# The Harrod-Domar model*

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The Harrod-Domar model[1] provided the initial 'impulse' which gave birth to (all stripes of) modern growth theory. Indeed, the Harrod model is the precursor of both the neoclassical growth models on which we will focus in this course and the so-called Post-Keynesian ones. It is actually possible (and quite insightful) to see different subsequent growth models as different ways to solve the problems highlighted by Harrod.

## 1 Overview

Harrod aimed to explore the dynamic implications of the newly born Keynesian theory, which Keynes had stated in mostly static terms in the General Theory few years earlier.

The model has two basic premises. The first is that any change in aggregate investment has a dual effect: a demand-side effect and a supply-side one. On the demand side, changes in investment determine changes in output through the multiplier. On the supply side, investment adds to the capital stock and thus determines the change in potential output (the productive capacity of the economy). Harrod wanted to study the interaction between these two effects.

The second premise concerns the determinants of aggregate investment. According to the accelerator effect, investment is driven by aggregate demand dynamics. Firms

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[1]See Harrod (1939) and Domar (1946). In what follows we will just call it the Harrod model, for brevity and because Harrod apparently had the idea first. If you are interested in the 'history of economics' aspect of this – including the debate on 'what Harrod really meant', and whether he would be happy with the currently prevailing interpretation of his work – you can take a look at a series of interesting papers by Daniele Besomi. A nice presentation of the Harrod model (on which these notes partly draw) can be found in Skott (1989, p. 18). A pared-down but helpful exposition focused on the mathematics of the model, based on the Domar version, can also be found in Section 16.4 of the fourth edition of Chiang's 'Fundamental Methods of Mathematical Economics'(which was a source of inspiration for these notes too).

want to expand their productive capacity when they experience increased demand for their products. The higher the growth rate of demand and output, the higher the investment rate.

Harrod found that the combination of these ingredients, in a stylized model of a closed economy with no government, results in a dynamic economic system with wildly unpleasant properties. The dynamic system has an equilibrium growth rate (the 'warranted' growth rate, as Harrod calls it) – a unique rate of growth which is compatible with the optimal rate of utilization of capital, and therefore does not induce further changes. However, this equilibrium growth rate has two disturbing properties: it does not guarantee full labor employment; it is not stable, therefore any deviation from equilibrium will be amplified in a self-reinforcing explosive or implosive pattern.

The resulting economic picture is reminescent of a classic Woody Allen joke (from Annie Hall), in which two elderly women are at a restaurant. One of them says, 'Boy, the food at this place is just terrible'. The other one replies, 'Yeah I know. And such small portions.' Similarly, in the Harrod model the equilibrium growth rate does not guarantee full (nor stable) employment, and is not likely to be reached anyway.

## 2 The Model

The model is quite simple, at least mathematically. Assume a one-good economy. This good can either be consumed or used as fixed capital for subsequent production. The economy is closed and there is no public sector. Also assume no technological progress and no depreciation (fixed capital is eternal) for simplicity.

The saving rate is given. Call it $s$. Total savings are thus $S(t) = sY(t)$, where $Y$ is output. The optimal output-capital ratio is also given, determined by the available technique of production. Call it $a$.

When firms utilize their capital stock at the planned (optimal) rate, they are able to achieve the optimal output-capital ratio $a$. Productive capacity, however, is flexible, at least in the short-run: based on the realized level of demand, firms can end up either under-utilizing their productive capacity or over-utilizing it.

We thus have $Y^*(t) = aK(t)$ where $Y^*$ is potential output: the output level that would be produced if the rate of utilization of the capital stock was the optimal one. The rate of capacity utilization $u(t)$ is defined as $u(t) = \frac{Y(t)}{Y^*(t)}$ . $u = 1$ corresponds to the normal (optimal) rate of utilization, so $u > 1$ implies a 'heated' economy in which demand outpaces productive capacity and firms over-utilize their machines, while $u < 1$ implies a depressed economy with idle machinery.

Investment decisions are driven by demand dynamics, consistent with the accelerator principle. The simplest way to represent this, in this context, is to write an investment function in which changes in the investment rate ($g_K = \frac{\dot{K}}{K} = \frac{I}{K}$) depend on the utilization rate, like the following:

$$\frac{\partial g_K(t)}{\partial t} = \dot{g}_K(t) = \alpha(u(t) - 1) \qquad \text{with } \alpha > 0 \tag{1}$$

When experiencing a shortage of productive capacity relative to demand ($u > 1$), firms will increase their investment rate. When experiencing under-utilization of their productive plants ($u < 1$), firms' investment will decrease.

