



Advanced Macroeconomics

Section 4 - Fluctuations (II): Keynesian and New-Keynesian theories

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'Old school' Keynesian theory

- ▶ Developed in the 1940s to formalise Keynes' ideas
- ▶ Was dominant and guided policy until the 1970s
- ▶ Simple models built up from sensible assumptions about relations between macroeconomic variables, but no explicit microfoundations
- ▶ IS-LM model + Phillips Curve
- ▶ Aggregate demand determines the level of output, inflation-unemployment trade-off

New Keynesian theory

- ▶ Micro-founded rational-expectations framework (like RBC)
- ▶ but introduces nominal rigidities (sticky prices/wages) and imperfect competition
- ▶ Baseline 3-equations DSGE model
 1. New Keynesian IS curve
 2. New Keynesian Phillips Curve
 3. Central Bank reaction function
- ▶ real effects of monetary policy (unlike RBC and *somewhat* similar to old Keynesian models)
- ▶ also the effects of other shocks (technology and fiscal policy) differ from the plain RBC model.

The plan

1. Old school IS-LM model and Lucas critique
2. New Keynesian IS-LM model
3. Phillips Curve(s)
4. IS-LM-PC: A simplified model in the spirit of New Keynesian macro
5. The canonical DSGE New Keynesian model

The 'old-school' IS-LM model

- ▶ Model of output determination in the short-run
- ▶ John Hicks (1937) formalisation of (his interpretation of) Keynes.
 - Neoclassical synthesis
- ▶ Became the dominant model of output determination since the 1940s and is still the model taught in intermediate classes.
- ▶ Notation:
 - Y = output
 - Z = aggregate demand
 - C = consumption
 - I = aggregate investment
 - G = government spending
 - τ = tax rate
 - i = nominal interest rate
 - r = real interest rate
 - M = quantity of money
 - P = price level

Goods market equilibrium

► **Definition:**

- o Aggregate demand $Z_t \equiv C_t + I_t + G_t$.

► **Behavioural equations:**

- o Consumption function: $C_t = c_0 + c_1(1 - \tau_t)Y_t$
- o Investment function: $I_t = a_0 - a_1r_t$.
- o G and τ taken as given: $G_t = G$, $\tau_t = \tau$.

► **Equilibrium:**

Equilibrium condition $Y = Z$ implies equilibrium output is

$$Y_t = \frac{1}{1 - c_1(1 - \tau)} [c_0 + (a_0 - a_1r) + G] = A - ar_t$$

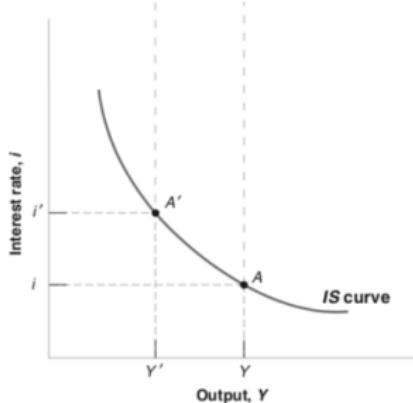
Where $A = \frac{c_0 + a_0 + G}{1 - c_1(1 - \tau)}$ and $a = \frac{a_1}{1 - c_1(1 - \tau)}$.

The 'old-school' IS-LM model

The old school IS curve

- goods' market equilibrium:

$$Y = A - ar \quad (\text{IS curve})$$



- A change in the interest rate is a movement along the IS curve
- A change in government spending or autonomous consumption shifts the IS curve up or down

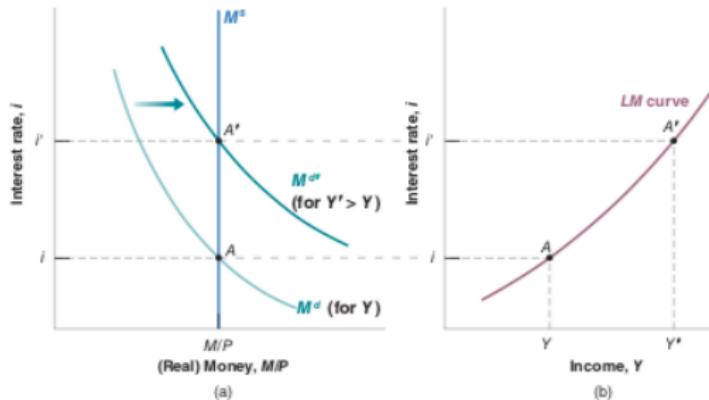
The 'old-school' IS-LM model

Money market equilibrium

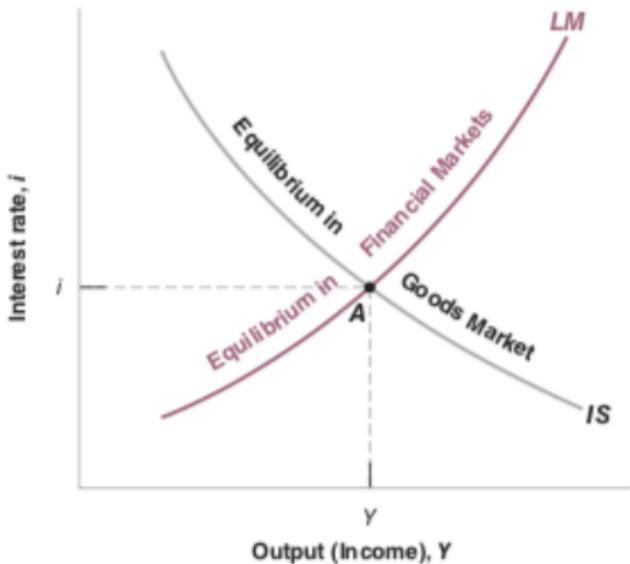
$$\frac{M_t}{P_t} = \alpha Y_t - \beta i_t \Rightarrow i_t = b Y_t - c \frac{M_t}{P_t} \quad (LM\ curve)$$

(Where $b = \alpha/\beta$ and $c = 1/\beta$)

- ▶ M and P exogenous constants ($P_t = P$, $M_t = M$).
- ▶ Higher $Y \rightarrow$ higher demand for $M \rightarrow$ higher equilibrium i



The 'old-school' IS-LM model



- ▶ Given fixed price assumption, $i = r$.
- ▶ Can be used to evaluate the effect of fiscal and monetary policy.
- ▶ Fiscal expansion (increase in G or decrease in τ) raises Y and i .
- ▶ Monetary expansion (increase in M) raises Y and lowers i .

