ALMA^3

the plAnetary Love nuMbers cAlculator

User Manual for version 3.11

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1 Introduction

This manual describes how to set up, configure and run ALMA, a Fortran 90 open-source program which computes the Love Numbers (both of tidal and loading types) of a spherically symmetric, radially stratified, incompressible self-gravitating planet. ALMA can evaluate time-dependent Love Numbers (hereinafter referred to as LNs) and their time derivatives, corresponding to step-wise and ramp-wise forcing time histories, suitable for modeling *e.g.* the planetary deformation in response to surface loads. In addition, ALMA can compute complex-valued LNs describing the response to periodic loads, which find a wide application in studying the tidal deformation of planets. The theoretical background underlying ALMA is discussed in a dedicated paper to which the reader is referred for details; this User Manual will only focus on the technicalities needed to run the program.

Copyright on ALMA is held by Daniele Melini and Giorgio Spada, 2021. ALMA is released under the BSD three-clause license, a copy if which is included in the distribution package and in §5.1. ALMA includes portions of the FMLIB multi-precision libraries, whose copyright is held by David M. Smith, 2008. FMLIB is released under the MIT license, a copy of which is distributed with ALMA package and reported in §5.2.

2 Setting up `ALMA`[3]

`ALMA`[3] can be installed on any system providing a standard command-line Unix environment, such as Linux and macOS. On Windows systems, ALMA can be used within either the Cygwin[1] environment or the Windows Subsystem for Linux[2] (WSL). To build ALMA, a Fortran compiler and a standard POSIX development toolchain shall be available on the system. ALMA can be successfully compiled by all recent versions of the freely available gfortran compiler[3], but it should be possible to build it with any reasonably modern Fortran compiler. ALMA requires the multi-precision FMLIB[4] libraries (Smith, 1991, 2003), which are included in the distribution package. While not strictly required in order to run ALMA, a graphics package like *e.g.* the Generic Mapping Tools (GMT) or GNUplot is useful to visualize the numerical results.

To install ALMA, it is sufficient to uncompress the distribution ZIP package in any directory chosen by the user. We will refer to the folder in which contains the files of the distribution package have been extracted as the *main ALMA directory*. The main ALMA directory contains the following subfolders:

- `src/`, containing the ALMA and FMLIB source files;
- `CONFIGS/`, containing sample ALMA configuration files (see §3.1), including those used in the worked examples illustrated in this User Manual (see §4);
- `MODELS/`, containing sample ALMA rheological model files (see §3.2), including those used in the examples of §4.

To build the ALMA executable, it is sufficient to type the `make` command inside the `src/` subfolder of the main ALMA directory. If the compilation is successful, the ALMA executable `alma.exe` shall be made available in the main ALMA directory. Once the executable is built, it can be copied to any directory included in the system search path, or it can be left in place if the user wishes to run ALMA only within its main directory.

[1] The Cygwin environment is freely available for download from `http://www.cygwin.com/`.
[2] WSL can be installed following the instructions at `https://docs.microsoft.com/en-us/windows/wsl/install`.
[3] gfortran is part of the GNU Compiler Collection, freely available at `https://gcc.gnu.org/`.
[4] FMLIB by D.M. Smith is available at `https://dmsmith.lmu.build/`.

3 Configuring ALMA

ALMA is invoked by typing at the system prompt the name of the alma executable (usually `alma.exe`) followed by the name of a *configuration file*, for instance:

```
$ ./alma.exe config.dat
```

The configuration file (`config.dat` in the example above) contains all the relevant parameters describing the job requested to ALMA. In addition, the configuration file contains a reference to the path of a *rheological model file* describing the structure of the planetary model for which ALMA shall compute the LNs of the requested type. In the following section, the structure of the configuration file and of the rheological model file will be discussed in detail.

3.1 Structure of the ALMA configuration file

An example ALMA configuration file is shown in Figure 1. While parsing the file, ALMA ignores all lines beginning with an exclamation mark character '`!`'; these lines may be used to annotate the configuration file with comments. In a similar fashion, portions of data lines after an exclamation mark are ignored and may be used to insert comments. The configuration file consists of several blocks corresponding to various aspects of the ALMA run. Here we will illustrate the meaning of the parameters and keywords appearing in the configuration file, following the example listed in Figure 1.

3.1.1 Arithmetic precision and order of the Gaver sequence

```
128                         ! number of digits
8                           ! order of the Gaver sequence
```

The first block sets the number of digits D to be used by the FMLIB multi-precision library ($D = 128$ in the example above) and the order M of the Gaver sequence employed to evaluate numerically the Laplace inverse transforms ($M = 8$ in the example above). As a general rule, higher values of D and M ensure numerical stability at the cost of increasing computational time (see, *e.g.* Spada, 2008). For most applications, the values $D = 128$, $M = 8$ are a good compromise between computational efficiency and numerical stability. However, when evaluating LNs at high harmonic degrees and/or using rheological models with a large number of distinct layers, the numerical convergence of the results should be carefully tested and higher values of D and M could be needed. Since the Post-Widder numerical Laplace inversion scheme is invoked only for the evaluation of time-domain LNs M is ignored when ALMA computes frequency-domain LNs.

