Macroeconomic Theory I

Section 5 - Fluctuations (II): New-Keynesian theory

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New-Keynesian theory

- micro-founded rational-expectations framework (like RBC);
- nominal rigidities (stickly prices/wages) and market imperfections;
- real effects of monetary policy;
- also the effects of other shocks (technology and fiscal policy) are altered.

The plan

1. Assume nominal rigidity (fixed prices/wages) and assess its effects in simple models;
2. Make nominal rigidity endogenous: how can it emerge from microfoundations?
3. Embed nominal rigidity into a micro-founded rational-expectations model of the economy (a DSGE).

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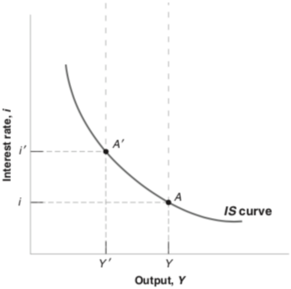
The ‘old-school’ IS-LM model

- goods’ market equilibrium:

$$Y = A - ar \qquad \textit{(IS curve)}$$



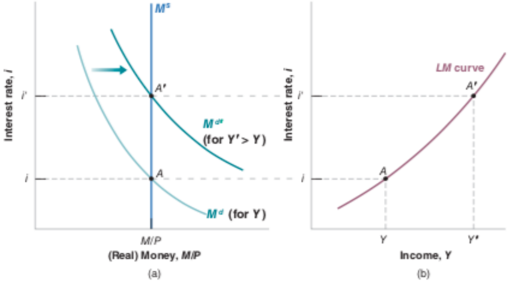
- (see IS-LM-PC lecture notes for details)

The ‘old-school’ IS-LM model

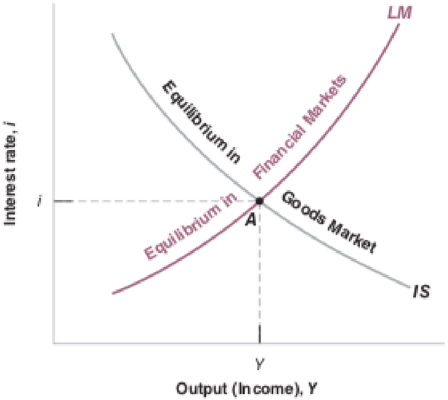
- money-market equilibrium:

$$\frac{M}{P} = \alpha Y - \beta r \quad \Rightarrow \quad r = bY - c\frac{M}{P} \qquad \textit{(LM curve)}$$

  - Higher Y → higher demand for (fixed) M → higher equilibrium r



The ‘old-school’ IS-LM model



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The New-Keynesian IS-LM model

- Production function: $Y = C = F(L); \quad F'(L) > 0; \quad F''(L) \leq 0$
- Representative household's lifetime utility:

$$U = \sum_{t=0}^{\infty} \beta^t \left[U(C_t) + \Gamma\left(\frac{M_t}{P_t}\right) - V(L_t) \right], \quad 0 < \beta < 1$$

  - $U'(.) > 0$ and $U''(.) < 0$;
  - $\Gamma'(.) > 0$ and $\Gamma''(.) < 0$;
  - $V' > 0$ and $V''(.) > 0$.

- Choice variables: C and M;
- L exogenous (for now);

Evolution of household's wealth

$$A_{t+1} = M_t + B_t(1+i_t) = M_t + (A_t + W_tL_t - P_tC_t - M_t)(1+i_t)$$

- A_{t+1} is wealth at the start of period $t+1$;
- M_t and B_t are money and bonds held during period t;

Household's behavior: Euler equation

- The infinite-horizon utility function implies

$$\ln C_t = \ln C_{t+1} - \frac{1}{\theta}\ln[(1+r_t)\beta]$$

$$\Downarrow$$

$$\ln Y_t = a + \ln Y_{t+1} - \frac{1}{\theta} r_t$$

(because $Y = C$ and $\ln(1+r) \approx r$, and with $a = -(\frac{1}{\theta})\ln\beta$)

The New-Keynesian IS curve

$$\ln Y_t = a + \ln Y_{t+1} - \frac{1}{\theta} r_t$$

- negative relation between Y_t and r_t;
- differences with old-school IS curve:
  - conceptual: driven by intertemporal substitution, not income effect;
  - practical: $\ln Y_{t+1}$ term
  - here, IS interpretation requires assuming fixed Y_{t+1}.

The New-Keynesian IS curve

- John Cochrane on the NK IS curve:

  This new-Keynesian model is an utterly and completely different mechanism and story [relative to the old-keynesian model]. (...)

  The marginal propensity to consume is exactly and precisely zero in the new-Keynesian model. There is no income at all on the right hand side [of the Euler equation]. (...)

The New-Keynesian IS curve

- John Cochrane on the NK IS curve (continued):

  The old-Keynesian model is driven completely by an income effect with no substitution effect. Consumers don't think about today vs. the future at all. The new-Keynesian model is based on the intertemporal substitution effect with no income effect at all. (...)

  [a lower r_t] induces consumers to spend their money today rather than in the future (...). Now, lowering consumption growth is normally a bad thing. But new-Keynesian modelers assume that the economy reverts to trend, so lowering growth rates is good, and raises the level of consumption today with no ill effects tomorrow.

  [from John Cochrane's 'New vs. Old Keynesian Stimulus' (on Moodle)]

Household's money demand

- Optimization requires that marginal increase in M_t/P_t (given total wealth) has no effect on utility;
- To leave wealth unchanged, $\Delta C_t = -\left(\frac{i}{1+i}\right)\Delta m$
- So in equilibrium:

$$\Gamma'\left(\frac{M_t}{P_t}\right)\Delta m = U'(C_t)\left(\frac{i_t}{1+i_t}\right)\Delta m$$

$$\Downarrow$$

$$\frac{M_t}{P_t} = Y_t^{\theta/\chi}\left(\frac{1+i_t}{i_t}\right)^{1/\chi}$$

New-Keynesian IS-LM

- Price of consumption good is assumed fixed:

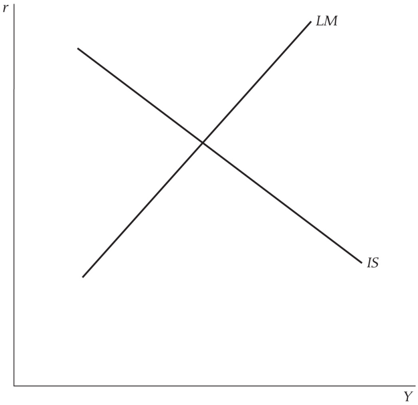
$$P_t = \bar{P} \quad \Rightarrow \quad i_t = r$$

- So both IS and money-demand are in terms of r and Y;
- M is also fixed (by CB), so money-demand implies r increasing in Y.

$$Y_t = f(r_t) \quad \text{with } f' < 0 \quad \text{(IS curve)}$$

$$r_t = g(Y_t) \quad \text{with } g' > 0 \quad \text{(LM curve)}$$

New-Keynesian IS-LM



but remember this is based on the assumption of unchanged (expectation of) Y_{t+1}!

