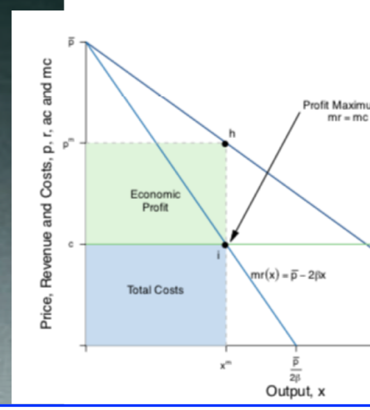
6 – Competition, rent-seeking & market equilibrium



Extended slides
this is an extended version (much more crowded with text and with additional explanations) of the slides I will project in class. You can use them as lecture notes.

Takeaways

- For a firm, maximizing profits means setting MR=MC (marginal revenue=marginal cost).
- Market power leads to lower output and higher prices, relative to 'social optimum'.
- Social efficiency reached when p=MC.
- Because firms have market power, usually p>MC.

The Plan

1. Basic ideas on firm behavior

2. The Cournot model of competition

3. Competition & social well-being

1- Basic ideas

- Assume that a firm chooses the quantity of the good to be produced (x).
- The price (p) is a function of the quantity produced: $p = p(x)$
  - *if you produce more, price goes down.*
- Revenues (r) equal quantity sold times price:

$$r = px = p(x)x$$

- Marginal revenue (MR): increase in revenues if you increase production by one unit

$$MR = \frac{\partial r}{\partial x}$$

1- Basic ideas

- The total cost of production (C) increases with the quantity produced (x):

$$C = C(x)$$

- Marginal cost (MC): The increase in total costs when you increase production by one unit

$$MC = \frac{\partial C}{\partial x}$$

1- Basic ideas

- Profits (π) are equal to revenues minus costs:

$$\pi(x) = r(x) - c(x)$$

- Because revenues and costs depend on the quantity produced (x), also profits depend on x.
- The firm chooses x in such a way as to maximize π
- This is found by setting $\frac{\partial \pi}{\partial x} = 0$ and solving for x.
- Shortcut: this is equivalent to setting MR=MC *(see lecture notes 'the principle of profit maximization' for the demonstration).*

1 – The Cournot Model

Figure 9.2: **Antoine Augustin Cournot (1801-1877)** developed the first mathematical models of the process of competition among firms ranging from two (duopoly), to a few (oligoploly), to many.

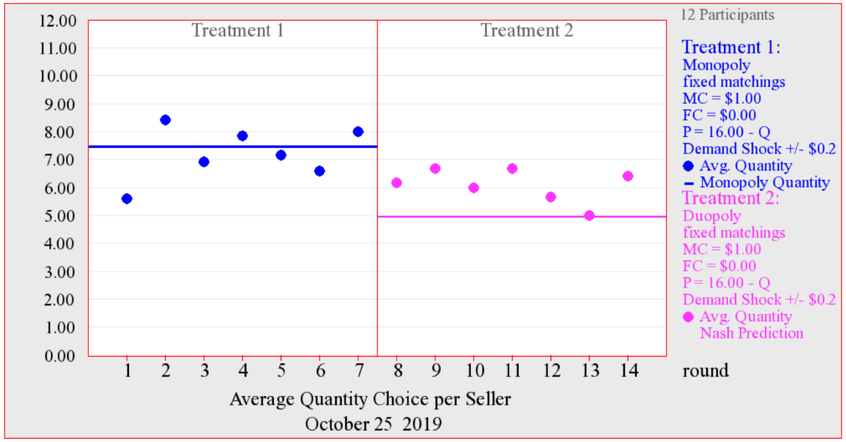
1 – The Cournot Model

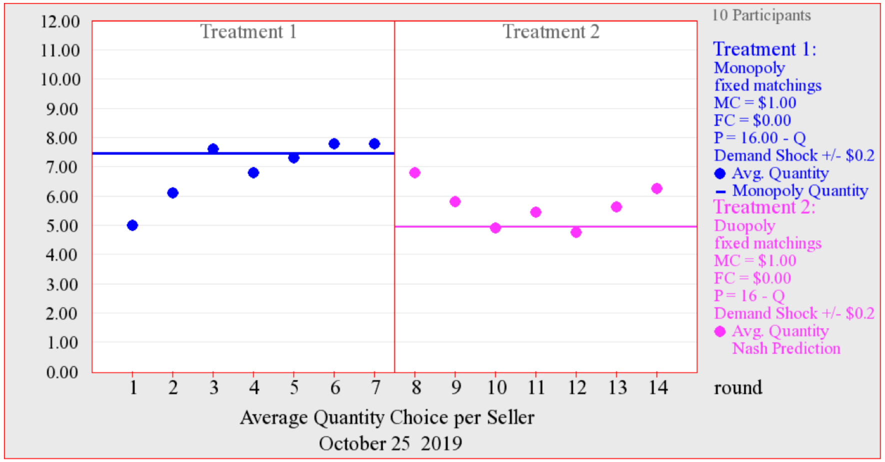
Cournot model of competition

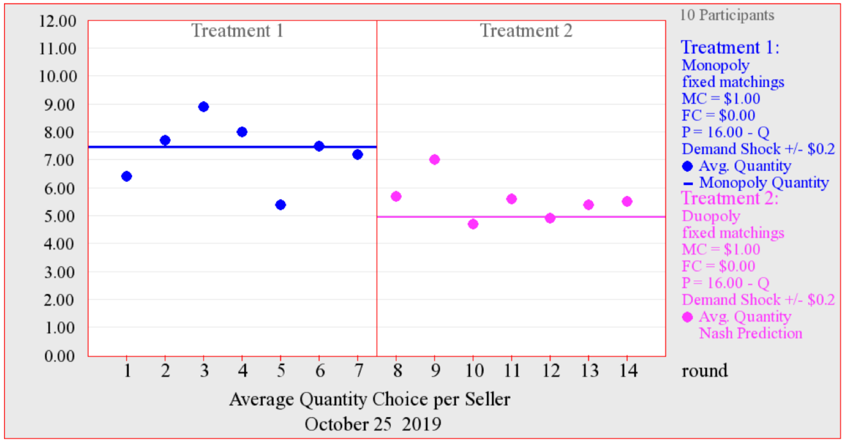
- Theoretical model to understand how quantity produced and price are determined in an industry.
- Take-away message: The number of firms present in the industry determines their market power, which in turn determines the market outcomes.
- We will analyze the model with:
  - only one firm (monopoly)
  - two firms (duopoly)
  - few firms (oligopoly)
  - many firms (competition).

1 – The Cournot Model

Last year's Econ 203 playing the Cournot game in discussion sections:







When passing from monopoly to duopoly, each of you produced slightly less, but each industry (=set of firms selling same product) produced more. Prices went down.

This is exactly what the Cournot model predicts!

1 – The Cournot Model

- Firms sell an identical (*standardized*) product.
- The *price* they can charge depends on the *total quantity* produced in the industry.
  - The higher the industry output, the lower the price.
- Each firm takes other firms' output as given.
- Each firm chooses its optimal level of output given the other firms' output (best-response).
- The best-response level of output is the one that makes MR=MC (*principle of profit-maximization* – see lecture notes on Moodle)
- Nash Equilibrium = mutual best-response
  - In a NE, each firm chooses its profit-maximizing output level, given the quantities produced by its competitors.

Demand function

- The *price* that can be charged for the product depends on the level of *industry output*
  - The higher the *quantity* of the good on the market, the lower the *price* that can be charged for it.
- Specifically, we assume this *inverse demand function*:

$$p(X) = \bar{p} - \beta X$$

X = industry output (sum of the output of all firms producing good *x*).

- *(called 'inverse' because in a demand function the quantity demanded is a function of the price, while here we have rearranged it, to have the price p as a function of the quantity X.)*

The Cost Function

- Production costs depend on the firm's output (x)
- Specifically, we assume the following *cost function*:

$$\mathrm{C}(x) = cx$$

(no fixed cost and constant marginal cost)

- We will assume these *(inverse) demand function* and *cost function* in all the versions of the Cournot model (1,2 and *n* firms) that we will analyze.

1 – The Cournot Model

- Inverse demand function:

$$p(X) = \bar{p} - \beta X$$

- Cost function:

$$\mathrm{C}(x) = cx$$

- Marginal cost $= \frac{dC}{dx} = c$

Case (1): Monopoly (n=1)

- 1 firm in the industry: industry output=firm's output (x=X)
- The monopolist maximizes profits subject to the demand curve.
- Profits are maximized when MR=MC.
- The monopolist chooses the output level which makes MR=MC
- It does not need to take into account what competitors are doing: market price depends only on its production decision.
  - No interdependence; no externalities.

