GARCH estimation for HSI - Hang Seng Index

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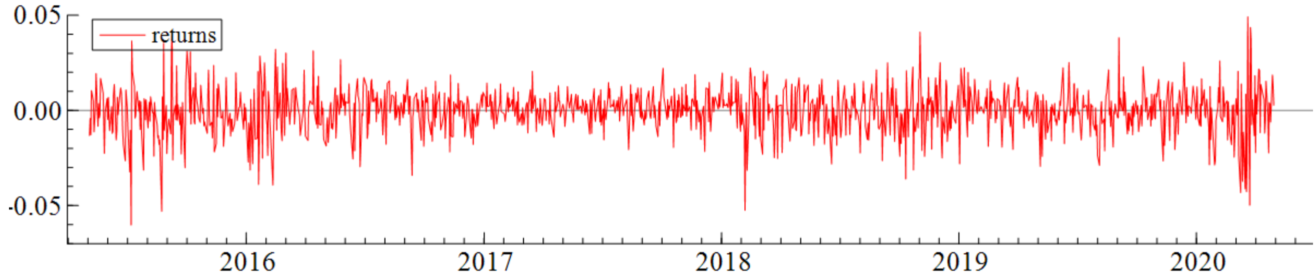
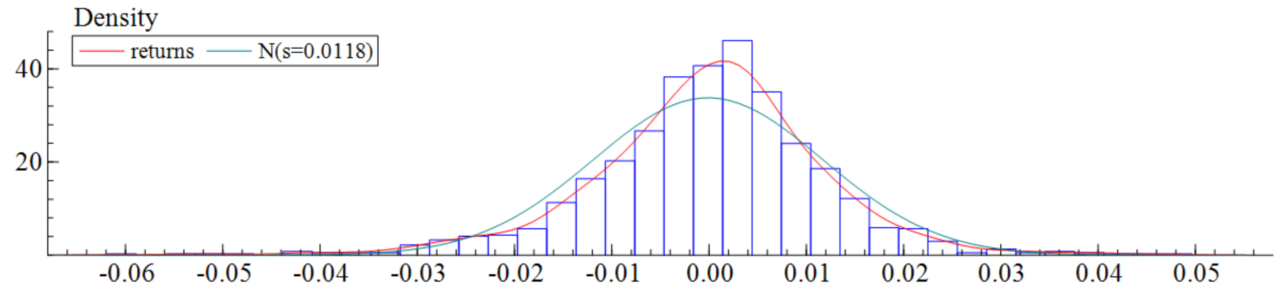
I will work on the daily data from the last 5 years for Hang Seng Index. The excel data source is comprehensive of opening prices, closing prices, highest prices and lowest prices, as well as volumes. One first thing to do is to calculate returns from closing prices and square returns using the Calculator tool.

After that is done, I would draw some graphs, starting from the Actual series for returns, so as to see its behaviour:



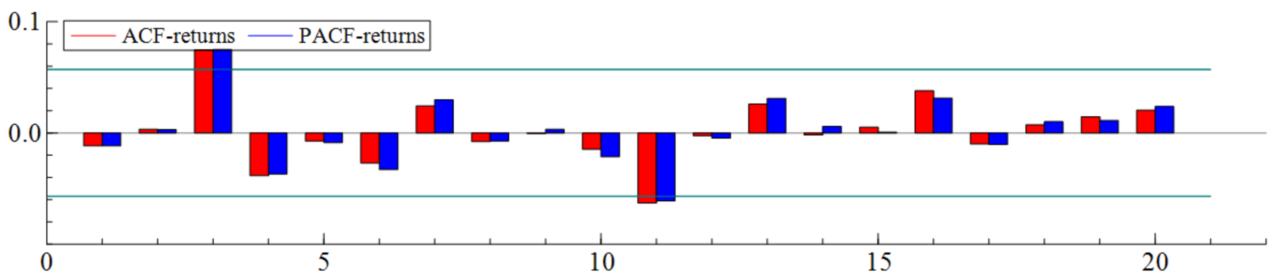
It seems that there are a few clusters of volatility, for example at the end of the sample (probably related with Coronavirus outbreak), namely there are periods of low volatility alternated with periods of higher volatility.

Now, let’s plot the distribution graph:



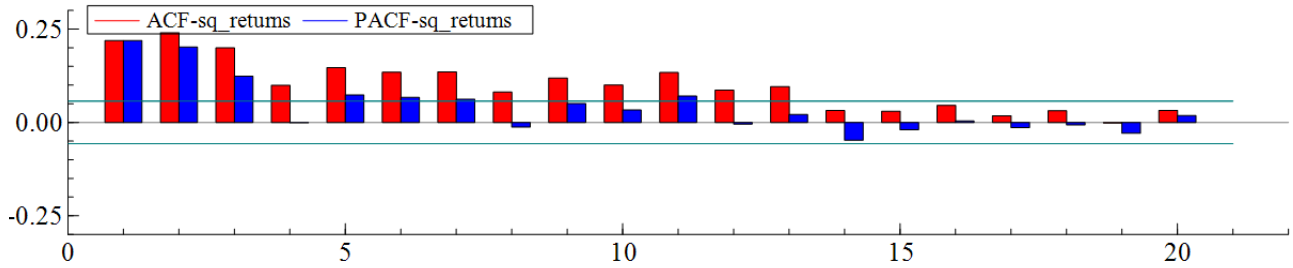
What can be seen in this graph is a small asymmetry on the left tail of the distribution (hence, negative skewness). Also, there is some excess kurtosis. It seems that the series doesn’t follow the normal distribution, however, **<u>it could be a good idea to check it.</u>**

Now, let’s draw time-series properties (ACF-PACF) of the returns series:



Significant autocorrelation is visible only at 3 and 11 lags, despite the latter is barely statistically significantly different from zero. **<u>The information that we can gather from this graph is that it could be a good idea to estimate an ARMA specification, just to see if at least lag 3 is considered important</u>**.

Now, let's plot the ACF-PACF graph for square returns series:



What can be seen here is that there is statistically significant autocorrelation at lags from 1 to 13. Also, there is partial autocorrelation in 1, 2, 3, 5, 6 and 11 lags in this series. **<u>Several GARCH(m, s) specifications could be modelled from here</u>**. Also, it is interesting to notice that not only the first lags are significant, but also some which are further. This might mean that there is a kind of **long memory** in the series. A long memory test will be necessary.

Now, I would check some descriptive statistics using G@RCH on returns and squared returns. I test for Normality (with no intercept and no time trend, lag length set to 3) and ARCH effect (**these two tests are important only for returns**), Box-Pierce on raw series, and for long memory, using Geweke and Porter-Hudak Test. There is no need to check a unit root test because returns can be assumed to be stationary. Here are the results of Normality Test, ARCH Test and Box-Pierce Test for returns series:

```
Series #1/2: returns
---------

Normality Test

                   Statistic       t-Test      P-Value
Skewness            -0.38961       5.5852   2.3346e-08
Excess Kurtosis       2.3842       17.103   1.4079e-65
Jarque-Bera           322.45         .NaN   9.5644e-71
---------------
ARCH 1-2 test:    F(2,1225) =   58.354 [0.0000]**
ARCH 1-5 test:    F(5,1219) =   28.896 [0.0000]**
ARCH 1-10 test:   F(10,1209)=   16.020 [0.0000]**
---------------
Box-Pierce Q-Statistics on Raw data
  Q(  5) =   8.99126    [0.1094133]
  Q( 10) =   10.9785    [0.3591972]
  Q( 20) =   19.5692    [0.4851545]
  Q( 50) =   52.2097    [0.3880979]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
```

The Normality Test highlights the negative skewness, which is statistically significant. There is also excess kurtosis, which is statistically significant as well. Jarque-Bera is statistically significant too. **This is the confirmation of the suspicion that the <u>return series does not follow a normal distribution</u>**. Next, ARCH Test, where we can see that **ARCH effect is present in all the considered lags** (2, 5 and 10).

