MGARCH

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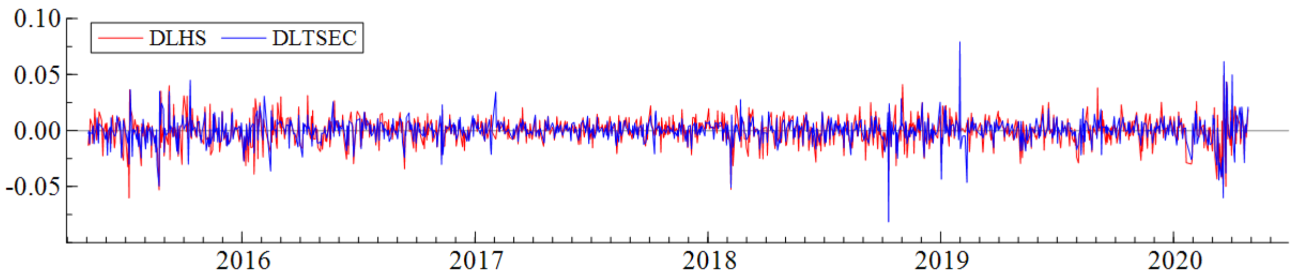
The goal of this project is to explore the correlation between chosen series and the possibility of constructing MGARCH specifications. The data that will be used corresponds to the closing prices from Hang Seng Index (HSI) from Hong Kong, which was previously used in the GARCH models' assignment. The additional series comes from the closing prices for Taiwan Capitalization Weighted Stock Index (TSEC a.k.a. TAIEX) from Taiwan. The timeframe equals approximately to 5 years of trading data (from 04.05.2015 to 27.04.2020).

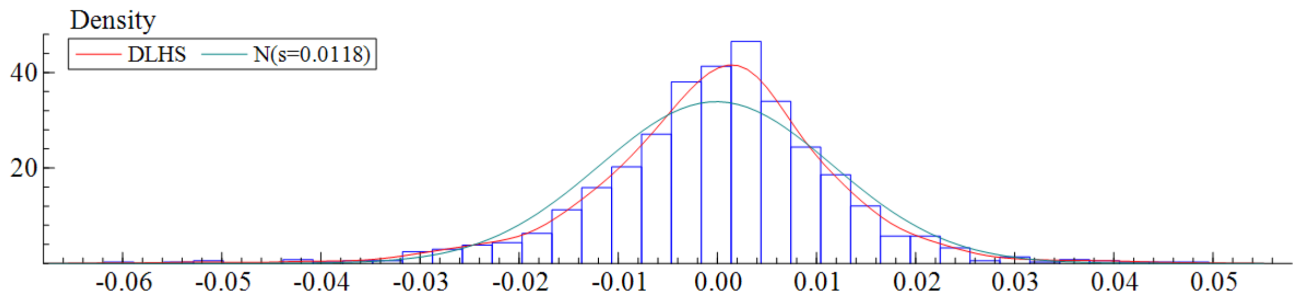
The first thing to do is to calculate returns using the Calculator tool (DLHS and DLTSEC). After that is done, we can plot Actual series for these returns:



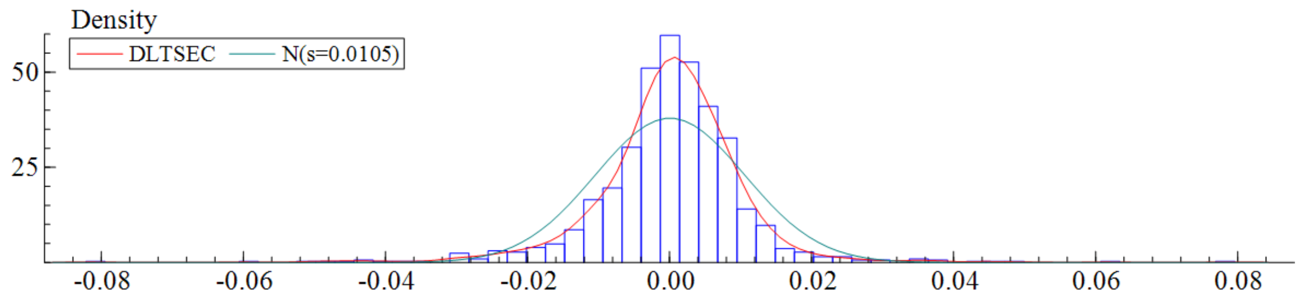
Here we can see that there are a few differences between the two series. The **returns from TSEC present more significantly big outliers**, while returns from HSI seem to be a little less volatile. Before starting to estimate with MGARCH specifications directly, we might be able to use a VAR specification and store the residuals.
Thanks to Descriptive Statistics using PcGive we can verify what is the correlation between the series:

```
Means, standard deviations and correlations
The sample is: 2015-05-05 - 2020-04-27 (1212 observations and 2 variables)
Means
          DLHS        DLTSEC
  -0.00012125   5.8411e-05
Standard deviations (using T-1)
          DLHS        DLTSEC
     0.011766     0.010526
Correlation matrix:
                     DLHS        DLTSEC
DLHS               1.0000       0.62201
DLTSEC            0.62201        1.0000
```

So, there is a correlation equal to 0.62201 between the two series. It is something that **lays between moderate and strong correlation**. I have tried to research for more correlated series, however, TSEC was just the nicest one from this point of view.
Now, we could plot the distributions, starting with the one from DLHS:



As we can see, there is a little asymmetry represented by negative skewness and some excess kurtosis. **The distribution won't be normal probably**, but we might check it.
If we look at the distribution from DLTSEC:

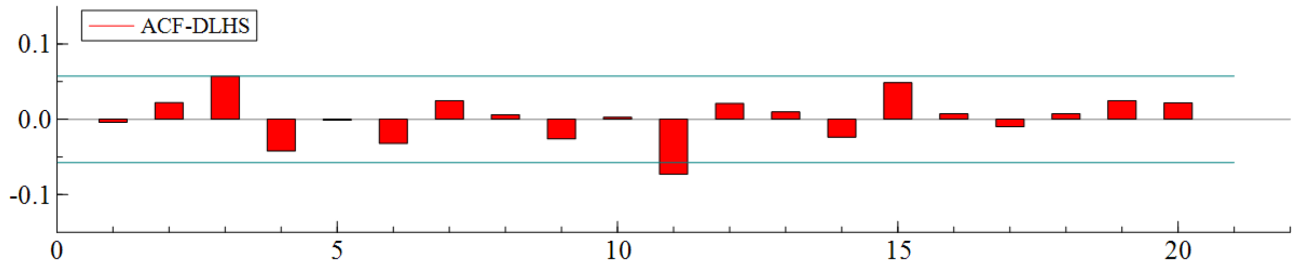


We can see that this distribution looks rather symmetric, but the excess kurtosis is even more than the one from DLHS, so **this distribution is not normal either**, perhaps. So, it would be better to make a Normality Test through Descriptive Statistics using G@RCH:

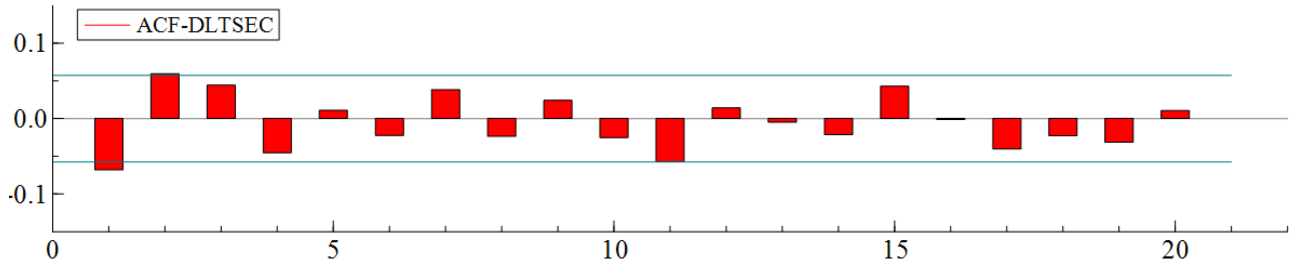
```
Series #1/2: DL_HS
---------

Normality Test

                   Statistic       t-Test       P-Value
Skewness            -0.38211       5.4375    5.4048e-08
Excess Kurtosis       2.3806       16.952    1.8606e-64
Jarque-Bera           315.68         .NaN    2.8188e-69
---------------

Series #2/2: DL_TSEC
---------

Normality Test

                   Statistic       t-Test       P-Value
Skewness            -0.35182       5.0065    5.5427e-07
Excess Kurtosis       9.9511       70.861       0.00000
Jarque-Bera           5025.7         .NaN       0.00000
---------------
```

And we can notice that for both series there is negative skewness and in general all P-values are statistically significant, hence, **<u>we can confirm the non-normality of the distribution</u>**.

