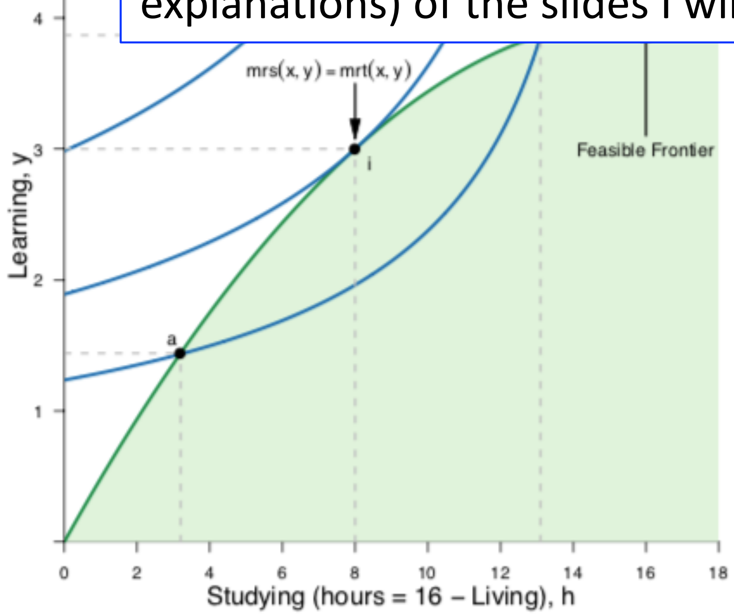
3 – Doing the Best You Can

Extended slides
this is an extended version (much more crowded with text and with additional explanations) of the slides I will project in class. You can use them as lecture notes.



3 – Doing the Best You Can

- *How can we use the 'preferences, beliefs and constraints' approach to formally model people's behavior?*
- *How can we take into account the constraints that people face in making their choices?*
- *How would people make choices if they were always doing the best they can, given the constraints they face?*

Study Materials for this Section:

- Textbook Chapter 3: "Doing the Best you Can: Constrained Optimization".

Additional Materials:

- Videos on deriving MRS and MRT (on Moodle, Section 3)

Section 3 - The key ideas

1. We can represent preferences using *utility functions* and *indifference curves*.
   - Both self-regarding and other-regarding preferences can be represented in this way.

2. *Feasible sets* are used to represents available choices (constraints).

3. *Doing the best you can* can be formalized as the problem of maximizing utility under the constraints imposed by the feasible set.
   - Utility is maximized when MRS=MRT.
   - MRS comes from preferences; MRT represents constraints.

3 –Doing the Best You Can

The Plan

1. Representing preferences: utility functions & indifference curves
2. Doing the best you can: feasibility & utility
3. Offer curves and demand functions
4. Representing *social* preferences.

Utility functions

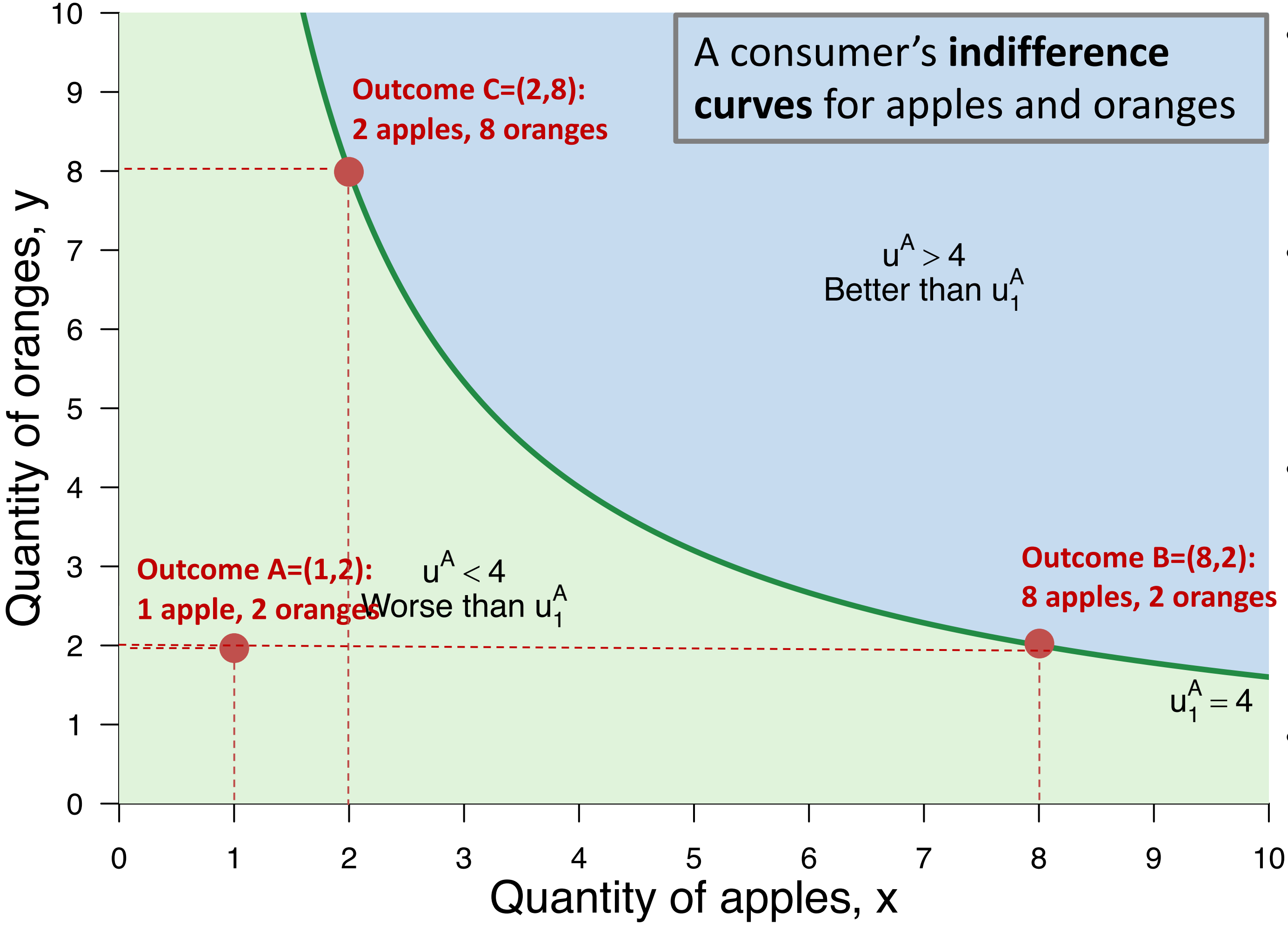
- $u(A)$ is the *utility* that you get from outcome A.
- Higher values of *u* are preferred to lower values:
  - If $u(A) > u(B)$, then $A \succ B$
  - If $u(A) = u(B)$, *then* $A \sim B$
  - If $u(A) < u(B)$, then $A \prec B$
- Utility is *not* a measure of well-being. It is just a way to represent people's choices mathematically.
  - You choose *A* over *B* $\Leftrightarrow u(A) > u(B)$

Utility functions

- Assume that an outcome is defined by two variables: *x* & *y*.
- For example, imagine a consumer who consumes just two goods: apples & oranges.
- An outcome is defined by the quantity of apples (x) and oranges (y) that this agent gets to consume.
- $u(x, y) > u(x', y')$ means that you prefer the combination (x,y) over the combination (x',y').
  - in simpler words, you prefer to consume x apples and y oranges, rather than x' apples and y' oranges.

Indifference Curves

- Indifference curves allow to visualize preferences over combinations (*bundles)* of goods.

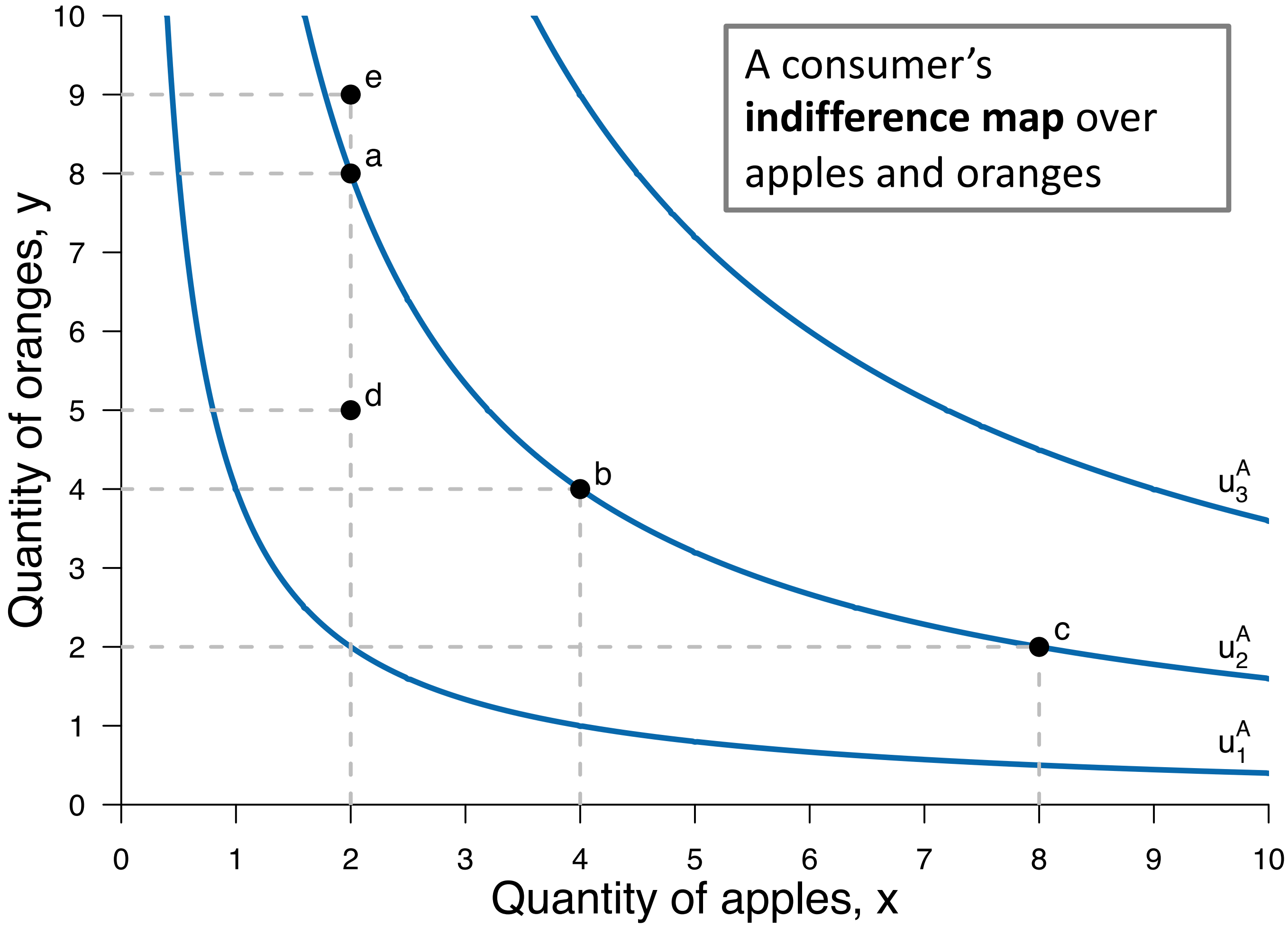


- Each point in the graph is a combination of apples and oranges (a *bundle*).
- *Indifference curve* (dark green line): connects different bundles that give the same utility.
- Consumer is indifferent between two points (two bundles) that lie on the same indifference curve.
- *Downward sloping* because the consumer likes both goods.