The Lucas (1976) critique

- ▶ Old-school Keynesian models lack microfoundations
- ▶ Relations between aggregates are assumed, without specifying how they arise from individual goal-oriented behavior.
- ▶ Policy evaluation might be flawed: policy change might change expectations & behaviour, altering aggregate relations.
- ▶ Example: In evaluating effect of fiscal expansion, old-Keynesian theory assumes a given propensity to save. But if stimulus is temporary, utility-maximizing agents might save most of it, so propensity to save is not stable.
- ▶ The equations of a macro model should be derived explicitly from a microeconomic model of individual behavior.

The New-Keynesian IS-LM model

- ▶ One-good economy with no K , large number of identical firms, and fixed number of identical infinitely lived households.

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- ▶ Production function: $Y = C = F(L)$; $F'(L) > 0$; $F''(L) \leq 0$

The New-Keynesian IS-LM model

- ▶ One-good economy with no K , large number of identical firms, and fixed number of identical infinitely lived households.
- ▶ Production function: $Y = C = F(L)$; $F'(L) > 0$; $F''(L) \leq 0$
- ▶ Representative household's lifetime utility:

$$U = \sum_{t=0}^{\infty} \beta^t \left[U(C_t) + \Gamma \left(\frac{M_t}{P_t} \right) - V(L_t) \right], \quad 0 < \beta < 1$$

- $U'(\cdot) > 0$ and $U''(\cdot) < 0$;
- $\Gamma'(\cdot) > 0$ and $\Gamma''(\cdot) < 0$;
- $V' > 0$ and $V''(\cdot) > 0$.

- ▶ Choice variables: C and M ;
- ▶ L exogenous (for now);

Evolution of household's wealth

- ▶ Two assets: Central Bank money M (gold coins) and a bond B (a claim on M).
- ▶ Evolution of household's wealth:

$$\begin{aligned} A_{t+1} &= M_t + B_t(1 + i_t) \\ &= M_t + (A_t + W_t L_t - P_t C_t - M_t)(1 + i_t) \end{aligned}$$

- A_{t+1} is wealth at the start of period $t + 1$;
- M_t and B_t are money and bonds held during period t ;

Household's behavior: Euler equation

- ▶ Assuming CRRA utility, the infinite-horizon utility function implies

$$\ln C_t = \ln C_{t+1} - \frac{1}{\theta} \ln[(1 + r_t)\beta]$$

↓

$$\ln Y_t = a + \ln Y_{t+1} - \frac{1}{\theta} r_t$$

(because $Y = C$ and $\ln(1 + r) \approx r$, and with $a = -(\frac{1}{\theta}) \ln \beta$)

- ▶ See demonstration in Romer Section 6.1

The New-Keynesian IS curve

$$\ln Y_t = a + \ln Y_{t+1} - \frac{1}{\theta} r_t$$

- ▶ negative relation between Y_t and r_t .
- ▶ differences with old-school IS curve:
 - conceptual: driven by intertemporal substitution, not income multiplier effect.
 - practical: $\ln Y_{t+1}$ term.
 - here, IS interpretation requires assuming fixed Y_{t+1} .

John Cochrane on the New Keynesian IS curve:

This new-Keynesian model is an utterly and completely different mechanism and story [relative to the old-keynesian model]. (...)

The marginal propensity to consume is exactly and precisely zero in the new-Keynesian model. There is no income at all on the right hand side [of the Euler equation]. (...)

John Cochrane on the NK IS curve (continued):

The old-Keynesian model is driven completely by an income effect with no substitution effect. Consumers don't think about today vs. the future at all. The new-Keynesian model is based on the intertemporal substitution effect with no income effect at all. (...)

[a lower r_t] induces consumers to spend their money today rather than in the future (...). Now, lowering consumption growth is normally a bad thing. But new-Keynesian modelers assume that the economy reverts to trend, so lowering growth rates is good, and raises the level of consumption today with no ill effects tomorrow.

[from John Cochrane's 'New vs. Old Keynesian Stimulus' (on Keats)]

Household's money demand

- ▶ Optimization requires that marginal increase in M_t/P_t by Δm (given total wealth) has no effect on utility.
- ▶ To leave wealth unchanged, $\Delta C_t = -\left(\frac{i}{1+i}\right)\Delta m$
- ▶ So in equilibrium:

$$\Gamma'\left(\frac{M_t}{P_t}\right)\Delta m = U'(C_t)\left(\frac{i_t}{1+i_t}\right)\Delta m$$

↓

$$\frac{M_t}{P_t} = Y_t^{\theta/\chi} \left(\frac{1+i_t}{i_t}\right)^{1/\chi}$$

- ▶ Real money demand is positive function of Y and negative function of i as in the old-Keynesian model.
- ▶ P and M are fixed, so implies i increasing function of Y .

New-Keynesian IS-LM

- Price of consumption good is assumed fixed:

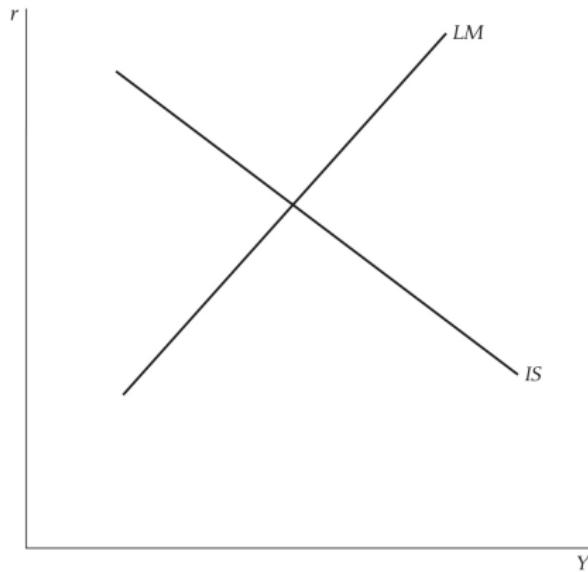
$$P_t = \bar{P} \Rightarrow i_t = r$$

- So both IS and money-demand are in terms of r and Y ;

$$Y_t = f(r_t) \text{ with } f' < 0 \quad (\text{IS curve})$$

$$r_t = g(Y_t) \text{ with } g' > 0 \quad (\text{LM curve})$$

New-Keynesian IS-LM



but remember this is based on the assumption of unchanged
(expectation of) Y_{t+1} !