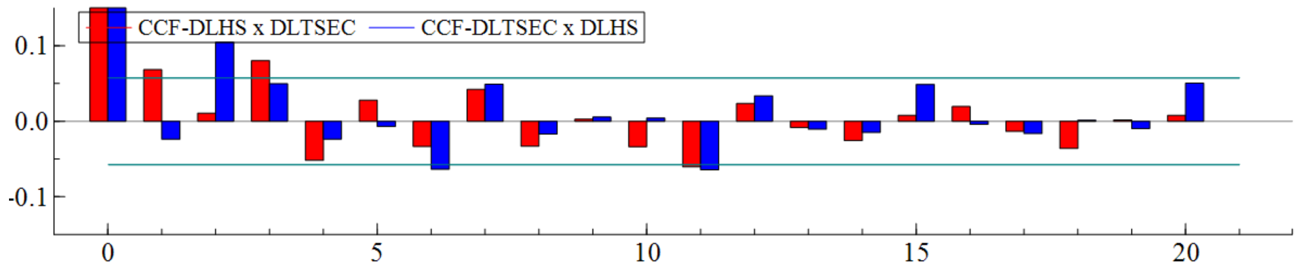
Now, we can plot the ACF graphs and the CCF. Let's start with the ACF for DLHS:



We can see that for DL_HS only the 3rd and the 11th lags are statistically significant (with the 3rd being on the edge). If we look at the ACF for DL_TSEC:



We can see that in these returns there are 1st, 2nd and 11th lags which are statistically significant (with both the 2nd and the 11th being on the edge). And when we look at the Cross-Correlation Function graph:



We can see **that there is only initially high correlation at lag 0**, but there is nothing such as a Cross-Correlation between Hang Seng at time t and TSEC at time $(t-1)$. **Using a VAR specification could be troublesome, <u>as it seems that only at 11 lags this is advised, but so many lags would probably over parametrise the model</u>**.

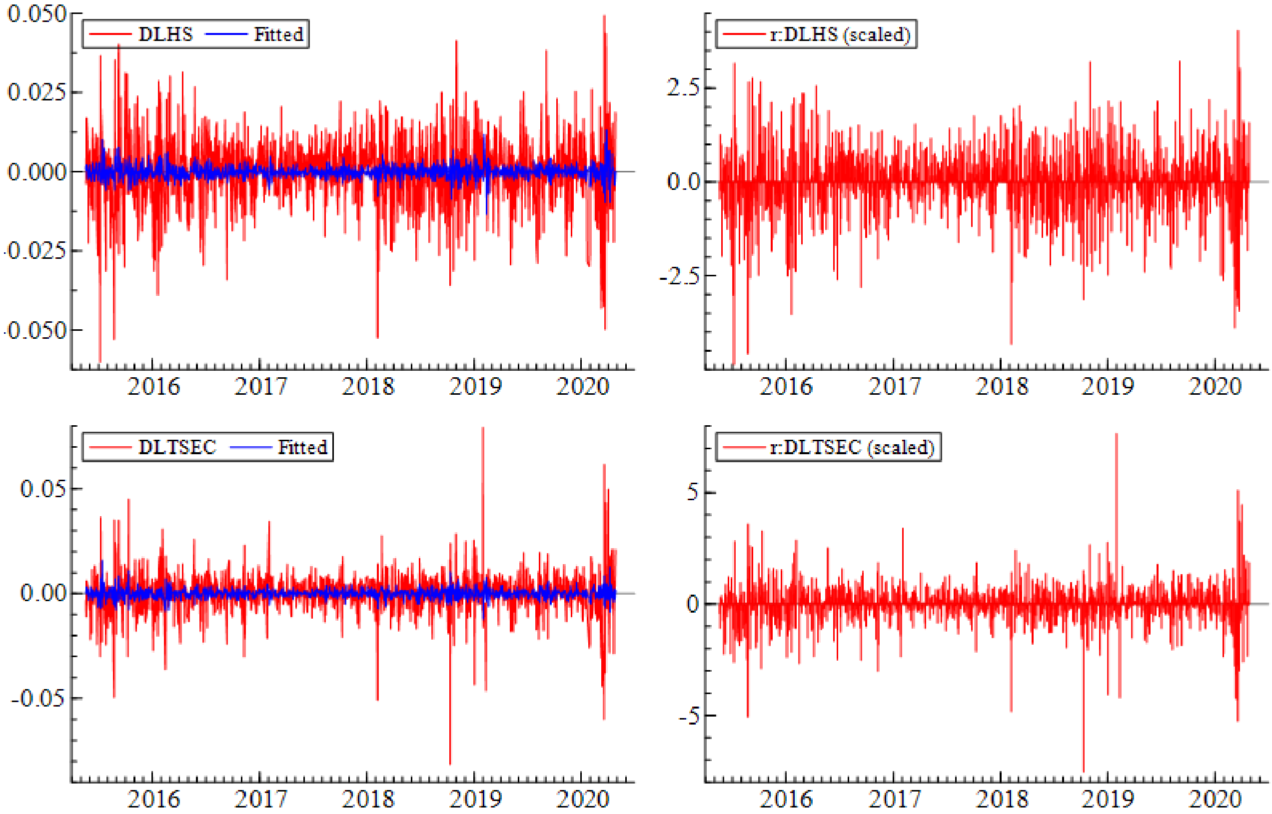
However, we can try VAR(11), selecting DLHS and DLTSEC series and choosing to estimate for the whole sample:

```
SYS( 1) Estimating the system by OLS
        The estimation sample is: 2015-05-20 - 2020-04-27

URF equation for: DLHS
                  Coefficient  Std.Error  t-value  t-prob
DLHS_1             0.00958907    0.03737    0.257  0.7975
DLHS_2             -0.0680972    0.03794    -1.79  0.0730
DLHS_3              0.0241719    0.03791    0.638  0.5239
DLHS_4             -0.0482851    0.03796    -1.27  0.2037
DLHS_5             -0.0119068    0.03797   -0.314  0.7539
DLHS_6             0.00791924    0.03788    0.209  0.8344
DLHS_7            -0.00447376    0.03788   -0.118  0.9060
DLHS_8              0.0213025    0.03784    0.563  0.5736
```

```
DLHS_9             -0.0433627    0.03789    -1.14  0.2527
DLHS_10           -0.00617524    0.03770   -0.164  0.8699
DLHS_11            -0.0535739    0.03752    -1.43  0.1535
DLTSEC_1           -0.0187967    0.04206   -0.447  0.6550
DLTSEC_2             0.165364    0.04256     3.89  0.0001
DLTSEC_3            0.0547088    0.04279     1.28  0.2013
DLTSEC_4            0.0161693    0.04277    0.378  0.7055
DLTSEC_5           -0.0234244    0.04285   -0.547  0.5847
DLTSEC_6           -0.0621238    0.04282    -1.45  0.1471
DLTSEC_7            0.0513268    0.04291     1.20  0.2318
DLTSEC_8           -0.0150735    0.04290   -0.351  0.7254
DLTSEC_9            0.0375713    0.04290    0.876  0.3813
DLTSEC_10          0.00381439    0.04296   0.0888  0.9293
DLTSEC_11          -0.0319038    0.04220   -0.756  0.4498
Constant      U  -0.000140556  0.0003383   -0.415  0.6779

sigma = 0.0116928   RSS = 0.1610581162

URF equation for: DLTSEC
                  Coefficient  Std.Error  t-value  t-prob
DLHS_1               0.165676    0.03321     4.99  0.0000
DLHS_2             -0.0101913    0.03372   -0.302  0.7625
DLHS_3              0.0777196    0.03370     2.31  0.0213
DLHS_4             -0.0225576    0.03374   -0.669  0.5039
DLHS_5              0.0318609    0.03375    0.944  0.3453
DLHS_6             -0.0290549    0.03367   -0.863  0.3883
DLHS_7              0.0172320    0.03367    0.512  0.6089
DLHS_8             -0.0298448    0.03363   -0.887  0.3750
DLHS_9             -0.0237009    0.03368   -0.704  0.4817
DLHS_10            -0.0331878    0.03350   -0.991  0.3221
DLHS_11            -0.0397879    0.03334    -1.19  0.2330
DLTSEC_1            -0.174175    0.03738    -4.66  0.0000
DLTSEC_2            0.0491359    0.03783     1.30  0.1942
DLTSEC_3          -0.00848798    0.03803   -0.223  0.8234
DLTSEC_4           -0.0382446    0.03801    -1.01  0.3146
DLTSEC_5           -0.0276019    0.03808   -0.725  0.4688
DLTSEC_6           0.00353255    0.03805   0.0928  0.9261
DLTSEC_7            0.0357946    0.03813    0.939  0.3481
DLTSEC_8            0.0103912    0.03813    0.273  0.7853
DLTSEC_9            0.0462622    0.03812     1.21  0.2252
DLTSEC_10        -0.000399080    0.03818  -0.0105  0.9917
DLTSEC_11          -0.0291794    0.03751   -0.778  0.4368
Constant      U   9.01699e-05  0.0003007    0.300  0.7643

sigma = 0.010392   RSS = 0.1272170589

log-likelihood      7742.2996  -T/2log|Omega|       11150.59
|Omega|        8.62291573e-09  log|Y'Y/T|        -18.4815175
R^2(LR)             0.0836208  R^2(LM)             0.0426657
no. of observations      1201  no. of parameters          46

F-test on regressors except unrestricted: F(44,2354) = 2.38769 [0.0000] **
F-tests on retained regressors, F(2,1177) =
      DLHS_1       19.2170 [0.000]**      DLHS_2       2.16636 [0.115]
      DLHS_3       3.19535 [0.041]*       DLHS_4      0.821752 [0.440]
      DLHS_5       1.12004 [0.327]        DLHS_6      0.834981 [0.434]
      DLHS_7      0.289403 [0.749]        DLHS_8       1.42442 [0.241]
      DLHS_9      0.654306 [0.520]       DLHS_10      0.662230 [0.516]
     DLHS_11       1.09196 [0.336]      DLTSEC_1       15.8844 [0.000]**
    DLTSEC_2       8.60470 [0.000]**    DLTSEC_3       1.68046 [0.187]
    DLTSEC_4       1.34298 [0.261]      DLTSEC_5      0.269420 [0.764]
    DLTSEC_6       1.87861 [0.153]      DLTSEC_7      0.744202 [0.475]
    DLTSEC_8      0.261501 [0.770]      DLTSEC_9      0.746573 [0.474]
   DLTSEC_10    0.00753699 [0.992]     DLTSEC_11      0.361583 [0.697]
    Constant U    0.344606 [0.709]

correlation of URF residuals (standard deviations on diagonal)
```

```
                      DLHS        DLTSEC
DLHS              0.011693       0.62687
DLTSEC             0.62687      0.010392
correlation between actual and fitted
             DLHS         DLTSEC
      0.18136        0.22020

Single-equation diagnostics using reduced-form residuals:
DLHS        : Portmanteau(12):  Chi^2(1)   =  0.76678 [0.3812]
DLHS        : AR 1-2 test:      F(2,1176) =  0.87779 [0.4160]
DLHS        : ARCH 1-1 test:    F(1,1199) =   39.384 [0.0000]**
DLHS        : Normality test:   Chi^2(2)   =   103.29 [0.0000]**
DLHS        : Hetero test:      F(44,1156)=   5.1326 [0.0000]**
DLTSEC      : Portmanteau(12):  Chi^2(1)   =  0.25491 [0.6136]
DLTSEC      : AR 1-2 test:      F(2,1176) =  0.67676 [0.5085]
DLTSEC      : ARCH 1-1 test:    F(1,1199) =   51.544 [0.0000]**
DLTSEC      : Normality test:   Chi^2(2)   =   1058.1 [0.0000]**
DLTSEC      : Hetero test:      F(44,1156)=   4.4806 [0.0000]**

Vector Portmanteau(12):  Chi^2(4)   =   1.8582 [0.7618]
Vector AR 1-2 test:      F(8,2346) =  0.55889 [0.8122]
Vector Normality test:   Chi^2(4)   =   1662.8 [0.0000]**
Vector Hetero test:      F(132,3458)=   4.0533 [0.0000]**
Vector RESET23 test:     F(8,2346) =  0.94862 [0.4749]
```

