Edge functional networks: what can we learn beyond what the covariance matrix already tells us?

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Defining networks from time series is enticing, can provide nice visualizations, and can possibly be scientifically useful. Still this procedure poses methodological challenges (how do we define and interpret a node, and more importantly an edge?), as well as interpretational issues connected to the degeneracy between the degrees of freedom of the data at hand and all the possible measures we can come up with (new, old, rebranded). These concerns are exacerbated by the natural tendency to map biological entities and the measures we use. This abstract focuses on the recently proposed "edge functional connectivity" (eFC) [1,2]. What we argue is that in this case there is little new information beyond the one already obtained by the covariance matrix. The most appropriate statistical test to assess the contribution of co-fluctuations beyond the covariance matrix is an approach in which the latter is preserved, and the time series are randomized, rather than an approach such as the circular shifting [3], which scrambles the covariance matrix. In figure 1 we show the circular shift based randomization, as well as two randomizations based on covariance matrix, one obtained simulating random noise with covariance matrix equal to the original one, and one obtained simulating a multivariate autoregressive model based on the original time series. We can notice two things: the first one is that the circular shift results in a null distribution which is farther away from the actual one, i.e., more frames result significant; the other one is that even in the absence of a meaningful internal dynamics the few selected points result in a eFC matrix with a structure resembling the averaged FC one. In nonlinear dynamical systems, the Takens theorem states that a few points suffice to describe an attractor. For linear systems, the covariance matrix is a representation of an attractor, and the ergodic theorem ensures that sample moments converge to their population counterparts. This is the case from the sum of individual terms of the covariance formula: they are meaningful only once the final outcome (in this case the correlation) is defined, and they are defined by it, rather than "driving" it.
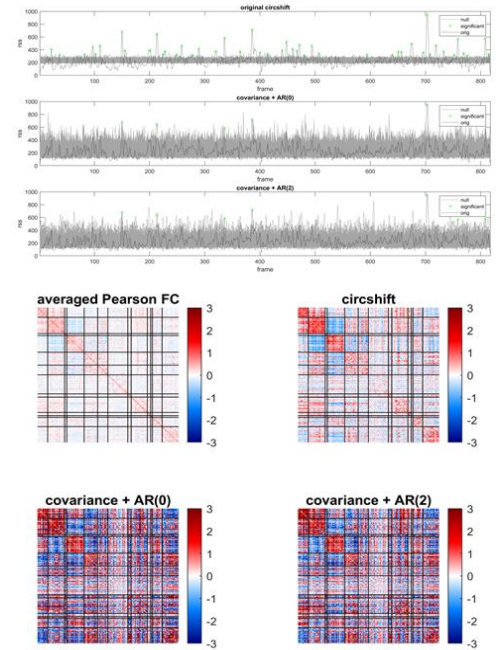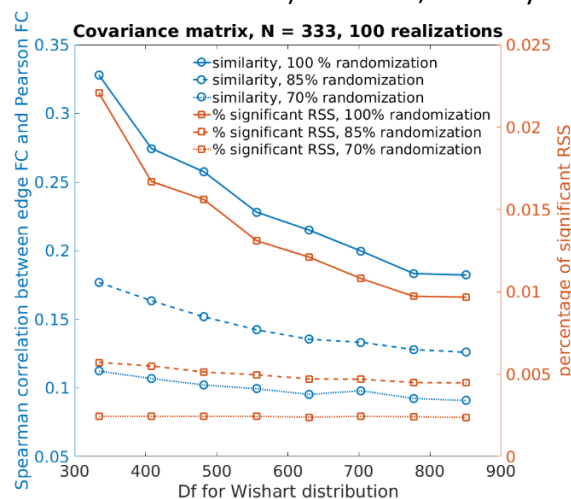

Figure 1: edge RSS and (e)FC matrices.

Figure 2: similarity between eFC and FC and number of significant RSS frames as a function of the degrees of freedom of the Wishart distribution generating the covariance matrix, for different levels of circular shift randomization.

As an additional test, we generated random covariance matrices from Wishart distributions with increasing degrees of freedom, and computed the similarity between the eFC and the FC, as well as the percentage of retained edges, for different levels of randomization with circular shift. The two quantities are correlated, and they decrease with the degrees of freedom (figure 2).

In conclusion, we think that in the case of eFC, is that static FC drives the eFC, not the other way around, and that the two approaches are not complementary from a fundamental point of view.

Code is available at
https://github.com/danielemarinazzo/event_detection

[1] Faskowitz, J., Esfahlani, F. Z., Jo, Y., Sporns, O., & Betzel, R. F. (2020). Edge-centric functional network representations of human cerebral cortex reveal overlapping system-level architecture. Nature neuroscience, 23(12), 1644-1654.
[2] Esfahlani, F. Z., Jo, Y., Faskowitz, J., Byrge, L., Kennedy, D. P., Sporns, O., & Betzel, R. F. (2020). High-amplitude cofluctuations in cortical activity drive functional connectivity. Proceedings of the National Academy of Sciences, 117(45), 28393-28401.
[3] Jo, Y., Faskowitz, J., Esfahlani, F. Z., Sporns, O., & Betzel, R. F. (2020). Subject identification using edge-centric functional connectivity. bioRxiv.