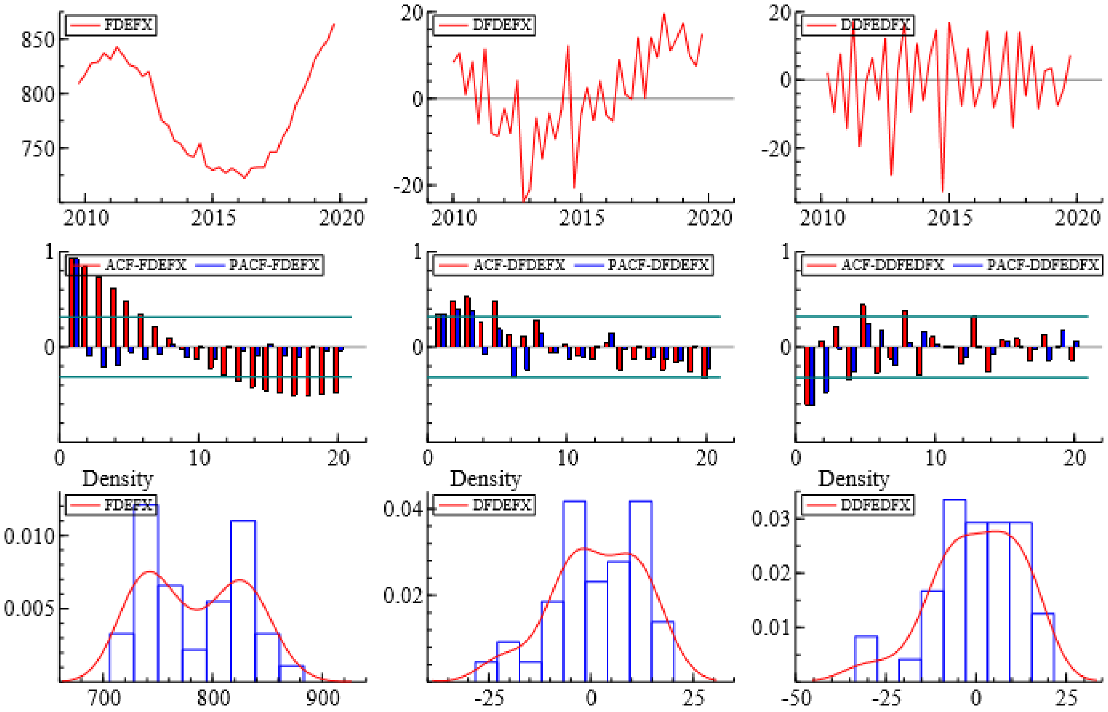
Modelling time series with AR, MA and ARMA models

Daniele Melotti

We are going to work with data related to National Defence excel spreadsheet. Before starting the work on OxMetrics, we can perform the first differences (DFDEFX) and the second differences (DDFDEFX). Then, we can upload the excel file to the console.

At first, we draw the plots for the data that we just elaborated, and we obtain:



The first row represents the Actual series, while the second one represents time series properties with ACF and PACF and the third one represents the distributions.
The results of descriptive statistics for mean, standard deviations and correlations are presented as well as ACF and Portmanteau statistics:

```
The sample is: 2010(2) - 2019(4) (39 observations and 3 variables)
Means
        FDEFX       DFDEFX      DDFEDFX
       781.99       1.1952      0.16582
Standard deviations (using T-1)
        FDEFX       DFDEFX      DDFEDFX
       45.152       10.853       12.189
Correlation matrix:
                    FDEFX       DFDEFX      DDFEDFX
FDEFX              1.0000      0.26892    -0.034008
DFDEFX            0.26892       1.0000      0.57505
DDFEDFX         -0.034008      0.57505       1.0000

DESCRIPTIVE STATISTICS FOR AUTOCORRELATION AND PORTMANTEAU
Autocorrelations (ACF) and Portmanteau statistic
The dataset is:
C:\Users\danie\Desktop\National_defense_consumption_expenditures_and_investment.xlsx
```

```
The sample is: 2011(2) - 2019(4) (39 observations and 3 variables)

FDEFX       : Sample Autocorrelation function (ACF) from lag 1 to 5:
      0.91424      0.82391      0.70979      0.57903      0.44539
Sample Partial autocorrelation function (PACF):
      0.91424    -0.072720     -0.19444     -0.16854    -0.085866
Portmanteau( 5):  Chi^2(5)  =   106.09 [0.0000]**

DFDEFX      : Sample Autocorrelation function (ACF) from lag 1 to 5:
      0.33917      0.47943      0.51660      0.28048      0.46361
Sample Partial autocorrelation function (PACF):
      0.33917      0.41176      0.38274    -0.050675      0.15509
Portmanteau( 5):  Chi^2(5)  =   38.362 [0.0000]**

DDFEDFX     : Sample Autocorrelation function (ACF) from lag 1 to 5:
     -0.60520     0.066171      0.21670     -0.34655      0.43725
Sample Partial autocorrelation function (PACF):
     -0.60520     -0.47355    -0.022134     -0.26818      0.24834
Portmanteau( 5):  Chi^2(5)  =   30.596 [0.0000]**
```

When modelling, we use DDFEDFX series only. The models are AR(1), AR(2), MA(1), MA(2), ARMA(3,1), ARMA(2,3), ARMA(2,1). Each of them is presented with graph analysis, test summary and forecast for 4 periods.

