Advanced Macroeconomics

Section 4 - Fluctuations (II): Keynesian and New-Keynesian theories

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‘Old school’ Keynesian theory

- Developed in the 1940s to formalise Keynes’ ideas
- Was dominant and guided policy until the 1970s
- Simple models built up from sensible assumptions about relations between macroeconomic variables, but no explicit microfoundations
- IS-LM model + Phillips Curve
- Aggregate demand determines the level of output, inflation-unemployment trade-off

New Keynesian theory

- Micro-founded rational-expectations framework (like RBC)
- but introduces nominal rigidities (sticky prices/wages) and imperfect competition
- Baseline 3-equations DSGE model
  1. New Keynesian IS curve
  2. New Keynesian Phillips Curve
  3. Central Bank reaction function
- real effects of monetary policy (unlike RBC and *somehow* similar to old Keynesian models)
- also the effects of other shocks (technology and fiscal policy) differ from the plain RBC model.

The plan

1. Old school IS-LM model and Lucas critique
2. New Keynesian IS-LM model
3. Phillips Curve(s)
4. IS-LM-PC: A simplified model in the spirit of New Keynesian macro
5. The canonical DSGE New Keynesian model

The 'old-school' IS-LM model

- Model of output determination in the short-run
- John Hicks (1937) formalisation of (his interpretation of) Keynes.
  - Neoclassical synthesis
- Became the dominant model of output determination since the 1940s and is still the model taught in intermediate classes.
- Notation:

- Y = output
- Z = aggregate demand
- C = consumption
- I = aggregate investment
- G = government spending
- τ = tax rate
- i = nominal interest rate
- r = real interest rate
- M = quantity of money
- P = price level

Goods market equilibrium

- **Definition**:
  - Aggregate demand $Z_t \equiv C_t + I_t + G_t$.
- **Behavioural equations**:
  - Consumption function: $C_t = c_0 + c_1(1-\tau_t)Y_t$
  - Investment function: $I_t = a_0 - a_1 r_t$.
  - G and τ taken as given: $G_t = G, \tau_t = \tau$.
- **Equilibrium**:
  Equilibrium condition $Y = Z$ implies equilibrium output is

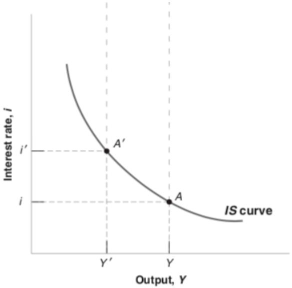
$$Y_t = \frac{1}{1-c_1(1-\tau)}[c_0 + (a_0 - a_1 r) + G] = A - ar_t$$

Where $A = \frac{c_0+a_0+G}{1-c_1(1-\tau)}$ and $a = \frac{a_1}{1-c_1(1-\tau)}$.

The old school IS curve

- goods’ market equilibrium:

$$Y = A - ar \qquad \textit{(IS curve)}$$



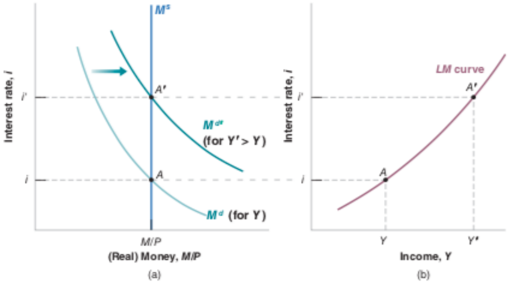
- A change in the interest rate is a movement along the IS curve
- A change in government spending or autonomous consumption shifts the IS curve up or down

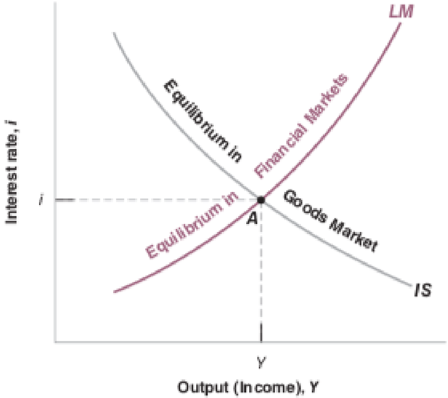
Money market equilibrium

$$\frac{M_t}{P_t} = \alpha Y_t - \beta i_t \quad \Rightarrow \quad i_t = bY_t - c\frac{M_t}{P_t} \qquad \textit{(LM curve)}$$

(Where $b = \alpha/\beta$ and $c = 1/\beta$)

- M and P exogenous constants ($P_t = P$, $M_t = M$).
- Higher Y → higher demand for M → higher equilibrium i





- Given fixed price assumption, $i = r$.
- Can be used to evaluate the effect of fiscal and monetary policy.
- Fiscal expansion (increase in G or decrease in τ) raises Y and i.
- Monetary expansion (increase in M) raises Y and lowers i.

The Lucas (1976) critique

- Old-school Keynesian models lack microfoundations
- Relations between aggregates are assumed, without specifying how they arise from individual goal-oriented behavior.
- Policy evaluation might be flawed: policy change might change expectations & behaviour, altering aggregate relations.
- Example: In evaluating effect of fiscal expansion, old-Keynesian theory assumes a given propensity to save. But if stimulus is temporary, utility-maximizing agents might save most of it, so propensity to save is not stable.
- The equations of a macro model should be derived explicitly from a microeconomic model of individual behavior.

The New-Keynesian IS-LM model

- One-good economy with no K, large number of identical firms, and fixed number of identical infinitely lived households.

