



## Advanced Macroeconomics

### Section 4 - Fluctuations (II): Keynesian and New-Keynesian theories

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# 'Old school' Keynesian theory

- ▶ Developed in the 1940s to formalise Keynes' ideas
- ▶ Was dominant and guided policy until the 1970s
- ▶ Simple models built up from sensible assumptions about relations between macroeconomic variables, but no explicit microfoundations
- ▶ IS-LM model + Phillips Curve
- ▶ Aggregate demand determines the level of output, inflation-unemployment trade-off

## New Keynesian theory

- ▶ Micro-founded rational-expectations framework (like RBC)
- ▶ but introduces nominal rigidities (sticky prices/wages) and imperfect competition
- ▶ Baseline 3-equations DSGE model
  1. New Keynesian IS curve
  2. New Keynesian Phillips Curve
  3. Central Bank reaction function
- ▶ real effects of monetary policy (unlike RBC and somehow similar to old Keynesian models)
- ▶ also the effects of other shocks (technology and fiscal policy) differ from the plain RBC model.

## The plan

1. Old school IS-LM model and Lucas critique
2. New Keynesian IS-LM model
3. Phillips Curve(s)
4. IS-LM-PC: A simplified model in the spirit of New Keynesian macro
5. The canonical DSGE New Keynesian model

## The 'old-school' IS-LM model

- ▶ Model of output determination in the short-run
- ▶ John Hicks (1937) formalisation of (his interpretation of) Keynes.
  - Neoclassical synthesis
- ▶ Became the dominant model of output determination since the 1940s and is still the model taught in intermediate classes.
- ▶ Notation:
  - $Y$  = output
  - $Z$  = aggregate demand
  - $C$  = consumption
  - $I$  = aggregate investment
  - $G$  = government spending
  - $\tau$  = tax rate
  - $i$  = nominal interest rate
  - $r$  = real interest rate
  - $M$  = quantity of money
  - $P$  = price level

## Goods market equilibrium

► **Definition:**

◦ Aggregate demand  $Z_t \equiv C_t + I_t + G_t$ .

► **Behavioural equations:**

◦ Consumption function:  $C_t = c_0 + c_1(1 - \tau_t)Y_t$

◦ Investment function:  $I_t = a_0 - a_1 r_t$ .

◦  $G$  and  $\tau$  taken as given:  $G_t = G, \tau_t = \tau$ .

► **Equilibrium:**

Equilibrium condition  $Y = Z$  implies equilibrium output is

$$Y_t = \frac{1}{1 - c_1(1 - \tau)} [c_0 + (a_0 - a_1 r) + G] = A - ar_t$$

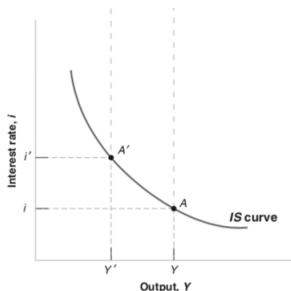
Where  $A = \frac{c_0 + a_0 + G}{1 - c_1(1 - \tau)}$  and  $a = \frac{a_1}{1 - c_1(1 - \tau)}$ .

# The 'old-school' IS-LM model

## The old school IS curve

- goods' market equilibrium:

$$Y = A - ar \quad (IS \text{ curve})$$



- A change in the interest rate is a movement along the IS curve
- A change in government spending or autonomous consumption shifts the IS curve up or down

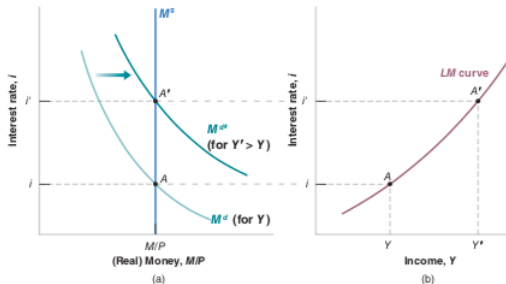
# The 'old-school' IS-LM model

## Money market equilibrium

$$\frac{M_t}{P_t} = \alpha Y_t - \beta i_t \Rightarrow i_t = b Y_t - c \frac{M_t}{P_t} \quad (LM \text{ curve})$$

