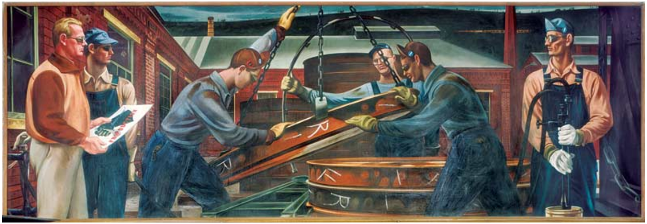


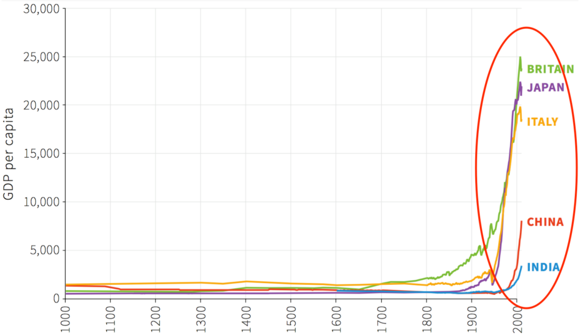
Advanced Macroeconomics

Section 2 - Growth (I): The mechanics of capital accumulation and growth

Daniele Girardi
King's College London

AY 2024-25, Semester I

The hockey stick of history



Section 2: Growth (I)

The Plan

1. Harrod-Domar
2. Solow
3. Ramsey-Cass-Koopmans
4. Diamond’s overlapping-generations (OLG)

- Variable t represents time
  - continuous (except in the OLG model)

Dynamic analysis

- Variable t represents time
  - continuous (except in the OLG model)
- $X(t) \rightarrow$ X can change over time.

- Variable t represents time
  - continuous (except in the OLG model)

- $X(t) \rightarrow$ X can change over time.

- $\frac{dX}{dt} \rightarrow$ rate of change of X.

Dynamic analysis

- Variable t represents time
  - continuous (except in the OLG model)

- $X(t) \rightarrow$ X can change over time.

- $\frac{dX}{dt} \rightarrow$ rate of change of X.

- $\dot{X}(t) \rightarrow$ shorthand for $\frac{dX}{dt}$

Dynamic analysis

- Variable t represents time
  - continuous (except in the OLG model)
- $X(t) \rightarrow$ X can change over time.
- $\frac{dX}{dt} \rightarrow$ rate of change of X.
- $\dot{X}(t) \rightarrow$ shorthand for $\frac{dX}{dt}$
- $\frac{\dot{X}(t)}{X(t)}$ = *growth rate* of X

Dynamic analysis

- Variable t represents time
  - continuous (except in the OLG model)
- $X(t) \rightarrow$ X can change over time.
- $\frac{dX}{dt} \rightarrow$ rate of change of X.
- $\dot{X}(t) \rightarrow$ shorthand for $\frac{dX}{dt}$
- $\frac{\dot{X}(t)}{X(t)}$ = *growth rate* of X
- g_X is a shorthand for $\frac{\dot{X}(t)}{X(t)}$

Intertemporal equilibrium

- Static analysis: equilibrium condition → equilibrium relations.
  - I=S → equilibrium level of Y
  - MRS=MRT → optimal quantity consumed
  - ...
- Growth theory is *dynamic*: intertemporal equilibrium.

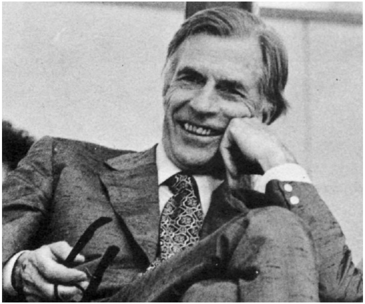
Intertemporal equilibrium

- Static analysis: equilibrium condition → equilibrium relations.
  - I=S → equilibrium level of Y
  - MRS=MRT → optimal quantity consumed
  - ...
- Growth theory is *dynamic*: intertemporal equilibrium.

Main concepts:

- Intertemporal equilibrium
- Steady state
- Dynamic stability

The Harrod-Domar model



There's an old joke. Two elderly women are at a Catskill restaurant. One of them says, 'Boy, the food at this place is just terrible.' The other one says, 'Yeah I know. And such small portions.'

(Woody Allen, 'Annie Hall')

The Harrod-Domar model

- ‘Grandfather’ of modern growth theory.
- Premise 1: aggregate investment has a dual effect
  1. multiplier effect (demand side)
  2. capacity-creating effect (supply side)
- Premise 2: investment depends on output (accelerator)

The Harrod-Domar model

- ‘Grandfather’ of modern growth theory.
- Premise 1: aggregate investment has a dual effect
  1. multiplier effect (demand side)
  2. capacity-creating effect (supply side)
- Premise 2: investment depends on output (accelerator)

- Main findings:
  - unique equilibrium path: $g_W = sa$ (*warranted rate*)
  - warranted rate does not guarantee full (nor stable) employment
  - instability: economy won’t converge to g_W, except by a fluke

Harrod-Domar: Assumptions

- One-good economy, closed, no government: $Y(t) = C(t) + I(t)$

Harrod-Domar: Assumptions

- One-good economy, closed, no government: $Y(t) = C(t) + I(t)$
- Costant saving rate: $S(t) = sY(t)$

Harrod-Domar: Assumptions

- One-good economy, closed, no government: $Y(t) = C(t) + I(t)$
- Costant saving rate: $S(t) = sY(t)$
- No depreciation: $\dot{K}(t) = I(t)$

