vehicles transferred *within* the station ($Y_{i,j,a}$) and the one transferred *from* the station ($Y_{i,a,j}$).

$$X_{i,a,1} - X_{i,a,2} = \sum_{j \in J} \left(Y_{i,a,j} - Y_{i,j,a}\right), \forall i \in I, a \in J \quad (15)$$

10) MAXIMUM TRANSFER DISTANCE: To avoid increasing workers' sense of fatigue, vehicles should not travel long distances during a possible transfer at the end of the shift.[7] Equation 16 requires that the distance $r_{a,b}$ that a vehicle should travel when transferring from a station a to a station b must be less than the determined threshold l_i.

$$r_{a,b} Y_{i,a,b} \leq l_i Y_{i,a,b}, \forall i \in \{1, 2, 3, 4\}, \{a, b\} \in J \quad (16)$$

IV. IMPLEMENTATION

The code attached to this document was written entirely in Python language, and is organized into the following `.ipynb` notebook files:

1) `data_exploration.ipynb`: In the file `data_exploration.ipynb` we have included the initial code for exploring the available data. In particular, we checked for differences between the populations and areas of the various neighborhoods, so that we could evaluate which criteria to use in the objective function of the model.

2) `neighborhoods_info.ipynb`: As depicted in the reference section, the objective function of our model incorporates certain factors not present in the dataset containing neighborhood information. Specifically, within the file is the code for extracting the following necessary information for each neighborhood n:

1) Population density p_n
2) Number of vehicles $e_{i,n,k}$ of each type i used for emergencies during shift k in the year 2019

To derive the population density of each neighborhood, we divided the population by the surface area, as outlined in equation 17.

$$p_n = \frac{population_n}{surface_n} \quad (17)$$

Next, the code calculates the number of times each vehicle type was used within a neighborhood in 2019. This data was calculated for both the first and second shifts, and involved using the information available in the `incidents2019.json` dataset. It is worth noting that not all neighborhoods experienced emergencies in 2019. For these unaffected neighborhoods, the count of vehicles used was designated as 0. At the end, we derived a total of 10 new statistical metrics for each neighborhood: one metric for each type of vehicle for both shifts.

3) `neighborhoods_serviceability.ipynb`: As outlined in section III, our model incorporates a crucial Boolean serviceability constant ($s_{i,j,n}$). This inclusion is indispensable, recognizing the impracticality of each station to covering all neighborhoods within the city.

To identify the neighborhoods a station can serve with a particular vehicle type, we must establish a unique maximum distance applicable to all stations for that vehicle type i. These thresholds allow us to reduce ambiguity in neighborhood serviceability, and limit the overlap of station service areas.

Algorithm 1 Serviceabily threshold algorithm

min_nta_to_cover $\leftarrow \left\lceil \frac{N}{v_i} \right\rceil$
for each j in J **do**
  sorted_nta_distances $\leftarrow$ SORT(distances[j][:])
  for each d in sorted_nta_distances **do**
    nta_distances $\leftarrow$ nta_distances $\cup$ d
  end for
  max_distance $\leftarrow$ MAX(nta_distances)
  min_distances $\leftarrow$ min_distances $\cup$ max_distance
end for
  return MAX(min_distances)

To fulfill these requirements, we devised a specialized algorithm. The functionality of the algorithm can be succinctly outlined in the following steps:

1) The algorithm determines the minimum number of neighborhoods `min_nta_to_cover` that a type-i vehicle should be able to cover from any given station.
2) For each station j, the algorithm executes the following procedure:
   a) Exploiting the distance dataset, the algorithm extracts from `sorted_nta_distances` the first `min_nta_to_cover` lower distances `nta_distances` (nearest neighborhoods)
   b) From the selected nearest neighborhoods distances `nta_distances`, the algorithm identifies the maximum distance `max_distance`.
3) The algorithm identifies the maximum distance among all those recorded in `min_distances`.

