Take-home assignment 3

Econ 705 - Spring 2021

Due: April 22 before 2pm (please send via email to Guilherme)

1. **IS-MP model** Consider an economy governed by the following two equations.

$$Y = C(Y - T) + I(r) + G; \qquad \frac{dI}{dr} < 0 \tag{1}$$

$$r = f(Y, \pi); \qquad \frac{df}{dY} > 0; \frac{df}{d\pi} > 0 \tag{2}$$

where Y is output; C is consumption; T is taxes (assumed to consist of an exogenous lump-sum tax equally shared among all agents); I is (private) investment; r is the real interest rate; G is government spending; π is inflation.

Equation 1 is an 'old-school' IS curve. Equation 2 is the monetary policy rule, or *MP equation*: it says that the interest rate is set by the Central Bank based on economic conditions.

i draw the IS-MP curves in the r-Y space (Y on the horizontal axis; r on the vertical axis), denoting the equilibrium level of output as $Y^\star$ and the equilibrium real interest rate as $r^\star$;

Now describe the effect on the IS and MP curves and on the equilibrium outcomes $r^\star$ and $Y^\star$ of...

ii a fall in taxes (assuming a given π);

iii a rise in π;

iv a rise in government spending G combined with a change in the monetary policy rule of the Central Bank, so that a higher interest rate is now being set for any given (Y, π) combination (assuming a given π);

v a decline in the sensitivity of investment to the real interest rate;

vi does government spending crowd-out private spending in this model? why does (or does not) that happen?

2. **Imperfections, rigidities and nominal shocks** Suppose that the economy is characterized by imperfect competition in the product market (similar to the model of Section 6.5), firms must pay small menu costs to change their prices, and the labor market is Walrasian (perfectly competitive). Is such an economy likely to feature real effects of nominal (monetary) shocks? Why or why not?
(max 300 words)

3. **Intratemporal labor supply condition in a New-Keynesian framework** Explain and show how the following labor-supply equilibrium condition (eq. 6.15 at p. 247 of the textbook)

$$C^{-\theta}\frac{W}{P} = V'(L) \tag{3}$$

can be derived from the following utility function (eq. 6.2-6.4 at p.239 of the textbook)

$$U = \sum_{t=0}^{\infty} \beta^t \left[U(C_t) + \Gamma\left(\frac{M_t}{P_t}\right) - V(L_t) \right], \quad 0 < \beta < 1$$

$$U(C_t) = \frac{C_t^{1-\theta}}{1-\theta}, \quad \theta > 0$$

$$\Gamma\left(\frac{M_t}{P_t}\right) = \frac{(M_t/P_t)^{1-\chi}}{1-\chi}, \quad \chi > 0 \tag{4}$$

4. **Menu costs with quadratic profit loss** Suppose that the economy is characterized by imperfect competition in the product market (similar to the model of Section 6.5). A producer must pay a menu cost Z in order to change its nominal price p_i. Denoting with π_{ADJ} the profits of a firm which sets its price equal to the optimal price ($p_i = p^\star$), with π_{FIX} the profits of a firm which keeps a price different from the optimal price ($p_i \neq p^\star$), and with C the profit loss from not adjusting the price, we have

$$C = \pi_{ADJ} - \pi_{FIX} = K(p_i - p^\star)^2, \qquad K > 0 \tag{5}$$

As in the model we studied in the textbook, we have $p^\star = p + \phi y$ and $y = m - p$. Assume that initially the economy is in a flexible-price equilibrium, with $m = 0$. All firms set the optimal price, so we have $y = 0$ and $p = m = 0$. Now suppose that the Central Bank changes the money supply from m to m'.

