A (very) simplified New Keynesian model*

Daniele Girardi†

1 Introduction

These notes present a simplified New Keynesian model. The model is simpler to derive than the canonical New Keynesian DSGE model and, unlike the latter, it is not completely microfounded and does not feature rational expectations. However, it does a good job in conveying the New Keynesian view of the economy. The model draws on Romer (2000), Carlin and Soskice (2005), and Blanchard and Johnson (2017).

I think it is fair to say that this model reflects the way in which mainstream policy-oriented macroeconomists (think of Olivier Blanchard, for example) think about the economy and economic policy, when it comes to applying macroeconomic insights to the real world. In a sense, we can see the more complex New-Keynesian DSGE models as attempts to derive implications broadly similar to those of this model from more complicated micro-founded rational-expectations models, which are seen as more rigorous in academic research.

The model is formed of three equations: the IS curve which determines short-run output; the (accelerationist) Phillips curve which determines inflation dynamics; and a Central Bank reaction function which describes the rule followed by the Central Banker in setting the interest rate. We will first derive each of the three equations, and then discuss qualitatively the economic mechanics that they generate.

*Lecture notes for 7SSPN402, Advanced Macroeconomics.

†Lecturer, Department of Political Economy, King's College London.

2 The demand side: IS curve

The model starts from a simple old-Keynesian determination of equilibrium in the product market. We assume a closed one-good economy. Aggregate demand Z is the sum of private consumption, housing investment and government spending: $Z_t \equiv C_t + I_t + G_t$.[1]

The behavioral equations that determine consumption, investment and government spending are as follows. 'Hand-to-mouth' consumers spend a constant fraction of their after-tax income. We denote output as Y and the tax rate as τ. So aggregate consumption is $C_t = c_0 + c_1(1 - \tau_t)Y_t$ (the consumption function).[2] Housing investment is negatively affected by the interest rate, and reacts with a lag to it: $I_t = a_0 - a_1 r_{t-1}$ (the investment function). Government spending G and the tax rate τ are exogenously set by the government, therefore taken as fixed: $G_t = G$ and $\tau_t = \tau$.

Imposing the equilibrium condition that output equals aggregate demand ($Y_t = Z_t$), we can derive the equilibrium level of output:

$$Y_t = \frac{1}{1 - c_1(1-\tau)}[c_0 + (a_0 - a_1 r_{t-1}) + G] = A - a r_{t-1} \tag{1}$$

Where $A = \frac{c_0 + a_0 + G}{1 - c_1(1-\tau)}$ and $a = \frac{a_1}{1 - c_1(1-\tau)}$.

3 The supply side: PC curve

The second building block of the model is the Phillips curve, which determines the rate of inflation. It is derived from a simple model of the labor market and of firms' pricing decisions.

Wage-setting The real wage that emerges from the wage-setting process is a negative function of the unemployment rate u. This can be seen as the outcome of either a

[1] I refer to housing investment for two reasons. First, in this model there is no business capital (labor is the only production input, as we will see). Second, the empirical evidence suggests that aggregate business investment is not much sensitive to the interest rate, while housing investment is.

[2] In the contemporary Macroeconomics jargon, 'hand-to-mouth' consumers are consumers who, at each point in time, just spend a fraction of their current income, independently of their expectations about the future. In contrast, the consumer of the typical contemporary macroeconomic model is much more sophisticated: in the initial period, she makes a correct prediction about the total income that she is going to earn in her entire lifetime, and then determines how to distribute her consumption across all future periods.

bargaining model or an efficiency-wage model of the labor market.[3]

When setting wages for period t, workers and firms do not observe the realized price level P yet, therefore they use their expectation P_t^e. So they set an *expected* real wage W_t/P_t^e.

Formally, we have $\frac{W_t}{P_t^e} = f(u_t)$ with $f'(u_t) < 0$. This implies the that nominal wage is $W_t = f(u_t)P_t^e$.

Let us assume a specific functional form for $f()$, and specifically $f(u_t) = 1 - \beta u_t$.