At the end, the algorithm detect a distance that enables the deployment of a vehicle type ensuring that it will cover a minimum number of neighborhoods from any station. As stated, this distance is independently calculated for each vehicle type. These thresholds were reformulated as the constant l_i and incorporated into multiple constraints, to establish specific distances that limit the movement of vehicles during shifts (see equation 16).

Vehicle types with limited availability, like Squad and Rescue, are tasked with covering a broader geographical range. Conversely, those with higher availability, such as Engine and Ladder, can afford to focus on a more restricted area.

[7] An exception was considered for the hazmat vehicle ($i = 5$), since we have only one unit of this type

Meanwhile, the Hazmat vehicle must serve the entire city, being available in only one unit.

TABLE II
MAXIMUM SERVICEABILITY DISTANCES FOR EACH TYPE OF VEHICLE

Vehicle type	**Minimum neighborhoods to serve**	**Minimum distance threshold**
Engine	1	800.24
Ladder	2	800.24
Squad	25	2339.26
Rescue	39	2675.74

After determining the distance thresholds for each vehicle type (see table II),[8] four serviceability tables were created (one for each vehicle type, excluding Hazmat). These tables indicate a Boolean True value only for neighborhoods located within the calculated threshold distance from a station.

4) `model_implementation.ipynb`: The model was implemented using the OR-Tools library,[9] an open-source toolkit developed by Google for solving optimization problems. The implementation is detailed in the file `model_implementation.ipynb`, which is organized into three main sections: 1) defining decision variables, 2) specifying constraints, and 3) formulating the objective function. Additionally, the file includes code cells for importing constant data and activating the solver.

The code was structured to allow easy variation of the factor weights in the objective function. In this way, it was possible to conduct several experiments with different weights configurations of the objective function factors.

5) `experimentation.ipynb`: The `experimentation.ipynb` file contains the code related to the model experimentation illustrated in the V section. In particular, maps are developed to visualize the positioning of vehicles by the model with different weight configurations. In addition, we created visualizations related to the distribution of vehicles in stations, and the performance of the various solvers tested.

V. EXPERIMENTS

The testing of the model is divided into 2 distinct phases: the analysis of the results and quality of the model, and the analysis of execution issues with different types of solvers:

1) Results analysis: The objective of the first part of experiments is to study the impact of the weight values in the final allocation solution. Concretely, how changing the weights modifies the distribution of vehicles of type i alongside the different stations j. There were done four experiments (see table IV), with a time limit of 30 minutes for the SCIP solver.

a) Engine and Ladder allocation analysis: As shown in Figures 2 and 3, the highly accessible vehicles, i.e., engines and ladders, are well allocated across the map, fulfilling the objective of equations 10 and 12.[10]

[8]Since we want a station to serve at least two neighborhoods, we set the distance threshold for Engine vehicle type to 800.24

[9]https://developers.google.com/optimization

[10]It is crucial to specify that points shown on the maps assess the presence of just one vehicle type inside a specific station.

TABLE IV
WEIGHT VALUES

Experiment	W_p	W_e	W_d
1	0.4	0.1	0.5
2	0.35	0.30	0.35
3	0.25	0.25	0.50
4	0.20	0.60	0.20

This has proven to be the case for the remaining experiments as well. However, there are some differences in the location of these vehicles between experiments 1 and 4, indicating that increasing the value for W_e has some impact on the distribution of vehicles throughout the stations. Indeed, the number of vehicles used in the past during emergencies is the only factor that shows variation between the first and second shifts. In contrast, the other factors remain the same.