- One-good economy, closed, no government: $Y(t) = C(t) + I(t)$
- Costant saving rate: $S(t) = sY(t)$
- No depreciation: $\dot{K}(t) = I(t)$
- Potential output $Y^*(t)$: output level if firms utilize their productive capacity at the planned/desired level

- One-good economy, closed, no government: $Y(t) = C(t) + I(t)$
- Costant saving rate: $S(t) = sY(t)$
- No depreciation: $\dot{K}(t) = I(t)$
- Potential output $Y^*(t)$: output level if firms utilize their productive capacity at the planned/desired level
- Linear technology: $Y^*(t) = aK(t)$

- One-good economy, closed, no government: $Y(t) = C(t) + I(t)$
- Costant saving rate: $S(t) = sY(t)$
- No depreciation: $\dot{K}(t) = I(t)$
- Potential output $Y^*(t)$: output level if firms utilize their productive capacity at the planned/desired level
- Linear technology: $Y^*(t) = aK(t)$
- Rate of capacity utilization: $u(t) = \frac{Y(t)}{Y^*(t)}$

Harrod-Domar: Assumptions

- One-good economy, closed, no government: $Y(t) = C(t) + I(t)$
- Costant saving rate: $S(t) = sY(t)$
- No depreciation: $\dot{K}(t) = I(t)$
- Potential output $Y^*(t)$: output level if firms utilize their productive capacity at the planned/desired level
- Linear technology: $Y^*(t) = aK(t)$
- Rate of capacity utilization: $u(t) = \frac{Y(t)}{Y^*(t)}$
- Investment rate: $\dot{g}_K(t) = \alpha(u(t) - 1)$ with $\alpha > 0$

Assumptions:

$$Y(t) = C(t) + I(t); \qquad S(t) = sY(t); \qquad Y^*(t) = aK(t); \qquad u(t) = \frac{Y(t)}{Y^*(t)}$$

$$g_K(t) = \frac{\dot{K}(t)}{K(t)} = \frac{I(t)}{K(t)}; \qquad \dot{g}_K(t) = \alpha(u(t) - 1) \qquad \text{with } \alpha > 0$$

Harrod-Domar: Intertemporal equilibrium

Assumptions:

$$Y(t) = C(t) + I(t); \qquad S(t) = sY(t); \qquad Y^*(t) = aK(t); \qquad u(t) = \frac{Y(t)}{Y^*(t)}$$

$$g_K(t) = \frac{\dot{K}(t)}{K(t)} = \frac{I(t)}{K(t)}; \qquad \dot{g}_K(t) = \alpha(u(t) - 1) \qquad \text{with } \alpha > 0$$

Investment rate at each point in time:

$$I(t) = S(t) \quad \rightarrow g_K(t) = sa[u(t)]$$

Assumptions:

$$Y(t) = C(t) + I(t); \qquad S(t) = sY(t); \qquad Y^*(t) = aK(t); \qquad u(t) = \frac{Y(t)}{Y^*(t)}$$

$$g_K(t) = \frac{\dot{K}(t)}{K(t)} = \frac{I(t)}{K(t)}; \qquad \dot{g}_K(t) = \alpha(u(t) - 1) \qquad \text{with } \alpha > 0$$

Investment rate at each point in time:

$$I(t) = S(t) \quad \rightarrow g_K(t) = sa[u(t)]$$

Intertemporal equilibrium (‘warranted’ growth rate):

$$\dot{g}_K(t) = 0 \quad \rightarrow u = 1 \quad \rightarrow g_W = sa$$

The equilibrium (‘warranted’) rate of growth

- Warranted vs natural growth rate:

$$g_Y = sa \neq n$$

The equilibrium (‘warranted’) rate of growth

- Warranted vs natural growth rate:

$$g_Y = sa \neq n$$

- Dynamic instability:

$$g_K = u(sa) > g_W = sa \quad \Rightarrow \quad u > 1 \quad \Rightarrow \quad \dot{g}_K > 0$$
$$g_K = u(sa) < g_W = sa \quad \Rightarrow \quad u < 1 \quad \Rightarrow \quad \dot{g}_K < 0$$

The equilibrium ('warranted') rate of growth

- Warranted vs natural growth rate:

$$g_Y = sa \neq n$$

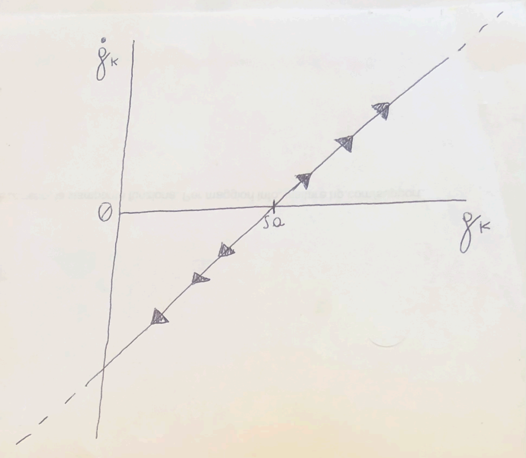
- Dynamic instability:

$$g_K = u(sa) > g_W = sa \quad \Rightarrow \quad u > 1 \quad \Rightarrow \quad \dot{g}_K > 0$$
$$g_K = u(sa) < g_W = sa \quad \Rightarrow \quad u < 1 \quad \Rightarrow \quad \dot{g}_K < 0$$

- More formally (by plugging I=S condition into investment function):

$$\dot{g}_K = \alpha \left[\frac{g_K}{g_W} - 1 \right]$$

$$\dot{g}_K = \alpha \left[\frac{g_K}{g_W} - 1 \right] \quad \text{with } \alpha > 0$$



- Phase diagram.
- positive slope $\rightarrow$ instability