Then Box-Pierce Test for the return series shows that **there is no autocorrelation within the considered lags** (5, 10, 20 and 50); the graph showed autocorrelation within 3 and 11 lags.
Now let's look at the results from Box-Pierce for the squared returns:

```
Series #2/2: sq_returns
---------
Box-Pierce Q-Statistics on Raw data
  Q(  5) =  218.907   [0.0000000]**
  Q( 10) =  302.448   [0.0000000]**
  Q( 20) =  353.484   [0.0000000]**
  Q( 50) =  422.394   [0.0000000]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
```

As we can see, there is **statistically significant autocorrelation at every considered lag**. This is one more reason why GARCH specifications will be required. And the long memory test for both series:

```
Series #1/2: returns

----  Log Periodogram Regression ----
d parameter         0.0318702 (0.0273717) [0.2443]
No of observations: 1230; no of periodogram points: 615

Series #2/2: sq_returns

----  Log Periodogram Regression ----
d parameter          0.192064 (0.0340467) [0.0000]
No of observations: 1230; no of periodogram points: 273
```

We can notice that **d parameter is close to zero in return series, and it is not statistically significant**; hence, there is **<u>no long memory in the series</u>**. So, regarding the Conditional Mean, if I will decide to estimate an ARFIMA specification, d parameter would almost certainly be statistically insignificant.
Regarding the squared returns, we can see that **d parameter is bigger and statistically significant**. This means that there seems to be long memory in the squared returns and a **<u>FIGARCH specification could work well</u>** later on, even if for now long memory was only tested in squared returns, which is a simple proxy of volatility, and not in Conditional Variance itself.

Let's create the first ARMA specification. Considering the time-series properties graph for returns, let's try ARMA(3,0) for the whole data sample:

```
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
The dataset is: C:\Users\danie\Documents\FINANCIAL ENGINEERING\SEMESTER 2\Financial Econometrics\8_29.04.2020 (online...)\Task 06.05\Hang Seng.xlsx

                  Coefficient  Std.Error  t-value  t-prob
AR-1               -0.0108062    0.02848   -0.379   0.704
AR-2               0.00502879    0.02849    0.177   0.860
AR-3                0.0760192    0.02852     2.67   0.008
Constant         -0.000107041  0.0003609   -0.297   0.767

log-likelihood     3702.11662
no. of observations      1230  no. of parameters           5
AIC.T             -7394.23324  AIC                -6.01157174
mean(returns)    -0.000107398  var(returns)       0.000139266
sigma               0.0117717  sigma^2            0.000138573

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
```

```
Used starting values:
    -0.011772    0.0039997     0.074964  -0.00010740
```

The value of AR-3 is positive as seen on the graph, and most importantly, it is significant. **This model with ARMA(3,0) specification should be a good one**. The results of a test summary will provide greater information:

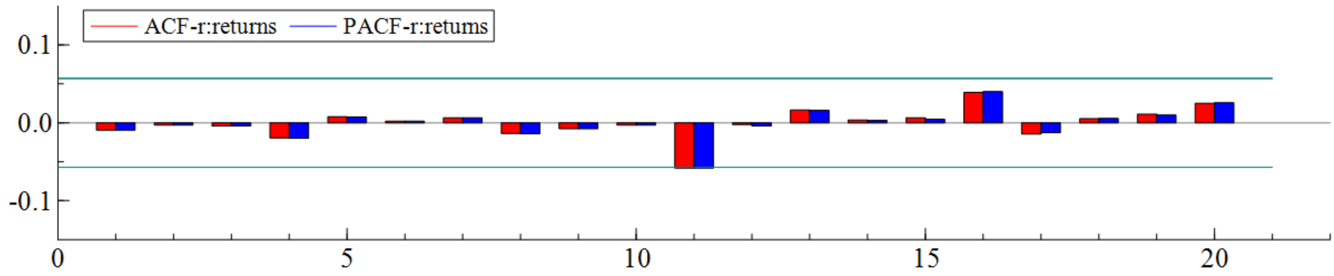
```
Descriptive statistics for residuals:
Normality test:   Chi^2(2)   =   138.23 [0.0000]**
ARCH 1-1 test:    F(1,1224) =    61.819 [0.0000]**
Portmanteau(35):  Chi^2(32) =    31.343 [0.4997]
```

The test summary provides greater certainty about what was supposed before, namely that the series does not follow a Normal distribution and that there is ARCH effect; moreover, **Portmanteau shows no autocorrelation left in the series after using our ARMA(3,0) specification**. Since ARCH effect is still there in the residuals, there is a **need to implement GARCH specifications**, but first, let's try to formulate an ARMA(3,3) model:

```
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
The dataset is: C:\Users\danie\Documents\FINANCIAL ENGINEERING\SEMESTER 2\Financial
Econometrics\8_29.04.2020 (online...)\Task 06.05\Hang Seng.xlsx

                  Coefficient  Std.Error  t-value  t-prob
AR-1                -0.213673     0.2584   -0.827   0.408
AR-2                -0.212532     0.1800    -1.18   0.238
AR-3                -0.460067     0.2046    -2.25   0.025
MA-1                 0.213591     0.2437    0.877   0.381
MA-2                 0.220218     0.1679     1.31   0.190
MA-3                 0.545586     0.1943     2.81   0.005
Constant          -0.000108457 0.0002801   -0.387   0.699

log-likelihood      3704.27197
no. of observations       1230  no. of parameters           8
AIC.T              -7392.54395  AIC                -6.01019833
mean(returns)     -0.000107398  var(returns)       0.000139266
sigma               0.00936192  sigma^2            8.76455e-05

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     -0.59110     -0.37366     -0.61163      0.99493      0.76413      0.35201  -0.00010740
```

As we can see, AR-3 and MA-3 are both statistically significant. Moreover, **the log-likelihood for this model is higher**.

We could prepare a small Graphical analysis after the model, and see what happened to those lags that were initially significant:



This model has removed autocorrelation from the series, there is just lag 11 which is on the edge. Let's see the results from test summary:

```
Descriptive statistics for residuals:
Normality test:   Chi^2(2)  =   132.58 [0.0000]**
ARCH 1-1 test:    F(1,1221) =   63.255 [0.0000]**
Portmanteau(35):  Chi^2(29) =   26.776 [0.5838]
```

The situation highlighted by these tests is similar to the one for ARMA(3,0). It is time to implement GARCH.

Let's start with an ARMA(3,3) with GARCH(0,1) specification and Student T distribution:

```
******************************
 ** G@RCH(1) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (3, 3) model.
No regressor in the conditional mean
Variance Equation:  GARCH (0, 1) model.
No regressor in the conditional variance
Student distribution, with 4.79314 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 3788.85
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000369 0.00030205    1.222  0.2220
AR(1)               -0.216783    0.10013   -2.165  0.0306
AR(2)               -0.608699    0.17470   -3.484  0.0005
AR(3)               -0.517223   0.078207   -6.613  0.0000
MA(1)                0.226148    0.10996    2.057  0.0399
MA(2)                0.612181    0.18039    3.394  0.0007
MA(3)                0.559376   0.091616    6.106  0.0000
Cst(V) x 10^4        1.201793   0.087674    13.71  0.0000
ARCH(Alpha1)         0.151354   0.044552    3.397  0.0007
Student(DF)          4.793141    0.65960    7.267  0.0000

No. Observations :      1230  No. Parameters  :        10
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3788.852  Alpha[1]+Beta[1]:   0.15135

The sample mean of squared residuals was used to start recursion.
The unconditional variance is 0.000141613
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.242019 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000369;-0.216783;-0.608699;-0.517223; 0.226148; 0.612181; 0.559376; 1.201793; 0.151354;
4.793146
Elapsed Time : 0.184 seconds (or 0.00306667 minutes).
```

