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ops = \{ QuantumOperator [{"R", \beta * \eta, "XZ"}, {1, 3}], \} QuantumOperator [
      {"R", \beta * \eta, "XZ"}, {2, 4}], * QuantumOperator [{"R", \beta * \gamma, "ZZZ"}, {1, 2, 5}],
    QuantumOperator \left[\left\{\text{"R", } \beta^2 * \frac{\eta * \gamma}{2}, \text{ "YZZ"}\right\}, \{1, 2, 6\}\right]
    © QuantumOperator \left[\left\{"R", \beta^2 * \frac{\eta * \gamma}{2}, "ZYZ"\right\}, \{1, 2, 7\}\right]\right\};
cz := ^{\circ} QuantumCircuitOperator [{"+" \rightarrow {3, 4, 5, 6, 7}, ops,
    ops, ops, ops, sx \rightarrow {3, 4, 5, 6, 7}}, "Parameters" \rightarrow {\eta, \gamma, \beta}]
cx := \bigcirc QuantumCircuitOperator [{"+" \rightarrow {3, 4, 5, 6, 7}, ops, ops, ops,
    ops, ops, "H" \rightarrow {1, 2}, sx \rightarrow {3, 4, 5, 6, 7}}, "Parameters" \rightarrow {\eta, \gamma, \beta}]
\Gamma = 0.785398163;
\Lambda = 0.785398163;
pos = Range[1, 128];
spins = 1 - 2 * IntegerDigits[pos - 1, 2, 7];
Fz[res_] := Sum[
   r * res[pos] * spins[pos] [1] * spins[pos] [2] * spins[pos] [5] / shots, {pos, 1, 128}]
Fx[res_] :=
 Sum[\(\lambda\) res[[pos]] * (spins[[pos]][3]] * spins[[pos]][1]] + spins[[pos]][4]] * spins[[pos]][2]) /
      shots, {pos, 1, 128}]
Hz = Table[cz(\langle | \eta \rightarrow 2\Lambda, \gamma \rightarrow 2\Gamma, \beta \rightarrow -i | \rangle)][inp], \{i, 0.01, 0.15, 0.01\}];
Hx = Table[cx[\langle | \eta \to 2\Lambda, \gamma \to 2\Gamma, \beta \to -i | \rangle][inp], \{i, 0.01, 0.15, 0.01\}];
shots := 10000
resz = Values | QuantumMeasurementSimulation | #,
          QuantumMeasurementOperator /@ {"ZZZZZZZ"}, shots]][1] & /@ Hz;
resx = Values QuantumMeasurementSimulation [#,
          QuantumMeasurementOperator /@ {"ZZZZZZZ"}, shots]][1] & /@ Hx;
approx = Prepend[Most[Map[Fz, resz] + Map[Fx, resx]], 0.]
\{0., -0.175615, -0.369294, -0.546794, -0.702303, -0.861111, -0.999498, -1.14621,
 -1.25884, -1.37209, -1.44388, -1.51362, -1.5298, -1.55996, -1.55226
X = \{\{0, 1\}, \{1, 0\}\};
Z = \{\{1, 0\}, \{0, -1\}\};
I2 = \{\{1, 0\}, \{0, 1\}\};
H = Γ KroneckerProduct[Z, Z] +
    Λ (KroneckerProduct[X, I2] + KroneckerProduct[I2, X]);
exact = Most[Table[Z = Tr[MatrixExp[-βH]];
       energy = Tr[H.MatrixExp[-βH]] / Z;
       \{\beta, N[energy]\}, \{\beta, Range[0, 0.15, 0.01] * 2 * 5\}][All, 2]];
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gs = Min[Eigenvalues[H]]

-1.7562

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p1 = Table [\{(i-1)*10^{-1}, approx[i]\}, \{i, 2, Length[approx]\}];
p2 = Table[{(i-1) * 10^{-1}, exact[i]}, {i, 2, Length[exact]}];
fit =
  FindFit[p1, yInf + (y0 - yInf) Exp[-\gamma x], {{yInf, -1.6}, {y0, 0}, {\gamma, 0.5}}, x];
f1[x_] = (yInf + (y0 - yInf) Exp[-\gamma x]) /. fit;
f2 = Interpolation[p2, InterpolationOrder → 2];
xmin = Min[Min[p1[[All, 1]]], Min[p2[[All, 1]]]];
xmax = Max[Max[p1[[All, 1]]], Max[p2[[All, 1]]]];
ymin = Min[Min[p1[[All, 2]]], Min[p2[[All, 2]]]];
ymax = Max[Max[p1[[All, 2]]], Max[p2[[All, 2]]]];
marginFactor = 0.15;
margin = marginFactor * Abs[gs] + 10^-1;
ylo = Min[ymin, gs - margin];
yhi = ymax;
Plot[{f1[x], f2[x], gs}, {x, xmin, xmax},
 PlotRange → {{xmin, xmax}, {ylo, yhi}}, PlotRangePadding → Scaled[.01],
 PlotStyle → {{Directive[Blue, Dashed, Opacity[0.5]]},
    {Directive[Red, Dashed, Opacity[0.5]]},
    {Directive[GrayLevel[0.25], Dashed, Opacity[0.4], AbsoluteThickness[2]]}},
 PlotLegends → Placed[{"approx", "exact", "ground"}, {0.8, 0.65}],
 AxesLabel \rightarrow \{ "\beta", " \setminus ! \setminus (\*SubscriptBox[\setminus (\H) \setminus), \setminus (\beta \setminus)] \setminus " \},
 GridLines → {Automatic, None}, LabelStyle → Directive[FontSize → 15],
 Epilog → {{Blue, Opacity[0.5], PointSize[0.015], Point[p1]},
    {Red, Opacity[0.5], PointSize[0.015], Point[p2]}}]
   \langle H \rangle_{\beta}
                                                                    ___β
      0.2
                            0.6
                                      8.0
                                                1.0
                                                          1.2
                  0.4
-0.5
                                                         approx
                                                         exact
-1.0
-1.5
-2.0
```