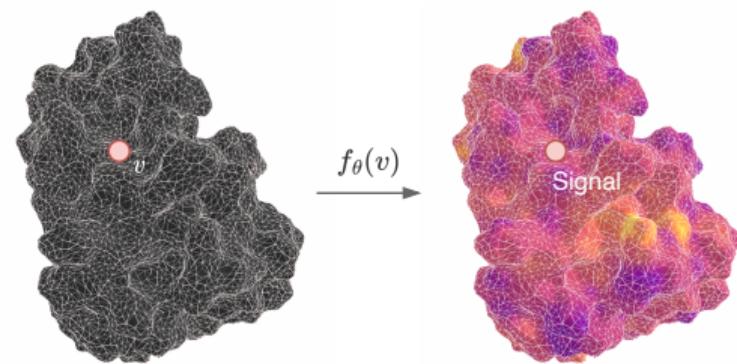


# Generalised Implicit Neural Representations

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Daniele Grattarola, Pierre Vandergheynst  
Neural Information Processing Systems 2022

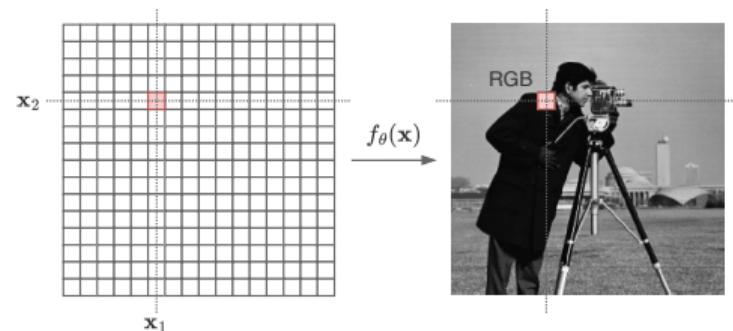


# Implicit neural representations

Given a signal or field  $f: \mathbb{R}^d \rightarrow \mathbb{R}^p$

Sample a regular grid  $\mathbf{y}_i = f(\mathbf{x}_i)$

Train neural network  $f_\theta: \mathbf{x}_i \mapsto \mathbf{y}_i$

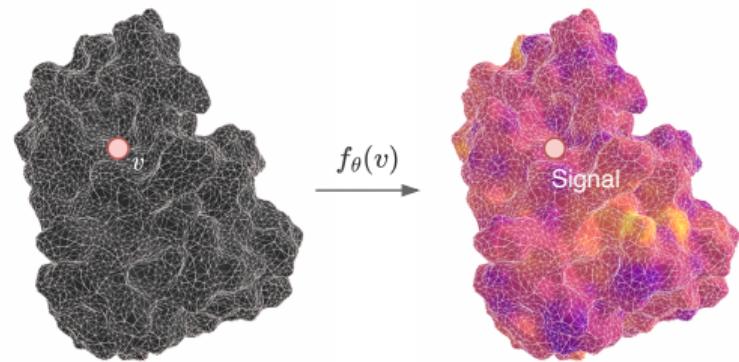


# Generalised implicit neural representations [1]

Given a signal or field  $f: \mathcal{T} \rightarrow \mathbb{R}^p$

Sample a graph signal  $\mathbf{y}_i = f(v_i)$