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The New-Keynesian IS-LM model

- One-good economy with no K, large number of identical firms, and fixed number of identical infinitely lived households.
- Production function: $Y = C = F(L); \quad F'(L) > 0; \quad F''(L) \leq 0$
- Representative household's lifetime utility:

$$U = \sum_{t=0}^{\infty} \beta^t \left[U(C_t) + \Gamma\left(\frac{M_t}{P_t}\right) - V(L_t) \right], \quad 0 < \beta < 1$$

  - $U'(.) > 0$ and $U''(.) < 0$;
  - $\Gamma'(.) > 0$ and $\Gamma''(.) < 0$;
  - $V' > 0$ and $V''(.) > 0$.
- Choice variables: C and M;
- L exogenous (for now);

Evolution of household's wealth

- Two assets: Central Bank money M (gold coins) and a bond B (a claim on M).
- Evolution of household's wealth:

$$\begin{aligned} A_{t+1} &= M_t + B_t(1+i_t) \\ &= M_t + (A_t + W_t L_t - P_t C_t - M_t)(1+i_t) \end{aligned}$$

  - A_{t+1} is wealth at the start of period $t+1$;
  - M_t and B_t are money and bonds held during period t;

Household's behavior: Euler equation

- Assuming CRRA utility, the infinite-horizon utility function implies

$$\ln C_t = \ln C_{t+1} - \frac{1}{\theta} \ln[(1 + r_t)\beta]$$

$$\Downarrow$$

$$\ln Y_t = a + \ln Y_{t+1} - \frac{1}{\theta} r_t$$

(because $Y = C$ and $\ln(1 + r) \approx r$, and with $a = -(\frac{1}{\theta}) \ln \beta$)

- See demonstration in Romer Section 6.1

The New-Keynesian IS curve

$$\ln Y_t = a + \ln Y_{t+1} - \frac{1}{\theta} r_t$$

- negative relation between Y_t and r_t.
- differences with old-school IS curve:
  - conceptual: driven by intertemporal substitution, not income multiplier effect.
  - practical: $\ln Y_{t+1}$ term.
  - here, IS interpretation requires assuming fixed Y_{t+1}.

John Cochrane on the New Keynesian IS curve:

This new-Keynesian model is an utterly and completely different mechanism and story [relative to the old-keynesian model]. (...)

The marginal propensity to consume is exactly and precisely zero in the new-Keynesian model. There is no income at all on the right hand side [of the Euler equation]. (...)

John Cochrane on the NK IS curve (continued):

The old-Keynesian model is driven completely by an income effect with no substitution effect. Consumers don't think about today vs. the future at all. The new-Keynesian model is based on the intertemporal substitution effect with no income effect at all. (...)

[a lower r_t] induces consumers to spend their money today rather than in the future (...). Now, lowering consumption growth is normally a bad thing. But new-Keynesian modelers assume that the economy reverts to trend, so lowering growth rates is good, and raises the level of consumption today with no ill effects tomorrow.

[from John Cochrane's 'New vs. Old Keynesian Stimulus' (on Keats)]

Household's money demand

- Optimization requires that marginal increase in M_t/P_t by Δm (given total wealth) has no effect on utility.
- To leave wealth unchanged, $\Delta C_t = -\left(\frac{i}{1+i}\right)\Delta m$
- So in equilibrium:

$$\Gamma'\left(\frac{M_t}{P_t}\right)\Delta m = U'(C_t)\left(\frac{i_t}{1+i_t}\right)\Delta m$$

$$\Downarrow$$

$$\frac{M_t}{P_t} = Y_t^{\theta/\chi}\left(\frac{1+i_t}{i_t}\right)^{1/\chi}$$

- Real money demand is positive function of Y and negative function of i as in the old-Keynesian model.
- P and M are fixed, so implies i increasing function of Y.

New-Keynesian IS-LM

- Price of consumption good is assumed fixed:

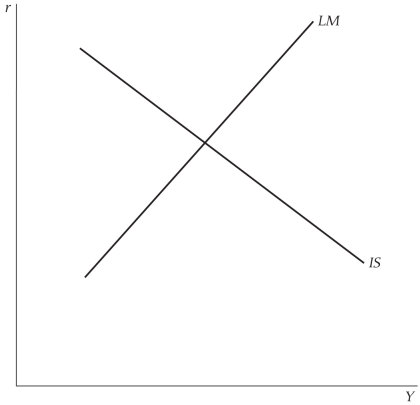
$$P_t = \bar{P} \quad \Rightarrow \quad i_t = r$$

- So both IS and money-demand are in terms of r and Y;

$$Y_t = f(r_t) \quad \text{with } f' < 0 \quad \text{(IS curve)}$$

$$r_t = g(Y_t) \quad \text{with } g' > 0 \quad \text{(LM curve)}$$

New-Keynesian IS-LM



but remember this is based on the assumption of unchanged (expectation of) Y_{t+1}!