3.1.2 Love Number type

```
Loading                     ! LN type ('Loading' or 'Tidal')
```

This line is used to specify whether LNs of loading or tidal type shall be computed. These are selected with the keywords `Loading` and `Tidal`, respectively. All the keywords in the configuration file are case-insensitive.

3.1.3 Range of harmonic degrees

```
2                          ! Minimum degree
10                         ! Maximum degree
1                          ! Step
```

This block defines the range of harmonic degrees of the LNs that ALMA shall compute, by specifying the minimum and maximum harmonic degree (n_{min} and n_{max}) and a sampling step n_{step}. In the above example, ALMA is scheduled to compute LNs for all harmonic degrees ($n_{step} = 1$) between $n_{min} = 2$ and $n_{max} = 10$. If $n_{step} > 1$, ALMA will compute LNs for all the harmonic degrees $n = n_{min} + kn_{step}$, with $k \geq 1$, for which the condition $n \leq n_{max}$ holds true.

3.1.4 Range of time steps or forcing periods

```
log                       ! Time scale ('log' / 'lin' / 'ext')
100                       ! Time points (minus one)
-4 4                      ! Time range  (10^(m1:m2) kyrs)
```

This section is used to define the range of time steps (for the time-domain case) or forcing periods (for the frequency-domain case) at which the requested LNs will be evaluated. The keyword on the first line specifies whether a linear time scale (`lin`) or a logarithmic time scale (`log`) shall be used, or if the user will supply to ALMA an external set of time points (`ext`). The second line specifies the number p of time steps minus one, *i.e.* ALMA will assume a number of time steps $N_t = p + 1$. If an external scale is selected with the `ext` keyword, the third line will be ignored and ALMA will read the N_t time steps from a data file named `time_steps.dat`, which shall be made available in the same directory from which ALMA is invoked. The time steps in file `time_steps.dat` must be specified in kyrs. Otherwise, ALMA will read the m_1 and m_2 parameters on the third line and automatically generate $p + 1$ time steps between 10^{m_1} and 10^{m_2} kyrs, either spaced linearly (`lin` keyword) or logarithmically (`log` keyword). Please note that ALMA cannot compute LNs for a time step $t = 0$ (in the time-domain case) or a forcing period $T = 0$ (in the frequency-domain case). If LNs in the "elastic limit" are needed, they can be computed by specifying the Hooke rheology in each layer of the model (see §3.2 and Table 2); in that case, the LNs will not depend upon time t or period T.

3.1.5 Time history of the forcing term

```
step                     ! Load fcn ( 'step' / 'ramp' )
1.0                      ! Ramp length (kyrs)
```

This block is used to specify the time-history of the forcing term. The keyword on the first line is used to select between a step-wise (`step`) or a ramp-wise (`ramp`) time-history. For a ramp-wise time-history, the second line defines the duration t_r of the loading phase (in kyrs); the value is ignored if a step-wise time-history is chosen. This block is used only in the evaluation of time-domain LNs, while it is ignored if frequency-domain LNs are requested (indeed, in that case a periodic forcing is assumed).

3.1.6 Rheological model

```
3                          ! Number of layers
rheological_profiles/GS00-maxwell.dat
```

This block contains information on the planetary model for which the requested LNs will be computed. The first line specifies the number of layers of the model (including the central core), while the second line contains the full path to the rheological model file, whose structure is described in §3.2. The number of data lines in the rheological model file must be consistent with the number of layers declared on the first line; if this is not the case, ALMA will abort with an error message. Please note that the model shall contain at least two layers; to simulate a homogeneous planet, it is advisable to define two layers (core and mantle) characterized by the same rheological parameters.

3.1.7 Log file

```
alma.log
```

This line specifies the name of the *log file*, where ALMA will write information about the scheduled job which can be useful to ensure that all parameters and keywords have been correctly processed or to debug eventual errors. In the example above, a log file named `alma.log` will be created.

3.1.8 Output settings

```
Real                    ! 'Real' or 'Complex' LNs or 'Rate' of change of LNs
!
ln_vs_t                 ! Output file format (LNs vs t or LNs vs n)
!
h.dat
l.dat
k.dat
```

The last block of the configuration file contains information about which outputs ALMA will write, the names of the output files and their formats. The keyword on the first line schedules ALMA to compute time-dependent real LNs (`Real`), their time derivatives (`Rate`) or frequency-domain complex LNs (`Complex`). The second line specifies whether LNs will be arranged in the output files with one harmonic degree per line (`ln_vs_n`) or with one time step per line (`ln_vs_t`). The last three lines specify the data files in which ALMA will write the h, l and k Love Numbers, which in the example above are `h.dat`, `l.dat` and `k.dat`, respectively. For complex-valued, frequency-domain LNs (`Complex` keyword), ALMA will output for each LN two adjacent numeric fields containing its real and imaginary parts. Table 1 shows how the output files are arranged for all combinations of the relevant keywords. If time derivatives of LNs are requested (`Rate` keyword), the time derivatives of LNs $\dot{h}$, $\dot{l}$ and $\dot{k}$ are expressed in units of kyr^{-1}.