Train neural network  $f_\theta : v_i \mapsto \mathbf{y}_i$



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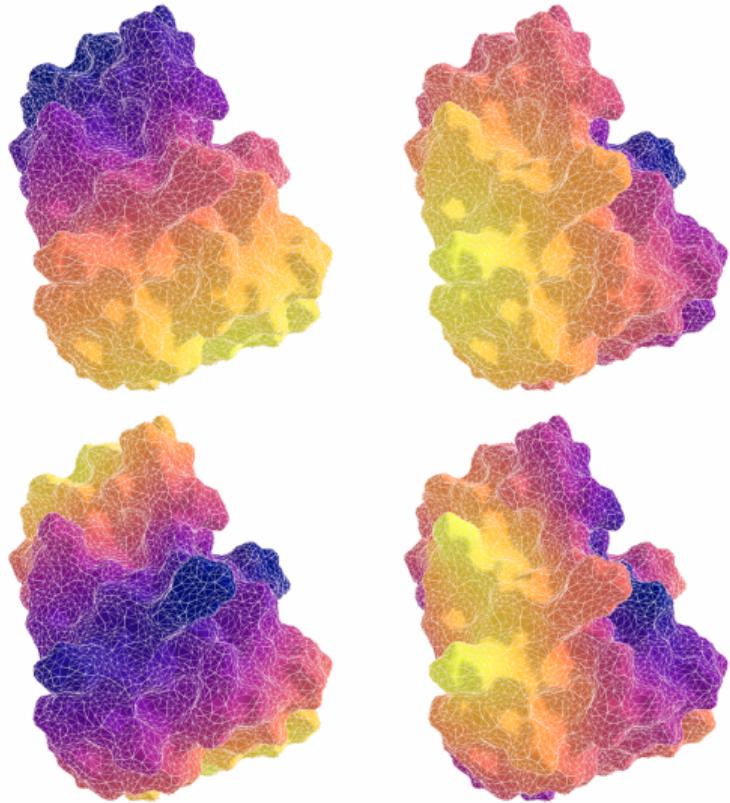
[1] D. Grattarola *et al.*, "Generalised implicit neural representations," *Advances in Neural Information Processing Systems*, 2022.

# Generalised implicit neural representations [1]

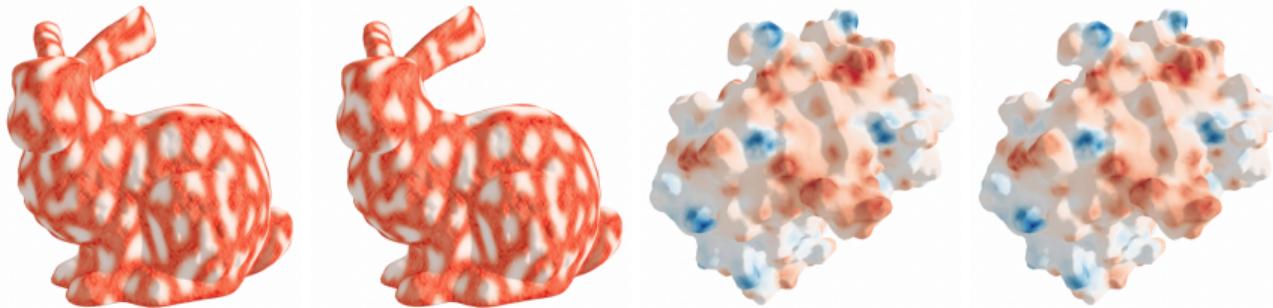
Use spectral positional encodings as coordinates:

$$\mathbf{e}_i = \sqrt{n} \underbrace{[\mathbf{u}_{1,i}, \dots, \mathbf{u}_{k,i}]^\top}_{\text{Laplacian eigenvectors}} \in \mathbb{R}^k$$

LB eigenfunctions for  $n \rightarrow \infty$ .



## Experiments

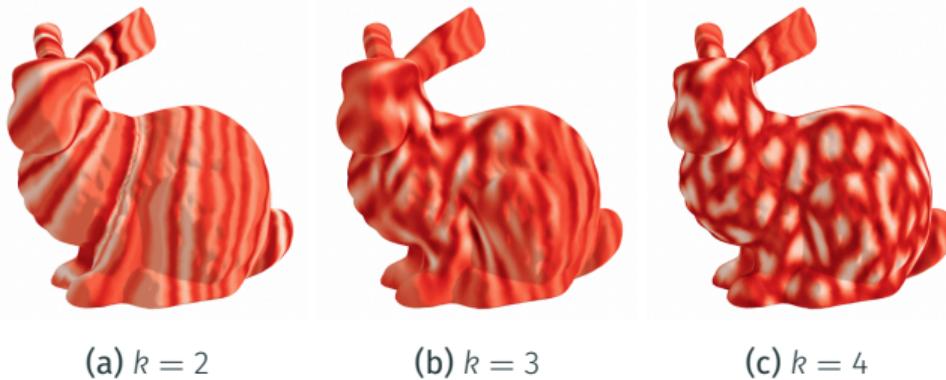
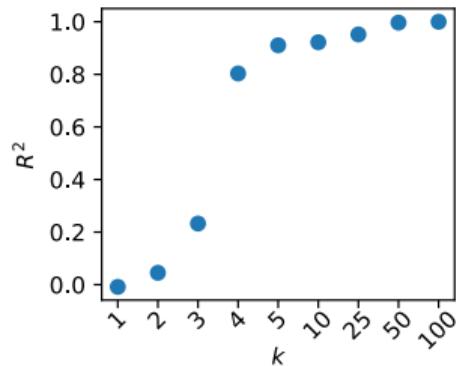