The **Constant in Mean is positive but not statistically significant**, while the **Conditional Variance is positive and significant**. Both AR-3 and MA-3 are significant.
Now, I will make some tests, more specifically: Box-Pierce on standardized residuals and on squared standardized residuals, Sign-Bias Test for Leverage effect, ARCH Test, Nyblom Test for parameters stability, and Adjusted Pearson Goodness-of-fit Test. Here is the outcome of the tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  --> P-values adjusted by 6 degree(s) of freedom
  Q( 10) =  3.71191   [0.4463963]
  Q( 20) =  12.0221   [0.6045288]
  Q( 50) =  41.9763   [0.5587134]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 1 degree(s) of freedom
  Q(  5) =  68.5390   [0.0000000]**
  Q( 10) =  107.764   [0.0000000]**
  Q( 20) =  136.803   [0.0000000]**
  Q( 50) =  200.898   [0.0000000]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.61621  0.53776
Negative Size Bias t-Test         0.68199  0.49524
Positive Size Bias t-Test         0.37854  0.70503
Joint Test for the Three Effects  1.89552  0.59437

---------------
ARCH 1-2 test:    F(2,1224) =   14.929 [0.0000]**
ARCH 1-5 test:    F(5,1218) =   12.513 [0.0000]**
ARCH 1-10 test:   F(10,1208)=   7.9873 [0.0000]**
---------------

Joint Statistic of the Nyblom test of stability: 3.28879

Individual Nyblom Statistics:
Cst(M)          0.19667
AR(1)           0.45558
AR(2)           0.26570
AR(3)           0.35296
MA(1)           0.45380
MA(2)           0.23813
MA(3)           0.38424
Cst(V) x 10^4   0.87035
ARCH(Alpha1)    0.58290
Student(DF)     0.43318

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       49.0894         0.129114          0.011299
    50       55.1220         0.254268          0.045082
    60       71.9512         0.119978          0.018014
Rem.: k = 10 = # estimated parameters
```

It is visible that **only ARCH is not enough**; there is autocorrelation in squared standardized residuals and there is ARCH effect too. Moreover, the joint statistic from Nyblom Test shows that such an approach, with the selected parameters, would not be ideal. Another detail, according to Pearson's Test the Student T distribution doesn't suit the model nicely.

So, let's try to formulate an ARMA(3,3) with GARCH(1,1) specification with a Student T distribution (gaussian distribution should rather not be considered, as the normality test and distribution graph initially showed):

```
******************************
 ** G@RCH(2) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (3, 3) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 6.03658 degrees of freedom.

Weak convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 3826.97
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000616 0.00028868    2.135  0.0329
AR(1)               -0.221807   0.081719   -2.714  0.0067
AR(2)               -0.603351    0.15670   -3.850  0.0001
AR(3)               -0.520987   0.085555   -6.090  0.0000
MA(1)                0.234534   0.085576    2.741  0.0062
MA(2)                0.599041    0.16052    3.732  0.0002
MA(3)                0.567562   0.083649    6.785  0.0000
Cst(V) x 10^4        0.015380   0.015538   0.9898  0.3225
ARCH(Alpha1)         0.053824   0.021418    2.513  0.0121
GARCH(Beta1)         0.937761   0.030020    31.24  0.0000
Student(DF)          6.036577     1.0400    5.805  0.0000

No. Observations :      1230  No. Parameters  :        11
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3826.967  Alpha[1]+Beta[1]:   0.99159

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000182779
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.997571 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000616;-0.221807;-0.603351;-0.520987; 0.234534; 0.599041; 0.567562; 0.015380; 0.053824;
0.937761; 6.036577
Elapsed Time : 0.208 seconds (or 0.00346667 minutes).
```

We can notice that the Constant in Mean (Cst(M)) is positive and statistically significant. **This means that there is a need for having the Constant in Mean in this model**.
The value of Conditional Variance is positive, but statistically insignificant. However, the **unconditional variance exists and is positive**, which is good. The values of **Alpha1 and Beta1 are positive and statistically significant. Their sum is lower than one**, however, very close. Also, **AR-3 and MA-3 are statistically significant**. The value of Alpha1 is 0.053824, which means that the squared residuals from the previous observation have this much impact on today's Conditional Variance. This value is not very high, therefore it indicates that only few squared residuals have an influence on today's Conditional Variance. The "previous" Conditional Variance has a value equal to Beta1 (0.937761).

Let’s make some tests, the same tests as for the previous model will be used pretty much all along this file:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  --> P-values adjusted by 6 degree(s) of freedom
  Q( 10) =  4.04843   [0.3994916]
  Q( 20) =  13.1032   [0.5184159]
  Q( 50) =  46.8673   [0.3556837]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  4.83029   [0.1846541]
  Q( 10) =  8.85674   [0.3545249]
  Q( 20) =  18.6392   [0.4143505]
  Q( 50) =  37.5292   [0.8618437]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.18854  0.85045
Negative Size Bias t-Test         0.91311  0.36118
Positive Size Bias t-Test         0.84394  0.39870
Joint Test for the Three Effects  2.68492  0.44280

---------------
ARCH 1-2 test:    F(2,1223) =  0.45078 [0.6372]
ARCH 1-5 test:    F(5,1217) =  0.91616 [0.4695]
ARCH 1-10 test:   F(10,1207)=  0.87088 [0.5602]
---------------

Joint Statistic of the Nyblom test of stability: 2.00892

Individual Nyblom Statistics:
Cst(M)          0.15980
AR(1)           0.28864
AR(2)           0.16632
AR(3)           0.15966
MA(1)           0.28446
MA(2)           0.14557
MA(3)           0.17804
Cst(V) x 10^4   0.16075
ARCH(Alpha1)    0.10840
GARCH(Beta1)    0.08684
Student(DF)     0.04334

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       50.1301         0.109166         0.006255
    50       57.3984         0.191951         0.022533
    60       60.3415         0.427056         0.108943
Rem.: k = 11 = # estimated parameters
```

Box-Pierce tests show that now there is **no autocorrelation neither in the standardized residuals nor in the squared standardized residuals**. If we look at the **Sign Bias Test, none of the P-values are showing any asymmetric relationship leading to Leverage effect**. The ARCH Test shows that there is **no more ARCH effect** in the series; this specification has captured all of it.
Looking at the outcome of Nyblom Test, **all the singular statistics are stable** (all the p-values are lower than the critical value for both 1% and 5% confidence), however, the joint statistic is quite higher than the 1% critical value. This is certainly related to the large amount of parameters used in this model (11 parameters).
When looking at the Adjusted Pearson Goodness-of-fit Test, we can see that if we divide the sample in 40 cells the p-value is smaller than the critical one, hence, the Student T distribution doesn't cope in the best way with a 40 cells division. The same works for a 50-cells division while it is fine if divide the sample in 60 cells. **This specification seems to be a good one in the overall**. It could be an idea to change distribution type for the next model into Skewed Student T, just to see if the fit will improve. Due to the fact that the sum of Alpha and Beta is really close to 1, another idea could be to formulate an IGARCH later on.