New-Keynesian IS-LM

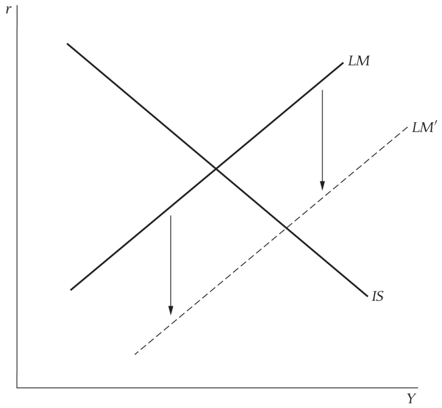


Figure: Effect of a temporary increase in money supply

Rigidities & imperfections

- Simple IS-LM story would not hold under perfect competition;
- Need nominal rigidity (fixed P) *and* imperfect competition to deliver the 'Keynesian' message
    - in the *labor market* and/or *product market*
- Different combinations of rigidites & imperfections –> different implications for unemployment, prices and wages;
- *4 stylized cases* within the NK IS-LM framework.

Case 1: Fixed W but perfectly-competitive goods market

- Nominal wage fixed above market-clearing level

$$W = \bar{W} > W^{eq}$$

- Competitive goods market

$$F'(L) = \frac{\bar{W}}{P}$$

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Case 1: Fixed W but perfectly-competitive goods market

- Nominal wage fixed above market-clearing level

$$W = \bar{W} > W^{eq}$$

- Competitive goods market

$$F'(L) = \frac{\bar{W}}{P}$$

- Effect of a positive demand shock:
  1. Initially: only P increases (firms don't expand given initial W/P);
  2. Then: increase in P brings $\frac{\bar{W}}{P}$ down, thus firms increase L and Y.

Case 1: Fixed W but perfectly-competitive goods market

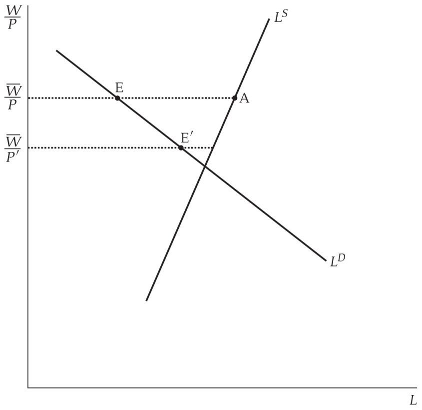

Figure: Effect of a positive demand shock

- Involuntary unemployment (EA distance);
- AD shock moves economy from E to E';
- countercyclical real wage in response to AD shocks;
- fluctuations are movements along a decreasing L^D curve;
- demand determines how much firms *want* to sell;

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Case 2: Perfectly-competitive labor market but fixed P

- Product price fixed and above marginal cost (market power);

$$P_t = \bar{P} > MC$$

- Increasing labor supply function:

$$L = L^S\left(\frac{W}{P}\right), \qquad L^{S\prime}(.) > 0$$

- Firms are demand-constrained (as long as $F'(L) > W/P$);
- *Effective labor demand:* labor demand just depends on aggregate demand for goods;

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Case 2: Perfectly-competitive labor market but fixed P

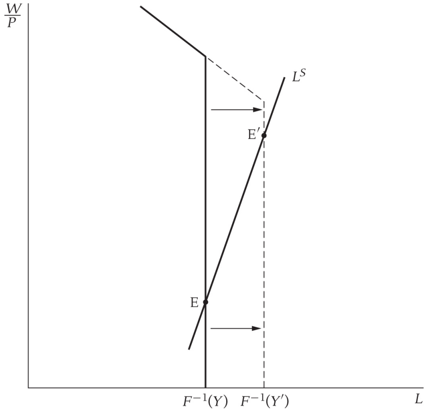

Figure: Effect of a positive demand shock

- As long as $W/P < MPL$, L^D inelastic: just depends on aggregate demand (Y);
- Labor supply elastic;
- No involuntary employment;
- Pro-cyclical real wage;
- Fluctuations are movements along increasing L^S curve;
- Counter-cyclical mark-up (increasing W & decreasing MPL);
- Demand determines how much firms *are able to* sell.

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Case 3: Non-Walrasian labor market and fixed P

- Product price fixed above marginal cost:

$$P_t = \bar{P} > MC$$

- Wage curve:

$$\frac{W}{P} = w(L) > \left(\frac{W}{P}\right)^{eq}, \qquad w'(.) \geq 0$$

Case 3: Non-Walrasian labor market and fixed P

- Product price fixed above marginal cost:

$$P_t = \bar{P} > MC$$

- Wage curve:

$$\frac{W}{P} = w(L) > \left(\frac{W}{P}\right)^{eq}, \qquad w'(.) \geq 0$$

- Unlike case 2, rigidity here is real (real wage) not only nominal.
- As long as $P > MC$, firms are demand-constrained;
- Effective labor demand determines employment *and unemployment*.

Case 3: Non-Walrasian labor market and fixed P

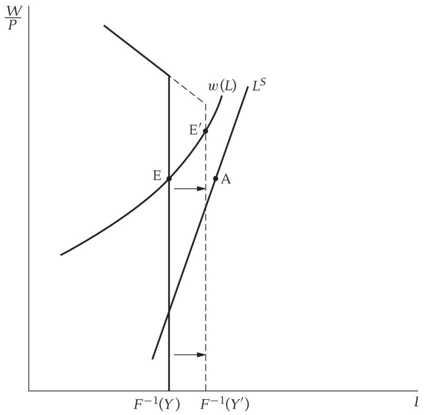

Figure: Effect of a positive demand shock

- Workers paid more than their reservation wage;
- fluctuations as movements along the wage curve;
- unemployment: horizontal distance EA;
- unemployment falls when demand raises (as long as $w(L)$ flatter than L^S);
- pro-cyclical real wage and counter-cyclical mark-up;

Case 4: Fixed W, imperfectly-competitive goods market

- Nominal wage fixed above market-clearing level;

$$W = \bar{W} > W^{eq}$$

- Imperfect competition in the goods market:

$$P = \mu(L)\frac{W}{F'(L)} \quad \Rightarrow \quad \frac{W}{P} = \frac{F'(L)}{\mu(L)}$$

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Case 4: Fixed W, imperfectly-competitive goods market

- Nominal wage fixed above market-clearing level;

$$W = \bar{W} > W^{eq}$$

- Imperfect competition in the goods market:

$$P = \mu(L)\frac{W}{F'(L)} \quad \Rightarrow \quad \frac{W}{P} = \frac{F'(L)}{\mu(L)}$$

- If μ constant or pro-cyclical, $\frac{W}{P}$ countercyclical (diminishing MPL);
- If μ sufficiently counter-cyclical, real wage acyclical or slightly pro-cyclical