3.2 Structure of the ALMA rheological model file

The rheological model file describes the structure of the planetary model which ALMA shall use to evaluate the LNs of the requested type. An example file is listed in Figure 2. The first four lines of the file are interpreted as headers and will not be parsed by ALMA. Each of the remaining data lines describe the properties of a rehologically distinct layer, from the outermost layer (first data line) to the inner core (last data line). Each line has the following structure:

```
r_top    density    rigidity    viscosity    rheology_kw   par_1 par_2
```

where `r_top` is the top radius of the layer (in km), `density` is its density in kg/m^3, `rigidity` its shear modulus in Pa and `viscosity` its viscosity in Pa·s. The keyword `rheology_kw` specifies the rheological law to be applied in the layer, among those listed in Table 2. Some rheological laws require additional parameters p_i (see Table 2) which, if needed, shall be specified after the rheology keyword (`par_1` and `par_2` above). The inviscid fluid rheology (`fluid` keyword) can be used only in the core (*i.e*, on the last data line); if this is not the case, ALMA will exit with an error message. The number of data lines in the rheological model file shall be consistent with the number of layers declared in the configuration file (see §3.1); otherwise ALMA will abort with an error. Please note that for some rheological laws the values of rigidity or viscosity are ignored (see Table 2).

```
! ==========================================================================
! ALMA configuration file
! ==========================================================================
!
! ~~~~~~~~~~~~~~~~~~~~~
! # General parameters
! ~~~~~~~~~~~~~~~~~~~~~
!
 128                        ! number of digits
 8                          ! order of the Gaver sequence
!
 Loading                    ! LN type ('Loading' or 'Tidal')
!
 2                          ! Minimum degree
 10                         ! Maximum degree
 1                          ! Step
!
!
 log                       ! Time scale ('log' / 'lin' / 'ext')
 100                       ! Time points (minus one)
 -4 4                      ! Time range  (10^(m1:m2) kyrs)
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Load time history
! ~~~~~~~~~~~~~~~~~~~~~
!
 step                     ! Load fcn ( 'step' / 'ramp' )
 1.0                      ! Ramp length (kyrs)
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Rheological model
! ~~~~~~~~~~~~~~~~~~~~~
!
 3                          ! Number of layers
 rheological_profiles/GS00-maxwell.dat
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Log file
! ~~~~~~~~~~~~~~~~~~~~~
!
alma.log
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Output files
! ~~~~~~~~~~~~~~~~~~~~~
!
Real                   ! 'Real' or 'Complex' LNs or 'Rate' of change of LNs
!
ln_vs_t                ! Output file format (LNs vs t or LNs vs n)
!
h.dat
l.dat
k.dat
!
! ==========================================================================
! End of ALMA configuration file
! ==========================================================================
```

Figure 1: Example ALMA configuration file

	Real or **Rate**					**Complex**					
ln_vs_n	n_1	$L_{n_1}(t_1)$	$L_{n_1}(t_2)$	$\dots$	$L_{n_1}(t_{p+1})$	n_1	$\mathrm{Re}(L_{n_1}(T_1))$	$\mathrm{Im}(L_{n_1}(T_1))$	$\dots$	$\mathrm{Re}(L_{n_1}(T_{p+1}))$	$\mathrm{Im}(L_{n_1}(T_{p+1}))$
	n_2	$L_{n_2}(t_1)$	$L_{n_2}(t_2)$	$\dots$	$L_{n_2}(t_{p+1})$	n_2	$\mathrm{Re}(L_{n_2}(T_1))$	$\mathrm{Im}(L_{n_2}(T_1))$	$\dots$	$\mathrm{Re}(L_{n_2}(T_{p+1}))$	$\mathrm{Im}(L_{n_2}(T_{p+1}))$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	n_{N_h}	$L_{n_{N_h}}(t_1)$	$L_{n_{N_h}}(t_2)$	$\dots$	$L_{n_{N_h}}(t_{p+1})$	n_{N_h}	$\mathrm{Re}(L_{n_{N_h}}(T_1))$	$\mathrm{Im}(L_{n_{N_h}}(T_1))$	$\dots$	$\mathrm{Re}(L_{n_{N_h}}(T_{p+1}))$	$\mathrm{Im}(L_{n_{N_h}}(T_{p+1}))$
ln_vs_t	t_1	$L_{n_1}(t_1)$	$L_{n_2}(t_1)$	$\dots$	$L_{n_{N_h}}(t_1)$	T_1	$\mathrm{Re}(L_{n_1}(T_1))$	$\mathrm{Im}(L_{n_1}(T_1))$	$\dots$	$\mathrm{Re}(L_{n_{N_h}}(T_1))$	$\mathrm{Im}(L_{n_{N_h}}(T_1))$
	t_2	$L_{n_1}(t_2)$	$L_{n_2}(t_2)$	$\dots$	$L_{n_{N_h}}(t_2)$	T_2	$\mathrm{Re}(L_{n_1}(T_2))$	$\mathrm{Im}(L_{n_1}(T_2))$	$\dots$	$\mathrm{Re}(L_{n_{N_h}}(T_2))$	$\mathrm{Im}(L_{n_{N_h}}(T_2))$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	t_{p+1}	$L_{n_1}(t_{p+1})$	$L_{n_2}(t_{p+1})$	$\dots$	$L_{n_{N_h}}(t_{p+1})$	T_{p+1}	$\mathrm{Re}(L_{n_1}(T_{p+1}))$	$\mathrm{Im}(L_{n_1}(T_{p+1}))$	$\dots$	$\mathrm{Re}(L_{n_{N_h}}(T_{p+1}))$	$\mathrm{Im}(L_{n_{N_h}}(T_{p+1}))$