To sum up, the nominal wage that emerges from the wage-setting process is

$$W_t = (1 - \beta u_t)P_t^e \tag{2}$$

Price-setting Assume that firms produce output using labor as the only input. So the production function is $Y_t = bN_t$ where N is employment and b is a parameter. To further simplify things, assume that $b = 1$, meaning that each worker produces one unit of output per period. The cost of producing one unit of output is therefore the cost of employing one worker for one period – that is, the wage W.

Imperfectly competitive firms set the price of output according to a mark-up m over the production cost This can be seen as the outcome of a Cournot model of imperfect competition. So we have $P_t = (1 + m)W_t$.

The equilibrium unemployment rate Together, the wage-setting and the price-setting equations determine the unemployment rate that will tend to prevail in the medium-run – the equilibrium (or 'natural') rate of unemployment. Equilibrium is defined here as a situation in with $P = P^e$, so that agents' expectations are correct and do not need to be revised. So the wage-setting relation becomes $\frac{W_t}{P_t} = 1 - \beta u_t$. Rearranging the price-setting equation ($P_t = (1 + m)W_t$), we obtain that the real wage must be equal to $\frac{W_t}{P_t} = \frac{1}{1+m}$. The equilibrium rate of unemployment ($u^\star$) is the one that satisfies both the wage-setting and the price-setting relations: $1 - \beta u^\star = \frac{1}{1+m}$. This

[3] In a bargaining model, higher unemployment undermines workers' bargaining power. In an efficiency-wage model, employers want to provide a higher wage when unemployment is lower to incentivise workers to put effort. Specifically, in an efficiency-wage model, a higher cost of job loss shifts up the workers' best response function: they will exert more effort for each given wage. This lowers the wage that employers must offer in order to obtain the desired intensity of effort from workers.

implies $u^\star = \frac{m}{\beta(1+m)}$.

Phillips curve Substituting the wage-setting equation for W in the price-setting equation, we get $P_t = P_t^e(1+m)(1-\beta u_t)$. In terms of the *inflation* rate (denoted by π), this implies (approximately) the following:[4] $\pi_t = \pi_t^e + m - \beta u_t$. Also this equation implicitly defines the equilibrium (or natural) unemployment rate. To find this, we set $\pi = \pi^e$, which implies $u^\star = \frac{m}{\beta}$.[5]

Rearranging the inflation equation as $\pi_t - \pi^e = \beta[u_t - \frac{m}{\beta}]$, and then substituting the formula for $u^\star$ into the equation, we obtain $\pi - \pi^e = -\beta(u_t - u^\star)$. Now assume that agents have adaptive expectations – that is, they form their inflation expectations based on past observed inflation. Specifically, assume $\pi^e = \pi_{t-1}$. Therefore the inflation rate

[4] Here is the demonstration:

- First rearrange the price level equation as follows:

$$P_t = P_t^e(1+m)(1-\beta u_t) \rightarrow \frac{P_t - P_{t-1} + P_{t-1}}{P_{t-1}} = \frac{P_t^e - P_{t-1} + P_{t-1}}{P_{t-1}}(1+m)(1-\beta u_t)$$

$$\rightarrow 1 + \pi_t = (1+\pi_t^e)(1+m)(1-\beta u_t) \rightarrow \frac{1+\pi_t}{(1+\pi_t^e)(1+m)} = 1 - \beta u_t$$