New-Keynesian IS-LM

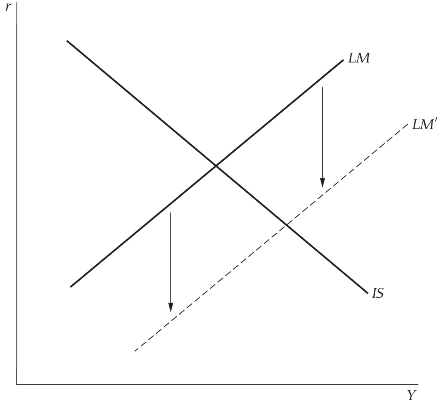
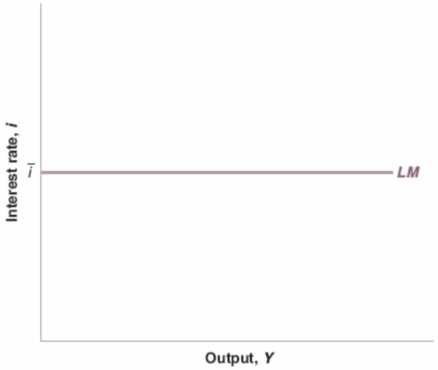


Figure: Effect of a temporary increase in money supply

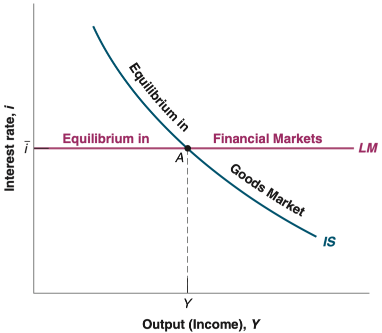
A more realistic LM "curve"

- In reality, money is endogenous and the Central Bank sets the interest rate.
- $i = \bar{i}$.



IS-LM with interest-rate setting

- IS relation: $Y_t = f(r_t) \quad$ with $f' < 0$
- LM relation: $r = i = \bar{i}$



- After adding a model of inflation (Phillips Curve), can be enriched by the Central Bank reaction function
- CB sets the interest rate based on inflation and output.

Phillips Curve(s)

- IS-LM framework (old or new) needs to be completed with a theory of inflation.
- *Phillips Curve*: A relation between inflation & unemployment/output.
- 'Traditional' Phillips Curve:

$$\pi_t = \alpha - \beta u_t$$

- 'Accelerationist' Phillips Curve:

$$\pi_t - \pi_{t-1} = \alpha - \beta u_t$$

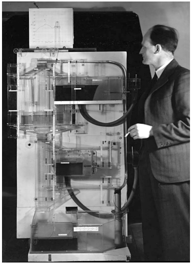
- New Keynesian Phillips Curve:

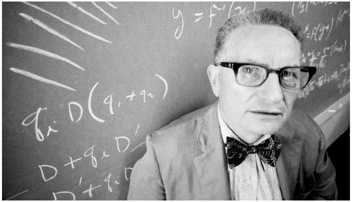
$$\pi_t = k y_t + \beta E_t \pi_{t+1}$$

- Very different implications for policy.

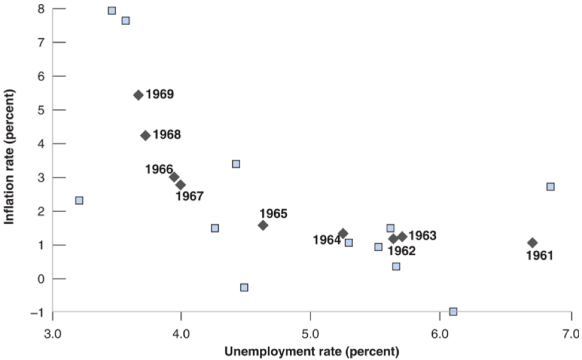
Historical origins of the Phillips Curve

- PC originally derived from empirical observation, not formal theory.
- *1958*: A.W. Phillips uncovers negative correlation between inflation and unemployment in UK 1861-1957 data.
- *1960*: Samuelson & Solow replicate it on 1900-1960 US data.
- In the 1970s the relation breaks down, which inspires the development of an ‘accelerationist’ Phillips Curve.



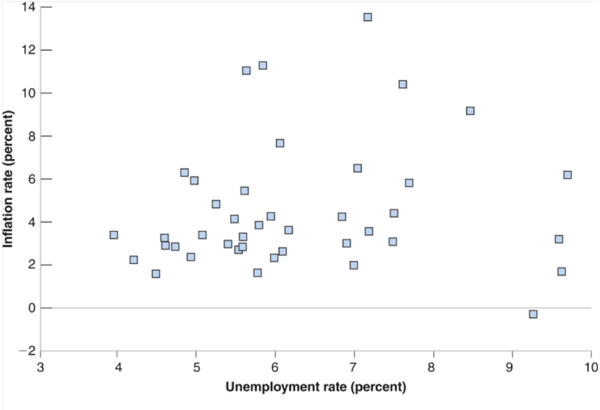


1948-1969: the ‘original’ Phillips Curve

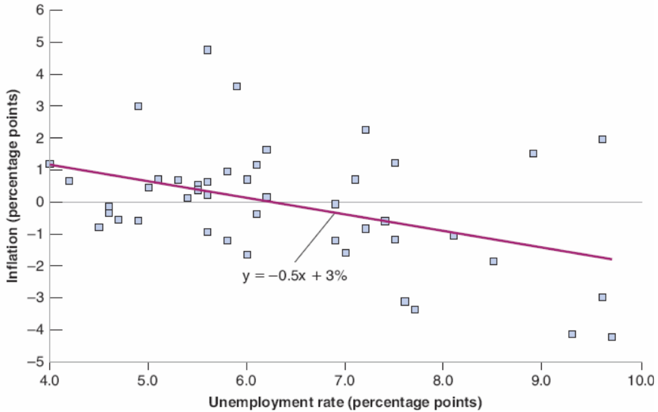


Source: Series UNRATE, CPIAUSCL Federal Reserve Economic Data (FRED) http://research.stlouisfed.org/fred2/

1970-2010: the disappearance of the ‘original’ PC



1970-2010: Accelerationist PC



Phillips Curve(s): Theoretical foundations

- Theoretical explanations of the PC focus on wage and price-setting processes.
- Models of wage and price-setting imply relations between π, $E(\pi)$ and u
- specific form of the PC depends on how agents form $E(\pi)$
  1. fixed (‘anchored’) expectations –> original PC
  2. adaptive expectations –> accelerationist PC
  3. rational expectations –> New-Keynesian PC
- Traditional & accelerationist PC can be derived from a simple macro model, while New Keynesian PC can be derived from the (more complicated) Calvo model of pricing.