Table 1: Arrangment of LNs in the ALMA output files according to different combinations of the (`Real`, `Rate`, `Complex`) and (`ln_vs_n`, `ln_vs_t`) keywords. Here, $L_n(t)$ represents any of the real-valued h, l, k LNs or of their time derivatives $\dot{h}$, $\dot{l}$, $\dot{k}$ for harmonic degree n and time t; $L_n(T)$ represents one of the complex-valued LNs $h(\omega)$, $l(\omega)$, $k(\omega)$ for harmonic degree n and frequency $\omega = 2\pi/T$, $p+1$ is the number of time steps and N_h is the number of harmonic degrees.

```
!------------------------------------------------------------------------
! radius,    density,        rigidity      viscosity
!  (m)       (kg/m^3)          (Pa)          (Pa.s)
!------------------------------------------------------------------------
  6371e3      3.037e3        0.50605e11        1.e21    elastic
  6301e3      3.438e3        0.70363e11        1.e21    burgers    1.0   0.1
  5951e3      3.871e3        1.05490e11        1.e21    maxwell
  5701e3      4.978e3        2.28340e11        2.e21    maxwell
  3480e3     10.750e3        0.e11             0.e21    fluid
```

Figure 2: Example ALMA rheological model file

Rheological law	Complex rigidity $\mu(s)$	Keyword	Additional parameters
Hooke	μ	`elastic`	–
Maxwell	$\dfrac{\mu s}{s+\mu/\eta}$	`maxwell`	–
Newton	$\eta\, s$	`newton`	–
Kelvin	$\mu+\eta s$	`kelvin`	–
Burgers	$\dfrac{\mu s\left(s+\dfrac{\mu_2}{\eta_2}\right)}{s^2+s\left(\dfrac{\mu}{\eta}+\dfrac{\mu+\mu_2}{\eta_2}\right)+\dfrac{\mu\,\mu_2}{\eta\,\eta_2}}$	`burgers`	$p_1=\dfrac{\mu_2}{\mu}$, $p_2=\dfrac{\eta_2}{\eta}$
Andrade	$\left[\frac{1}{\mu}+\frac{1}{\eta s}+\Gamma(\alpha+1)\frac{1}{\mu}\left(\frac{\eta s}{\mu}\right)^{-\alpha}\right]^{-1}$	`andrade`	$p_1=\alpha$
Inviscid fluid	–	`fluid`	–

Table 2: Rheological constitutive laws implemented in ALMA with their complex rigidities $\mu(s)$, the corresponding keywords for the rheological model file and the list of additional parameters p_i, where applicable. Here, μ is the elastic rigidity, η is the newtonian viscosity, μ_2 and η_2 are the rigidity and viscosity of the transient element in the bi-viscous Burgers rheology, respectively. In the Andrade rheological law, α is the creep parameter while $\Gamma(x)$ denotes the Euler Gamma function.

4 Worked examples

In this section we will guide the user through two examples of the ALMA usage.

4.1 Loading Love Numbers for the Earth

In this first example, we schedule ALMA to compute time-domain loading LNs $h_n(t)$, $l_n(t)$, $k_n(t)$ suitable for describing the response of the Earth to a step-wise loading. We will compute LNs for harmonic degrees from $n = 2, 4, \ldots, 500$ and at four time steps $t = 10^{-2}, 10^{-1}, 1, 10$ kyr. The configuration file corresponding to this example is `config.EarthLLNs.dat`, stored in the `CONFIGS/` folder of the main ALMA directory. Its contents is reported below.

```
! ============================================================================
! ALMA configuration file
! ============================================================================
!
! ~~~~~~~~~~~~~~~~~~~~~
! # General parameters
! ~~~~~~~~~~~~~~~~~~~~~
!
 128                         ! number of digits
 8                           ! order of the Gaver sequence
!
 Loading                     ! LN type ('Loading' or 'Tidal')
!
 2                           ! Minimum degree
 500                         ! Maximum degree
 2                           ! Step
!
 log                        ! Time scale ('log' / 'lin' / 'ext')
 3                          ! Time points (minus one)
 -2 1                       ! Time range  (10^(m1:m2) kyrs)
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Load time history
! ~~~~~~~~~~~~~~~~~~~~~
!
 step                      ! Load fcn ( 'step' / 'ramp' )
 1.0                       ! Ramp length (kyrs)
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Rheological model
! ~~~~~~~~~~~~~~~~~~~~~
!
 6                           ! Number of layers
 MODELS/VM5a.dat
!
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Log file
! ~~~~~~~~~~~~~~~~~~~~~
!
```

```
 alma.log
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Output files
! ~~~~~~~~~~~~~~~~~~~~~
!
 Real                    ! 'Real' or 'Complex' LNs
!
 ln_vs_n                      ! Output file format (LNs vs t or LNs vs n)
!
 h_EarthLLN.dat
 l_EarthLLN.dat
 k_EarthLLN.dat
!
! ============================================================================
! End of ALMA configuration file
! ============================================================================
```