Ground truth signals vs. signals predicted by the GINR.

|       | Bunny                | Protein               | US Election          |
|-------|----------------------|-----------------------|----------------------|
| $R^2$ | 1.000                | 1.000                 | 0.999                |
| MSE   | $9.14 \cdot 10^{-8}$ | $1.17 \cdot 10^{-10}$ | $1.45 \cdot 10^{-3}$ |

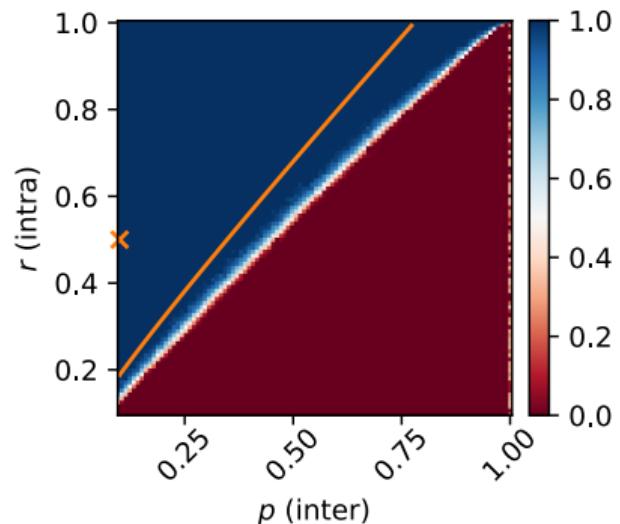
Approximation error

## Experiments - Size of embeddings

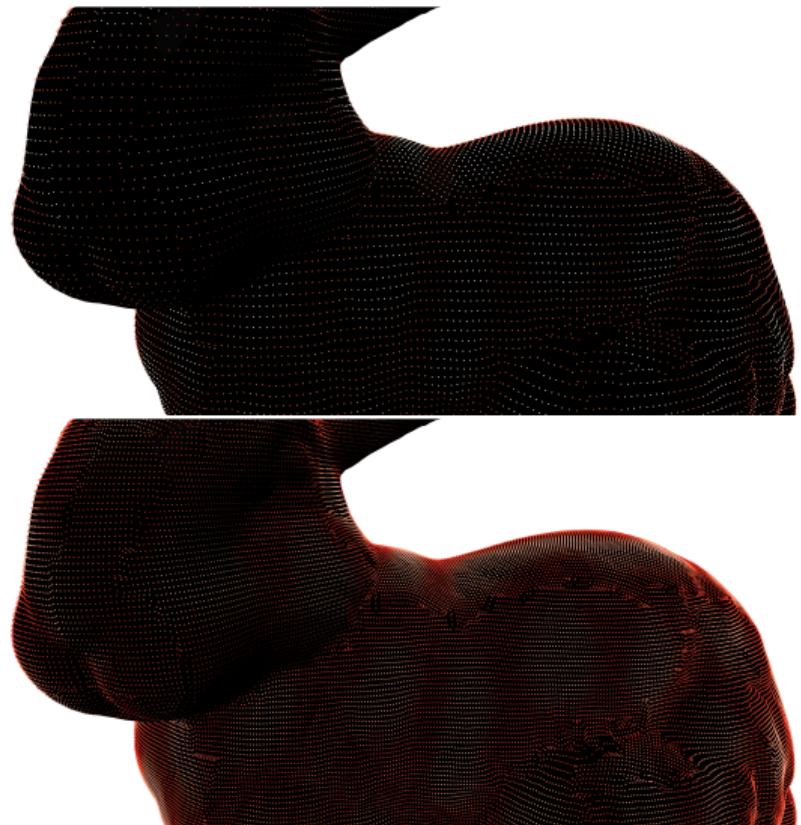


Left:  $R^2$  vs.  $k$ ; Right: signals learned by the INR for  $k = 2, 3, 4$ .