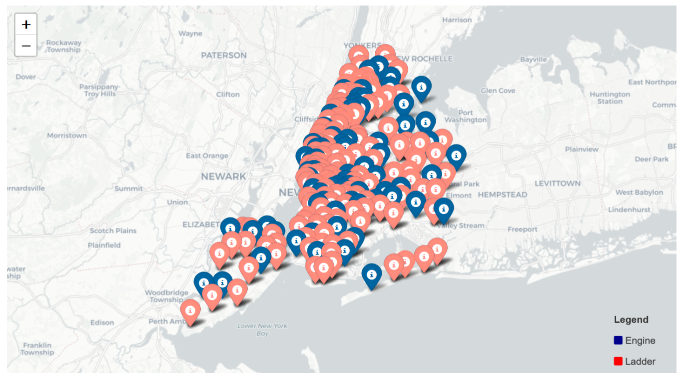

Fig. 2. Distribution of Engine and Ladder vehicles for the first shift in experiment 1

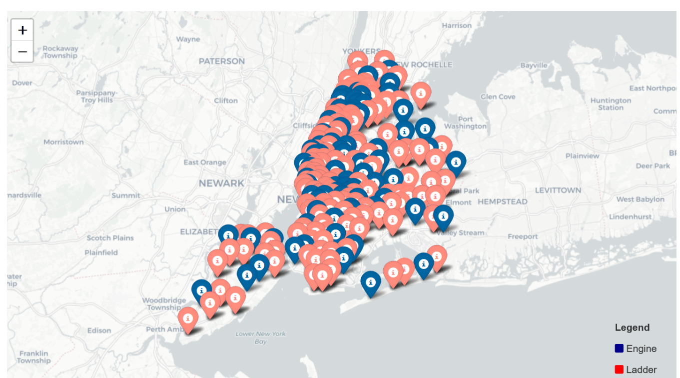

Fig. 3. Distribution of Engine and Ladder vehicles for the first shift in experiment 4

b) Rescue and Squad allocation analysis: Figure 4 and 5 show the allocation of Rescue and Squad type vehicles in the city, respectively. The black points indicate that a vehicle is allocated in the same station in more than one experiment.

As shown in Figure 4, there are remarkable similarities in the distribution of Rescue vehicles despite different weighting. Only when the distance weight assess itself as the most

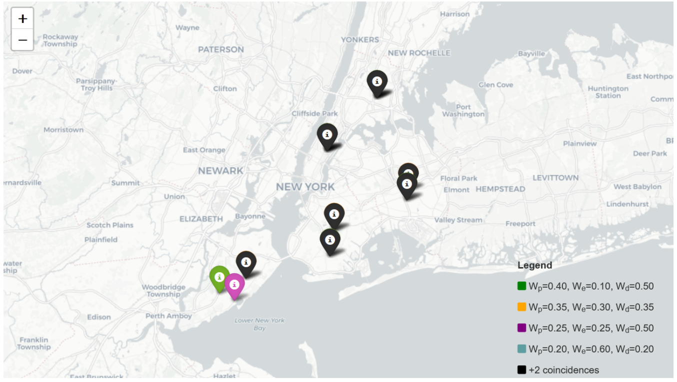

Fig. 4. Distribution of Rescue vehicles for the first shift

important factor, we are able to detect cases of unique vehicle locations (no coincidences).

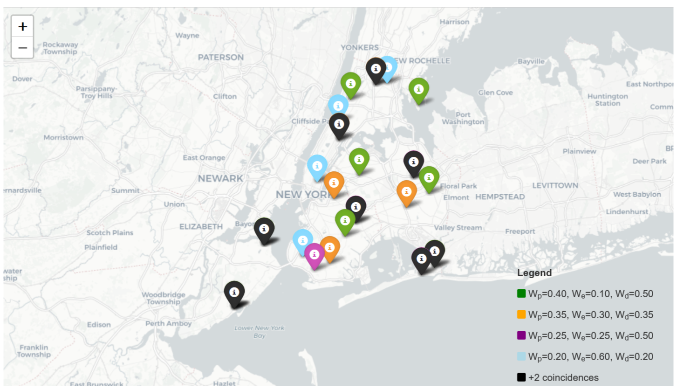

Fig. 5. Distribution of Squad vehicles for the first shift

On the other hand, the distribution of squad vehicles varies to a greater extent, influenced by changes in weighting (Figure 5). Nevertheless, even in this case a relatively even distribution is observable, indicating that equations 8 and 13 properly works.

c) Vehicles distribution analysis: Figure 6 provides information about the distribution of vehicles within the stations. The majority of the stations contain only one or two vehicles, proving a tending uniform distribution of the vehicles among the city territory. The model is capable to ensure a fair allocation, and a correct exploiting of the station capacities.

