REAR

a Regional ElAstic Rebound calculator

User Manual for version 1.5

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Contents

List of Figures

List of Tables

1 Introduction

REAR (acronym of "Regional ElAstic Rebound calculator") is a Fortran 90 program for computing the response of a SNREI (Solid, non-Rotating, Elastic, Isotropic) Earth model to surface loading. REAR is free software and is released under the GPLv3 license (see http://www.gnu.org/copyleft/gpl.html for details).

REAR has been specifically designed for studying the regional deformations induced by variations of the cryospheric loads. As such, it only accounts for the deformation induced by the surface load (*e.g.*, a cryospheric load), and does not solve the Sea Level Equation [Farrell and Clark, 1976]. On a global scale, this approximation could lead to significant errors and the use of a more realistic tool as a Sea Level Equation solver is recommended (see *e.g.,* program SELEN[4] by Spada and Melini [2019], available from http://geodynamics.org/cig/software/selen/).

While other tools exist for the computation of the elastic response of the Earth (see in particular the TABOO calculator of Spada [2003], available from http://github.com/danielemelini/TABOO), REAR offers the advantage of being much more flexible, versatile and user-friendly. Furthermore, it is optimized for the particularly challenging task of the computation of deformations of very high harmonic degrees, which is essential for the interpretation of geodetic observations in regions subject to small-scale surface mass variations.

REAR obtains the solution in two distinct steps.

In the first step, REAR computes the response to a finite-sized disk load of unitary thickness with a Heaviside load history, or, equivalently, the surface rates of displacement associated to a unit rate of mass variation. In order to compute these "Green Functions" (GFs), REAR needs a set of load-deformation coefficients (LDCs). Some sample LDCs sets are provided with REAR. Of course, the user can employ any other preferred set of LDCs, appropriate for describing the rheological structure of the considered region. For an arbitrary, spherically symmetric rheological profile, LDCs can be easily obtained with the ALMA[3] code [Melini et al., 2022], available at https://github.com/danielemelini/ALMA3. ALMA[3] can compute time-dependent Love Numbers for a set of incompressible viscoelastic rheologies of geophysical relevance, enabling the possibility of modeling with REAR the time-dependent response to a surface load.

In the second step, by means of the superposition principle, REAR combines the GFs computed in the fist step in order to obtain maps of geodetic variables associated to a user-supplied mass balance model. This model must be discretized in disk-shaped elements with the same radius of those used to construct the GF. In the current version (1.5), REAR can compute vertical and horizontal surface displacements and geoid height variations (or their rates of variation). If uncertainties on the input mass balance model are known, REAR can propagate them in order to obtain an estimate of uncertainties on the geodetic observables. Due to its simple structure, REAR can be easily modified to deal with additional geodetic variables.

For the complete theory background of REAR, the reader is referred to Farrell [1972], Spada [2003] and Spada et al. [2011]. A condensed description of the theory is given below in Section 5. The method of computation of the Love numbers and LDCs is given by Gegout et al. [2010]. A previous version of REAR has been recently employed by Spada et al. [2012] in order to solve the "Regional Elastic Rebound" problem for the present melting of the Greenland ice sheet, based on the results of Sørensen et al. [2011]. More recently, the current version of REAR has employed to obtain estimates of elastic uplift in response to ice mass loss in Antarctica and Greenland within the context of the IMBIE3 exercise (Ice sheet Mass Balance Inter-comparison Exercise, see http://www.imbie.org/)

2 Acknowledgements

REAR has been originally developed in the framework of the International Association of Geodesy (IAG) Sub-Commission 3.4: Cryospheric Deformation and has been further updated in the context of the Ice sheet Mass Balance Inter-comparison Exercise (IMBIE, http://www.imbie.org/). Matt King and Ben Marzeion have greatly contributed to the development of REAR 1.0, upon which the current version is based. We have benefited from discussion with a number of colleagues, particularly within the ES0701 COST Action and the European Commission's 7^{th} Framework Programme ice2sea project. We are indebted with Mike Bevis, Karina Nielsen, Louise Sandberg Sørensen, Nicolaj Hansen, Anthony Memin and Valentina Barletta for advice and encouragement. This work is partly supported by a research grant of Dipartimento di Scienze di Base e Fondamenti (DiSBeF, Urbino University), by a grant of Dipartimento di Fisica e Astronomia (DIFA, Alma Mater Studiorum Università di Bologna), by Programma Nazionale di Ricerche in Antartide (CUP D32I14000230005) and by the INGV project "Pianeta Dinamico" 2021-22 Tema 4 KINDLE (CUP D53J19000170001), funded by the Italian Ministry of University and Research "Fondo finalizzato al rilancio degli investimenti delle amministrazioni centrali dello Stato e allo sviluppo del Paese, legge 145/2018". Preliminary development of REAR has been performed thanks to a CINECA grant under the ISCRA (Italian SuperComputing Resource Allocation) initiative. REAR includes portions from the SHTOOLS package by Wieczorek and Meschede [2018].

3 Attribution

The seminal references to REAR are:

D. Melini, P. Gegout, M. King, B. Marzeion and G. Spada. On the rebound: Modeling Earth's ever-changing shape, *Eos*, 96, doi:10.1029/2015EO033387, 2015.

A. Consorzi, D. Melini, P. Gegout and G. Spada. REAR: a Regional Elastic Rebound Calculator, submitted to *Journal of Open Source Software*, 2022.

4 Copyright statement

5 Theory

The elastic response of the Earth to a disk load is computed in a straightforward way according to the theory presented by Farrell [1972]. Throughout this manual, the response to this disk load will be referred to as "Green's Function" (GF), although strictly speaking this name denotes the solution for a perfectly localized and impulsive load (see Farrell [1972] and Bevis et al. [2016]).

Assuming that the load is turned on at time $t = 0$ and its amplitude is kept constant, `REAR` implements the formula:

$$\left\{ \begin{array}{c} u \\ v \\ n \end{array} \right\} (\psi) = 3 \left(\frac{\rho}{\rho_e} \right) H \sum_{\ell=\ell_{min}}^{\ell_{max}} \left\{ \begin{array}{c} h'_\ell \\ l'_\ell \\ 1 + k'_\ell \end{array} \right\} \frac{f_\ell(\alpha)}{2\ell + 1} \left\{ \begin{array}{c} 1 \\ \frac{\partial}{\partial\psi} \\ 1 \end{array} \right\} P_\ell(\cos\psi), \tag{1}$$

where ψ is the angular separation of the observed with respect to the axis of symmetry of the disk load, $u(\psi)$ and $v(\psi)$ are the GFs for vertical and horizontal displacement, $n(\psi)$ is the GF for the geoid height variation, defined as:

$$n(\psi) = \frac{\Phi(\psi)}{\gamma}, \tag{2}$$

where Φ is the total variation of the Earth's gravity potential, γ is the reference gravity field at the surface, and the LDCs (h'_ℓ, l'_ℓ, k'_ℓ) are the proportionality factors that connect the forcing (*i.e.,* the surface load) to the Earth's surface response in terms of gravitational potential (k'_ℓ), radial (h'_ℓ) and horizontal displacement (l'_ℓ), where ℓ is the harmonic degree. In Eq. (1), ψ is the angular separation between the centre of the disk load and the observer, ρ is the mass density of the surface load (for ice loads, $\rho = \rho_i$, where ρ_i is the ice density), H is the change of the thickness of the surface load, ρ_e is the Earth's average density, $P_\ell(\cos\psi)$ is the Legendre polynomial of degree ℓ, and f_ℓ is a non-dimensional shape factor depending on the load geometry, described below. It is important to note that in (1) u is positive upward, and that positive (negative) values of v denote horizontal displacement *away from* (*towards*) the center of the load, respectively.

As discussed by Bevis et al. [2016], the disk load can be defined in two alternative ways, which differ in how the problem of mass conservation is addressed. The first possibility is to define a disk load uniformly distributed for $0 \leq \psi \leq \alpha$, and exactly compensated by a complementary load of constant thickness acting on $\alpha < \psi \leq \pi$ so that there is no net variation of the total mass of the Earth [Spada et al., 2011]. In Spada [2003], this particular surface load is dubbed *compensated disk load* and is described by the shape factors

$$f_\ell = \begin{cases} 0 & \text{if } \ell = 0 \\ -\dfrac{P_{\ell+1}(\cos\alpha) - P_{\ell-1}(\cos\alpha)}{1 + \cos\alpha} & \text{if } \ell \geq 1\,. \end{cases} \tag{3}$$

As pointed by Bevis et al. [2016], a second possibility is to consider only the primary disk load without introducing a complementary disk load. In this case, which we refer to as *uncompensated disk load*, the shape factors are given by

$$f_\ell = \frac{1}{2} \begin{cases} 1 - \cos\alpha & \text{if } \ell = 0 \\ -\left(P_{\ell+1}(\cos\alpha) - P_{\ell-1}(\cos\alpha)\right) & \text{if } \ell \geq 1\,. \end{cases} \tag{4}$$

Of course, for an uncompensated disk load, mass conservation is not automatically ensured. In this case, the user can ensure that the total mass change is zero by introducing one or more uncompensated disk loads on the basis of some specific geophysical intuition; for example, if the primary disk load represents an ice cap, the compensating ones could be distributed over the ocean basins to fit the shape of the coastlines. It is worth to note that, while for a compensated surface load there is no response of the Earth at degree $\ell = 0$ since $f_0 = 0$ in (3), for an uncompensated load there is a non-vanishing shape factor at degree $\ell = 0$.