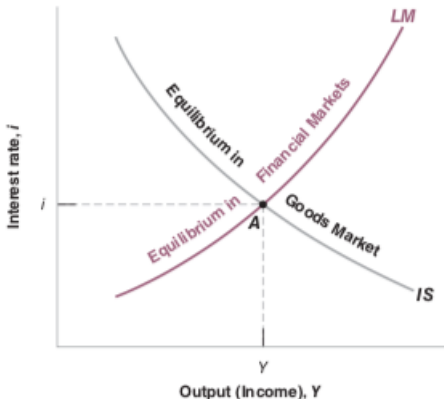
(Where  $b = \alpha/\beta$  and  $c = 1/\beta$ )

- $M$  and  $P$  exogenous constants ( $P_t = P$ ,  $M_t = M$ ).
- Higher  $Y \rightarrow$  higher demand for  $M \rightarrow$  higher equilibrium  $i$





## The 'old-school' IS-LM model



- ▶ Given fixed price assumption,  $i = r$ .
- ▶ Can be used to evaluate the effect of fiscal and monetary policy.
- ▶ Fiscal expansion (increase in  $G$  or decrease in  $\tau$ ) raises  $Y$  and  $i$ .
- ▶ Monetary expansion (increase in  $M$ ) raises  $Y$  and lowers  $i$ .

## The Lucas (1976) critique

- ▶ Old-school Keynesian models lack microfoundations
- ▶ Relations between aggregates are assumed, without specifying how they arise from individual goal-oriented behavior.
- ▶ Policy evaluation might be flawed: policy change might change expectations & behaviour, altering aggregate relations.
- ▶ Example: In evaluating effect of fiscal expansion, old-Keynesian theory assumes a given propensity to save. But if stimulus is temporary, utility-maximizing agents might save most of it, so propensity to save is not stable.
- ▶ The equations of a macro model should be derived explicitly from a microeconomic model of individual behavior.

## The New-Keynesian IS-LM model

- ▶ One-good economy with no  $K$ , large number of identical firms, and fixed number of identical infinitely lived households.

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## The New-Keynesian IS-LM model

- ▶ One-good economy with no  $K$ , large number of identical firms, and fixed number of identical infinitely lived households.
- ▶ Production function:  $Y = C = F(L)$ ;  $F'(L) > 0$ ;  $F''(L) \leq 0$
- ▶ Representative household's lifetime utility:

$$U = \sum_{t=0}^{\infty} \beta^t \left[ U(C_t) + \Gamma \left( \frac{M_t}{P_t} \right) - V(L_t) \right], \quad 0 < \beta < 1$$

- $U'(\cdot) > 0$  and  $U''(\cdot) < 0$ ;
- $\Gamma'(\cdot) > 0$  and  $\Gamma''(\cdot) < 0$ ;
- $V' > 0$  and  $V''(\cdot) > 0$ .
- ▶ Choice variables:  $C$  and  $M$ ;
- ▶  $L$  exogenous (for now);

## Evolution of household's wealth

- ▶ Two assets: Central Bank money  $M$  (gold coins) and a bond  $B$  (a claim on  $M$ ).
- ▶ Evolution of household's wealth:

$$\begin{aligned}A_{t+1} &= M_t + B_t(1 + i_t) \\ &= M_t + (A_t + W_t L_t - P_t C_t - M_t)(1 + i_t)\end{aligned}$$

- $A_{t+1}$  is wealth at the start of period  $t + 1$ ;
- $M_t$  and  $B_t$  are money and bonds held during period  $t$ ;

## Household's behavior: Euler equation

- ▶ Assuming CRRA utility, the infinite-horizon utility function implies

$$\ln C_t = \ln C_{t+1} - \frac{1}{\theta} \ln[(1 + r_t)\beta]$$

↓

$$\ln Y_t = a + \ln Y_{t+1} - \frac{1}{\theta} r_t$$

(because  $Y = C$  and  $\ln(1 + r) \approx r$ , and with  $a = -(\frac{1}{\theta}) \ln \beta$ )

- ▶ See demonstration in Romer Section 6.1

## The New-Keynesian IS curve

$$\ln Y_t = a + \ln Y_{t+1} - \frac{1}{\theta} r_t$$

- ▶ negative relation between  $Y_t$  and  $r_t$ .
- ▶ differences with old-school IS curve:
  - conceptual: driven by intertemporal substitution, not income multiplier effect.
  - practical:  $\ln Y_{t+1}$  term.
  - here, IS interpretation requires assuming fixed  $Y_{t+1}$ .