New-Keynesian IS-LM

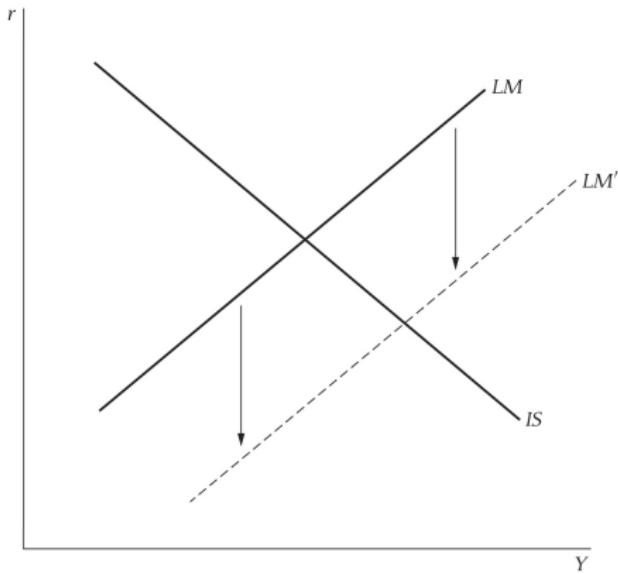
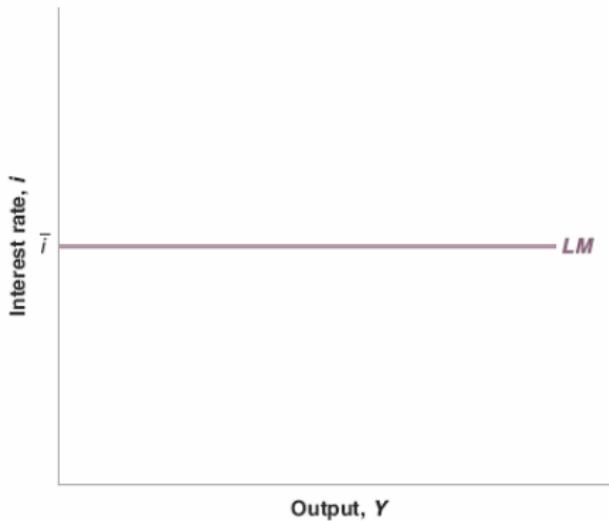


Figure: Effect of a temporary increase in money supply

IS-LM with interest-rate setting

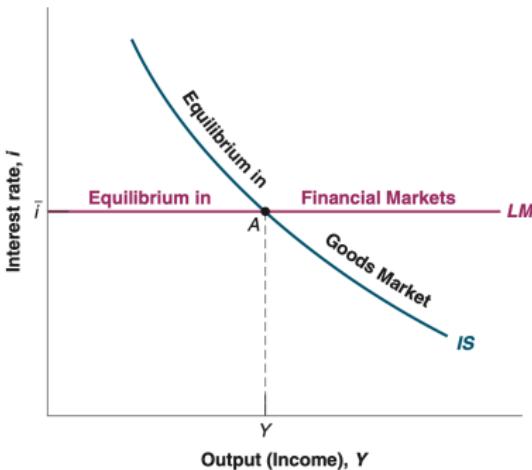
A more realistic LM "curve"

- ▶ In reality, money is endogenous and the Central Bank sets the interest rate.
- ▶ $i = \bar{i}$.



IS-LM with interest-rate setting

- ▶ IS relation: $Y_t = f(r_t)$ with $f' < 0$
- ▶ LM relation: $r = i = \bar{i}$



- ▶ After adding a model of inflation (Phillips Curve), can be enriched by the Central Bank reaction function
- ▶ CB sets the interest rate based on inflation and output.

Phillips Curve(s)

- ▶ IS-LM framework (old or new) needs to be completed with a theory of inflation.
- ▶ *Phillips Curve*: A relation between inflation & unemployment/output.
- ▶ ‘Traditional’ Phillips Curve:

$$\pi_t = \alpha - \beta u_t$$

- ▶ ‘Accelerationist’ Phillips Curve:

$$\pi_t - \pi_{t-1} = \alpha - \beta u_t$$

- ▶ New Keynesian Phillips Curve:

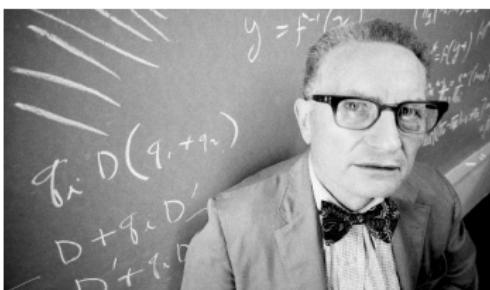
$$\pi_t = k y_t + \beta E_t \pi_{t+1}$$

- ▶ Very different implications for policy.