Plenty of parameters are estimated, and we can see that **in the single equation for DLHS only the 2nd lag for DLTSEC is statistically significant**! Instead, **in the single equation for DLTSEC, the 1st and 3rd lags of DLHS are significant and the 1st lag from DLTSEC itself**. When looking at the diagnostics we can see **several issues**; there is ARCH effect, non-normality, and the residuals are heteroscedastic. One positive aspect is that **there is no autocorrelation left**. If we look at the graphical representation for this model:



We can see that logically there is room for improvement, as **the empirical values are very different from the fitted values**. **<u>The whole dynamics of the series are not taken into account</u>**, because only autocorrelation has been considered, but there is more to be captured within the model. **The scaled residuals look quite similar to the series**, **<u>which is not good</u>**, as we would like them to be IID and it is quite clear that there is clustering of volatility. We could store these residuals in the datasheet under the name of resHS and resTSEC.

Now, we can try to build a MGARCH specification modelling on the base of the stored residuals. We can do this through Multivariate GARCH Models using G@RCH. We will start with a CCC (Constant Correlation is assumed) specification using a plain vanilla GARCH(1,1) without ARMA, as we have seen that there is no autocorrelation left in the series. And we can set the Student T as a distribution, since we saw that the series don't follow a Gaussian distribution. Also, we will include the Constants in Mean equation and in the Variance equation as well:

```
  ********************
 **   FIRST STEP   **
*******************

---------------------------Estimating       the       univariate       GARCH       model       for
resHS---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: resHS
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3711.05
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000406 0.00031930    1.272  0.2036
Cst(V) x 10^4        0.013071   0.014724   0.8877  0.3749
ARCH(Alpha1)         0.045533   0.019714    2.310  0.0211
GARCH(Beta1)         0.946032   0.028479    33.22  0.0000

No. Observations :      1201  No. Parameters  :         4
Mean (Y)         :   0.00000  Variance (Y)    :   0.00013
Skewness (Y)     :  -0.41875  Kurtosis (Y)    :   4.99060
Log Likelihood   :  3711.046  Alpha[1]+Beta[1]:   0.99157

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000154968
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.987349 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000406; 0.013071; 0.045533; 0.946032
Elapsed Time : 0.044 seconds (or 0.000733333 minutes).
```

```
---------------------------Estimating the univariate GARCH model for resTSEC---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: resTSEC
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3912.45
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000360 0.00026674    1.350  0.1773
Cst(V) x 10^4        0.127511   0.053277    2.393  0.0168
ARCH(Alpha1)         0.221946   0.077605    2.860  0.0043
GARCH(Beta1)         0.676215   0.080986    8.350  0.0000

No. Observations :      1201  No. Parameters  :         4
Mean (Y)         :  -0.00000  Variance (Y)    :   0.00011
Skewness (Y)     :  -0.40449  Kurtosis (Y)    :  12.32520
Log Likelihood   :  3912.448  Alpha[1]+Beta[1]:   0.89816

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000125209
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.905213 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000360; 0.127511; 0.221946; 0.676215
Elapsed Time : 0.054 seconds (or 0.0009 minutes).


  ********************
 **  SECOND STEP   **
*******************


  *************
 ** SERIES  **
*************
#1: resHS
#2: resTSEC

The estimation sample is:  2015-05-20 - 2020-04-27

 ******************************
 ** MG@RCH(1) SPECIFICATIONS **
 ******************************
Conditional Variance : Constant Correlation Model
Multivariate Student distribution, with 5.37659 degrees of freedom.

Strong convergence using numerical derivatives
```

```
Log-likelihood = 8050.33
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                 Coefficient  Std.Error  t-value  t-prob
rho_21              0.627707   0.018501    33.93  0.0000
df                  5.376593    0.50121    10.73  0.0000
No. Observations :      1201  No. Parameters  :        10
No. Series       :         2  Log Likelihood  :  8050.332
Elapsed Time : 0.031 seconds (or 0.000516667 minutes).
```

Looking at the outcome from the 1st step, **starting with the residuals from Hang Seng**, we see that Alpha1, Beta1 and the Constant are statistically significant; moreover, the positivity constraint is observed as well as the condition for the existence of the fourth moment. **The model is fine**. **Regarding DL_TSEC**, we can see that again Alpha1, Beta1 and the Constant are statistically significant, this time together with the Constant in the Variance equation. Even here the positivity constraint is observed as well as the condition for the existence of the fourth moment. So, **here the model is acceptable too**. Interestingly, despite the fact that the Student T distribution was chosen for the estimation, in this 1st step the normal distribution has been used, as there is no information regarding the degrees of freedom.