The rheological model specified in the configuration file is a 6-layer model based upon the VM5a viscosity profile (Argus et al., 2014, Peltier et al., 2015), which includes an elastic lithosphere, 4 mantle layers with a Maxwell viscoelastic rheology and a fluid inviscid core. The density and rigidity of each layer are volume averages of the PREM reference model (Dziewonski and Anderson, 1981). The corresponding rheological model file is stored inside the `MODELS/` subfolder with name `VM5a.dat` and reported below.

```
!-------------------------------------------------------------
! radius,    density,      rigidity     viscosity
!  (m)       (kg/m^3)        (Pa)         (Pa.s)
!-------------------------------------------------------------
 6371.0e3    3.02751e3     5.2429e10      1.00e99      elastic
 6311.0e3    3.37472e3     6.7446e10      1.00e22      maxwell
 6271.0e3    3.63687e3     8.9212e10      5.00e20      maxwell
 5701.0e3    4.54824e3     1.8321e11      1.57e21      maxwell
 5111.0e3    5.10252e3     2.4446e11      3.23e21      maxwell
 3485.5e3    1.09626e4     0.0000e00      0.00e00      fluid
```

ALMA can be run with the setup described above by typing `alma.exe` in the main directory, followed by the path of the configuration file. A transcript of the terminal session is reported below. The symbol '`$`' represents the system prompt, the text in boldface is the command typed by the user and part of the program output has been omitted for brevity.

```
$ ./alma.exe CONFIGS/config.EarthLLNs.dat
                                                      ___
                                                   ,o88888
                                                ,o8888888'
                          ,:o:o:oooo.       ,8O88Pd8888"
                      ,.::.::o:ooooOoOoO. ,oO8O8Pd888'"
                    ,.:.::o:ooOoOoOO8O8OOo.8OOPd8O8O"
                   , ..:.::o:ooOoOOOO8OOOOo.FdO8O8"
                  , ..:.::o:ooOoOO8O888O8O,COCOO"
                 , . ..:.::o:ooOoOOOO8OOOOCOCO"
                  . ..:.::o:ooOoOoOO8O8OCCCC"o
                     . ..:.::o:ooooOoCoCCC"o:o
                     . ..:.::o:o:,cooooCo"oo:o:
                  `   . . ..:.:cocoooo"'o:o:::'
                  .`   . ..::ccccoc"'o:o:o:::'
                 :.:.    ,c:cccc"':.:.:.:.:.'
               ..:.:"'`::::c:"'..:.:.:.:.:.'
             ...:.'.:.::::"'    . . . . .'
            .. . ....:."' `   .  . . ''
          . . . ...."'
          .. . ."'
         .


  ****
  **** This is ALMA
  **** (the plAnetary Love nuMbers cAlculator)
  ****

  - Parsing configuration file: CONFIGS/config.EarthLLNs.dat
  - Initializing the multi-precision libraries
  - Opening the log file 'alma.log'
  - Building the rheological model
  - Building the time steps
  - Harmonic degree n =            2 (  0.671000004     s)
  - Harmonic degree n =            4 (  0.671999931     s)
  - Harmonic degree n =            6 (  0.672000170     s)
  - Harmonic degree n =            8 (  0.687000036     s)
  - Harmonic degree n =           10 (  0.702999830     s)
  ...
  ...
  - Harmonic degree n =          496 (  0.734008789     s)
  - Harmonic degree n =          498 (  0.750000000     s)
  - Harmonic degree n =          500 (  0.750000000     s)
  - Writing output files 'h_EarthLLN.dat', 'l_EarthLLN.dat', 'k_EarthLLN.dat'
  - Closing the log file 'alma.log'
  - ALMA job completed. Time elapsed:    183.826996     s
```

After execution, ALMA stores the $h_n(t)$, $l_n(t)$ and $k_n(t)$ LNs into data files named `h_EarthLLN.dat`, `h_EarthLLN.dat` and `k_EarthLLN.dat`, respectively. Since the keyword `ln_vs_n` is specified in the configuration file, each of these files is arranged with one harmonic degree per line and one column per time step (see Table 1). The first few lines of data file `h_EarthLLN.dat` are reported below.

```
# ----------------------------------------------------------
# h load Love number
# Created by ALMA on 2021-10-12 15:09:24
# ----------------------------------------------------------
#
   2     -0.45895716E+00    -0.53057515E+00    -0.10103710E+01    -0.19017933E+01
   4     -0.47552003E+00    -0.58593382E+00    -0.14735430E+01    -0.35945671E+01
   6     -0.54567519E+00    -0.70252759E+00    -0.19779187E+01    -0.54316880E+01
   8     -0.64724064E+00    -0.85634676E+00    -0.25474846E+01    -0.73431047E+01
  10     -0.74839642E+00    -0.10091285E+01    -0.31213066E+01    -0.92982440E+01
  12     -0.84127319E+00    -0.11504238E+01    -0.36727925E+01    -0.11255150E+02
  14     -0.92396395E+00    -0.12770698E+01    -0.41868015E+01    -0.13172869E+02
  16     -0.99614938E+00    -0.13880353E+01    -0.46524094E+01    -0.15015365E+02
  18     -0.10581614E+01    -0.14832951E+01    -0.50620781E+01    -0.16750919E+02
  ...
  ...
```