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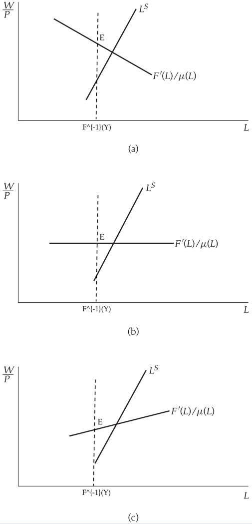
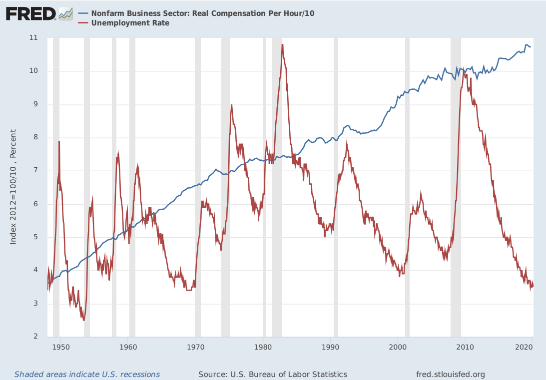
Case 4: Fixed W, imperfectly-competitive goods market



- Demand determines Y & L [vertical dotted L^D curve];
- equilibrium = intersection between (vertical) demand level and W/P curve;
- unemployment=horizontal difference between L^S and $\frac{W}{P}$ curve;
- Fluctuations are movements along the W/P curve, which can be increasing, decreasing or horizontal;

The cyclical behavior of the real wage

- On aggregate: average real wage acyclical or moderately procyclical.



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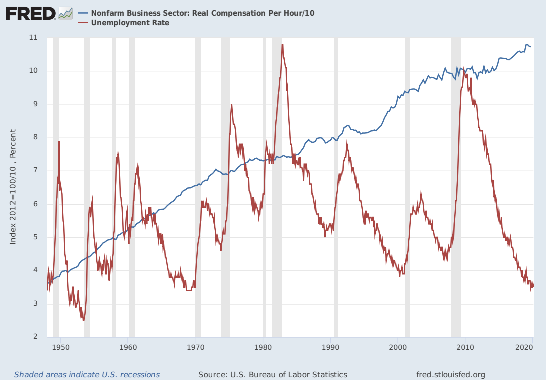


- mix of wage-effects & skill-composition effects;
- employment more cyclical for low-wage workers;
- % of low-skill jobs up in booms, down in downturns

  → wage cyclicality is underestimated;

The cyclical behavior of the real wage

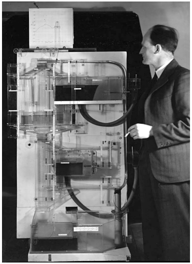
- Solon, Barsky and Parker (1994, QJE):

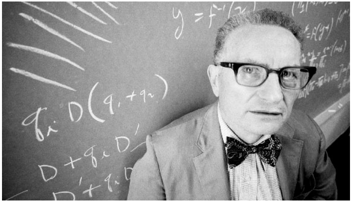
$$\Delta \ln w_{it} = \beta_1 + \beta_2 \Delta u_t + \beta_3 X_{it} + \epsilon_{it}$$

- includes only people employed both in $t-1$ and t;
- after netting-out skill-composition, real wages are twice as pro-cyclical as in the aggregate;
- Fluctuations as movements along a labor supply curve (Walrasian labor market) or a wage-curve (efficiency wages)?
- Implausibly high labor supply elasticity required to explain SBP results, so non-Walrasian explanations may be more appropriate.

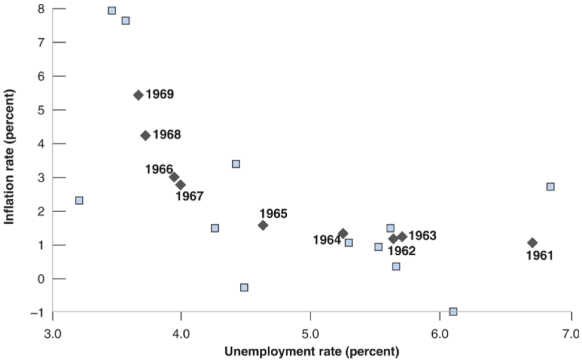
The Phillips Curve and its mutations

- *Phillips Curve*: low unemployment associated with high inflation.
- *1958*: A.W. Phillips uncovers negative correlation between inflation and unemployment in UK 1861-1957 data;
- *1960*: Samuelson & Solow replicate it on 1900-1960 US data;



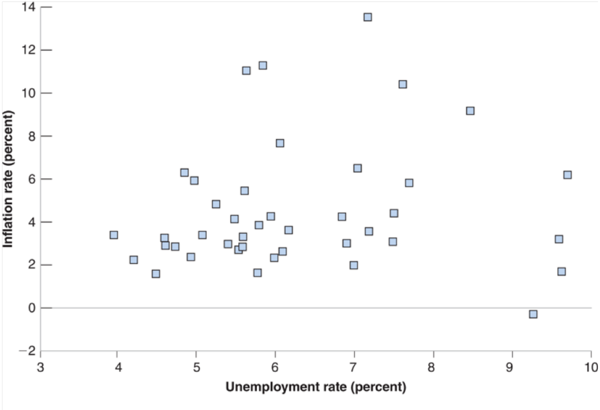


1948-1969: the ‘original’ Phillips Curve

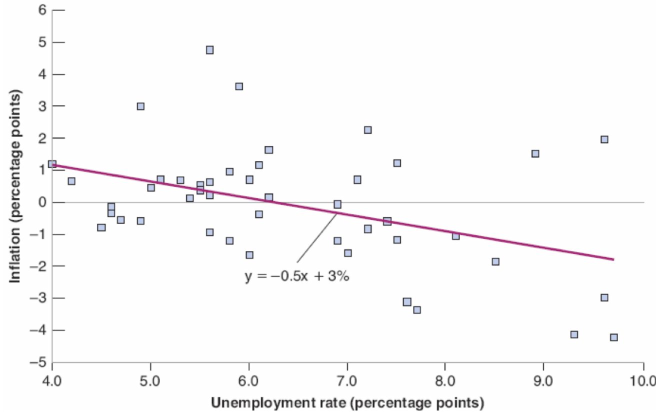


Source: Series UNRATE, CPIAUSCL Federal Reserve Economic Data (FRED) http://research.stlouisfed.org/fred2/

1970-2010: the disappearance of the ‘original’ PC



1970-2010: Accelerationist PC



The PC and its mutations

- Theoretical explanations of the PC focus on labor market dynamics;
- labor market models imply relations between π, $E(\pi)$ and u;
- specific form of the PC depends on how people form $E(\pi)$
  1. fixed expectations –> original PC
  2. adaptive expectations –> accelerationist PC
  3. rational expectations –> New-Keynesian PC

The PC and its mutations

- Central idea:

  lower $u_t \Rightarrow$ higher $W_t \Rightarrow$ increase in P_t & π_t.