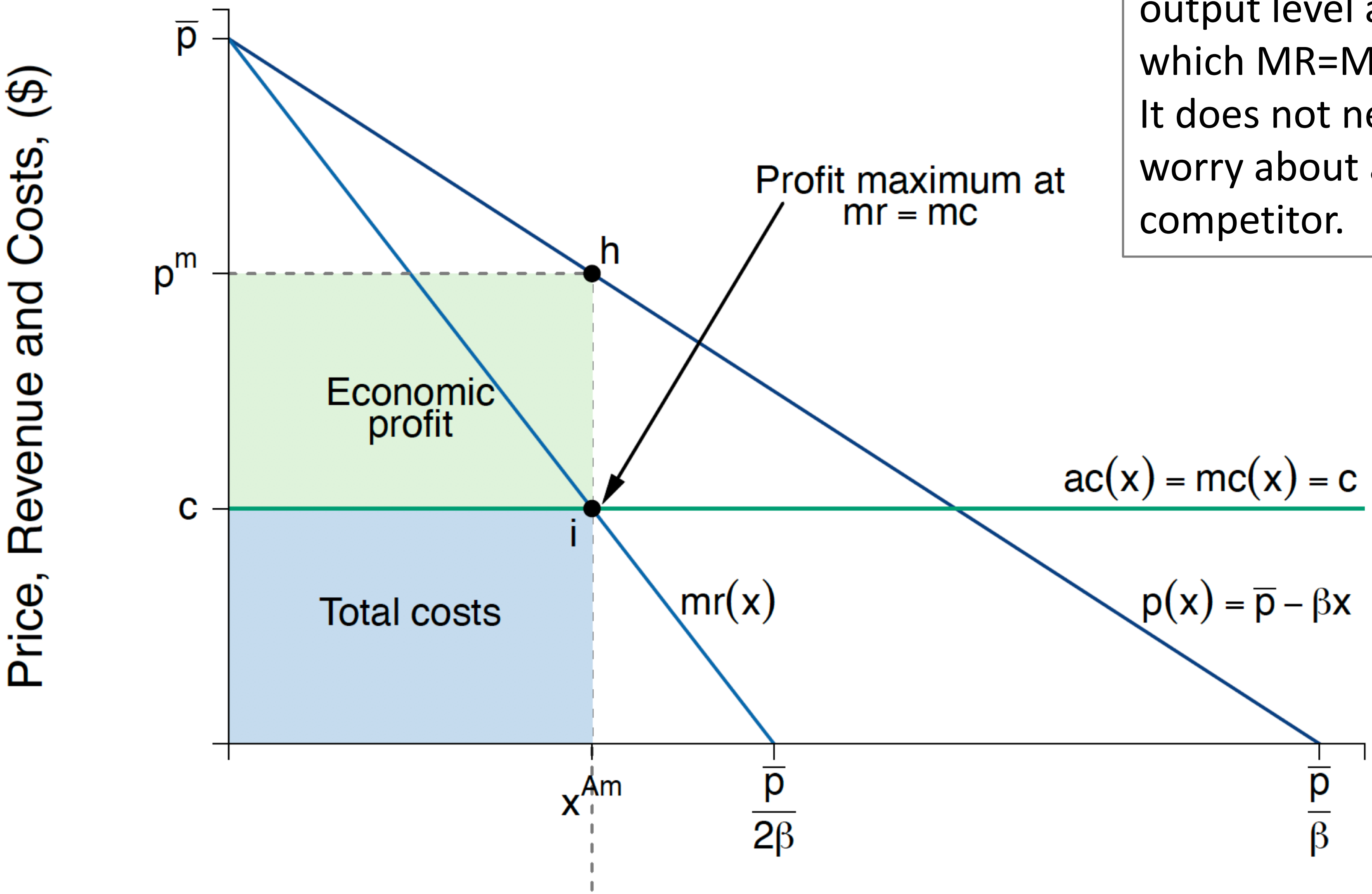
Revenues: $r^{mon} = p(x) * x = (\bar{p} - \beta x)x$

Profits: $\pi^{mon} = r(x) - C(x) = (\bar{p} - \beta x)x - cx$

Profit maximization: MR=MC → $\bar{p} - 2\beta x = c$ → $x^{mon} = \frac{\bar{p}-c}{2\beta}$

$$p^{mon} = c + \frac{1}{2}(\bar{p} - c)$$

1 – The Cournot Model



A monopolist maximizes profits by choosing the output level at which MR=MC. It does not need to worry about any competitor.

Case (2): Duopoly (n=2)

- Total output is the sum of the output of two firms (A and B)

Duopoly market output: $X = x^A + x^B$

- Each firm must take into account what the other firm is doing
  - > the output level of both firms will influence the product's price.
  - > so the revenues of A depend also on B's output (and vice-versa).
  - > each firm inflicts a negative externality on the other.
- Each firm chooses the output level that makes MR=MC.
- But now the marginal revenue of firm A depends not only on x^A, but also on x^B (and vice-versa).
- Now MR=MC leads to a *Best Response Function*.
  - > A's optimal output is a function of B's output (and vice-versa).

Best-response output in a duopoly

- A's profit: $\pi^A = r(x^A, x^B) - c(x^A) =$
$$= p(X) * x^A - cx^A =$$
$$= [\bar{p} - \beta(x^A + x^B)]x^A - cx^A$$

- Maximizing this profit function w.r.t. (own) output implies setting MR=MC:

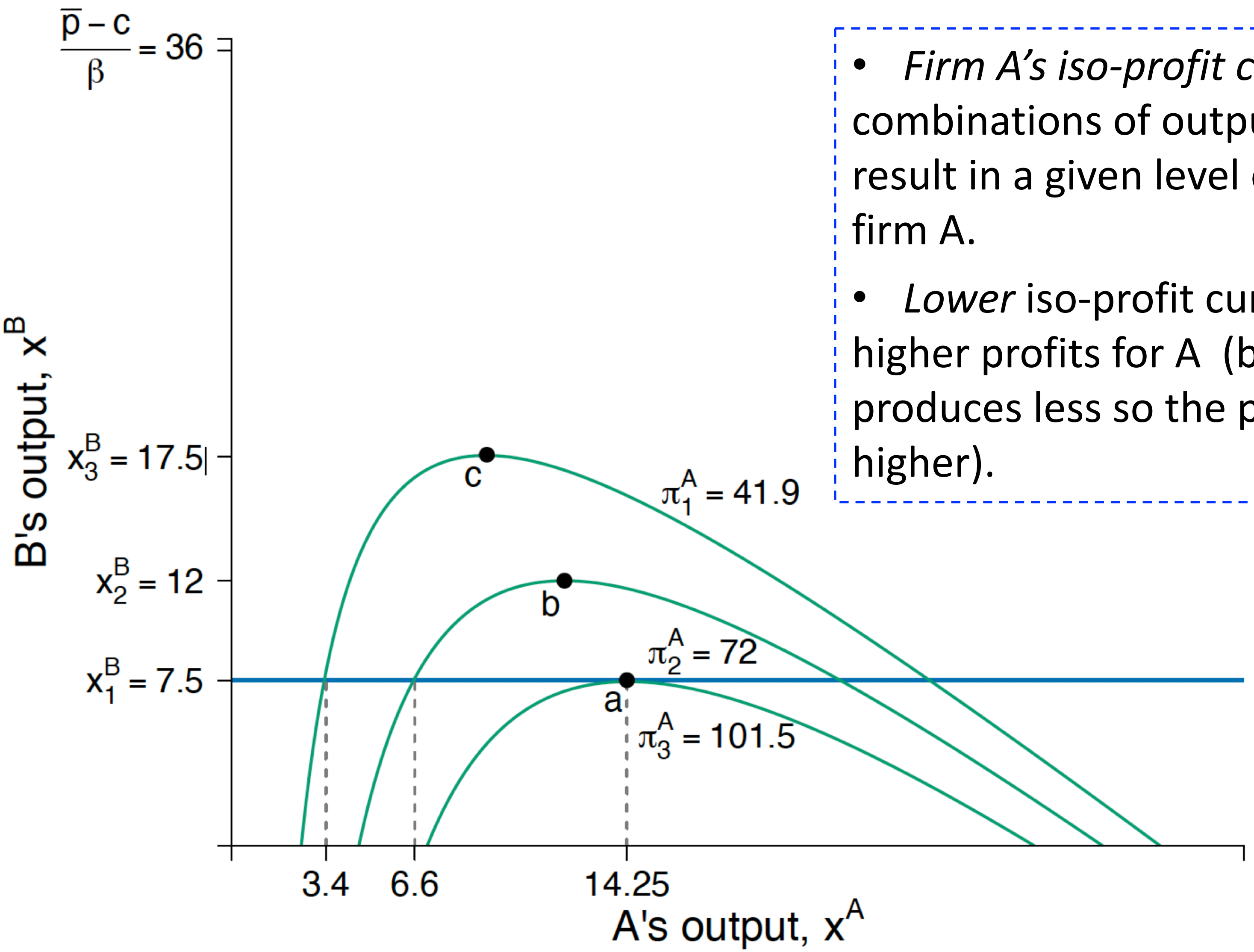
$$(\bar{p} - \beta x^B) - 2\beta x_A = c$$