While `REAR` implements disk-shaped loads of uniform thickness, other choices of load geometry are possible, for which shape factors in analytical form are available. Since they are illustrated in detail in Spada [2003] and in Spada et al. [2011], the implementation in `REAR` is straightforward.

Modeling the response of the Earth to a disk-shaped load requires some attention to ensure the convergence of the harmonic sum in Eq. (1). Indeed, as shown in detail by Bevis et al. [2016], to accurately represent a disk element of angular radius α the maximum harmonic degree must satisfy the "rule of thumb":

$$\ell_{max} \gtrsim \overline{\ell} = \frac{360^\circ}{\alpha}\,, \tag{5}$$

so that, for instance, for a disk element of radius 10 km (corresponding to $\alpha \simeq 0.1^\circ$), the harmonic sum in (1) shall be performed at least up to $\overline{\ell} \simeq 4,000$; as discussed by Bevis et al. [2016], a more defensive choice would be setting $\ell_{max} = 2\overline{\ell}$. If LDCs are not known up to the needed harmonic degree, it may be useful to take advantage of their asymptotic approximations. Indeed, for large values of ℓ the following asymptotic expressions are valid

$$h'_\ell \simeq h'_\infty \tag{6}$$

$$l'_\ell \simeq \frac{l'_\infty}{\ell} \tag{7}$$

$$k'_\ell \simeq \frac{k'_\infty}{\ell}\,, \tag{8}$$

where h'_∞, l'_∞ and k'_∞ are the limits of h'_ℓ, $\ell\, l'_\ell$ and $\ell\, k'_\ell$ for $\ell \to \infty$, respectively [Farrell, 1972]. Furthermore, for large ℓ values the LDCs become increasingly sensitive to the values of the elastic constants of the Earth's outmost layers and the asymptotic dependence in (6) matches the solution of the Boussinesq problem, valid for a flat and homogeneous Earth model having the same elastic properties of the superficial layer. `REAR` can automatically extend the available set of LDCs to higher harmonic degrees according to Eqs. (6-8), estimating the constants h'_∞, l'_∞ and k'_∞ from the user-supplied database of LDCs.

Since the three GFs (1) scale with the variation of the load thickness H (this is a direct consequence of the assumption of elasticity), rates of change of the GFs for relevant geodetic variables ($\dot{u}, \dot{v}, \dot{n}$) are simply obtained from Eq. (1), with H substituted by its time-derivative $\dot{H}$. Therefore, `REAR` can equivalently compute absolute variation of observables or their rate of variation. By convention, a value $H > 0$ indicates an instantaneous increase in the load thickness; while $\dot{H} > 0$ denotes a positive rate of variation. Of course, the source of `REAR` can be easily modified in order to build other GFs of interest, such as for example those pertaining to free-air gravity field anomalies ot tilts, using appropriate combinations of LDCs [Farrell, 1972].

In `REAR`, the response of the Earth to the load exerted by a realistic surface mass distribution is constructed by a superposition of GFs, each corresponding to one of many disks that discretize the load itself. Accordingly, at a given location of co-latitude θ and longitude λ on the Earth's surface ($r = a$), the total response is

$$\left\{ \begin{array}{c} u_r \\ u_\theta \\ u_\lambda \\ n \end{array} \right\} (\theta, \lambda) = \sum_{k=1}^{N_d} H_k \left\{ \begin{array}{c} u_k \\ \cos\delta\, v_k \\ \sin\delta\, v_k \\ n_k \end{array} \right\} (\psi_k), \tag{9}$$

where $(u_r, u_\theta, u_\lambda)$ are the components of surface displacement in spherical coordinates, n is the geoid height variation, (r, θ, λ) denote the radius, co-latitude and longitude, N_d is the number of disk elements, (u_k, v_k, n_k) are the GFs relative to a elementary surface load of unit height (these are given by Eq. (1), assuming that the load thickness changes by $H = 1$), H_k is the load thickness change of the k-th disk element and ψ_k is the co-latitude of the observer at (θ, λ) relative to the center of the k-th disk. According to usual conventions in spherical geometry, u_r is positive upward, while u_θ and u_λ are positive southward and eastward, respectively. With the aid of the cosine and sine theorems in spherical geometry, it is easily established that

$$\begin{aligned} \cos\delta &= \frac{\cos\theta_k - \cos\theta\cos\psi_k}{\sin\theta\sqrt{1-\cos^2\psi_k}} \\ \sin\delta &= \frac{\sin(\lambda-\lambda_k)\sin\theta_k}{\sqrt{1-\cos^2\psi_k}}, \end{aligned} \tag{10}$$

with

$$\cos\psi_k = \cos\theta\cos\theta_k + \sin\theta\sin\theta_k\cos(\lambda-\lambda_k), \tag{11}$$

where θ_k and λ_k are the co-latitude and the longitude of the k-th disk, respectively (for details, the reader is referred to page 78 of Spada [2003]).

If an estimate of the uncertainty σ_{H_k} on the thickness change H_k of each load element is known, the corresponding uncertainties on the deformation and geoid height variation can be evaluated by summing the errors in quadrature:

$$\left\{ \begin{array}{c} \sigma^2_{u_r} \\ \sigma^2_{u_\theta} \\ \sigma^2_{u_\lambda} \\ \sigma^2_n \end{array} \right\} (\theta, \lambda) = \sum_{k=1}^{N_d} \sigma^2_{H_k} \left\{ \begin{array}{c} u_k \\ \cos\delta\, v_k \\ \sin\delta\, v_k \\ n_k \end{array} \right\} (\psi_k), \tag{12}$$

where σ_{u_r}, σ_{u_θ}, σ_{u_λ} and σ_n are uncertainties upon u_r, u_θ, u_λ and n, respectively.

In `REAR`, all the computations rely upon input data expressed in terms of latitude and longitude, hence all formulas above are directly implemented. Load models that are given on projected coordinate systems need to be un-projected to geographical coordinates before running `REAR`. Raw data may therefore need re-sampling in order to ensure that loading and unloading data have consistent geometry (e.g. disk sizes). This procedure may introduce an additional source of epistemic uncertainty in the model outputs; indeed, especially in proximity of the load, the output fields may be sensitive to the chosen disk size. In this case it may be useful to perform a set of simulations using different values of the disk size, in order to obtain an estimate of the model uncertainty associated with the choice of the spatial discretization of the load. To minimize overlapping and interstices between adjacent disc elements, it may be convenient to discretize the ice load on the icosahedral grid proposed by Tegmark [1996]; Fortran codes to generate icosahedral grids at arbitrary resolutions are available on the Max Tegmark webpage at `https://space.mit.edu/home/tegmark/icosahedron.html`.

6 Running REAR

6.1 Installation and prerequisites

REAR can be run on any UNIX-like system (including Linux and macOS); on Windows computers, it can be run within the Cygwin environment[1] or using the Windows Subsystem for Linux[2]. The computationally intensive portions of REAR employ multi-threaded parallelism, so the code can take advantage of modern multi-core CPUs.

REAR has been tested with both the open-source GNU `gfortran` compiler[3] and the commercial Intel Fortran compiler; however, REAR can be used with any standards-compliant Fortran 90 compiler.

The Generic Mapping Tools (GMT) public domain mapping software[4] [Wessel and Smith, 1998], while not strictly needed to run REAR, it is useful to reproduce the outputs of the example runs provided with REAR and discussed in Section 7.

REAR includes routines from the Fortran version of SHTOOLS by Wieczorek and Meschede [2018] for the numerical evaluation of the Legendre polynomials. It also takes advantage of the icosahedron-based method[5] for pixelizing the sphere introduced by Tegmark [1996].