AR(1) model

```
                  Coefficient  Std.Error  t-value  t-prob
AR-1                -0.588842     0.1362    -4.32   0.000
Constant             0.106166      1.079   0.0984   0.922

log-likelihood    -125.430623
no. of observations        35  no. of parameters           3
AIC.T              256.861246  AIC                7.33889275
mean(DDFEDFX)        0.155486  var(DDFEDFX)          157.728
sigma                  10.037  sigma^2               100.741

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     -0.61611      0.15549
```

<u>Test summary:</u>

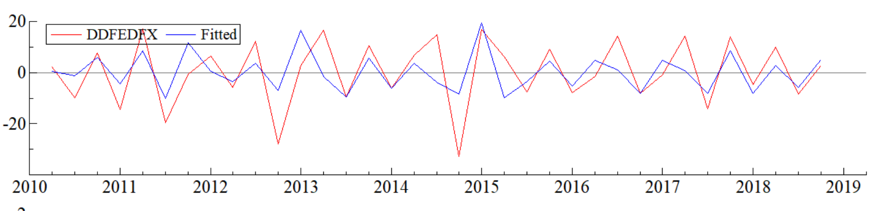
```
Descriptive statistics for residuals:
Normality test:   Chi^2(2)  =  0.94384 [0.6238]
ARCH 1-1 test:    F(1,31)   =  0.93389 [0.3413]
Portmanteau(12):  Chi^2(11) =   28.797 [0.0024]**
```

Observing the statistics, we can notice that:

- In normality test the residuals are normal p-value greater than a significance level (the distribution is still normal);
- In ARCH 1-1 test there is no ARCH effect as the p-value is greater than the significance level (0,05);
- However, Portmanteau statistics tells us that there is autocorrelation because the p-value is lesser than the significance level.

After these considerations, we can say that this model doesn't satisfy the 3 criteria, hence is not efficient.

<u>Graph analysis:</u>



Forecast:


```
Forecasts from 2019(1)
Horizon     Forecast      (SE)       Actual       Error  Naive forc      (SE)
      1      -1.4330     10.04       3.5040      4.9370     -1.4330     10.04
      2       1.0125     11.65      -7.4440     -8.4565      1.0125     11.65
      3     -0.42751     12.16      -2.2780     -1.8505    -0.42751     12.16
      4      0.42041     12.33       7.2430      6.8226     0.42041     12.33
   mean(Error) =       0.36315     RMSE =       6.0386
   SD(Error)   =        6.0276     MAPE =       808.86
```

AR(2) model

```
                  Coefficient  Std.Error  t-value  t-prob
AR-1                -0.869710     0.1547    -5.62   0.000
AR-2                -0.440124     0.1537    -2.86   0.007
Constant             0.183336     0.6617    0.277   0.784

log-likelihood     -121.96532
no. of observations        35  no. of parameters           4
AIC.T              251.930641  AIC                7.19801831
mean(DDFEDFX)        0.155486  var(DDFEDFX)          157.728
sigma                 8.84538  sigma^2               78.2408

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     -0.90600      -0.47053      0.15549
```

Test summary:

```
Descriptive statistics for residuals:
Normality test:   Chi^2(2)  =   1.6901 [0.4295]
ARCH 1-1 test:    F(1,30)   =   3.6459 [0.0658]
Portmanteau(12):  Chi^2(10) =   13.101 [0.2181]
```

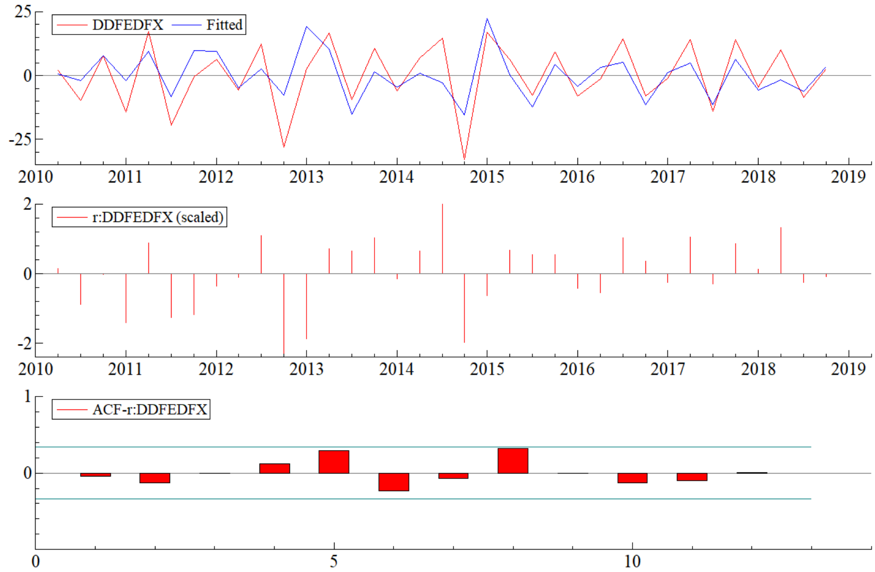
Observing the statistics, we can notice that:

- In normality test the residuals are normal p-value greater than a significance level (the distribution is still normal);
- In ARCH 1-1 test there is no ARCH effect;

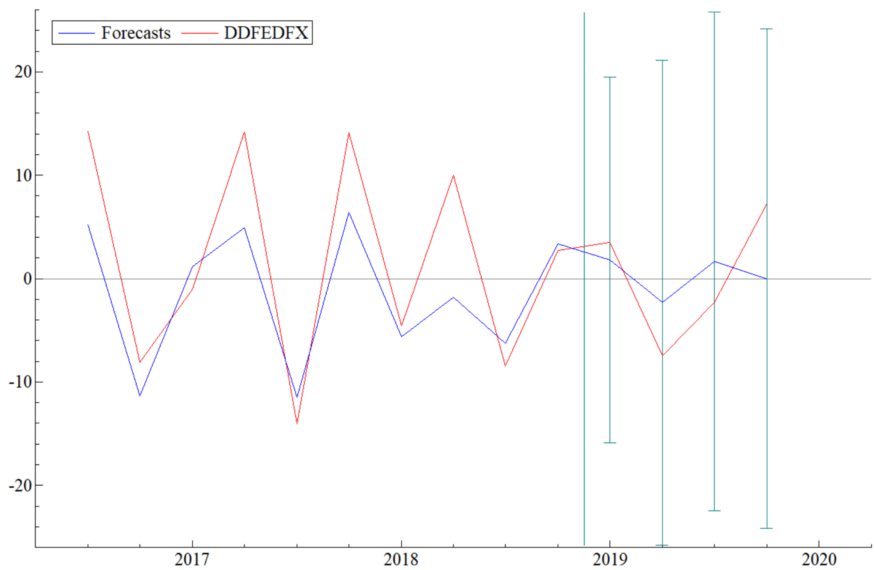
- Portmanteau statistics tells us that there is no autocorrelation because the p-value is greater than the significance level.

After these considerations, we can say that this model satisfies the 3 criteria, hence is efficient.

Graph analysis:



Forecast:



Forecasts from 2019(1)

Horizon	Forecast	(SE)	Actual	Error	Naive forc	(SE)
1	1.7831	8.845	3.5040	1.7209	1.7831	8.845
2	-2.3244	11.72	-7.4440	-5.1196	-2.3244	11.72
3	1.6603	12.05	-2.2780	-3.9383	1.6603	12.05
4	0.0025513	12.09	7.2430	7.2404	0.0025513	12.09

mean(Error) = -0.024120 RMSE = 4.9271

SD(Error) = 4.9271 MAPE = 71088.

MA(1) model

```
                  Coefficient  Std.Error  t-value  t-prob
MA-1                -0.657523     0.1117    -5.89   0.000
Constant             0.214624     0.5846    0.367   0.716

log-likelihood    -124.413699
no. of observations        35  no. of parameters           3
AIC.T              254.827398  AIC                7.28078279
mean(DDFEDFX)        0.155486  var(DDFEDFX)          157.728
sigma                 9.53427  sigma^2               90.9024

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     -0.22199      0.15549
```

<u>Test summary:</u>

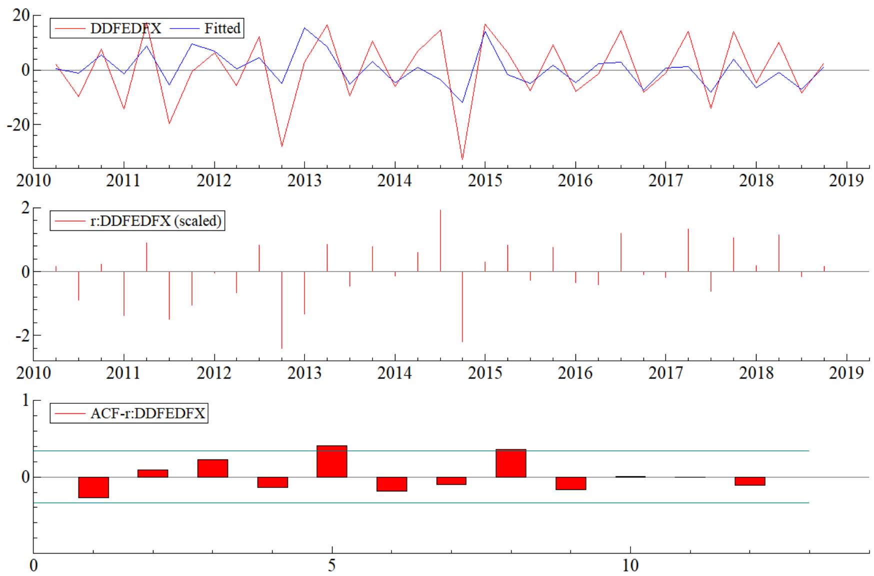
```
Descriptive statistics for residuals:
Normality test:    Chi^2(2)  =    1.5822 [0.4534]
ARCH 1-1 test:     F(1,31)   =  0.74066 [0.3961]
Portmanteau(12):   Chi^2(11) =    22.007 [0.0243]*
```