$$\Downarrow$$

$$x_A = \frac{\bar{p}-c}{2\beta} - \frac{1}{2}x^B \quad \text{(A's BRF)}$$

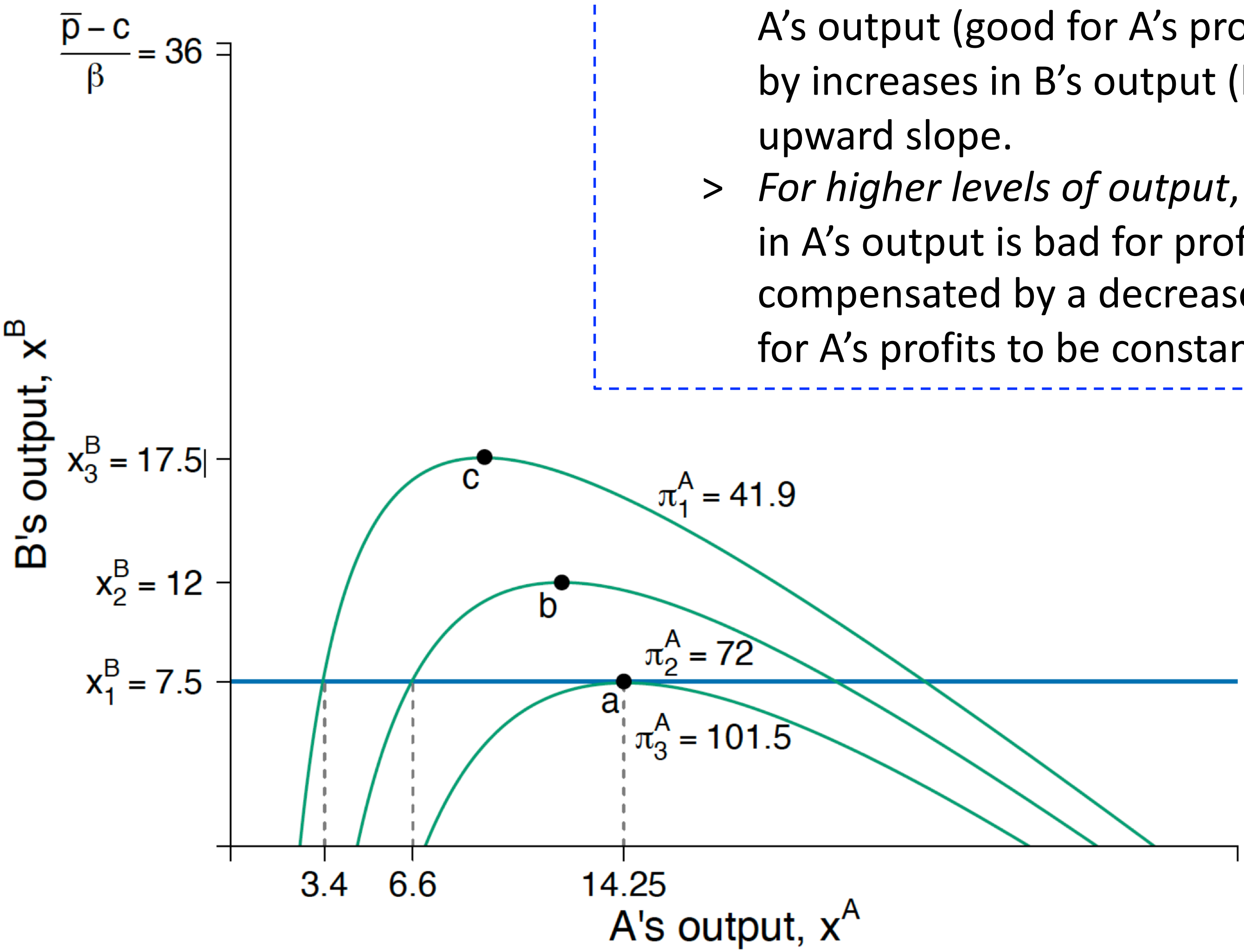
Symmetrically, $x_B = \frac{\bar{p}-c}{2\beta} - \frac{1}{2}x^A$ (B's BRF)

Iso-profits curves in a duopoly



- *Firm A's iso-profit curves*: combinations of outputs (x^A, x^B) that result in a given level of profits for firm A.
- *Lower* iso-profit curves imply higher profits for A (because B produces less so the product price is higher).

- *inverted U-shape* because of the demand function
  - *For lower levels of output*, MR>MC. So increases in A's output (good for A's profits) are compensated by increases in B's output (bad for A's profits): upward slope.
  - *For higher levels of output*, MR<MC, so the increase in A's output is bad for profits and must be compensated by a decrease in B's output, in order for A's profits to be constant: downward slope.

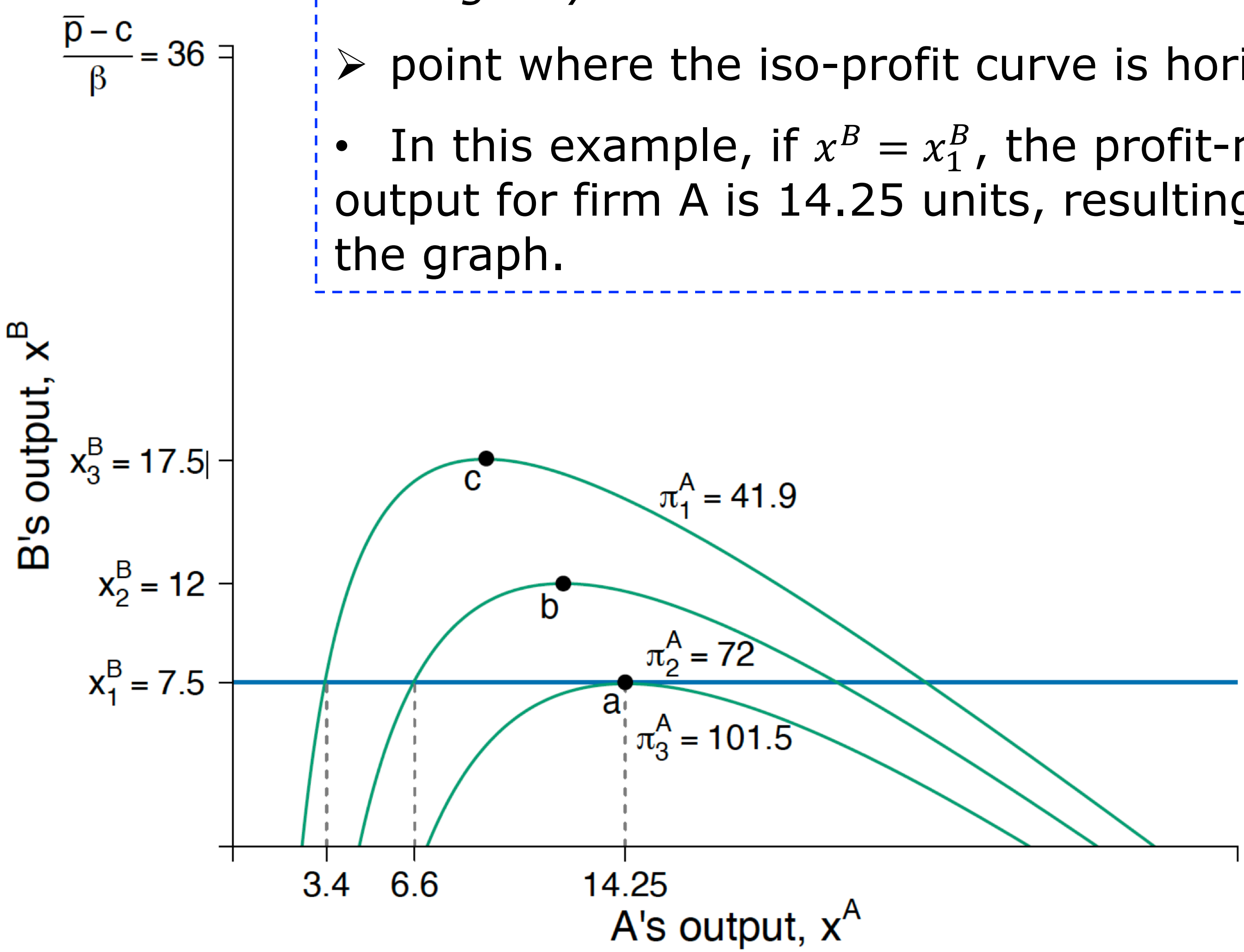


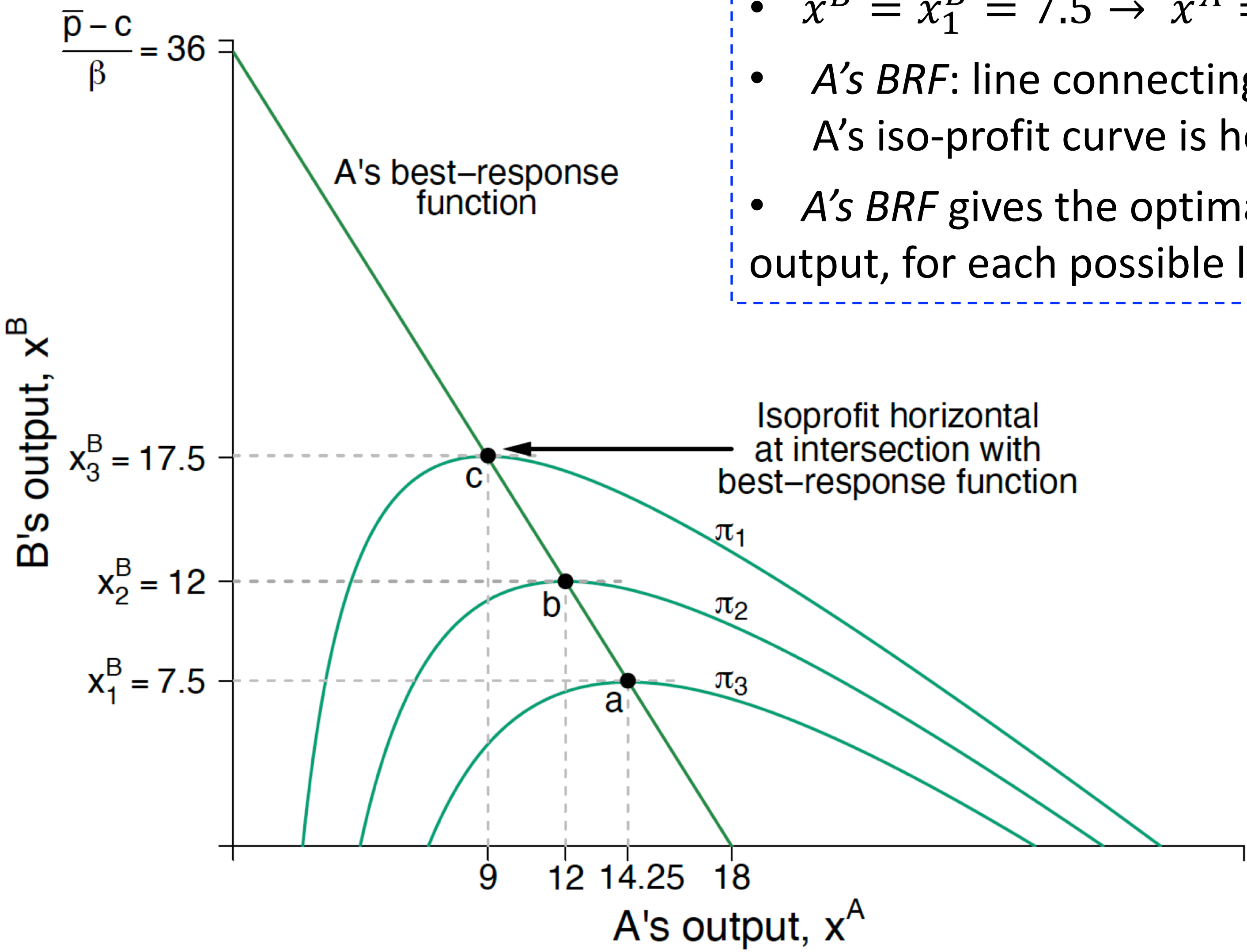
- For a given level of x^B (horizontal line), firm A wants to be on its lowest possible iso-profit curve.