Let's formulate an ARMA(3,3) with GARCH(1,1) and Skewed Student T distribution this time:

```
******************************
 ** G@RCH(3) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (3, 3) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 6.62244 degrees of freedom.
and asymmetry coefficient (log xi) -0.130077.

Strong convergence using numerical derivatives
Log-likelihood = 3832.48
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000282 0.00030746   0.9185  0.3585
AR(1)               -0.216668   0.084675   -2.559  0.0106
AR(2)               -0.595405    0.14627   -4.071  0.0000
AR(3)               -0.523744   0.077560   -6.753  0.0000
MA(1)                0.230431   0.090224    2.554  0.0108
MA(2)                0.591850    0.15125    3.913  0.0001
MA(3)                0.572036   0.083368    6.862  0.0000
Cst(V) x 10^4        0.016250   0.014364    1.131  0.2582
ARCH(Alpha1)         0.052915   0.018423    2.872  0.0041
GARCH(Beta1)         0.936776   0.026886    34.84  0.0000
Asymmetry           -0.130077   0.037187   -3.498  0.0005
Tail                 6.622442     1.2219    5.420  0.0000

No. Observations :      1230  No. Parameters  :        12
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3832.478  Alpha[1]+Beta[1]:   0.98969

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000157634
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.992063 and should be < 1.
```

```
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000282;-0.216668;-0.595405;-0.523744; 0.230431; 0.591850; 0.572036; 0.016250; 0.052915;
0.936776;-0.130077; 6.622447
Elapsed Time : 0.285 seconds (or 0.00475 minutes).
```

As we can see, there has been an increase in log-likelihood. However, **the Constant in Mean is no more statistically significant now**. The Conditional Variance is also not significant, but like before, the unconditional variance exists. AR-3 and MA-3 are still significant. Alpha1 and Beta1 are significant too and their sum is lower than 1, even if it is still quite close. Let's make tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  --> P-values adjusted by 6 degree(s) of freedom
  Q( 10) =  3.79871   [0.4339321]
  Q( 20) =  12.9939   [0.5270017]
  Q( 50) =  46.3780   [0.3745234]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  4.73671   [0.1921235]
  Q( 10) =  9.04698   [0.3383486]
  Q( 20) =  18.8097   [0.4036298]
  Q( 50) =  37.6517   [0.8585684]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.05984  0.95228
Negative Size Bias t-Test         0.90952  0.36308
Positive Size Bias t-Test         0.74112  0.45862
Joint Test for the Three Effects  2.76120  0.42993

---------------
ARCH 1-2 test:    F(2,1223) =  0.45869 [0.6322]
ARCH 1-5 test:    F(5,1217) =  0.89877 [0.4811]
ARCH 1-10 test:   F(10,1207)=  0.89377 [0.5384]
---------------

Joint Statistic of the Nyblom test of stability: 2.15439

Individual Nyblom Statistics:
Cst(M)           0.17051
AR(1)            0.25524
AR(2)            0.13421
AR(3)            0.16070
MA(1)            0.24682
MA(2)            0.11654
MA(3)            0.17817
Cst(V) x 10^4    0.13566
ARCH(Alpha1)     0.09258
GARCH(Beta1)     0.07320
Asymmetry        0.10972
Tail             0.04808

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------
```

```
Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       37.5122         0.537777         0.085897
    50       48.3740         0.498416         0.099820
    60       52.1463         0.724121         0.280685
Rem.: k = 12 = # estimated parameters
```

As we can see, there is no autocorrelation in residuals and squared residuals and no ARCH effect, which is good. Neither there is Leverage effect. In the Nyblom test, the joint statistic has worsened a bit, but there is a big improvement in Pearson's Test, as now all the p-values are greater than 0.05, hence, the **Skewed Student T distribution fits better to this specification than the Student T distribution**.

Considering that the sum of Alpha and Beta was again really close to 1, we can try to formulate an IGARCH(1,1), still keeping ARMA(3,3) as well, with Student T distribution:

```
******************************
 ** G@RCH(5) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (3, 3) model.
No regressor in the conditional mean
Variance Equation:  IGARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 5.67718 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 3826.42
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000628 0.00028813    2.180  0.0294
AR(1)               -0.223510   0.081952   -2.727  0.0065
AR(2)               -0.601995    0.15510   -3.881  0.0001
AR(3)               -0.521122   0.087457   -5.959  0.0000
MA(1)                0.234590   0.085996    2.728  0.0065
MA(2)                0.597925    0.15844    3.774  0.0002
MA(3)                0.567242   0.085159    6.661  0.0000
Cst(V) x 10^4        0.007739  0.0059823    1.294  0.1960
ARCH(Alpha1)         0.053027   0.020159    2.631  0.0086
Student(DF)          5.677178    0.96444    5.887  0.0000
GARCH(Beta1)         0.946973

No. Observations :      1230  No. Parameters  :        10
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3826.416

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
 0.000628;-0.223510;-0.601995;-0.521122; 0.234590; 0.597925; 0.567242; 0.007739; 0.053027;
5.677183
Elapsed Time : 0.162 seconds (or 0.0027 minutes).
```

As we can see, the Constant in Mean is positive and statistically significant, while the Conditional Variance is not statistically significant. AR-3 and MA-3 are statistically significant and Alpha1 is positive and significant. The sum of Alpha1 and Beta1 is assumed to be equal to 1 in IGARCH models. Let's make the usual tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  --> P-values adjusted by 6 degree(s) of freedom
  Q( 10) =  4.47039   [0.3460731]
  Q( 20) =  13.6881   [0.4731987]
  Q( 50) =  48.0857   [0.3108827]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  4.91142   [0.1783995]
  Q( 10) =  8.97913   [0.3440596]
  Q( 20) =  19.4777   [0.3629780]
  Q( 50) =  38.3708   [0.8384572]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.20789  0.83531
Negative Size Bias t-Test         0.87985  0.37894
Positive Size Bias t-Test         0.94014  0.34715
Joint Test for the Three Effects  2.83979  0.41699

---------------
ARCH 1-2 test:    F(2,1223) =  0.44494 [0.6410]
ARCH 1-5 test:    F(5,1217) =  0.93100 [0.4598]
ARCH 1-10 test:   F(10,1207)=  0.87966 [0.5518]
---------------

Joint Statistic of the Nyblom test of stability: 1.85649

Individual Nyblom Statistics:
Cst(M)           0.16283
AR(1)            0.28174
AR(2)            0.17445
AR(3)            0.15358
MA(1)            0.27705
MA(2)            0.15295
MA(3)            0.17196
Cst(V) x 10^4    0.08741
ARCH(Alpha1)     0.05200
Student(DF)      0.04141

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       43.2358         0.295225          0.043295
    50       63.0894         0.085050          0.008630
    60       66.4878         0.234861          0.048742
Rem.: k = 10 = # estimated parameters
```

There is no autocorrelation and no ARCH effect thanks to this specification. Also, there is no Leverage effect, and the individual Nyblom statistics are all lower than the 1% critical value. Only the **Joint statistic is still quite high, but smaller than the one from previous models**. The **Student T distribution is not the best one for this model** in any division we might make.

Therefore, we can try to formulate again the ARMA(3,3) with IGARCH(1,1), with a Skewed Student T distribution this time:

```
*****************************
 ** G@RCH(7) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (3, 3) model.
No regressor in the conditional mean
Variance Equation:  IGARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 6.11223 degrees of freedom.
and asymmetry coefficient (log xi) -0.127859.

Strong convergence using numerical derivatives
Log-likelihood = 3831.58
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000280 0.00030876   0.9061  0.3651
AR(1)               -0.217668   0.085166   -2.556  0.0107
AR(2)               -0.595003    0.14340   -4.149  0.0000
AR(3)               -0.523638   0.077899   -6.722  0.0000
MA(1)                0.229582   0.090866    2.527  0.0116
MA(2)                0.591941    0.14756    4.011  0.0001
MA(3)                0.571309   0.083656    6.829  0.0000
Cst(V) x 10^4        0.007136  0.0050436    1.415  0.1574
ARCH(Alpha1)         0.052662   0.017472    3.014  0.0026
Asymmetry           -0.127859   0.038129   -3.353  0.0008
Tail                 6.112230     1.1197    5.459  0.0000
GARCH(Beta1)         0.947338

No. Observations :      1230  No. Parameters   :        11
Mean (Y)         :  -0.00011  Variance (Y)     :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)     :   5.38423
Log Likelihood   :  3831.579

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
 0.000280;-0.217668;-0.595003;-0.523638; 0.229582; 0.591941; 0.571309; 0.007136;
0.052662;-0.127859; 6.112235
Elapsed Time : 0.252 seconds (or 0.0042 minutes).
```