Indifference Map

- Many indifference curves on the same axes.

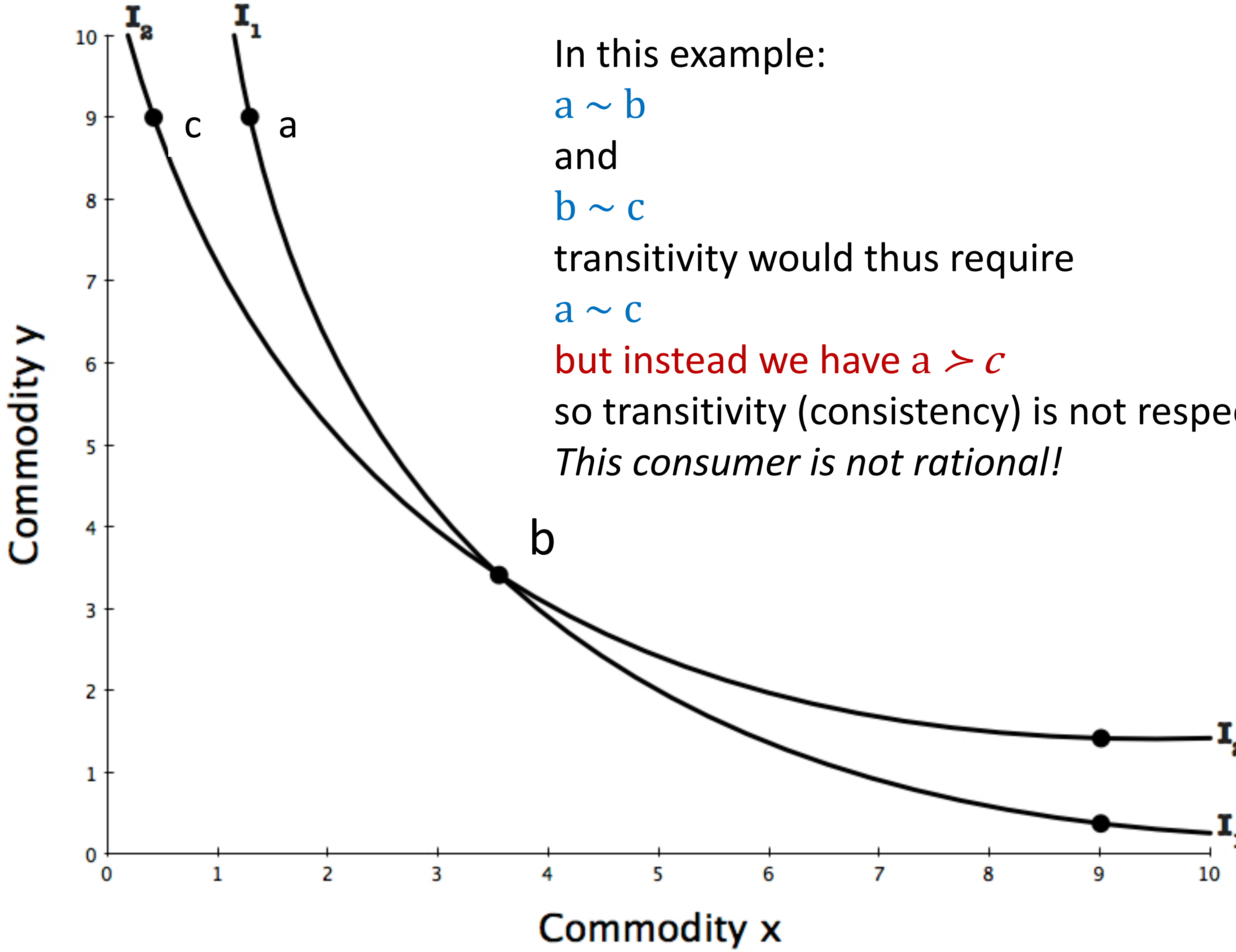
(only 3 shown here, but imagine there is an infinite number of them, all parallel.)



- Bundles that lie on higher indifference curves give more utility.

- Indifference curves represent preferences: visualization of consumer's ranking of outcomes.

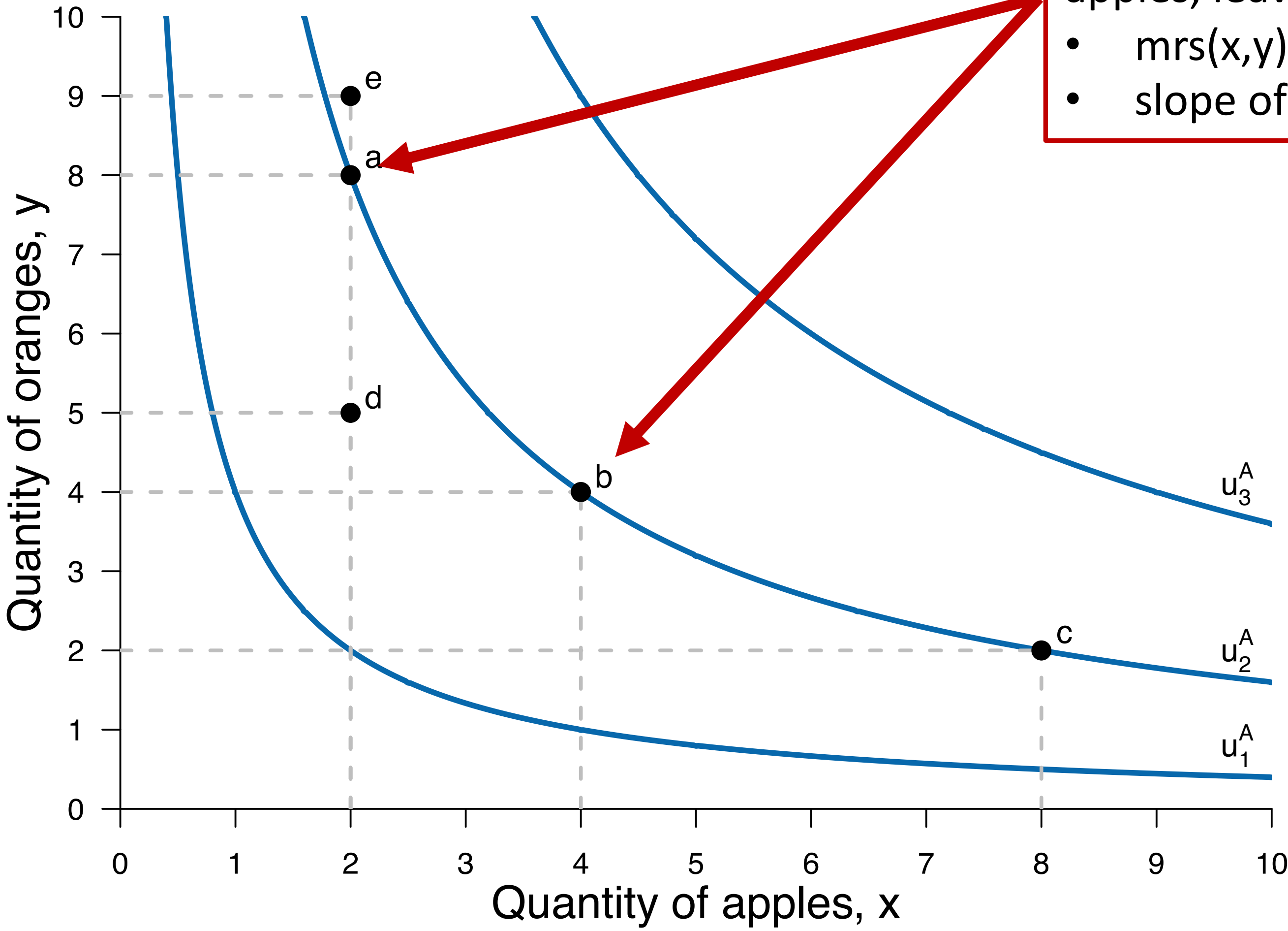
- *Transitivity is respected when indifference curves are parallel (they don't cross).*



In this example:
a ~ b
and
b ~ c
transitivity would thus require
a ~ c
but instead we have a $\succ c$
so transitivity (consistency) is not respected.
This consumer is not rational!

Definitions: Marginal utility & MRS

- **Marginal utility of x**
  - The change in utility caused by having one additional unit of x.
  - It is the first derivative of the utility function: u_x or equivalently $\frac{\partial u}{\partial x}$
- **Marginal rate of substitution between x and y: mrs(x,y)**
  - max amount of y that you can give up to get one more unit of x, without lowering your utility.
  - If you receive MRS units of y in exchange for one unit of x, your utility will stay the same.
  - It represents your *willingness to pay (WTP):* max amount of *y* that you are willing to give up in order to obtain another unit of *x*.
  - It is related to *indifference curves*: when you move along an indifference curve, what you are doing is exactly *trading-off* some units of x for some units of y, in such a way as to leave total utility unchanged.



<u>Consider a movement from **a** to **b**:</u>
Consumer loses 4 oranges but gains 2 apples, leaving total utility unchanged.