Phillips Curve(s)

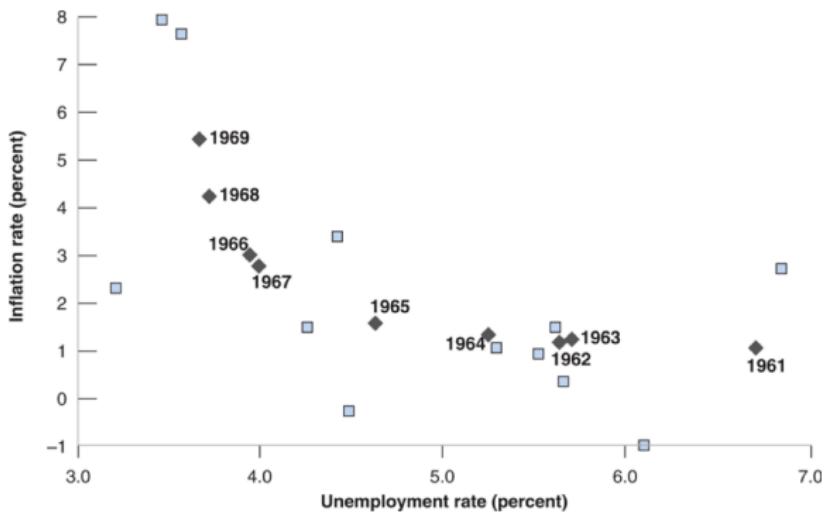
Historical origins of the Phillips Curve

- ▶ PC originally derived from empirical observation, not formal theory.
- ▶ 1958: A.W. Phillips uncovers negative correlation between inflation and unemployment in UK 1861-1957 data.
- ▶ 1960: Samuelson & Solow replicate it on 1900-1960 US data.
- ▶ In the 1970s the relation breaks down, which inspires the development of an 'accelerationist' Phillips Curve.



Phillips Curve(s)

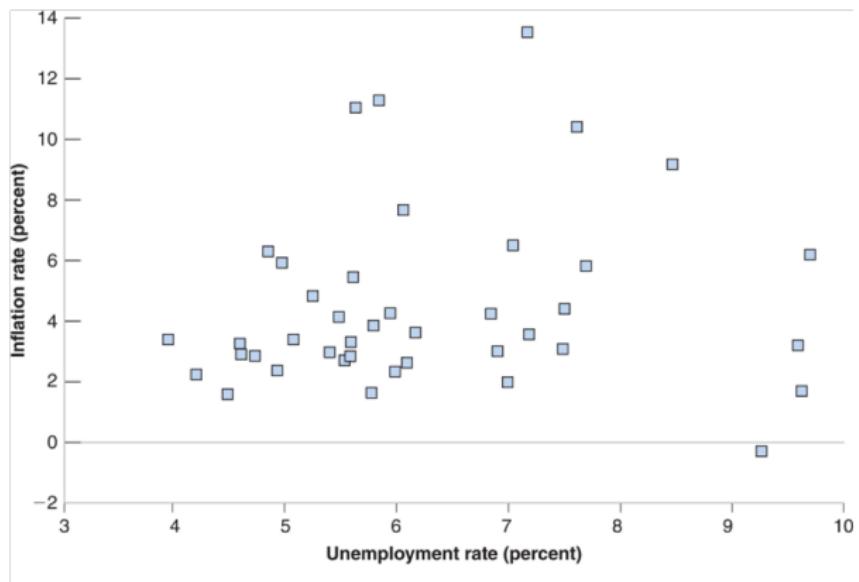
1948-1969: the 'original' Phillips Curve



Source: Series UNRATE,
CPIAUSCL Federal Reserve Eco-
nomic Data (FRED) [http://research.
stlouisfed.org/fred2/](http://research.stlouisfed.org/fred2/)

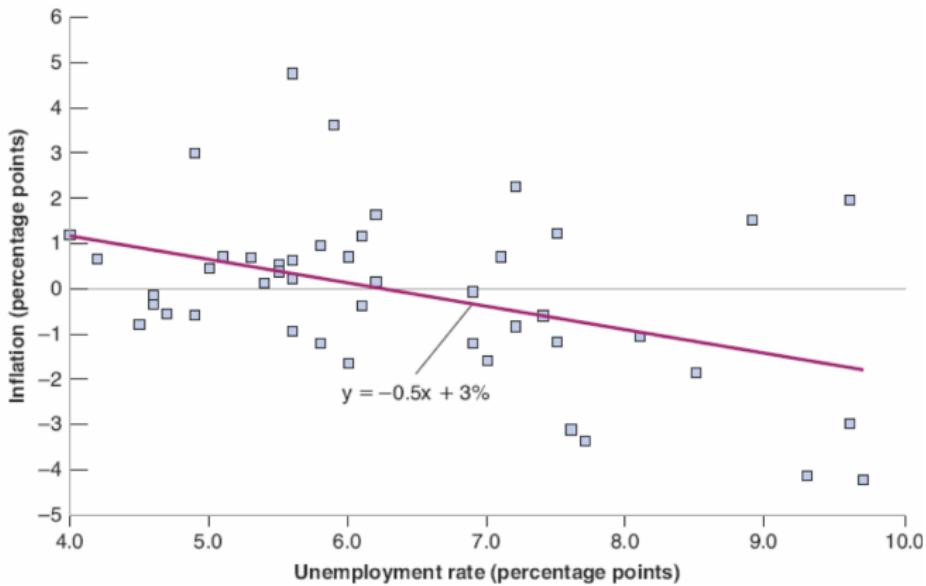
Phillips Curve(s)

1970-2010: the disappearance of the 'original' PC



Phillips Curve(s)

1970-2010: Accelerationist PC



Phillips Curve(s): Theoretical foundations

- ▶ Theoretical explanations of the PC focus on wage and price-setting processes.
- ▶ Models of wage and price-setting imply relations between π , $E(\pi)$ and u
- ▶ specific form of the PC depends on how agents form $E(\pi)$
 1. fixed ('anchored') expectations -> original PC
 2. adaptive expectations -> accelerationist PC
 3. rational expectations -> New-Keynesian PC
- ▶ Traditional & accelerationist PC can be derived from a simple macro model, while New Keynesian PC can be derived from the (more complicated) Calvo model of pricing.

