## Take-home assignment 4

Econ 705 - Spring 2021

Due: May 4th before 11.59pm (please send via email to Guilherme)

1. Efficiency wages (1) Consider an efficiency-wage model, slightly different (and simpler) than the Stiglitz-Shapiro one. The model abstracts from dynamics and just considers one single period. The representative worker has the following utility function

$$u(y,e) = y - a\left(\frac{1}{1-e} - e\right),$$
 with  $a > 0$ 

where y is income (equal to the wage w if the worker is employed) and e is the effort that the worker exerts in doing the job, with  $0 \le e \le 1$ .

The employer's profits are simply equal to  $\Pi = e/w$ , that is, worker's effort per dollar spent in wages.

The employer offers a wage to the worker. The worker can either accept and get the job, or reject and be unemployed. The job contract cannot specify the amount of effort that the worker will have to provide. The employer, however, can imprecisely observe effort, and can fire the worker (not paying her the wage) if she detects a low effort level. Therefore the probability that a worker is fired (and doesn't get the job wage)  $P_F$  is negatively related to the worker's effort level ( $P_F = P_F(e)$ ). In particular, assume the following functional form:

$$P_F(e) = 1 - e$$

The government has committed an overall sum equal to B to unemployment benefits, independent of how many people are unemployed. This means that if unemployed (either because she refused the job or because she was fired), the worker receives an unemployment benefit equal to  $\frac{B}{s}$ , where s is the number of unemployed people. This is the worker's reservation wage. Define the cost of being unemployed (or cost of job loss)  $\hat{c}$  as the

difference between the wage on the job and the reservations wage

$$\hat{c} = \left(w - \frac{B}{s}\right)$$

Assume that  $w > \frac{B}{s}$ , so the cost of job loss is strictly positive.

Based on the assumptions above, the value of the job for the worker (that is, the expected utility from accepting the job) is

$$v(w, e) = [1 - P_F(e)]w + P_F(e)[w - \hat{c}] - a\left(\frac{1}{1 - e} - e\right)$$

- i Explain the meaning of each of the three terms in the v(w,e) function.
- ii Derive the workers' best-response function, or effort function. This function gives the optimal effort level  $(e^*)$  of an employed worker as a function of things that the worker takes as exogenous.
- iii Calculate the derivative of the optimal effort level  $(e^*)$  with respect to the cost of job loss  $(\hat{c})$  and with respect to the parameter a. What is the sign of these derivatives? Explain the intuition behind your results.
- iv Show that the employer will offer a wage which satisfies the condition  $w^* = \frac{e(w)}{e_w(w)}$

2. Efficiency-wages (2) The representative firm has the following profit function

$$\pi = Y - wh$$

where Y is output, w is the wage and h is the amount of labor it hires, measured in hours. The firm's output is given by the following production function

$$Y = \frac{(eh)^{\alpha}}{\alpha}$$

where e denotes workers' effort. In turn, workers' effort is given by

$$e = \begin{cases} \left(\frac{w - \bar{b}}{\bar{b}}\right)^{\beta} & \text{if } w > \bar{b} \\ 0 & \text{otherwise} \end{cases}$$

where  $\bar{b}$  are unemployment benefits, and with  $0 < \beta < 1$ .

Workers are represented in the wage-bargaining process by a union with the following objective function

$$U = (w - \bar{b})h$$

The firm and the union bargain over the wage. They choose w to maximize  $U^{\gamma}\pi^{1-\gamma}$ .  $\gamma$  is a measure of workers' bargaining power, and we have  $0 < \gamma < \alpha < 1$ .

After the wage is determined in the bargaining process, the firm chooses h, taking w as given.

- i What value of h does the firm choose, given w? What is the resulting level of profits?
- ii What is the wage level that will emerge from the wage-bargaining process? Assume that  $w > \bar{b}$  always holds.

[Hint: remember that maximizing ln(f) can sometimes be easier than maximizing f and gives the same result]

iii Now set  $\beta = 0$ , meaning that wage-efficiency considerations are absent. Answer the following questions: do efficiency-wage considerations raise wages? is the proportional impact of workers' bargaining power on wages greater with efficiency wages than without?

- 3. Inflationary bias Consider a one-good economy, in which output Y is produced by means of labor L only. Specifically, the production function is  $Y_t = L_t$ . Because of imperfect competition, the price of the good is a mark-up over production costs:  $P_t = (1+m)W_t$ , with m>0. The (expected) real wage that emerges from wage-setting is a positive function of employment. Specifically, we have  $W_t/P_t^e = \beta L_t$ , where  $W_t$  is the nominal wage,  $P_t^e$  is the expected price of the good based on information available at the beginning of period t (before the actual price is observed),  $\beta$  is a constant, and  $1 > \beta > 0$ . Further assume that  $\ln(1+m) + \ln \beta = 0$ .
  - i Show that this economy exhibits a 'supply curve' of the following form:

$$y_t = \pi_t - \pi_t^e \tag{1}$$

where y is the log of Y,  $\pi$  is the inflation rate  $(\pi_t = ln(P_t) - ln(P_{t-1}))$ , and  $\pi_t^e$  is expected inflation  $(\pi_t^e = ln(P_t^e) - ln(P_{t-1}))$ .

ii The government is able to affect the inflation rate through monetary policy. For simplicity, assume that the government just sets the inflation rate  $\pi_t$  at each time t, after the inflation expectation  $\pi_t^e$  has been determined and incorporated in wage-setting. More precisely, at each t, the government sets the inflation rate  $\pi_t$  in such a way as to minimize the following loss function, taking inflation expectations as given:

$$L = \frac{1}{2}(\pi_t - \bar{\pi})^2 + \frac{1}{2}(y_t - \bar{y})^2 \quad \text{with } \bar{\pi} \ge 0 \text{ and } \bar{y} > 0$$
 (2)

where  $\bar{\pi}$  is the government's preferred inflation rate and  $\bar{y}$  is the (log of the) government's preferred output level.

Suppose that inflation expectations are exogenous and fixed:  $\pi_t^e = \alpha \ge 0$  at all t. Show that in this case output y and inflation  $\pi$  are both positive functions of target output  $\bar{y}$  and target inflation  $\bar{\pi}$ .

iii Now suppose that wage-setters are rational, know that the government can manipulate the inflation rate, and know the government loss function. Therefore they are able to correctly and exactly predict the inflation rate when they set wages at the beginning of each period. Show that in this case inflation will be always higher than target inflation  $(\pi > \bar{\pi})$ , but real output will be unaffected by monetary policy.

iv With reference to the rational expectations case  $(\pi_t^e = \pi_t)$ , suppose that the government has made the following announcement:

'Given that we cannot affect real output, we will just set inflation equal to target inflation  $(\pi = \bar{\pi})$  at all points in time. Knowing this, wage-setters will expect  $\pi^e = \bar{\pi}$ . In this way, the economy will enjoy lower inflation at no cost for output. You should trust that we will always stick to this commitment, because our loss function implies that we are better off in this way, relative to the case of exercise (iii) above, when we were unconstrained and we always overshoot our inflation target.'

Will rational wage-setters believe the government and actually expect  $\pi^e = \bar{\pi}$ ?

4. **Identified moments** With reference to the paper 'Identification in Macroeconomics' (Nakamura and Steinsson, Journal of Economic Perspectives, Summer 2018), explain *in your own words* what the authors mean by 'identified moments', how 'identified moments' can be useful in guiding macroeconomic theory, and how 'identified moments' compare with traditional 'unconditional moments' (as used for example in calibrating RBC models). [Max 350 words]