## John Cochrane on the New Keynesian IS curve:

*This new-Keynesian model is an utterly and completely different mechanism and story [relative to the old-keynesian model]. (...)*

*The marginal propensity to consume is exactly and precisely zero in the new-Keynesian model. There is no income at all on the right hand side [of the Euler equation]. (...)*

## John Cochrane on the NK IS curve (continued):

*The old-Keynesian model is driven completely by an income effect with no substitution effect. Consumers don't think about today vs. the future at all. The new-Keynesian model is based on the intertemporal substitution effect with no income effect at all. (...)*

*[a lower  $r_t$ ] induces consumers to spend their money today rather than in the future (...). Now, lowering consumption growth is normally a bad thing. But new-Keynesian modelers assume that the economy reverts to trend, so lowering growth rates is good, and raises the level of consumption today with no ill effects tomorrow.*

[from John Cochrane's 'New vs. Old Keynesian Stimulus' (on Keats)]

## Household's money demand

- ▶ Optimization requires that marginal increase in  $M_t/P_t$  (given total wealth) has no effect on utility.
- ▶ To leave wealth unchanged,  $\Delta C_t = -\left(\frac{i}{1+i}\right) \Delta m$
- ▶ So in equilibrium:

$$\Gamma' \left( \frac{M_t}{P_t} \right) \Delta m = U'(C_t) \left( \frac{i_t}{1+i_t} \right) \Delta m$$

↓

$$\frac{M_t}{P_t} = Y_t^{\theta/\chi} \left( \frac{1+i_t}{i_t} \right)^{1/\chi}$$

- ▶ Real money demand is positive function of  $Y$  and negative function of  $i$  as in the old-Keynesian model.
- ▶  $P$  and  $M$  are fixed, so implies  $i$  increasing function of  $Y$ .

## New-Keynesian IS-LM

- ▶ Price of consumption good is assumed fixed:

$$P_t = \bar{P} \Rightarrow i_t = r$$

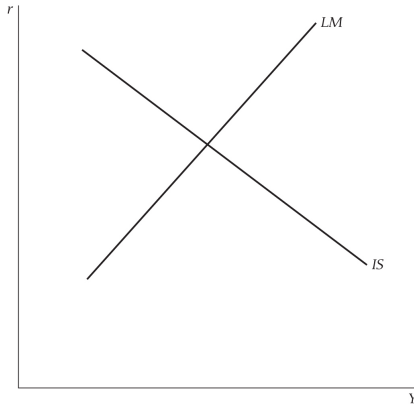
- ▶ So both IS and money-demand are in terms of  $r$  and  $Y$ ;

$$Y_t = f(r_t) \quad \text{with } f' < 0 \quad (\text{IS curve})$$

$$r_t = g(Y_t) \quad \text{with } g' > 0 \quad (\text{LM curve})$$

## New Keynesian IS-LM

## New-Keynesian IS-LM



but remember this is based on the assumption of unchanged (expectation of)  $Y_{t+1}$ !

## New-Keynesian IS-LM

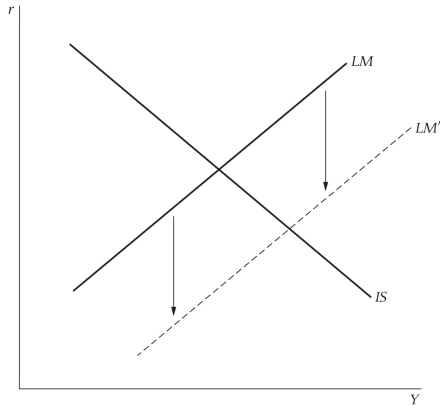
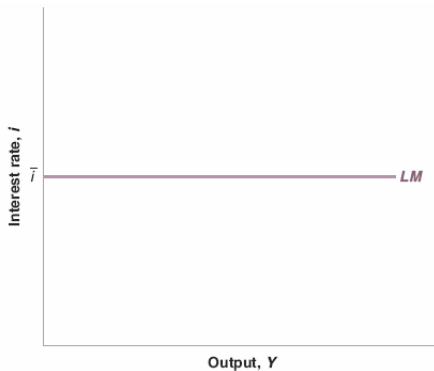


Figure: Effect of a temporary increase in money supply

## IS-LM with interest-rate setting

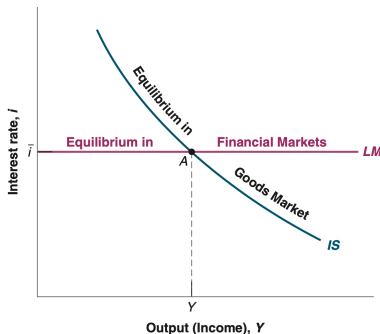
## A more realistic LM "curve"

- ▶ In reality, money is endogenous and the Central Bank sets the interest rate.
- ▶  $i = \bar{i}$ .