Now, neither Cst(M) nor Cst(V) are statistically significant, while Alpha1 is positive and statistically significant. The log-likelihood has improved, if we compare this model to the one made with a Student T distribution. AR-3 and MA-3 are statistically significant too. Let’s check the tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  --> P-values adjusted by 6 degree(s) of freedom
  Q( 10) =  4.29119   [0.3680300]
  Q( 20) =  13.6942   [0.4727333]
  Q( 50) =  47.7913   [0.3214160]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  4.72035   [0.1934578]
  Q( 10) =  9.03956   [0.3389693]
```

```
  Q( 20) =  19.7173   [0.3489603]
  Q( 50) =  38.4732   [0.8354691]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.07804  0.93780
Negative Size Bias t-Test         0.84394  0.39870
Positive Size Bias t-Test         0.87044  0.38406
Joint Test for the Three Effects  2.91310  0.40522

---------------
ARCH 1-2 test:    F(2,1223) =  0.43754 [0.6457]
ARCH 1-5 test:    F(5,1217) =  0.89559 [0.4832]
ARCH 1-10 test:   F(10,1207)=  0.89078 [0.5412]
---------------

Joint Statistic of the Nyblom test of stability: 1.97172

Individual Nyblom Statistics:
Cst(M)           0.19461
AR(1)            0.24866
AR(2)            0.14041
AR(3)            0.15345
MA(1)            0.24038
MA(2)            0.12243
MA(3)            0.17097
Cst(V) x 10^4    0.06054
ARCH(Alpha1)     0.06153
Asymmetry        0.10431
Tail             0.04163

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       45.7073         0.213488          0.018683
    50       52.1138         0.353764          0.063302
    60       57.8049         0.519639          0.157037
Rem.: k = 11 = # estimated parameters
```

There is **no autocorrelation** neither in the residuals nor in the squared residuals. There is no Leverage effect and no ARCH effect. The **joint statistic for Nyblom keeps being high**, and this time **Pearson Test shows an improvement** at 50 and 60 cells thanks to the adoption of the Skewed Student T distribution.

Let's check what would be the efficiency of a GARCH(1,1) with ARMA(0,0) and a Skewed Student T distribution. Perhaps, the reduction of parameters will improve the outcome of Nyblom Test:

```
******************************
 ** G@RCH(9) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 6.60404 degrees of freedom.
```

```
and asymmetry coefficient (log xi) -0.123323.

Strong convergence using numerical derivatives
Log-likelihood = 3824.58
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000271 0.00030462   0.8899  0.3737
Cst(V) x 10^4        0.015084   0.012810    1.178  0.2392
ARCH(Alpha1)         0.050233   0.016810    2.988  0.0029
GARCH(Beta1)         0.939983   0.024123    38.97  0.0000
Asymmetry           -0.123323   0.035429   -3.481  0.0005
Tail                 6.604036     1.1713    5.638  0.0000

No. Observations :      1230  No. Parameters  :         6
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3824.577  Alpha[1]+Beta[1]:   0.99022

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000154171
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.991853 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000271; 0.015084; 0.050233; 0.939983;-0.123323; 6.604041
Elapsed Time : 0.113 seconds (or 0.00188333 minutes).
```

First of all, **removing the ARMA(3,3) process has reduced the log-likelihood** value. The Constant in Mean is positive but not statistically significant and the Conditional Variance is positive but not significant either. Unconditional Variance exists and is positive, and both Alpha1 and Beta1 are positive and statistically significant. Their sum is lower than one. Let's verify the tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  Q(  5) =  8.60668   [0.1258188]
  Q( 10) =  10.0584   [0.4353811]
  Q( 20) =  18.6959   [0.5416671]
  Q( 50) =  52.4296   [0.3799572]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  6.03977   [0.1096913]
  Q( 10) =  9.69385   [0.2871744]
  Q( 20) =  18.9547   [0.3946190]
  Q( 50) =  39.6923   [0.7976639]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.40352  0.68656
Negative Size Bias t-Test         1.02578  0.30500
Positive Size Bias t-Test         1.00701  0.31393
Joint Test for the Three Effects  2.98998  0.39317
```

```
---------------
ARCH 1-2 test:    F(2,1223) =  0.51638 [0.5968]
ARCH 1-5 test:    F(5,1217) =   1.1511 [0.3314]
ARCH 1-10 test:   F(10,1207)=  0.95851 [0.4782]
---------------

Joint Statistic of the Nyblom test of stability: 0.806074

Individual Nyblom Statistics:
Cst(M)             0.19145
Cst(V) x 10^4      0.13339
ARCH(Alpha1)       0.09618
GARCH(Beta1)       0.07558
Asymmetry          0.12618
Tail               0.05437

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       38.6829          0.484195          0.228427
    50       44.8780          0.640865          0.393061
    60       64.0488          0.303926          0.142249
Rem.: k = 6 = # estimated parameters
```

What we can see from these tests is that there is no autocorrelation, ARCH effect and Leverage effect. The **joint statistic for Nyblom has drastically reduced**, however, it is not yet lower than the 1% critical value. According to Pearson's Test, now the **empirical distribution is following the theoretical one**.

Let's try to formulate a GARCH(2,1) with Student T distribution:

```
******************************
 ** G@RCH(10) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (2, 1) model.
No regressor in the conditional variance
Student distribution, with 6.10109 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 3818.68
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000575 0.00028455    2.022  0.0434
Cst(V) x 10^4        0.016516   0.013614    1.213  0.2253
ARCH(Alpha1)         0.071548   0.019469    3.675  0.0002
GARCH(Beta1)         0.100746    0.13310   0.7569  0.4492
GARCH(Beta2)         0.819093    0.13953    5.870  0.0000
Student(DF)          6.101094     1.0384    5.876  0.0000

No. Observations :      1230  No. Parameters  :         6
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3818.676  Alpha[1]+Beta[1]:   0.99139
```

```
The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (2,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000191763
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.

Estimated Parameters Vector :
 0.000575; 0.016516; 0.071548; 0.100746; 0.819093; 6.101099
Elapsed Time : 0.098 seconds (or 0.00163333 minutes).
```

It is noticeable that the **Constant in Mean is positive and statistically significant** here. The Conditional Variance is positive but not significant, however, the unconditional variance exists and is positive. Alpha1 and Beta2 are positive and statistically significant, while the sum of Alphas and Betas is lower than 1. **The log-likelihood for this model is quite low**, but let's see the outcome of tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  Q(  5) =  8.61825   [0.1252945]
  Q( 10) =  10.3284   [0.4121708]
  Q( 20) =  19.5754   [0.4847532]
  Q( 50) =  54.5205   [0.3066589]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 3 degree(s) of freedom
  Q(  5) =  9.34511   [0.0093484]**
  Q( 10) =  14.2358   [0.0471423]*
  Q( 20) =  22.6049   [0.1625618]
  Q( 50) =  44.6794   [0.5691832]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.37922  0.70452
Negative Size Bias t-Test         0.88766  0.37473
Positive Size Bias t-Test         1.08300  0.27881
Joint Test for the Three Effects  2.86066  0.41361

---------------
ARCH 1-2 test:    F(2,1222) =   3.4352 [0.0325]*
ARCH 1-5 test:    F(5,1216) =   1.8198 [0.1061]
ARCH 1-10 test:   F(10,1206)=   1.3901 [0.1791]
---------------

Joint Statistic of the Nyblom test of stability: 1.04093

Individual Nyblom Statistics:
Cst(M)           0.21019
Cst(V) x 10^4    0.20532
ARCH(Alpha1)     0.14950
GARCH(Beta1)     0.15596
GARCH(Beta2)     0.14138
Student(DF)      0.04929

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test
```

```
# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       40.7642        0.392764         0.165949
    50       58.3740        0.168808         0.058978
    60       59.0732        0.472834         0.263380
Rem.: k = 6 = # estimated parameters
```

As we can see, **this model is not good as we have some statistically significant autocorrelation and some ARCH effect**.

If we try to change distribution, hence create a GARCH(2,1) with Skewed Student T distribution:

```
******************************
 ** G@RCH(11) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (2, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 6.59185 degrees of freedom.
and asymmetry coefficient (log xi) -0.123519.