Observing the statistics, we can notice that:

- In normality test the residuals are normal p-value greater than a significance level (the distribution is still normal);
- In ARCH 1-1 test there is no ARCH effect;
- Portmanteau statistics tells us that there is autocorrelation because the p-value is smaller than the significance level.

After these considerations, we can say that this model doesn't satisfy all the criteria, hence is not efficient.

<u>Graph analysis:</u>



Forecast:


```
Forecasts from 2019(1)
Horizon     Forecast      (SE)       Actual        Error  Naive forc       (SE)
      1     -0.81072     9.534       3.5040       4.3147    -0.81072      9.534
      2      0.21462     11.41      -7.4440      -7.6586     0.21462      11.41
      3      0.21462     11.41      -2.2780      -2.4926     0.21462      11.41
      4      0.21462     11.41       7.2430       7.0284     0.21462      11.41
   mean(Error)  =       0.29796     RMSE =       5.7637
   SD(Error)    =        5.7560     MAPE =       2134.2
```

MA(2) model

```
                  Coefficient  Std.Error  t-value  t-prob
MA-1                -0.974687     0.1752    -5.56   0.000
MA-2                 0.426809     0.1581     2.70   0.011
Constant             0.169721     0.6776    0.250   0.804

log-likelihood    -121.847556
no. of observations        35  no. of parameters          4
AIC.T              251.695111  AIC               7.19128889
mean(DDFEDFX)        0.155486  var(DDFEDFX)         157.728
sigma                 8.80185  sigma^2              77.4726

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     -0.86818      0.035112      0.15549
```

Test summary:

```
Descriptive statistics for residuals:
Normality test:    Chi^2(2)  =    1.2890 [0.5249]
ARCH 1-1 test:     F(1,30)   =    3.2510 [0.0814]
Portmanteau(12):   Chi^2(10) =    11.719 [0.3043]
```

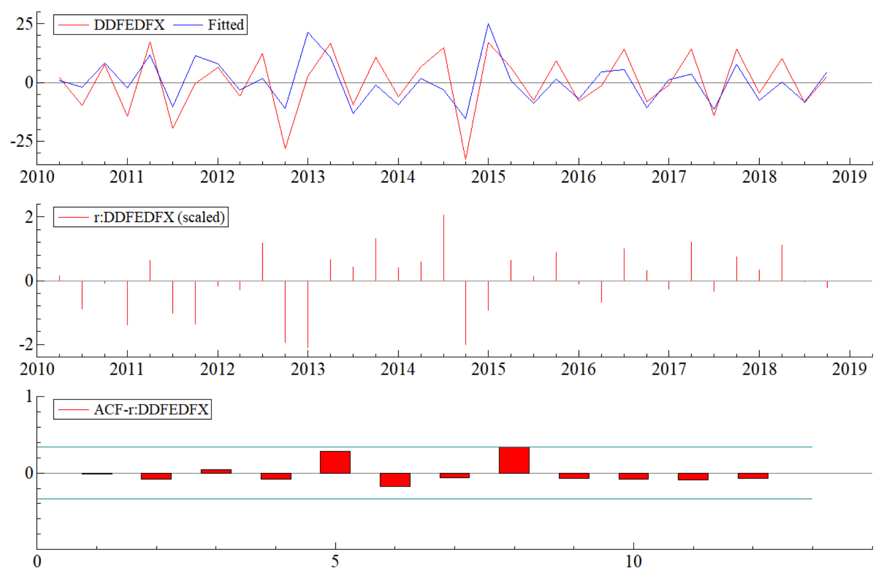
Observing the statistics, we can notice that:

- In normality test the residuals are normal p-value greater than a significance level (the distribution is still normal);

