Advanced Macroeconomics – 7SSPN402 – Week 4 Seminar

1. Consider the overlapping-generations (OLG) economy we studied in the third week. Time is discrete and runs to infinity: $t = 0, 1, 2, \ldots$. There is one good in the economy, which can be either consumed or used as capital. An individual lives for 2 periods, supplies one unit of labour in the first period ('youth') and does not work in the second period ('old age'). The labour income earned during youth is partly consumed in the same period, partly saved. During old age, an individual rents out their capital (ie, the amount they saved during youth) and consumes the proceeds. Instantaneous utility is logarithmic. Individuals maximize the present value of their lifetime utility.

   Formally, let C_{1t} and C_{2t} denote the consumption in period t of young and old individuals, let ρ denote the discount rate, and r the interest rate. The present value of lifetime utility for an individual born at time t is given by:

   $$U_t = \ln(C_{1t}) + \frac{1}{1+\rho}\ln\left(C_{2,t+1}\right) \ with\ \rho \succ 1$$

   The budget constraint of an individual born at time t is

   $$C_{1t} + \frac{1}{1+r_{t+1}}C_{2,t+1} = A_t W_t$$

   a) Show formally that the consumption pattern of an individual over time follows the Euler equation $\frac{C_{2,t+1}}{C_{1t}} = \frac{1+r_{t+1}}{1+\rho}$. Make sure to show all the steps in your derivation.

      See page 67 in the lecture slides for Section 2.

   b) Explain in your own words what the Euler equation means and the economic intuition behind it.

      See page 44 in the lecture slides for Section 2.

   c) Do you think that the Euler equation provides a useful guide to predict consumption patterns in actual economies? Briefly explain why or why not.

      This is an opinion, so there is no universally correct answer. It might be true that when interest rates increase, people have more incentive to save, and therefore postpone consumption. And it might be true that with an interest rate decrease, people might anticipate consumption given the lower returns to savings. However, it could be objected that, in reality, people are constrained by the liquidity they have access to, so they can't freely substitute consumption today for consumption tomorrow. Moreover, people have consumption habits that tend to be rather stable and independent of interest rates. After all, you need to eat and dress and consume electricity also when the interest rate goes up! It might also be unrealistic to think that people are able to optimize their lifetime consumption patterns to maximize the present value of lifetime utility, as that would require a lot of ability to predict the future and to perform complicated computations.

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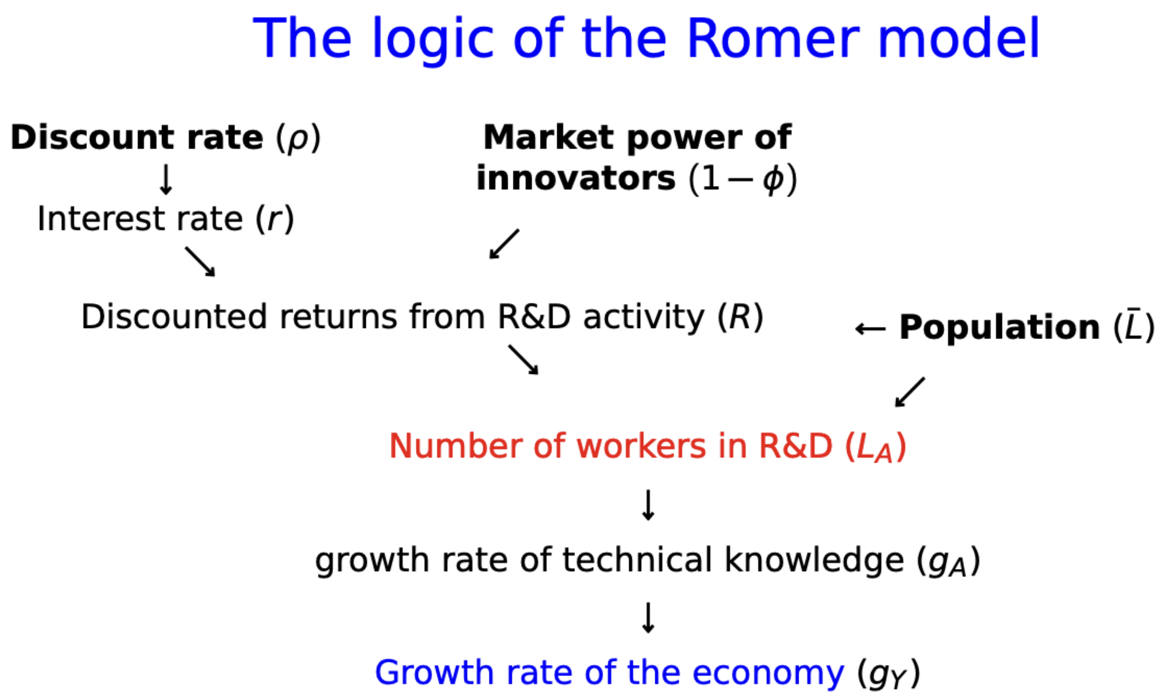
2. With the help of the diagram below, explain the logic of the Romer (1990) model. Explain how the exogenous variables (in bold font in the diagram) determine the equilibrium growth rate of the economy. Make sure to clarify and explain the key causal connections and how they determine equilibrium output.