Numerical values output by ALMA can be plotted with any standard visualization package. For instance, Figure 3 shows plots of $h_n(t)$ and $k_n(t)$ as a function of the harmonic degree n fot $t = 10$ yr and $t = 1000$ yr (corresponding to the second and fourth columns of the output files, respectively). The `MODELS/` subfolder of the ALMA main directory contains additional data files with variations of the VM5a model, in which the upper mantle is modeled with transient rheologies according to the Burgers (`VM5a-BG.dat`) or Andrade (`VM5a-AD.dat`) constitutive laws.

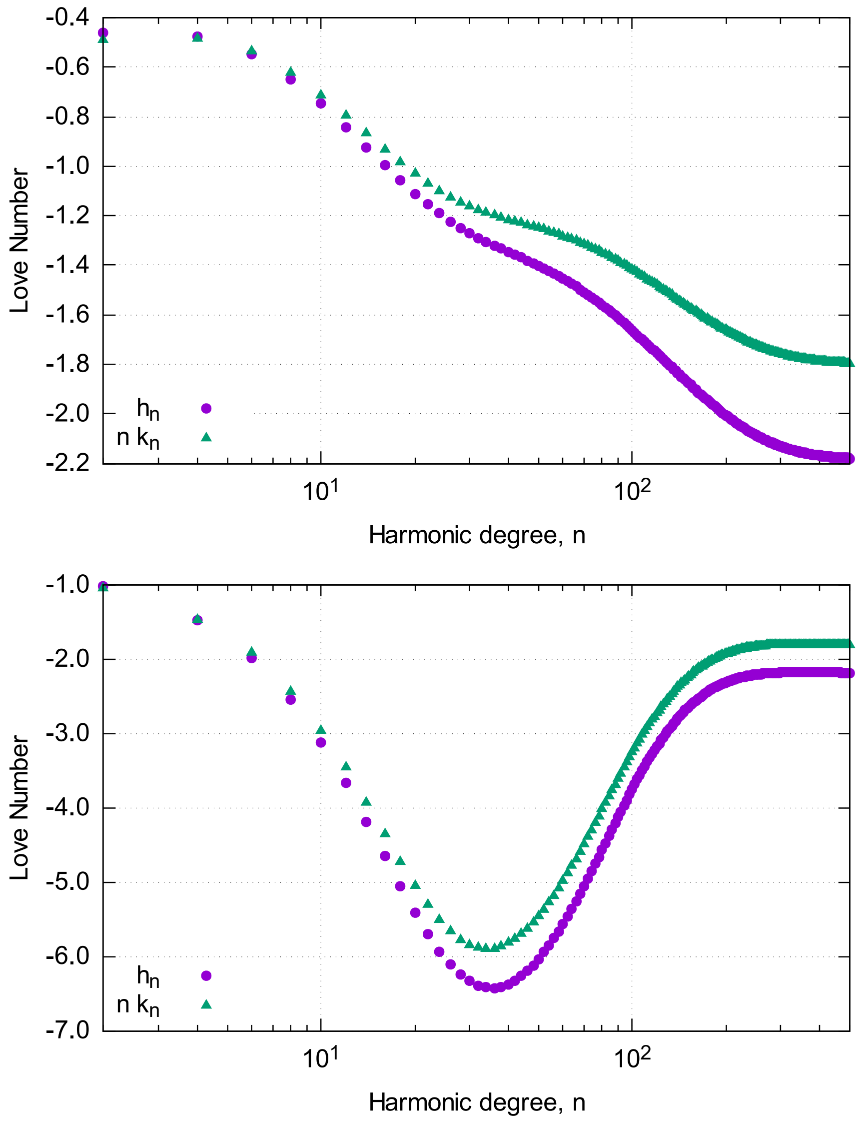

Figure 3: Time-domain loading Love Numbers $h_n(t)$ and $k_n(t)$ for Earth model VM5a assuming a step-wise load time history. Top and bottom panels correspond to times $t = 10$ yr and $t = 1000$ yr, respectively.