In addition, we can observe only a slight difference in the distribution through the different experiments: when w_e is increased, more stations host four vehicles compared to the rest of the experiments. In contrast, with a more minor w_e, we can observe more stations with three vehicles.

d) Shift vehicles transfer analysis: Figure 7 shows vehicle changes between the first and second shifts for experiments 1 and 4. The allocation of vehicles changes between the two shifts, as can be seen. For experiment 1, six stations change engine and ladder vehicles. On contrast, for experiment 4, there are 20 stations with changes in engine vehicles, 38 with

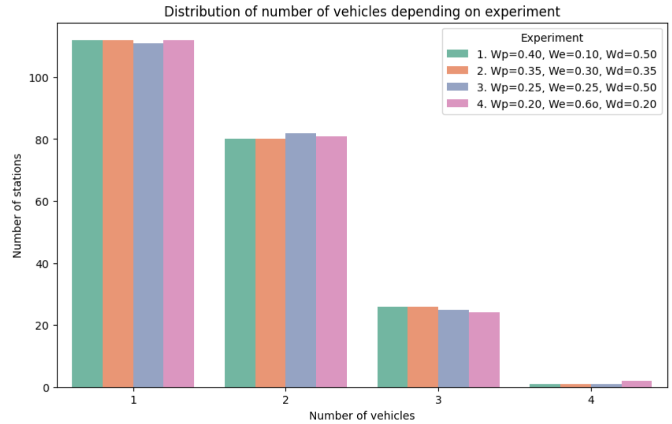

Fig. 6. Distribution of the number of vehicles depending on weight configurations

changes in ladder vehicles, 4 with changes in Rescue vehicles and 14 with changes in Squad vehicles.

Consequently, we can state that w_e has a more significant impact than the rest of the weights since increasing its value (0.6 VS 0.1) leads to more vehicle changes between the shifts.

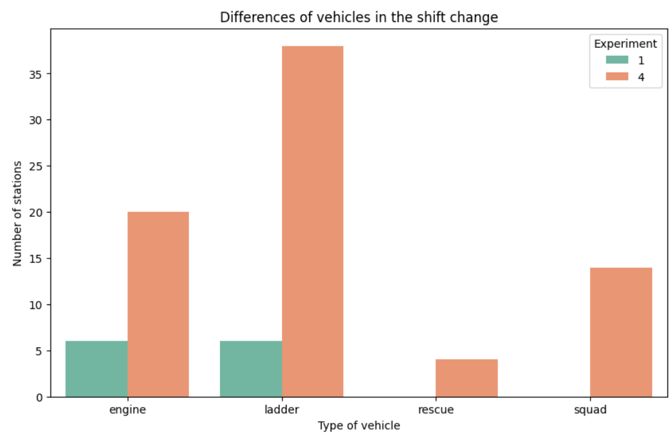

Fig. 7. Differences of vehicles in the shift change for experiments 1 and 4

2) Solvers comparison: Figure 8 provides a comprehensive comparison of different solvers' execution times. This comparison lets us understand whether solvers achieve a solution within the set time. Particularly, we computed the average of the execution time with a 5-iteration loop. This evaluation becomes critical, especially when faced with real-world scenarios where timely solutions are imperative.