REAR does not require an installation procedure; it is sufficient to extract the REAR distribution package into any folder and build the executables from the program sources. The contents of the distribution package is described in Table 1. The REAR executables can be compiled by typing the `make` command inside the `src/` T1 subfolder of the REAR distribution package. The `Makefile` provided with REAR is configured to invoke the GNU `gfortran` compiler, and should be appropriate in most cases. If the user wishes to use a different Fortran compiler, the `Makefile` must be edited accordingly. If the build process is successfully, the two executables `make_gf.exe` and `make_map.exe` will be made available in the main REAR directory. Those program units can be run in place or can be moved to any filesystem location which is included in the system search path.

In Section 7, the setup and the execution of REAR will be illustrated by means of an example.

6.2 Configuration and execution

REAR consists of the two program units: `make_gf.exe`, which computes the GFs for a disk load of assigned size and for a given set of LDCs, and `make_map.exe`, which combines the GFs and the load model to obtain predictions of deformation and geoid height change on a set of user-supplied coordinates. The various options appropriate for each program unit are specified in a configuration file, which is passed as a command-line argument when the program is invoked.

In the following sections, we give the configuration data for the regional rebound due ice mass loss in Greenland, and discuss the meaning of the various options of REAR.

[1] The Cygwin environment can be obtained at `http://www.cygwin.org/`

[2] For details about the Windows Subsystem for Linux, see `https://docs.microsoft.com/en-us/windows/wsl/`.

[3] The `gfortran` compiler is part of the GNU compiler collection, available at `http://gcc.gnu.org/`

[4] GMT can be downloaded from `http://gmt.soest.hawaii.edu/`

[5] The theory and a the code are available from: `http://space.mit.edu/home/tegmark/icosahedron.html`

6.3 Computing the Green's Functions

The configuration file for the REAR module make_gf.exe contains options controlling the calculation of the GFs. Below, we briefly illustrate the meaning of each section of this file, using as an example the file config_gf.dat provided in the EXAMPLES/ folder of the REAR package.

6.3.1 Elementary surface load

```
!
! ##### disk load parameters
!
 917.0                      ! Ice density (kg/m^3)
 0.02537d0                  ! Half-amplitude of the load (deg)
 1                          ! Uncompensated (0) or compensated (1) load
!
```

This first section of the GF configuration file contains the definition of the disk load parameters for the computation of the GFs: the load mass density ρ and the half-amplitude of the disk load α, expressed in decimal degrees. Note that REAR employs SI units throughout. The value of α given here must match the one employed to discretize the ice model in Section 6.4 below. A compensated or uncompensated disk can be chosen by setting the corresponding flag to values 0 or 1, respectively (see Section 5 for details). When parsing the configuration lines, information following the exclamation mark symbol '!' are interpreted as a comment and ignored by REAR. In the example above, we configure REAR to use a compensated disk load of half-amplitude $\alpha = 0.02537°$ and density equal to the ice density $\rho_i = 917\,\mathrm{kg}\cdot\mathrm{m}^{-3}$. This choice for the load density is appropriate if we want to model the response to a mass balance given in terms of ice thickness change; if the load model is in terms of equivalent water height, the disc mass density needs to be set to the water density $\rho_w = 1000\,\mathrm{kg}\cdot\mathrm{m}^{-3}$. The particular value of α chosen here ensures that the disk load has an area of 25 km^2 over the Earth's surface.

6.3.2 Green's function grid

```
! #### 1D Grid (two options available)
!
! Grid type (1-->Uniform, 2-->Stepsize increases
 1
!
! Minimum and maximum colatitude
!
 0.   60.
!
! Grid stepsize and number of points.
! For gridtype #1, only the (constant) stepsize is used.
! For gridtype #2, stepsize is the minimum spacing.
! *Negative steps are interpreted as alpha/(-step)*
!
 -5.0    1001
!
```

In this input section, various options for the computation of the GFs are specified. The 1D grid of angular distances on which the GFs will be evaluated can be defined in two ways: if the grid type switch is set to 1, the GFs will be computed on equally-spaced points; otherwise, if the switch is set to 2, REAR will build a grid in which the spacing between adjacent points increases linearly with distance from the axis of symmetry of the disk element. This second option is useful to save computation time by sampling the GF with a finer grid only near the load, where the geodetic variables are expected to show significant gradients.

For both grid options, the user can specify the minimum and maximum angular distances θ_{min} and θ_{max} for which the GF will be evaluated and a grid spacing $\Delta\theta$. For a uniform grid, these three parameters completely define the grid geometry; for a variable-spacing grid, $\Delta\theta$ is interpreted as the *minimum* grid spacing (*i.e.*, the distance between the two grid points closest to θ_{min}), and the number of grid points n_{grid} shall also be specified (this parameter will be ignored if a uniform grid is selected).

Note that for a variable-spacing grid, the parameters θ_{min}, θ_{max}, $\Delta\theta$ and n_{grid} must satisfy the relation $\Delta\theta_{min} < (\theta_{max} - \theta_{min})/(n_{grid} - 1)$. If this condition is not met, the execution of `make_gf.exe` stops with an error message.

If in place of the grid spacing $\Delta\theta$ the configuration file contains a negative value $p < 0$, `REAR` will assume $\Delta\theta = \alpha/(-p)$, where α is the angular half-amplitude of the disk load. This option is useful to define a 1D grid that adequately samples the near-field of the disk load, where the GFs are expected to have the largest gradients.

In the input example above, a uniformly spaced grid is configured between the center of the load and an angular distance of $60°$, with points spaced by $\alpha/5$, where α is the load half-amplitude. When configuring the grid parameters, it is important to set θ_{min} and θ_{max} in such a way that, for all load-observer angular distances θ, the relation $\theta_{min} \leq \theta \leq \theta_{max}$ is satisfied (otherwise the `make_map.exe` program will fail).

6.3.3 LDCs and harmonic synthesis

```
!
! #### Harmonic degrees
!
! Min/max harmonic degrees
   1     32768
!
!
! #### Asymptotic approximation of LNs
!
 0        ! Extend LNs using asymptotic expressions? (1->Yes, 0->No)
 10       ! Number of harmonic degrees used to compute the asymptotic limit
 100000   ! Maximum harmonic degree for the asymptotic extension
!
! ##### File name for Load-deformation coefficients (input)
!
./DATA/REF_6371_loading_love_numbers.txt
!
!
```

This section of the configuration file defines the parameters for the harmonic synthesis of the GFs. First, the range of harmonic degrees is set through the minimum and maximum values ℓ_{min} and ℓ_{max} (1 and 32768, respectively, in the example above). Note that, since we are simulating the response to a load of size $\alpha = 0.02537°$, according to the rule of thumb in Eq. (5) we need to include harmonic terms at least to degree $\ell_{max} \simeq 14\ 190$, so the range of harmonic degrees specified in the example above ensures an accurate spectral representation of the load. At runtime, `REAR` verifies that the chosen values of α and ℓ_{max} satisfy the relation in Eq. (5) and, if it is not the case, a warning message is printed to the user. Then, some options controlling the asymptotic extension of LDCs are given. The first switch controls whether an asymptotic extension should be performed (flag set to 1) or not (flag set to 0). Then, the number of harmonic degrees n_a which will be used to estimate the h'_∞, l'_∞

and k'_∞ is specified; REAR will estimate these asymptotic constants according to:

$$h'_\infty = \frac{1}{n_a}\sum_{\ell=\ell_1}^{\ell_{max}} h'_\ell \tag{13}$$

$$l'_\infty = \frac{1}{n_a}\sum_{\ell=\ell_1}^{\ell_{max}} (\ell\, l'_\ell) \tag{14}$$

$$k'_\infty = \frac{1}{n_a}\sum_{\ell=\ell_1}^{\ell_{max}} (\ell\, k'_\ell)\,, \tag{15}$$

where $\ell_1 = \ell_{max} - n_a + 1$. Last, the maximum harmonic degree to which the LDCs will be expanded using the asymptotic expressions is specified.

At the end of this input section, the name of the multi-column ASCII text file containing the LDCs is provided. Here, the user can specify the name of one of the LDC databases distributed with REAR (see Table 1) or any other LDCs set. T1

The sets of LDCs made available in folder `DATA/` in the REAR package are expressed in the reference frame of the center of mass (CM) of the whole Earth (this implies that the gravity potential LDC is $k'_1 = -1$). From these, the users can easily derive sets of LDCs referred to the system of the center of mass of the solid Earth (CE). These are related to those written in the CM by $h'^{CE}_1 = 1 + h_1$, $l'^{CE}_1 = 1 + l_1$, $k'^{CE}_1 = 1 + k_1 = 0$, and $h'^{CE}_\ell = h'_\ell$, $l'^{CE}_\ell = l'_\ell$, $k'^{CE}_\ell = k'_1 = 0$ for $\ell > 1$ [see, *e.g.*, Greff-Lefftz and Legros, 1997].