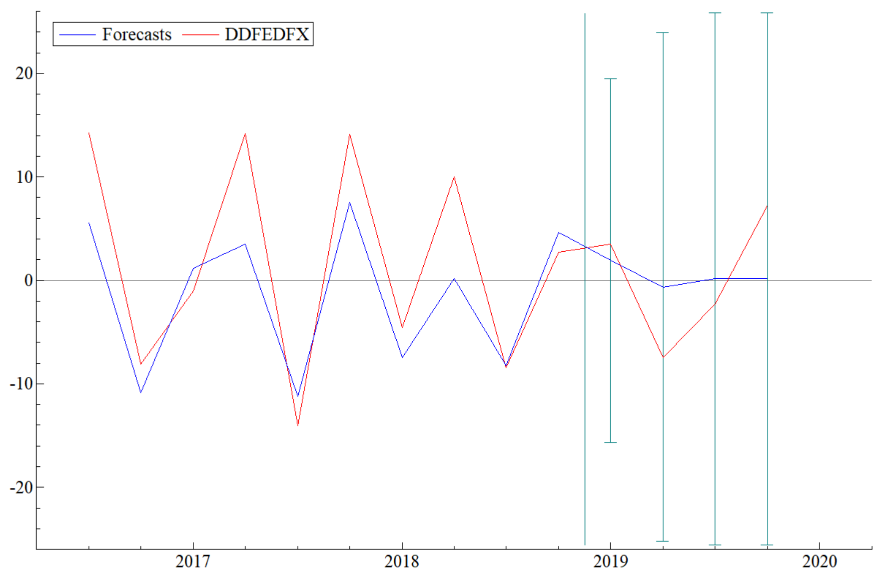
- In ARCH 1-1 test there is no ARCH effect, as the p-value is greater than the significance level;
- Portmanteau statistics tells us that there is no autocorrelation because the p-value is greater than the significance level.

After these considerations, we can say that this model satisfies the 3 criteria, hence is efficient.

Graph analysis:



Forecast:




```
Forecasts from 2019(1)
Horizon     Forecast      (SE)      Actual       Error  Naive forc      (SE)
      1       1.9194     8.802      3.5040      1.5846      1.9194     8.802
      2     -0.64269     12.29     -7.4440     -6.8013    -0.64269     12.29
      3      0.16972     12.85     -2.2780     -2.4477     0.16972     12.85
      4      0.16972     12.85      7.2430      7.0733     0.16972     12.85
   mean(Error) =      -0.14780    RMSE =       5.1184
```

```
SD(Error)    =       5.1162     MAPE =       1687.7
```

ARMA(3,1) model

```
                  Coefficient  Std.Error  t-value  t-prob
AR-1                 -1.66316     0.1801    -9.24   0.000
AR-2                 -1.06250     0.2916    -3.64   0.001
AR-3                -0.219501     0.1884    -1.17   0.253
MA-1                 0.999975     0.1155     8.66   0.000
Constant             0.246054     0.7188    0.342   0.735

log-likelihood    -120.669042
no. of observations         35  no. of parameters            6
AIC.T               253.338083  AIC                 7.23823095
mean(DDFEDFX)         0.155486  var(DDFEDFX)           157.728
sigma                  8.23766  sigma^2                 67.859

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     -0.91562     -0.48904    -0.020438    -0.0018167      0.15549
```

<u>Test summary:</u>

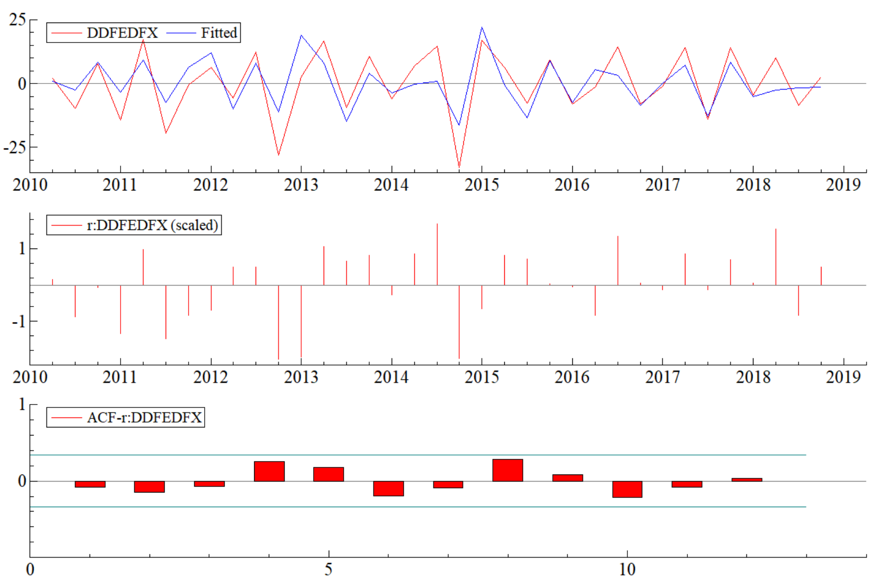
```
Descriptive statistics for residuals:
Normality test:    Chi^2(2)   =    2.4011 [0.3010]
ARCH 1-1 test:     F(1,28)    =    2.5846 [0.1191]
Portmanteau(12):   Chi^2(8)   =    13.564 [0.0939]
```

Observing the statistics, we can notice that:

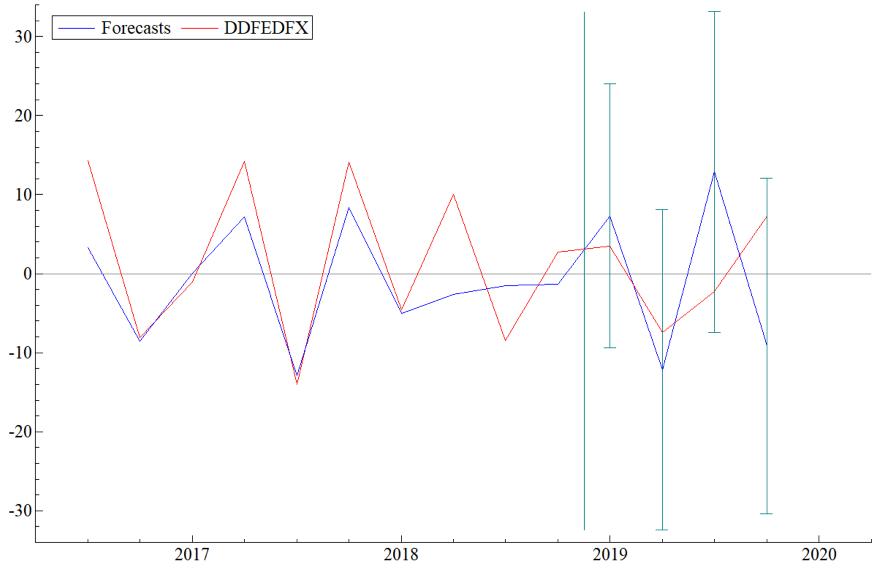
- In normality test the residuals are normal p-value greater than a significance level (the distribution is still normal);
- In ARCH 1-1 test there is no ARCH effect, as the p-value is greater than the significance level;
- Portmanteau statistics tells us that there is no autocorrelation because the p-value is greater than the significance level.

After these considerations, we can say that this model satisfies the 3 criteria, hence is efficient.

<u>Graph analysis:</u>



Forecast:




```
Forecasts from 2019(1)
Horizon     Forecast      (SE)       Actual        Error  Naive forc      (SE)
      1       7.2814     8.345       3.5040      -3.7774      13.691     8.238
      2      -12.172     10.13      -7.4440       4.7276     -22.831     9.885
      3       12.881     10.15      -2.2780      -15.159      23.799     9.890
      4      -9.1176     10.61       7.2430       16.361     -17.358     10.47
   mean(Error)  =      0.53807     RMSE  =      11.555
   SD(Error)    =       11.542     MAPE  =      96.961
```

ARMA(2,3) model

```
                  Coefficient  Std.Error  t-value  t-prob
AR-1                -0.922807     0.5009    -1.84   0.076
AR-2                -0.221528     0.4829   -0.459   0.650
MA-1                 0.127295     0.5352    0.238   0.814
MA-2                -0.487177     0.2275    -2.14   0.041
MA-3                 0.385490     0.3803     1.01   0.319
Constant             0.238968     0.6664    0.359   0.723

log-likelihood     -120.322372
no. of observations         35  no. of parameters           7
AIC.T               254.644744  AIC                7.27556411
mean(DDFEDFX)         0.155486  var(DDFEDFX)          157.728
sigma                  8.16495  sigma^2               66.6664

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     -0.90600     -0.47053    -0.0049574     -0.083965    0.0071368      0.15549
```

Test summary:

```
Descriptive statistics for residuals:
Normality test:    Chi^2(2)   =    1.9351 [0.3800]
ARCH 1-1 test:     F(1,27)    =   0.77282 [0.3871]
Portmanteau(12):   Chi^2(7)   =    6.9112 [0.4382]
```

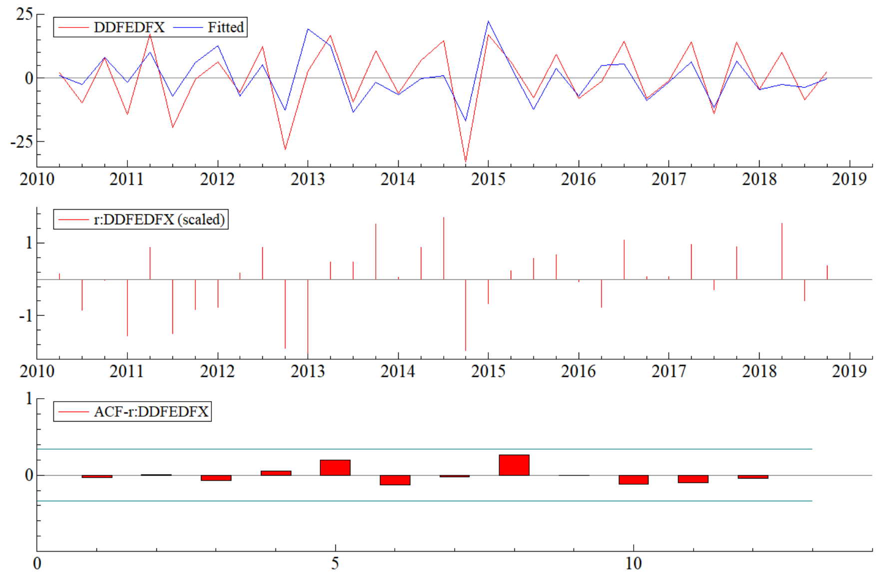
Observing the statistics, we can notice that:

- In normality test the residuals are normal p-value greater than a significance level (the distribution is still normal);

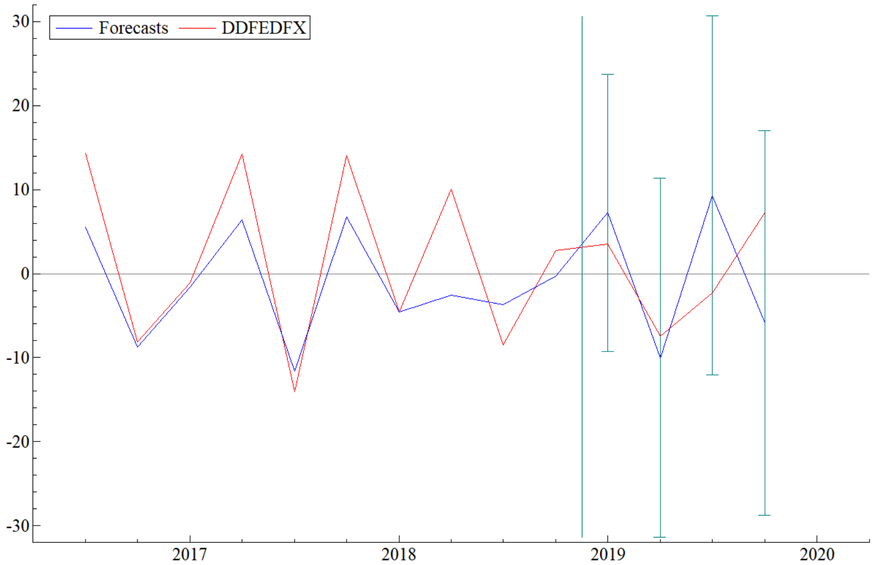
- In ARCH 1-1 test there is no ARCH effect, as the p-value is greater than the significance level;
- Portmanteau statistics tells us that there is no autocorrelation because the p-value is greater than the significance level.

After these considerations, we can say that this model satisfies the 3 criteria, hence is efficient.

Graph analysis:



Forecast:



Forecasts from 2019(1)

Horizon	Forecast	(SE)	Actual	Error	Naive forc	(SE)
1	7.2415	8.265	3.5040	-3.7375	11.972	8.165
2	-10.014	10.68	-7.4440	2.5699	-18.507	10.43
3	9.3005	10.69	-2.2780	-11.578	17.913	10.44
4	-5.8518	11.44	7.2430	13.095	-11.918	11.32

mean(Error) = 0.087171 RMSE = 9.0292
SD(Error) = 9.0288 MAPE = 106.39

ARMA(2,1) model

```
                  Coefficient  Std.Error  t-value  t-prob
AR-1                 -1.30973     0.2629    -4.98   0.000
AR-2                -0.679103     0.1582    -4.29   0.000
MA-1                 0.608055     0.3656     1.66   0.107
Constant             0.211642     0.8094    0.261   0.796

log-likelihood     -121.561472
no. of observations         35  no. of parameters           5
AIC.T               253.122945  AIC                7.23208414
mean(DDFEDFX)         0.155486  var(DDFEDFX)          157.728
sigma                   8.7693  sigma^2               76.9007

BFGS using numerical derivatives (eps1=0.0001; eps2=0.005):
Strong convergence
Used starting values:
     -0.87218     -0.44969    -0.018046      0.15549
```