Phillips Curve: a simple framework

- ▶ Traditional and accelerationist PC can be derived from a very simple macro model.
- ▶ Central idea:
 $lower u_t \Rightarrow higher W_t \Rightarrow increase in P_t \& \pi_t.$
- ▶ if it stops here, we have the 'original' PC

Phillips Curve: a simple framework

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- ▶ Central idea:
 $lower u_t \Rightarrow higher W_t \Rightarrow increase in P_t \& \pi_t.$
- ▶ if it stops here, we have the ‘original’ PC
- ▶ BUT with adaptive expectations, inflationary spiral:
 $lower u_t \Rightarrow higher W_t \Rightarrow increase in P_t \& \pi_t \Rightarrow increase in E(\pi_{t+1}) \Rightarrow increase in W_{t+1} \Rightarrow \dots$
- ▶ ‘accelerationist’ PC

Traditional and accelerationist Phillips Curves

- Basic model:

$$Y_t = N_t$$

$$P_t = (1 + m)W_t$$

$$\frac{W_t}{E(P_t)} = 1 - \beta u_t \quad \Rightarrow \quad W_t = E(P_t)(1 - \beta u_t)$$

- Y = output;
- N = employment;
- W = nominal wage;
- P = price of the good;
- m = mark-up;
- $u = 1 - \frac{N}{L}$ = unemployment rate;

Traditional and accelerationist Phillips Curves

- ▶ Combine price-setting & wage-setting:

$$P_t = E(P_t)(1 + m)(1 - \beta u_t)$$

- ▶ rewrite (approximately) in terms of π :

$$\pi_t = E(\pi_t) + m - \beta u_t$$

- ▶ What determines $E(\pi_t)$?

Traditional and accelerationist Phillips Curves

- ▶ ‘Generic’ Phillips Curve:

$$\pi_t = E(\pi_t) + m - \beta u_t$$

- ▶ Assume fixed expectations

$$E(\pi) = \bar{\pi}$$

- ▶ Then we have

$$\pi_t = \alpha - \beta u_t \quad (\text{with } \alpha = \bar{\pi} + m)$$

- ▶ ‘original’ (old-Keynesian) Phillips curve
- ▶ Inflation-unemployment trade-off for policy.

The PC and its mutations

- ▶ ‘Generic’ Phillips Curve:

$$\pi_t = E(\pi_t) + m - \beta u_t$$

- ▶ Assume adaptive expectations

$$E(\pi) = \pi_{t-1}$$

- ▶ ‘Accelerationist’ PC:

$$\pi_t - \pi_{t-1} = \alpha - \beta u_t$$

- ▶ Lower unemployment leads to higher *change* in the inflation rate (like in the 1970s).

Traditional and accelerationist Phillips Curves

An interpretation of inflation in industrialized economies

1948-1969

- ▶ inflation not persistent;
- ▶ wage-setters assumed inflation would revert to mean $\bar{\pi}$;
- ▶ $E(\pi) \approx \bar{\pi} \Rightarrow$ Original PC.

after 1970

- ▶ inflation became persistent (oil shocks);
- ▶ wage-setters started taking persistence into account;
- ▶ $E(\pi_t) \approx \pi_{t-1} \Rightarrow$ accelerationist PC.



The equilibrium unemployment rate

In this model, a unique unemployment rate makes inflation equal expected inflation:

$$\pi_t = \mathbb{E}(\pi_t) \rightarrow u_t^* = \frac{m}{\beta}$$

Implications for traditional PC:

- ▶ Possible to sustain $u < u_t^*$ only as long as $\pi > \mathbb{E}(\pi)$.
- ▶ But if $\pi > \mathbb{E}(\pi)$ is persistent, wage-setters would surely update their expectations!
- ▶ Traditional PC with anchored expectations unlikely to be stable unless $u = u^*$.

The equilibrium unemployment rate

In this model, a unique unemployment rate makes inflation equal expected inflation:

$$\pi_t = \mathbb{E}(\pi_t) \rightarrow u_t^* = \frac{m}{\beta}$$

Implications for accelerationist PC:

- ▶ When $u = u^*$, inflation is stable over time ($\pi_t = \pi_{t-1}$).
- ▶ $u < u^*$ leads to accelerating inflation (increasing over time).
- ▶ $u > u^*$ leads to deflation (decreasing over time).
- ▶ Disinflation is painful: to bring down π , you need $u > u^*$ for a period of time.

Calvo price setting model

- ▶ New Keynesian PC is derived from a more complex model of dynamic price setting.
- ▶ Calvo (1983) "Staggered prices in a utility-maximizing framework".
- ▶ Sticky prices: they cannot be adjusted in all periods.
- ▶ Opportunities to change prices arrive randomly.
 - *Poisson process*: same probability of price adjustment in every period.
- ▶ A bit arbitrary: chosen as the baseline model of prices not because realistic, but because it happens to deliver a convenient PC that works well in a DSGE model.

Calvo model

Framework (1/3)

- ▶ A monopolistic competition model
- ▶ Production function

$$Y_t = L_t$$

- ▶ Closed economy with no government and no capital:

$$C_t = Y_t$$

- ▶ Exogenous nominal expenditure (aggregate demand)

$$M_t = Y_t P_t$$

- ▶ Labor supply curve

$$\frac{W_t}{P_t} = BY_t^{\theta+\gamma-1}$$

- ▶ Monopolistic pricing

$$\frac{P_{it}^*}{P_t} = \frac{\eta}{\eta-1} \frac{W_t}{P_t}$$

Framework (2/3)

Time-dependent price-adjustment:

- ▶ Firms cannot adjust their prices in all periods.
- ▶ P_i set at time 0 has probability $q_t \geq 0$ of remaining in effect at time $t > 0$.
- ▶ $p_t \equiv \ln(P_t)$.
- ▶ firm sets p_i as a weighted average of expected future p_t^* 's:

$$p_i = \sum_{t=0}^{\infty} \tilde{\omega}_t E[p_t^*] \quad \text{with} \quad \tilde{\omega}_t \equiv \frac{\beta^t q_t}{\sum_{\tau=0}^{\infty} \beta^{\tau} q_{\tau}}$$

Framework (3/3)