- mrs(x,y) = Δoranges/Δapples = 4/2 = 2
- slope of the curve = ΔY/ΔX = -4/2 = -2

This is a general rule:
The *marginal rate of substitution (mrs)* is the negative of the slope of the indifference curve.

- As we have just seen, the MRS is equal to the negative of the slope of the indifference curve

  - mrs(x,y) = - slope

- Another Very Important Relation:
  The MRS is equal to the ratio of marginal utilities:

  - mrs(x,y) = u_x/u_y

(read the proof in M-Note 3.2, p.113 of the book, and make sure you understand it)

The Cobb-Douglas utility function

$$u(x,y) = x^{\alpha} y^{(1-\alpha)} \quad (\text{with } 0<\alpha<1)$$

- Properties:
  - Utility is positive as long as the agent has some goods.
  - An additional unit of a good increases utility, but at a decreasing rate (*diminishing marginal utility*).
  - The size of α indicates the consumer's intensity of preferences for good x relative to good y.
- Why would we assume a Cobb-Douglas utility function?
  - simple to work with;
  - reasonable properties (positive and diminishing marginal utility);
  - not because people really follow this particular utility function rather than another!
  - just a conventional simplified way to represent preferences.

A useful dirty trick

When the utility function is Cobb-Douglas, the mrs(x,y) is equal to the ratio of the intensities of the preferences for each good (α/1- α) multiplied by the ratio of the quantity of the goods that the consumer holds (y/x).

Formally:

if $u(x,y) = x^{\alpha}y^{(1\text{-}\alpha)}$ *(with $0<\alpha<1$)*

then $mrs(x,y)=(\frac{\alpha}{1-\alpha})\,\frac{y}{x}$

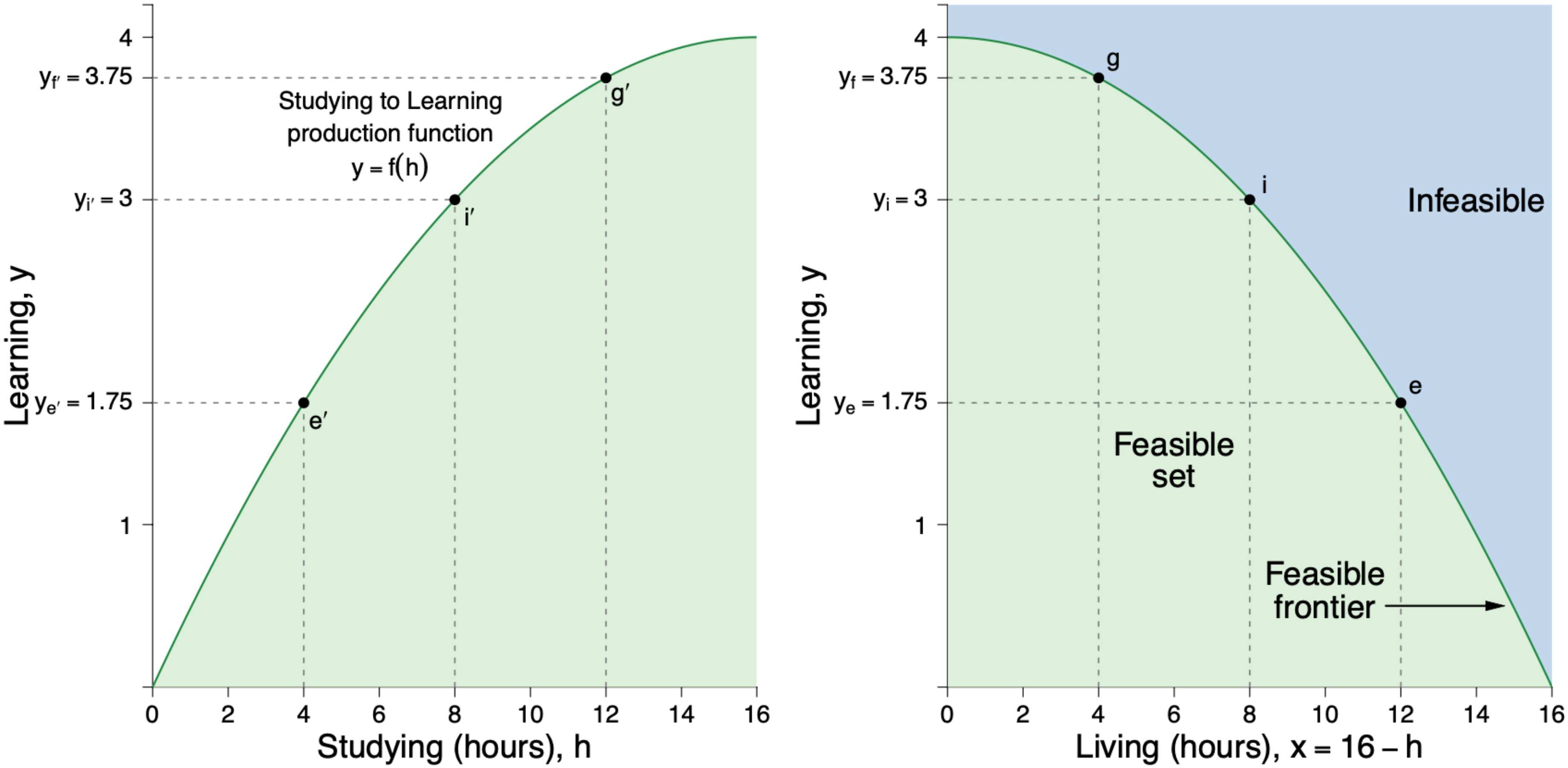
Note: if a constant k was multiplied to the function, as in $u(x,y)=k\,x^{\alpha}y^{(1\text{-}\alpha)}$*, this formula for mrs(x,y) would still hold.*

2 – Doing the best you can: Feasibility & utility

- Preferences are not sufficient to predict/explain behavior: we need to introduce *constraints*.
- Rational agents will choose the outcomes they prefer, *given what is feasible*.
- In other words, people will maximize their utility, given the constraints they face.
- *Examples of constraints*:
  - Your budget allows you to buy only combinations of goods that you can afford (budget constraint).
  - There are only 24h in the day, so you cannot do all the activities you want (time constraint).
  - …

Feasibility: a time constraint

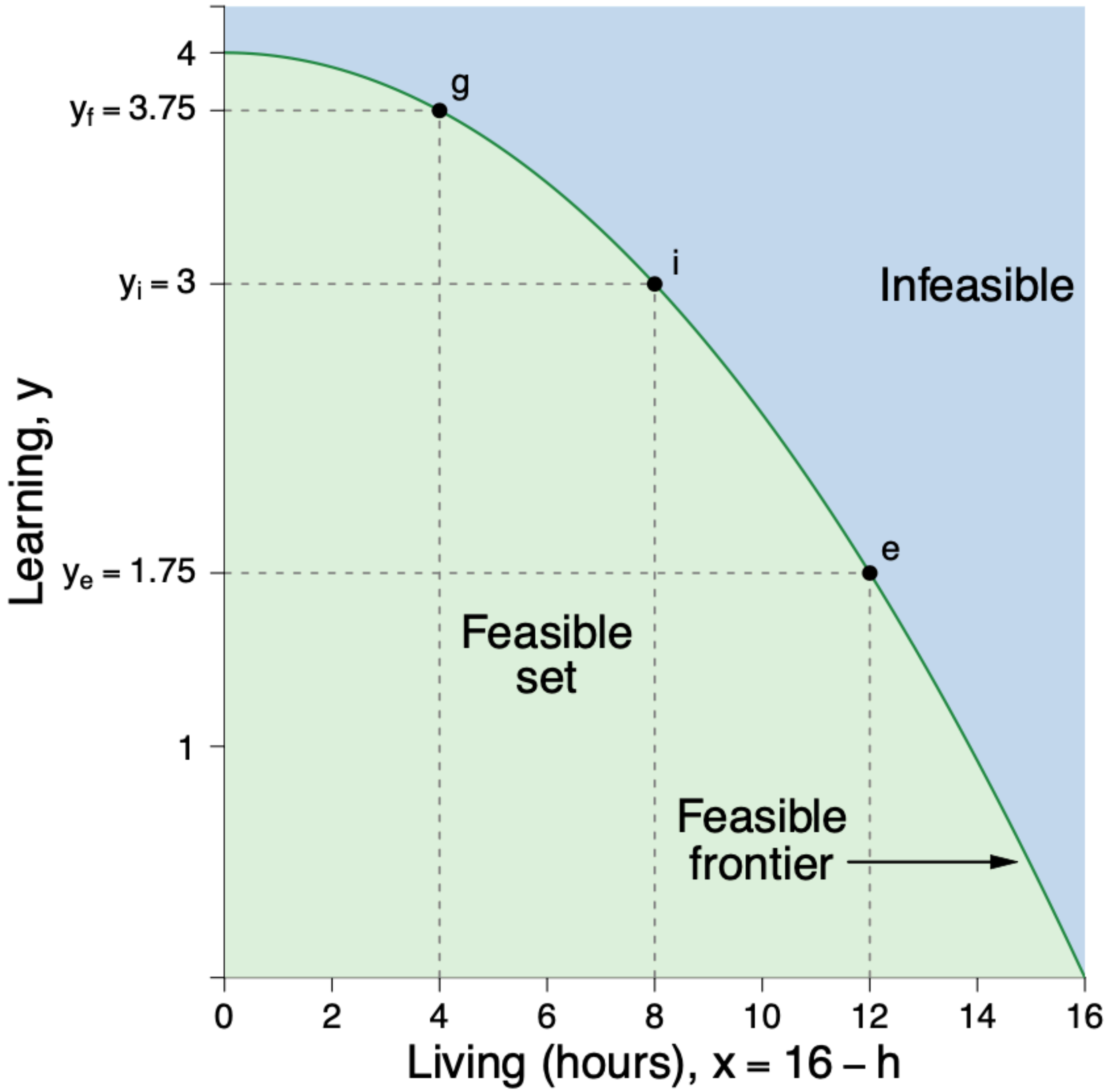
- Keiko has to allocate her time (16hrs per day) among leisure (‘living’) and study (‘learning’).
- Study hours (waking hours not dedicated to leisure) determine her learning.
- An outcome (x,y) is a combination of amount of Learning (y) and hours spent living (x).
- Keiko likes both Learning (y) and Living (x), but because of the time constraint (16hrs per day) there are only some combinations (x,y) that are *feasible*
  – you cannot get a huge amount of learning and a huge amount of ‘living’ at the same time.