Looking at the outcome from the 2nd step, where Constant Correlation is estimated, we see that this **correlation equals to 0.627707**. Let's make some tests for this model, namely the usual Box-Pierces on standardized residuals and on squared standardized residuals, and a new test which would be Hosking' Portmanteau Test on standardized residuals and on squared standardized residuals. Also, we could introduce Constant Correlation Tests of Tse and of Engle and Sheppard. These last 2 tests would allow us to understand if it is reasonable to consider models where it is assumed that Correlation is constant. So, here are the outputs:

```
  ***********
 ** TESTS **
***********
Q-Statistics on Standardized Residuals

 Series: resHS
  Q(  5) =  1.98296   [0.8514983]
  Q( 10) =  4.26708   [0.9344951]
  Q( 20) =  8.21127   [0.9903752]
  Q( 50) =  37.4193   [0.9056085]

 Series: resTSEC
  Q(  5) =  7.93344   [0.1599423]
  Q( 10) =  9.71098   [0.4662058]
  Q( 20) =  13.2554   [0.8661572]
  Q( 50) =  35.3594   [0.9417074]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals

 Series: resHS
  Q(  5) =  6.69269   [0.2445167]
  Q( 10) =  14.1036   [0.1683168]
  Q( 20) =  31.3560   [0.0506621]
  Q( 50) =  50.7541   [0.4436653]

 Series: resTSEC
  Q(  5) = 0.999440   [0.9626109]
  Q( 10) =  1.79524   [0.9976822]
  Q( 20) =  2.19739   [0.9999997]
  Q( 50) =  12.4363   [1.0000000]
```

```
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hosking's Multivariate Portmanteau Statistics on Standardized Residuals
  Hosking(  5) =  14.6786   [0.7944953]
  Hosking( 10) =  22.4475   [0.9886729]
  Hosking( 20) =  48.8394   [0.9976689]
  Hosking( 50) =  150.035   [0.9966260]
---------------

Hosking's Multivariate Portmanteau Statistics on Squared Standardized Residuals
  Hosking(  5) =  10.8440   [0.9008591]
  Hosking( 10) =  39.1790   [0.4167396]
  Hosking( 20) =  74.3196   [0.5970873]
  Hosking( 50) =  135.106   [0.9998006]
Warning: P-values have been corrected by 2 degrees of freedom
---------------
```

As for Box-Pierces tests and for Hosking's, we can see that **there is no autocorrelation at all**, which is very good. We are just **"on the edge" at 20 lags for the Box-Pierce Test on squared standardized residuals for resHS**. Then, looking at the Constant Correlation Tests:

```
LM Test for Constant Correlation of Tse (2000), JoE
---------------------------------------------------
LMC:  21.6838 [0.0000032]
---------------------------------------------------
P-value in brackets. LMC~X²(N*(N-1)/2)) under H0: CCC model, with N=#series
```

The hypotheses for the "Constant Correlation Test of TSE" are:

$$H_0: Constant\ \ Correlation\ is\ true$$

$$p - value > 0.05$$

$$H_1: Constant\ \ Correlation\ is\ not\ true$$

$$p - value < 0.05$$

So, in our case we can see that we must reject the null hypothesis, hence **there is no Constant Correlation**, according to this test.

```
Engle and Sheppard Test for dynamic correlation
-----------------------------------------------
   E-S Test( 5) =  8.94961   [0.1764320]
   E-S Test(10) =  11.5034   [0.4021045]
-----------------------------------------------
P-values in brackets. E-S Test(j)~X²(j+1) under H0: CCC model
```

In the case of "Constant Correlation Test of Engle and Sheppard" we have two different tests for 5 and 10 lags. **The hypotheses are the same**, so according to this test the **Correlation is constant at both number of lags**. Despite this, we will verify other types of models too.

So, we can compare what try to make another CCC model, but we will work on the **returns series** this time. We will keep GARCH(1,1) and attempt an AR(11), so as to make a comparison with our first VAR(11) model. However, **<u>we should take into account that this will hardly be better than our last model</u>**, especially if we consider the number of parameters that we are going to estimate:

```
  ********************
 **   FIRST STEP   **
*******************

---------------------------Estimating the univariate GARCH model for
DLHS---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: DLHS
Mean Equation:  ARMA (11, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3712.3
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000341 0.00031012    1.099  0.2720
AR(1)                0.028295   0.031518   0.8977  0.3695
AR(2)                0.009516   0.031289   0.3041  0.7611
AR(3)                0.070961   0.030980    2.291  0.0222
AR(4)               -0.050264   0.029202   -1.721  0.0855
AR(5)               -0.029808   0.032466  -0.9182  0.3587
AR(6)               -0.015633   0.031069  -0.5032  0.6149
AR(7)               -0.004752   0.032330  -0.1470  0.8832
AR(8)                0.031250   0.029497    1.059  0.2896
AR(9)               -0.004770   0.031864  -0.1497  0.8810
AR(10)              -0.021987   0.028588  -0.7691  0.4420
AR(11)              -0.059244   0.028902   -2.050  0.0406
Cst(V) x 10^4        0.033723   0.053771   0.6272  0.5307
ARCH(Alpha1)         0.069385   0.047672    1.455  0.1458
GARCH(Beta1)         0.907321   0.085661    10.59  0.0000

No. Observations :      1201  No. Parameters  :        15
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38841  Kurtosis (Y)    :   5.39439
Log Likelihood   :  3712.305  Alpha[1]+Beta[1]:   0.97671

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000144769
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.963583 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000341; 0.028295; 0.009516; 0.070961;-0.050264;-0.029808;-0.015633;-0.004752;
0.031250;-0.004770;-0.021987;-0.059244; 0.033723; 0.069385; 0.907321
Elapsed Time : 0.271 seconds (or 0.00451667 minutes).

---------------------------Estimating the univariate GARCH model for
DLTSEC---------------------------
```

```
 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: DLTSEC
Mean Equation:  ARMA (11, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3907.69
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000454 0.00025988    1.745  0.0812
AR(1)               -0.052598   0.040606   -1.295  0.1955
AR(2)                0.036610   0.039449   0.9280  0.3536
AR(3)                0.044254   0.043785    1.011  0.3124
AR(4)               -0.032597   0.038221  -0.8528  0.3939
AR(5)               -0.007017   0.031598  -0.2221  0.8243
AR(6)                0.007175   0.030260   0.2371  0.8126
AR(7)                0.052582   0.032289    1.628  0.1037
AR(8)               -0.002174   0.029229 -0.07439  0.9407
AR(9)               -0.002642   0.031421 -0.08409  0.9330
AR(10)              -0.028906   0.033668  -0.8585  0.3908
AR(11)              -0.057182   0.030913   -1.850  0.0646
Cst(V) x 10^4        0.128853   0.047525    2.711  0.0068
ARCH(Alpha1)         0.245523   0.080846    3.037  0.0024
GARCH(Beta1)         0.661116   0.071521    9.244  0.0000

No. Observations :      1201  No. Parameters  :        15
Mean (Y)         :   0.00007  Variance (Y)    :   0.00011
Skewness (Y)     :  -0.35407  Kurtosis (Y)    :  12.92069
Log Likelihood   :  3907.687  Alpha[1]+Beta[1]:   0.90664

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000138014
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.942555 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000454;-0.052598; 0.036610; 0.044254;-0.032597;-0.007017; 0.007175;
0.052582;-0.002174;-0.002642;-0.028906;-0.057182; 0.128853; 0.245523; 0.661116
Elapsed Time : 0.262 seconds (or 0.00436667 minutes).


  ********************
 **  SECOND STEP   **
*******************


  *************
 ** SERIES  **
*************
#1: DLHS
#2: DLTSEC

The estimation sample is:  2015-05-20 - 2020-04-27
```

```
 ******************************
 ** MG@RCH(2) SPECIFICATIONS **
 ******************************
Conditional Variance : Constant Correlation Model
Multivariate Student distribution, with 5.05341 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 8032.2
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                 Coefficient  Std.Error  t-value  t-prob
rho_21              0.609416   0.019351    31.49  0.0000
df                  5.053414    0.45078    11.21  0.0000
No. Observations :      1201  No. Parameters  :        32
No. Series       :         2  Log Likelihood  :  8032.200
Elapsed Time : 0.03 seconds (or 0.0005 minutes).
```

