Econ 203 - Spring 2021 - Solutions to selected exercises from assignments 4 and 5

Assignment 5, Exercises 14 to 26

(a) It is a duopoly (there are only two firms in the industry);

(b) First, we have to transform the demand function into the inverse demand function:

$X = \frac{210-p}{2} \rightarrow p = 210 - 2X = 210 - 2(x_A + x_B)$

Then we can write down the profit function of firm A:

$\pi^A = px_A - c_1 x_A = [210 - 2(x_A + x_B)]x_A - 10x_A$

$= (210 - 2x_A - 2x_B)x_A - 10x_A$

(c) The procedure for finding the profit function of firm B is the same;

(d) By maximizing the profit function (or, equivalently, setting marginal cost = marginal revenue) we find the BRF of firm A:

$\frac{\partial \pi^A}{\partial x_A} = 210 - 4x_A - 2x_B - 10 = 0 \quad \Rightarrow \quad x_A = 50 - \frac{1}{2}x_B$

the procedure for finding the BRF of firm B is the same.

(e) To find the Nash Equilibrium output of each firm, we solve the system given by the two BRFs. We can do this by substitution (substituting B's

BRF into A's BRF):

$x_A = 50 - \frac{1}{2}x_B = 50 - \frac{1}{2}(50 - \frac{1}{2}x_A) \quad \Rightarrow \quad x_A = 33.33$

(f) Because the problem is symmetric, it must be that $x_B = x_A = 33.33$;

(g) We find the product price by using the inverse demand function. First note that total output in the industry is equal to $X = x_A + x_B = 66.66$. So we have

$p = 210 - 2X = 210 - 2(66.66) = 76.67$

(h) Profits of A are found using the profit function of A:

$\pi_A = px_A - c_1x_A = 76.67(33.33) - 10(33.33) = 2,222.1$

(i) The procedure for finding the profits of B is the same

(j) Now the industry is a monopoly. Total output is just the output of the monopolist ($X = x_C$). The profit function of the monopolist is:

$\pi_C = px_C - c_1x_C = (210 - 2x_C)x_C - 10x_C$

Maximization of the profit function (or, equivalently, setting marginal cost = marginal revenue) gives us the profit-maximizing output level that the monopolist will select:

$\frac{\partial \pi_C}{\partial x_C} = 200 - 4x_C = 0 \quad \Rightarrow x_C = 50$

(k) We find the price by plugging $X = x_C = 50$ in the inverse demand function: $p = 210 - 2X = 210 - 2(50) = 110$

(l) To find profits, plug $x_c = X = 50$ and $p = 110$ into the profit function:

$\pi_C = px_C - c_1 x_C = 110(50) - 10(50) = 5,000$

(m) Consumer surplus decreases because the price is now higher (110 versus 76.7).

Assignment 4 - Exercises 2 to 11

2. Re-write utility in terms of effort levels:

   $u^M = 10e^M - \frac{1}{2}(e^M e^N) - (e^M)^2$

   $u^N = 10e^N - \frac{1}{2}(e^M e^N) - (e^N)^2$

3. Find the BRF of McKenna, by maximizing her utility (u^M) with respect to her effort level (e_M)

   $\frac{\partial u^M}{\partial e^M} = 10 - \frac{1}{2}e^N - 2e^M = 0 \Rightarrow 2e^M = 10 - \frac{1}{2}e^N \Rightarrow e^M = 5 - \frac{1}{4}e^N$

   Symmetrically, the BRF of Natasha is

   $e^N = 5 - \frac{1}{4}e^M$

4. To find the N.E. level of effort, we solve (by substitution) the system given by the two BRFs. (We could also use a shortcut: because the problem is symmetric, we know that we must have $e^M = e^N$, so we can just plug $e^M = e^N$ into the BRF of one of the two players. Also in this way we would get the correct result).

