Advanced Macroeconomics – 7SSPN402 – Week 2 Seminar – Answer Keys

1. Consider the Solow model, as described in the textbook and lecture slides. Derive the fundamental equation of the model, which says that the change in capital per unit of effective labor over time equals investment per unit of effective labor minus breakeven investment. Start by taking the derivative of k=K/AL with respect to time. (Try to do the derivation by yourself, without copying it from the textbook and slides!)

The derivation is as follows:

$$\dot{k}(t) = \frac{d\left(\frac{K}{AL}\right)}{dt} = \frac{\dot{K}AL - (\dot{A}L + A\dot{L})K}{(AL)^2} =$$

$$= \frac{\dot{K}}{AL} - \frac{\dot{A}L}{AL}\frac{K}{AL} - \frac{A\dot{L}}{AL}\frac{K}{AL} =$$

$$= \frac{sY - \delta K}{AL} - kg - kn =$$

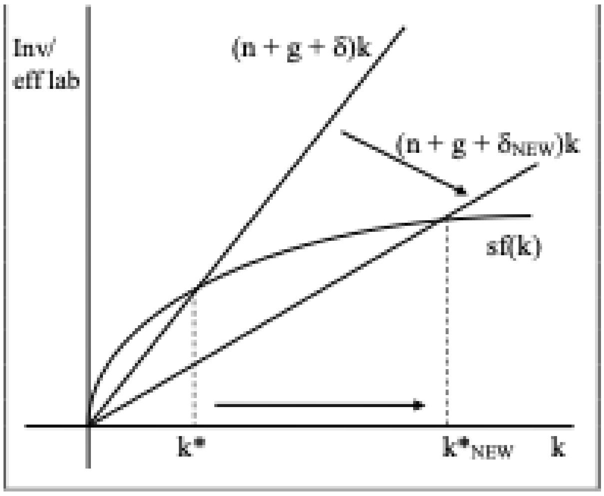
$$= \; sf(k) - (n + g + n)k$$

2. Describe how, if at all, each of the following developments affects the break-even and actual investment lines in our basic diagram for the Solow model:
   a. The rate of depreciation falls.
   b. The rate of technological progress rises.
   c. The production function is Cobb Douglas, $f(k) = k^{\alpha}$, and capital's share, α, rises.
   d. Workers exert more effort, so that output per unit of effective labor for a given value of capital per unit of effective labor is higher than before.

(a) The slope of the break-even investment line is given by $(n + g + \delta)$ and thus a fall in the rate of depreciation, δ, decreases the slope of the break-even investment line.

The actual investment curve, sf(k) is unaffected.

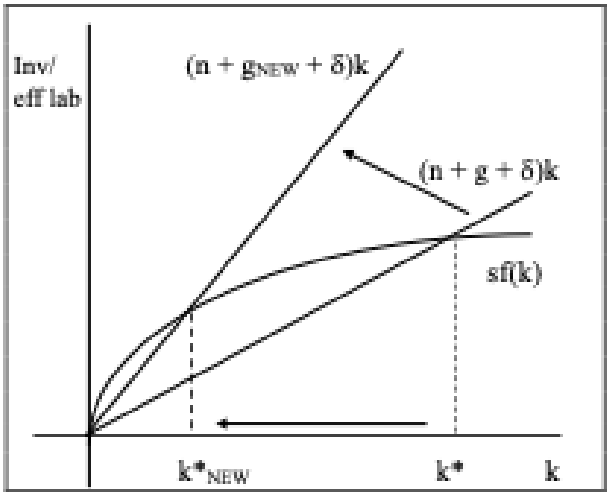
From the figure at right we can see that the balanced-growth-path level of capital per unit of effective labor rises from k^* to k^*_{NEW} .



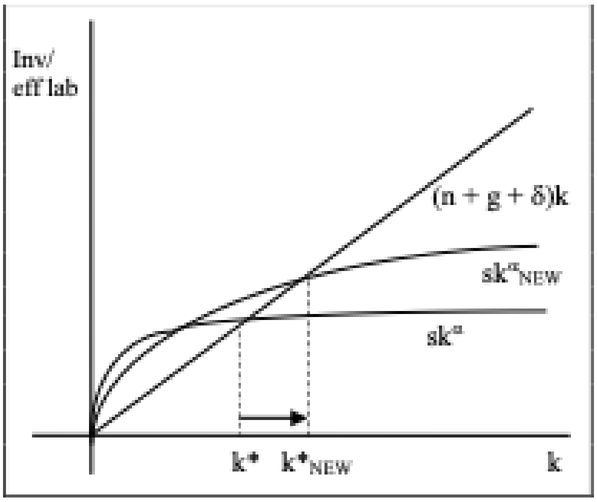
(b) Since the slope of the break-even investment line is given by $(n + g + \delta)$, a rise in the rate of technological progress, g, makes the break-even investment line steeper.