Note that, when a compensated disk load is selected, REAR ensures that the mass of the system (Earth+Load) is conserved (see Eq. 3), so the results are unaffected by the value of the degree $\ell = 0$ LDCs and the minimum harmonic degree effectively available for the computations is $\ell = 1$.

LDCs must be stored in a free-format text file, with each row containing the harmonic degree, vertical, horizontal and gravity potential LDCs, respectively ($\ell, h'_\ell, l'_\ell, k'_\ell$). Lines at the beginning of the file which start with an hash character ('#') are interpreted as header lines and ignored.

The set of LDCs in file `REF_6371_loading_love_numbers.txt`, distributed with REAR in the `DATA/` folder, corresponds to the seismological "REF model" STW105 [Kustowski et al., 2008], which updates the PREM model of Dziewonski and Anderson [1981]. The elastic constants are evaluated from the velocity of the seismic waves and the density profile, with the top three kilometers of oceanic water replaced by underneath rock materials. For reference, the structure of the REF model is shown in Figure 1, while Figure 2 shows the LDCs as a function of harmonic degree ℓ. Some numerical values of the LFDCs are given in Table 2. F1,F2 T2

6.3.4 Output file for the GFs

```
!
! ##### File name for gridded Green Function (output)
gf_Greenland.dat
!
```

This last section of the configuration file specifies the name of the output file that will be created by `make_gf.exe`. By default, REAR will provide the outputs in multi-column ASCII text files. As from the configuration lines above, the GFs will be stored into file `gf_Greenland.dat`.

6.4 Creating deformation maps

The configuration file for the REAR module make_map.exe contains options controlling for the calculation of geodetic variables. Below, we briefly illustrate its contents, using as an example the file config_map.dat provided in the EXAMPLES/ folder of the REAR package.

6.4.1 Load model

```
!
!
! >>>> ICE MODEL
!
./DATA/greeM3R.dat
 0                              ! Ice model includes uncertainties (0->no, 1->yes)
!
```

The first block of the configuration file contains the definition of the ice model. Here, the user provides the ASCII file containing the ice model, which in the example above is DATA/greeM3R.dat, derived from the M3 present-day ice balance model for Greenland by Sørensen et al. [2011].

The ice model file must be a free-format text file, with each row containing k, λ_k, ϕ_k, α_k, h_k (ice element number, longitude and latitude of its center, angular half-amplitude of the disk, thickness variation). Lines at the beginning of the file starting with the '#' symbol are automatically ignored. The disk half-amplitudes α_k must match the value used to compute the GFs in Section 6.3.1 above. The make_map.exe program checks the values of disk half-amplitudes in the ice model and GFs, and exits with an error message if inconsistent values are found.

After the name of the ice model file, a flag specifies whether the ice model contains uncertainties on the thickness variation (flag set to 1) or not (flag set to 0). If the flag is set to 1, REAR expects to find an additional field in the ice model file with uncertainties σ_{H_k} on the thickness variation H_k (see Eqs. 12).

6.4.2 Green's functions

```
!
! >>>> GREEN'S FUNCTION
!
gf_Greenland.dat
 1                           ! grid style (1=uniform, 2=variable spacing)
!
```

In this section the user defines the GFs to be used to compute the geodetic quantities. The first line reports the name the GFs file (gf_Greenland.dat in the example above); the flag is the second line contains the type of grid spacing (uniform or variable spacing).

6.4.3 Output options

```
!
!
! >>>> SELECTION of the OUTPUT <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
!
! 1 ---> Rates of displacement are on a 2D grid
```

```
! 2 ---> Rates of displacement on isolated points
!
 1
!
! --------------------------------------------------------------
! >>>> PARAMETERS for IWHERE = 1 (2D grid) <<<<<<<<<<<<<<<<<<<<
! --------------------------------------------------------------
!
!
 2            ! Grid type (1=Tegmark, 2=lon/lat)
!
 100.0        ! Pixel area for GRD_TYPE=1 (km^2)
 0.5          ! Grid spacing for GRD_TYPE=2 (deg)
!
 285.0    350.0   ! Region boundaries (lon) (for both types)
 58.0      86.0   ! Region boundaries (lat)
!
! Output file
 uvg.dat
!
! --------------------------------------------------------------
! >>>> PARAMETERS for IWHERE = 2 (isolated points) <<<<<<<<<<<<
! --------------------------------------------------------------
!
!
 GPS-points.dat
!
!
 GPS-out.dat
!
!
!
!
```

Here the user defines where `make_map.exe` computes predictions of surface displacements and geoid changes. The option `IWHERE` controls whether the program will use a 2-D grid (suitable for plotting a map), or a set of user-provided isolated points (suitable, for example, to compute predictions at the location of GPS or tide gauge sites).

For `IWHERE=1`, `REAR` will compute geodetic variables on a 2D grid. In this case, the user can choose between a grid registered on a icosahedron-based equal-area pixelization of the sphere (grid type equal to 1 see Tegmark [1996] for details) or on a regular grid (grid type equal to 2). For the icosahedron-based grid, the user provides the pixel area in km^2, while for a regular grid the spacing in both latitude and longitude is specified (the output will be on a pixel-registered grid). For both choices, the extents of the grid are defined by providing the range of longitudes and latitudes. Finally, the name of the output file that will be generated by `make_map.exe` is specified. In the example above, `REAR` is configured for a regular grid spacing of $0.5°$ and extents $285° \leq \lambda \leq 350°$, $58° \leq \phi \leq 86°$ in longitude and in latitude, respectively. The geodetic variables shall be reported on the `uvg.dat` file.

For `IWHERE=2`, `REAR` will compute geodetic quantities on a set of user-supplied points. The name of the file containing the points is specified (`GPS-points.dat` in the example above) and must contain, for each point, a line with an index number, longitude, latitude and (optionally) station name. Lines at the beginning of the file starting with the '`#`' symbol are automatically skipped. An example is in the `DATA` folder (see Table 1). The T1
name of the output file is specified in the following line (`GPS-out.dat` in the example above).

For both options IWHERE=1 and IWHERE=2, the output file will contain the following columns: longitude and latitude of the observation point, displacement (or velocity) along the radial, co-latitude and longitude directions, and geoid change (or variation rate). For IWHERE=2, a column with station name is also present. If the input load model contains also estimates of uncertainties (see Section 6.4.1), each field will be followed by an estimate of its uncertainty, computed according to Eq. (12).

7 A guided example

By a simple example, here we will demonstrate the use of REAR to compute the present-day elastic rebound of Greenland using the ice mass balance ”M3” of Sørensen et al. [2011]. The corresponding configuration files for this case have been illustrated in Section 6 and are stored in the EXAMPLES/ folder (config_gf.dat and config_map.dat, see Table 1).

First, we need to build the REAR executables, if this task has not yet been executed. This can be done by typing at the shell prompt the make command inside the src/ folder:

```
$ cd src
$ make
gfortran -fopenmp -O -c make_gf.f90 -o make_gf.o
gfortran -fopenmp -O -c utils.f90 -o utils.o
gfortran -fopenmp -O -c plegendre.f90 -o plegendre.o
gfortran -fopenmp -O -w -c icosahedron.f -o icosahedron.o
gfortran -fopenmp -O make_gf.o utils.o plegendre.o icosahedron.o -o ../make_gf.exe
gfortran -fopenmp -O -c make_map.f90 -o make_map.o
gfortran -fopenmp -O make_map.o utils.o icosahedron.o -o ../make_map.exe
$ cd ..
```

If the build process is succesful, the REAR executables make_gf.exe and make_map.exe will be placed in the main REAR directory. We are now ready to compute the GFs by invoking the make_gf.exe executable at the shell prompt and passing to it on the command line the full path to the config_gf.dat configuration file, which is stored in the EXAMPLES/ folder:

```
$ ./make_gf.exe EXAMPLES/config_gf.dat

 <<<<<<<<  \texttt{REAR}: a Regional ElAstic Rebound calculator  >>>>>>>>

 make_gf: Using configuration file: config_gf.dat
 make_gf: Reading the Love numbers from file: ./DATA/REF_6371_loading_love_numbers.txt
 make_gf: No asymptotic extension of LNs has been requested
 make_gf: Computing the harmonic coefficients of the load ...
 make_gf: The load is compensated
 make_gf: Computing displacements and geoid height at the grid points ...
 make_gf: The Green function grid has a uniform spacing
 make_gf: The Green functions are written on file: gf_Greenland.dat
```