4.2 Tidal dissipation in a uniform sphere

In this second example, we will configure ALMA to compute the frequency-domain, complex-valued LNs of degree $n = 2$ for a periodic forcing, suitable for describing the tidal dissipation. We will assume a uniform planetary model with a Maxwell viscoelastic rheology (the so-called "Kelvin sphere"), which is of particular relevance since for that simplified model closed-form analytical expressions of the LNs are available, so a benchmark of numerical solutions is possible. We will compute tidal LNs $h_2(\omega)$, $l_2(\omega)$ and $k_2(\omega)$ for frequencies $\omega = 2\pi/T$ corresponding to logarithmically spaced periods T ranging from $T = 10^{-5}$ kyr to $T = 10^5$ kyr. The corresponding configuration file is `config.KS-complexTLN.dat`, and it is stored in the `CONFIGS/` subfolder. Its contents are listed below.

```
! ===========================================================================
! ALMA configuration file
! ===========================================================================
!
! ~~~~~~~~~~~~~~~~~~~~~
! # General parameters
! ~~~~~~~~~~~~~~~~~~~~~
!
 128                        ! number of digits
 8                          ! order of the Gaver sequence
!
 Tidal                      ! LN type ('Loading' or 'Tidal')
!
 2                          ! Minimum degree
 2                          ! Maximum degree
 1                          ! Step
!
 log                       ! Time scale ('log' / 'lin' / 'ext')
 100                       ! Time points (minus one)
 -5 5                      ! Time range  (10^(m1:m2) kyrs)
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Load time history
! ~~~~~~~~~~~~~~~~~~~~~
!
 step                     ! Load fcn ( 'step' / 'ramp' )
 1.0                      ! Ramp length (kyrs)
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Rheological model
! ~~~~~~~~~~~~~~~~~~~~~
!
 2                          ! Number of layers
 MODELS/uniform-sphere.dat
!
! ~~~~~~~~~~~~~~~~~~~~~
! # Log file
! ~~~~~~~~~~~~~~~~~~~~~
!
 alma-KS.log
!
```

```
! ~~~~~~~~~~~~~~~~~~~~~
! # Output files
! ~~~~~~~~~~~~~~~~~~~~~
!
 Complex                    ! 'Real' or 'Complex' LNs
!
 ln_vs_t                    ! Output file format (LNs vs t or LNs vs n)
!
 h_complex_KS.dat
 l_complex_KS.dat
 k_complex_KS.dat
!
! ===========================================================================
! End of ALMA configuration file
! ===========================================================================
```

Since ALMA requires at least 2 layers in the rheological model file, we simulate a uniform sphere as a two-layer model including a core and a mantle with the same rheological parameters. We assume a sphere of radius $a = 6371$ km, density $\rho = 5.514 \times 10^3$ kg/m^3, rigidity $\mu = 1.46 \times 10^{11}$ Pa and Maxwell viscosity $\eta = 10^{21}$ Pa·s. The core radius is set to $c = 3480$ km; however, since the core and the mantle have the same rheological parameters, the value of c has no effect on the model output. The corresponding rheological model file is `uniform-sphere.dat` and it is stored into the `MODELS/` folder. Its contents are listed below.

```
!-------------------------------------------------------------
! radius,    density,      rigidity     viscosity
!  (m)       (kg/m^3)        (Pa)         (Pa.s)
!-------------------------------------------------------------
 6371e3       5.514e3       1.46e11       1.e21      maxwell
 3480e3       5.514e3       1.46e11       1.e21      maxwell
```

To compute LNs with this setup, we type at the system prompt the name of the ALMA executable followed by the full path of the configuration file `config.KS-complexTLN.dat`. Below is a transcript of the ALMA session.

```
$ ./alma.exe CONFIGS/config.KS-complexTLN.dat
                                                  ___
                                               ,o88888
                                            ,o8888888'
                      ,:o:o:oooo.        ,8O88Pd8888"
                  ,.::.::o:ooooOoOoO. ,oO8O8Pd888'"
                ,.:.::o:ooOoOoOO8O8OOo.8OOPd8O8O"
               , ..:.::o:ooOoOOOO8OOOOo.FdO8O8"
              , ..:.::o:ooOoOO8O888O8O,COCOO"
             , . ..:.::o:ooOoOOOO8OOOOCOCO"
              . ..:.::o:ooOoOoOO8O8OCCCC"o
                 . ..:.::o:ooooOoCoCCC"o:o
                 . ..:.::o:o:,cooooCo"oo:o:
              `   . . ..:.:cocoooo"'o:o:::'
              .`   . ..::ccccoc"'o:o:o:::'
             :.:.    ,c:cccc"':.:.:.:.:.'
           ..:.:"'`::::c:"'..:.:.:.:.:.'
         ...:.'.:.::::"'    . . . . .'
        .. . ....:."' `   .  . . ''
```

```
         . . . ...."'
         .. . ."'
        .

  ****
  **** This is ALMA
  **** (the plAnetary Love nuMbers cAlculator)
  ****

  - Parsing configuration file: CONFIGS/config.KS-complexTLN.dat
  - Initializing the multi-precision libraries
  - Opening the log file 'alma-KS.log'
  - Building the rheological model
  - Building the time steps
  - Harmonic degree n =            2 (  0.530999959     s)
  - Writing output files 'h_complex_KS.dat', 'l_complex_KS.dat', 'k_complex_KS.dat'
  - Closing the log file 'alma-KS.log'
  - ALMA job completed. Time elapsed:   0.561999977     s
```