- if it stops here, we have the 'original' PC

The PC and its mutations

- Central idea:

  lower $u_t \Rightarrow$ higher $W_t \Rightarrow$ increase in P_t & π_t.

- if it stops here, we have the 'original' PC

- BUT with adaptive expectations, inflationary spiral:

  lower $u_t \Rightarrow$ higher $W_t \Rightarrow$ increase in P_t & $\pi_t \Rightarrow$ increase in $E(\pi_{t+1}) \Rightarrow$ increase in $W_{t+1} \Rightarrow$ ...

- 'accelerationist' PC

The PC and its mutations

- Basic model:

$$Y_t = N_t$$

$$P_t = (1+m)W_t$$

$$\frac{W_t}{E(P_t)} = 1 - \beta u_t \quad \Rightarrow \quad W_t = E(P_t)(1 - \beta u_t)$$

  - Y = output;
  - N = employment;
  - W = nominal wage;
  - P = price of the good;
  - m = mark-up;
  - $u = 1 - \frac{L}{N}$ = unemployment rate;
- details in lecture notes ‘a (very) simplified new-synthesis model’

The PC and its mutations

- Combine price-setting & wage-setting:

$$P_t = E(P_t)(1+m)(1-\beta u_t)$$

- rewrite (approximately) in terms of π:

$$\pi_t = E(\pi_t) + m_t - \beta u_t$$

- What determines $E(\pi_t)$?

The PC and its mutations

- 'Generic' Phillips Curve:

$$\pi_t = E(\pi_t) + m_t - \beta u_t$$

- Assume fixed expectations

$$E(\pi) = \bar{\pi}$$

- Then we have

$$\pi_t = \alpha - \beta u_t \qquad \text{(with } \alpha = \bar{\pi} + m\text{)}$$

- *'original' Phillips curve*

The PC and its mutations

- 'Generic' Phillips Curve:

$$\pi_t = E(\pi_t) + m_t - \beta u_t$$

- Assume adaptive expectations

$$E(\pi) = \pi_{t-1}$$

- *'Accelerationist' PC*:

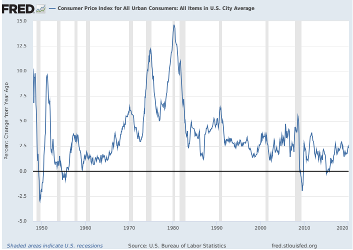
$$\pi_t - \pi_{t-1} = \alpha - \beta u_t$$

- Lower unemployment leads to higher *change* in the inflation rate (like in the 1970s).

An interpretation of the history of inflation in the US

1948-1969

- inflation not persistent;
- wage-setters assumed inflation would revert to mean $\bar{\pi}$;
- $E(\pi) \approx \bar{\pi} \Rightarrow$ Original PC.



after 1970

- inflation became persistent (oil shocks);
- wage-setters started taking persistence into account;
- $E(\pi_t) \approx \pi_{t-1} \Rightarrow$ accelerationist PC.

The PC & the NAIRU

- under 'accelerationist' PC, there is a unique sustainable unemployment rate (NAIRU);
  - see lecture notes 'a (very) simplified new-synthesis model';
  - any level of π can be sustained, but for π to fall you need $u > u^N$ for some time;

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A model of monopolistic competition

- Imperfect competition + nominal rigidities can produce real effects of nominal (monetary) shocks;
- Menu-costs as sources of nominal rigidity
  - an alternative: imperfect information (Lucas model).
- Plan:
  1. A model of monopolistic competition;
  2. then add menu costs;

Assumptions (1): product & labor markets:

- Continuum of differentiated goods $i \in [0, 1]$;
- monopolistic producers;
- production function:

$$Y_i = L_i$$

- goods are imperfect substitutes;
- Walrasian labor market;

Assumptions (2): households & preferences:

- Continuum of identical households $i \in [0, 1]$;
- Each household owns a (monopolistic) firm, gets w & π;

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Assumptions (2): households & preferences:

- Continuum of identical households $i \in [0, 1]$;
- Each household owns a (monopolistic) firm, gets w & π;
- Utility:

$$U = C - \frac{1}{\gamma} L^{\gamma} \quad \text{with} \quad C = \left[\int_{i=0}^{1} C_i^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}};$$

  - $\gamma > 1$ and $\eta > 1$;
  - Constant MU in overall C, but diminishing in individual C_i;
  - $C_i = \bar{C}$ for all $i \Rightarrow C = C_i = \bar{C}$;

Assumptions (3): Macroeconomy

- Closed economy without K and G

$$Y \equiv C$$

- Output equals aggregate demand

$$Y = \frac{M}{P}$$

  - $\frac{M}{P}$ = real money holdings = real expenditure (no savings and no utility from holding cash);
  - M = exogenous money supply = nominal expenditure.

Demand function for goods:

- Demand function for an individual good *i*:

$$C_i = \left(\frac{P_i}{P}\right)^{-\eta} C$$

- (derived from the utility function assuming a given budget, but don't worry about the technicalities of this derivation);

Labor supply curve L^S

- Households consume their income

$$CP = WL + R \quad \Rightarrow \quad C = \frac{WL + R}{P}$$

- So the maximization problem for choosing L is

$$\max_{L} \frac{WL + R}{P} - \frac{1}{\gamma} L^{\gamma}$$

- FOC:

$$\frac{W}{P} - L^{\gamma - 1} = 0 \quad \Rightarrow \quad L = \left(\frac{W}{P} \right)^{\frac{1}{\gamma - 1}}$$

Firm pricing behavior:

- Monopolistic mark-up pricing:

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} \frac{W}{P}$$

- Mark-up depends on the elasticity of demand η;
- P = average price level in the economy (CPI index);
- (simply derived from profit-maximization).

Equilibrium (1)

- By symmetry (w/ households/producers normalized to 1),

$$P = P_i; \quad L_i = L; \quad C_i = C = Y = L;$$

- From labor supply curve

$$L = \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}} \Rightarrow \frac{W}{P} = Y^{\gamma-1}$$

  - Higher W/P needed to elicit higher L, which is necessary to increase Y;
  - Pro-cyclical real wage.