   $e^M = 5 - \frac{1}{4}[5 - \frac{1}{4}e^M] \Rightarrow e^M = 5 - \frac{5}{4} + \frac{1}{16}e^M \Rightarrow e^M = 4$

   The utility of of McKenna at the N.E. is given by the utility function:

   $u = 10(4) - \frac{1}{2}(4 \times 4) - (4)^2 = 32 - 16 = 16$

The number of fishes she catches is given by the production function:

$y = 10(4) - \frac{1}{2}(4 \times 4) = 40 - 8 = 32$

The problem is symmetric, so the same results hold also for Natasha

5. The marginal rates of substitution are as follows:

$mrs^M(e^M, e^N) = \frac{u^M_{e^M}}{u^M_{e^N}} = \frac{10 - \frac{1}{2}e^N - 2e^M}{-\frac{1}{2}e^M} = \frac{10 - \frac{1}{2}\times 4 - 2\times 4}{-\frac{1}{2}\times 4} = \frac{0}{-2} = 0$

$mrs^N(e^M, e^N) = \frac{u^N_{e^M}}{u^N_{e^N}} = \frac{-\frac{1}{2}e^N}{10 - \frac{1}{2}e^M - 2e^N} = \frac{-\frac{1}{2}\times 4}{10 - \frac{1}{2}\times 4 - 2\times 4} = \frac{-2}{0} =$ undefined

6. They will look just like the graphs in the book for Abdul and Bridget;

7. A social planner that cared equally for McKenna and Natasha would maximize the following social welfare function

$W = u^M + u^N = 10e^M - \frac{1}{2}(e^M e^N) - (e^M)^2 + 10e^N - \frac{1}{2}(e^M e^N) - (e^N)^2$

First order conditions would be as follows

$$\frac{dW}{de^M} = 10 - \frac{1}{2}e^N - 2e^M - \frac{1}{2}e^N = 0 \Rightarrow e^M = 5 - \frac{1}{2}e^N$$

$$\frac{dW}{de^N} = -\frac{1}{2}e^M + 10 - \frac{1}{2}e^M - 2e^N = 0 \Rightarrow e^N = 5 - \frac{1}{2}e^M$$

Solving by substitution the system given by the two f.o.c., we obtain the levels of effort that the social planned would pick

$e^M = 5 - \frac{1}{2}[5 - \frac{1}{2}e^M] \Rightarrow e^M = 5 - \frac{5}{2} + \frac{1}{4}e^M \Rightarrow \frac{3}{4}e^M = \frac{5}{2} \Rightarrow e^M = \frac{5}{2} \times \frac{4}{3} = \frac{10}{3} = 3.33$

Because the problem is symmetric, we have also $e^N = 3.33$

The level of utility of each player at the Social Planner Nash Equilibrium would be

$u = 10 \times \frac{10}{3} - \frac{1}{2}(\frac{10}{3} \times \frac{10}{3}) - (\frac{10}{3})^2 = \frac{100}{3} - \frac{100}{18} - \frac{100}{9} = \frac{300}{18} = 16.67$

8. If McKenna has first-mover advantage, she sets her fishing time (e^M) in such a way as to maximize her utility, taking into account Natasha's best response function (BRF). Mathematically, the problem is:

$$\max u^M = 10e^M - \frac{1}{2}(e^M e^N) - (e^M)^2 \quad \text{s.t.} \quad e^N = 5 - \frac{1}{4}e^M$$

Substituting the constraint (the BRF) into the utility function, we get:

$$\max u^M = 10e^M - \frac{1}{2}(e^M[5 - \frac{1}{4}e^M]) - (e^M)^2$$

The first order condition gives us the optimal effort level e^M that McKenna will choose:

$$\frac{\partial u^M}{\partial e^M} = 10 - \frac{5}{2} + \frac{1}{4}e^M - 2e^M = 0 \Rightarrow e^M = \frac{30}{7} = 4.2857$$