We can see how among the single equations for the series there is merely **only the 11th lag in DLHS equation which is significant**. And if we look at the 2nd step we see that now the **log-likelihood of this model has decreased** if compared to the previous one. **The correlation indicated by** rho_21 **has decreased too**, by about 2%. This was definitely not a good model, as expected. And if we make Box-Pierces and Hoskings' tests:

```
  ***********
 ** TESTS **
***********
Q-Statistics on Standardized Residuals

 Series: DLHS
  Q(  5) =  3.57400   [0.6122218]
  Q( 10) =  5.66423   [0.8426405]
  Q( 20) =  9.75212   [0.9724471]
  Q( 50) =  34.8382   [0.9490182]

 Series: DLTSEC
  Q(  5) =  13.2864   [0.0208376]
  Q( 10) =  15.5995   [0.1116866]
  Q( 20) =  22.9734   [0.2901026]
  Q( 50) =  49.5785   [0.4902259]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals

 Series: DLHS
  Q(  5) =  3.87703   [0.5672535]
  Q( 10) =  7.25578   [0.7010968]
  Q( 20) =  20.3435   [0.4366359]
  Q( 50) =  43.3082   [0.7369922]

 Series: DLTSEC
  Q(  5) =  1.07738   [0.9560859]
  Q( 10) =  1.88154   [0.9971700]
  Q( 20) =  2.46341   [0.9999993]
  Q( 50) =  15.1178   [0.9999996]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hosking's Multivariate Portmanteau Statistics on Standardized Residuals
  Hosking(  5) =  70.0476   [0.0000000]
  Hosking( 10) =  87.7429   [0.0000001]
  Hosking( 20) =  120.357   [0.0001286]
  Hosking( 50) =  223.512   [0.0435325]
Warning: P-values have been corrected by 11 degrees of freedom
```

```
--------------

Hosking's Multivariate Portmanteau Statistics on Squared Standardized Residuals
  Hosking( 5) =  8.17478   [0.9759196]
  Hosking( 10) =  22.8752   [0.9750311]
  Hosking( 20) =  48.8653   [0.9960251]
  Hosking( 50) =  114.374   [0.9999997]
Warning: P-values have been corrected by 2 degrees of freedom
--------------
```

We can see that **now there is autocorrelation within 5 lags according to Box-Pierce on standardized residuals for DLTSEC**. Also, there has been a big step backwards **if we look at Hosking's on standardized residuals**, as now **<u>there is autocorrelation within all the considered lags</u>**.

So, if we are not in need to show that there are some cross relations in mean, we may go straightforward to Multivariate specifications, but if we can see that there are some cross correlations between series in different lags and they are strong enough; then we might consider using a VAR specification, saving the residuals from it and putting them into the Multivariate GARCH specification, but then leave this specification for the Conditional Mean clear.

We shall try to reformulate the model without the AR(11) process, hence, we set back to ARMA(0,0):

```
  ********************
 **   FIRST STEP   **
*******************

---------------------------Estimating the univariate GARCH model for DLHS---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: DLHS
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Weak convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 3699.97
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000301 0.00032150   0.9371  0.3489
Cst(V) x 10^4        0.020370   0.028104   0.7248  0.4687
ARCH(Alpha1)         0.052887   0.028641    1.847  0.0651
GARCH(Beta1)         0.933555   0.047420    19.69  0.0000

No. Observations :      1201  No. Parameters  :         4
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38841  Kurtosis (Y)    :   5.39439
Log Likelihood   :  3699.966  Alpha[1]+Beta[1]:   0.98644

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000150253
```

```
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.978663 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000301; 0.020370; 0.052887; 0.933555
Elapsed Time : 0.053 seconds (or 0.000883333 minutes).

---------------------------Estimating the univariate GARCH model for
DLTSEC---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: DLTSEC
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3892.7
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000441 0.00025904    1.704  0.0886
Cst(V) x 10^4        0.132665   0.055240    2.402  0.0165
ARCH(Alpha1)         0.228275   0.074424    3.067  0.0022
GARCH(Beta1)         0.672217   0.072952    9.215  0.0000

No. Observations :      1201  No. Parameters  :         4
Mean (Y)         :   0.00007  Variance (Y)    :   0.00011
Skewness (Y)     :  -0.35407  Kurtosis (Y)    :  12.92069
Log Likelihood   :  3892.696  Alpha[1]+Beta[1]:   0.90049

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000133322
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.915106 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000441; 0.132665; 0.228275; 0.672217
Elapsed Time : 0.048 seconds (or 0.0008 minutes).


  ********************
 **  SECOND STEP   **
*******************


  *************
 ** SERIES  **
*************
#1: DLHS
#2: DLTSEC
```

```
The estimation sample is:  2015-05-20 - 2020-04-27

 *******************************
 ** MG@RCH(3) SPECIFICATIONS **
 *******************************
Conditional Variance : Constant Correlation Model
Multivariate Student distribution, with 5.03076 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 8021.27
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                 Coefficient  Std.Error  t-value  t-prob
rho_21              0.614543   0.019011    32.33  0.0000
df                  5.030764    0.43030    11.69  0.0000
No. Observations :      1201  No. Parameters  :        10
No. Series       :         2  Log Likelihood  :  8021.269
Elapsed Time : 0.01 seconds (or 0.000166667 minutes).
```

As we see in the 1st step in the equation for DLHS, Alpha1 is not statistically significant but the positivity constraint is observed, which makes the model reliable. Then, in the equation for DLTSEC, both Alpha1 and Beta1 are statistically significant, which is good. In the 2nd step, we see that the **log-likelihood of the Multivariate model has decreased again**, while the correlation has increased from the same model estimated with AR(11). **In this specification, we do not take into account any linear dependencies and cross dependencies, we are just focusing on the dependence in volatility**. Let's make the usual tests:

```
  ***********
 ** TESTS **
***********
Q-Statistics on Standardized Residuals

 Series: DLHS
  Q(  5) =  4.56885   [0.4707241]
  Q( 10) =  8.50596   [0.5795405]
  Q( 20) =  15.6102   [0.7404936]
  Q( 50) =  44.7610   [0.6829881]

 Series: DLTSEC
  Q(  5) =  9.95698   [0.0764634]
  Q( 10) =  13.1861   [0.2134524]
  Q( 20) =  14.9481   [0.7793687]
  Q( 50) =  35.2631   [0.9431115]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals

 Series: DLHS
  Q(  5) =  4.88912   [0.4295614]
  Q( 10) =  8.83391   [0.5479356]
  Q( 20) =  22.5820   [0.3097801]
  Q( 50) =  41.7911   [0.7890092]

 Series: DLTSEC
  Q(  5) =  1.32026   [0.9328334]
  Q( 10) =  2.09992   [0.9955154]
  Q( 20) =  2.54698   [0.9999990]
  Q( 50) =  12.6823   [1.0000000]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hosking's Multivariate Portmanteau Statistics on Standardized Residuals
```

```
  Hosking(  5) =  56.0774   [0.0000283]
  Hosking( 10) =  75.0178   [0.0006614]
  Hosking( 20) =  105.239   [0.0308323]
  Hosking( 50) =  207.649   [0.3405593]
---------------

Hosking's Multivariate Portmanteau Statistics on Squared Standardized Residuals
  Hosking(  5) =  9.13229   [0.9565990]
  Hosking( 10) =  23.1142   [0.9726944]
  Hosking( 20) =  51.4499   [0.9912270]
  Hosking( 50) =  110.426   [0.9999999]
Warning: P-values have been corrected by 2 degrees of freedom
---------------
```

The situation has definitely improved, as now there is autocorrelation according to Hosking's test on standardized residuals only within 5 and 10 lags.