Weak convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 3824.84
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000276 0.00030439   0.9076  0.3643
Cst(V) x 10^4        0.012812   0.011232    1.141  0.2542
ARCH(Alpha1)         0.040237   0.013051    3.083  0.0021
GARCH(Beta1)         1.223129    0.23000    5.318  0.0000
GARCH(Beta2)        -0.271711    0.23157   -1.173  0.2409
Asymmetry           -0.123519   0.035420   -3.487  0.0005
Tail                 6.591850     1.1644    5.661  0.0000

No. Observations :      1230  No. Parameters  :         7
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3824.838  Alpha[1]+Beta[1]:   0.99165

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (2,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance does not exist and/or is not positive.
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.

Estimated Parameters Vector :
 0.000276; 0.012812; 0.040237; 1.223129;-0.271711;-0.123519; 6.591850
Elapsed Time : 0.176 seconds (or 0.00293333 minutes).
```

We can see that log-likelihood rises, but **Beta2 is not statistically significant**, so this model should not be considered. Also, Conditional Variance is not significant, and there is no unconditional variance.

Now, we can try to formulate IGARCH(1,1) with Student T distribution:

```
******************************
 ** G@RCH(14) SPECIFICATIONS **
 ******************************
```

```
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  IGARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 5.70831 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 3818.92
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000605 0.00028272    2.141  0.0325
Cst(V) x 10^4        0.006981  0.0052391    1.332  0.1830
ARCH(Alpha1)         0.050670   0.018261    2.775  0.0056
Student(DF)          5.708312    0.93169    6.127  0.0000
GARCH(Beta1)         0.949330

No. Observations :      1230  No. Parameters  :         4
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3818.922

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
 0.000605; 0.006981; 0.050670; 5.708317
Elapsed Time : 0.051 seconds (or 0.00085 minutes).
```

The Constant in Mean is positive and statistically significant, while the Conditional Variance is positive but not significant. Alpha1 is positive and statistically significant. Here are the tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  Q(  5) =  8.80979   [0.1168960]
  Q( 10) =  10.6588   [0.3847101]
  Q( 20) =  19.4539   [0.4925200]
  Q( 50) =  54.0889   [0.3211322]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  6.19761   [0.1023822]
  Q( 10) =  9.62083   [0.2926535]
  Q( 20) =  19.5859   [0.3566100]
  Q( 50) =  40.6536   [0.7651137]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.31239  0.75474
Negative Size Bias t-Test         0.85601  0.39199
Positive Size Bias t-Test         1.09260  0.27457
Joint Test for the Three Effects  2.98569  0.39384

---------------
ARCH 1-2 test:    F(2,1223) =  0.44520 [0.6408]
ARCH 1-5 test:    F(5,1217) =   1.1813 [0.3161]
ARCH 1-10 test:   F(10,1207)=  0.94774 [0.4880]
---------------
```

```
Joint Statistic of the Nyblom test of stability: 0.506921

Individual Nyblom Statistics:
Cst(M)           0.18169
Cst(V) x 10^4    0.08769
ARCH(Alpha1)     0.04921
Student(DF)      0.03959

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       50.0650         0.110336          0.047487
    50       82.4390         0.001968          0.000558
    60       89.7073         0.006112          0.002161
Rem.: k = 4 = # estimated parameters
```

Thanks to this model there is no ARCH effect, no autocorrelation and no Leverage effect. The joint statistic for Nyblom is lower than the 1% critical value for the first time, but the distribution doesn't seem appropriate. That's why we can try to formulate an IGARCH(1,1) with Skewed Student T distribution:

```
******************************
 ** G@RCH(15) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  IGARCH (1, 1) model.
No regressor in the conditional variance
Skewed Student distribution, with 6.09863 degrees of freedom.
and asymmetry coefficient (log xi) -0.120799.

Strong convergence using numerical derivatives
Log-likelihood = 3823.66
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000269 0.00030620   0.8793  0.3794
Cst(V) x 10^4        0.006518  0.0045491    1.433  0.1522
ARCH(Alpha1)         0.050493   0.016135    3.129  0.0018
Asymmetry           -0.120799   0.036333   -3.325  0.0009
Tail                 6.098633     1.0721    5.688  0.0000
GARCH(Beta1)         0.949507

No. Observations :      1230  No. Parameters   :         5
Mean (Y)         :  -0.00011  Variance (Y)     :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)     :   5.38423
Log Likelihood   :  3823.659

The sample mean of squared residuals was used to start recursion.

Estimated Parameters Vector :
 0.000269; 0.006518; 0.050493;-0.120799; 6.098638
Elapsed Time : 0.077 seconds (or 0.00128333 minutes).
```

The first change that can be noticed from the previous model is that **Constant in Mean is now not significant**. The other parameters are quite similar. Let's verify tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  Q(  5) =  8.95828   [0.1107399]
  Q( 10) =  10.6720   [0.3836367]
  Q( 20) =  19.5989   [0.4832578]
  Q( 50) =  54.0855   [0.3212463]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  5.94259   [0.1144370]
  Q( 10) =  9.54996   [0.2980427]
  Q( 20) =  19.7120   [0.3492629]
  Q( 50) =  40.5820   [0.7676159]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.42594  0.67015
Negative Size Bias t-Test         0.93274  0.35095
Positive Size Bias t-Test         1.14899  0.25056
Joint Test for the Three Effects  3.12882  0.37219

---------------
ARCH 1-2 test:    F(2,1223) =  0.44664 [0.6399]
ARCH 1-5 test:    F(5,1217) =   1.1324 [0.3411]
ARCH 1-10 test:   F(10,1207)=  0.94500 [0.4905]
---------------

Joint Statistic of the Nyblom test of stability: 0.607065

Individual Nyblom Statistics:
Cst(M)           0.21475
Cst(V) x 10^4    0.06414
ARCH(Alpha1)     0.05378
Asymmetry        0.12146
Tail             0.04562

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       48.5691         0.140099          0.050329
    50       71.7073         0.018880          0.005211
    60       70.5854         0.143612          0.064326
Rem.: k = 5 = # estimated parameters
```

As we can see, the Nyblom joint statistic has risen a little bit, but now the **distribution seems appropriate, apart from when we divide the sample in 50 cells**.

We could also try to use ARFIMA(0,d,0) with GARCH(1,1) and a Student T distribution:

```
******************************
 ** G@RCH(17) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
```

```
Mean Equation:  ARFIMA (0, d, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 6.06495 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 3819.49
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000594 0.00027887    2.130  0.0333
d-Arfima            -0.000147   0.021428-0.006864  0.9945
Cst(V) x 10^4        0.014042   0.013531    1.038  0.2996
ARCH(Alpha1)         0.050896   0.019263    2.642  0.0083
GARCH(Beta1)         0.941198   0.026547    35.45  0.0000
Student(DF)          6.064949     1.0125    5.990  0.0000

No. Observations :      1230  No. Parameters  :         6
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3819.489  Alpha[1]+Beta[1]:   0.99209

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000177604
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.996957 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000594;-0.000147; 0.014042; 0.050896; 0.941198; 6.064954
Elapsed Time : 0.524 seconds (or 0.00873333 minutes).
```

We can see that **d parameter is not significant**, hence **ARFIMA cannot be considered as a good model here**. This was just to show the correct supposition initially made after the Geneke and Porter-Hudak Test.

Models that deal with Leverage effect such as GJR, EGARCH and ARCH-in-mean won't be effective, since we have seen that there is not such effect. I made just one attempt with a GJR specification but Alpha1 parameter was negative, therefore, the model would not be considered. APARCH is not a good specification either, as there was no convergence when I tried to estimate it.

In the beginning, I said that there could be a long memory in the squared returns. Squared returns' time series properties showed statistically significant autocorrelation even at high lags, so it might be time to use a FIGARCH-BBM(1,1) with Student T distribution:

```
******************************
 ** G@RCH(21) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  FIGARCH (1, d, 1) model estimated with BBM's method (Truncation order :
1000).
No regressor in the conditional variance
Student distribution, with 6.02758 degrees of freedom.
```

```
Strong convergence using numerical derivatives
Log-likelihood = 3821.61
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000640 0.00028116    2.275  0.0231
Cst(V) x 10^4        0.073807   0.046087    1.601  0.1095
d-Figarch            0.415924    0.11033    3.770  0.0002
ARCH(Phi1)           0.116303    0.11227    1.036  0.3004
GARCH(Beta1)         0.518417    0.17208    3.013  0.0026
Student(DF)          6.027577    0.99441    6.061  0.0000

No. Observations :      1230  No. Parameters  :         6
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3821.613

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is
observed (0.102493<0.116303<0.528025 and -0.0730923<0.00715958 valid).
  => See Bollerslev and Mikkelsen (1996) for more details.