Equilibrium (2)

- Combining labor supply curve & pricing

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} Y^{\gamma - 1}$$

  - higher Y makes each producer want higher P_i/P, to compensate higher W/P;

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- Combining labor supply curve & pricing

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} Y^{\gamma - 1}$$

  - higher Y makes each producer want higher P_i/P, to compensate higher W/P;

- Equilibrium output (by symmetry)

$$P = P_i \quad \Rightarrow \quad Y = \left(\frac{\eta - 1}{\eta}\right)^{\frac{1}{\gamma - 1}}$$

  - increasing in η (EoS) and decreasing in γ (disutility of labor);

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Equilibrium (2)

- Combining labor supply curve & pricing

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} Y^{\gamma - 1}$$

  - higher Y makes each producer want higher P_i/P, to compensate higher W/P;

- Equilibrium output (by symmetry)

$$P = P_i \quad \Rightarrow \quad Y = \left(\frac{\eta - 1}{\eta} \right)^{\frac{1}{\gamma - 1}}$$

  - increasing in η (EoS) and decreasing in γ (disutility of labor);

- Equilibrium price level

$$Y = \frac{M}{P} \quad \Rightarrow \quad P = \frac{M}{Y} = \frac{M}{\left(\frac{\eta - 1}{\eta} \right)^{\frac{1}{\gamma - 1}}}$$

Takeaway 1: Inefficiency

1. *Equilibrium output is below the socially-optimal level*
   - Social efficiency

$$\max_{\bar{L}} \bar{L} - (1/\gamma)\bar{L}^{\gamma} \quad \Rightarrow \quad \bar{L}^{opt} = 1 > \frac{\eta - 1}{\eta}^{\frac{1}{\gamma - 1}}$$

   - Market power causes inefficiency (gap decreasing in η);
   - $Y > Y^*$ good for welfare, recession very costly;

Takeaway 2: Aggregate demand externality

2. *Pricing decisions have (negative) externalities*

- Everyone would be better-off with $P < P^*$...
  - Demand for goods would increase with lower P
- ...but individually no one has an incentive to set $P_i < P^*$
- Coordination failure

Takeaway 3: Money is neutral

3. *Real output depends only on η and γ*

    - change in M affects only nominal prices and wages;
    - imperfect competition in goods' market alone not sufficient to get non-neutrality of money.

Adding frictions: menu costs & real rigidities

- *Menu costs*
  - Printing new catalogs/menus;
  - marketing costs;
  - cost of disseminating information;
  - risk of alienating customers;
  - ...
- Consider a flex-price imperfect-competition equilibrium;
- a demand shock changes the equilibrium price;
- *under what conditions is the gain from adjusting lower than a plausible (ie small) menu cost, conditional on other firms not adjusting either?*
- Real rigidities help make the cost of non-adjustment small. ‘

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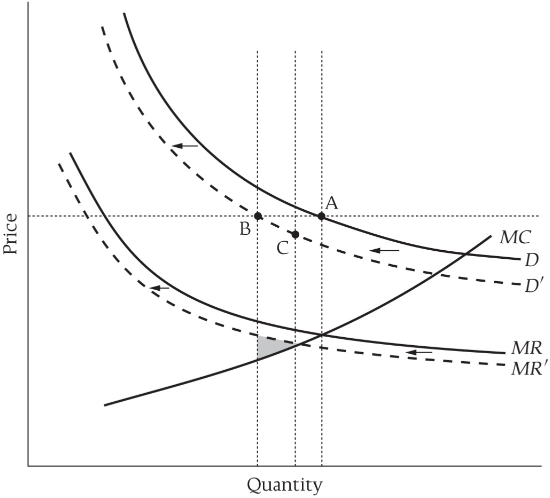
Representative firm's price-setting problem



- A=initial equilibrium;
- Negative demand shock;
- No adjustment –> B;
- Adjustment –> C;
- Shaded triangle=profit loss;
- *What can make this triangle small enough to be outweighted by modest menu costs?*

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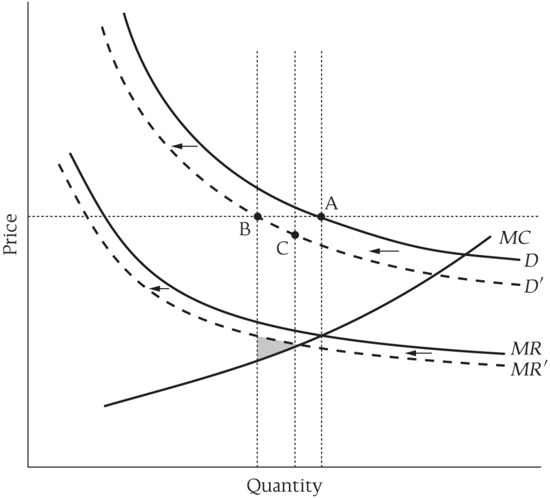
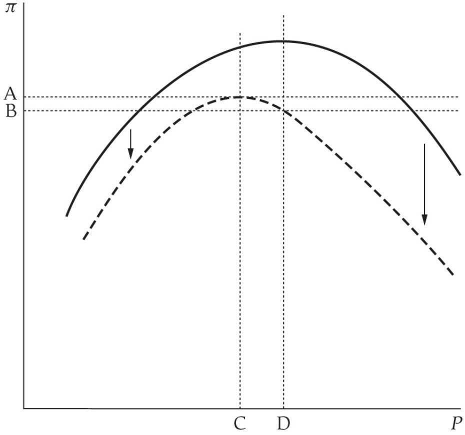


- Assume Walrasian labor market
- and plausible (low) L^S elasticity;
- large fall in W/P would occur, to restore full employment;
- MC curve would shift down substantially;
- this would make the triangle large;
- ‘don’t adjust’ unlikely to be a Nash Equilibrium.
- see textbook (pp.277-278) for a ‘quantitative example’;

What may make $(\pi_{ADJ} - \pi_{FIXED})$ small?

Representative firm's profit function



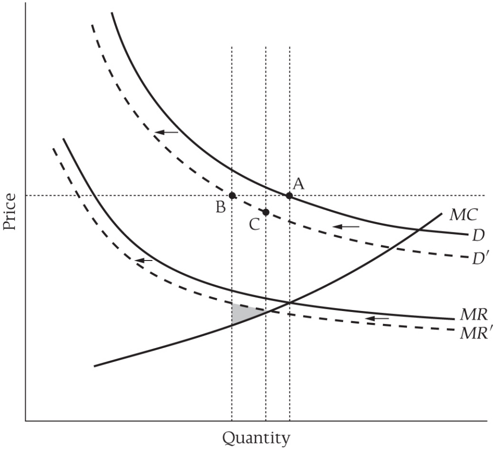
- $\pi = \pi(P)$;
- $AB = \pi_{ADJ} - \pi_{FIXED}$
- AB depends on CD and the slope of $\pi(p)$;
- *real rigidity*
  - small CD
  - small ϕ in

$$p^{\star} - p = c + \phi y;$$

- *insensitivity of $\pi(P)$*
  - smaller π loss for a given CD;
  - flat $\pi(P)$ curve;

What may make $(\pi_{ADJ} - \pi_{FIXED})$ small?

Representative firm's price-setting



- Little response of MC to Y
  - real rigidity: downward shift in MC curve would move C to the right;
- Flat MC curve
  - real rigidity: moves C left;
  - insensitivity: reduces shaded area for given C.
- Large effect of Y on MR
  - real rigidity: moves C left
- Steep MR curve [for a given leftward shift]
  - real rigidity

Possible sources of real rigidity and/or insensitivity of $\pi(P)$

- *Sticky wages*: MC curve both less steep *and* less responsive to fall in demand;
- *Countercyclical mark-up:* MR curve both more steep *and* more responsive to fall in demand;
- (see quantitative example in the textbook (pp.285-286).