Phillips Curve: a simple framework

- Traditional and accelerationist PC can be derived from a very simple macro model.
- Central idea:

  lower $u_t \Rightarrow$ higher $W_t \Rightarrow$ increase in P_t & π_t.
- if it stops here, we have the 'original' PC

Phillips Curve: a simple framework

- Traditional and accelerationist PC can be derived from a very simple macro model.
- Central idea:

  lower $u_t \Rightarrow$ higher $W_t \Rightarrow$ increase in P_t & π_t.
- if it stops here, we have the 'original' PC
- BUT with adaptive expectations, inflationary spiral:

  lower $u_t \Rightarrow$ higher $W_t \Rightarrow$ increase in P_t & $\pi_t \Rightarrow$ increase in $E(\pi_{t+1}) \Rightarrow$ increase in $W_{t+1} \Rightarrow \ldots$
- 'accelerationist' PC

- Basic model:

$$Y_t = N_t$$

$$P_t = (1+m)W_t$$

$$\frac{W_t}{E(P_t)} = 1 - \beta u_t \quad \Rightarrow \quad W_t = E(P_t)(1-\beta u_t)$$

  - Y = output;
  - N = employment;
  - W = nominal wage;
  - P = price of the good;
  - m = mark-up;
  - $u = 1 - \frac{L}{N}$ = unemployment rate;

Traditional and accelerationist Phillips Curves

- Combine price-setting & wage-setting:

$$P_t = E(P_t)(1+m)(1-\beta u_t)$$

- rewrite (approximately) in terms of π:

$$\pi_t = E(\pi_t) + m - \beta u_t$$

- What determines $E(\pi_t)$?

Traditional and accelerationist Phillips Curves

- 'Generic' Phillips Curve:

$$\pi_t = E(\pi_t) + m - \beta u_t$$

- Assume fixed expectations

$$E(\pi) = \bar{\pi}$$

- Then we have

$$\pi_t = \alpha - \beta u_t \qquad \text{(with } \alpha = \bar{\pi} + m\text{)}$$

- *'original' (old-Keynesian) Phillips curve*
- Inflation-unemployment trade-off for policy.

The PC and its mutations

- 'Generic' Phillips Curve:

$$\pi_t = E(\pi_t) + m - \beta u_t$$

- Assume adaptive expectations

$$E(\pi) = \pi_{t-1}$$

- *'Accelerationist' PC*:

$$\pi_t - \pi_{t-1} = \alpha - \beta u_t$$

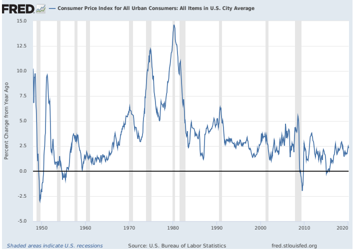
- Lower unemployment leads to higher *change* in the inflation rate (like in the 1970s).

Traditional and accelerationist Phillips Curves

An interpretation of inflation in industrialized economies

1948-1969

- inflation not persistent;
- wage-setters assumed inflation would revert to mean $\bar{\pi}$;
- $E(\pi) \approx \bar{\pi} \Rightarrow$ Original PC.



after 1970

- inflation became persistent (oil shocks);
- wage-setters started taking persistence into account;
- $E(\pi_t) \approx \pi_{t-1} \Rightarrow$ accelerationist PC.

The equilibrium unemployment rate

In this model, a unique unemployment rate makes inflation equal expected inflation:

$$\pi_t = \mathbb{E}(\pi_t) \rightarrow u_t^\star = \frac{m}{\beta}$$

Implications for traditional PC:

- Possible to sustain $u < u_t^\star$ only as long as $\pi > \mathbb{E}(\pi)$.
- But if $\pi > \mathbb{E}(\pi)$ is persistent, wage-setters would surely update their expectations!
- Traditional PC with anchored expectations unlikely to be stable unless $u = u^\star$.

The equilibrium unemployment rate

In this model, a unique unemployment rate makes inflation equal expected inflation:

$$\pi_t = \mathbb{E}(\pi_t) \rightarrow u_t^\star = \frac{m}{\beta}$$

Implications for accelerationist PC:

- When $u = u^\star$, inflation is stable over time ($\pi_t = \pi_{t-1}$).
- $u < u^\star$ leads to accelerating inflation (increasing over time).
- $u > u^\star$ leads to deflation (decreasing over time).
- Disinflation is painful: to bring down π, you need $u > u^\star$ for a period of time.

Calvo price setting model

- New Keynesian PC is derived from a more complex model of dynamic price setting.
- Calvo (1983) "Staggered prices in a utility-maximizing framework".
- Sticky prices: they cannot be adjusted in all periods.
- Opportunities to change prices arrive randomly.
  - *Poisson process*: same probability of price adjustment in every period.
- A bit arbitrary: chosen as the baseline model of prices not because realistic, but because it happens to deliver a convenient PC that works well in a DSGE model.