Estimated Parameters Vector :
 0.000640; 0.073807; 0.415924; 0.116303; 0.518417; 6.027582
Elapsed Time : 0.964 seconds (or 0.0160667 minutes).
```

As we can see, the Constant in Mean is positive and statistically significant. The d parameter is statistically significant as well which means that the **Conditional Variance of returns presents long memory**. Phi1 and Conditional Variance are not statistically significant, however, the **positivity constraint for FIGARCH is observed**. This seems to be not a very good specification. Let's verify tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  Q(  5) =  7.20899   [0.2055554]
  Q( 10) =  8.45202   [0.5847734]
  Q( 20) =  16.7009   [0.6723028]
  Q( 50) =  50.5106   [0.4532078]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  3.31328   [0.3457990]
  Q( 10) =  4.98825   [0.7588310]
  Q( 20) =  10.8645   [0.9000176]
  Q( 50) =  37.0672   [0.8737836]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.25036  0.80231
Negative Size Bias t-Test         1.43971  0.14995
Positive Size Bias t-Test         0.66742  0.50450
Joint Test for the Three Effects  4.02825  0.25843

---------------
ARCH 1-2 test:    F(2,1223) =  0.12990 [0.8782]
ARCH 1-5 test:    F(5,1217) =  0.67336 [0.6437]
ARCH 1-10 test:   F(10,1207)=  0.52049 [0.8767]
---------------
```

```
Joint Statistic of the Nyblom test of stability: 0.914997

Individual Nyblom Statistics:
Cst(M)           0.16309
Cst(V) x 10^4    0.24056
d-Figarch        0.21316
ARCH(Phi1)       0.17017
GARCH(Beta1)     0.22365
Student(DF)      0.05430

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       53.9024         0.056610          0.012260
    50       60.8130         0.119972          0.037919
    60       87.6585         0.009104          0.001934
Rem.: k = 6 = # estimated parameters
```

As we can see, there is no autocorrelation, no ARCH effect and no Leverage effect. Nyblom Test is good, apart from the joint statistic which is still a little higher than 0.75. According to Pearson's Test, we should change the distribution.
So, if we formulate a FIGARCH-BBM(1,1) with Skewed Student T distribution:

```
******************************
 ** G@RCH(22) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  FIGARCH (1, d, 1) model estimated with BBM's method (Truncation order :
1000).
No regressor in the conditional variance
Skewed Student distribution, with 6.48049 degrees of freedom.
and asymmetry coefficient (log xi) -0.118049.

Strong convergence using numerical derivatives
Log-likelihood = 3826.22
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000326 0.00030446    1.070  0.2850
Cst(V) x 10^4        0.066105   0.042108    1.570  0.1167
d-Figarch            0.405903    0.10835    3.746  0.0002
ARCH(Phi1)           0.144354    0.10582    1.364  0.1728
GARCH(Beta1)         0.532951    0.16247    3.280  0.0011
Asymmetry           -0.118049   0.035859   -3.292  0.0010
Tail                 6.480494     1.1422    5.674  0.0000

No. Observations :      1230  No. Parameters  :         7
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3826.224

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is
observed (0.127049<0.144354<0.531366 and -0.0619792<0.00922288 valid).
  => See Bollerslev and Mikkelsen (1996) for more details.

Estimated Parameters Vector :
```

```
 0.000326; 0.066105; 0.405903; 0.144354; 0.532951;-0.118049; 6.480499
Elapsed Time : 1.296 seconds (or 0.0216 minutes).
```

As we can see, now even the Constant in Mean is not statistically significant. The positivity constraint for the FIGARCH is observed. Tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  Q(  5) =  7.49777   [0.1861732]
  Q( 10) =  8.68127   [0.5625958]
  Q( 20) =  17.0125   [0.6521609]
  Q( 50) =  50.7290   [0.4446452]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  3.28603   [0.3495919]
  Q( 10) =  5.16215   [0.7401134]
  Q( 20) =  11.3732   [0.8778553]
  Q( 50) =  37.0798   [0.8734675]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  0.36540  0.71481
Negative Size Bias t-Test         1.48516  0.13750
Positive Size Bias t-Test         0.72424  0.46892
Joint Test for the Three Effects  4.03598  0.25760

---------------
ARCH 1-2 test:    F(2,1223) = 0.080412 [0.9227]
ARCH 1-5 test:    F(5,1217) =  0.66375 [0.6510]
ARCH 1-10 test:   F(10,1207)=  0.53731 [0.8645]
---------------

Joint Statistic of the Nyblom test of stability: 1.06189

Individual Nyblom Statistics:
Cst(M)           0.16557
Cst(V) x 10^4    0.21225
d-Figarch        0.23095
ARCH(Phi1)       0.18315
GARCH(Beta1)     0.25352
Asymmetry        0.11917
Tail             0.06190

Rem: Asymptotic 1% critical value for individual statistics = 0.75.
     Asymptotic 5% critical value for individual statistics = 0.47.
---------------

Adjusted Pearson Chi-square Goodness-of-fit test

# Cells(g)  Statistic      P-Value(g-1)     P-Value(g-k-1)
    40       39.8537         0.431954          0.160327
    50       55.3659         0.247036          0.081012
    60       56.6341         0.563224          0.306239
Rem.: k = 7 = # estimated parameters
```

The only changes that happened consist in the fact that now according to Pearson's Test the distribution is good; however, this had a "price" as the joint statistic for Nyblom has risen.

Now, let's try to make a FIGARCH-CHUNG(1,1) with Student T distribution:

```
******************************
 ** G@RCH(24) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-05-05 - 2020-04-29
The dependent variable is: returns
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  FIGARCH (1, d, 1) model estimated with Chung's method.
No regressor in the conditional variance
Student distribution, with 6.01405 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 3821.75
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000643 0.00028147    2.284  0.0225
Cst(V) x 10^4        1.934430    0.74710    2.589  0.0097
d-Figarch            0.408519   0.091635    4.458  0.0000
ARCH(Phi1)           0.116459    0.11375    1.024  0.3061
GARCH(Beta1)         0.511604    0.16204    3.157  0.0016
Student(DF)          6.014052    0.96582    6.227  0.0000

No. Observations :      1230  No. Parameters  :         6
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38961  Kurtosis (Y)    :   5.38423
Log Likelihood   :  3821.754

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the FIGARCH (1,d,1) is
not observed.
  => See Chung (1999), Appendix A, for more details.