The actual investment curve, sf(k), is unaffected.

From the figure at right we can see that the balanced-growth-path level of capital per unit of effective labor falls from k^* to k^*_{NEW} .



(c) The break-even investment line, $(n + g + \delta)k$, is unaffected by the rise in capital's share, α.



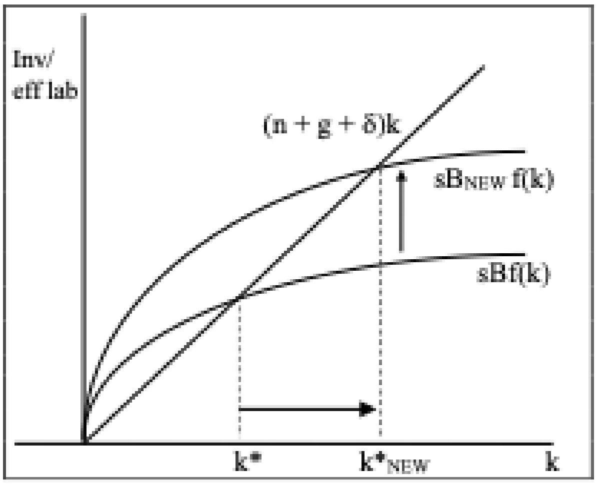
The effect of a change in α on the actual investment curve, sk^{α}, can be determined by examining the derivative $\partial(sk^{\alpha})/\partial\alpha$. It is possible to show that

$$(1) \quad \frac{\partial sk^{\alpha}}{\partial\alpha} = sk^{\alpha}\ln k .$$

For $0 < \alpha < 1$, and for positive values of k, the sign of $\partial(sk^{\alpha})/\partial\alpha$ is determined by the sign of lnk. For $\ln k > 0$, or $k > 1$, $\partial sk^{\alpha}/\partial\alpha > 0$ and so the new actual investment curve lies above the old one. For $\ln k < 0$ or $k < 1$, $\partial sk^{\alpha}/\partial\alpha < 0$ and so the new actual investment curve lies below the old one. At $k = 1$ so that $\ln k = 0$, the new actual investment curve intersects the old one.

In addition, the effect of a rise in α on k* is ambiguous and depends on the relative magnitudes of s an $(n + g + \delta)$. It is possible to show that a rise in capital's share, α, will cause k* to rise if $s > (n + g + \delta)$ This is the case depicted in the figure above.

(d) Suppose we modify the intensive form of the production function to include a non-negative constant, B, so that the actual investment curve is given by $sBf(k)$, $B > 0$.



Then workers exerting more effort, so that output per unit of effective labor is higher than before, can be modeled as an increase in B. This increase in B shifts the actual investment curve up.

The break-even investment line, $(n + g + \delta)k$, is unaffected.

From the figure at right we can see that the balanced-growth-path level of capital per unit of effective labor rises from k* to k^*_{NEW} .

3. Suppose that the production function is Cobb Douglas:

$Y = F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$, and therefore $f(k) = k^{\alpha}$.

Find expressions for k∗, y∗, and c∗ as functions of the parameters of the model, s, n, δ, g, and α.

(a) The equation describing the evolution of the capital stock per unit of effective labor is given by

(1) $\dot{k} = sf(k) - (n + g + \delta)k$.

Substituting in for the intensive form of the Cobb-Douglas, $f(k) = k^{\alpha}$, yields

(2) $\dot{k} = sk^{\alpha} - (n + g + \delta)k$.

On the balanced growth path, $\dot{k}$ is zero; investment per unit of effective labor is equal to break-even investment per unit of effective labor and so k remains constant. Denoting the balanced-growth-path value of k as k*, we have $sk^{*\alpha} = (n + g + \delta)k^*$. Rearranging to solve for k* yields

(3) $k^* = [s/(n + g + \delta)]^{1/(1-\alpha)}$.

To get the balanced-growth-path value of output per unit of effective labor, substitute equation (3) into the intensive form of the production function, $y = k^{\alpha}$:

(4) $y^* = [s/(n + g + \delta)]^{\alpha/(1-\alpha)}$.

Consumption per unit of effective labor on the balanced growth path is given by $c^* = (1 - s)y^*$. Substituting equation (4) into this expression yields

(5) $c^* = (1 - s)[s/(n + g + \delta)]^{\alpha/(1-\alpha)}$.