(a) The production of Learning by studying

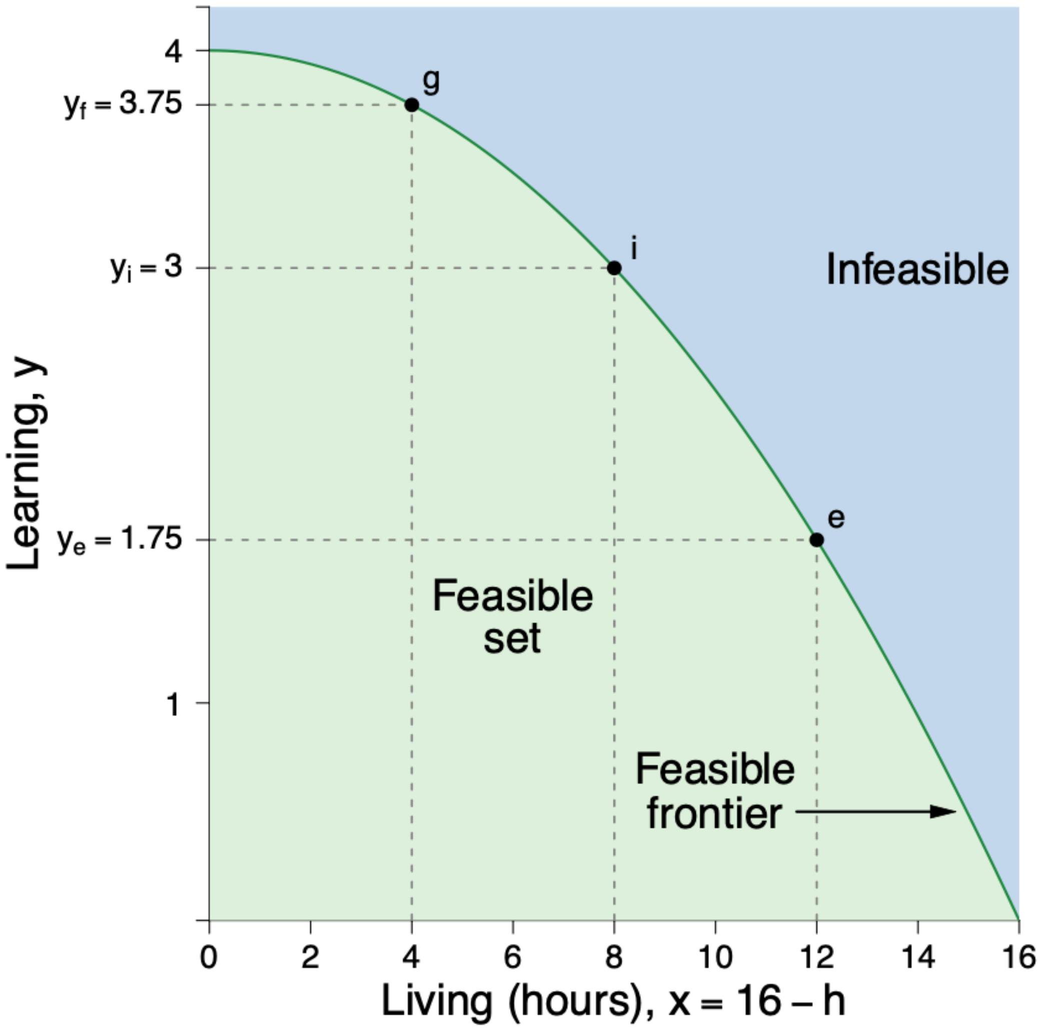
(b) The feasible frontier of Living and Learning

- These graphs assume decreasing marginal productivity of study hours: the more you study and learn, the more it takes to increase learning by another unit.



(b) The feasible frontier of Living and Learning

- *Feasible set*: contains outcomes that are feasible.
- *Feasible frontier* (dark green line): the max. learning she can obtain, given the amount of 'living' she does.
- *Opportunity cost*: the more hours dedicated to 'living', the lower the learning she can obtain (decreasing line).
- *Feasible frontier is concave*: increases in learning are more and more costly in terms of leisure (because of decreasing marginal productivity of study hours)



(b) The feasible frontier of Living and Learning

- *Marginal Rate of Transformation mrt(x,y):* the rate at which the agent can sacrifice y to have more x
- *mrt(x,y):* how many units of *y* you lose if you increase *x* by one unit (*opportunity cost* of x)
- In this case, how much learning Keiko loses if she grants herself one extra hour of living.
- *mrt(x,y)* is the negative of the slope of the feasible frontier:
  - *mrt(x,y)=-slope(ff)*
- *mrt* is *increasing* here (increasing opportunity cost) because of diminishing productivity of study hours.

Calculating mrt(x,y)

- You calculate mrt(x,y) from the feasible frontier, according to the formula:

$$\text{mrt(x,y)} = -\frac{dy}{dx}$$

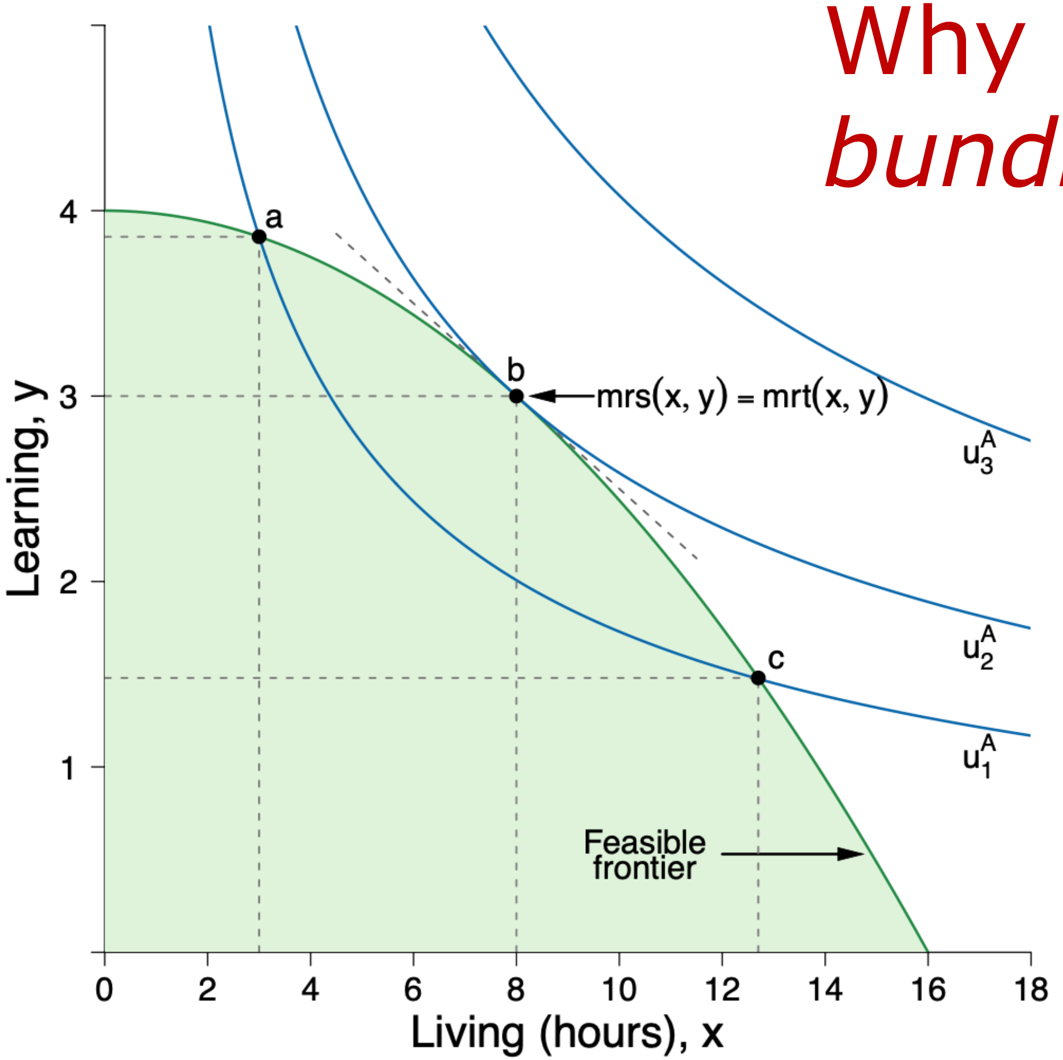
- Imagine that the feasible frontier is described by the equation $y = 4 - \frac{1}{64}x^2$
- Then we have

$$\text{mrt(x,y)} = -\frac{dy}{dx} = \frac{1}{32}x$$

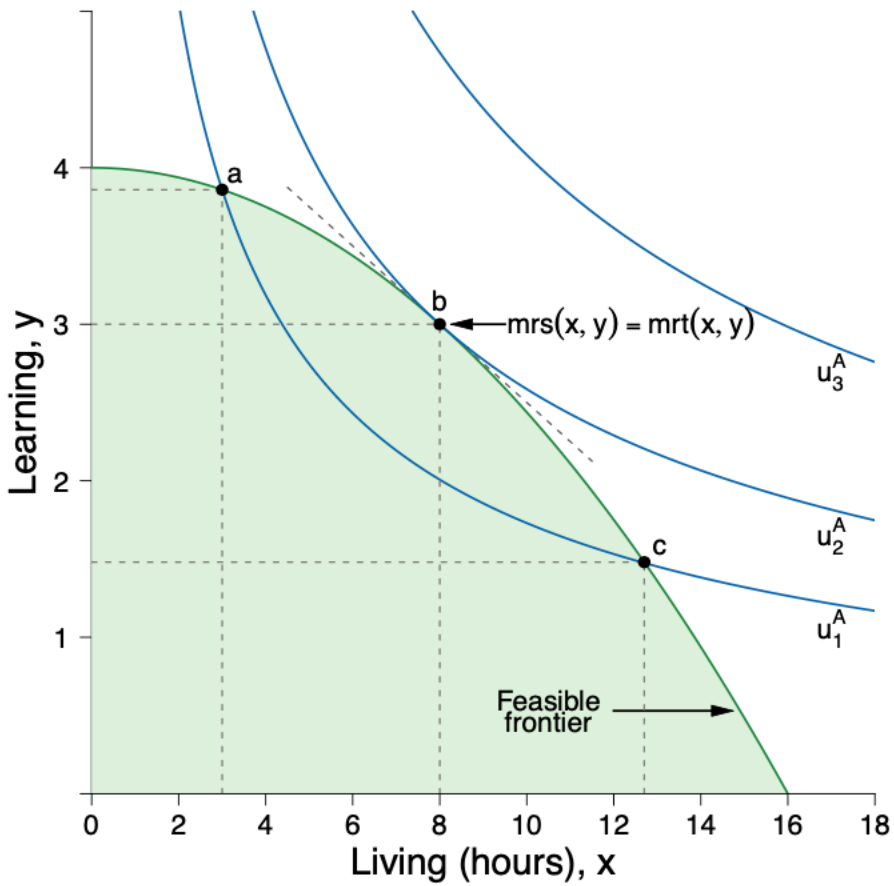
- Note: *mrt is a function of x*: for example here when x is higher, obtaining an additional unit of x is more costly.
- Tip: before calculating the mrt, always make sure that the feasible frontier is expressed as above, with y on the left and a function of x on the right. If it is not, rearrange to have that form.