## IS-LM with interest-rate setting

- ▶ IS relation:  $Y_t = f(r_t)$  with  $f' < 0$
- ▶ LM relation:  $r = i = \bar{i}$



- ▶ After adding a model of inflation (Phillips Curve), can be enriched by the Central Bank reaction function
- ▶ CB sets the interest rate based on inflation and output.



## Phillips Curve(s)

- ▶ IS-LM framework (old or new) needs to be completed with a theory of inflation.
- ▶ *Phillips Curve*: A relation between inflation & unemployment/output.
- ▶ 'Traditional' Phillips Curve:

$$\pi_t = \alpha - \beta u_t$$

- ▶ 'Accelerationist' Phillips Curve:

$$\pi_t - \pi_{t-1} = \alpha - \beta u_t$$

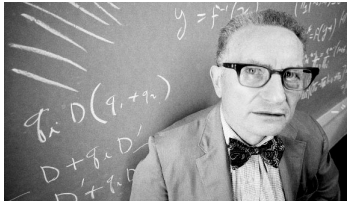
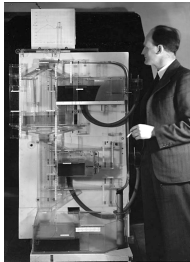
- ▶ New Keynesian Phillips Curve:

$$\pi_t = ky_t + \beta E_t \pi_{t+1} \quad (1)$$

- ▶ Very different implications for policy.

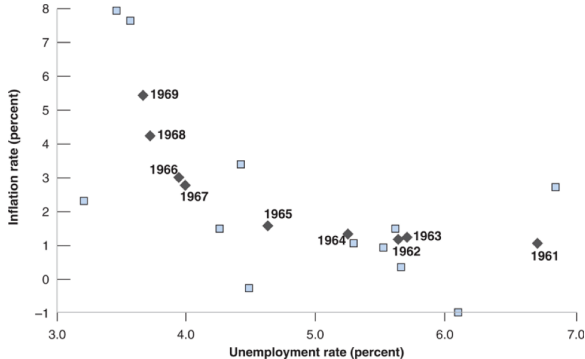
## Historical origins of the Phillips Curve

- ▶ PC originally derived from empirical observation, not formal theory.
- ▶ 1958: A.W. Phillips uncovers negative correlation between inflation and unemployment in UK 1861-1957 data.
- ▶ 1960: Samuelson & Solow replicate it on 1900-1960 US data.
- ▶ In the 1970s the relation breaks down, which inspires the development of an 'accelerationist' Phillips Curve.



# Phillips Curve(s)

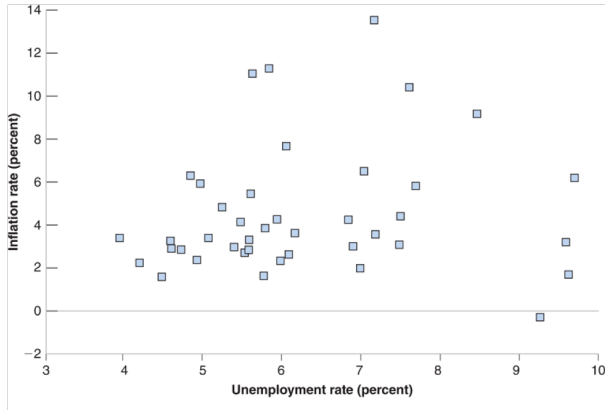
## 1948-1969: the 'original' Phillips Curve



Source: Series UNRATE,  
CPIAUSCL Federal Reserve Eco-  
nomic Data (FRED) <http://research.stlouisfed.org/fred2/>

