



Advanced Macroeconomics

Section 6 - Financial Markets

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Diamond-Dybvig (1983) model

- ▶ A model of banking.
- ▶ Financial instability (bank runs).
- ▶ *Maturity mismatch*: long-term assets vs short-term liabilities.
- ▶ Justifies deposit insurance schemes.
- ▶ 2022 Nobel Prize (with Ben Bernanke).



III. Niklas Elmehed © Nobel Prize Outreach
Douglas W. Diamond
Prize share: 1/3



III. Niklas Elmehed © Nobel Prize Outreach
Philip H. Dybvig
Prize share: 1/3

Assumptions

- ▶ One-good economy with a continuum of agents.
- ▶ Three periods (0, 1 and 2).
- ▶ Each agent endowed with 1 good in period 0.
- ▶ If 1 good invested until period 2, yields $R > 1$ goods.
- ▶ Two types of agents
 - Type-a: values only consumption in period 1
 - Type-b: values consumption in both periods
- ▶ Share of type-a = θ
- ▶ Individual learns its type in period 1, but not visible to others.

Utility functions

Utility of a type-a individual:

$$U^a = \ln c_1^a$$

Utility of a type-b individual:

$$U^b = \rho \ln(c_1^b + c_2^b)$$

where $0 < \rho < 1$ and $\rho R > 1$.

Benchmark case 1: Autarchy

- ▶ Agents cannot trade, each just manages her own wealth.
- ▶ Optimal choice of type-a agents:
 - Consume 1 in period 1 ($c_1^a = 1$)
 - Consume 0 in period 2 ($c_2^a = 0$)
- ▶ Optimal choice of type-b agents:
 - Consume 0 in period 1 ($c_1^b = 0$)
 - Consume R in period 2 ($c_2^b = R$)
- ▶ Ex-ante expected utility (computed at period 0):

$$\begin{aligned}E(U^{AUTARCHY}) &= \theta \ln 1 + (1 - \theta)\rho \ln R \\&= (1 - \theta)\rho \ln R\end{aligned}$$

- ▶ Type-b gets more consumption than type-a.

Benchmark case 2: Social planner

- ▶ A social planner can dictate $c_1^a, c_2^a, c_1^b, c_2^b$.
- ▶ They want to maximize ex-ante expected utility.
- ▶ Will obviously set $c_2^a = 0$ and $c_1^b = 0$.

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- ▶ Will obviously set $c_2^a = 0$ and $c_1^b = 0$.
- ▶ Share of total endowments liquidated early: θc_1^a .
- ▶ Share of total endowments kept invested: $1 - \theta c_1^a$.
- ▶ So, the social planner problem is:

$$\max_{c_1^a, c_2^b} E(U) = \theta \ln c_1^a + (1 - \theta) \rho \ln c_2^b$$

$$\text{s.t. } (1 - \theta)c_2^b = (1 - \theta c_1^a)R$$

Socially-optimal allocation

- ▶ Substituting the resource constraint into expected utility:

$$\max_{c_1^a} E(U) = \theta \ln c_1^a + (1 - \theta) \rho \left[\ln(1 - \theta c_1^a) + \ln R - \ln(1 - \theta) \right]$$

- ▶ Solution:

$$c_1^{a*} = \frac{1}{\theta + (1 - \theta)\rho} > 1$$

$$c_2^{b*} = \frac{\rho R}{\theta + (1 - \theta)\rho} < R$$

- ▶ c_1^a higher than in autarchy case, c_2^b lower.
- ▶ Social planner transfers some resources in favor of type-a.
- ▶ Reason: type-a likes consumption more ($\rho < 1$).
- ▶ But still $c_2^{*b} > c_1^{*a}$.

Case 3: A bank

- ▶ At time 0, individuals deposit their endowments in the bank.
- ▶ Can withdraw $c_1^{a*} > 1$ at period 1.
- ▶ Funds that are not withdrawn are invested by the bank.
- ▶ All returns are divided up between remaining depositors in period 2:

$$c_2 = \frac{(1 - \phi c_1^{a*})R}{1 - \phi}, \text{ where } \phi \text{ is the share of agents withdrawing early}$$

- ▶ So the return from holding until maturity depends (negatively) on the share that withdraws early.
- ▶ If bank cannot meet all period 1 withdrawals, it gives c_1^{a*} to some and 0 to the rest.

The choice to withdraw or hold

- ▶ Will depositors withdraw or hold?
- ▶ Type-a will always want to withdraw in period 1.
- ▶ Type-b will hold if she expects $c_2 > c_1^{a*}$.
- ▶ So a type-b agent will hold if the following is expected to be true

$$c_2 = \frac{(1 - \phi c_1^{a*})R}{1 - \phi} > c_1^{a*}$$

- ▶ This holds if ϕ is not too high.
- ▶ Type-b agent will hold if they expect enough other people to hold too.

The ‘good’ equilibrium

- ▶ The following is an equilibrium outcome:
 - Type-a individuals withdraw and all get c^{a*}
 - Type-b individuals hold and receive $c_2 = \frac{1-\theta c_1^a}{1-\theta} R = c_2^{b*}$.
 - Therefore $\phi = \theta$.
- ▶ Nash Equilibrium: If type-b expects $\phi = \theta$, best response is holding.
 - because $c_2^{b*} > c_1^{a*}$.

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- ▶ Nash Equilibrium: If type-b expects $\phi = \theta$, best response is holding.
 - because $c_2^{b*} > c_1^{a*}$.
- ▶ The bank *can* realize the social optimum (maximize the expected utility of a representative agent).
- ▶ Basically, bank provides insurance against the possibility that you might end up being type-a.

The ‘bank run’ equilibrium

- ▶ However, also the following is an equilibrium:
 - Everybody tries to withdraw
 - Since $c_1^{a*} > 1$, bank cannot meet all withdrawals.
- ▶ Nash equilibrium: if you expect everybody else to try to withdraw, you are better off trying to withdraw too.
 - ▶ with ϕ close to one, c_2 is close to zero.

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- ▶ Nash equilibrium: if you expect everybody else to try to withdraw, you are better off trying to withdraw too.
 - ▶ with ϕ close to one, c_2 is close to zero.
- ▶ Bank run can be a self-fulfilling prophecy: the expectation that everyone will withdraw causes everyone to withdraw.
- ▶ Many examples in history (recently Northern Rock 2007, Silicon Valley Bank 2023)

Diamond-Dybvig (1983) model

- ▶ *Liquidity mismatch* is the source of the problem: the bank uses short-term borrowing to finance long-term investment.
- ▶ A bank run can happen even if the bank does nothing wrong, just because of self-fulfilling expectations.
- ▶ Some possible solutions to eliminate the bad equilibrium:
 1. *Suspension of payments*: bank pays c_1^{a*} to a maximum of θ agents, the others are forced to wait.
 2. *Deposit insurance*: If bank doesn't have enough funds to pay c_2^{b*} , government will pay them (financed by taxes on type-a agents).
 3. *Lender of last resort*: CB lends resources to the bank in case $\phi > \theta$, so the bank can pay all depositors and keep a share θ of its investments, which will be used to repay depositors who hold and the CB loan.