Now, we can try to estimate a DCC(Engle) model, which is a type of model where the Correlation is considered dynamic, keeping GARCH(1,1) and Student T distribution. We keep the returns for estimation:

```
  ********************
 **   FIRST STEP   **
*******************

---------------------------Estimating the univariate GARCH model for
DLHS---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: DLHS
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Weak convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 3699.97
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                 Coefficient  Std.Error  t-value  t-prob
Cst(M)              0.000301 0.00032150   0.9371  0.3489
Cst(V) x 10^4       0.020370   0.028104   0.7248  0.4687
ARCH(Alpha1)        0.052887   0.028641    1.847  0.0651
GARCH(Beta1)        0.933555   0.047420    19.69  0.0000

No. Observations :      1201  No. Parameters  :         4
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38841  Kurtosis (Y)    :   5.39439
Log Likelihood   :  3699.966  Alpha[1]+Beta[1]:   0.98644

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000150253
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.978663 and should be < 1.
```

```
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000301; 0.020370; 0.052887; 0.933555
Elapsed Time : 0.053 seconds (or 0.000883333 minutes).

---------------------------Estimating the univariate GARCH model for
DLTSEC---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: DLTSEC
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3892.7
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000441 0.00025904    1.704  0.0886
Cst(V) x 10^4        0.132665   0.055240    2.402  0.0165
ARCH(Alpha1)         0.228275   0.074424    3.067  0.0022
GARCH(Beta1)         0.672217   0.072952    9.215  0.0000

No. Observations :      1201  No. Parameters  :         4
Mean (Y)         :   0.00007  Variance (Y)    :   0.00011
Skewness (Y)     :  -0.35407  Kurtosis (Y)    :  12.92069
Log Likelihood   :  3892.696  Alpha[1]+Beta[1]:   0.90049

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000133322
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.915106 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000441; 0.132665; 0.228275; 0.672217
Elapsed Time : 0.072 seconds (or 0.0012 minutes).


  ********************
 **  SECOND STEP   **
*******************


  *************
 ** SERIES  **
*************
#1: DLHS
#2: DLTSEC

The estimation sample is:  2015-05-20 - 2020-04-27

 ********************************
 ** MG@RCH(4) SPECIFICATIONS **
```

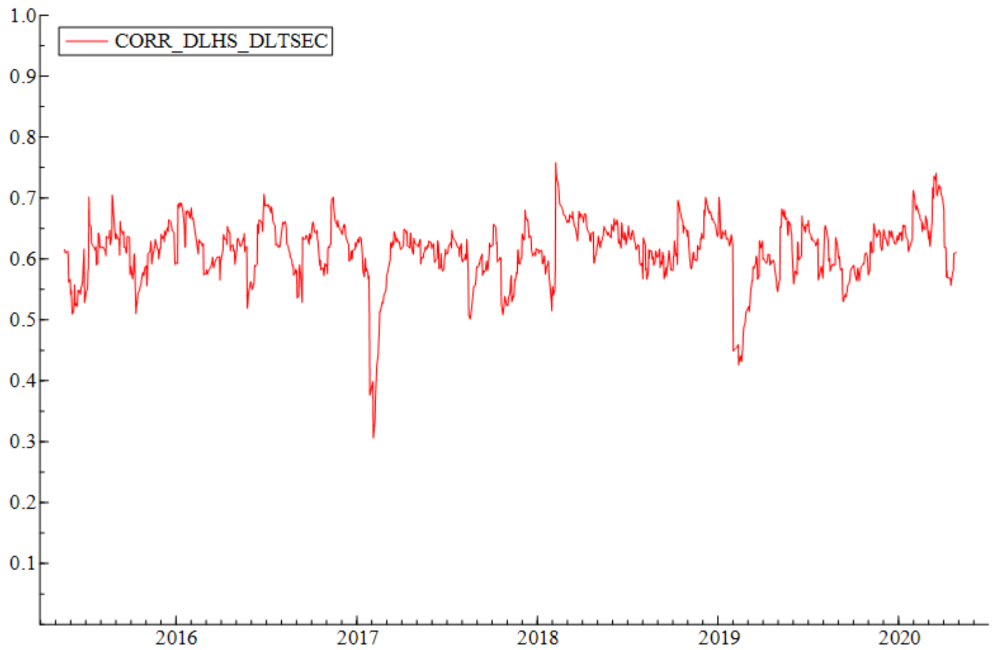
```
 ******************************
Conditional Variance : Dynamic Correlation Model (Engle)
Multivariate Student distribution, with 5.05279 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 8024.41
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
rho_21               0.614088   0.024543    25.02  0.0000
alpha                0.027868   0.014811    1.882  0.0601
beta                 0.879307   0.053822    16.34  0.0000
df                   5.052795    0.43862    11.52  0.0000
No. Observations :      1201  No. Parameters  :        12
No. Series       :         2  Log Likelihood  :  8024.412
Elapsed Time : 0.047 seconds (or 0.000783333 minutes).
```

As we see in the 1st step in the equation for DLHS, Alpha1 is not statistically significant but the positivity constraint is observed, which makes the model reliable. Then, in the equation for DLTSEC, both Alpha1 and Beta1 are statistically significant, which is good. In the 2nd step, we can see that log-likelihood has risen a bit, while rho_21 is very close to the one estimated in our last CCC model. If we look at the tests:

```
  ***********
 ** TESTS **
***********
Q-Statistics on Standardized Residuals

 Series: DLHS
  Q(  5) =  4.72256   [0.4506685]
  Q( 10) =  8.74198   [0.5567536]
  Q( 20) =  15.8754   [0.7243214]
  Q( 50) =  45.0667   [0.6712138]

 Series: DLTSEC
  Q(  5) =  9.38624   [0.0946151]
  Q( 10) =  12.8775   [0.2305995]
  Q( 20) =  14.8131   [0.7869997]
  Q( 50) =  35.2727   [0.9429723]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals

 Series: DLHS
  Q(  5) =  5.55213   [0.3522634]
  Q( 10) =  10.7926   [0.3739001]
  Q( 20) =  24.6384   [0.2156171]
  Q( 50) =  44.0416   [0.7101721]

 Series: DLTSEC
  Q(  5) = 0.994833   [0.9629814]
  Q( 10) =  1.75590   [0.9978918]
  Q( 20) =  2.18450   [0.9999998]
  Q( 50) =  12.3748   [1.0000000]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hosking's Multivariate Portmanteau Statistics on Standardized Residuals
  Hosking(  5) =  54.1582   [0.0000548]
  Hosking( 10) =  73.2547   [0.0010380]
  Hosking( 20) =  104.108   [0.0364139]
  Hosking( 50) =  207.665   [0.3402816]
---------------
```

```
Hosking's Multivariate Portmanteau Statistics on Squared Standardized Residuals
  Hosking(  5) =  8.17158   [0.9759715]
  Hosking( 10) =  21.1513   [0.9876843]
  Hosking( 20) =  54.2040   [0.9816050]
  Hosking( 50) =  113.252   [0.9999998]
Warning: P-values have been corrected by 2 degrees of freedom
---------------
```

We can see that not much has changed here. According to Hosking's test on standardized residuals, there is still statistically significant autocorrelation within 5 and 10 lags.
Since in this model we do not assume the Correlation to be constant in time, we can try to make a graphical analysis and see how this Correlation varies within the 5 years interval:



As we can see, the **correlation is not very constant**, we can see that there are at least two important outliers, at the beginning of February 2017 and in January 2019.