Doing the best you can

- What will a rational agent do? We need to combine the ideas of preferences & feasibility.
- Consider Keiko’s preferences through her indifference curves.
- Combine that with Keiko’s *feasible frontier*.
- *Constrained optimization*: maximize the utility you get, subject to the constraint you face.



Why will Keiko choose *bundle* **b**?



- Keiko wants to be on her highest attainable indifference curve.
- But she can only choose points in her feasible set.
- At point **b** she gets the highest possible utility, given what is feasible: it is the point in the feasible set which yields the highest possible indifference curve.
- Among all feasible combinations, **b** is the one which gives most utility (highest indifference curve).

The principle of utility maximization

At Keiko's utility maximizing (or 'optimal') choice, the following will occur:

- The indifference curve she is on is *tangent* to her feasible frontier (otherwise there would be an higher feasible indifference curve)
- So, slope of feasible frontier = slope of indifference curve.
- This means that **mrs(x,y)=mrt(x,y)**.
- This is the principle of utility maximization:

MRS (willingness to pay)= MRT (opportunity cost)

The principle of utility maximization

WTP (mrs) = Opportunity costs (mrt)

- *mrs(x,y)* represents willingness to pay (WTP) or *tradeoff*: how many units of *y* is one unit of *x* worth, in terms of utility?
- *mrt(x,y)* represents *opportunity costs*: how many units of *y* you actually have to sacrifice, to obtain another unit of *x [it* comes from feasibility constraints*]*.
- Keiko stops at the point where the *opportunity cost* of another hour of leisure is equal to her *tradeoff* (or WTP) for another hour of leisure.

- If MRS(x,y)>MRT(x,y), you could increase utility by increasing the quantity of x
  - the max amount of *y* that you are willing to give up for another unit of *x* is higher than what you really have to give up.
- If MRT(x,y)>MRS(x,y), you can increase utility by decreasing the quantity of x
  - the amount of *y* you are giving up for the 'last' unit of *x* is higher than what you are willing to give up.
- Only when MRT=MRS, you cannot do anything to further increase your utility.
- *Make sure you study M-Note 3.7 in the textbook, to learn to use MRT=MRS to identify the optimal choice numerically.*

The principle of utility maximization: Summing up

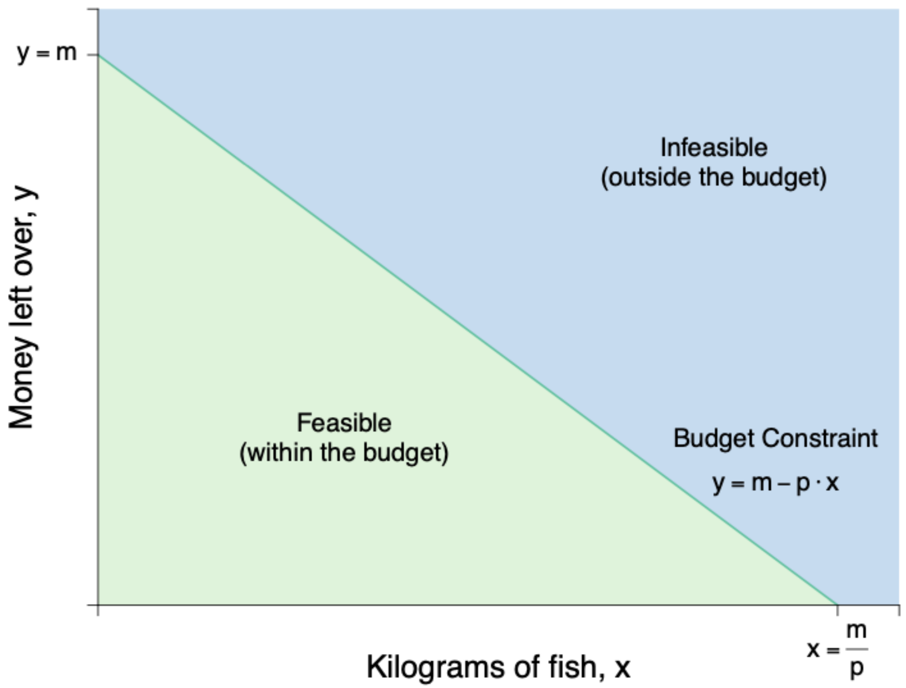
Slope of feasible frontier	=	*Slope of indifference curve*
which requires that:		
Marginal rate of transformation (mrt)	=	*Marginal rate of substitution (mrs)*
Or, what is the same thing,		
Opportunity cost of x	=	*Willingness to pay for x*

3 –Offer curve & demand function

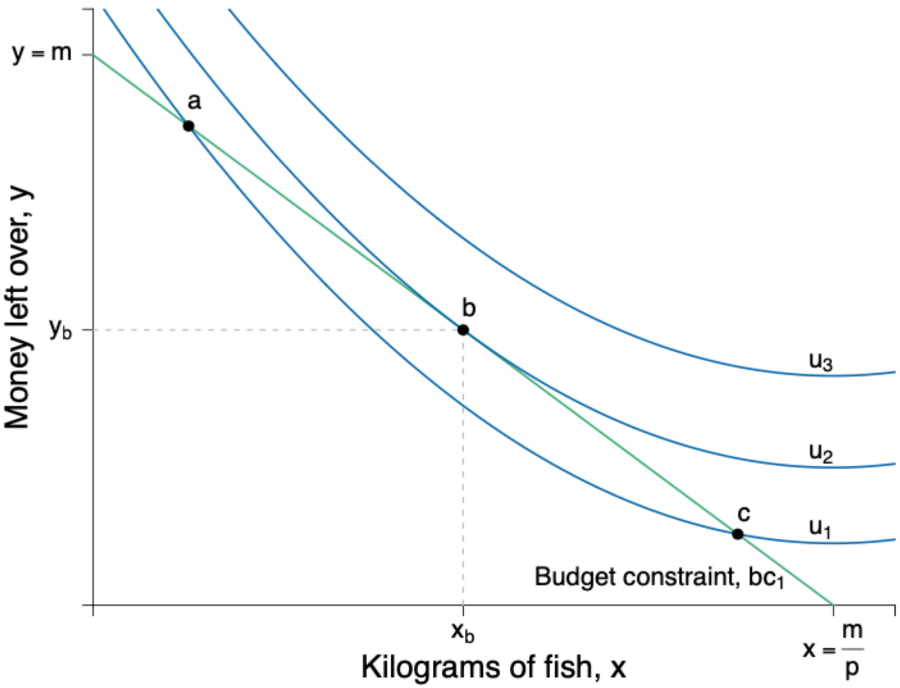
- What's a *price* in a world of apples and oranges?
  - a *ratio of exchange* between apples and oranges.
  - price of apples (x) in terms of oranges (y) = MRT(x,y)
    [MRT(x,y) = How many oranges you have to give up, to get one apple]
- Now think of one of the two goods as being *money*.
  - fish & money: you get utility both by consuming fish and by saving money for other purposes.
- *Price* of fish: the amount of money you have to give up in order to get one fish → *MRT(fish, money)*
- A feasible frontier over fish and money, with constant MRT, is called a *price line*.
- Then we can find the quantity of fish that the consumer would consume for each given price.
- Doing it for all possible prices, we get the *offer curve* and the *demand function*, which gives quantity demanded as a function of price.

A budget constraint

- When displaying preferences over money and fish (or some other good) with a given budget, the MRT is the price of fish, and the feasible frontier is a *price line.*
- A *price line* shows all the combinations of money and fish that the consumer can obtain by buying/selling fish at that given price, given the available budget.



