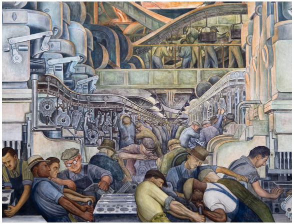

Advanced Macroeconomics

Section 7 - Labor market

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Labor market models

- Key stylized facts:
  - persistent and substantial involuntary unemployment
  - limited pro-cyclicality of wages
  - strong pro-cyclicality of employment
  - at odds with a plain neoclassical demand-supply model
- *Efficiency-wages* [Bowles-Stiglitz-Shapiro]
- *Search-and-matching* [Diamond-Mortensen-Pissarides]
- *Monopsony* [Manning 2003, Dube et al., 2018, Azar et al. 2019, ...]

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- Cannot specify exactly what the worker should do in any possible situation,
- nor how much *effort* the worker must exert on the job.
- Effort is hard to observe, measure and prove in court.

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- nor how much *effort* the worker must exert on the job.
- Effort is hard to observe, measure and prove in court.
- With an incomplete contract, power matters
- Employers use their position of power (the threat of the sack) to obtain effort from workers.
- But for employers to be in a position of power, they have to pay an *efficiency wage*, not a market-clearing wage.

Equilibrium unemployment as a discipline device

- Unemployment emerges in equilibrium as a *discipline device*.
- Crucial for the functioning of the labor market: it makes it costly for workers to lose their job, thus inducing adequate work effort.
- *Wage curve*: the lower the unemployment rate, the higher the equilibrium wage.

A very simplified efficiency-wage model

- Abstract from dynamics and focus on one single period.
- A representative firm hires a representative worker.
- Worker chooses how much effort to exert
- Firm can imperfectly observe the effort level of the worker.
- Firm chooses the wage to offer and a *termination schedule*.
- The termination schedule relates the probability of employment termination to its (imperfect) observation of the worker's effort.

Assumptions

- Employer wants to maximise the effort-wage ratio:

$$\Pi = e/w \quad \text{with } 0 \leq e \leq 1$$

- Worker gets utility from income and disutility from effort:

$$u(y, e) = y - \frac{a}{1-e} \quad \text{with } a > 0$$

- Worker income equals the wage w if not terminated.
- If terminated, worker gets unemployment benefit B/s, where s is the number of unemployed workers in the economy.
- Termination schedule determines probability of termination P_F:

$$P_F = P_F(e) = 1 - e$$

The worker choice of effort level

- Worker expected utility

$$E(U) = [1 - P_F(e)]w + P_F(e)\left(\frac{B}{s}\right) - \frac{a}{1-e}$$

- Defining the cost of job loss $\hat{c} \equiv w - \frac{B}{s}$,

$$E(U) = [1 - P_F(e)]w + P_F(e)\,(w - \hat{c}) - \frac{a}{1-e}$$

- Expected utility maximization implies:

$$e^{\star} = 1 - \left(\frac{a}{\hat{c}}\right)^{\frac{1}{2}}$$

The worker optimal effort function

$$e^{\star} = 1 - \left(\frac{a}{\hat{c}}\right)^{\frac{1}{2}} \quad \text{with} \quad \hat{c} \equiv w - \frac{B}{s}$$

- Worker effort is an increasing function of the cost of job loss.
- Higher wage $\longrightarrow$ more effort.
- Higher generosity of unemployment benefits B $\longrightarrow$ less effort.
- Higher unemployment s $\longrightarrow$ more effort
- Higher disutility of effort a $\longrightarrow$ less effort.

The firm choice of a wage offer

- Firm knows the worker optimal effort function, so maximizes

$$\Pi = e^{\star}(w)/w$$

- So their optimal wage offer is

$$w^{\star} = \frac{e^{\star}(w)}{e^{\star}_{w}(w)}$$

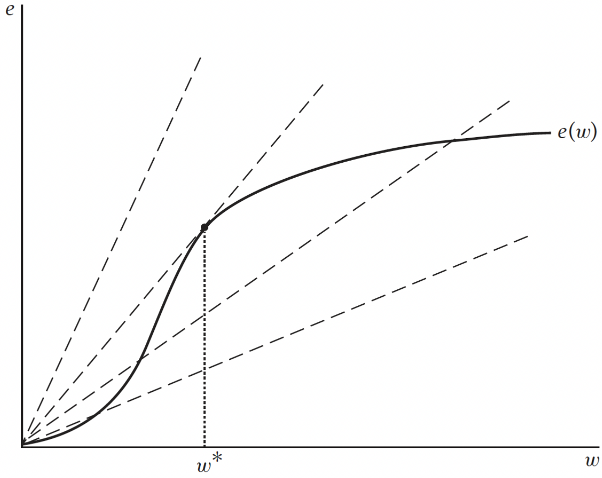
  - $\max_w \frac{e(w)}{w} \Rightarrow \frac{\partial \frac{e(w)}{w}}{\partial w} = 0 \Rightarrow w = \frac{e(w)}{e_w(w)}$

- This also implies that in equilibrium

$$\frac{e}{w} = e_w$$

At the optimal wage offer, the slope of the iso-profit curve equals the slope of the worker optimal effort function (graph next slide).

Equilibrium wage and effort



The wage curve

- An increase in unemployment s shifts the worker optimal effort function up (higher effort for each given wage).
- Therefore it leads to a lower equilibrium wage.
- Intuition: unemployment raises the cost of job loss, so a lower wage is necessary to induce adequate effort.
- *Wage curve:* The wage is a negative function of unemployment s.
- This provides a micro-foundation for the wage-setting curve in our simplified New Keynesian model!