After execution, the GFs are stored in the text file gf_Greenland.dat, which can be examined with a text editor:

```
 - Love numbers from file:./DATA/REF_6371_loading_love_numbers.txt
 - Green functions for MAX degree:       32768
 - Ice density (kg/m^3):   917.00000000000000
```

```
 - Earth density (kg/m^3):   5505.0609557847711
 - Load half-amplitude (deg):   2.5370000000000000E-002
 - Ice thickness variation: +1 m/year
 - ======= Grid properties (the grid has costant spacing) =======
 - Min and Max colatitudes (deg):   0.0000000000000000        60.000000000000000
 - Colatitude increment(deg):   5.0740000000000004E-003
 - Number of grid points:       11825
#
#   Line        colat, deg        vert. vel., mm/yr    hor. vel., mm/yr     geoid vel., mm/yr
#
      1       0.00000000E+00      -0.50871721E+00       0.00000000E+00       0.10955602E+00
      2       0.50740000E-02      -0.50733845E+00      -0.17094634E-01       0.10926704E+00
      3       0.10148000E-01      -0.49089914E+00      -0.36009384E-01       0.10581934E+00
      4       0.15222000E-01      -0.45985367E+00      -0.52474962E-01       0.99308204E-01
      5       0.20296000E-01      -0.41580245E+00      -0.71728450E-01       0.90069331E-01
      6       0.25370000E-01      -0.32076983E+00      -0.84531181E-01       0.70136357E-01
      7       0.30444000E-01      -0.22986821E+00      -0.74395911E-01       0.51069059E-01
[...]
```

The header section of `gf_Greenland.dat` contains, for the User's convenience, a summary of the options used for the generation of the GFs. Starting from line 14, the file contains the response to a unit load rate, computed along the user-specified grid. Columns represent, respectively: line number, co-latitude (i.e., angular distance from the center of the disk), velocities along the radial and co-latitude directions, geoid variation rate. This set of GFs can be plotted with the `plot_greenfunctions.gmt` GMT script, which is provided with `REAR` in the `EXAMPLES/` folder[6]. The resulting plot is shown in Figure 3. Consistently with a surface loading scenario, F3 the elastic response corresponds to a subsidence below the disk, while the geoid is shifted upwards because of the mass excess associated with the load, which is not fully compensated by the flexure of the external surface. Horizontal velocities reach their maximum values at the disk edge and directed towards the disk center.

In order to check that `make_gf.exe` has run correctly, the GFs file produced by this example can be compared with a "reference" file `gf_Greenland_REF6371.dat`, distributed with the `REAR` package in the `EXAMPLES/` folder. The results can be conveniently compared with one the `compare_gf.*` scripts, stored in the `EXAMPLES/` folder. Please note that here we use the `compare_gf.gmt`, suitable for version 4 of GMT; for versions 5 and 6, the `compare_gf.gmt5` script shall be used.

```
$ EXAMPLES/compare_gf.gmt gf_Greenland.dat EXAMPLES/gf_Greenland_REF6371.dat

Comparing gf_Greenland.dat and EXAMPLES/gf_Greenland_REF6371.dat ...

Min/max ABSOLUTE differences:
  - Vertical component:     -1.0000e-11 /   1.0000e-11
  - Horizontal component:   -1.0000e-12 /   1.0000e-11
  - Geoid:        -1.0000e-10 /   1.0000e-11

Min/max RELATIVE differences:
  - Vertical component:     -9.4029e-08 /   9.9015e-08
  - Horizontal component:   -7.6562e-07 /   4.6191e-07
  - Geoid:        -4.0250e-07 /   2.3291e-06
```

In this example, the negligible differences between the two files are likely the result of numerical roundoffs, so the `make_gf.exe` program is running correctly.

[6]The `plot_greenfunctions.gmt` script is compatible with version 4.5 of GMT. Users of GMT version 5 should use the `plot_greenfunctions.gmt5` script.

Once the GFs are computed, program `make_map.exe` can be used to compute geodetic variables corresponding to a given ice model on a map or on a set of discrete points. We invoke the `make_map.exe` at the shell prompt, passing on the command line the path to the `config_map.dat` configuration file, which configures the program to compute observables on a geographical grid spanning the Greenland region:

```
$ ./make_map.exe EXAMPLES/config_map.dat

 <<<<<<<<  \texttt{REAR}: a Regional ElAstic Rebound calculator  >>>>>>>>

 make_map: Using configuration file: config_map.dat
 make_map: Reading ice model information
 make_map: Assuming NO uncertainties on ice model
 make_map: Found        70850  ice elements in file ./DATA/greeM3R.dat
 make_map: Reading the Green Function file: gf_Greenland.dat
 make_map: Reading        11826  points from the Green Function file
 make_map: Grid data: min colat =    0.0000000000000000
 make_map: Grid data: max colat =    60.000050000000002
 make_map: Number of points: =        11826
 make_map: Option 1: Gridded output
 make_map: Using a uniform lon/lat grid
 make_map: Spacing of the 2D GRID of observers (deg):   0.50000000000000000
 make_map: Number of GRID points within the region:         7467
 make_map: Number of pixels (observers) within the region:         7467
 make_map: Pre-computing trigonometric functions
 make_map: Computing displacements at the requested points
 make_map: END of computation
 make_map: Writing the output
 make_map: The output is reported on file: uvg.dat
```

After the execution, predictions of geodetic variables on the grid points are stored in the text file `uvg.dat`:

```
# Created by REAR 1.5 on 2022-11-03 15:56:59
# Green function from file: gf_Greenland.dat
# All rates are in units of mm/yr
#
#   Lon        Lat          Up          South         East        Geoid
# --------  ----------  ------------  ------------  ------------  ------------
  285.0000     58.0000  0.466614E+00  0.481741E-01  -.434961E-01  -.179143E+00
  285.5000     58.0000  0.471237E+00  0.496603E-01  -.447466E-01  -.184496E+00
  286.0000     58.0000  0.475925E+00  0.511707E-01  -.460061E-01  -.189964E+00
  286.5000     58.0000  0.480686E+00  0.527076E-01  -.472700E-01  -.195552E+00
  287.0000     58.0000  0.485549E+00  0.542693E-01  -.485383E-01  -.201266E+00
  287.5000     58.0000  0.490501E+00  0.558645E-01  -.498212E-01  -.207109E+00
  288.0000     58.0000  0.495520E+00  0.574904E-01  -.511125E-01  -.213078E+00
  288.5000     58.0000  0.500638E+00  0.591489E-01  -.524120E-01  -.219183E+00

[...]
```

The file contains an header section with the name of the GFs file used to compute the observables. Data columns correspond, respectively, to longitude, latitude, surface velocity along the radial (r), co-latitudinal (θ) and longitudinal (λ) directions, and geoid variation rate. For each of these observables, it is possible to obtain a map using the GMT script `plot_greenland_maps.gmt`, provided with the `REAR` package in the `EXAMPLES/` folder[7]. As a default, the script draws the map of the uplift rate which is shown in Figure 4. Using the provided GMT F4

[7] The `plot_greenland_maps.gmt` script is compatible with version 4.5 of GMT. Users of GMT version 5 should use the `plot_greenland_maps.gmt5` script.

scripts as a template, it is easy to draw maps also for the horizontal components of the velocity and for the rate of geoid change; for reference, maps for those fields are shown in Figure 5. F5

As shown for the computation of GFs, it is possible to validate the results of `make_map` by comparing the `uvg.dat` file produced by this run with the `uvg_REF6371.dat` file located in the `EXAMPLES/` folder. The comparison can be done either manually or through one of the `compare_map.*` scripts, distributed with `REAR` in the `EXAMPLES/` folder:

```
$ EXAMPLES/compare_map.sh uvg.dat  EXAMPLES/uvg_REF6371.dat

Comparing uvg.dat and EXAMPLES/uvg_REF6371.dat ...

Min/max ABSOLUTE differences:
  - UrDOT:        0.0000e+00 /   0.0000e+00
  - UthetaDOT:   -1.0000e-07 /   1.0000e-07
  - UphiDOT:     -1.0000e-06 /   1.0000e-10
  - NDOT:         0.0000e+00 /   0.0000e+00

Min/max RELATIVE differences:
  - UrDOT:        0.0000e+00 /   0.0000e+00
  - UthetaDOT:    0.0000e+00 /   4.9830e-06
  - UphiDOT:     -1.9613e-06 /   1.5299e-06
  - NDOT:         0.0000e+00 /   0.0000e+00
```

A negligible difference between the two files indicates that `REAR` is working correctly.