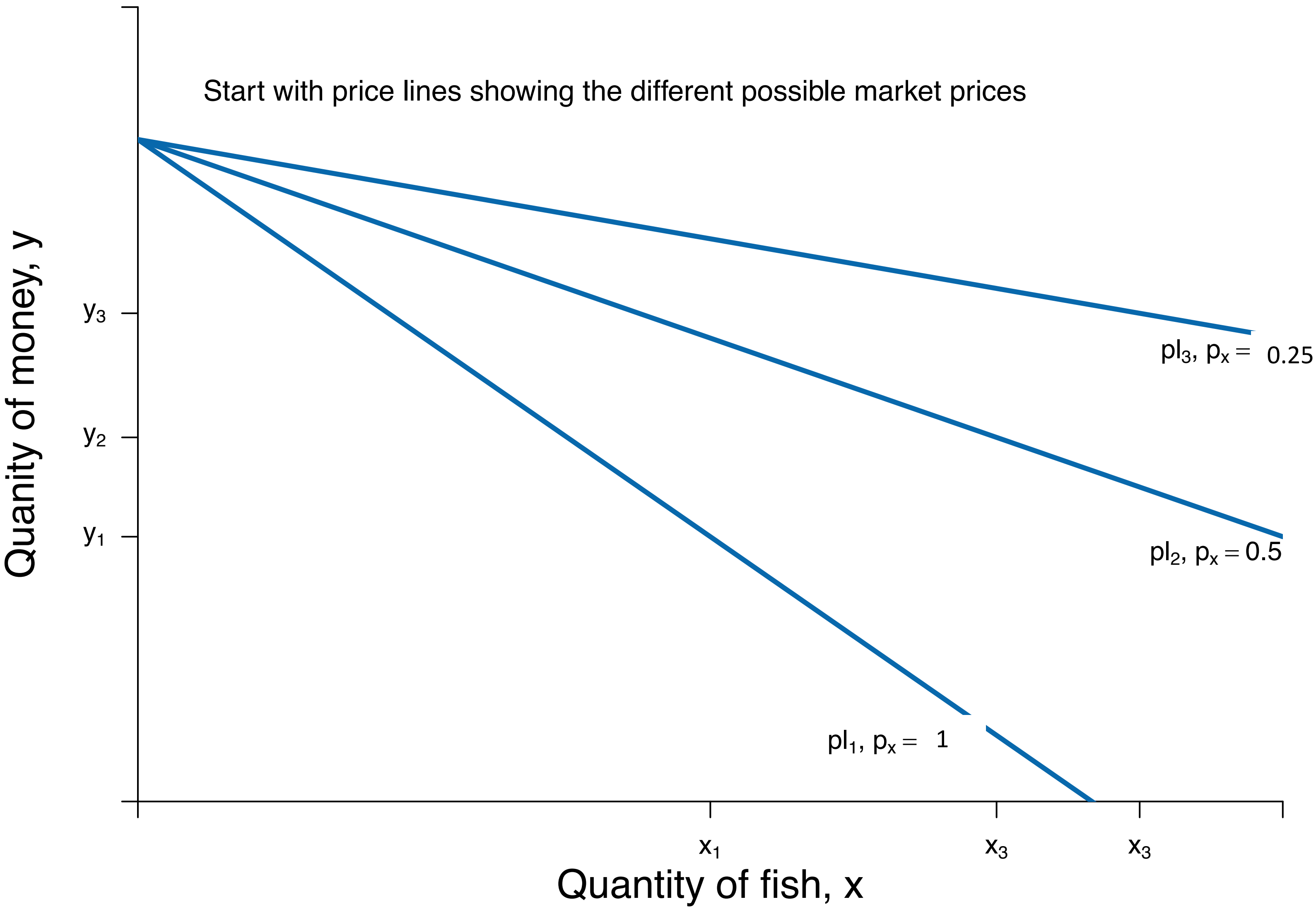
(a) The budget constraint



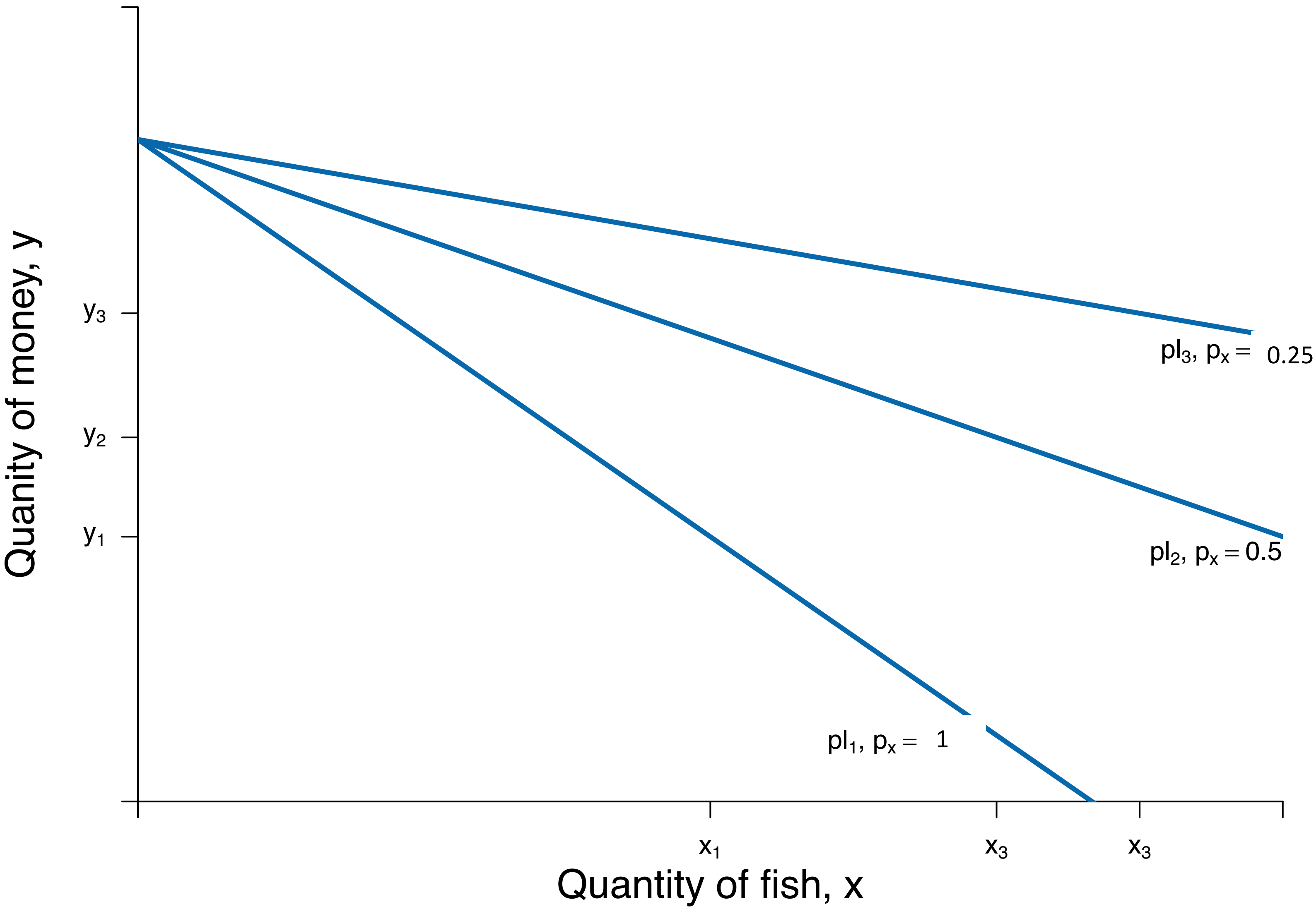
(b) Utility-maximizing choice

STEP 1

- Each possible price can be represented by a *price line*.
- *intercept* = available monetary budget; *slope* = price.