Framework (1/3)

- A monopolistic competition model
- Production function

$$Y_t = L_t$$

- Closed economy with no government and no capital:

$$C_t = Y_t$$

- Exogenous nominal expenditure (aggregate demand)

$$M_t = Y_t P_t$$

- Labor supply curve

$$\frac{W_t}{P_t} = BY_t^{\theta+\gamma-1}$$

- Monopolistic pricing

$$\frac{P_{it}^{\star}}{P_t} = \frac{\eta}{\eta-1}\frac{W_t}{P_t}$$

Framework (2/3)

Time-dependent price-adjustment:

- Firms cannot adjust their prices in all periods.
- P_i set at time 0 has probability $q_t \geq 0$ of remaining in effect at time $t > 0$.
- $p_t \equiv ln(P_t)$.
- firm sets p_i as a weighted average of expected future $p_t^{\star}$'s:

$$p_i = \sum_{t=0}^{\infty} \tilde{\omega}_t E[p_t^{\star}] \quad \text{with} \quad \tilde{\omega}_t \equiv \frac{\beta^t q_t}{\sum_{\tau=0}^{\infty} \beta^{\tau} q_{\tau}}$$

Framework (3/3)

- Profit-maximizing price is a mark-up over the wage

$$\frac{P^{\star}_{it}}{P_t} = \frac{\eta}{\eta-1}\frac{W_t}{P_t} \quad \Rightarrow \quad p^{\star}_t = \ln\left[\frac{\eta}{\eta-1}\right] + w_t$$

- Substitute in the (log of the) labor supply curve

$$w_t = p_t + \ln B + (\theta+\gamma-1)y_t \quad \Rightarrow \quad p^{\star} = p + \ln\frac{\eta}{\eta+1} + \ln B + (\theta+\gamma-1)y_t$$

- Given that $m = y + p$, and assuming for simplicity $\ln\frac{\eta}{\eta-1} + \ln B = 0$,

$$p^{\star}_t = \phi m_t + (1-\phi)p_t \quad \text{with } \phi = (\theta+\gamma-1)$$

- optimal 'sticky' price to set at time 0:

$$p_i = \sum_{t=0}^{\infty} \tilde{\omega}_t E_0[\phi m_t + (1-\phi)p_t]$$

Deriving π

- Each period share α of firms, randomly chosen, adjusts prices

$$\textit{aggregate price level:} \quad p_t = \alpha x_t + (1-\alpha)p_{t-1}$$

$$\textit{inflation:} \quad \pi_t = p_t - p_{t-1} = \alpha(x_t - p_{t-1})$$

Deriving π

- Each period share α of firms, randomly chosen, adjusts prices

$$\textit{aggregate price level:} \quad p_t = \alpha x_t + (1-\alpha)p_{t-1}$$

$$\textit{inflation:} \quad \pi_t = p_t - p_{t-1} = \alpha(x_t - p_{t-1})$$

- optimal 'sticky' prices:

$$x_t = \sum_{j=0}^{\infty} \tilde{\omega}_j E(p^{\star}_{t+j}) \quad \text{with} \quad \tilde{\omega}_j = \frac{\beta^j q_j}{\sum_{k=0}^{\infty} \beta^{\kappa} q_k}$$

- Poisson process implies $q_j = (1-\alpha)^j$
- $\rightarrow \sum_{k=0}^{\infty} \beta^{\kappa} q_k = \sum_{k=0}^{\infty} \beta^k (1-\alpha)^k = \frac{1}{1-\beta(1-\alpha)}$

Calvo model - deriving π

- ...plugging in:

$$x_t = [1-\beta(1-\alpha)]\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j E_t p^\star_{t+j}$$

- Rewrite in terms of $p_t^\star$ and $E_t x_{t+1}$:

$$x_t = [1-\beta(1-\alpha)]\left(p_t^\star + \beta(1-\alpha)\left[\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j E_t p^\star_{t+1+j}\right]\right) =$$

$$= [1-\beta(1-\alpha)]p_t^\star + \beta(1-\alpha)[1-\beta(1-\alpha)]\left[\sum_{j=0}^{\infty}\beta^j(1-\alpha)^j E_t p^\star_{t+1+j}\right] =$$

$$= [1-\beta(1-\alpha)]\mathbf{p_t^\star} + \beta(1-\alpha)\mathbf{E_t x_{t+1}}$$

Deriving π

$$x_t = [1-\beta(1-\alpha)]p_t^\star + \beta(1-\alpha)E_t x_{t+1}$$

- Express in terms of π_t, using $\pi_t = \alpha(x_t - p_{t-1})$ and $p^\star = \phi m_t + (1-\phi)p_t$

$$\pi_t = k y_t + \beta E_t \pi_{t+1} \quad \text{with} \quad k = \frac{\alpha[1-(1-\alpha)\beta]\phi}{1-\alpha}$$

- New-Keynesian Phillips Curve
- Inflation depends on expected inflation & output (as in all PCs);
- Difference: it is the expectation of *future* inflation.
- *Intuition*: higher output → higher costs (wages); higher future inflation → higher optimal future prices.

3 Phillips Curves and their implications

1. *Old-Keynesian PC*: $\pi_t = \alpha + \lambda y_t$
   - *output-inflation trade-off*: disinflation requires permanently lower y;
2. *Accelerationist PC*: $\pi_t = \pi_{t-1} + \lambda(y_t - y_t^\star)$
   - painful disinflation: requires $y < y^\star$ for some time (*inflation inertia*);
3. *New-Keynesian PC*: $\pi_t = ky_t + \beta E_t \pi_{t+1}$
   - expansionary disinflation: $E_t(\pi_{t+1})$ down $\rightarrow$ y_t up.

