Equivalent circuit method to determine the complex permittivity from Reflection coefficient

Introduction

A coaxial transmission line can have multiple applications and in this project, we will review an application used for the measurement of the complex permittivity. A typical setup includes a coaxial transmission line used as an antenna to irradiate the material under test. A schematic of this technique is presented in Figure 1. The input reflection coefficient of the probe is measured with a vector network analyser, thus measured experimentally. However, the complex permittivity is determined using an approximate circuit model for the probe admittance.

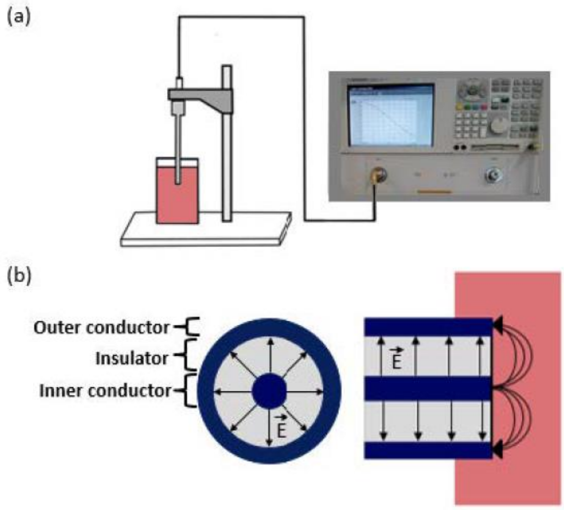

Figure 1 Schematics of open-ended coaxial probe technique: (a) Schematised measurement set-up, including the Vector Network Analyser (on the right), the cable connecting one port of the VNA to the coaxial probe, the probe bracket, and the liquid sample being measured; (b) top and side cross-sections of the coaxial probe, with electric field orientation indicated.

In this project you will be deriving a radiating antenna model that approximates a coaxial transmission line. The method has been proposed by Marsland and Evans in the paper,

Marsland, T. P., & Evans, S. (1987, August). Dielectric measurements with an open-ended coaxial probe. In *IEE Proceedings H (Microwaves, Antennas and Propagation)* (Vol. 134, No. 4, pp. 341-349). IET Digital Library. (see Appendix 1)

Background

The reflection and the transmission coefficients of antennas, filters and transmission lines with n ports can be fully represented by complex valued scattering parameters (S-parameters). It is usual to characterize them in a square n x n scattering matrix S = [Sij] where I, j = 1,…., n:

$$S = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1n} \\ S_{21} & S_{22} & \dots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \dots & S_{nn} \end{bmatrix}$$

The first index *i* represents the "output" port, where the scattered electromagnetic energy emerges and index *j* the "input" port of the incident wave. Therefore, $(i \neq j)$ expresses the transmission at the different ports and $(i = j)$ the reflection. For a network with two ports $(n = 2)$, the matrix reduces to:

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

Since S-parameters are ratios of the incidence (a_j) and the reflected (b_i) signal and are dimensionless. In particular, $S_{11} = b_1/a_1$ is also known as the reflection coefficient and $S_{21} = b_2/a_1$ as the transmission coefficient. For the purpose of measuring the complex permittivity using a coaxial transmission line, a one port system S = [S11] is technically sufficient.

A vector network analyzer is used to measure the reflection coefficient from the probe aperture/material interface. The inverse model is then used to compute the permittivity from the reflection coefficient. In a first order approximation, the interface between probe aperture and sample can be modelled as a lumped-element equivalent circuit. The equivalent circuit relates the permittivity of the material under test to the admittance and the reflection coefficient.

The model proposed by Marsland and Evans is based on a radiating antenna model and approximates the coaxial probe as a radiating antenna.

The Admittance Y is defined as

$$Z = \frac{1}{Y} = Z_0 \frac{1 - \Gamma}{1 + \Gamma}$$

where Z is the impedance of the probe, Z_0 is the characteristic impedance of the probe and Γ the reflection coefficient.

The admittance normalised to the characteristic impedance $(y = YZ_0)$ is

$$y\left(\omega, \varepsilon_r\right) = G_0 Z_0 \varepsilon_r^{5/2} + j\omega Z_0 \left(\varepsilon_r C_0 + C_f\right)$$

and presented by the lumped-element equivalent circuit in Figure 2. G_0 is the free-space radiation conductance and the capacitance Cf represents the fringing field in the dielectric of the cable, that is independent of the dielectric sample. C_0 is the capacitance that depends on effects of the fringing-fields outside the probe tip and ω is the angular frequency.

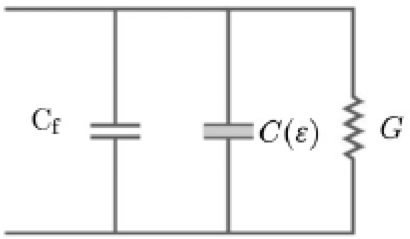

Figure 2 Radiating antenna equivalent circuit model.

Tasks

1. Describe the method used to compensate for systematic errors as explained in the paper by Marsland and Evans.
2. By following the methodology used in Marsland and Evans, derive the following simplified model;

$$\boxed{\varepsilon_m = -\frac{\Delta_{m2}\Delta_{13}}{\Delta_{m1}\Delta_{32}}\varepsilon_3^* - \frac{\Delta_{m3}\Delta_{21}}{\Delta_{m1}\Delta_{32}}\varepsilon_2^*}$$

ε_m .... unknown complex permittivity of MUT
ε_1^* .... known permittivity of a short-circuit termination
ε_2^* .... known permittivity of air (open)
ε_2^* .... known permittivity of a liquid (Di-Water or Methanol)

$\Delta_{ij} = \rho_i - \rho_j$ .... difference of two reflection-coefficients

3. Use the S_{11} data for short-circuit, open and DI water to compute the complex permittivity of two unknown material, Methanol and 0.1 Sodium Chloride as a function of frequency.
4. Compare the complex permittivity values computed in 3 to those provided in the excel sheet.