- ▶ Profit-maximizing price is a mark-up over the wage

$$\frac{P_{it}^*}{P_t} = \frac{\eta}{\eta-1} \frac{W_t}{P_t} \Rightarrow p_t^* = \ln \left[\frac{\eta}{\eta-1} \right] + w_t$$

- ▶ Substitute in the (log of the) labor supply curve

$$w_t = p_t + \ln B + (\theta + \gamma - 1)y_t \Rightarrow p^* = p + \ln \frac{\eta}{\eta+1} + \ln B + (\theta + \gamma - 1)y_t$$

- ▶ Given that $m = y + p$, and assuming for simplicity $\ln \frac{\eta}{\eta-1} + \ln B = 0$,

$$p_t^* = \phi m_t + (1 - \phi)p_t \quad \text{with } \phi = (\theta + \gamma - 1)$$

- ▶ optimal 'sticky' price to set at time 0:

$$p_i = \sum_{t=0}^{\infty} \tilde{\omega}_t E_0[\phi m_t + (1 - \phi)p_t]$$

Deriving π

- ▶ Each period share α of firms, randomly chosen, adjusts prices

aggregate price level: $p_t = \alpha x_t + (1 - \alpha)p_{t-1}$

inflation: $\pi_t = p_t - p_{t-1} = \alpha(x_t - p_{t-1})$

Deriving π

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aggregate price level: $p_t = \alpha x_t + (1 - \alpha)p_{t-1}$

inflation: $\pi_t = p_t - p_{t-1} = \alpha(x_t - p_{t-1})$

- ▶ optimal 'sticky' prices:

$$x_t = \sum_{j=0}^{\infty} \tilde{\omega}_j E(p_{t+j}^*) \quad \text{with} \quad \tilde{\omega}_j = \frac{\beta^j q_j}{\sum_{k=0}^{\infty} \beta^k q_k}$$

- ▶ Poisson process implies $q_j = (1 - \alpha)^j$

$$\rightarrow \sum_{k=0}^{\infty} \beta^k q_k = \sum_{k=0}^{\infty} \beta^k (1 - \alpha)^k = \frac{1}{1 - \beta(1 - \alpha)}$$

Calvo model - deriving π

- ...plugging in:

$$x_t = [1 - \beta(1 - \alpha)] \sum_{j=0}^{\infty} \beta^j (1 - \alpha)^j E_t p_{t+j}^*$$

- Rewrite in terms of p_t^* and $E_t x_{t+1}$:

$$\begin{aligned} x_t &= [1 - \beta(1 - \alpha)] \left(p_t^* + \beta(1 - \alpha) \left[\sum_{j=0}^{\infty} \beta^j (1 - \alpha)^j E_t p_{t+1+j}^* \right] \right) = \\ &= [1 - \beta(1 - \alpha)] p_t^* + \beta(1 - \alpha) [1 - \beta(1 - \alpha)] \left[\sum_{j=0}^{\infty} \beta^j (1 - \alpha)^j E_t p_{t+1+j}^* \right] = \\ &= [1 - \beta(1 - \alpha)] \mathbf{p}_t^* + \beta(1 - \alpha) \mathbf{E}_t \mathbf{x}_{t+1} \end{aligned}$$

Deriving π

$$x_t = [1 - \beta(1 - \alpha)]p_t^* + \beta(1 - \alpha)E_t x_{t+1}$$

- ▶ Express in terms of π_t , using $\pi_t = \alpha(x_t - p_{t-1})$ and $p^* = \phi m_t + (1 - \phi)p_t$

$$\pi_t = ky_t + \beta E_t \pi_{t+1} \quad \text{with} \quad k = \frac{\alpha[1 - (1 - \alpha)\beta]\phi}{1 - \alpha}$$

- ▶ New-Keynesian Phillips Curve
- ▶ Inflation depends on expected inflation & output (as in all PCs);
- ▶ Difference: it is the expectation of *future* inflation.
- ▶ *Intuition:* higher output → higher costs (wages);
higher future inflation → higher optimal future prices.

3 Phillips Curves and their implications

1. *Old-Keynesian PC:* $\pi_t = \alpha + \lambda y_t$

- ▶ *output-inflation trade-off:* disinflation requires permanently lower y ;

2 *Accelerationist PC:* $\pi_t = \pi_{t-1} + \lambda(y_t - y_t^*)$

- ▶ painful disinflation: requires $y < y^*$ for some time (*inflation inertia*);

3 *New-Keynesian PC:* $\pi_t = k y_t + \beta E_t \pi_{t+1}$

- ▶ expansionary disinflation: $E_t(\pi_{t+1})$ down $\rightarrow y_t$ up.

New Keynesian models of fluctuations

- ▶ IS curve & Phillips curve are the key building blocks of Keynesian & New Keynesian macroeconomics.
- ▶ They can be integrated to build dynamic models of fluctuations.
- ▶ We will consider two:
 1. A simplified New Keynesian model
 2. The canonical New Keynesian DSGE model

A (very) simplified New-Keynesian model

- ▶ IS + PC + Central Bank reaction function
- ▶ Simpler than the canonical New Keynesian DSGE model and not microfounded
- ▶ But captures the New-Keynesian perspective on fluctuations well.
- ▶ Most mainstream policy discussions are implicitly based on this model
- ▶ *Romer (2000), Carlin & Soskice (2005), Blanchard (2017).*

A (very) simplified New-Keynesian model

A 3-equations economy

- ▶ IS Curve:

$$Y_t = A - ar_{t-1} \quad (1)$$

- ▶ Accelerationist PC:

$$\pi_t = \pi_{t-1} + \alpha(Y_t - Y^*) \quad (2)$$

- ▶ Central Bank reaction function:

$$r_t = r^* + \psi(\pi_t - \pi^T) \quad (3)$$

y = output; π = inflation rate; Y^* = potential output; r = interest rate;
 r^* = equilibrium interest rate; π^T = target interest rate;