New Keynesian models of fluctuations

- IS curve & Phillips curve are the key building blocks of Keynesian & New Keynesian macroeconomics.
- They can be integrated to build dynamic models of fluctuations.
- We will consider two:
  1. A simplified New Keynesian model
  2. The canonical New Keynesian DSGE model

A (very) simplified New-Keynesian model

- IS + PC + Central Bank reaction function
- Simpler than the canonical New Keynesian DSGE model and not microfounded
- But captures the New-Keynesian perspective on fluctuations well.
- Most mainstream policy discussions are implicitly based on this model
- *Romer (2000), Carlin & Soskice (2005), Blanchard (2017).*

A 3-equations economy

- IS Curve:

$$Y_t = A - ar_{t-1} \tag{1}$$

- Accelerationist PC:

$$\pi_t = \pi_{t-1} + \alpha(Y_t - Y^\star) \tag{2}$$

- Central Bank reaction function:

$$r_t = r^\star + \psi(\pi_t - \pi^T) \tag{3}$$

$y =$ output; π = inflation rate; $Y^\star$=potential output; r=interest rate; $r^\star$=equilibrium interest rate; π^T=target interest rate;

Old-Keynesian IS Curve

- Output:

$$Y_t = C_t + I_t + \bar{G}$$

- Consumption:

$$C_t = c_0 + c_1(1 - \bar{\tau})Y_t$$

- Housing investment:

$$I_t = a_0 - a_1 r_{t-1}$$

- Short-run equilibrium output:

$$Y_t = A - a r_{t-1}$$

$$\text{where } A = \frac{c_0 + a_0 + \bar{G}}{1 - c_1(1 - \bar{\tau})} \text{ and } a = \frac{a_1}{1 - c_1(1 - \bar{\tau})}$$

Accelerationist Phillips Curve (1/2)

- Wage setting

$$\frac{W_t}{P_t^e} = 1 - \beta u_t \quad \Rightarrow \quad W_t = P^e(1 - \beta u_t)$$

- Price setting

$$Y_t = N_t \quad \Rightarrow \quad P_t = (1+m)W_t$$

- Inflation rate

$$P_t = P_t^e(1+m)(1-\beta u_t) \quad \Rightarrow \quad \pi_t = \pi_t^e + m - \beta u_t$$

- Medium-run equilibrium unemployment rate

$$\pi = \pi^e \quad \Rightarrow u^\star = \frac{m}{\beta} \quad \Rightarrow \quad \pi - \pi^e = -\beta(u_t - u^\star)$$

Accelerationist Phillips Curve (2/2)

- Phillips curve

$$\pi - \pi^e = -\beta(u_t - u^\star)$$

- Assuming adaptive expectations

$$\pi^e = \pi_{t-1} \quad \Rightarrow \pi_t = \pi_{t-1} - \beta(u_t - u^\star)$$

- Rewrite in terms of output

$$\pi_t = \pi_{t-1} + \alpha(Y_t - Y^\star)$$

- Define equilibrium (‘natural’) interest rate:

$$Y^\star = A - ar^\star \quad \Rightarrow \quad Y_t - Y^\star = -a(r_{t-1} - r^\star)$$

Central Bank reaction function

- CB minimizes a loss function

$$\min_r \ell = (Y_t - Y^\star)^2 + \gamma(\pi - \pi^T)^2$$

- CB's desired output gap

$$Y_t - Y^\star = -\alpha\gamma(\pi_t - \pi^T)$$

- CB choice of interest rate (*Monetary policy rule*)

$$r_t = r^\star + \psi(\pi_t - \pi^T)$$

$$\text{with } \psi = \frac{1}{a(\alpha + \frac{1}{\alpha\gamma})}$$

Equilibrium of the 3-equations economy

- IS Curve:

$$Y_t = A - a r_{t-1} \tag{1}$$

- Accelerationist PC:

$$\pi_t = \pi_{t-1} + \alpha(Y_t - Y^\star) \tag{2}$$

- Central Bank reaction function:

$$r_t = r^\star + \psi(\pi_t - \pi^T) \tag{3}$$

- Equilibrium:

$$\pi = \pi^T; \quad y = y^\star; \quad u = u^\star; \quad r = r^\star.$$

Stability

- Focus on inflation
- Define deviation from target: $\tilde{\pi} = \pi - \pi^T$
- We want $\tilde{\pi}_t$ as a function of $\tilde{\pi}_{t-1}$.