- Now consider the denominator of the left side $(1+\pi_t^e)(1+m) = (1+m+\pi_t^e+\pi_t^e m)$. If both m and π_t^e are small, as they generally are, the interaction term $\pi_t^e m$ will be sufficiently small that we can ignore it, and approximate this as $1+m+\pi_t^e$. We thus can write

$$\frac{1+\pi_t}{1+m+\pi_t^e} \approx 1 - \beta u_t$$

- Consider the expression $(1+\pi_t-m-\pi_t^e)(1+m+\pi^e) = (1+m+\pi_t^e+\pi_t+m\pi_t+\pi_t\pi_t^e-m-m^2-m\pi_t^e-\pi_t^e-m\pi_t^e-(\pi_i^e)^2)$. Again, we can ignore all the interaction terms and the squared terms, because they will generally be very small, and write $(1+\pi_t-m-\pi_t^e)(1+m+\pi^e) \approx (1+\pi_t)$. Dividing both sides by $(1+m+\pi^e)$, we can now write

$$\frac{1+\pi_t}{1+m+\pi_t^e} \approx 1+\pi_t-m-\pi_t^e$$

- Finally, we can combine the approximations derived in the previous two steps in order to conclude that

$$\pi_t \approx \pi_t^e + m - \beta u_t$$

[5] You may note an inconsistency here: the equilibrium unemployment rate derived from the inflation formula ($u^\star = \frac{m}{\beta}$) is not exactly the same as the one derived from the price level formula ($u^\star = \frac{m}{\beta(1+m)}$). It is actually not an inconsistency, but an approximation. Because the inflation formula is actually an approximation (see footnote 4 above), also the resulting formula for the natural unemployment rate is an approximation, in contrast with the one derived from the price level formula, which is exact. You can try plugging in plausible values of β and m to convince yourself that the discrepancy is not very large for reasonable values of the equilibrium unemployment rate (for example with a 10% mark-up ($m = 0.10$) and $\beta = 2$, $u^\star$ is 5% from the approximated formula, and 4.8% from the exact one).

is determined by the following 'accelerationist' Phillips curve:[6]

$$\pi_t = \pi_{t-1} - \beta(u_t - u^\star)$$

The last step is to express the Phillips curve in terms of output instead of unemployment. To do that, note that the unemployment rate is equal to $u = \frac{U}{L} = \frac{L-N}{L} = 1 - \frac{N}{L}$ (where U is unemployment and L is the labor force). Rearranging, we obtain that employment is equal to $N = L(1-u)$. Substituting this into the production function, we have $Y_t = L_t(1-u_t)$.

Now we can define equilibrium (or potential) output as the output level consistent with equilibrium unemployment: $Y^\star = L(1-u^\star)$. The output gap – the difference between actual and potential output – is therefore $Y_t - Y^\star = L[(1-u_t) - (1-u^\star)] = -L(u_t - u^\star)$. We can now write the Phillips Curve in terms of the output gap:

$$\pi_t = \pi_{t-1} + \alpha(Y_t - Y^\star) \tag{3}$$

where $\alpha = \frac{\beta}{L}$.

4 The Central Bank's reaction function

The task of the Central Bank is to set the real interest rate.[7] In doing that, the Central Bank is guided by the target it has established for the inflation rate (call it π^T), but cares also about output. It wants the inflation rate to be as close as possible to the target, and output to be as close as possible to potential. In particular, the Central Bank's preferences are given by the following loss function: $\ell = (Y_t - Y^\star)^2 + \gamma(\pi_t - \pi^T)^2$. The smaller the loss ℓ, the happier the Central Banker.

The Central Bank is aware of the Philips curve which governs inflation dynamics (equation 3) and the IS curve which determines output (equation 1). So at each point

[6]It is 'accelerationist' because it is the change (not the level) of the inflation rate that depends on the unemployment rate. Therefore if unemployment stays constant but under the equilibrium rate for some time, the inflation rate would increase period after period: we would observe not only high, but *accelerating* inflation.

[7]We are assuming for simplicity that the Central Bank sets directly the real rate, rather than (more realistically) setting the nominal rate given inflation expectations.

in time it sets the interest rate which minimizes the loss function, subject to the Phillips curve and the IS curve.

Let us first find the level of output that the Central Bank would like to see realized. Then we will identify the interest rate that it must set, in order to reach that particular output level.