Estimated Parameters Vector :
 0.000643; 1.934430; 0.408519; 0.116459; 0.511604; 6.014057
Elapsed Time : 1.366 seconds (or 0.0227667 minutes).
```

Unfortunately, the **positivity constraint is not observed**, so this model cannot be considered. The same outcome happens with a Skewed Student T distribution.

Up to now, many models have been tried. It is very difficult to find the "perfect" specification for the returns series, however, **some better models have been created**, such as the ARMA(3,3) with GARCH(1,1) with a Skewed Student T distribution, or the IGARCH(1,1) with Skewed Student T distribution.

If we use the Progress tool:

Progress to date

Model	T	p		log-likelihood	SC	HQ	AIC
G@RCH(2)	1230	11	BFGS	3826.9674	-6.1591	-6.1876	-6.2048
G@RCH(3)	1230	12	BFGS	3832.4783	-6.1623	-6.1934	-6.2122
G@RCH(5)	1230	10	BFGS	3826.4156	-6.1640	-6.1899	-6.2056
G@RCH(7)	1230	11	BFGS	3831.5790	-6.1666	-6.1951	-6.2123<
G@RCH(9)	1230	6	BFGS	3824.5773	-6.1841	-6.1997	-6.2091
G@RCH(10)	1230	6	BFGS	3818.6760	-6.1745	-6.1901	-6.1995
G@RCH(14)	1230	4	BFGS	3818.9217	-6.1865	-6.1969	-6.2031
G@RCH(15)	1230	5	BFGS	3823.6594	-6.1884<	-6.2014<	-6.2092
G@RCH(21)	1230	6	BFGS	3821.6134	-6.1793	-6.1949	-6.2042
G@RCH(22)	1230	7	BFGS	3826.2239	-6.1810	-6.1992	-6.2101

Just to refresh, this is the list of models corresponding to those shown in the tool:

- ARMA(3,3) with GARCH(1,1) with a Student T distribution;
- ARMA(3,3) with GARCH(1,1) with a Skewed Student T distribution;
- ARMA(3,3) with IGARCH(1,1) with a Student T distribution;
- ARMA(3,3) with IGARCH(1,1) with a Skewed Student T distribution;
- GARCH(1,1) with a Student T distribution;
- GARCH(2,1) with a Student T distribution;
- IGARCH(1,1) with a Student T distribution;
- IGARCH(1,1) with a Skewed Student T distribution;
- FIGARCH-BBM(1,1) with a Student T distribution;
- FIGARCH-BBM(1,1) with a Skewed Student T distribution.

If we look for the **highest log-likelihood**, the best model is the **ARMA(3,3) with GARCH(1,1) with a Skewed Student T distribution**. Instead, according to the Schwarz Criterion, this model is almost the worst; this is strongly related with the high number of parameters used in the model (12 parameters). Also, according to Akaike Information Criterion, this model is the second best one.
When looking for the **lowest SC**, we can see that **IGARCH(1,1) with a Skewed Student T distribution** is the best one; this information is strongly influenced by the low number of parameters used in this model (5 parameters). This model holds the **lowest HQ** value too.
When looking for the **lowest AIC**, the best model is the **ARMA(3,3) with IGARCH(1,1) with a Skewed Student T distribution**. This model also holds the second-highest log-likelihood, and in general it is easy to notice that the 4 models where ARMA process is included hold the 4 highest likelihoods and the 4 worst Schwarz Criteria, due to the high amount of parameters estimated.
Another interesting observation is that the Skewed Student T distribution is the dominant one.

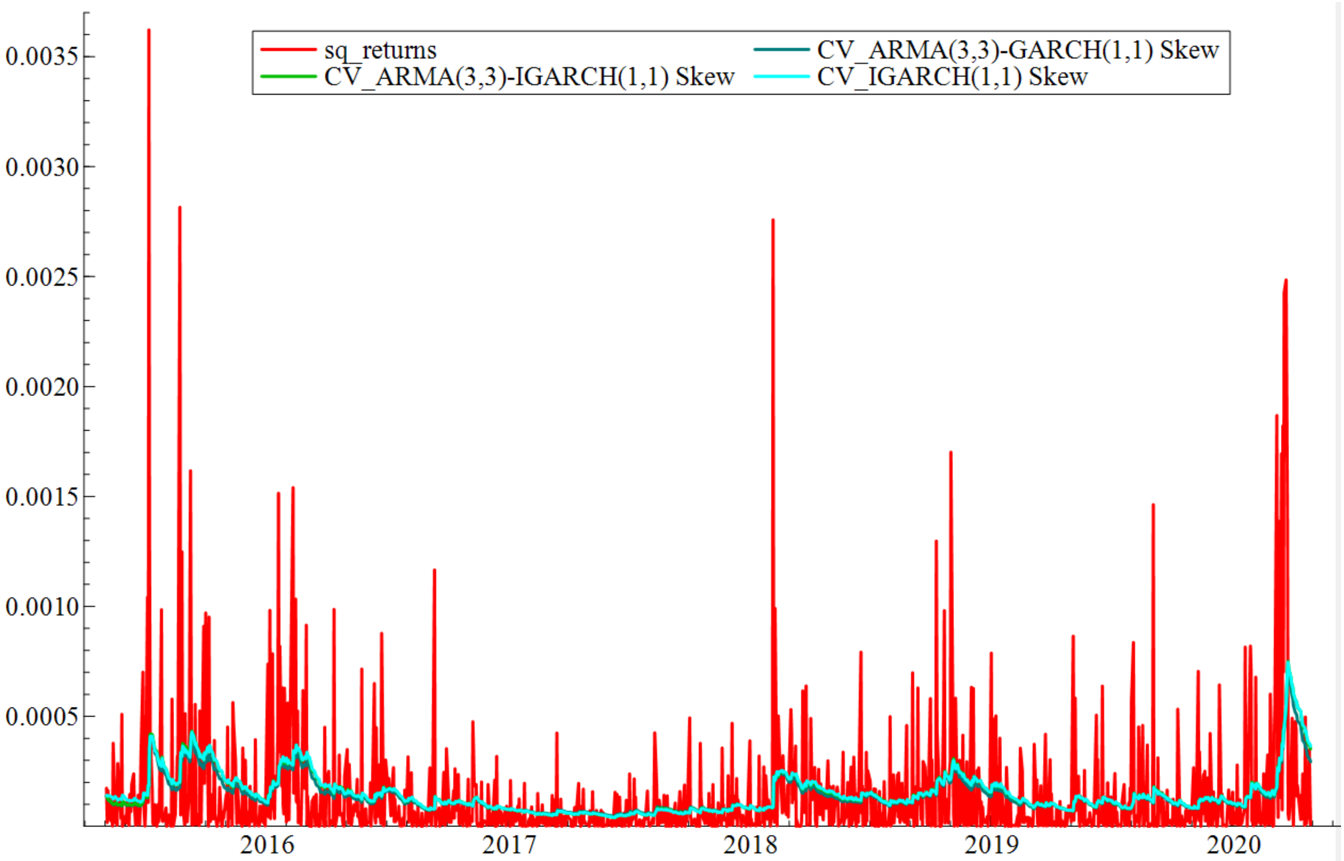
These models had different test outcomes, so I created a table to better understand if we can decide for a best model:

Specification	Autocorrelation in residuals and squared residuals	Leverage Effect	ARCH effect	Nyblom joint statistic	Goodness of fit
ARMA(3,3) with GARCH(1,1) with a Skewed Student T distribution	No	No	No	2.15439	Acceptable at 40, 50 and 60 cells
ARMA(3,3) with IGARCH(1,1) with a Skewed Student T distribution	No	No	No	1.97172	Acceptable at 40, 50 and 60 cells
IGARCH(1,1) with a Skewed Student T distribution	No	No	No	0.607065	Acceptable at 40 and 60 cells

It is difficult to decide upon the best model, as the models with ARMA have very high Nyblom joint statistic , but the goodness of fit of the distribution is ideal, while IGARCH(1,1) has a very low Nyblom joint statistic and not ideally fitted distribution.
Perhaps, **ARMA(3,3) with IGARCH(1,1) with a Skewed Student T distribution could be the best one**, since it has the best AIC, and among the processes with ARMA it has the second best SC, HQ and log-likelihood. Between the two best models with ARMA specification, this one has the

lowest value of Nyblom joint statistic (hence, it is a little more stable) and the empirical distribution follows the theoretical one.

We can plot the Conditional Variance from these models onto the squared returns, just to see how the estimated Conditional Variances follow almost the same path:



I decided to plot only the 3 better models because plotting 10 Conditional Variances on the same graph would not look clear.