The `make_map.exe` program can evaluate geodetic variables also on a discrete set of points; this would be appropriate, for instance, to obtain predictions of horizontal and vertical velocities at locations of GPS stations. The `config_gps.dat` example configuration file schedules `make_map.exe` to use the same set of GFs obtained above to evaluate geodetic predictions at the locations of some of the GPS stations considered by Bevis et al. [2012]. The coordinates of the sites are tabulated in the data file `GPS_Bevis2012.dat`, stored in the `DATA/` subfolder. We invoke the `make_map.exe` program with the path to the `config_gps.dat` configuration file on the command line:

```
$ ./make_map.exe EXAMPLES/config_gps.dat

 <<<<<<<<  REAR: a Regional ElAstic Rebound calculator  >>>>>>>>

 make_map: Using configuration file: EXAMPLES/config_gps.dat
 make_map: Reading ice model information
 make_map: Assuming NO uncertainties on ice model
 make_map: Found        70850  ice elements in file ./DATA/greeM3R.dat
 make_map: Reading the Green Function file: gf_Greenland.dat
 make_map: Reading        11826  points from the Green Function file
 make_map: Grid data: min colat =    0.0000000000000000
 make_map: Grid data: max colat =    60.000050000000002
 make_map: Number of points: =        11826
 make_map: Option 2: Sparse points
 make_map: Reading           53  sparse points from file DATA/GPS-Bevis2012.dat
 make_map: Pre-computing trigonometric functions
 make_map: Computing displacements at the requested points
 make_map: END of computation
 make_map: Writing the output
 make_map: The output is reported on file: GPS-out.dat
```

Modeled geodetic observables at the specified sites can be found in the `GPS-out.dat` data file, whose first lines are reported below:

```
# Created by REAR 1.5 on 2022-11-03 16:23:57
# Green function from file: gf_Greenland.dat
# All rates are in units of mm/yr
#
#    Lon          Lat            Up          South          East         Geoid      Site
# ----------  ------------  ------------  ------------  ------------  ------------  ----
  291.175000     76.537000  0.283675E+01  -.117575E-01  -.770214E+00  -.155371E+01  THU2
  298.322000     76.352000  0.902346E+01  -.182867E+00  -.415989E+00  -.326147E+01  DKSG
  301.743000     75.726000  0.798510E+01  -.677201E+00  -.747838E+00  -.335825E+01  ASKY
  302.773000     74.581000  0.481857E+01  0.318808E+00  -.821870E+00  -.258220E+01  KULL
  305.606000     72.911000  0.724735E+01  0.685586E+00  -.104070E+01  -.336733E+01  SRMP
  303.872000     72.788000  0.373029E+01  0.414978E+00  -.908476E+00  -.220387E+01  UPVK
[...]
```

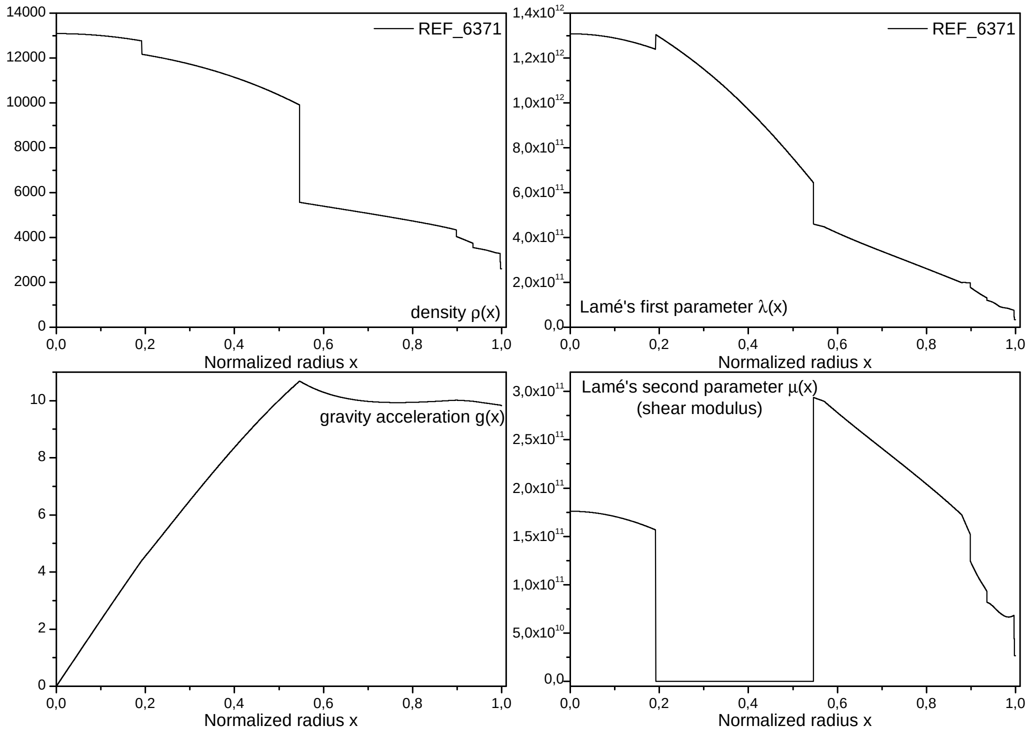

Figure 1: Structure of the REF6371 model, showing density (in units of kg m^{-3}), the first Lamé parameter (λ, in units of Pa), gravity acceleration (m s^{-2}), and the second Lamé parameter (μ, Pa) as a function of $x = r/a$, where r is radius and a is Earth's radius.

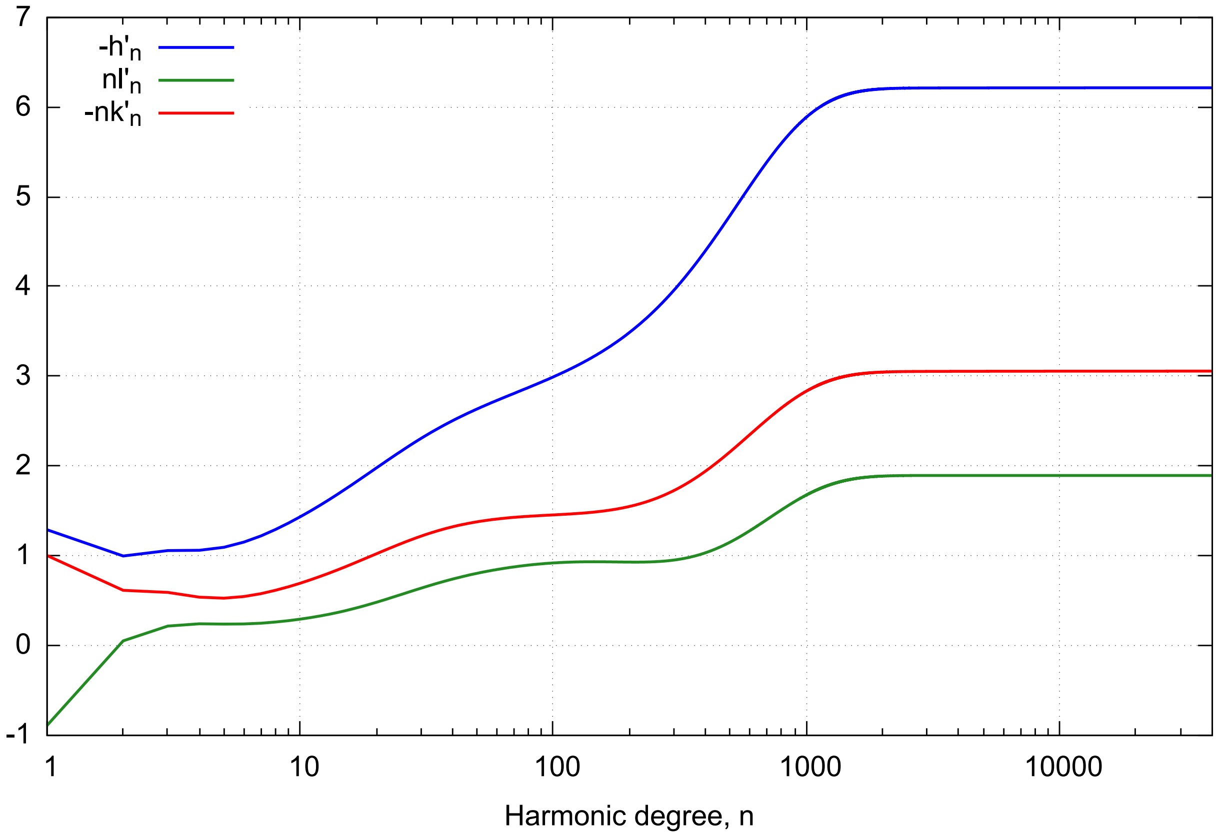

Figure 2: LDCs $(h'_\ell, l'_\ell, k'_\ell)$ according to the SNREI Earth model REF6371, as a function of harmonic degree (note that in this chart the harmonic degree is denoted by $n = \ell$).