Takeaways

- The model provides a possible explanation for unemployment
  - Efficiency wage > market-clearing wage.
- Moreover, the equilibrium outcome is inefficient for firm and worker
  - It can be shown that it would be possible to increase both the worker utility and the firm profit by choosing a higher wage and higher effort.
  - Coordination failure.
- *Power matters:* Firm uses the threat of termination to discipline workers into exerting effort on the job.

Search-and-matching

- No Walrasian centralized market clearing.
- Workers & firms meet in decentralized one-on-one matches.
- Costly and time-consuming search process produces ‘frictional’ unemployment.
- Diamond-Mortensen-Pissarides model (2010 Nobel Prize).

Assumptions about the economy

- Continuum of workers of mass 1.
- Firms open vacancies and then search for workers.
- Maintaining a job (filled or unfilled) costs c to the firm.
- Firm's payoff per period from a job:
  - $y - w(t) - c$ if filled.
  - $-c$ if unfilled.
- Worker's payoff per period:
  - w if employed.
  - b if unemployed.
- $y > b + c$, so there is always positive surplus from filling a job.

Assumptions about job matching

- At each point in time $M(t)$ job matches occur.
- Matching function:

$$M(t) = M[U(t), V(t)], \qquad M_U > 0; \quad M_V > 0$$

- Jobs end at an exogenous rate λ.
- Employment change:

$$\dot{E}(t) = M(U(t), V(t)) - \lambda E(t)$$

- Share ϕ of surplus from filling a vacancy goes to the worker (bargaining power).

Search-and-matching

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- Job-finding rate:

$$a(t) = \frac{M(t)}{U(t)} = m[\theta(t)]$$

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- Specifically, Cobb-Douglas matching function:

$$M(U, V) = kU^{1-\gamma}V^{\gamma}; \qquad m(\theta) = k\theta^{\gamma}$$

Solving the model

- We are after the *inter-temporal equilibrium* (steady state); we'll ignore disequilibrium dynamics & stability issues;
- Strategy:

1. Figure out the value (= expected lifetime utility) of each state for each agent
   - V_E, V_U, V_F, V_V
2. Impose intertemporal equilibrium conditions (constant V's, E, a, α).
3. Find V_V as a function of E and exogenous parameters;
4. Impose equilibrium condition $V_V = 0$ to determine the equilibrium values of E, a and α.

1 - Value of each possible state

- Value of being employed:

$$rV_E(t) = w(t) - \lambda[V_E(t) - V_U(t)] + \dot{V}_E(t)$$

- Value of being unemployed:

$$rV_U(t) = b + a(t)[V_E(t) - V_U(t)] + \dot{V}_U(t)$$

- Value of a filled job:

$$rV_F(t) = [y - w(t) - c] - \lambda[V_F(t) - V_V(t)] + \dot{V}_F(t)$$

- Value of a vacancy:

$$rV_V(t) = -c + \alpha(t)[V_F(t) - V_V(t)] + \dot{V}_V(t)$$

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2 - Steady-state conditions

- $\dot{E} = \dot{\alpha} = \dot{a} = \dot{V}_E = \dot{V}_U = \dot{V}_F = \dot{V}_V = 0;$

Step 3 - Find V_V as a function of E

- Model equations + steady-state conditions imply (after some algebra)

$$rV_V = -c + \frac{[(1-\phi)\alpha(E)](y-b)}{\phi a(E) + (1-\phi)\alpha(E) + \lambda + r}, \quad a_E > 0, \alpha_E < 0 \quad \Rightarrow \quad \frac{\partial V_V}{\partial E} < 0$$

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Step 4 - Free-entry condition pins down equilibrium E

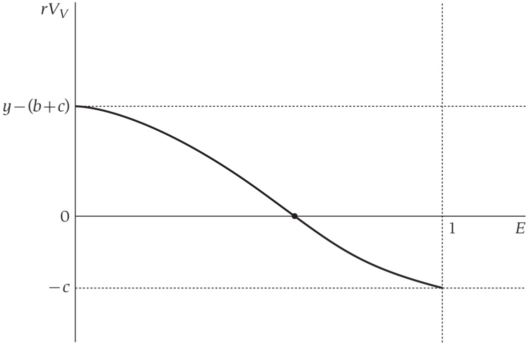
- Free-entry implies $V_V = 0$

$$rV_V = -c + \frac{[(1-\phi)\alpha(E)](y-b)}{\phi a(E) + (1-\phi)\alpha(E) + \lambda + r} = 0$$

- This implicitly defines the equilibrium values of E, a and α.

The equilibrium employment rate

- The equilibrium unemployment rate is implicitly defined by
  $\frac{\partial V_V}{\partial E} < 0 \quad \& \; V_V = 0$



Takeaways

- Equilibrium unemployment could be explained by search frictions
  - (but evidence on long-term unemployment suggests otherwise; moreover, unemployment could seem frictional for the individual worker, while not being so on aggregate).
- cyclical increase in profitability of a filled job (y up, no change in c and b) brings to large wage increase and modest increase in employment and vacancies
  - no wage rigidity!
  - (increase in wages reduces the incentive to create new vacancies);
- decentralized equilibrium is generally not efficient
  - We'll skip the proof.
  - see stylized example in the book if you are interested in knowing more.