Take-home assignment 1

Econ 705 - Spring 2021

Due: Thursday February 25 before 2pm (send via email to Guilherme)

1. Briefly describe and discuss the main similarities and differences between the Ramsey-Cass-Koopmans growth model and
   (a) the Solow model
   (b) Diamond's OLG model
   (c) the Harrod-Domar model

2. Describe how, if at all, each of the following developments affect the break-even and actual investment lines in our basic diagram for the Solow model. Then figure out the effect on the equilibrium growth rate of output per worker (Y/L) and the equilibrium *levels* of capital and output per unit of effective labor ($k = K/AL$ and $y = Y/AL$).
   (a) The rate of depreciation falls.
   (b) The rate of technological progress rises.
   (c) The production function is Cobb-Douglas, $f(k) = k^{\alpha}$, and the capital share, α, rises. Assume $k > 1$ and $s > (n + g + \delta)$.
   (d) Workers exert more effort, so they produce a higher y for any given k.

3. Consider a Solow economy that is on its balanced growth path. Assume for simplicity that there is no technological progress. Now suppose that the rate of population growth falls.

(a) What happens to the balanced-growth-path values of capital per worker, output per worker, and consumption per worker? Sketch the paths of these variables as the economy moves to its new balanced growth path.

(b) Describe the effect of the fall in population growth on the path of output (that is, total output, not output per worker).

4. Consider the standard Ramsey-Cass-Koopmans model. Assuming no depreciation and using the same notation as in the Romer's textbook (and in the class slides), the key dynamic equations are:

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \rho - \theta g}{\theta}$$

$$\dot{k} = f(k(t)) - c(t) - (n+g)k(t)$$

(a) Briefly describe the economic intuition behind both these equations and where they come from.

(b) Assuming that the economy is initially in a dynamic equilibrium (steady state), use a phase diagram to illustrate and describe the trajectories of k and c following:

i. a sudden unexpected increase in the growth rate of the labor force;

ii. a sudden, unexpected and temporary alien attack that destroys 20 percent of the capital stock but leaves the labor force unchanged;

iii. a sudden, unexpected and temporary alien attack that destroys 20 percent of the capital stock, leaves the labor force unchanged, but also affects households' confidence by raising the discount rate.

5. Assume that the aggregate production function is given by

$$Y = K^{\alpha} H^{1-\alpha}; \quad 0 < \alpha < 1$$

where Y is output; K is physical capital; H is human capital. The amount of human capital is given by $H = uh(t)L$, where u is the proportion of the workforce that is employed in final production and h is the average human capital per worker. The labor force L is constant. Also u is constant.
The proportion $(1-u)$ of the workforce is in school, and the changes in average human capital are determined by

$$\dot{h} = (1-u)h - \delta_h h$$

where δ_h is the rate of depreciation of human capital. Changes in physical capital are given by

$$\dot{K} = sY - \delta_k K$$

where s is the (fixed) saving rate.

(a) Show that for a given value of u the model has the same formal structure (in terms of the characteristics and stability properties of the intertemporal equilibrium) as a standard Solow model with a constant rate of technical progress.

(b) Assume that initially the economy is at the steady state associated with some given values of s and u. Analyze the effects of: (i) a permanent rise in s, keeping u constant; (ii) a permanent decrease in u, keeping s constant.

(c) Briefly discuss the intuition behind the results in (b)

6. Discuss the two main empirical exercises inspired by neoclassical growth theory: 'growth accounting' and 'convergence regressions'. How are they related to the Solow model? Express your opinion on the usefulness and/or potential shortcomings of these exercises.

7. Suppose that an individual's utility depends only on her consumption. In particular, assume a logarithmic utility function: $U(C(t)) = ln(C(t))$. The individual is endowed with an initial stock of wealth (or capital), K_0. Wealth produces an income stream according to the formula $Y(t) = rK(t)$, where r is the prevailing interest rate. This individual does not earn any wage. At each point in time, she allocates her income between consumption (which provides utility) and investment (which adds to the capital stock). There is no depreciation, so the stock of wealth of the individual evolves according to the following equation: $\dot{K} = I = Y - C = rK - C$. The individual is going to live until time $t = T$, and knows this for sure from the initial period. Her budget constraint implies that she cannot die with negative wealth (she has to repay debts). She is forward-looking and maximizes utility over her lifetime, applying a positive discount rate δ.

   The consumer's optimization problem can thus be written as:

$$\text{Maximize } \int_0^T ln(C(t))e^{-\delta t}dt \tag{1}$$

$$\text{Subject to: } \dot{K} = rK(t) - C(t);\ K(0) = K_0; \quad K(T) \geq 0 \tag{2}$$

   Write down the Hamiltonian for this problem, and derive the optimal dynamics of consumption (that is, an expression for $\frac{\dot{C}}{C}$).