## Experiments - Transferability

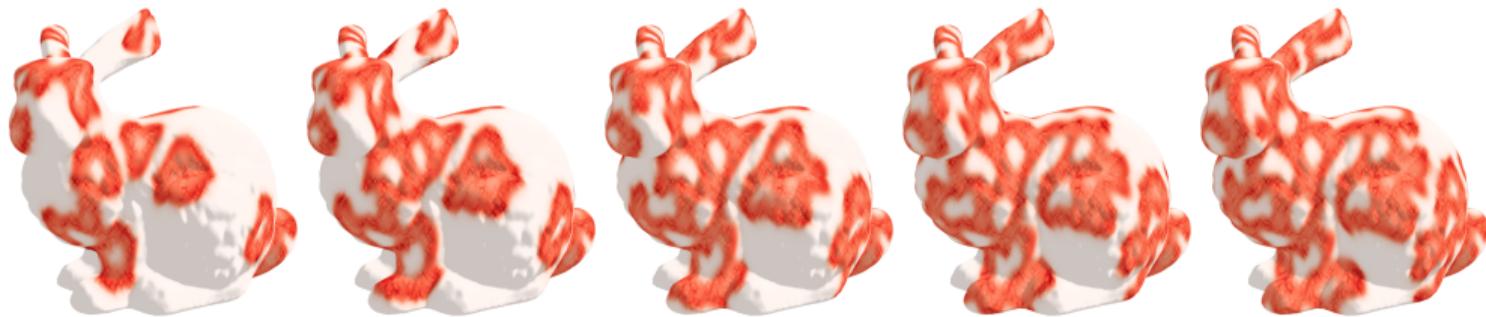


Toy problem with SBM.



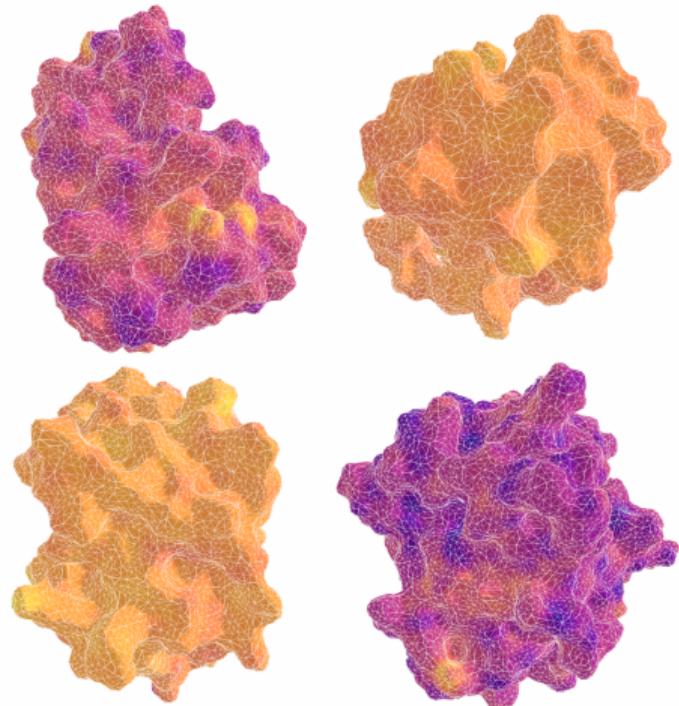
## Experiments - Conditional GINRs

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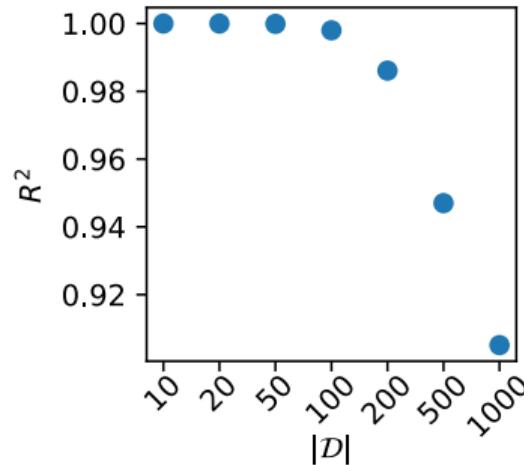