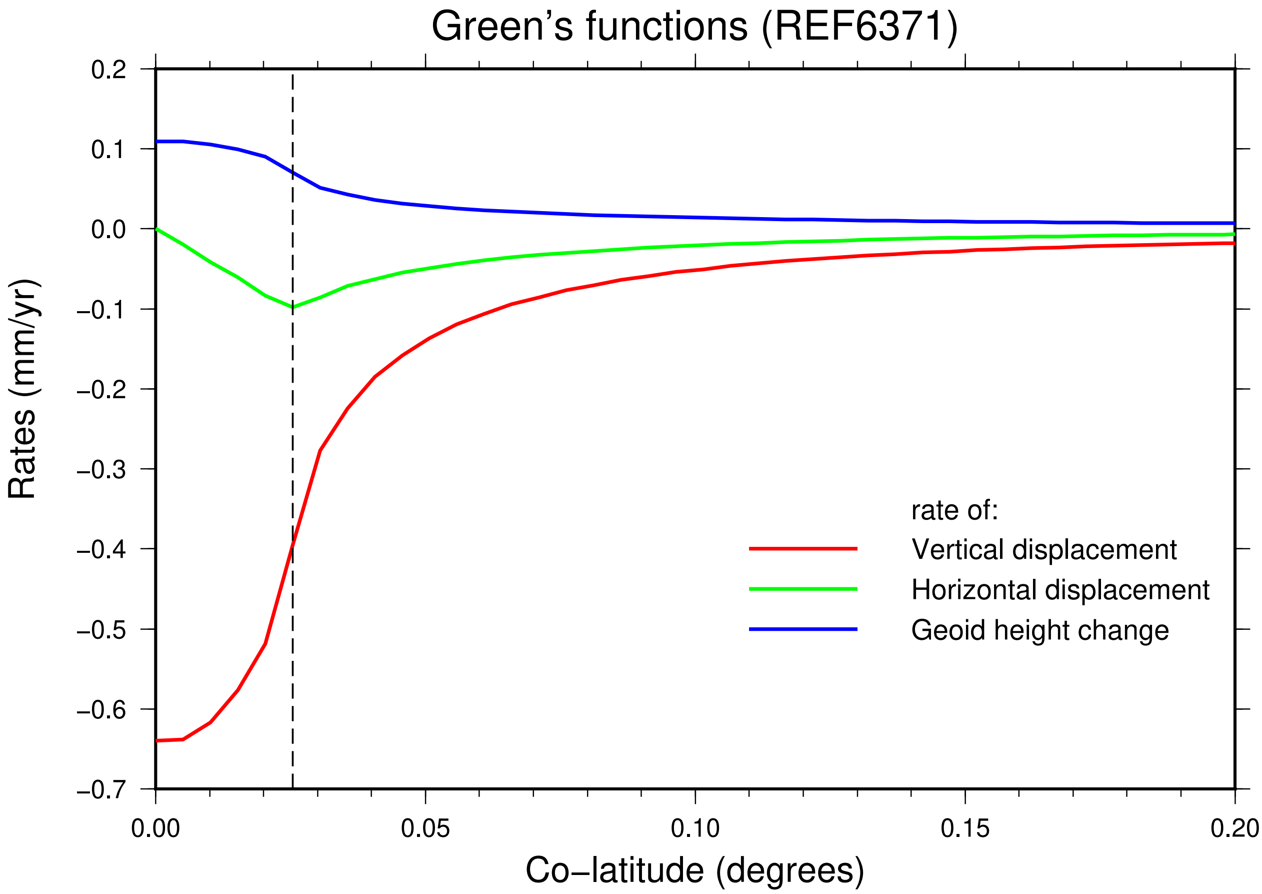

Figure 3: GFs computed with the example configuration discussed in Section 7. Plotted quantities show the GFs pertaining to the rates of vertical displacement ($\dot{u}$, red), of horizontal displacement ($\dot{v}$, green) and of the rate of geoid height variation ($\dot{n}$, blue) for a disk load of angular with half-amplitude $\alpha = 0.02537°$, with a thickness changing at the rate $\dot{H} = 1$ m/yr. With the chosen value of α, the area of the disk load is 25 km^2 over the Earth's surface. The dashed line marks the margin of the disk.

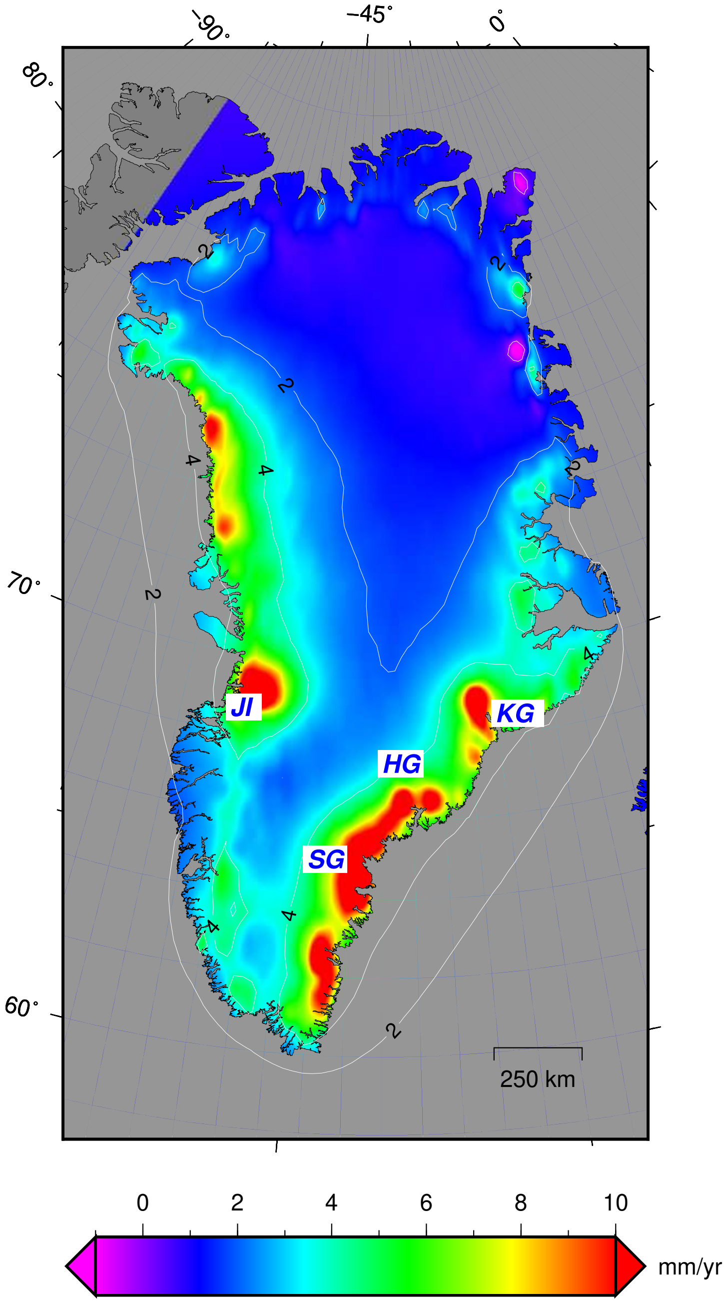

Figure 4: Map showing the rate of vertical uplift (in mm/yr) across the Greenland ice sheet according to the mass balance model M3 derived by ICESat (Ice, Cloud, and land Elevation SATellite) data by Sørensen et al. [2011], for the time period 2003-2008. The map is obtained following the REAR guided example described in the text. Labels mark the location of the outlet glaciers mostly contributing to mass loss: the Kangerdlugssuaq Glacier (KG), Jakobshavn Isbræ (JI), the Helheim Glacier (HG) and the Southeast Glaciers (SG).