➢ *tangency* between x^B level and A's iso-profit curve.

➢ point where the iso-profit curve is horizontal (slope=0)

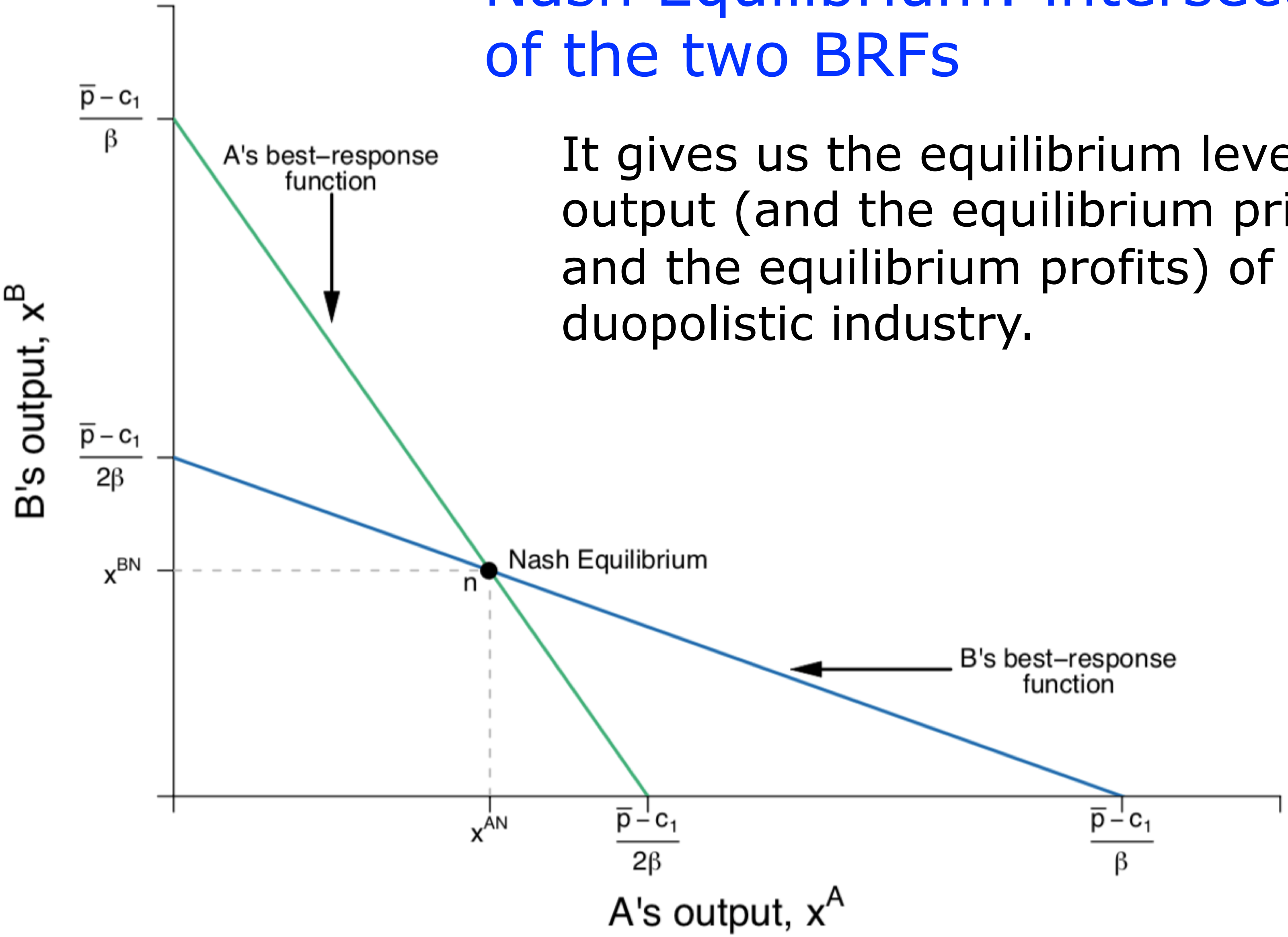
- In this example, if $x^B = x_1^B$, the profit-maximizing level of output for firm A is 14.25 units, resulting in outcome *a* in the graph.





- $x^B = x_3^B = 17.5 \rightarrow\ x^A = 9\ \rightarrow$ point c
- $x^B = x_2^B = 12 \rightarrow\ x^A = 12\ \rightarrow$ point b
- $x^B = x_1^B = 7.5 \rightarrow\ x^A = 14.25\ \rightarrow$point a
- *A's BRF*: line connecting all points where A's iso-profit curve is horizontal.
- *A's BRF* gives the optimal level of A's output, for each possible level of B's output.

Nash Equilibrium: intersection of the two BRFs



It gives us the equilibrium levels of output (and the equilibrium price and the equilibrium profits) of the duopolistic industry.

- We find the duopoly Nash Equilibrium by solving the system given by the two BRFs. We obtain:
  - $x_A^N = x_B^N = \frac{\bar{p}-c}{3\beta}$
  - $p^N = c + \frac{1}{3}(\bar{p} - c)$
- Compare them with the equilibrium p and x of the monopolistic industry that we found before:
  - $x^{mon} = \frac{\bar{p}-c}{2\beta}$
  - $p^{mon} = c + \frac{1}{2}(\bar{p} - c)$

Under duopoly, total industry output is higher and price lower, relative to monopoly.
(check for yourself by looking at the formulas we found!)

Summing up what we have so far:

- *A monopolist* sets the level of output at which MR=MC.

MONOPOLIST OUTPUT: $x^{mon} = \frac{\bar{p}-c}{2\beta}$

MONOPOLIST PRICE: $p^{mon} = c + \frac{1}{2}(\bar{p} - c)$

- In *duopoly*, each firm's optimal level of output depends on the output of the other firm (best-response function).

BRFs: $x_A = \frac{\bar{p}-c}{2\beta} - \frac{1}{2}x_B$; $x_B = \frac{\bar{p}-c}{2\beta} - \frac{1}{2}x_A$

The duopolistic Nash Equilibrium is the intersection of the best-response functions

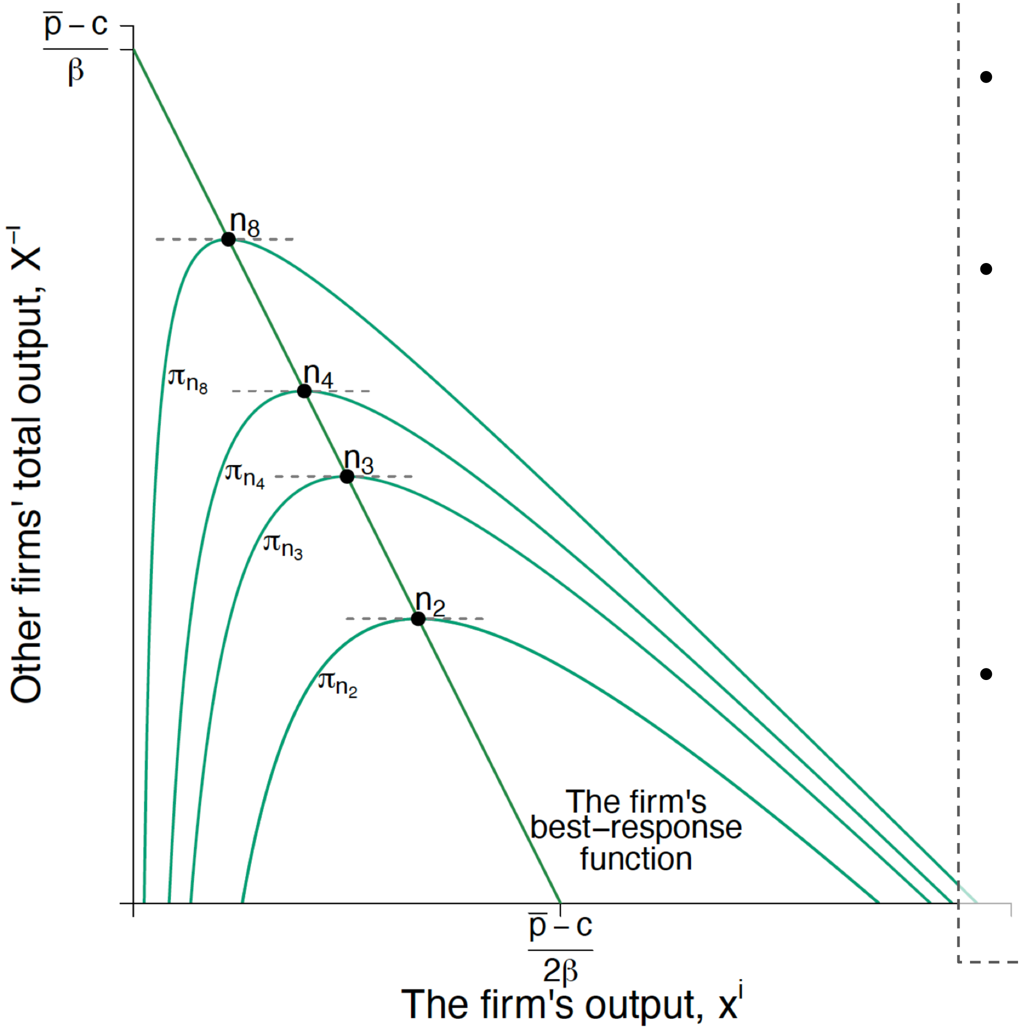
DUOPOLY OUTPUTS: $x_A^N = x_B^N = \frac{\bar{p}-c}{3\beta}$