By substituting equation 3 (the Phillips curve) into the loss function, differentiating with respect to Y_t, and setting the derivative equal to zero, we have

$$\frac{\delta \ell}{\delta Y_t} = 2(Y_t - Y^\star) + 2\alpha\gamma[\pi_{t-1} + \alpha(Y_t - Y^\star) - \pi^T] = 0$$

which implies $Y_t - Y^\star = -\alpha\gamma[\pi_{t-1} + \alpha(Y_t - Y^\star) - \pi^T]$.

Given that the loss function is convex, this first order condition defines the unique minimum. Substituting the PC (eq. 3) back into this equation gives the Central bank rule in terms of output:

$$Y_t - Y^\star = -\alpha\gamma(\pi_t - \pi^T)$$

Now that we know what output level (and output gap) the Central Bank wants to see realized, we can use the IS curve (eq.1) to establish the interest rate that it should set.

Start by defining the equilibrium (or 'natural') interest rate $r^\star$: the interest rate that makes realized output equal to potential, thus causing the inflation rate to be stable. This is defined by the equation $Y^\star = A - ar^\star$. Subtracting this from the IS equation, we can express the IS curve in terms of the output gap: $Y_t - Y^\star = -a(r_{t-1} - r^\star)$.

Now take the Central bank rule in terms of output, and rearrange it as follows: $\pi_t - \pi^T = \frac{Y_t - Y^\star}{-\alpha\gamma}$. Substituting for π_t using the Phillips curve (eq. 3), we obtain the following:

$$\pi_{t-1} + \alpha(Y_t - Y^\star) - \pi^T = \frac{Y_t - Y^\star}{-\alpha\gamma} \rightarrow \pi_{t-1} - \pi^T = -(\alpha + \frac{1}{\alpha\gamma})(Y_t - Y^\star)$$

Now, if we substitute for $(Y_t - Y^\star)$ using the IS curve in terms of output gap that we derived above, we obtain the Central Bank's reaction function:

$$r_t = r^\star + \psi(\pi_t - \pi^T) \tag{4}$$

where $\psi = \frac{1}{a(\alpha + \frac{1}{\alpha\gamma})}$.

5 A three-equations economy

We now have the three equations which govern this economy:

$$Y_t = A - ar_{t-1} \tag{1}$$

$$\pi_t = \pi_{t-1} + \alpha(Y_t - Y^\star) \tag{3}$$

$$r_t = r^\star + \psi(\pi_t - \pi^T) \tag{4}$$

In equilibrium, output is at potential, unemployment is at its equilibrium rate, inflation is stable and equal to the Central Bank inflation target, and the interest rate is equal to its equilibrium ('natural') rate. In this situation, the three equations tell us that the system would be stationary, and all the endogenous variables would remain equal to their 'starred' values. This is what makes these values the equilibrium of the system.

What happens out of equilibrium? We will just analyze this qualitatively by means of an example, guided by the three equations.

Suppose that this economy is in equilibrium, when a demand shock occurs. For example, suppose that at some point the propensity to save decreases permanently, increasing the parameter A in the IS curve.

In the short-run, this positive demand shock will result in a 'booming' economy, with output above potential and unemployment below the equilibrium rate. Indeed, the immediate effect of the increase in A is an increase in actual output and a decrease in actual unemployment, while equilibrium unemployment ($u^\star = (\frac{m}{\beta(1+m)})$) and potential output ($Y^\star = [L(1 - \frac{m}{\beta(1+m)})]$) remain the same. Note that the equilibrium interest rate *does* increase as a result of this demand shock: it is given by $r^\star = (\frac{A - Y^\star}{a})$, so it goes up as a result of the increase in A. The new equilibrium interest rate is higher than the initial one.

The booming economy, however, will shortly become a 'overheated' one: the PC curve implies that the inflation rate will start rising, thus surpassing the Central Bank target.

Equation 4 tells us that this will induce the Central Bank to increase the real interest rate. In particular, the real interest rate will rise *above* the new equilibrium rate for some time, generating an economic downturn – with output below potential and unemployment above its equilibrium rate – making the inflation rate decrease. As the decreasing inflation rate gets closer to its target value π^T, the Central Bank will decrease the interest rate. At the moment in which inflation is back to its target level, the interest rate will be equal to its new equilibrium level (higher than the initial level, but lower than it was during the 'disinflation' process), and it will be kept there. With the interest rate at its equilibrium level, output will be back to potential.