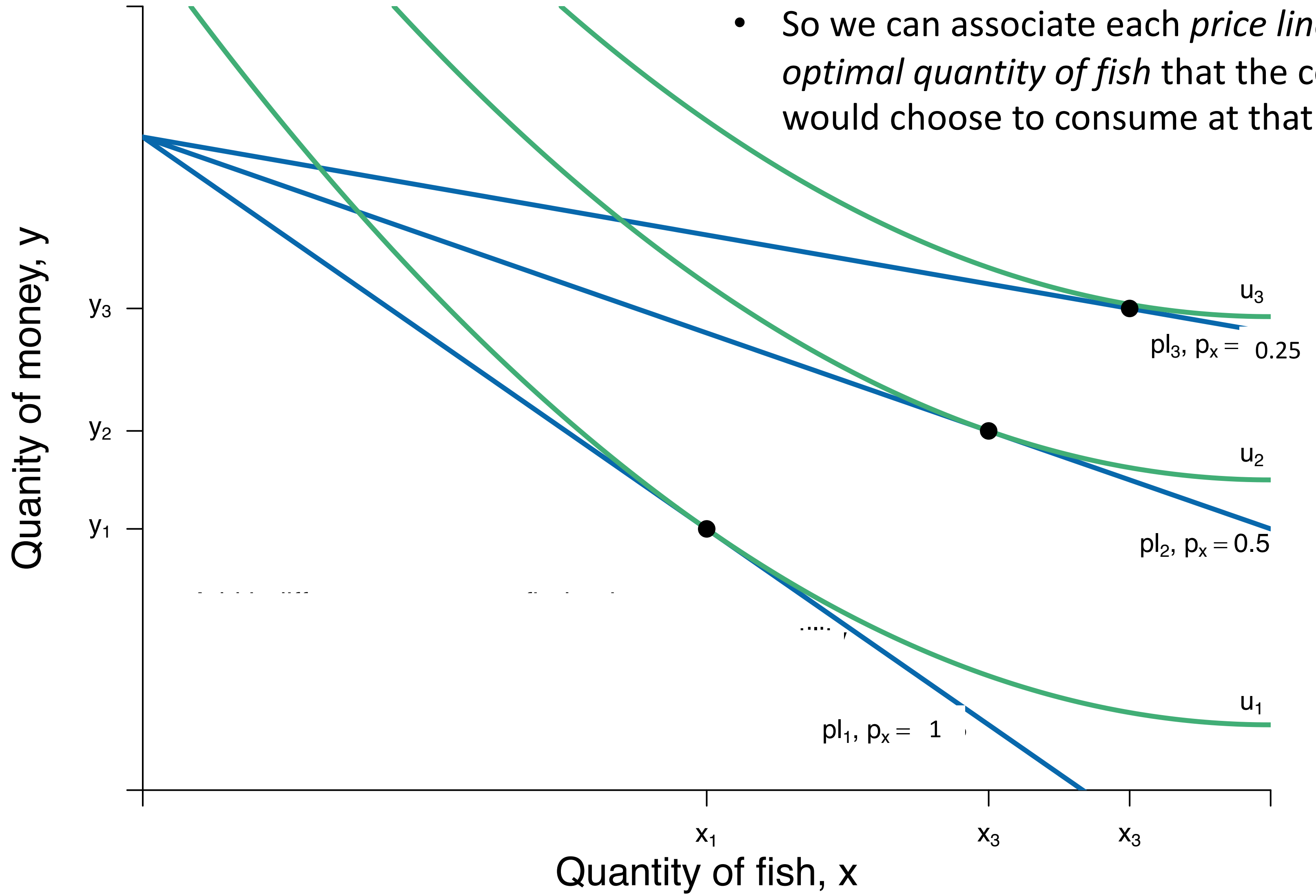
STEP 1



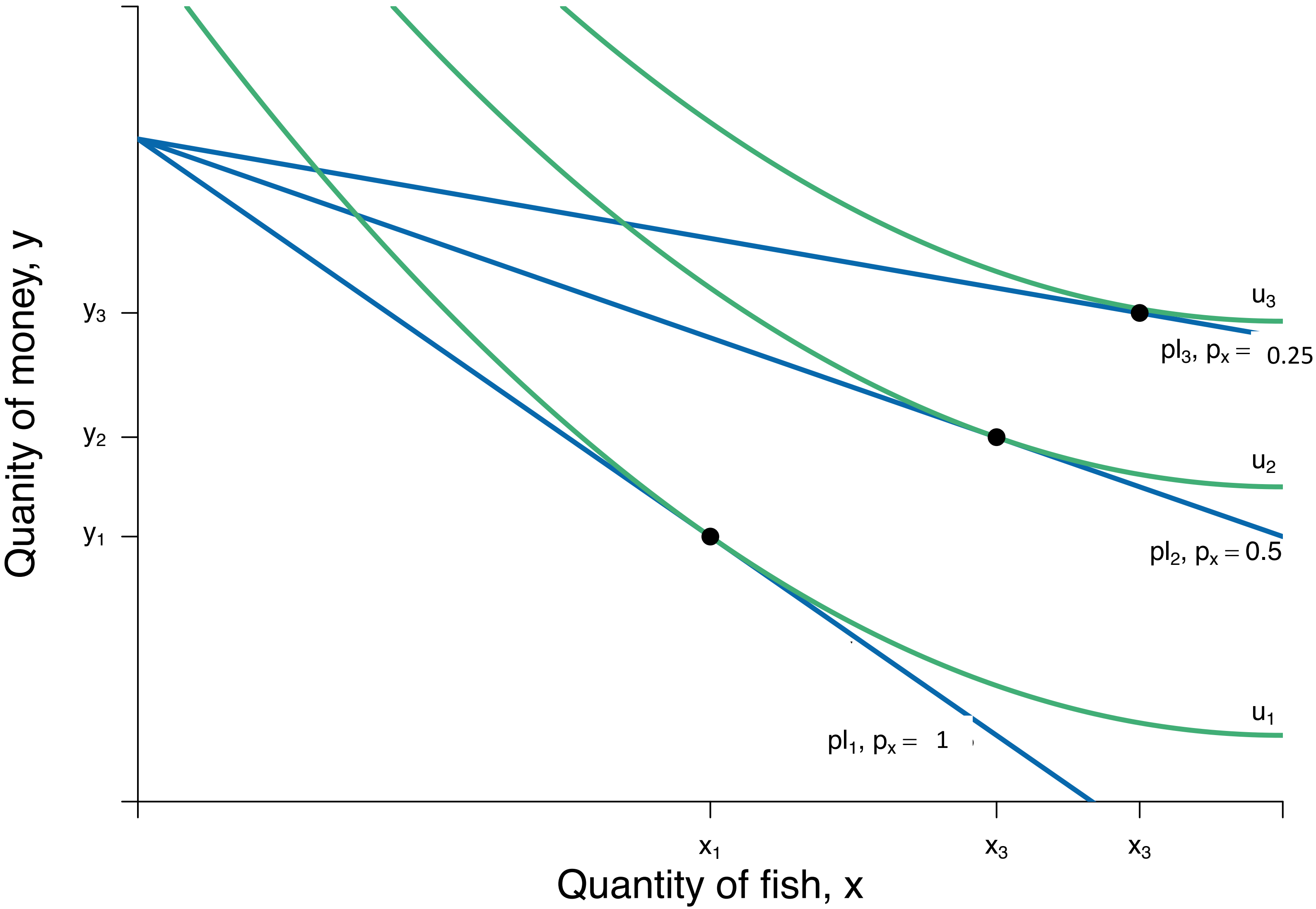
Quanity of money, y
y3
y2
y1
pl3, px = 0.25
pl2, px = 0.5
pl1, px = 1
x1
x3
x3
Quantity of fish, x

STEP 2

- Given the outcomes made possible by the given *price line*, the consumer chooses the one that yields the best (highest) possible *indifference curve*.
- So we can associate each *price line* with a *optimal quantity of fish* that the consumer would choose to consume at that price.



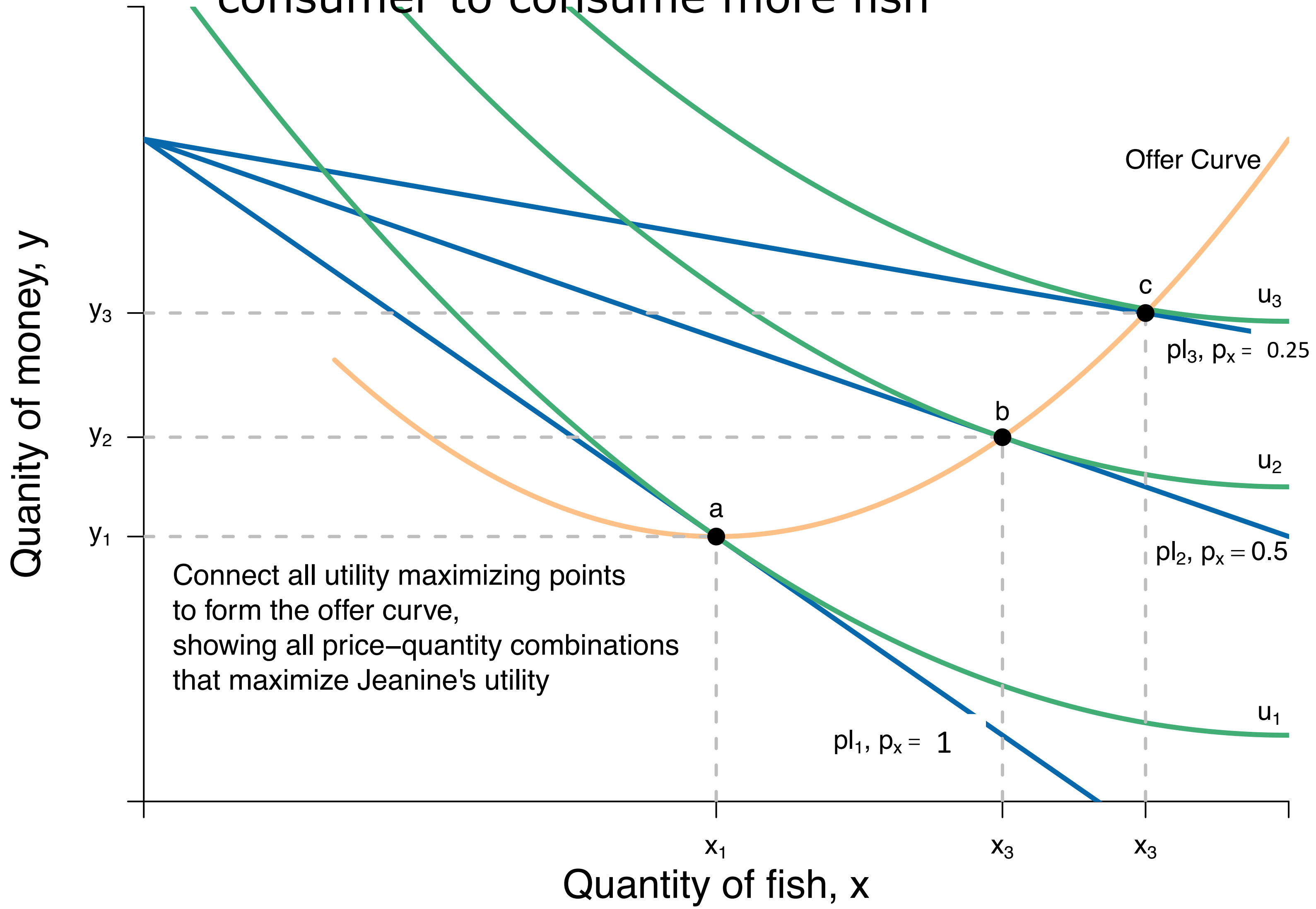
STEP 2



Quanity of money, y
y3
y2
y1
u3
u2
u1
pl3, px = 0.25
pl2, px = 0.5
pl1, px = 1
x1
x3
x3
Quantity of fish, x

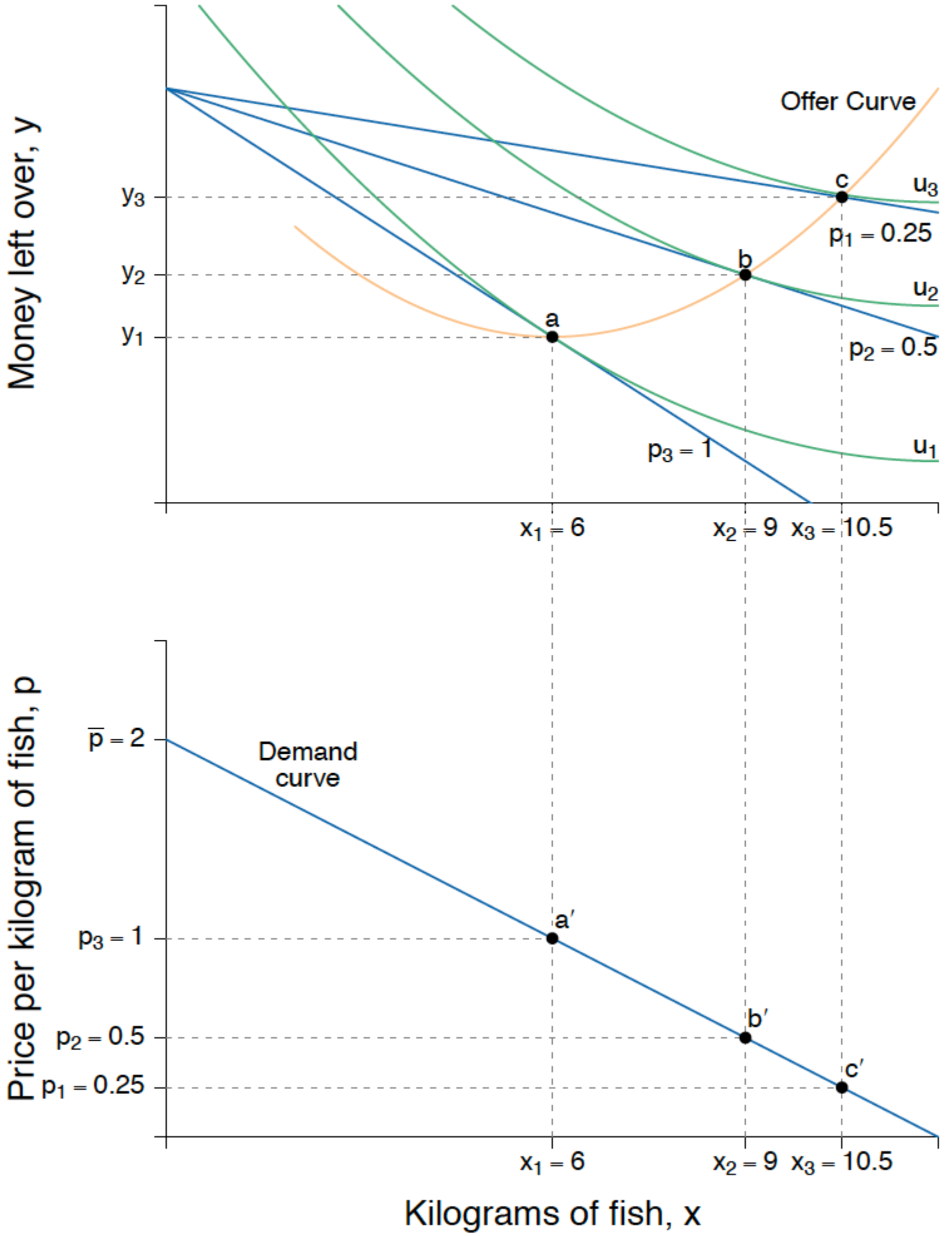
STEP 3

- The *offer curve* (orange line) shows you the quantity of fish that the consumer will consume at each market price.
- A lower price of fish (=a flatter price line) induces the consumer to consume more fish



The demand curve

- From the *offer curve* we obtain the *demand curve (or demand function).*
- Price on vertical axis.
- Quantity of fish on horizontal axis.
- Draw a line that connects all price-quantity combinations that are on the offer curve.
- This line is the demand curve: it tells you how much fish will the consumer buy, for each possible price.
- Here the higher the price, the lower the quantity of fish you want to buy.