DUOPOLY PRICE: $p^N = c + \frac{1}{3}(\bar{p} - c)$

Case (3): Multiple firms (*n* firms)

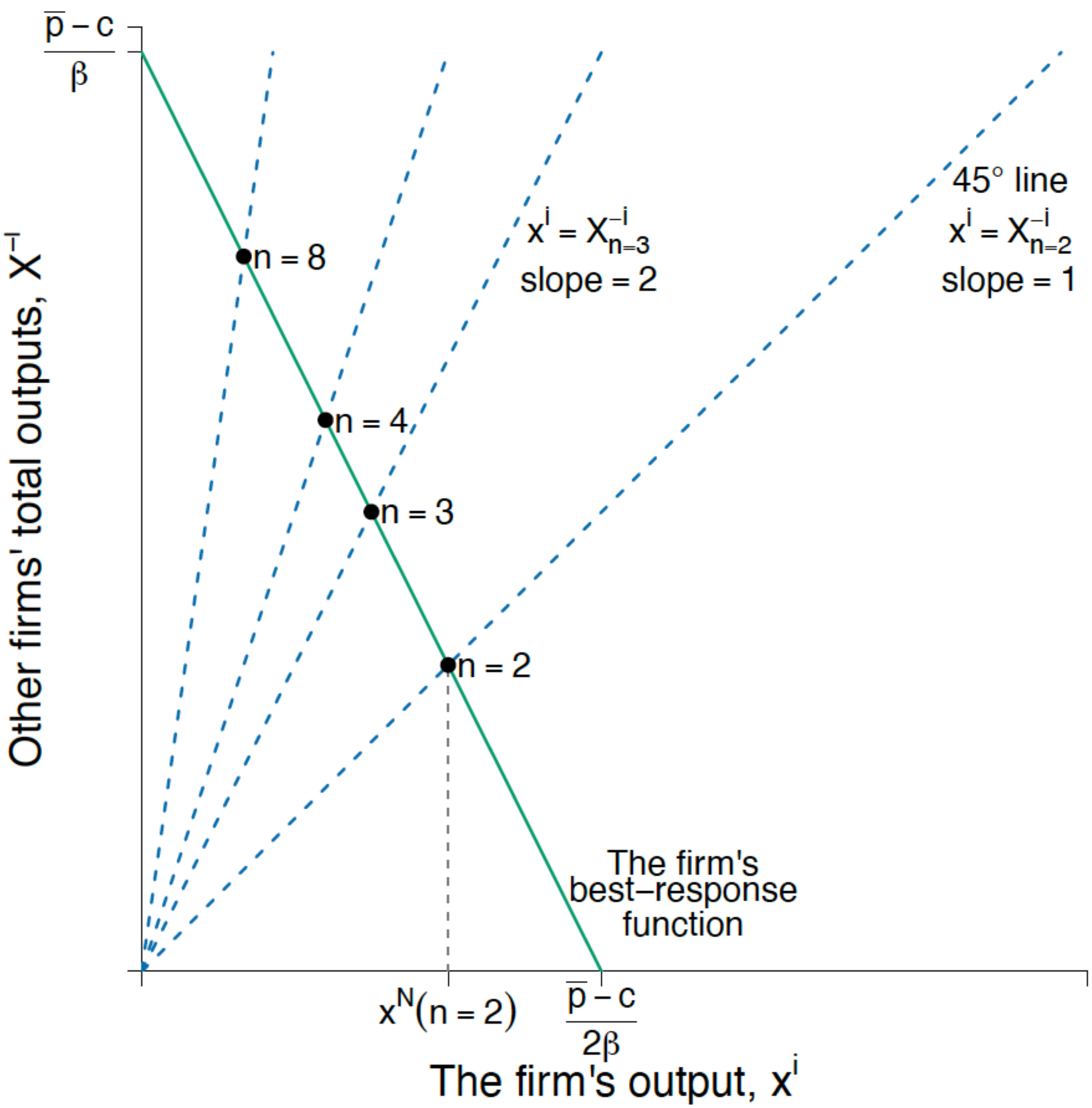
- Can be *oligopoly* (few firms) or *unlimited competition* (very many firms), or something in between.
- Multiple firms in the same industry: they all share the demand.
- Each firm best-responds to the output level of the rest of the firms.
- As we assume that all firms are identical, in the Nash Equilibrium the output of all firms must be equal (*symmetric equilibrium*).
- In the Nash Equilibrium, the output of all firms is equal and all firms are best responding.
- Let us analyze the behavior of the typical firm…

Iso-profit curves & BRF for the typical individual firm



- *Horizontal axis*: output chosen by the individual firm under consideration (x^i).
- *Vertical axis*: total output of all other firms (X^{-i})
- Individual firm's *iso-profit curves*
  - Inverted-U shaped.
  - Lower is better (more profits).
- For a given level of other firms' output, the firm's optimal output is at the point of *tangency* between output by other firms (horizontal line) and its iso-profit curve
  - *slope* of the iso-profit curve is zero (because tangent to a horizontal line)
- *Firm's BRF*: connects all points where the iso-profit curves have slope zero.
  - Gives us the firm's optimal output level, as a function of the output level of all other firms.

Nash Equilibrium with n firms



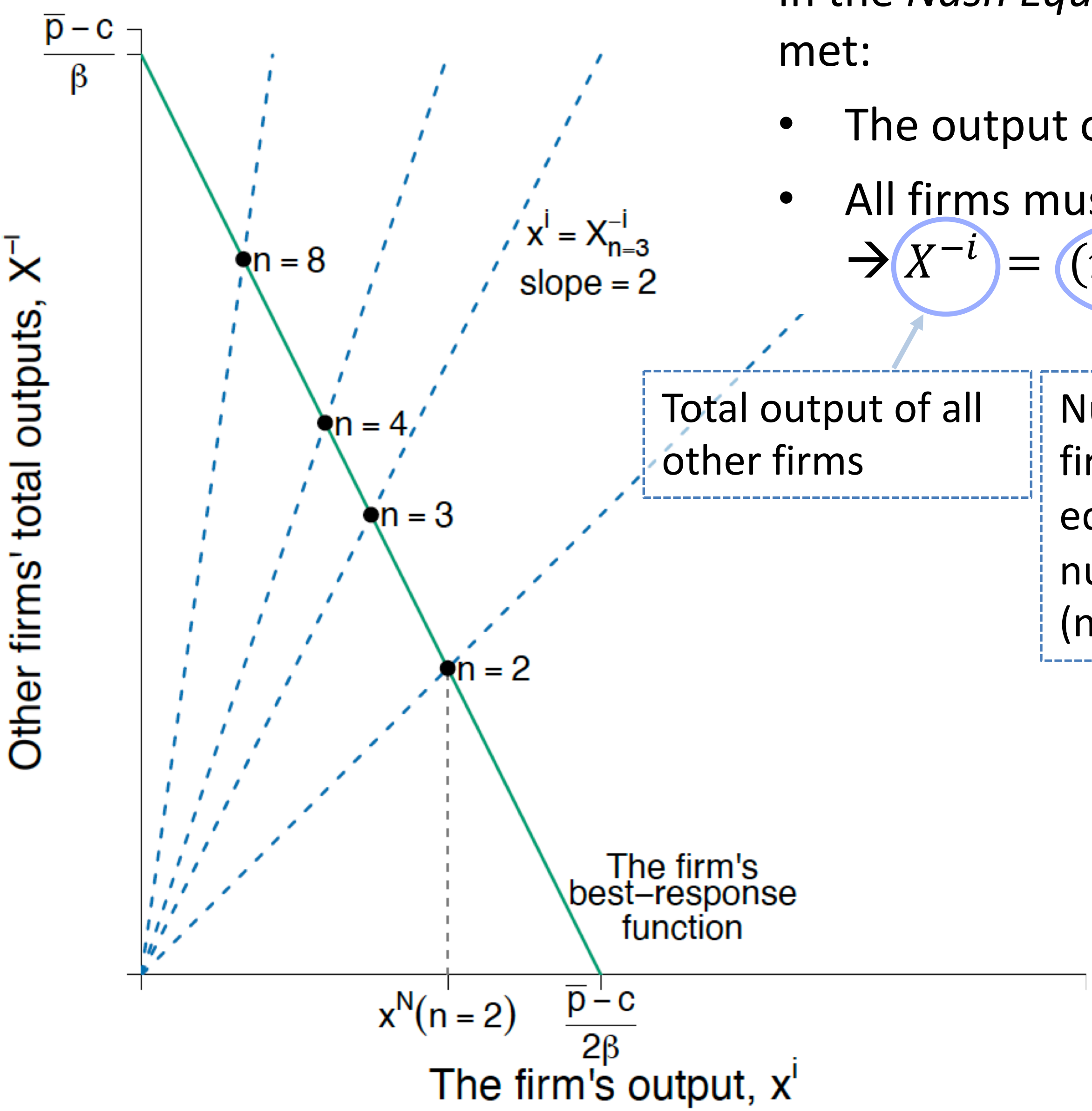
In the *Nash Equilibrium* two conditions must be met:

1. The output of the firm must be on its BRF
2. All firms must produce the same output level
   → $X^{-i} = (n-1)x^i$

> NE is intersection between the firm's BRF (*green decreasing line*) and the $X^{-i} = (n-1)x^i$ line (*dotted blue increasing line*).

> The $X^{-i} = (n-1)x^i$ line has slope (n-1)…

> …so it gets *steeper* as *n* (the number of firms in the industry) increases.