<u>Test summary:</u>

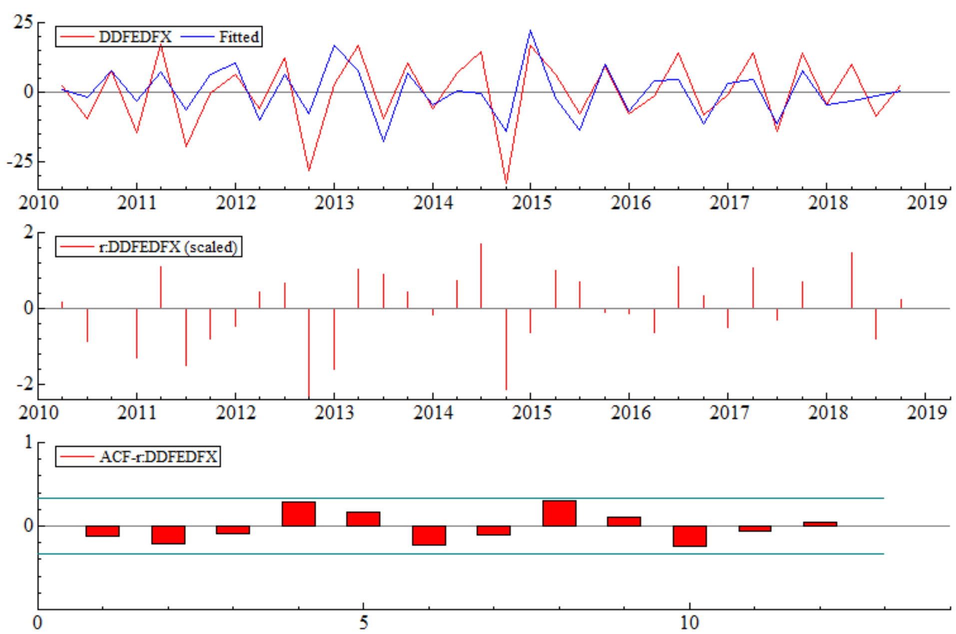
```
Descriptive statistics for residuals:
Normality test:   Chi^2(2)  =   2.1796 [0.3363]
ARCH 1-1 test:    F(1,29)   =   1.7397 [0.1975]
Portmanteau(12):  Chi^2(9)  =   17.222 [0.0454]*
```

Observing the statistics, we can notice that:

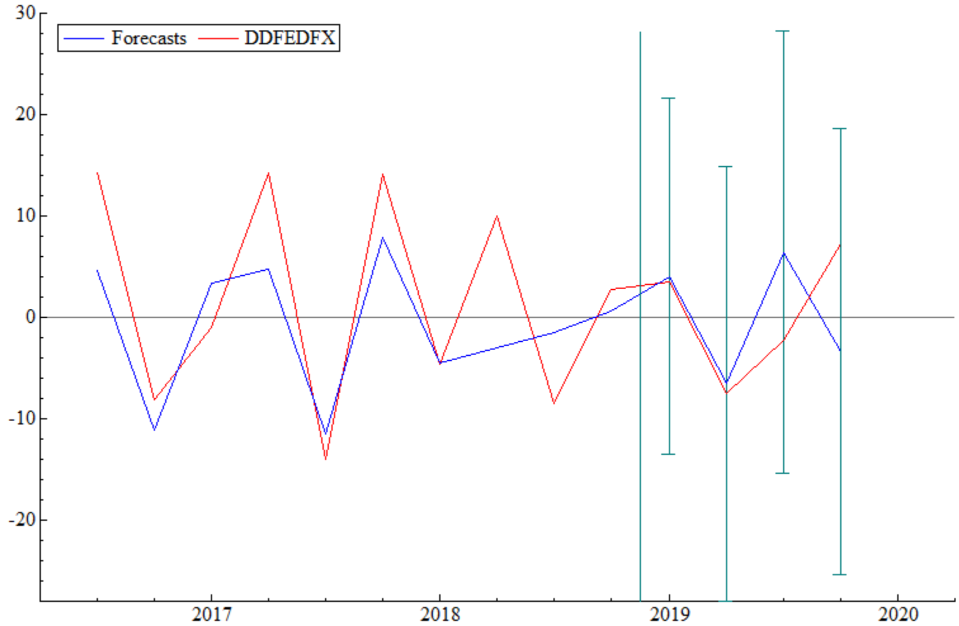
- In normality test the residuals are normal p-value greater than a significance level (the distribution is still normal);
- In ARCH 1-1 test there is no ARCH effect, as the p-value is greater than the significance level;
- Portmanteau statistics tells us that there is autocorrelation because the p-value is smaller than the significance level.

After these considerations, we can say that this model does not satisfy the 3 criteria, hence is not efficient.

<u>Graph Analysis:</u>



Forecast:



Finally, after creating the models, we can try to make a comparison and see which one is looks best. Using Progress tool on OxMetrics for the three ARMA models we obtain:

```
Progress to date
Model            T    p         log-likelihood          SC          HQ          AIC
Arfima( 1)      35    6  MPL        -120.66904      7.5049      7.3303       7.2382
Arfima( 2)      35    7  MPL        -120.32237      7.5866      7.3829       7.2756
Arfima( 3)      35    5  MPL        -121.56147      7.4543<     7.3088<      7.2321<
```

As we can see, the last model ARMA(2,1) appears to be better as it has lower values of SC, HQ and AIC. However, we must remember that descriptive statistics told us that ARMA(2,1) was not an efficient model as it had a p-value smaller than the significance level as for Portamanteau statistic. So, the next "best" model would be ARMA(3,1), which is the first one from the Progress tool's outcome.
Then we could look at the log-likelihood of models that were considered efficient:

```
log-likelihood    -121.96532  (AR(2))
log-likelihood    -121.847556 (MA(2))
log-likelihood    -120.669042 (ARMA(3,1))
log-likelihood    -120.322372 (ARMA(2,3))
```

The greatest log-likelihood belongs to the ARMA(2,3) model, however it is not considered because of the insignificance of the parameters.