Dynamic models of price adjustment

- Time-dependent vs state-dependent;
- Baseline time-dependent models
  - Prices reviewed on a multi-period basis;
  - *Fischer*: prices pre-determined but not fixed;
  - *Taylor*: prices pre-determined & fixed;
  - *Calvo*: random opportunities to change (fixed) prices;

Dynamic models of price adjustment

- Time-dependent vs state-dependent;
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  - *Fischer*: prices pre-determined but not fixed;
  - *Taylor*: prices pre-determined & fixed;
  - *Calvo*: random opportunities to change (fixed) prices;
- *Simplification*: in the original models it is wages that are sticky, here directly the p_i's.
- *Takeaway:* gradual adjustment of P -> temporary real effects of M shocks.

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General framework (1/3)

- A dynamic version of the monopolistic competition model
- Production function
$$Y_t = L_t$$
- Closed economy without government
$$C_t = Y_t = L_t$$
- Exogenous nominal expenditure (aggregate demand)
$$M_t = Y_t P_t$$
- Labor supply curve
$$\frac{W_t}{P_t} = B Y_t^{\theta+\gamma-1}$$
- Monopolistic pricing
$$\frac{P_t^\star}{P_t} = \frac{\eta}{\eta-1}\frac{W_t}{P_t}$$

General framework (2/3)

Time-dependent price-adjustment:

- Prices set on a multi-period basis;
- p_i set at time 0 has probability $q_t \geq 0$ of remaining in effect at time $t > 0$;
- firm sets p_i as a weighted average of expected future $p_t^{\star}$'s:

$$p_i = \sum_{t=0}^{\infty} \tilde{\omega}_t E[p_t^{\star}] \quad \text{with} \quad \tilde{\omega}_t \equiv \frac{\beta^t q_t}{\sum_{\tau=0}^{\infty} \beta^{\tau} q_{\tau}}$$

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General framework (3/3)

- Profit-maximizing price is a mark-up over the wage

$$\frac{P_t^\star}{P_t} = \frac{\eta}{\eta - 1}\frac{W_t}{P_t} \quad \Rightarrow \quad p_t^\star = \ln\left[\frac{\eta}{\eta-1}\right] + w_t$$

- Substitute in the (log of the) labor supply curve

$$w_t = p_t + \ln B + (\theta + \gamma - 1)y_t \quad \Rightarrow \quad p^\star = p + \ln\frac{\eta}{\eta+1} + \ln B + (\theta + \gamma - 1)y_t$$

- Given that $m = y + p$, and assuming for simplicity $\ln\frac{\eta}{\eta-1} + \ln B = 0$,

$$p_t^\star = \phi m_t + (1-\phi)p_t \quad \text{with } \phi = (\theta + \gamma - 1)$$

- optimal 'sticky' price to set at time 0:

$$p_i = \sum_{t=0}^{\infty} \tilde{\omega}_t E_0[\phi m_t + (1-\phi)p_t]$$

Fischer model

- *Pre-determined:* Each firm sets p_i every other period for the next two periods
  - in period 0 set prices for 1 & 2;
  - in period 2 set prices for 3 & 4.
- *Flexible*: you can set two different prices for the two periods.
- *Staggered*: In any period, 1/2 of the firms are setting prices.

Fischer model

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  - in period 0 set prices for 1 & 2;
  - in period 2 set prices for 3 & 4.
- *Flexible*: you can set two different prices for the two periods.
- *Staggered*: In any period, 1/2 of the firms are setting prices.
- Average price level at any t:

$$p_t = \frac{1}{2}(p_t^1 + p_t^2) \qquad [p_t^i = \text{ price for period } t \text{ set in } t-i]$$

- Takeaways:
  - Prices cannot (fully) react to m_t shocks not known yet in $t-2$;
  - m_t shocks not known at $t-2$ affect real output;
  - m_t changes already known at m_{t-2} have no effect on real output;

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Fischer model

- $p_t = \frac{1}{2}(p_t^1 + p_t^2)$
- Firms set prices equal to expected optimal prices:

$$p_t^1 = E_{t-1}(p_t^\star) = E_{t-1}[\phi m_t + (1-\phi)p_t]$$
$$= \phi E_{t-1}(m_t) + (1-\phi)\frac{1}{2}(p_t^1 + p_t^2);$$

..and

$$p_t^2 = E_{t-2}(p_t^\star) = E_{t-2}[\phi m_t + (1-\phi)p_t]$$
$$= \phi E_{t-2}(m_t) + (1-\phi)\frac{1}{2}(E_{t-2}(p_t^1) + p_t^2);$$

- Solving the system:

$$p_t^1 = E_{t-2}m_t + \frac{2\phi}{1+\phi}[E_{t-1}(m_t) - E_{t-2}(m_t)];$$
$$p_t^2 = E_{t-2}(m_t).$$

Fischer model

- Equilibrium price level

$$p = \frac{p^1 + p^2}{2} \quad \rightarrow \quad p_t = E_{t-2}(m_t) + \frac{\phi}{1+\phi}[E_{t-1}(m_t) - E_{t-2}(m_t)]$$

- Equilibrium output:

$$y = m - p \quad \rightarrow \quad y_t = \frac{1}{1+\phi}\underbrace{[E_{t-1}(m_t) - E_{t-2}(m_t)]}_{\swarrow} + \underbrace{[m_t - E_{t-1}(m_t)]}_{\downarrow}$$

surprise about m_t revealed in $t-1$

surprise about m_t revealed in t

- lower ϕ (greater real rigidity) $\rightarrow$ higher importance of m_t surprise revealed in $t-1$.

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Taylor model – Assumptions

- Prices predetermined & fixed;
- price set in t holds in t and $t+1$;
- staggered: in any period, 1/2 of firms set prices.
- money supply is a random walk:

$$m_t = m_{t-1} + u_t \quad \Rightarrow \quad E_{t-1}(m_t) = m_{t-1}$$

- Aggregate price level:

$$p_t = \frac{1}{2}(x_t + x_{t-1})$$

(x_i=price set by firms which set price in time i);

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Taylor model – Finding x_t

- Recall

$$p_i = \sum_{t=0}^{\infty} \omega_t E[P_t^\star] \quad \& \quad p_t^\star = \phi m_t + (1-\phi) p_t$$

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Taylor model – Finding x_t

- Recall

$$p_i = \sum_{t=0}^{\infty} \omega_t E[P_t^\star] \quad \& \quad p_t^\star = \phi m_t + (1-\phi) p_t$$

- So

$$x_t = \frac{1}{2}[p_t^\star + E_t(p_{t+1}^\star)] =$$

$$= \frac{1}{2}\{[\phi m_t + (1-\phi)p_t] + [\phi E_t(m_{t+1}) + (1-\phi)E_t(p_{t+1})]\}$$