In the *Nash Equilibrium* two conditions must be met:

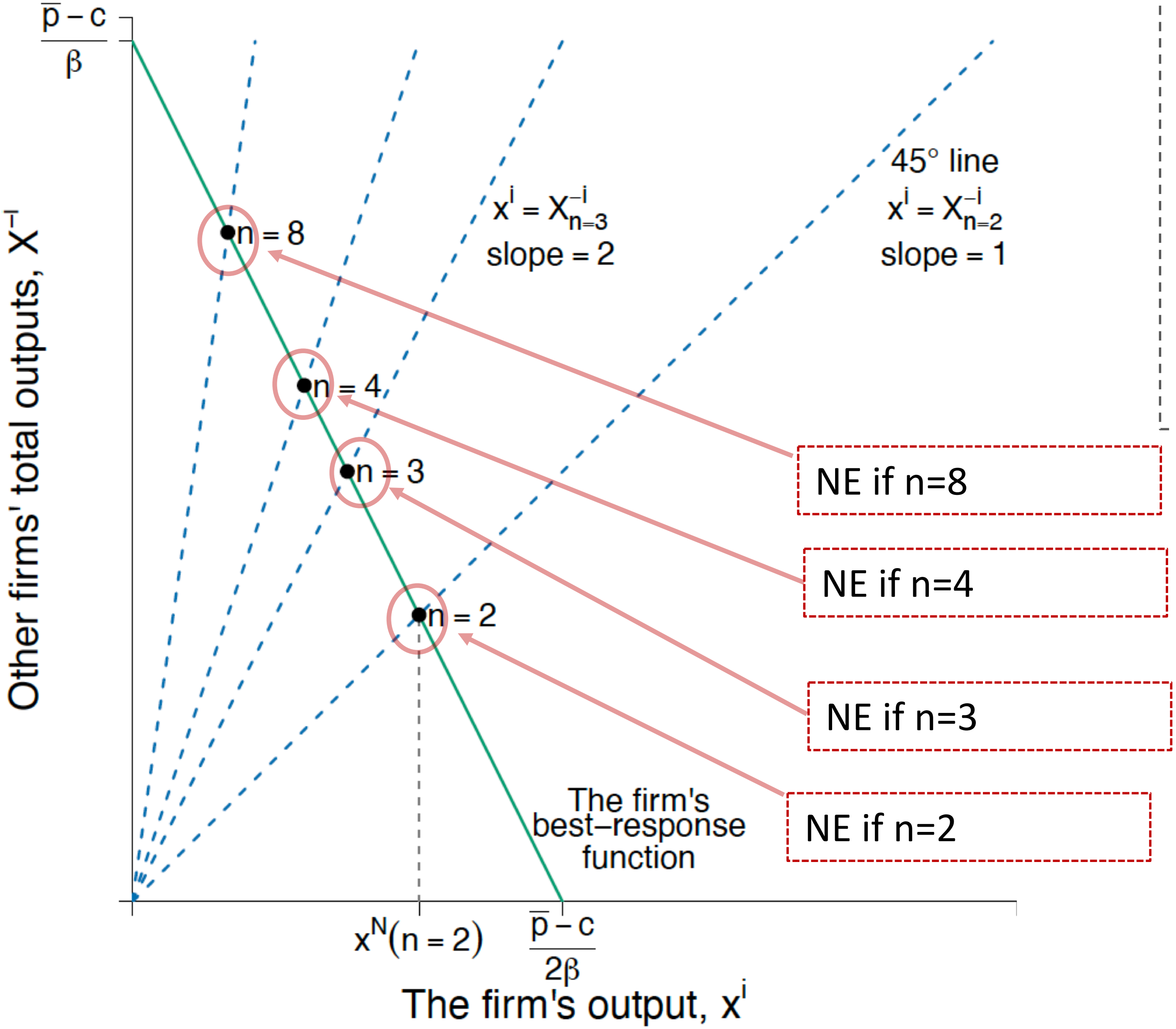
- The output of the firm must be on its BRF
- All firms must produce the same output level

$\rightarrow X^{-i} = (n-1)\ x^i$

Total output of all other firms

Number of other firms, which is equal to the total number of firms (n) minus one.

Output of the individual firm under consideration.



This graph shows how, for a given demand function and cost function, the NE output of a firm varies as the number of firms in the industry increases.

As the number of firms increases, the output produced by each firm decreases but (as we will demonstrate in the next slides) total output in the industry increases, and therefore price goes down.

Equilibrium output & price with *n* firms

- Demand curve for the typical firm: $p\left(x^i, X^{-i}\right) = \bar{p} - \beta\left(x^i + X^{-i}\right)$
- Revenues of the typical firm: $r\left(x^i, X^{-i}\right) = px^i = \left[\bar{p} - \beta\left(x^i + X^{-i}\right)\right] x^i$
- Production cost for the typical firm: $c\left(x^i\right) = cx^i$
- The *typical firm BRF* is found by setting MR=MC and solving for x^i :

$$x^i = \frac{\bar{p} - c}{2\beta} - \frac{1}{2} X^{-i}$$

- Set $X^{-i} = (n-1)x^i$ and solve for x^i to find equilibrium output per firm.

$$x^i = \frac{(\bar{p} - c)}{(n+1)\beta}$$

See M-Notes 9.5 and 9.6 in the textbook for the details on this derivation.

- Equilibrium output per firm: $x^i = \frac{(\bar{p}-c)}{(n+1)\beta}$
  - > General result: with n=1 you would get the monopolistic N.E.; with n=2 the duopolistic N.E.
- Total output in the industry: $X = nx^i = \frac{n}{n+1}\frac{(\bar{p}-c)}{\beta}$
- The term $\frac{n}{n+1}$ gets bigger as n increases, therefore total industry output increases as the number of firms increases.
  - > *As the number of firms increases, each firms produces less, but total output in the industry increases.*
- *Equilibrium price* found by using the demand curve

$$p = \bar{p} - \beta X = \bar{p} - \beta\frac{n}{n+1}\frac{(\bar{p}-c)}{\beta} = c + \frac{1}{n+1}(\bar{p}-c)$$

  - > *Price goes down as the number of firms n goes up.*

The case of unlimited competition (very large number of firms)

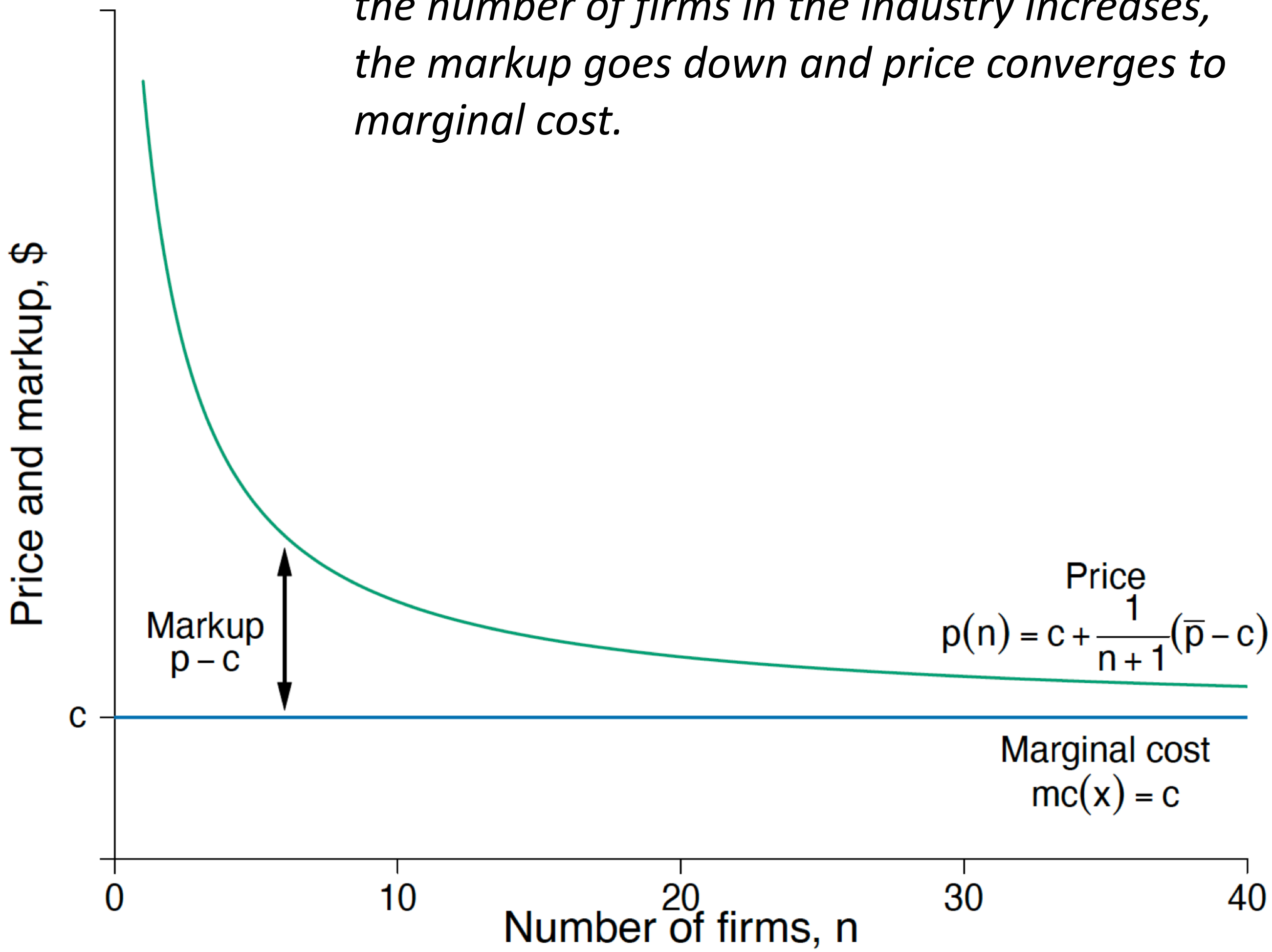
- Cournot Nash Equilibrium with n firms:

  output per firm: $x^i = \frac{(\bar{p}-c)}{(n+1)\beta}$; price: $p = c + \frac{1}{n+1}(\bar{p} - c)$

- What if the number of firms in the industry gets *really* large? (mathematically, what happens when *n* approaches infinity?)
  - As n → ∞, x → 0: quantity produced by each firm approaches zero.
  - As n → ∞, p → c: product price tends to equal marginal cost.
- Super-important result: under *unlimited competition* (extremely large number of very small firms), price converges to marginal cost.
- In general (with any number of firms), price is equal to marginal cost plus a mark-up $[\frac{1}{n+1}(\bar{p} - c)]$. The mark-up goes down as *n* increases.