4 – Representing Social Preferences

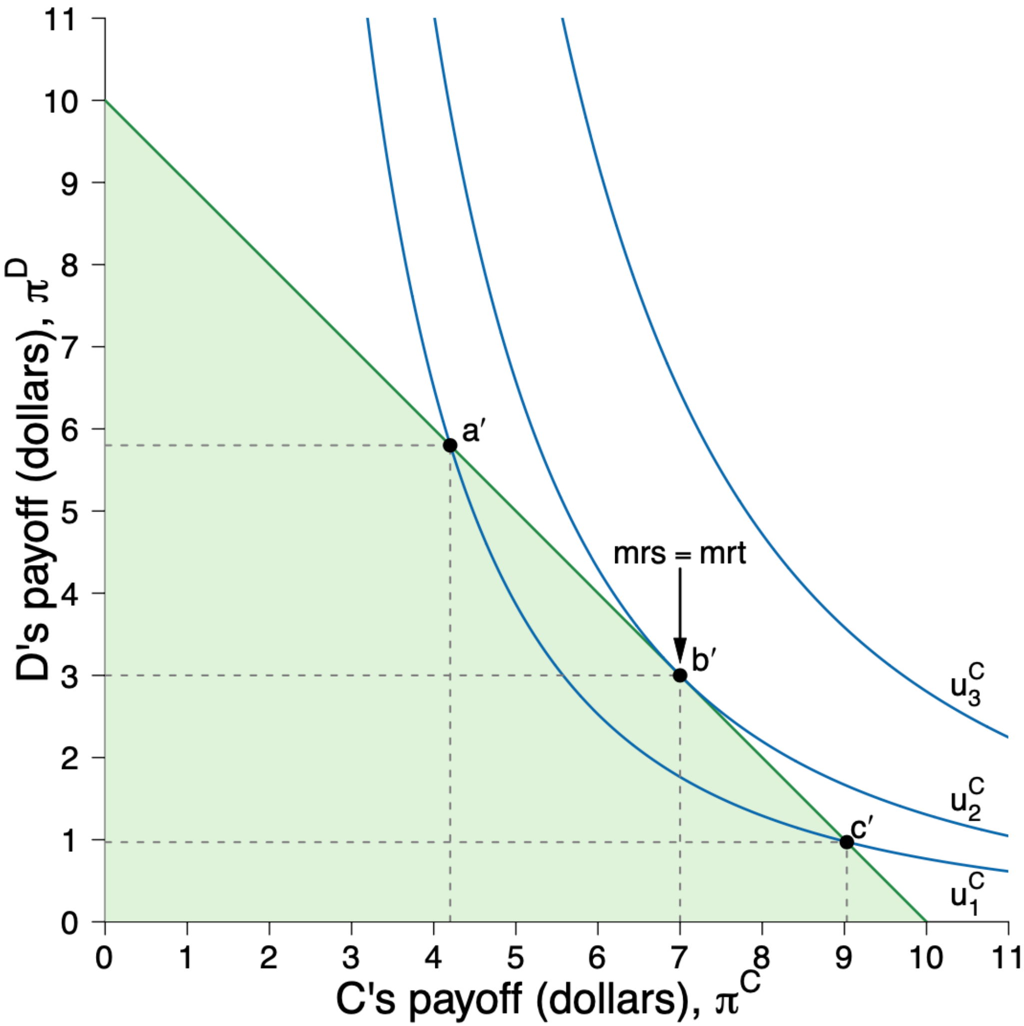
- When studying a choice that impacts also other people, *social preferences* can be important.
  - Someone else's payoff can enter your utility function.
- Anmei and Chen play as Proposers in a Dictator Game, with an endowment of $10.
  - Anmei (A, proposer) paired with Ben (B, responder).
  - Chen (C, proposer) paired with Diane (D, responder).

○ *Budget constraint*: $\pi^B + \pi^A = 10$

○ *Feasible frontier*: $\pi^B = 10 - \pi^A$

○ *Anmei's utility*: $u^A(\pi^A, \pi^B) = (\pi^A)^1(\pi^B)^0$

○ *Chen's utility*: $u^C(\pi^C, \pi^D) = (\pi^C)^{0.7}(\pi^D)^{0.3}$



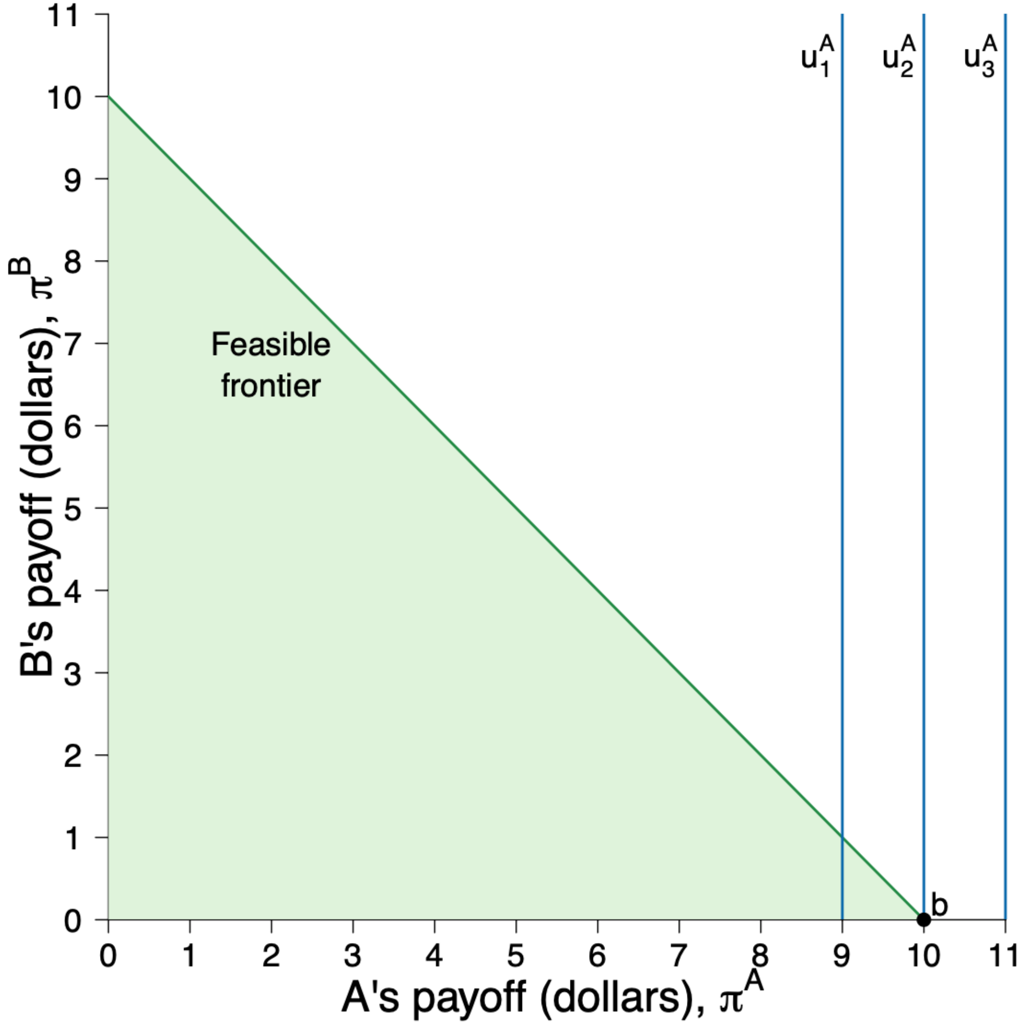
(b) Chen is altruistic, offering a (7, 3) split

- Chen has *altruistic preferences*.
- She gets utility from his own payoff, but also from Diane's payoff.
- With Cobb-Douglas function, diminishing marginal utility: convex indifference curve.
- We can use our usual criterion to find the highest feasible indifference curve:

tangency ←→ $\mathrm{mrs}(\pi^C, \pi^D) = mrt(\pi^C, \pi^D)$

- Given that she attributes a higher weight to his own payoff (0.7) than to Diane's payoff (0.3), he's offering a (7,3) split.

4 – Representing Social Preferences



(a) Self-interested Anmei offers a (10, 0) split

- Anmei is *homo economicus (self-regarding preferences)*.
- She cares only about her own payoff: Ben's payoff has zero weight in her utility function.
- *Vertical* indifference curves.
- Her mrs is undefined (infinite) → cannot use the usual *mrs=mrt* criterion!
- But we can still figure out her choice by picking the highest feasible indifference curve (u_2^A).
- Unsurprisingly, she will just keep the entire $10!

Doing the best we can: some concluding remarks

- The models we have studied in this chapter are all about agents optimally allocating a scarce resource: the *economics of scarcity*.
- There are *no externalities* in this story, and thus utility maximization leads to efficient allocations.
- But scarcity is only *one* part of economics!
- Often, we are instead in the *economics of coordination failures*: people individually doing their best produce inefficient social outcomes.
- In those cases, the main problem is not scarcity, but *coordination*: we have the resources to produce better outcomes but fail to do so.