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$$= \frac{1}{2}\{[\phi m_t + (1-\phi)p_t] + [\phi E_t(m_{t+1}) + (1-\phi)E_t(p_{t+1})]\}$$

- Use $p_t = \frac{1}{2}(x_t + x_{t-1})$ and $E_t(m_{t+1}) = m_t$ and solve for x_t:

$$\Downarrow$$

$$x_t = A[x_{t-1} + E_t(x_{t+1})] + (1-2A)m_t \quad \text{with} \quad A = \frac{1}{2}\frac{1-\phi}{1+\phi}$$

Taylor model – Finding x_t

$$x_t = A[x_{t-1} + E_t(x_{t+1})] + (1 - 2A)m_t \quad \text{with} \quad A = \frac{1}{2}\frac{1-\phi}{1+\phi}$$

Firms adjusting at t set x_t as a function of:

1. money supply (m_t);
2. current prices of other firms (x_{t-1});
3. expectation of what prices other firms will set next period ($E_t(x_{t+1})$)

Taylor model – Finding x_t

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To pin down x_t, we need to get rid of $E_t(x_{t+1})$:

- $E_t(x_{t+1})$ must be based on stuff known at time t: m_t and x_{t-1};
- so x_t ultimately a function of m_t and x_{t-1} only;

Taylor model – Finding x_t

- use method of undetermined coefficients;
- educated guess:

$$x_t = \mu + \lambda x_{t-1} + \nu m_t$$

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Taylor model – Finding x_t

- use method of undetermined coefficients;
- educated guess:
$$x_t = \mu + \lambda x_{t-1} + \nu m_t$$
- flex-price equilibrium: $p_t^* = m_t$;
- so a situation in which $x_t = x_{t-1} = m_t$ must be an equilibrium;
- and so the following must hold
$$\mu + \lambda m_t + \nu m_t = m_t \quad \Rightarrow \quad \lambda + \nu = 1 \;\&\; \mu = 0$$
- These restrictions on the parameters imply
$$x_t = \lambda x_{t-1} + (1 - \lambda) m_t \tag{1}$$
- Now we need another restriction from the model, to determine λ.

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Taylor model – Finding x_t

- The needed additional restriction comes from the model equation

$$x_t = A[x_{t-1} + E_t(x_{t+1})] + (1 - 2A)m_t$$

(derived earlier)

- Combining it with

$$x_t = \lambda x_{t-1} + (1 - \lambda)m_t \tag{2}$$

we get two possible solutions:

$$\lambda_1 = \frac{1 - \sqrt{\phi}}{1 + \sqrt{\phi}} \qquad \& \qquad \lambda_2 = \frac{1 + \sqrt{\phi}}{1 - \sqrt{\phi}} \tag{3}$$

- But the second $\lambda_2 > 1$ would imply instability ($|\lambda| > 1$);
- assume stability and focus on λ_1 only.

Taylor model: implications for output dynamics

- Real output:

$$y_t = m_t - p_t$$

$$= m_t - \frac{x_{t-1} + x_t}{2}$$

$$= m_t - \frac{1}{2}\{\lambda x_{t-2} + (1-\lambda)m_{t-1}] + [\lambda x_{t-1} + (1-\lambda)m_t]\}$$

- Use $m_t = m_{t-1} + u_t$ & $(x_{t-1} + x_{t-2})/2 = p_{t-1}$ to rewrite as:

$$y_t = \lambda y_{t-1} + \frac{1-\lambda}{2} u_t$$

Taylor model: takeaways

$$y_t = \lambda y_{t-1} + \frac{1-\lambda}{2} u_t$$

- Persistent (if $\lambda > 0$) but temporary ($\lambda < 1$) real effects of m shocks;
- $\lambda > 0$ is necessary for persistence and requires $\phi < 1$, which implies that $p^\star$ is increasing in p;
- Incomplete nominal adjustment produces real effects of monetary shocks.
- Effect can last more than two periods because real rigidity (low ϕ) produces persistence (as in the Fischer model).

Calvo model - overview

- Prices predetermined & fixed;
- opportunities to change prices arrive stochastically;
  - n. of periods a price will be in effect is random;
- *Poisson process*: same probability of price adjustment in every period;

Calvo model - overview

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- opportunities to change prices arrive stochastically;
  - n. of periods a price will be in effect is random;
- *Poisson process*: same probability of price adjustment in every period;

Takeaways:

- As in Taylor & Fischer, gradual adjustment of the price level;
- it implies the *NK Phillips curve*.

Calvo model - deriving π

- Each period share α of firms, randomly chosen, adjusts prices

$$\textit{aggregate price level:} \quad p_t = \alpha x_t + (1-\alpha)p_{t-1}$$

$$\textit{inflation:} \quad \pi_t = p_t - p_{t-1} = \alpha(x_t - p_{t-1})$$

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Calvo model - deriving π

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- optimal 'sticky' prices with discounting:

$$x_t = \sum_{j=0}^{\infty} \tilde{\omega}_j E(p^\star_{t+j}) \quad \text{with} \quad \tilde{\omega}_j = \frac{\beta^j q_j}{\sum_{k=0}^{\infty} \beta^\kappa q_k}$$

- Poisson process implies $q_j = (1-\alpha)^j$
- $\rightarrow \sum_{k=0}^{\infty} \beta^\kappa q_k = \sum_{k=0}^{\infty} \beta^k (1-\alpha)^k = \frac{1}{1-\beta(1-\alpha)}$

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Calvo model - deriving π

- ...plugging in:

$$x_t = [1-\beta(1-\alpha)]\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j E_t p^\star_{t+j}$$

- Rewrite as:

$$x_t = [1-\beta(1-\alpha)]E_t(p^\star_t)+\beta(1-\alpha)[1-\beta(1-\alpha)]\left[\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j E_t p^\star_{t+1+j}\right]$$

$$= [1-\beta(1-\alpha)]p^\star_t + \beta(1-\alpha)E_t x_{t+1}$$

[took $p^\star_t$ out of the summation, and used the fact that the subsequent terms are equal to discounted value of x_{t+1}.]

Calvo model - deriving π

$$x_t = [1-\beta(1-\alpha)]p_t^{\star} + \beta(1-\alpha)E_t x_{t+1}$$

- Express in terms of π_t, using $\pi_t = \alpha(x_t - p_{t-1})$ and $p^{\star} = \phi m_t + (1-\phi)p_t$

$$\pi_t = ky_t + \beta E_t \pi_{t+1} \quad \text{with} \quad k = \frac{\alpha[1-(1-\alpha)\beta]\phi}{1-\alpha}$$

- New-Keynesian Phillips Curve
- Inflation depends on expected inflation & output (as in all PCs);
- Difference: it is $E_t\pi_{t+1}$ that matters here: expectation of *future* inflation.