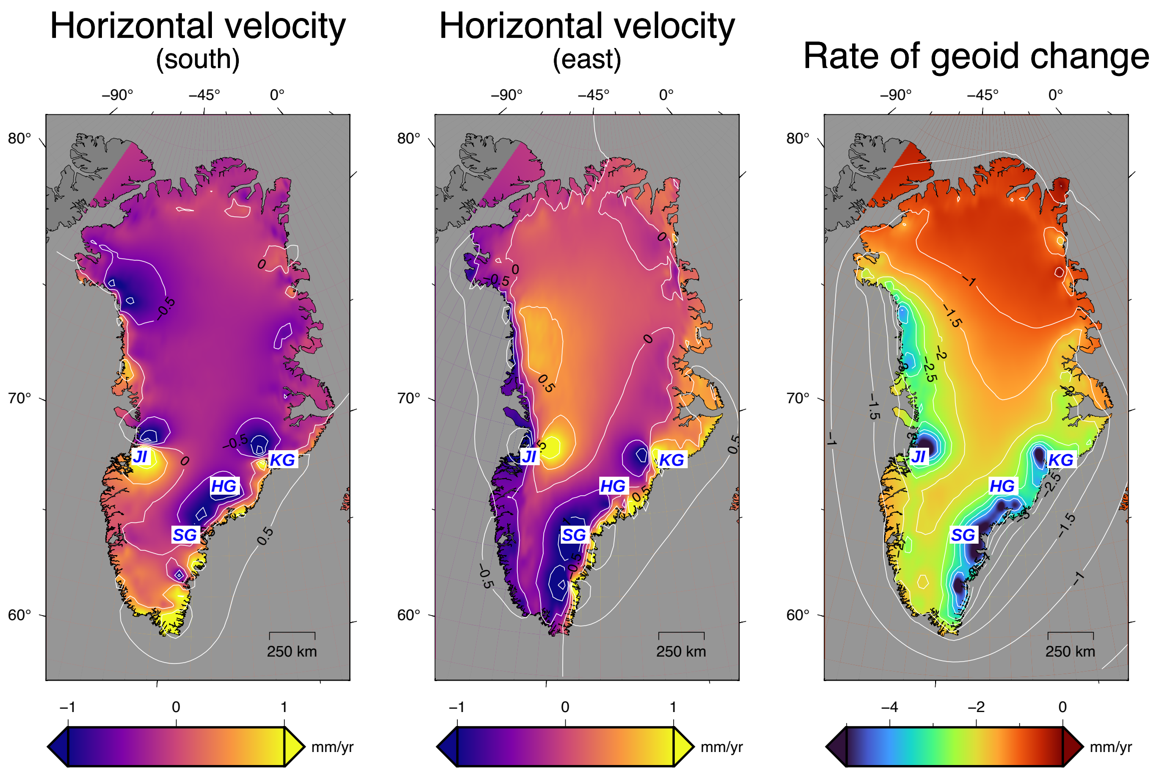

Figure 5: From the left to the right, maps of horizontal velocities (south and east components) and rate of geoid change across Greenland, obtained using the mass balance M3 by Sørensen et al. [2011]. Please note that in the first figure on the left, positive values represent southward displacements, while in the second one, positive values stand for eastward displacements. As in Figure 4, the labels mark the glaciers that mostly contribute to the ice loss.

File	Description
REAR-User-Guide.pdf	This document
LICENSE.txt	A copy of the GPLv3 license
src/	
Makefile	"Makefile" for the compilation of REAR programs
make_gf.f90	Source code for computation of GFs
make_map.f90	Source code for computation of geodetic variables
utils.f90	Source code for several utility functions
icosahederon.f	Icosahedron pixelixation package by Tegmark [1996]
plegendre.f90	Source code for the PLegendre function from the SHTOOLS package by Wieczorek and Meschede [2018]
SHTOOLS_LICENSE.txt	A copy of the license for the SHTOOLS package
EXAMPLES/	
compare_gf.gmt	GMT4 script for Green's Functions comparison
compare_map.gmt	GMT4 script for geodetic variables comparison
compare_gf.gmt5	GMT5/6 script for Green's Functions comparison
compare_map.gmt5	GMT5/6 script for geodetic variables comparison
config_gf.dat	Example configuration file for make_gf.exe
config_map.dat	Example configuration file for make_map.exe
config_gps.dat	Example configuration file for make_map.exe
gf_Greenland_REF6371.dat	Green's Functions for the Greenland example
uvg_REF6371.dat	Geodetic observables for the Greenland example
GPS-out_REF6371.dat	Geodetic observables at GPS sites from Bevis et al. [2012] for the Greenland example
plot_greenfunctions.gmt	Example GMT4 script for the Green's Functions plot
plot_greenland_maps.gmt	Example GMT4 script for the Greenland uplift map
plot_greenfunctions.gmt5	Example GMT5 script for the Green's Functions plot
plot_greenland_maps.gmt5	Example GMT5 script for the Greenland uplift map
DATA/	
Load_Love2_CM.dat	LDCs from the NASA Atmospheric Pressure Loading Service ($0 \leq \ell \leq 1,024$)
REF_6371_loading_love_numbers.txt	LDCs for model REF6371 ($0 \leq \ell \leq 40,000$).
greeM3R.dat	Greenland ice mass balance model from Sørensen et al. [2011]
GPS-Bevis2012.dat	Coordinates of Greenland GPS sites from Bevis et al. [2012]

Table 1: Contents of the REAR distribution package.

ℓ	h'_ℓ	l'_ℓ	k'_ℓ	$-h'_\ell$	$\ell l'_\ell$	$-\ell k'_\ell$
0	-0.216000	0.000000	0.000000	0.216000	0.000000	0.000000
1	-1.287113	-0.895496	-1.000000	1.287113	-0.895496	1.000000
2	-0.994587	0.024113	-0.305770	0.994587	0.048225	0.611541
3	-1.054653	0.070855	-0.196272	1.054653	0.212565	0.588817
4	-1.057784	0.059587	-0.133791	1.057784	0.238349	0.535162
5	-1.091186	0.047026	-0.104762	1.091186	0.235131	0.523809
6	-1.149254	0.039408	-0.090350	1.149254	0.236449	0.542097
7	-1.218363	0.034994	-0.082057	1.218363	0.244958	0.574401
8	-1.290474	0.032251	-0.076523	1.290474	0.258010	0.612188
9	-1.361848	0.030386	-0.072393	1.361848	0.273471	0.651536
10	-1.430982	0.029023	-0.069078	1.430982	0.290226	0.690777
20	-1.975466	0.023899	-0.050927	1.975466	0.477972	1.018545
30	-2.304458	0.021158	-0.040503	2.304458	0.634734	1.215100
40	-2.502887	0.018419	-0.032997	2.502887	0.736753	1.319881
50	-2.633749	0.016026	-0.027515	2.633749	0.801321	1.375768
60	-2.730019	0.014065	-0.023449	2.730019	0.843906	1.406926
70	-2.807682	0.012470	-0.020363	2.807682	0.872915	1.425409
80	-2.874634	0.011164	-0.017966	2.874634	0.893121	1.437272
90	-2.935055	0.010080	-0.016064	2.935055	0.907204	1.445727
100	-2.991305	0.009169	-0.014526	2.991305	0.916862	1.452572
200	-3.486737	0.004632	-0.007734	3.486737	0.926495	1.546760
300	-3.958810	0.003164	-0.005749	3.958810	0.949282	1.724819
400	-4.401433	0.002577	-0.004853	4.401433	1.030908	1.941392
500	-4.793152	0.002299	-0.004315	4.793152	1.149354	2.157259
600	-5.123408	0.002132	-0.003919	5.123408	1.278922	2.351472
700	-5.391418	0.002003	-0.003594	5.391418	1.402423	2.515550
800	-5.602517	0.001889	-0.003310	5.602517	1.511004	2.648362
900	-5.764919	0.001780	-0.003058	5.764919	1.601553	2.752584
1000	-5.887537	0.001674	-0.002832	5.887537	1.674308	2.832483
2000	-6.203847	0.000943	-0.001523	6.203847	1.885420	3.045266
3000	-6.213711	0.000631	-0.001018	6.213711	1.892336	3.052948
4000	-6.214465	0.000473	-0.000763	6.214465	1.892413	3.053918
5000	-6.214822	0.000378	-0.000611	6.214822	1.892376	3.054435
6000	-6.215059	0.000315	-0.000509	6.215059	1.892350	3.054778
7000	-6.215229	0.000270	-0.000436	6.215229	1.892332	3.055023
8000	-6.215356	0.000237	-0.000382	6.215356	1.892318	3.055207
9000	-6.215454	0.000210	-0.000339	6.215454	1.892307	3.055350
10000	-6.215533	0.000189	-0.000306	6.215533	1.892299	3.055465
20000	-6.215889	0.000095	-0.000153	6.215889	1.892261	3.055980
30000	-6.216008	0.000063	-0.000102	6.216008	1.892248	3.056152
40000	-6.216068	0.000047	-0.000076	6.216068	1.892241	3.056238

Table 2: Numerical values of the LDCs $(h'_\ell, l'_\ell, k'_\ell)$ for model REF6371, for some values of harmonic degree ℓ.

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