See textbook Chapter 3.

3. **Learning-by-doing** Consider a one-good economy in which output Y is produced using capital K, labor L and technology A, according to the following Cobb-Douglas production function

$$\mathrm{Y(t) = K(t)^{\alpha}[A(t)L(t)]^{1-\alpha}} \tag{1}$$

L is constant and equal to $\bar{L}$. Assume no depreciation of physical capital, so that $\dot{K}(t) = sY(t)$. Technical progress occurs as a side-effect of goods production:

$$\dot{A}(t) = BY(t) \tag{2}$$

a. Derive expressions for the growth rates of K and A (call them g_K and g_A) in terms of the parameters of the model and the levels of A and K;

$$g_K = \frac{\dot{K}}{K} = \frac{\mathrm{sY}}{K} = s\left(\frac{A\bar{L}}{K}\right)^{1-\alpha}$$

$$g_A = \frac{\dot{A}}{A} = \frac{\mathrm{BY}}{A} = \frac{BK^{\alpha}\bar{L}^{1-\alpha}}{A^{\alpha}}$$

b. Derive an expression for the growth rate of Y (call it g_Y) as a function of g_K and g_A and the parameters of the model (*hint*: you only need to use equation 1).

$$g_Y = \alpha g_K + (1-\alpha) g_A$$

c. Focusing on g_K and g_A, derive the intertemporal equilibrium condition of the model (*hint*: focus on the growth rates of g_K and g_A, that is, $\frac{\dot{g_K}}{g_K}$ and $\frac{\dot{g_A}}{g_A}$; what you need to eventually find is that in equilibrium $\frac{K}{A}$ is fixed and equal to $\frac{s}{B}$).

The equation above implies that for the growth rate of the economy to be constant, also the growth rates of K and A have to be constant. Therefore to characterize the intertemporal equilibrium we can start by setting

$$\frac{\dot{g_K}}{g_K} = \frac{\dot{g_A}}{g_A} = 0$$

To find $\frac{\dot{g_K}}{g_K}$ we can take the log of the expression for g_K and then its derivative with respect to time (exploiting the fact that the derivative of the log of a variable w.r.t. time equals the growth rate of the variable).

$$\ln g_K = \ln s + (1-\alpha)(\ln A + \ln L - \ln K)$$

derivating wrt time:

$$\frac{\dot{g}_K}{g_K} = (1-\alpha)(g_A - g_K),$$

which implies that in equilibrium $g_A = g_K$. Using the expressions found in part (a), we thus have that in equilibrium

$$s\left(\frac{AL}{K}\right)^{1-\alpha} = \frac{BK^{\alpha}L^{1-\alpha}}{A^{\alpha}} \rightarrow \frac{\mathrm{K}}{\mathrm{A}} = \frac{\mathrm{s}}{\mathrm{B}}$$

d. Find the equilibrium growth rate of the economy, g^*. According to conventional definitions, is growth in this model fully endogenous, semi-endogenous or exogenous?

$$g_K^* = g_A^* = \frac{BK^{\alpha}L^{1-\alpha}}{A^{\alpha}} = B\bar{L}^{1-\alpha}\left(\frac{s}{B}\right)^{\alpha} = s^{\alpha}\, B^{1-\alpha}\,\bar{L}^{1-\alpha}$$

growth is endogenous: the saving rate affects the growth rate of the economy. Also population size affects the growth rate.