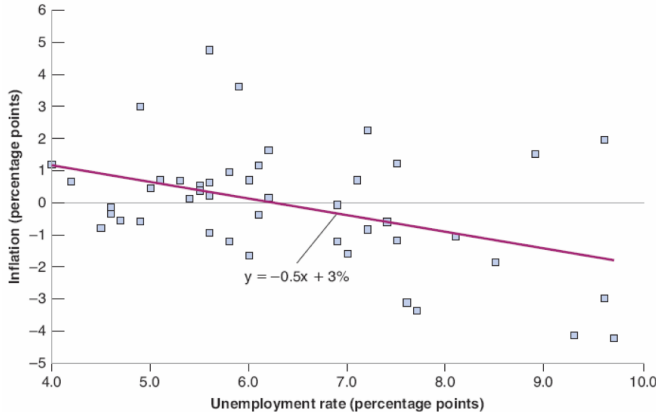
# Phillips Curve(s)

## 1970-2010: the disappearance of the 'original' PC



# Phillips Curve(s)

## 1970-2010: Accelerationist PC



## Phillips Curve(s): Theoretical foundations

- ▶ Theoretical explanations of the PC focus on wage and price-setting processes.
- ▶ Models of wage and price-setting imply relations between  $\pi$ ,  $E(\pi)$  and  $u$
- ▶ specific form of the PC depends on how agents form  $E(\pi)$ 
  1. fixed ('anchored') expectations  $\rightarrow$  original PC
  2. adaptive expectations  $\rightarrow$  accelerationist PC
  3. rational expectations  $\rightarrow$  New-Keynesian PC
- ▶ Traditional & accelerationist PC can be derived from a simple macro model, while New Keynesian PC can be derived from the (more complicated) Calvo model of pricing.

## Traditional and accelerationist Phillips Curves

### Phillips Curve: a simple framework

- ▶ Traditional and accelerationist PC can be derived from a very simple macro model.
- ▶ Central idea:  
lower  $u_t \Rightarrow$  higher  $W_t \Rightarrow$  increase in  $P_t$  &  $\pi_t$ .
- ▶ if it stops here, we have the 'original' PC

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lower  $u_t \Rightarrow$  higher  $W_t \Rightarrow$  increase in  $P_t$  &  $\pi_t$ .
- ▶ if it stops here, we have the 'original' PC
- ▶ BUT with adaptive expectations, inflationary spiral:  
lower  $u_t \Rightarrow$  higher  $W_t \Rightarrow$  increase in  $P_t$  &  $\pi_t \Rightarrow$  increase in  $E(\pi_{t+1}) \Rightarrow$  increase in  $W_{t+1} \Rightarrow \dots$
- ▶ 'accelerationist' PC



## Traditional and accelerationist Phillips Curves

► Basic model:

$$Y_t = N_t$$

$$P_t = (1 + m)W_t$$

$$\frac{W_t}{E(P_t)} = 1 - \beta u_t \quad \Rightarrow \quad W_t = E(P_t)(1 - \beta u_t)$$

- $Y$  = output;
- $N$  = employment;
- $W$  = nominal wage;
- $P$  = price of the good;
- $m$  = mark-up;
- $u = 1 - \frac{L}{N}$  = unemployment rate;

## Traditional and accelerationist Phillips Curves

- ▶ Combine price-setting & wage-setting:

$$P_t = E(P_t)(1 + m)(1 - \beta u_t)$$

- ▶ rewrite (approximately) in terms of  $\pi$ :

$$\pi_t = E(\pi_t) + m - \beta u_t$$

- ▶ What determines  $E(\pi_t)$ ?

## Traditional and accelerationist Phillips Curves

- ▶ 'Generic' Phillips Curve:

$$\pi_t = E(\pi_t) + m - \beta u_t$$

- ▶ Assume fixed expectations

$$E(\pi) = \bar{\pi}$$

- ▶ Then we have

$$\pi_t = \alpha - \beta u_t \quad (\text{with } \alpha = \bar{\pi} + m)$$

- ▶ '*original*' (old-Keynesian) Phillips curve
- ▶ Inflation-unemployment trade-off for policy.

## The PC and its mutations

- ▶ 'Generic' Phillips Curve:

$$\pi_t = E(\pi_t) + m - \beta u_t$$

- ▶ Assume adaptive expectations

$$E(\pi) = \pi_{t-1}$$

- ▶ 'Accelerationist' PC:

$$\pi_t - \pi_{t-1} = \alpha - \beta u_t$$

- ▶ Lower unemployment leads to higher *change* in the inflation rate (like in the 1970s).