Price and markup over production cost

For a given demand curve and cost function, as the number of firms in the industry increases, the markup goes down and price converges to marginal cost.

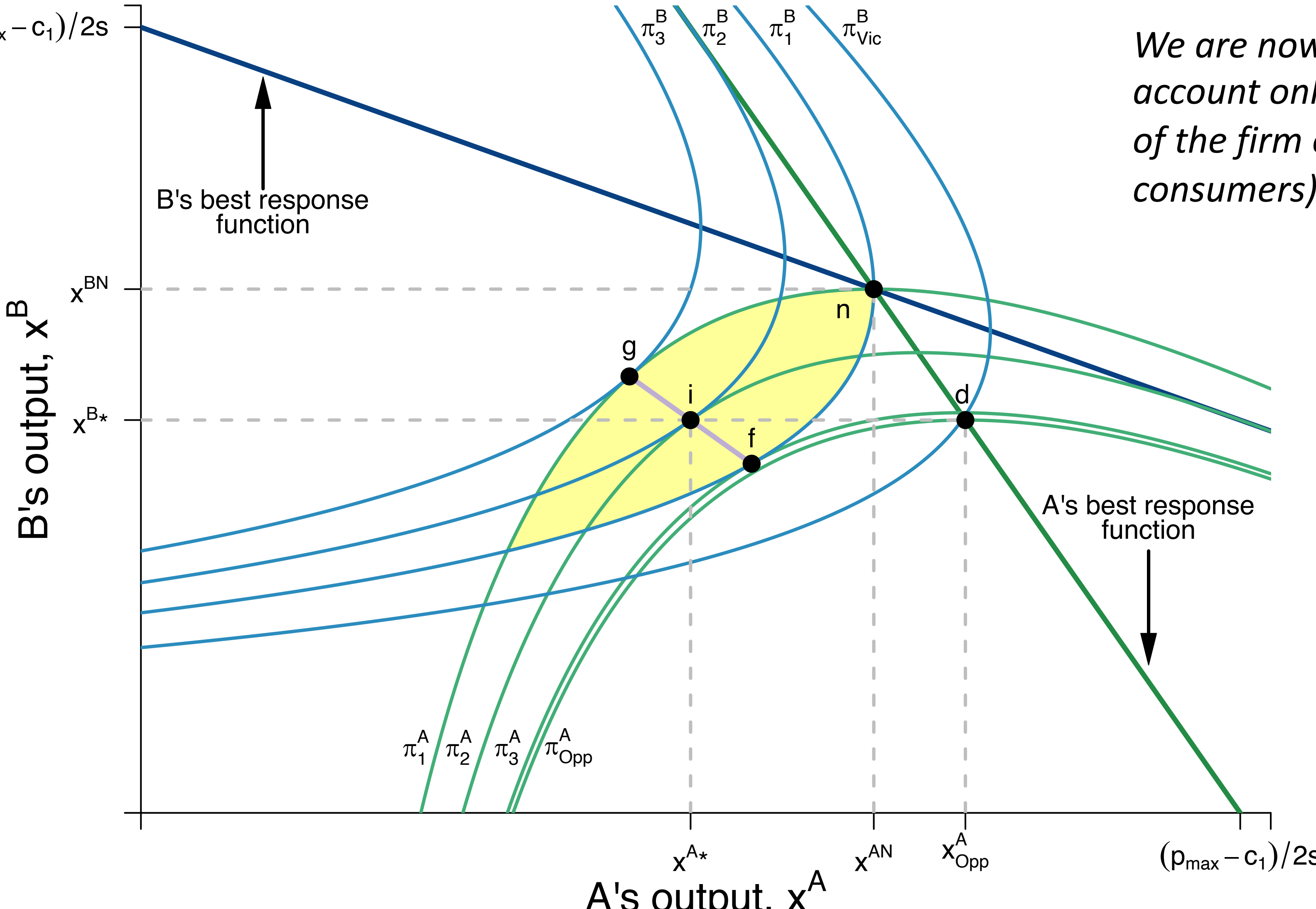


- *Horizontal axis*: number of firms in the industry.
- *Vertical axis*: product price.
- *Blue line*: marginal cost of production (here assumed to be constant, therefore horizontal).
- *Green line*: price.
- Distance between green line and blue line: markup (p-c) $(p - c = \frac{1}{n+1}(\bar{p} - c))$

2 – Competition and Social Well-Being

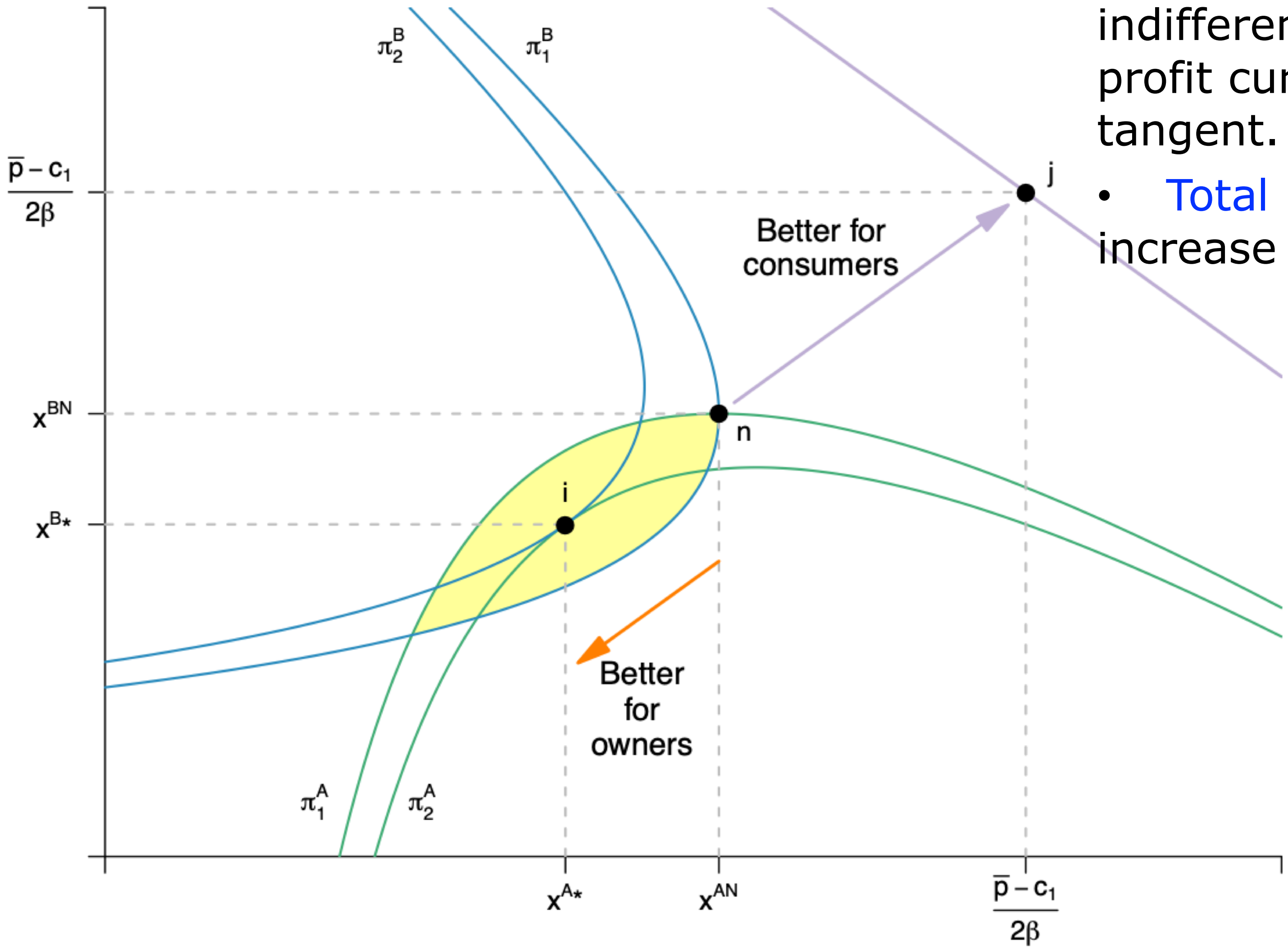
- More competition (more firms producing the same product) leads to lower prices and higher output
  - although each firm produces a smaller amount, the whole industry output is higher with larger number of firms.
- But what are the implications for social welfare?
- First, consider only the welfare of firms
- Then, overall social welfare (firms + consumers).

Consider a duopoly. From the point of view of the two firms, the N.E. is *Pareto-inefficient*: they would make more profits by decreasing output and increasing the price.



We are now taking into account only the welfare of the firm owners (not of consumers).

- Light blue curves = B’s iso-profit curves (they increase going to the left).
- Light green curves = A’s iso-profit curves (they increase going south)
- N.E. = intersection of the BRFs (not shown)



- Here Pareto-improving lens is defined by only considering firms’ profits (not consumer welfare).
- N.E. not Pareto-efficient for the two firms: indifference curves (=iso-profit curves) are not tangent.
- Total profits would increase with lower output and higher price.