This economy will thus gravitate around its equilibrium, with irregular oscillations determined by exogenous shocks, like the change in the propensity to save that we used as an example. Output will fluctuate irregularly but symmetrically around potential output, with 'booms' compensated by subsequent downturns, and in the medium-run will always go back to equilibrium.

In this simple model we assumed constant technology and labor force. What if we allowed technological progress (a positive rate of change of the parameter b in the production function) and population growth (a positive rate of growth of L)? With exogenous technical change like in the Solow model, potential output will increase at a constant rate, equal to the sum of the growth rates of technological progress and population. Intuitively and simplifying, the equilibrium will now be a dynamic one: output will still fluctuate randomly and symmetrically around potential output, but both will have an underlying stable increasing trend. In other words, short-run fluctuations like the one described above will happen on top of an underlying long-term trend.

6 Challenges for the canonical New-Keynesian model

Let us briefly mention some challenges for this view of the economy. In particular, let us focus on possible ways in which the adjustment mechanism described in the model may fail to restore full (or equilibrium) employment in the medium-run.

Recent literature, based on the experience of the US and European economies after 2008, has stressed the 'zero lower bound' (ZLB) problem: the Central Bank cannot set a substantially negative nominal interest rate. When inflation is low, this means that it cannot set a substantially negative *real* interest rate either. So it may be unable to bring output back to potential after a negative demand shock which makes the natural interest rate substantially negative (Bernanke, 2017; Fischer, 2016).

More in general, in order for the Central Bank reaction function to bring the system to equilibrium, the Central Bank must have a reliable and relatively precise estimate of the specific shape of the Phillips curve, and of the equilibrium unemployment rate (or of potential output). It is not clear, however, that available estimates of these theoretical objects are reliable and precise enough, especially considering that they may change in time. For example, in the example above, the Central Bank should quickly figure out that the increase in output and employment is due to a decreased propensity to save (as opposed to, for example, an increase in productivity or a downward shift in the parameters of the wage curve) and estimate the corresponding new equilibrium interest rate.

Another challenge comes from the empirical evidence suggesting *asymmetric effects of monetary policy*. While interest rate increases seem to be effective in curbing economic activity and possibly reducing inflation, interest rate decreases do not seem to be very effective in increasing output (Angrist et al., 2018). If firms and households are eager to invest, rising the interest rate may make it difficult for them to get the necessary credit, so investment (and consumer credit) may decrease. But if agents do not want to invest because they have very negative expectations (or are too indebted), lowering the cost of borrowing may not matter much for them. In other words, you can make a horse less hydrated by not bringing it to the fountain. However, when you would want to hydrate a reluctant horse you may fail, because 'you can lead a horse to water, but you cannot make it drink'.

A further important challenge comes from *Hysteresis.* It has been hypothesized (mostly based on suggestive but quite consistent empirical evidence) that periods of high unemployment tend to reduce potential output and increase the equilibrium unemployment rate. This would indicate a failure of a crucial premise of the New Keynesian model: the assumption that *potential output and equilibrium unemployment are independent of aggregate demand dynamics* (Fatás and Summers, 2016; Yellen, 2016). If demand shocks affect potential output or, in other words, the supply side of the economy is influenced by the demand side, a whole different scenario – and different policy prescriptions – are opened.

Why would hysteresis happen? Different hypotheses have been suggested. Prolonged unemployment may lead people to lose their skills and employability, or to become discouraged and stop looking for a job (Blanchard and Summers, 1986 is a classic on this topic). A major channel, traditionally explored by non-mainstream schools of thought but recently emphasized by mainstream authors too, is the effect of demand dynamics on capital accumulation (Haltmaier, 2013; Rowthorn, 1995).