(Notation is the same as in the textbook.)

i Assume that fraction f of firms adjust their prices after the monetary shock. So now fraction f charges $p^\star$, while fraction $1 - f$ keeps charging 0. This implies $p = fp^\star$. Find p, y and $p^\star$ as a function of m' and f.

ii Find the first and second derivatives of C with respect to f ($\frac{\delta C}{\delta f}$ and $\frac{\delta^2 C}{\delta f^2}$), where C is the profit loss from not adjusting the price after the monetary shock we are studying [so $C = K(0 - p^\star)^2 = Kp^{\star 2}$].

iii Assume $0 < \phi < 1$. Plot the cost from not adjusting [$C = K(0 - p^\star)^2 = Kp^{\star 2}$] as a function of f. Include in the graph a horizontal line that represents the menu cost Z, assuming $Km'^2\phi^2 < Z < Km'^2$.

iv Make the same plot, but assuming $\phi > 1$ and $Km'^2\phi^2 > Z > Km'^2$.

v Explain the economic story conveyed by the two graphs. You can choose what to emphasize, but make sure to employ the concepts of monetary neutrality and equilibrium/equilibria. Explain the economic meaning of ϕ begin lower or higher than 1. (max 600 words.)

5. **Lucas imperfect information model** Consider the Lucas imperfect information model.

   i Explain *in your own words* the relation between the model, the Phillips curve, and the so-called *Lucas critique* (max 400 words);

   ii Consider the problem facing an individual in the Lucas model when P_i is observed but P_i/P is not. The individual chooses L_i to maximize the *expectation of* U_i; U_i is given by

   $$U_i = C_i - \frac{1}{\gamma}L_i^{\gamma} = \frac{P_i}{P}Y_i - \frac{1}{\gamma}Y_i^{\gamma}$$

   We are thus keeping all the assumptions in the Lucas model in the textbook, except one: we are *not* assuming *certainty-equivalence* behavior; we are instead assuming that individuals maximize expected utility.

   Find the first-order condition for Y_i, and rearrange it to obtain an expression for Y_i in terms of $E[P_i/P]$. Take logs of this expression to obtain an expression for y_i [as usual we are defining $y_i = log(Y_i)$].

   iii How does the amount of labor the individual supplies if he or she follows the *certainty-equivalence* rule (which you don't need to derive from scratch, but can just take from the model in the textbook) compare with the optimal amount derived in part (ii)? [*Hint: You will need to apply a result called* **Jensen's inequality**, *which says that if X is a random variable and ϕ is a concave function, then $\phi(E(X)) > E(\phi(X))$.*]

6. **Phillips Curve(s)** Consider the four types of Phillips Curves (PCs) that we have encountered: the traditional PC; the accelerationist PC; Lucas' supply curve and the New-Keynesian PC. Write down and briefly discuss each one, explaining its meaning, its main implications, and where it comes from (from what model and assumptions it has been derived).

Extra-credits **Using the Bellman equation to derive optimal consumption dynamics**

An infinitely-lived consumer derives utility from consumption. She does not work but she holds some wealth, invested in in a safe asset, which rate of return is certain and constant over time. Each period she receives capital income and decides how much to consume; if she saves her wealth will increase. She aims to maximize her lifetime utility, and she applies a discount rate ρ. The model is in discrete time. Her maximization problem is:

$$\max_{\{C_{t+i};\ i=0,1,\ldots\}} \left[U_t = \sum_{i=0}^{\infty} \left(\frac{1}{1+\rho} \right)^i U(C_{t+i}) \right] \tag{6}$$

Instantaneous utility is given by

$$U(C_{t+i}) = \ln(C_{t+i}) \tag{7}$$

Her wealth evolves according to the following equation:

$$W_{t+i+1} = (1+r)(W_{t+i} - C_{t+i}) \tag{8}$$

Where W_{t+i} is the wealth available at the beginning of period $t+i$.

Let $V_t(W_t)$ be the *value function* which summarizes the solution of this problem. The problem can be written in terms of the following *Bellman equation:*

$$V_t(W_t) = \max_{C_t} \left(U(C_t) + \frac{1}{1+\rho} V_{t+1}(W_{t+1}) \right) \tag{9}$$

i Explain *in your own words* the meaning of the value function $V(W)$. Provide also a formal definition.

ii Explain *in your own words* the meaning of the Bellman equation.

iii Use the Bellman equation to derive the Euler equation which describes the optimal consumption path (that is, an expression for $\frac{C_{t+1}}{C_t}$)

[Hint: no need to use the method of undetermined coefficients here (although using it would allow to provide additional results, but the question does not ask for those)]