Take-home assignment 2

Econ 705 - Spring 2021

Due: Thursday March 18 before 2pm (send via email to Guilherme)

1. With reference to the long-run fundamental determinants of economic development and cross-country income differences, discuss and compare the geographical (Jared Diamond) and the institutional (Daron Acemoglu) hypotheses. After briefly outlining the two approaches, take a stance on which of them (if any) is most compelling in your view, briefly explaining why.

   Maximum length: 500 words (this is an upper limit – less is fine too)

2. Consider the learning-by-doing model (pp.119-120 of the Romer textbook). Derive, describe and discuss the steady-state of the model under the assumption that $\phi < 1$ and $n > 0$.

3. *OLG model with social security* Consider the Diamond OLG model. Assume $g = 0$, logarithmic utility and Cobb-Douglas production.

   a *Pay-as-you-go social security.* Suppose the government taxes each young individual an amount T and uses the proceeds to pay benefits to old individuals; thus each old person receives $(1 + n)T$.

      (i) How, if at all, does this change affect the equation of motion for k (2.61 in the textbook)?

      (ii) How, if at all, does this change affect the balanced-growth-path value of k?

      (iii) If the economy is initially on a balanced growth path that is dynamically efficient, how does a marginal increase in T affect the welfare of current and future generations? What happens if the initial balanced growth path is dynamically inefficient?

b *Fully-funded social security.* Suppose the government taxes each young person an amount T and uses the proceeds to purchase capital. Individuals born at t therefore receive $(1 + r_{t+1})T$ when they are old.

(i) How, if at all, does this change affect the equation of motion for k (2.61 in the textbook)?

(ii) How, if at all, does this change affect the balanced-growth-path value of k?

4. Consider the Solow model augmented with human capital presented in Section 4.1 of the Romer textbook, with the assumption that $G(E)$ takes the form $G(E) = e^{\phi E}$.

(a) Find an expression that characterizes the value of E that maximizes the level of output per person on the balanced growth path (the *golden-rule* level of education). Are there cases where this value equals 0? Are there cases where it equals T?

[hint: remember our discussion in class about taking logs in order to more conveniently derive a multiplicative expression]

(b) Assuming an interior solution, describe how, if at all, the golden-rule level of E is affected by each of the following changes:

(i) Rise in T.

(ii) Fall in n.

(c) Now suppose that E, rather than being constant, grows steadily: $\dot{E}(t) = m$, with $m > 0$. Assume that, despite the steady increase in the amount of education people are getting, the growth rate of the number of workers remains constant and equal to n.

(i) With this change in the model, what is the long-run growth rate of output per worker?

(ii) In the United States over the past century, if we measure E as years of schooling, we have $\phi \approx 0.1$ and $m \approx 1/15$. Overall growth of output per worker has been about 2 percent per year. In light of your answer above, approximately what fraction of this overall growth has been due to increasing education, according to the human-capital model we are analyzing?

(iii) Can $\dot{E}(t)$ continue to equal $m > 0$ forever? Explain

5. Assume that production is described by the following Cobb-Douglas function:

$$Y = K^{\alpha}(AL)^{1-\alpha}; \qquad 0 < \alpha < 1 \tag{1}$$

where Y is final output and K is the amount of capital used in the production of final output. Employment L is constant and for simplicity we normalize so that $L = 1$. Increases in technical knowledge are due to public $R\&D$ investment according to the following knowledge production function:

$$\dot{A} = G^{\beta}A^{1-\beta}; \qquad 0 < \beta < 1 \tag{2}$$

where G is government spending on $R\&D$, which is financed by a constant tax rate τ. We thus have

$$G = \tau Y \tag{3}$$

The rate of change of the capital stock is equal to investment, which is assumed to be determined by savings, with a constant saving rate:

$$\dot{K} = sY \tag{4}$$

(a) Equation 2 implicitly expresses a vision of technological progress and of the nature of technical knowledge. Explain in less than 50 words.

(b) Derive expressions for the growth rates of K, A and $\frac{A}{K}$ in terms of the parameters of the model and the levels of A and K. Derive an expression for the growth rate of Y in terms of the growth rates of K and A.

(c) Show that the model has an intertemporal equilibrium with a constant $\frac{A}{K}$ ratio, and determine whether it is stable.

(d) Find the equilibrium growth rates of Y, A and K.

(e) What can we say about the effect of s and τ on the balanced growth path of this economy?

(f) In class you were told that to get stable but endogenous growth, some sort of linearity assumption is almost always needed. You need to have constant MP of some produced factor of production. Does this model confirm the rule? Or is it an exception? Explain briefly.

(g) Modify one equation of the model, in such a way as to turn it into a semi-endogenous growth model. A formal analysis of the modified model is not necessarily required (but is appreciated if you want to provide it): you can just explain what you would change and why it would deliver semi-endogenous growth.