Note that also in the presence of hystheresis we may observe that output in the medium-run reverts to potential output. However in this case, potential output would have adapted to realized output, rather than the other way around.

Another issue that has been debated recently is the possibility that interest rate decreases, also when necessary to bring output back to potential, may have bad side-effects. According to some, low interest rates may induce the creation of too much debt (mostly household debt: consumer credit and mortgages) and in general lead to higher risk-taking by financial institutions, which could possibly lead to financial instability (Cecchetti et al., 2017; Lian et al., 2017; Powell, 2017).

Increasing acknowledgement of (at least some of) the challenges mentioned, is leading some economists to rethink the ideas (1) that we should rely only on monetary policy and (2) that recessions have no medium-run effects. Some ideas that are currently 'in the air' are the following. Monetary policy may be the best tool to curb inflation when the economy is above potential; but fiscal policy maybe more appropriate than monetary policy to fight recessions: it is more effective and it does not increase private debts (Furman, 2016). If expansionary fiscal policy can increase potential output (because of hysteresis), then it does not necessarily increase public debt: tax revenues can increase because of increase in Y and $Y^\star$ (DeLong et al., 2012). If recessions have persistent (perhaps permanent) negative effects because of hysteresis, while expansions may have long-lasting positive effects, then we should be a bit more inclined to risk some higher inflation, rather than sacrifice growth in order to make sure to keep inflation low. In other words, the costs of keeping inflation low may be higher than previously thought.

References

Angrist, J D, Ò Jordà, and G M Kuersteiner (2018). "Semiparametric estimates of monetary policy effects: string theory revisited". In: *Journal of Business & Economic Statistics* 36.3, pp. 371–387.

Bernanke, B (2017). *How big a problem is the zero lower bound on interest rates?*

Blanchard, O and D R Johnson (2017). "Macroeconomics, 7th Edition". In: *London usw.: Prentice-Hall International Inc.*

Blanchard, O J and L H Summers (1986). *Hysteresis in unemployment.*

Carlin, W and D Soskice (2005). "Macroeconomics: imperfections, institutions, and policies". In: *OUP Catalogue.*

Cecchetti, S, M. Griffoli, and M Narita (2017). *Does prolonged monetary policy easing increase financial vulnerability?* International Monetary Fund.

DeLong, J B, L H Summers, M Feldstein, and V A Ramey (2012). "Fiscal policy in a depressed economy [with comments and discussion]". In: *Brookings Papers on Economic Activity*, pp. 233–297.

Fatás, Antonio and Lawrence H Summers (2016). "Hysteresis and fiscal policy during the Global Crisis". In: *VOX, CEPR's. Policy Portal* 12.

Fischer, Stanley (2016). "Monetary policy, financial stability, and the zero lower bound". In: *American Economic Review* 106.5, pp. 39–42.

Furman, Jason (2016). "The new view of fiscal policy and its application". In: *VoxEU. org.*

Haltmaier, J (2013). "Do recessions affect potential output?" In.

Lian, C, Y Ma, and C Wang (2017). "Low interest rates and risk taking: Evidence from individual investment decisions". In.

Powell, J H (2017). *Low Interest Rates and the Financial System: a speech at the 77th Annual Meeting of the American Finance Association, Chicago, Illinois, January 7, 2017.* Tech. rep. Board of Governors of the Federal Reserve System (US).

Romer, D H (2000). "Keynesian macroeconomics without the LM curve". In: *Journal of economic perspectives* 14.2, pp. 149–169.

Rowthorn, R (1995). "Capital formation and unemployment". In: *Oxford Review of Economic Policy* 11.1, pp. 26–39.

Yellen, JL (2016). "Macroeconomic research after the crisis: a speech at the elusive 'great'recovery: causes and implications for future business cycle dynamics". In: *60th annual economic conference sponsored by the Federal Reserve Bank of Boston. Boston, Massachusetts.* URL: `https://www.federalreserve.gov/newsevents/speech/yellen20161014a.htm`.