: Daniele Girardi

Competition as a coordination failure for firms

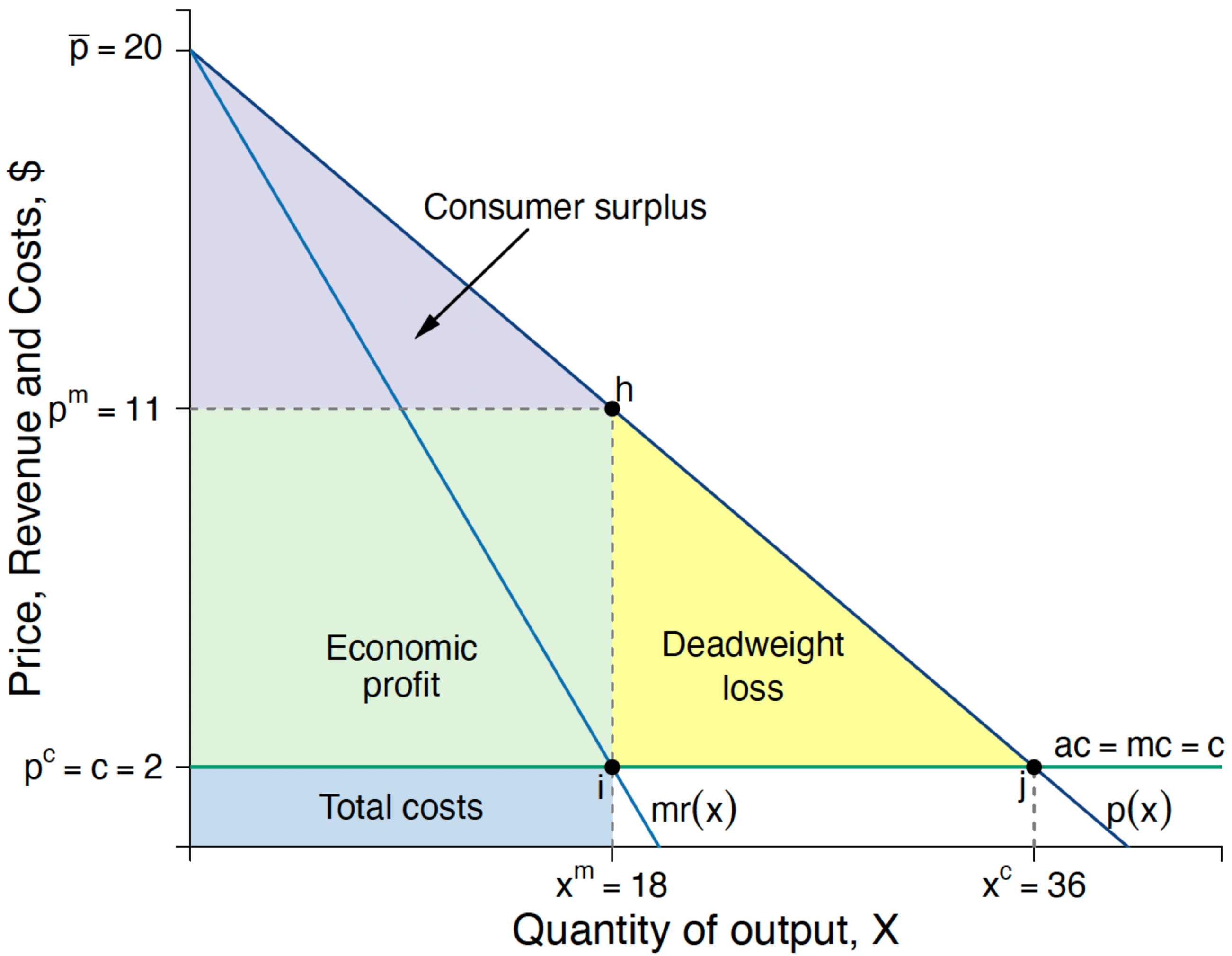
- From the point of view of firms, duopoly is a prisoners' dilemma:
  - both would do better (higher profits) if both cut production
  - but if one cuts production (cooperates), the other wants to keep its production high to enjoy the higher price (defect).
  - This coordination failure (for firms) comes from an externality: when firm A increases its output, it inflicts a negative externality on firm B (lower price).
- The same holds with oligopoly or competition: total industry profits are lower than under monopoly, because of higher quantity produced.
- From the point of view of firms, competition is a coordination failure: they might all do better by agreeing to cut production to raise prices.
- Firms sometimes try to solve the coordination problem though cartels or mergers. But also product differentiation (think of smartphones) and advertising
  - But this is no good for consumers (and society)….

Competition and social welfare

- To introduce *consumers' welfare* in the picture, we need to define and measure it (see lecture notes 'demand and consumer surplus' on Moodle).
- Consumers' surplus is the difference between their maximum willingness to pay for a quantity of good, and what they actually pay.
- Consumers' surplus is higher the lower the price (quite intuitive!).
- Social welfare = producer surplus(profits) + consumer surplus
- Social welfare is maximized when *p=MC* (which happens under unlimited competition) and all economic surplus goes to consumers.
- So more competition (larger number of firms) increases social welfare.
- Market power causes an increase in firms' profits but a greater decrease in consumers' surplus: this is called the *deadweight loss* (of welfare) due to market power.
- We will demonstrate this graphically in reference to monopoly vs. perfect competition.

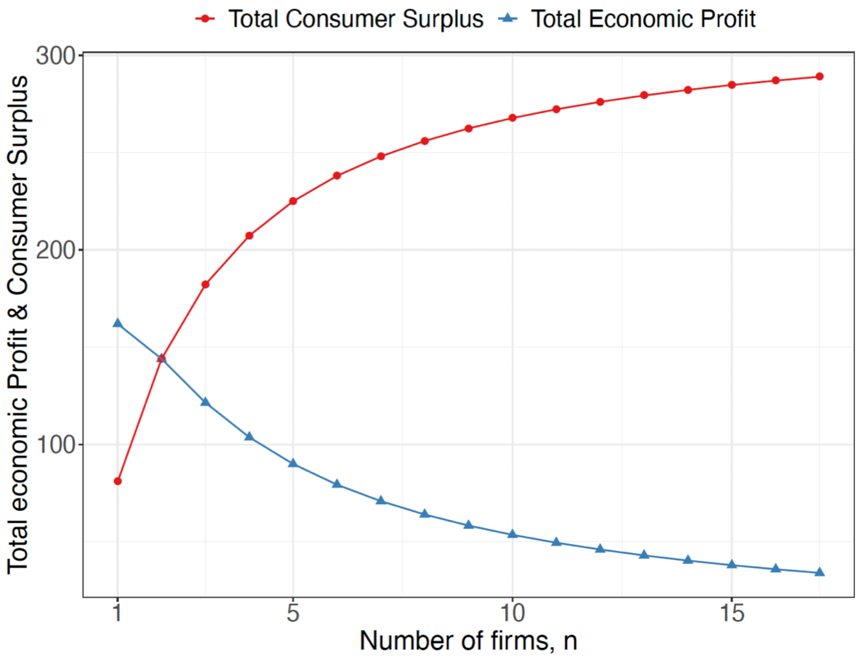
Unlimited competition ($n \to \infty$):

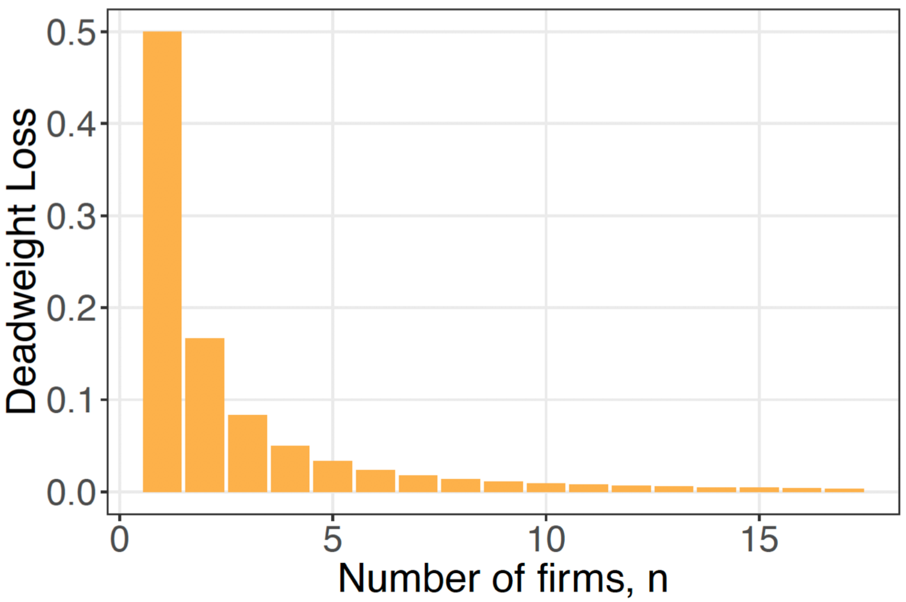
- MC = p → point *j* in the graph
- Firms just recover costs (with the constant MC we assumed): no profits.
- Social surplus = consumer surplus = green area + violet area + yellow area



Monopoly (n=1):

- MR=MC<p
- Point *h*
- Profits = green area
- Consumer surplus=violet area
- Social surplus=consumer surplus + profits = violet area + green area.
- Yellow area gets lost (*deadweight loss*)


Total Consumer Surplus
Total Economic Profit
Total economic Profit & Consumer Surplus
300
200
100
1
5
10
15
Number of firms, n


Deadweight Loss
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
Number of firms, n

Conclusions

- Cournot shows that more intense competition means lower profits, lower prices and higher production.
- A product's price is equal to a mark-up over the marginal cost of production. The higher the number of firms, the lower this mark-up.
- Welfare analysis shows that more intense competition reduces producers' surplus (profits) but increases consumers' surplus by more.
- Market power (monopoly, duopoly, oligopoly) implies a deadweight loss for society: not all the welfare lost by consumers goes to firms (some gets lost along the way).
- Social Welfare maximized when p=MC. Cournot shows this would happen in the utopian case of perfect competition (infinite number of very small price-taking firms).