Signals predicted by the conditional GINR  $f_{\theta}(\mathbf{e}_i, t)$

## Experiments - Conditional GINRs

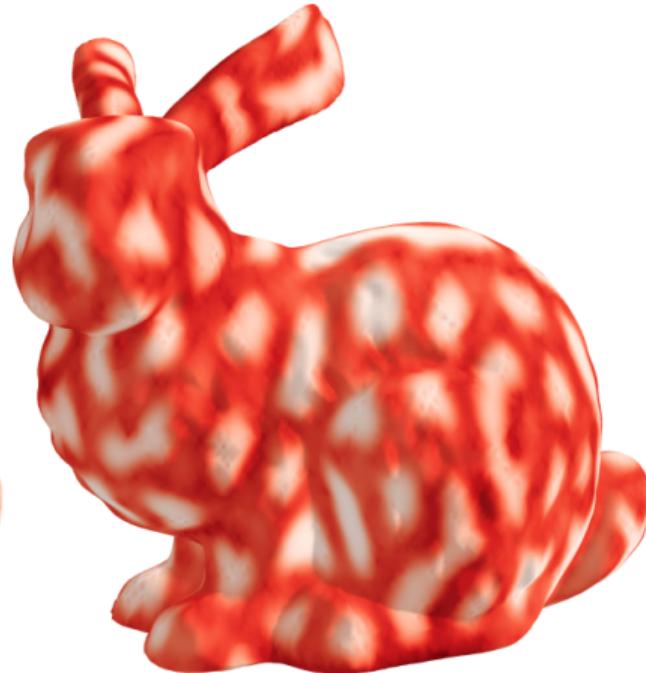


$f_{\theta}(\mathbf{e}_i, \mathbf{z}_d)$  for node  $i$  and protein  $d$ .

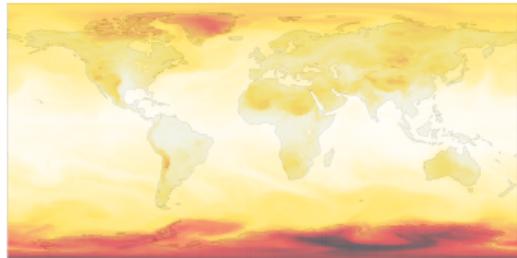


Good performance up to 100s of proteins.

## Experiments - Solving differential equations



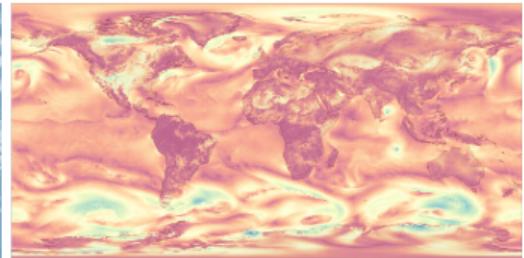
# Experiments - Weather modelling



(a) Temperature



(b) Clouds



(c) Wind

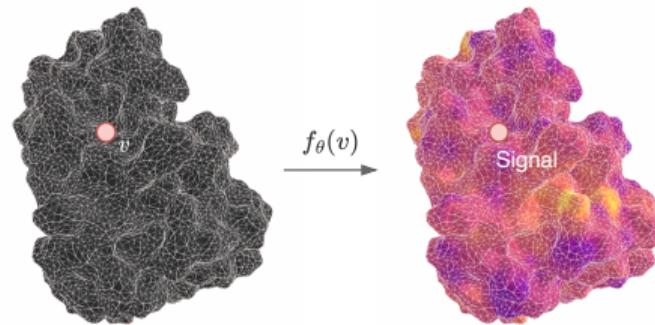


Data from the National Oceanic and Atmospheric Administration (NOAA).



Generalised Implicit Neural Representations  
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[github.com/danielegrattarola/GINR](https://github.com/danielegrattarola/GINR)  
[arxiv.org/abs/2205.15674](https://arxiv.org/abs/2205.15674)



Get in touch: [daniele.grattarola@gmail.com](mailto:daniele.grattarola@gmail.com) or [@riceasphalt](https://riceasphalt.com)

## References i

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- [1] D. Grattarola and P. Vandergheynst, “Generalised implicit neural representations,” *Advances in Neural Information Processing Systems*, 2022.