A (very) simplified New-Keynesian model

Old-Keynesian IS Curve

- ▶ Output:

$$Y_t = C_t + I_t + \bar{G}$$

- ▶ Consumption:

$$C_t = c_0 + c_1(1 - \bar{\tau})Y_t$$

- ▶ Housing investment:

$$I_t = a_0 - a_1 r_{t-1}$$

- ▶ Short-run equilibrium output:

$$Y_t = A - ar_{t-1}$$

$$\text{where } A = \frac{c_0 + a_0 + \bar{G}}{1 - c_1(1 - \bar{\tau})} \text{ and } a = \frac{a_1}{1 - c_1(1 - \bar{\tau})}$$

A (very) simplified New-Keynesian model

Accelerationist Phillips Curve (1/2)

- ▶ Wage setting

$$\frac{W_t}{P_t^e} = 1 - \beta u_t \Rightarrow W_t = P_t^e(1 - \beta u_t)$$

- ▶ Price setting

$$Y_t = N_t \Rightarrow P_t = (1 + m)W_t$$

- ▶ Inflation rate

$$P_t = P_t^e(1 + m)(1 - \beta u_t) \Rightarrow \pi_t = \pi_t^e + m - \beta u_t$$

- ▶ Medium-run equilibrium unemployment rate

$$\pi = \pi^e \Rightarrow u^* = \frac{m}{\beta} \Rightarrow \pi - \pi^e = -\beta(u_t - u^*)$$

A (very) simplified New-Keynesian model

Accelerationist Phillips Curve (2/2)

- ▶ Phillips curve

$$\pi - \pi^e = -\beta(u_t - u^*)$$

- ▶ Assuming adaptive expectations

$$\pi^e = \pi_{t-1} \Rightarrow \pi_t = \pi_{t-1} - \beta(u_t - u^*)$$

- ▶ Rewrite in terms of output

$$\pi_t = \pi_{t-1} + \alpha(Y_t - Y^*)$$

- ▶ Define equilibrium ('natural') interest rate:

$$Y^* = A - ar^* \Rightarrow Y_t - Y^* = -a(r_{t-1} - r^*)$$

A (very) simplified New-Keynesian model

Central Bank reaction function

- CB minimizes a loss function

$$\min_r \ell = (Y_t - Y^*)^2 + \gamma(\pi - \pi^T)^2$$

- CB's desired output gap

$$Y_t - Y^* = -\alpha\gamma(\pi_t - \pi^T)$$

- CB choice of interest rate (*Monetary policy rule*)

$$r_t = r^* + \psi(\pi_t - \pi^T)$$

$$\text{with } \psi = \frac{1}{a(\alpha + \frac{1}{\alpha\gamma})}$$

A (very) simplified New-Keynesian model

A 3-equations economy

- ▶ IS Curve:

$$Y_t = A - ar_{t-1} \quad (1)$$

- ▶ Accelerationist PC:

$$\pi_t = \pi_{t-1} + \alpha(Y_t - Y^*) \quad (2)$$

- ▶ Central Bank reaction function:

$$r_t = r^* + \psi(\pi_t - \pi^T) \quad (3)$$

- ▶ Equilibrium:

$$y = y^*; \quad u = u^*; \quad r = r^*; \quad \pi = \pi^T$$

A (very) simplified New-Keynesian model

Out of equilibrium dynamics

- ▶ suppose $y = y^*$, $r = r^*$ and $\pi = \pi^T$ initially
- ▶ a positive demand shock occurs, eg $c_0 \uparrow$

1 Economic boom:

$$y > y^*, \quad u < u^*; \quad r^* \uparrow;$$

2 Accelerating inflation:

$$\pi > \pi^T \text{ and rising}$$

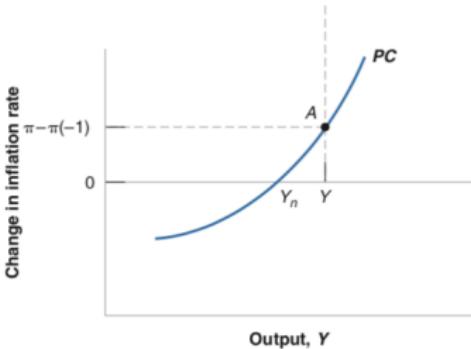
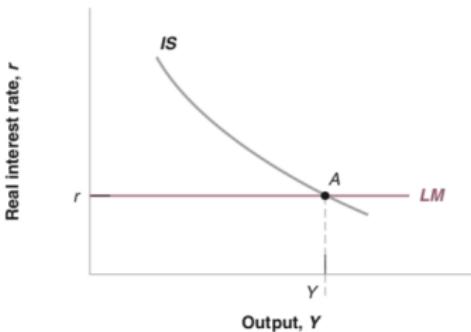
3 CB reaction and downturn:

$$r \uparrow; r > r^* \Rightarrow Y \downarrow; Y < Y^*.$$

4 Stabilization:

$$\pi = \pi^T; \quad r = r^*; \quad Y = Y^*$$

A short-run equilibrium with output above potential



Challenges

Challenges for the simplified New Keynesian model

Five critical and potentially problematic assumptions:

1. Monetary policy always effective in increasing output;
2. Policy-makers have a good estimate of a well-defined u^* and other key parameters;
3. Low unemployment always translates in higher wages & prices;
4. The level of potential output is unaffected by changes in demand;
5. Low interest rates have no negative side-effects

The baseline New Keynesian DSGE model

- ▶ New-Keynesian IS curve

$$y_t = E_t[y_{t+1}] - \frac{1}{\theta} r_t + u_t^{IS} \quad \text{with} \quad \theta > 0$$

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- ▶ New-Keynesian Phillips Curve

$$\pi_t = \beta E_t[\pi_{t+1}] + k y_t + u_t^\pi \quad \text{with } 0 < \beta < 1, k > 0$$

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- ▶ Monetary policy rule

$$r_t = \phi_\pi E_t[\pi_{t+1}] + \phi_y E_t[y_{t+1}] + u_t^{MP} \quad \text{with } \phi_\pi > 0, \phi_y \geq 0$$

The canonical NK model

The baseline New Keynesian DSGE model

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- ▶ no constants: deviations from steady-state, normalized to 0