We tested four different solvers:

- *GLOP* demonstrates a remarkable degree of consistency and stability in its performance. The average execution times associated with different parameter configurations for GLOP are similar, hovering within a narrow range of approximately 159-162 seconds. The tightly clustered run times underscore GLOP's ability to withstand parameter fluctuations. This stability provides predictable

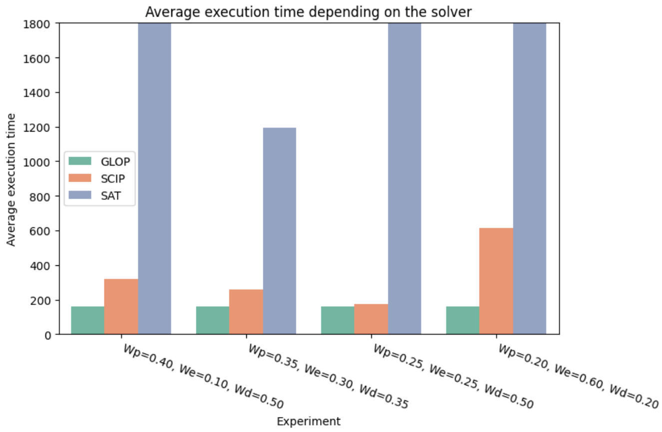

Fig. 8. Solver comparison depending on weight configurations

and reliable performance regardless of the parameters configuration used.

- *SCIP* has a more pronounced variability in the average run time than GLOP. The time execution for experiment 4 (w_p=0.20, w_e=0.60, w_d=0.20) prominently stands out, showing a significantly higher execution time than the other experiments. This substantial difference in run times suggests that SCIP may show greater sensitivity to specific parameter configurations than the more stable GLOP.
- *SAT* shows relative variability in execution time, as indicated by comparing the experiments. In particular, the experiment performed with the first, third, and fourth parameter configurations has a longer execution time than the second one. Like the SCIP solver, the SAT solver is sensitive to varying parameter configurations.

VI. Conclusions

In this paper a linear optimization model has been designed to allocate firefighting vehicles in New York City. The model maximize the population and emergency coverage as well as minimizing distances to neighborhoods. Several constraints were made for ensuring a technical and ethical coherence of the vehicle allocation.[11]

Various observations of the model have been made by performing several experiments with different weightings related to distance, population, and previous incidents.

Firstly, we saw that low and high-availability vehicles are well distributed in all the model configurations. The results indicated that the distribution of vehicles is minimally influenced by variations in the weight values. This is most likely due to strict constraints that limit the possibility of imbalances in vehicle allocation, showcasing the system's robustness across different weighting scenarios. Furthermore, most stations only contain one or two vehicles, indicating that the model achieves a fair distribution. Finally, it was also observed that the more importance given to past emergencies, the more changes we observed during the two shifts.

In terms of solvers, while GLOP was found to be the most efficient, it is inoperable for this problem, as the values for the optimal solution obtained are floats $\mathbb{R}$, and the decision variables are integers $\mathbb{Z}$. For that reason, we used SCIP.

A. Future Improvements

To further expand the applicability and accuracy of the proposed model, we can explore future research directions for a better fit to real dynamics.

The model can be enriched by integrating additional data sources, such as detailed traffic information, particularly, by introducing constraints that take into account peak hours during the day. This could be done by exploring the possibility of dynamically adjusting the distribution of vehicles based on variations in emergency requests and traffic levels. Adopting this strategy could ensure optimal response even at the most critical times. Fitting the model to traffic dynamics could allow optimal distribution of vehicles in response to daily, weekly, or seasonal variations in coverage needs.

Another improvement could be considering the economic aspect of the problem, by investigating the possibility of incorporating constraints on the budget available for each type of vehicle. To customize the model in line with specific financial policies of a locality or other urban jurisdiction, financial constraints can be implemented in line with local government spending policies and budget targets. This approach will achieve an optimal balance between maximizing operational effectiveness and optimizing the use of available financial resources.

Further improvement could include converting the model from linear to nonlinear. In this way, it would be possible to include a serviceability variable to optimize the area of station operations. Such an approach would further limit the overlap between vehicle operating areas and ambiguities in service.

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B. Annex

Eva Cantín Larumbe reviewed [1]
Francesco Pio Capoccello reviewed [2]
Mikel Baraza Vidal reviewed [3]
Eva Teisberg reviewed [4]
Daniele Borghesi reviewed [5]

[11] All the materials related to the project can be found at https://github.com/danieleborghe/optimization-project-1-UPV