To find the effort level of Natasha, plug this result ($e^M = 4.2857$) into Natasha's BRF

$$e^N = 5 - \frac{1}{4}(4.2857) = 3.928$$

To find how many fishes each gets, plug these numbers into the production functions:

$$y^M = 10e^M - \frac{1}{2}(e^M e^N) = 10(4.2857) - \frac{1}{2}(4.2857 \times 3.9286) = 34.43$$

$$y^N = 10e^N - \frac{1}{2}(e^M e^N) = 10(3.9286) - \frac{1}{2}(4.2857 \times 3.9286) = 30.86$$

Now we can plug our results into the utility function, to calculate how much utility each obtains:

$$u^M = y^M - (e^M)^2 = 34.43 - (4.2857)^2 = 16.07$$

$$u^N = y^N - (e^N)^2 = 30.86 - (3.928)^2 = 15.43$$

9. With taxes, the utility functions would change as follows:

   $u^M = 10e^M - \frac{1}{2}(e^M e^N) - (e^M)^2 - \tau e^M$

   $u^N = 10e^N - \frac{1}{2}(e^M e^N) - (e^N)^2 - \tau e^N$

10. The BRF of McKenna with taxes is obtained by maximizing utility with respect to effort

    $\frac{du^M}{de^M} = 10 - \frac{1}{2}e^N - 2e^M - \tau = 0 \Rightarrow 2e^M = 10 - \frac{1}{2}e^N - \tau \Rightarrow e^M = 5 - \frac{1}{4}e^N - \frac{1}{2}\tau$

    Because the interaction is symmetric, the BRF of Natasha is $e^N = 5 - \frac{1}{4}e^M - \frac{1}{2}\tau$

11. Solving by substitution the system given by the two BRFs (or using the shortcut that because the problem is symmetric, we must have $e^M = e^M$), we find the Nash Equilibrium with the tax:

    $e^M = 5 - \frac{1}{4}(5 - \frac{1}{4}e^M - \frac{1}{2}\tau) - \frac{1}{2}\tau \Rightarrow e^M = 5 - \frac{5}{4} + \frac{1}{16}e^M + \frac{1}{8}\tau - \frac{1}{2}\tau$

    $e^M = \frac{20-5}{4} + \frac{1}{16}e^M + \frac{1-4}{8}\tau \Rightarrow \frac{15}{16}e^M = \frac{15}{4} - \frac{3}{8}\tau \Rightarrow e^M = \frac{15}{4}(\frac{16}{15}) - \frac{3}{8}(\frac{16}{15})\tau$

    $e^M = 4 - \frac{2}{5}\tau$

Because of symmetry, the N.E. effort level of Natasha will be equal: $e^N = 4 - \frac{2}{5}\tau$

12. To find the tax that would yield the effort levels that maximize social welfare (defined as above), we just have to plug the desired effort levels into the BRF of one of the two players:

    $\frac{10}{3} = 5 - \frac{1}{4}(\frac{10}{3}) - \frac{1}{2}\tau \Rightarrow \frac{10}{3} = 5 - \frac{10}{12} - \frac{1}{2}\tau \Rightarrow \frac{10}{3} = \frac{60-10}{12} - \frac{1}{2}\tau$

    $\frac{1}{2}\tau = \frac{50}{12} - \frac{10}{3} \Rightarrow \frac{1}{2}\tau = \frac{50-40}{12} \Rightarrow \frac{1}{2}\tau = \frac{10}{12} \Rightarrow \tau = \frac{20}{12} = \frac{5}{3} = 1.67$

    We could also have used a shortcut to find this result: we could have plugged the desired effort levels directly in the Nash Equilibrium effort of one of the two players:

    $\frac{10}{3} = 4 - \frac{2}{5}\tau \Rightarrow \frac{2}{5}\tau = 4 - \frac{10}{3} \Rightarrow \frac{2}{5}\tau = \frac{2}{3} \Rightarrow \tau = \frac{5}{3} = 1.67$