# Traditional and accelerationist Phillips Curves

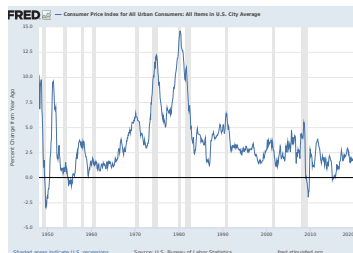
## An interpretation of the history of inflation in the US

### 1948-1969

- ▶ inflation not persistent;
- ▶ wage-setters assumed inflation would revert to mean  $\bar{\pi}$ ;
- ▶  $E(\pi) \approx \bar{\pi} \Rightarrow$  Original PC.

### after 1970

- ▶ inflation became persistent (oil shocks);
- ▶ wage-setters started taking persistence into account;
- ▶  $E(\pi_t) \approx \pi_{t-1} \Rightarrow$  accelerationist PC.



## The equilibrium unemployment rate

In this model, a unique unemployment rate makes inflation equal expected inflation:

$$\pi_t = \mathbb{E}(\pi_t) \rightarrow u_t^* = \frac{m}{\beta}$$

### Implications for traditional PC:

- ▶ Possible to sustain  $u < u_t^*$  only as long as  $\pi > \mathbb{E}(\pi)$ .
- ▶ But if  $\pi > \mathbb{E}(\pi)$  is persistent, wage-setters would surely update their expectations!
- ▶ Traditional PC with anchored expectations unlikely to be stable unless  $u = u^*$ .

## The equilibrium unemployment rate

In this model, a unique unemployment rate makes inflation equal expected inflation:

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### Implications for accelerationist PC:

- ▶ When  $u = u^*$ , inflation is stable over time ( $\pi_t = \pi_{t-1}$ ).
- ▶  $u < u^*$  leads to accelerating inflation (increasing over time).
- ▶  $u > u^*$  leads to deflation (decreasing over time).
- ▶ Disinflation is painful: to bring down  $\pi$ , you need  $u > u^*$  for a period of time.

## Calvo price setting model

- ▶ New Keynesian PC is derived from a more complex model of dynamic price setting.
- ▶ Calvo (1983) "Staggered prices in a utility-maximizing framework".
- ▶ Sticky prices: they cannot be adjusted in all periods.
- ▶ Opportunities to change prices arrive randomly.
  - *Poisson process*: same probability of price adjustment in every period.
- ▶ A bit arbitrary: chosen as the baseline model of prices not because realistic, but because it happens to deliver a convenient PC that works well in a DSGE model.



## Framework (1/3)

- ▶ A monopolistic competition model

- ▶ Production function

$$Y_t = L_t$$

- ▶ Closed economy with no government and no capital:

$$C_t = Y_t$$

- ▶ Exogenous nominal expenditure (aggregate demand)

$$M_t = Y_t P_t$$

- ▶ Labor supply curve

$$\frac{W_t}{P_t} = B Y_t^{\theta + \gamma - 1}$$

- ▶ Monopolistic pricing

$$\frac{P_{it}^*}{P_t} = \frac{\eta}{\eta - 1} \frac{W_t}{P_t}$$

## Framework (2/3)

*Time-dependent price-adjustment:*

- ▶ Firms cannot adjust their prices in all periods.
- ▶  $P_i$  set at time 0 has probability  $q_t \geq 0$  of remaining in effect at time  $t > 0$ .
- ▶  $p_t \equiv \ln(P_t)$ .
- ▶ firm sets  $p_i$  as a weighted average of expected future  $p_t^*$ 's:

$$p_i = \sum_{t=0}^{\infty} \tilde{\omega}_t E[p_t^*] \quad \text{with} \quad \tilde{\omega}_t \equiv \frac{\beta^t q_t}{\sum_{\tau=0}^{\infty} \beta^{\tau} q_{\tau}}$$

## Framework (3/3)

- ▶ Profit-maximizing price is a mark-up over the wage

$$\frac{p_{it}^*}{p_t} = \frac{\eta}{\eta - 1} \frac{W_t}{p_t} \Rightarrow p_t^* = \ln \left[ \frac{\eta}{\eta - 1} \right] + w_t$$

- ▶ Substitute in the (log of the) labor supply curve

$$w_t = p_t + \ln B + (\theta + \gamma - 1)y_t \Rightarrow p_t^* = p_t + \ln \frac{\eta}{\eta - 1} + \ln B + (\theta + \gamma - 1)y_t$$