After the execution of the ALMA job, the frequency-domain tidal LNs are stored in data files named `h_complex_KS.dat`, `l_complex_KS.dat` and `k_complex_KS.dat`. With the configuration discussed above, in these files each line will contain a column with the period T (in kyrs) followed by two columns with $\mathrm{Re}\, L_2(T)$ and $\mathrm{Im}\, L_2(T)$, where L_2 is any of the h_2, l_2 and k_2 LNs. The first lines of data file containing $h_2(T)$ (`h_complex_KS.dat`) are tabulated below.

```
# ----------------------------------------------------------
# h tidal Love number
# Created by ALMA on 2021-10-12 16:27:37
# ----------------------------------------------------------
#
     0.10000000E-04     0.49798330E+00    -0.29242831E-05
     0.12589254E-04     0.49798330E+00    -0.36814544E-05
     0.15848932E-04     0.49798330E+00    -0.46346765E-05
     0.19952623E-04     0.49798330E+00    -0.58347120E-05
     0.25118864E-04     0.49798330E+00    -0.73454672E-05
     0.31622777E-04     0.49798330E+00    -0.92473953E-05
     0.39810717E-04     0.49798330E+00    -0.11641781E-04
     0.50118723E-04     0.49798330E+00    -0.14656134E-04
     0.63095734E-04     0.49798330E+00    -0.18450979E-04
     0.79432823E-04     0.49798330E+00    -0.23228407E-04
     ...
     ...
```

The dependence on the forcing period of the real and imaginary parts of the LNs can be easily visualized with any plotting package; for instance, Figure 4 shows the real and imaginary parts of $h_2(T)$ and $k_2(T)$ as a function of T. Other quantities of interest for describing the tidal dissipation, such as the phase lag ϕ and the quality factor Q, can be easily obtained from these tabulated values.

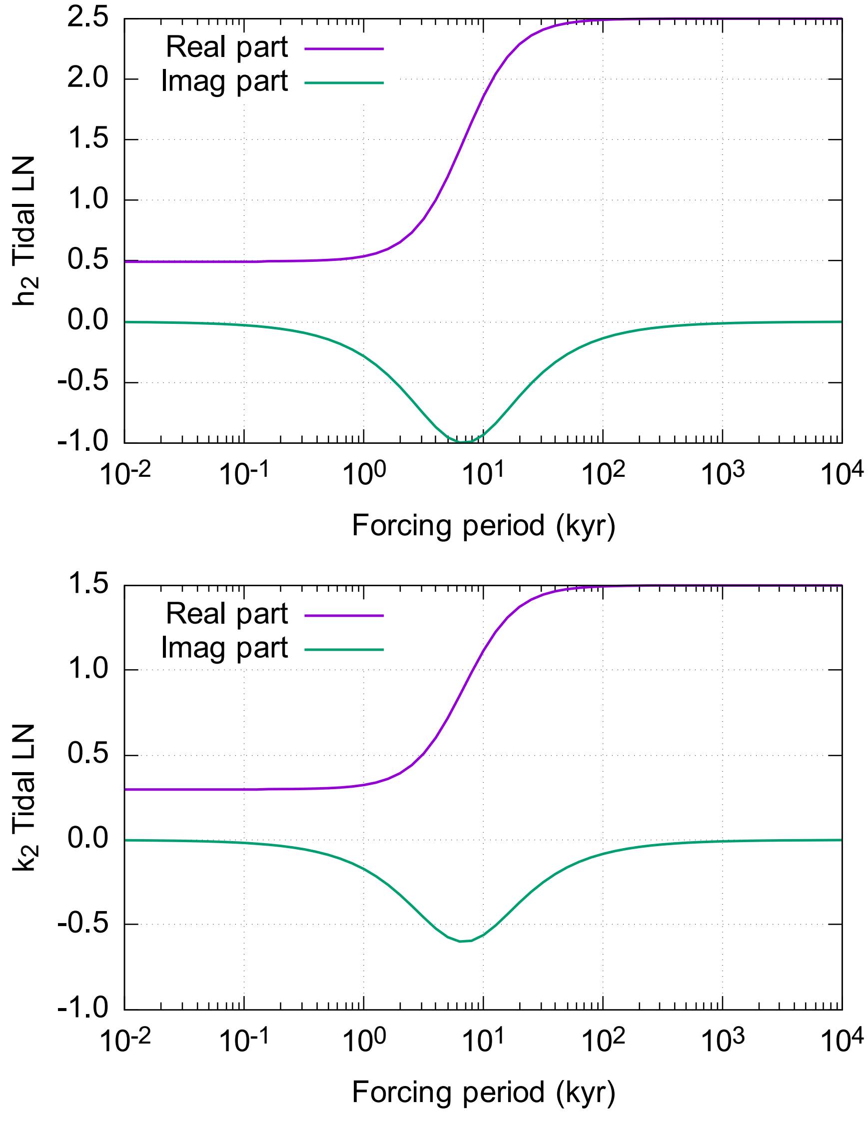

Figure 4: Frequency-domain tidal loading Love Numbers h_2 (top) and k_2 (bottom) for a homogeneous Earth model with Maxwell viscoelastic rheology as a function of the period of the tidal forcing.

5 Copyright statements

5.1 Copyright statement for ALMA

5.2 Copyright statement for FMLIB

References

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