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4 Phillips Curves and their implications

1. *Old-Keynesian PC*: $\pi_t = \alpha + \lambda y_t$
   - *output-inflation trade-off*: disinflation requires permanently lower y;
2. *Accelerationist PC*: $\pi_t = \pi_{t-1} + \lambda(y_t - y_t^\star)$
   - painful disinflation: requires $y < y^\star$ for some time (*inflation inertia*);
3. *Lucas 'supply curve'*: $\pi_t = E_{t-1}\pi_t + \lambda(y_t - y_t^\star)$
   - costless disinflation: just alter $E_{t-1}\pi_t$ with no output implications;
4. *New-Keynesian PC*: $\pi_t = k y_t + \beta E_t \pi_{t+1}$
   - expansionary disinflation: $E_t(\pi_{t+1})$ down $\rightarrow$ y_t up.

State-dependent pricing

- Fixed cost of adjusting prices;
- share of firms that adjust depends on $\pi_{ADJ} - \pi_{FIX}$;
- $\pi_{ADJ} - \pi_{FIX}$ depends on economic conditions;
- *faster adjustment* of $p \rightarrow$ shorter-lived real effects of m shocks (relative to time-dependent models);

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- $\pi_{ADJ} - \pi_{FIX}$ depends on economic conditions;
- *faster adjustment* of p → shorter-lived real effects of m shocks (relative to time-dependent models);
- *frequency effect*: the larger the m shock, the higher the number of firms which adjust (Caplin-Spulber model)
- *Selection effect*: ‘adjusters’ have higher ($\pi_{ADJ} - \pi_{FIX}$), so they make larger price changes (Danziger-Golosov-Lucas: firm-specific shocks, heterogeneity)
- (Will not do the models: just know they exist and the general ideas.)

The canonical New Keynesian 3-equations model

NK IS curve: $y_t = E_t[y_{t+1}] - \frac{1}{\theta} r_t + u_t^{IS}$ with $\theta > 0$

NK PC: $\pi_t = \beta E_t[\pi_{t+1}] + k y_t + u_t^{\pi}$ with $0 < \beta < 1, k > 0$

MP rule: $r_t = \phi_\pi E_t[\pi_{t+1}] + \phi_y E_t[y_{t+1}] + u_t^{MP}$ with $\phi_\pi > 0, \phi_y \geq 0$

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- no constants: deviations from steady-state, normalized to 0
- shocks structure:

$$u_t^{IS} = \rho_{IS} u_{t-1}^{IS} + e_t^{IS}, \qquad -1 < \rho_{IS} < 1$$
$$u_t^{\pi} = \rho_{\pi} u_{t-1}^{\pi} + e_t^{\pi}, \qquad -1 < \rho_{\pi} < 1$$
$$u_t^{MP} = \rho_{MP} u_{t-1}^{MP} + e_t^{MP}, \qquad -1 < \rho_{MP} < 1$$

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Solving the 3-equations model

- Express the model in terms only of shocks and expectations;
- plug the MP rule into the IS curve:

$$y_t = -\frac{\phi_\pi}{\theta} E_t[\pi_{t+1}] + \left(1 - \frac{\phi_y}{\theta}\right) E_t[y_{t+1}] + u_t^{IS} - \frac{1}{\theta} u_t^{MP}$$

- plug the equation above into the NK PC:

$$\pi_t = \left(\beta - \frac{\phi_\pi k}{\theta}\right) E_t[\pi_{t+1}] + \left(1 - \frac{\phi_y}{\theta}\right) k E_t[y_{t+1}] + k u_t^{IS} + u_t^{\pi} - \frac{k}{\theta} u_t^{MP}$$

Special case: no serial correlation in shocks

- Assume $\rho_{IS} = \rho_{\pi} = \rho_{MP} = 0$;
- this implies $E_t[y_{t+1}] = E_t[\pi_{t+1}] = 0$;
- So we have:

$$y_t = u_t^{IS} - \frac{1}{\theta} u_t^{MP}$$

$$\pi_t = k u_t^{IS} + u_t^{\pi} - \frac{k}{\theta} u_t^{MP}$$

$$r_t = u_t^{MP}$$

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$$r_t = u_t^{MP}$$

- shows effect of demand, monetary policy and inflation shocks;
- *no internal propagation mechanisms*: without assuming serial correlation in shocks, we don't get any persistence (just like RBC).

The general case

- Method of undetermined coefficients;
- Educated guess:

$$y_t = a_{IS}u_t^{IS} + a_\pi u_t^\pi + a_{MP}u_t^{MP}$$

$$\pi_t = b_{IS}u_t^{IS} + b_\pi u_t^\pi + b_{MP}u_t^{MP}$$

- Plug these into the y_t and π_t functions derived earlier;
- solve the resulting system of equations to get the a's and b's;
- we will skip the algebra and directly discuss implications for the effects of shocks;

Implications of the general case

- Assumptions:
  - A period is a quarter;
  - $\theta = 1$ in utility function;
  - $k = 0.172$ & $\beta = 0.99$ in PC;
  - $\phi_\pi = 0.5$ & $\phi_y = 0.125$ in MP;
  - $\rho = 0.5$ for all shocks.

- Effect of *MP* shock:
  - $y_t = -1.54 u_t^{MP}$;
  - $\pi_t = -0.53 u_t^{MP}$;
  - $r_t = 0.77 u_t^{MP}$

- Effect of *IS* shock:
  - $y_t = 1.54 u_t^{IS}$;
  - $\pi_t = 0.53 u_t^{IS}$;
  - $r_t = 0.23 u_t^{IS}$.

- Effect of π shock:
  - $y_t = -0.76 u_t^{\pi}$;
  - $\pi_t = 1.72 u_t^{\pi}$;
  - $r_t = 0.38 u_t^{\pi}$.

Application:

Monetary policy rules and macroeconomic stability: Evidence and some theory

by Clarida, Gali and Gertler (2000)

Open issues & extensions

- Standard NK DSGE model produces very weird predictions about the effect of 'forward guidance';
- the implications of the NK PC for the effect of disinflation are also quite embarrassing;

Open issues & extensions

- Standard NK DSGE model produces very weird predictions about the effect of ‘forward guidance’;
- the implications of the NK PC for the effect of disinflation are also quite embarrassing;
- popular extension: some source of π inertia (like indexation);
- include (exogenous) government spending and taxes;
- open economy extensions;
- introduce (a share of) hand-to-mouth consumers
- include investment, possibly with adjustment costs;
- credit market imperfections: financial sector intermediates between saving and investment, with possible frictions;

DSGE models: optimistic vs pessimistic views

The optimistic view:

- DSGE describe reasonably well the behavior of macro aggregates...
- ... and are micro-founded so their parameters are plausibly policy-invariant;
- Extensions are making them more realistic, and technology allows analysis of ever more sophisticated versions (including HANK);
- we should all be working on further improving DSGE models.

Pessimistic view:

- The baseline model actually produces embarrassing predictions...
- ...and only large ad-hoc modifications just designed to make the models' implications more reasonable attenuate that;
- we should all be working on seeking radically different alternatives (back to old-school Keynesian? agent-based models? no all-encompassing model at all? a type of model that has not been conceived yet?).