The baseline New Keynesian DSGE model

$$\text{NK IS curve: } y_t = E_t[y_{t+1}] - \frac{1}{\theta} r_t + u_t^{IS} \quad \text{with } \theta > 0$$

$$\text{NK PC: } \pi_t = \beta E_t[\pi_{t+1}] + k y_t + u_t^\pi \quad \text{with } 0 < \beta < 1, k > 0$$

$$\text{MP rule: } r_t = \phi_\pi E_t[\pi_{t+1}] + \phi_y E_t[y_{t+1}] + u_t^{MP} \quad \text{with } \phi_\pi > 0, \phi_y \geq 0$$

► shocks structure:

$$u_t^{IS} = \rho_{IS} u_{t-1}^{IS} + e_t^{IS}, \quad -1 < \rho_{IS} < 1$$

$$u_t^\pi = \rho_\pi u_{t-1}^\pi + e_t^\pi, \quad -1 < \rho_\pi < 1$$

$$u_t^{MP} = \rho_{MP} u_{t-1}^{MP} + e_t^{MP}, \quad -1 < \rho_{MP} < 1$$

Solving the 3-equations model

- ▶ Express the model in terms only of shocks and expectations;
- ▶ plug the MP rule into the IS curve:

$$y_t = -\frac{\phi_\pi}{\theta} E_t[\pi_{t+1}] + \left(1 - \frac{\phi_y}{\theta}\right) E_t[y_{t+1}] + u_t^{IS} - \frac{1}{\theta} u_t^{MP}$$

- ▶ plug the equation above into the NK PC:

$$\pi_t = \left(\beta - \frac{\phi_\pi k}{\theta}\right) E_t[\pi_{t+1}] + \left(1 - \frac{\phi_y}{\theta}\right) k E_t[y_{t+1}] + k u_t^{IS} + u_t^\pi - \frac{k}{\theta} u_t^{MP}$$

The canonical NK model

Special case: no serial correlation in shocks

- ▶ Assume $\rho_{IS} = \rho_\pi = \rho_{MP} = 0$.
- ▶ So the following is a solution:

$$E_t[y_{t+1}] = E_t[\pi_{t+1}] = 0$$

$$y_t = e_t^{IS} - \frac{1}{\theta} e_t^{MP}$$

$$\pi_t = k e_t^{IS} + e_t^\pi - \frac{k}{\theta} e_t^{MP}$$

$$r_t = e_t^{MP}$$

The canonical NK model

Special case: no serial correlation in shocks

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$$y_t = e_t^{IS} - \frac{1}{\theta} e_t^{MP}$$

$$\pi_t = k e_t^{IS} + e_t^\pi - \frac{k}{\theta} e_t^{MP}$$

$$r_t = e_t^{MP}$$

- ▶ shows effect of demand, monetary policy and inflation shocks;
- ▶ *no internal propagation mechanisms*: without assuming serial correlation in shocks, we don't get any persistence (just like RBC).

The general case

- ▶ Method of undetermined coefficients;
- ▶ Educated guess:

$$y_t = a_{IS} u_t^{IS} + a_\pi u_t^\pi + a_{MP} u_t^{MP}$$

$$\pi_t = b_{IS} u_t^{IS} + b_\pi u_t^\pi + b_{MP} u_t^{MP}$$

- ▶ Plug these into the y_t and π_t functions derived earlier;
- ▶ solve the resulting system of equations to get the a 's and b 's;
- ▶ we will skip the algebra and directly discuss implications for the effects of shocks;

Implications of the general case

► Assumptions:

- A period is a quarter;
- $\theta = 1$ in utility function;
- $k = 0.172$ & $\beta = 0.99$ in PC;
- $\phi_\pi = 0.5$ & $\phi_y = 0.125$ in MP;
- $\rho = 0.5$ for all shocks.

► Effect of *MP* shock:

- $y_t = -1.54u_t^{MP}$;
- $\pi_t = -0.53u_t^{MP}$;
- $r_t = 0.77u_t^{MP}$

► Effect of *IS* shock:

- $y_t = 1.54u_t^{IS}$;
- $\pi_t = 0.53u_t^{IS}$;
- $r_t = 0.23u_t^{IS}$.

► Effect of π shock:

- $y_t = -0.76u_t^\pi$;
- $\pi_t = 1.72u_t^\pi$;
- $r_t = 0.38u_t^\pi$.

Application:

*Monetary policy rules and macroeconomic stability:
Evidence and some theory*

by Clarida, Gali and Gertler (2000)

- ▶ Uses the canonical NK model to explain disinflation in the US in the 1980s
- ▶ Argues that a change in the conduct of monetary policy explains the stabilization of inflation.
- ▶ Available on Keats

The canonical NK model

- ▶ All kinds of extensions in the literature, but this remains the basic model
- ▶ Some problems:
 - ▶ No unemployment (workers are on their supply curve)
 - ▶ All consumers are forward-looking and unconstrained by liquidity.
 - ▶ No internal propagation mechanisms (effects of shocks are not persistent except by assumption).
 - ▶ Implications of the NK PC about effect of anticipated disinflation are wildly unrealistic.
 - ▶ *Forward guidance puzzle*: announced temporary interest rate reduction in the distant future has an enormous effect on inflation today (pretty weird)

The canonical NK model

DSGE models: optimistic vs pessimistic views

The optimistic view:

- ▶ DSGE describe reasonably well the behavior of macro aggregates...
- ▶ ... and are micro-founded so their parameters are plausibly policy-invariant;
- ▶ Extensions are making them more realistic, and technology allows analysis of ever more sophisticated versions (including HANK);
- ▶ macroeconomists should all focus on further improving DSGE models.

Pessimistic view:

- ▶ The baseline model actually produces embarrassing predictions...
- ▶ ...and only large ad-hoc modifications just designed to make the models' implications more reasonable attenuate that;
- ▶ macroeconomists should seek radically different alternatives (back to old-school Keynesian? agent-based models? no all-encompassing model at all? a type of model that has not been conceived yet?).