- ▶ Given that  $m = y + p$ , and assuming for simplicity  $\ln \frac{\eta}{\eta - 1} + \ln B = 0$ ,

$$p_t^* = \phi m_t + (1 - \phi)p_t \quad \text{with } \phi = (\theta + \gamma - 1)$$

- ▶ optimal 'sticky' price to set at time 0:

$$p_i = \sum_{t=0}^{\infty} \tilde{\omega}_t E_0[\phi m_t + (1 - \phi)p_t]$$

Deriving  $\pi$ 

- ▶ Each period share  $\alpha$  of firms, randomly chosen, adjusts prices

*aggregate price level:*  $p_t = \alpha x_t + (1 - \alpha)p_{t-1}$

*inflation:*  $\pi_t = p_t - p_{t-1} = \alpha(x_t - p_{t-1})$

## Deriving $\pi$

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*inflation:*  $\pi_t = p_t - p_{t-1} = \alpha(x_t - p_{t-1})$

- ▶ optimal 'sticky' prices:

$$x_t = \sum_{j=0}^{\infty} \tilde{\omega}_j E(p_{t+j}^*) \quad \text{with} \quad \tilde{\omega}_j = \frac{\beta^j q_j}{\sum_{k=0}^{\infty} \beta^k q_k}$$

- ▶ Poisson process implies  $q_j = (1 - \alpha)^j$
- ▶  $\rightarrow \sum_{k=0}^{\infty} \beta^k q_k = \sum_{k=0}^{\infty} \beta^k (1 - \alpha)^k = \frac{1}{1 - \beta(1 - \alpha)}$

## Calvo model - deriving $\pi$

- ...plugging in:

$$x_t = [1 - \beta(1 - \alpha)] \sum_{j=0}^{\infty} \beta^j (1 - \alpha)^j E_t p_{t+j}^*$$

- Rewrite in terms of  $p_t^*$  and  $E_t x_{t+1}$ :

$$\begin{aligned} x_t &= [1 - \beta(1 - \alpha)] \left( p_t^* + \beta(1 - \alpha) \left[ \sum_{j=0}^{\infty} \beta^j (1 - \alpha)^j E_t p_{t+1+j}^* \right] \right) = \\ &= [1 - \beta(1 - \alpha)] p_t^* + \beta(1 - \alpha) [1 - \beta(1 - \alpha)] \left[ \sum_{j=0}^{\infty} \beta^j (1 - \alpha)^j E_t p_{t+1+j}^* \right] = \\ &= [1 - \beta(1 - \alpha)] p_t^* + \beta(1 - \alpha) E_t x_{t+1} \end{aligned}$$

Deriving  $\pi$ 

$$x_t = [1 - \beta(1 - \alpha)]p_t^* + \beta(1 - \alpha)E_t x_{t+1}$$

- Express in terms of  $\pi_t$ , using  $\pi_t = \alpha(x_t - p_{t-1})$  and  $p_t^* = \phi m_t + (1 - \phi)p_t$

$$\pi_t = ky_t + \beta E_t \pi_{t+1} \quad \text{with} \quad k = \frac{\alpha[1 - (1 - \alpha)\beta]\phi}{1 - \alpha}$$

- New-Keynesian Phillips Curve
- Inflation depends on expected inflation & output (as in all PCs);
- Difference: it is  $E_t \pi_{t+1}$  that matters here: expectation of *future* inflation.

### 3 Phillips Curves and their implications

1. *Old-Keynesian PC*:  $\pi_t = \alpha + \lambda y_t$

- ▶ *output-inflation trade-off*: disinflation requires permanently lower  $y$ ;

2 *Accelerationist PC*:  $\pi_t = \pi_{t-1} + \lambda(y_t - y_t^*)$

- ▶ painful disinflation: requires  $y < y^*$  for some time (*inflation inertia*);

3 *New-Keynesian PC*:  $\pi_t = ky_t + \beta E_t \pi_{t+1}$

- ▶ expansionary disinflation:  $E_t(\pi_{t+1})$  down  $\rightarrow y_t$  up.



## New Keynesian models of fluctuations

- ▶ IS curve & Phillips curve are the key building blocks of Keynesian & New Keynesian macroeconomics.
- ▶ They can be integrated to build dynamic models of fluctuations.
- ▶ We will consider two:
  1. A simplified New Keynesian model
  2. The canonical New Keynesian DSGE model