Now, let's try to formulate a DCC(Tse and TSUI), with the same settings as so far:

```
  ********************
 **   FIRST STEP   **
*******************

---------------------------Estimating the univariate GARCH model for
DLHS---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: DLHS
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Weak convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 3699.97
```

```
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000301 0.00032150   0.9371  0.3489
Cst(V) x 10^4        0.020370   0.028104   0.7248  0.4687
ARCH(Alpha1)         0.052887   0.028641    1.847  0.0651
GARCH(Beta1)         0.933555   0.047420    19.69  0.0000

No. Observations :      1201  No. Parameters  :         4
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38841  Kurtosis (Y)    :   5.39439
Log Likelihood   :  3699.966  Alpha[1]+Beta[1]:   0.98644

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000150253
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.978663 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000301; 0.020370; 0.052887; 0.933555
Elapsed Time : 0.126 seconds (or 0.0021 minutes).

---------------------------Estimating the univariate GARCH model for
DLTSEC---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: DLTSEC
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3892.7
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000441 0.00025904    1.704  0.0886
Cst(V) x 10^4        0.132665   0.055240    2.402  0.0165
ARCH(Alpha1)         0.228275   0.074424    3.067  0.0022
GARCH(Beta1)         0.672217   0.072952    9.215  0.0000

No. Observations :      1201  No. Parameters  :         4
Mean (Y)         :   0.00007  Variance (Y)    :   0.00011
Skewness (Y)     :  -0.35407  Kurtosis (Y)    :  12.92069
Log Likelihood   :  3892.696  Alpha[1]+Beta[1]:   0.90049

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000133322
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.915106 and should be < 1.
```

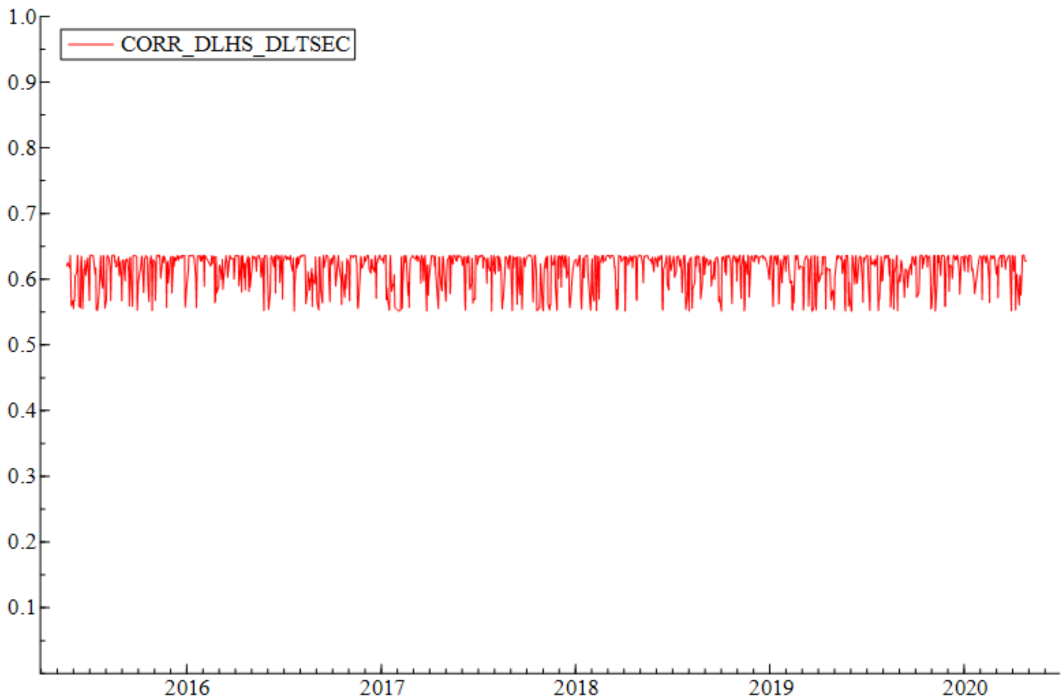
```
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000441; 0.132665; 0.228275; 0.672217
Elapsed Time : 0.062 seconds (or 0.00103333 minutes).


  ********************
 **  SECOND STEP   **
*******************


  *************
 ** SERIES  **
*************
#1: DLHS
#2: DLTSEC

The estimation sample is:  2015-05-20 - 2020-04-27

 ******************************
 ** MG@RCH(5) SPECIFICATIONS **
 ******************************
Conditional Variance : Dynamic Correlation Model (Tse and Tsui) with M = 2.
Multivariate Student distribution, with 5.05528 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 8022.24
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
rho_21               0.620385   0.019767    31.39  0.0000
alpha                0.042485    0.10100   0.4206  0.6741
beta                 0.000000     2.5693     0.00  1.0000
df                   5.055284    0.44614    11.33  0.0000
No. Observations :      1201  No. Parameters  :        12
No. Series       :         2  Log Likelihood  :  8022.244
Elapsed Time : 0.057 seconds (or 0.00095 minutes).
```

In the single equation for DLHS Alpha1 is not significant, but the positivity constraint is observed, which is good. In the single equation for DLTSEC both Alpha1 and Beta1 are statistically significant and their sum is lower than zero. In the 2nd step we can see that the correlation has risen a little bit. But if we do tests:

```
  ***********
 ** TESTS **
***********
Q-Statistics on Standardized Residuals

 Series: DLHS
  Q(  5) =  4.50073   [0.4797861]
  Q( 10) =  8.49556   [0.5805485]
  Q( 20) =  15.6034   [0.7409025]
  Q( 50) =  44.7381   [0.6838669]

 Series: DLTSEC
  Q(  5) =  10.0476   [0.0738969]
  Q( 10) =  13.2891   [0.2079515]
  Q( 20) =  15.2202   [0.7636710]
  Q( 50) =  35.5714   [0.9385307]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals
```

```
 Series: DLHS
  Q(  5) =  4.43187   [0.4890504]
  Q( 10) =  8.55044   [0.5752314]
  Q( 20) =  22.2313   [0.3280910]
  Q( 50) =  41.9985   [0.7822058]

 Series: DLTSEC
  Q(  5) =  1.32218   [0.9326328]
  Q( 10) =  2.11207   [0.9954068]
  Q( 20) =  2.55160   [0.9999990]
  Q( 50) =  12.7636   [1.0000000]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hosking's Multivariate Portmanteau Statistics on Standardized Residuals
  Hosking(  5) =  56.0537   [0.0000285]
  Hosking( 10) =  74.9805   [0.0006678]
  Hosking( 20) =  106.004   [0.0274909]
  Hosking( 50) =  209.130   [0.3145730]
---------------

Hosking's Multivariate Portmanteau Statistics on Squared Standardized Residuals
  Hosking(  5) =  8.41411   [0.9718130]
  Hosking( 10) =  21.5106   [0.9855956]
  Hosking( 20) =  50.7718   [0.9928043]
  Hosking( 50) =  110.807   [0.9999999]
Warning: P-values have been corrected by 2 degrees of freedom
---------------
```

We can see that **Hosking's test keeps being the "Achilles heel" of the specifications where we model the returns series**. Interestingly, if we plot the Correlation from the Graphical analysis:



We can see that the **Correlation is always ranging between approximately 0.55 and 0.64**.

