```
In[\circ]:= sx := QuantumOperator["SX", "Label" \rightarrow "\sqrt{X}"]
         inp := @ QuantumState ["UniformMixture"[2]]
  In[0]:= example := | ** QuantumCircuitOperator |
            \{"+" \rightarrow \{3, 4, 5, 6, 7\}, \text{ @ QuantumOperator } [\{"R", \beta * \eta, "XZ"\}, \{1, 3\}],
               \circ QuantumOperator [{"R", \beta * \eta, "XZ"}, {2, 4}],
              \blacksquare QuantumOperator [{"R", \beta * \gamma, "ZZZ"}, {1, 2, 5}],
              © QuantumOperator \left[\left\{"R", \beta^2 * \frac{\eta * \gamma}{2}, "YZZ"\right\}, \{1, 2, 6\}\right]
              "WireLabels" \rightarrow {""}, "MeasurementWireLabel" \rightarrow "", "Parameters" \rightarrow {\eta, \gamma, \beta}
         example["Diagram"]
Out[0]=
                       R_X(\beta \eta)
                      R_Z(\beta \eta)
                                                                      \sqrt{X}
                                R_Z(\beta \eta)
                                         R_Z(\beta \gamma)
         ops = {    QuantumOperator [{"R", \beta * \eta, "XZ"}, {1, 3}],   QuantumOperator [
                {"R", \beta * \eta, "XZ"}, {2, 4}], • QuantumOperator [{"R", \beta * \gamma, "ZZZ"}, {1, 2, 5}],
              © QuantumOperator \left[\left\{\text{"R"}, \beta^2 * \frac{\eta * \Upsilon}{2}, \text{"YZZ"}\right\}, \{1, 2, 6\}\right]
              QuantumOperator \left[\left\{"R", \beta^2 * \frac{\eta * \gamma}{2}, "ZYZ"\right\}, \{1, 2, 7\}\right]\right\};
  ln[\cdot]:= cz :=  QuantumCircuitOperator [\{"+" \rightarrow \{3, 4, 5, 6, 7\}, ops, 
              ops, ops, ops, ops, sx \rightarrow {3, 4, 5, 6, 7}}, "Parameters" \rightarrow {\eta, \gamma, \beta}]
         cx := \[ \] QuantumCircuitOperator \[ \] {3, 4, 5, 6, 7}, ops, ops, ops,
              ops, ops, "H" \rightarrow {1, 2}, sx \rightarrow {3, 4, 5, 6, 7}}, "Parameters" \rightarrow {\eta, \gamma, \beta}]
 In[\circ]:= \Gamma = 0.785398163;
         \Lambda = 0.785398163;
```

In[o]:= Needs["Wolfram`QuantumFramework`"]

```
pos = Range[1, 128];
       spins = 1 - 2 * IntegerDigits[pos - 1, 2, 7];
       Fz[res_] := Sum[
          res[pos] * spins[pos] [1] * spins[pos] [2] * spins[pos] [5] / shots, {pos, 1, 128}]
       Fx[res ] :=
         Sum[\Lambda * res[pos] * (spins[pos][3] * spins[pos][1] + spins[pos][4] * spins[pos][2]) /
             shots, {pos, 1, 128}]
       Hz = Table[cz[\langle | \eta \rightarrow 2 \Lambda, \gamma \rightarrow 2 \Gamma, \beta \rightarrow -i | \rangle][inp], {i, 0.01, 0.15, 0.01}];
       Hx = Table[cx[\langle | \eta \to 2\Lambda, \gamma \to 2\Gamma, \beta \to -i | \rangle][inp], \{i, 0.01, 0.15, 0.01\}];
 In[ ]:= shots := 10000
       resz = Values QuantumMeasurementSimulation [#,
                 QuantumMeasurementOperator /@ {"ZZZZZZZ"}, shots ] [1] & /@ Hz;
       resx = Values QuantumMeasurementSimulation [#,
                 QuantumMeasurementOperator /@ {"ZZZZZZZ"}, shots]] [1] & /@ Hx;
 In[0]:= approx = Prepend[Most[Map[Fz, resz] + Map[Fx, resx]], 0.]
Out[0]=
       \{0., -0.175615, -0.369294, -0.546794, -0.702303, -0.861111, -0.999498, -1.14621,
        -1.25884, -1.37209, -1.44388, -1.51362, -1.5298, -1.55996, -1.55226
 ln[ \circ ] := X = \{ \{ 0, 1 \}, \{ 1, 0 \} \};
       Z = \{\{1, 0\}, \{0, -1\}\};
       I2 = \{\{1, 0\}, \{0, 1\}\};
       H = Γ KroneckerProduct[Z, Z] +
           Λ (KroneckerProduct[X, I2] + KroneckerProduct[I2, X]);
```

 $\{\beta, N[energy]\}, \{\beta, Range[0, 0.15, 0.01] * 2 * 5\}] [All, 2]];$

exact = Most[Table[Z = Tr[MatrixExp[-β H]];
energy = Tr[H.MatrixExp[-β H]] / Z;

In[0]:= gs = Min[Eigenvalues[H]]

-1.7562

Out[0]=

```
ln[\cdot]:= p1 = Table[{(i-1) * 10^{-1}, approx[i]]}, {i, 2, Length[approx]}];
     p2 = Table[{(i-1) * 10^{-1}, exact[i]}, {i, 2, Length[exact]}];
     fit =
        FindFit[p1, yInf + (y0 - yInf) Exp[-\gamma x], {{yInf, -1.6}, {y0, 0}, {\gamma, 0.5}}, x];
     f1[x_] = (yInf + (y0 - yInf) Exp[-\gamma x]) /. fit;
     f2 = Interpolation[p2, InterpolationOrder → 2];
     xmin = Min[Min[p1[All, 1]], Min[p2[All, 1]]];
     xmax = Max[Max[p1[[All, 1]]], Max[p2[[All, 1]]]];
     ymin = Min[Min[p1[[All, 2]]], Min[p2[[All, 2]]]];
     ymax = Max[Max[p1[[All, 2]]], Max[p2[[All, 2]]]];
     marginFactor = 0.15;
     margin = marginFactor * Abs[gs] + 10 ^ -1;
     ylo = Min[ymin, gs - margin];
     yhi = ymax;
     Plot[{f1[x], f2[x], gs}, {x, xmin, xmax},
      PlotRange → {{xmin, xmax}, {ylo, yhi}}, PlotRangePadding → Scaled[.01],
      PlotStyle → {{Directive[Blue, Dashed, Opacity[0.5]]},
         {Directive[Red, Dashed, Opacity[0.5]]},
         {Directive[GrayLevel[0.25], Dashed, Opacity[0.4], AbsoluteThickness[2]]}},
      PlotLegends → Placed[{"approx", "exact", "ground"}, {0.8, 0.65}],
      AxesLabel \rightarrow {"\beta", "\!\(\*SubscriptBox[\(\(\(\(H\)\)\), \(\beta\)]\)"},
      GridLines → {Automatic, None}, LabelStyle → Directive[FontSize → 15],
      Epilog → {{Blue, Opacity[0.5], PointSize[0.015], Point[p1]},
         {Red, Opacity[0.5], PointSize[0.015], Point[p2]}}]
```