## 2.1 The warranted rate of growth

The actual growth rate of the capital stock $g_K$ must be consistent with the usual condition for product market equilibrium, according to which ex-post savings equal investment. We thus have:

$$g_K(t) = \frac{\dot{K}(t)}{K(t)} = \frac{I(t)}{K(t)} = \frac{S(t)}{K(t)} = s\frac{Y(t)}{K(t)} = s\frac{Y^*(t)}{K(t)}\frac{Y(t)}{Y^*(t)} = sa(u(t)) \tag{2}$$

The equilibrium growth path – or 'warranted' rate, as Harrod famously called it – is the one corresponding to $\dot{g}_K = 0$. According to equation 1, this requires $u = 1$. The warranted rate ($g_W$) is therefore equal to:

$$g_W = sa \tag{3}$$

On such an equilibrium path, the actual output-capital ratio would stay constant (and equal to its optimal value $a$), which implies that $g_W$ is the equilibrium growth rate of both capital stock and output: in equilibrium $g_Y = g_K = g_W$.

As long as the economy grows at the warranted rate, aggregate demand and productive capacity grow at the same pace, and the rate of utilization of the capital stock stays stable at its optimal value ($u = 1$). The warranted path can be seen as a rational expectations equilibrium: firms' investment plans turn out to be based on correct demand expectations, allowing them to reach precisely their target rate of utilization.[2]

## 2.2 Warranted vs. natural growth rate

One major implication of the Harrod model is that there is no reason for the warranted rate to guarantee full or stable employment. Assume that the labor force grows at some given rate $n$. With no technical progress and a given technique of production, employment is proportional to output, and grows at the same rate. A necessary condition for full employment is thus that output grows at rate $n$. This is actually necessary not only for full employment, but for the unemployment rate to be stable at all: if the growth rate of output and population do not coincide over long periods of time, this will lead the economy to eventually run out of labor (if $g_Y > n$) or to an ever-rising unemployment rate (if $g_Y < n$). For this reason, $n$ is called the 'natural' rate of growth of this economy.[3]

The problem is that the condition for dynamic equilibrium ($g_K = g_Y = sa$) is completely independent from the condition for a stable unemployment rate ($g_Y = n$). $s$, $a$ and $n$ are all exogenous in this model, and they come from different sources: there is no reason for the economy to fully employ labor, and not even to display a stable unemployment rate. Even if we could guarantee that the economy converged to the warranted growth path (and we will see that we actually can't), the pattern of the unemployment rate would in all likelihood be a concerning one.

[2] Harrod describes the warranted rate as *"that rate of growth which, if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount"* (Harrod, 1939, p. 16).

[3] With technological progress making labor productivity grow at some rate $m$, the natural rate of growth would be $n + m$.

## 2.3 Harrodian instability

$g_K = g_W = sa$ is a dynamic equilibrium: as long as the growth rate is exactly equal to $g_W$, we will have $u = 1$ and so the accumulation rate (governed by equation 1) will stay constant at its warranted rate.

But what happens out of equilibrium? Will the system tend to converge towards the warranted rate? Quite the contrary. Imagine a situation in which $g_K > g_W$. This implies $u > 1$. So firms will *increase* their investment rate further, in order to try to address the shortage of productive capacity that they are experiencing. This will increase even more the discrepancy between $g_K$ and $g_W$, between $Y$ and $Y^*$ and between $u$ and 1. In turn, this will lead to a further increase in the investment rate, in an explosive pattern that would make the investment rate and the rate of utilization tend to infinity. The intuition is that each individual firm is expanding its capacity to meet the excess demand; collectively, however, this results in a multiplier effect of aggregate investment which is stronger than the capacity-generating effect, so that excess demand grows even faster. Similarly, if you start from a situation of under-utilization ($g_K < g_W$), this will lead firms to make the *wrong* kind of adjustment, and the investment rate and the utilization rate would follow a path of collapse.

A more formal way to see the instability problem is to use equations 1 and 2 to obtain $\dot{g}_K = \alpha[\frac{g_K}{g_W} - 1]$. This implies that the change in the growth rate is a *positive* function of the discrepancy between the actual and the warranted rate. When the growth rate is above equilibrium ($\frac{g_K}{g_W} > 1$), it will tend to increase even more. When it is below equilibrium, it will tend to decrease further. Stability of the equilibrium would require $\alpha < 0$, but it is very implausible that firms would increase their investment when their productive capacity is under-utilized.

## 2.4 Takeaways

What to take away from this? One possible interpretation is that there must be something wrong with the Harrod model: we do not observe this kind of explosive instability in the real world, and we observe relatively stable unemployment rates (at least most of the time). This interpretation underlies the subsequent development of mainstream neoclassical growth theory. As we will see, the neoclassical growth model assumes that the economy is always at full employment (Say' s law), with investment passively adapting to savings, and the optimal output-capital ratio is flexible. In this way the parameter $a$ adjusts to ensure that $g_K = sa = n$. This interpretation underlies also some subsequent post-Keynesian proposals, in which it would be income distribution (the saving rate) which adjusts to ensure equilibrium.

Another possible interpretation is that the Harrod model captures a fundamental source of instability that comes from the (private) goods market of the economy. But it leaves out very important parts of the economy, like the labor market, monetary policy, the fiscal sector and the external sector. Stabilizing forces could come from (some of) these other parts of the economy, and this may be why we do not generally observe extreme instability of the Harrodian type.

## References


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