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$$Y_t = A - a[r^* + \psi(\pi_{t-1} - \pi^T)]$$

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- Subtract Y^* from both sides

$$Y_t - Y^* = -a\psi(\pi_{t-1} - \pi^T)$$

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- Subtract Y^* from both sides

$$Y_t - Y^* = -a\psi(\pi_{t-1} - \pi^T)$$

- Substitute this into the Phillips Curve

$$\pi_t = \pi_{t-1} - \alpha a\psi(\pi_{t-1} - \pi^T)$$

$$\tilde{\pi}_t = (1 - \alpha a\psi)\tilde{\pi}_{t-1}$$

Stability

- We now have $\tilde{\pi}_t$ as a function of $\tilde{\pi}_{t-1}$ only:

$$\tilde{\pi}_t = (1 - \alpha a \psi)\tilde{\pi}_{t-1}$$

- The dynamic system is stable iff

$$|1 - \alpha a \psi| < 1 \quad \rightarrow \quad 0 < \alpha a \psi < 2$$

  (discrepancies disappear with time)

- Recall we defined $\psi = \frac{1}{a(\alpha + \frac{1}{\alpha\gamma})}$
- Therefore $\alpha a \psi = \alpha a \cdot \frac{1}{a(\alpha + \frac{1}{\alpha\gamma})} = \frac{\alpha}{\alpha + \frac{1}{\alpha\gamma}} = \frac{\alpha^2\gamma}{\alpha^2\gamma + 1}$
- Since $\alpha^2\gamma > 0$, we have $0 < \frac{\alpha^2\gamma}{\alpha^2\gamma+1} < 1$
- So the stability condition is satisfied.

Out of equilibrium dynamics

- suppose $y = y^\star$, $r = r^\star$ and $\pi = \pi^T$ initally
- a positive demand shock occurs, eg $c_0 \uparrow$

1. Economic boom:

$$y > y^\star, \quad u < u^\star; \quad r^\star \uparrow;$$

2. Accelerating inflation:

$$\pi > \pi^T \text{ and rising}$$

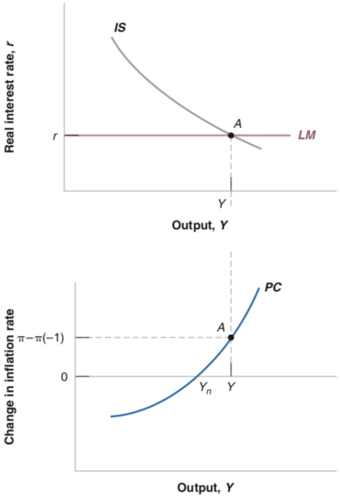
3. CB reaction and downturn:

$$r \uparrow; r > r^\star \quad \Rightarrow \quad Y \downarrow; Y < Y^\star.$$

4. Stabilization:

$$\pi = \pi^T; \quad r = r^\star; \quad Y = Y^\star$$

A short-run equilibrium with output above potential



Challenges for the simplified New Keynesian model

Five critical and potentially problematic assumptions:

1. Monetary policy always effective in increasing output;
2. Policy-makers have a good estimate of a well-defined $u^{\star}$ and other key parameters;
3. Low unemployment always translates in higher wages & prices;
4. The level of potential output is unaffected by changes in demand;
5. Low interest rates have no negative side-effects

The baseline New Keynesian DSGE model

- New-Keynesian IS curve

$$y_t = E_t[y_{t+1}] - \frac{1}{\theta} r_t + u_t^{IS} \quad \text{with} \quad \theta > 0$$

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- Monetary policy rule

$$r_t = \phi_\pi E_t[\pi_{t+1}] + \phi_y E_t[y_{t+1}] + u_t^{MP} \quad \text{with} \quad \phi_\pi > 0, \quad \phi_y \geq 0$$

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- no constants: deviations from steady-state, normalized to 0

The baseline New Keynesian DSGE model

$$\text{NK IS curve:} \quad y_t = E_t[y_{t+1}] - \frac{1}{\theta} r_t + u_t^{IS} \quad \text{with} \quad \theta > 0$$

$$\text{NK PC:} \quad \pi_t = \beta E_t[\pi_{t+1}] + k y_t + u_t^{\pi} \quad \text{with} \quad 0 < \beta < 1, k > 0$$

$$\text{MP rule:} \quad r_t = \phi_\pi E_t[\pi_{t+1}] + \phi_y E_t[y_{t+1}] + u_t^{MP} \quad \text{with} \quad \phi_\pi > 0, \phi_y \geq 0$$

- shocks structure:

$$u_t^{IS} = \rho_{IS} u_{t-1}^{IS} + e_t^{IS}, \qquad -1 < \rho_{IS} < 1$$
$$u_t^{\pi} = \rho_{\pi} u_{t-1}^{\pi} + e_t^{\pi}, \qquad -1 < \rho_{\pi} < 1$$
$$u_t^{MP} = \rho_{MP} u_{t-1}^{MP} + e_t^{MP}, \qquad -1 < \rho_{MP} < 1$$

Solving the 3-equations model

- Express the model in terms only of shocks and expectations;
- plug the MP rule into the IS curve:

$$y_t = -\frac{\phi_\pi}{\theta} E_t[\pi_{t+1}] + \left(1 - \frac{\phi_y}{\theta}\right) E_t[y_{t+1}] + u_t^{IS} - \frac{1}{\theta} u_t^{MP}$$

- plug the equation above into the NK PC:

$$\pi_t = \left(\beta - \frac{\phi_\pi k}{\theta}\right) E_t[\pi_{t+1}] + \left(1 - \frac{\phi_y}{\theta}\right) k E_t[y_{t+1}] + k u_t^{IS} + u_t^{\pi} - \frac{k}{\theta} u_t^{MP}$$

Special case: no serial correlation in shocks

- Assume $\rho_{IS} = \rho_{\pi} = \rho_{MP} = 0$.
- So the following is a solution (intertemporal equilibrium):

$$E_t[y_{t+1}] = E_t[\pi_{t+1}] = 0$$

$$r_t = e_t^{MP}$$

$$y_t = e_t^{IS} - \frac{1}{\theta} e_t^{MP}$$

$$\pi_t = k e_t^{IS} + e_t^{\pi} - \frac{k}{\theta} e_t^{MP}$$

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$$\pi_t = k e_t^{IS} + e_t^{\pi} - \frac{k}{\theta} e_t^{MP}$$

- shows effect of demand, monetary policy and inflation shocks;
- *no internal propagation mechanisms*: without assuming serial correlation in shocks, we don't get any persistence (just like RBC).