When using the first differences series we can see that it is a little complex to obtain feasible results. We can try to take into account a GJR specification because we had seen some asymmetry on the distributions and we are not able to take this into account using a Student T distribution. So here is a DCC(Tse and Tsui) with GJR(1,1):

```
  ********************
 **   FIRST STEP   **
*******************

---------------------------Estimating the univariate GARCH model for
DLHS---------------------------


 *******************
 ** SPECIFICATIONS **
 *******************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: DLHS
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GJR (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Weak convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 3722.92
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000015 0.00029817  0.05172  0.9588
Cst(V) x 10^4        0.078574   0.034115    2.303  0.0214
ARCH(Alpha1)        -0.038434   0.013926   -2.760  0.0059
GARCH(Beta1)         0.883403   0.043980    20.09  0.0000
GJR(Gamma1)          0.177077   0.055153    3.211  0.0014

No. Observations :      1201  No. Parameters  :         5
Mean (Y)         :  -0.00011  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38841  Kurtosis (Y)    :   5.39439
Log Likelihood   :  3722.921

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is alpha(1) + beta(1) + k gamma(1) < 1 (with k = 0.5 with this distribution.)
In this estimation, this sum equals 0.933508.
The condition for existence of the fourth moment of the GJR is observed.
The constraint equals 0.899975 (should be < 1).  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000015; 0.078574;-0.038434; 0.883403; 0.177077
Elapsed Time : 0.224 seconds (or 0.00373333 minutes).

---------------------------Estimating the univariate GARCH model for
DLTSEC---------------------------


 *******************
 ** SPECIFICATIONS **
 *******************
The estimation sample is:  2015-05-20 - 2020-04-27
The dependent variable is: DLTSEC
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GJR (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
```

```
Log-likelihood = 3896.49
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000108 0.00024467   0.4397  0.6603
Cst(V) x 10^4        0.028648   0.088678   0.3231  0.7467
ARCH(Alpha1)         0.025111    0.11150   0.2252  0.8219
GARCH(Beta1)         0.887139    0.22297    3.979  0.0001
GJR(Gamma1)          0.132502   0.090966    1.457  0.1455

No. Observations :      1201  No. Parameters  :         5
Mean (Y)         :   0.00007  Variance (Y)    :   0.00011
Skewness (Y)     :  -0.35407  Kurtosis (Y)    :  12.92069
Log Likelihood   :  3896.488

The sample mean of squared residuals was used to start recursion.
The condition for existence of the second moment of the GJR is observed.
This condition is alpha(1) + beta(1) + k gamma(1) < 1 (with k = 0.5 with this distribution.)
In this estimation, this sum equals 0.978501.
The condition for existence of the fourth moment of the GJR is observed.
The constraint equals 0.987327 (should be < 1).  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000108; 0.028648; 0.025111; 0.887139; 0.132502
Elapsed Time : 0.233 seconds (or 0.00388333 minutes).


  ********************
 **  SECOND STEP   **
*******************


  *************
 ** SERIES  **
*************
#1: DLHS
#2: DLTSEC

The estimation sample is:  2015-05-20 - 2020-04-27

 *******************************
 ** MG@RCH(6) SPECIFICATIONS **
 *******************************
Conditional Variance : Dynamic Correlation Model (Tse and Tsui) with M = 2.
Multivariate Student distribution, with 5.36107 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 8024.85
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
rho_21               0.620076   0.022736    27.27  0.0000
alpha                0.026387   0.032269   0.8177  0.4137
beta                 0.685962    0.60044    1.142  0.2535
df                   5.361068    0.49249    10.89  0.0000
No. Observations :      1201  No. Parameters  :        14
No. Series       :         2  Log Likelihood  :  8024.855
Elapsed Time : 0.108 seconds (or 0.0018 minutes).
```

As we can see, there has been an improvement in log-likelihood. Moreover, beta is quite big, which indicates the **presence of a kind of Leverage effect**. If we look at tests:

```
  ***********
 ** TESTS **
```

```
***********
Q-Statistics on Standardized Residuals

 Series: DLHS
  Q(  5) =  3.38473   [0.6408967]
  Q( 10) =  5.87720   [0.8254723]
  Q( 20) =  14.3973   [0.8097922]
  Q( 50) =  37.9066   [0.8953082]

 Series: DLTSEC
  Q(  5) =  12.7637   [0.0256966]
  Q( 10) =  14.8550   [0.1374385]
  Q( 20) =  17.8276   [0.5987646]
  Q( 50) =  34.2260   [0.9567343]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals

 Series: DLHS
  Q(  5) =  1.11118   [0.9531096]
  Q( 10) =  3.23303   [0.9753959]
  Q( 20) =  9.93921   [0.9692608]
  Q( 50) =  31.3967   [0.9817027]

 Series: DLTSEC
  Q(  5) =  9.90033   [0.0781091]
  Q( 10) =  12.2333   [0.2697480]
  Q( 20) =  15.7692   [0.7308370]
  Q( 50) =  33.4829   [0.9649035]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hosking's Multivariate Portmanteau Statistics on Standardized Residuals
  Hosking(  5) =  61.1377   [0.0000047]
  Hosking( 10) =  76.7177   [0.0004246]
  Hosking( 20) =  112.347   [0.0099702]
  Hosking( 50) =  209.704   [0.3047608]
---------------

Hosking's Multivariate Portmanteau Statistics on Squared Standardized Residuals
  Hosking(  5) =  17.2448   [0.5063378]
  Hosking( 10) =  36.2015   [0.5528145]
  Hosking( 20) =  63.7176   [0.8785146]
  Hosking( 50) =  142.533   [0.9989164]
Warning: P-values have been corrected by 2 degrees of freedom
---------------
```

We can see that the **<u>same issue</u>** remains in the Hosking's test on standardized residuals.

Now, we could try to increase the number of lags; for example, let's try a DCC(Engle) with GARCH(2,1):

```
  ********************
 **   FIRST STEP   **
*******************

---------------------------Estimating the univariate GARCH model for
DLHS---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-05 - 2020-04-27
```

```
The dependent variable is: DLHS
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (2, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3734.58
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000291 0.00031923   0.9123  0.3618
Cst(V) x 10^4        0.021671   0.027933   0.7758  0.4380
ARCH(Alpha1)         0.054862   0.024348    2.253  0.0244
GARCH(Beta1)         0.907769    0.30315    2.994  0.0028
GARCH(Beta2)         0.022943    0.31343  0.07320  0.9417

No. Observations :      1212  No. Parameters  :         5
Mean (Y)         :  -0.00012  Variance (Y)    :   0.00014
Skewness (Y)     :  -0.38211  Kurtosis (Y)    :   5.38058
Log Likelihood   :  3734.580  Alpha[1]+Beta[1]:   0.98557

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (2,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000150226
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.

Estimated Parameters Vector :
 0.000291; 0.021671; 0.054862; 0.907769; 0.022943
Elapsed Time : 0.077 seconds (or 0.00128333 minutes).

---------------------------Estimating the univariate GARCH model for
DLTSEC---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-05-05 - 2020-04-27
The dependent variable is: DLTSEC
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (2, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3931.14
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000419 0.00026596    1.576  0.1153
Cst(V) x 10^4        0.131997   0.048089    2.745  0.0061
ARCH(Alpha1)         0.225576   0.081372    2.772  0.0057
GARCH(Beta1)         0.672168    0.29049    2.314  0.0208
GARCH(Beta2)         0.001813    0.23921 0.007579  0.9940

No. Observations :      1212  No. Parameters  :         5
Mean (Y)         :   0.00006  Variance (Y)    :   0.00011
Skewness (Y)     :  -0.35182  Kurtosis (Y)    :  12.95105
Log Likelihood   :  3931.142  Alpha[1]+Beta[1]:   0.89956
```

```
The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (2,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000131416
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.

Estimated Parameters Vector :
 0.000419; 0.131997; 0.225576; 0.672168; 0.001813
Elapsed Time : 0.155 seconds (or 0.00258333 minutes).


  ********************
 **  SECOND STEP   **
*******************


  *************
 ** SERIES  **
*************
#1: DLHS
#2: DLTSEC

The estimation sample is:  2015-05-05 - 2020-04-27

 *******************************
 ** MG@RCH(18) SPECIFICATIONS **
 *******************************
Conditional Variance : Dynamic Correlation Model (Engle)
Multivariate Student distribution, with 5.11176 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 8095.31
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                 Coefficient  Std.Error  t-value  t-prob
rho_21              0.609106   0.026405    23.07  0.0000
alpha               0.031735   0.014767    2.149  0.0318
beta                0.887247   0.049014    18.10  0.0000
df                  5.111760    0.45090    11.34  0.0000
No. Observations :      1212  No. Parameters  :        14
No. Series       :         2  Log Likelihood  :  8095.305
Elapsed Time : 0.038 seconds (or 0.000633333 minutes).
```

As we can see, increasing the number of lags has **improved the value of log-likelihood**, however, rho_21 has decreased. In general, if we compare the models created so far using the Progress tool:

```
Progress to date
Model          T    p          log-likelihood         SC         HQ        AIC
MG@RCH(1)    1201   2  MaxSQP      8050.3325     -13.394<    -13.400<    -13.403<
MG@RCH(2)    1201   2  MaxSQP      8032.2003     -13.364     -13.369     -13.373
MG@RCH(3)    1201   2  MaxSQP      8021.2687     -13.346     -13.351     -13.354
MG@RCH(4)    1201   4  MaxSQP      8024.4118     -13.339     -13.350     -13.356
MG@RCH(5)    1201   4  MaxSQP      8022.2442     -13.336     -13.346     -13.353
MG@RCH(6)    1201   4  MaxSQP      8024.8547     -13.340     -13.351     -13.357
MG@RCH(18)   1212   4  MaxSQP      8095.3053     -13.335     -13.346     -13.352
```

We see that the best model is the one created using the residuals from VAR(11)! Actually, despite trying to change specification and number of lags when using the first differences series, the **MG@RCH(1) model remains the best**. The issue related to the autocorrelation indicated by Hosking's test on standardized residuals was never resolved.