The general case

- Method of undetermined coefficients;
- Educated guess:

$$y_t = a_{IS} u_t^{IS} + a_\pi u_t^\pi + a_{MP} u_t^{MP}$$

$$\pi_t = b_{IS} u_t^{IS} + b_\pi u_t^\pi + b_{MP} u_t^{MP}$$

- Plug these into the y_t and π_t functions derived earlier;
- solve the resulting system of equations to get the a's and b's;
- we will skip the algebra and directly discuss implications for the effects of shocks;

Implications of the general case

- Assumptions (from Gali, 2015):
  - $\theta = 1$ in utility function;
  - $k = 0.172$ & $\beta = 0.99$ in PC;
  - $\phi_\pi = 0.5$ & $\phi_y = 0.125$ in MP;
  - $\rho = 0.5$ for all shocks.

- Effect of *MP* shock:
  - $y_t = -1.54 u_t^{MP}$;
  - $\pi_t = -0.53 u_t^{MP}$;
  - $r_t = 0.77 u_t^{MP}$

- Effect of *IS* shock:
  - $y_t = 1.54 u_t^{IS}$;
  - $\pi_t = 0.53 u_t^{IS}$;
  - $r_t = 0.23 u_t^{IS}$.

- Effect of π shock:
  - $y_t = -0.76 u_t^{\pi}$;
  - $\pi_t = 1.72 u_t^{\pi}$;
  - $r_t = 0.38 u_t^{\pi}$.

Effect of a unit *IS* shock:

- $y_t = 1.54 u_t^{IS}$;
  - -> Stronger than in the special case without persistence
  - -> output now increases more than one-for-one
  - -> The shock is persistent, so also $E(y_{t+1})$ goes up, which amplifies the effect on y.
- $\pi_t = 0.53 u_t^{IS}$;
  - -> Stronger than in the special case without persistence
  - -> Now also $E(y_{t+1})$ and $E(\pi_{t+1})$ are positively affected, which amplifies the effect on current inflation.
- $r_t = 0.23 u_t^{IS}$.
  - -> It was zero with white-noise shocks (no reaction)
  - -> But now $E(y_{t+1})$ and $E(\pi_{t+1})$ are affected (because of persistence), and so monetary policy will react.
  - -> But less than one-for-one, because the shock dissipates in time.

Effect of a unit *MP* shock:

- $y_t = -1.54 u_t^{MP}$;
  - -> Stronger than in the special case without persistence
  - -> Being persistent, shocks affects also future output. So y_t is affected both directly and through $E(u_{t+1}^{MP})$ and $E(y_{t+1})$

- $\pi_t = -0.53 u_t^{MP}$;
  - -> Stronger than in the special case without persistence
  - -> Because the output effect is now stronger.
  - -> And also through $E(\pi_{t+1})$.

- $r_t = 0.77 u_t^{MP}$.
  - -> *Weaker* than in the special case without persistence.
  - -> Interest rate now increases less than one-for-one
  - -> Mitigated by the decrease in $E(y_{t+1})$ and $E(\pi_{t+1})$, which reduces r through the MP rule.

Effect of a unit π shock:

- $y_t = -0.76 u_t^{\pi}$;
  - -> It was zero without persistence
  - -> Now there is an effect because also future inflation is expected to be higher now, triggering contractionary MP reaction.
- $\pi_t = 1.72 u_t^{\pi}$;
  - -> Stronger than in the special case without persistence
  - -> Now π increases more than one-for-one
  - -> Because also $E(\pi_{t+1})$ goes up, which has a positive feedback effect on current inflation.
- $r_t = 0.38 u_t^{\pi}$.
  - -> It was zero before
  - -> Because of persistence, now future inflation is affected, which triggers a monetary-policy response.

Application:

Monetary policy rules and macroeconomic stability:
Evidence and some theory

by Clarida, Gali and Gertler (2000)

- Uses the canonical NK model to explain disinflation in the US in the 1980s
- Argues that a change in the conduct of monetary policy explains the stabilization of inflation.
- Available on Keats

- All kinds of extensions in the literature, but this remains the basic model
- Some problems:
  - No unemployment (workers are on their supply curve)
  - All consumers are forward-looking and unconstrained by liquidity.
  - No internal propagation mechanisms (effects of shocks are not persistent except by assumption).
  - Implications of the NK PC about effect of anticipated disinflation are wildly unrealistic.
  - *Forward guidance puzzle*: announced temporary interest rate reduction in the distant future has an enormous effect on inflation today (pretty weird)

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DSGE models: optimistic vs pessimistic views

The optimistic view:

- DSGE describe reasonably well the behavior of macro aggregates...
- ... and are micro-founded so their parameters are plausibly policy-invariant;
- Extensions are making them more realistic, and technology allows analysis of ever more sophisticated versions (including HANK);
- macroeconomists should all focus on further improving DSGE models.

Pessimistic view:

- The baseline model actually produces embarrassing predictions...
- ...and only large ad-hoc modifications just designed to make the models' implications more reasonable attenuate that;
- macroeconomists should seek radically different alternatives (back to old-school Keynesian? agent-based models? no all-encompassing model at all? a type of model that has not been conceived yet?).