Unit Root Testing

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We are going to work with data related to National Defence excel spreadsheet. After performing first and second differences (DFDEFX and DDFDEFX respectively).

Before starting our process on OxMetrics, we should consider p_t as the price of an asset, where:

$$p_t = \phi_1 p_{t-1} + a_t$$
$$p_t = \phi_0 + \phi_1 p_{t-1} + a_t$$

The result from p_t are only predictable if p_t doesn't have a unit root (the process is stationary).

PcGive

Let's try to implement a Dickey-Fuller test, which means that we will consider 0 lags for now, and form the following hypothesis:

$H_0: \emptyset_1 = 1$ □ The process is non-stationary.
$H_1: \emptyset_1 < 1$ □ The process is stationary.

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Unit-root tests

The sample is: 2011(2) - 2019(4) (36 observations and 3 variables)

FDEFX: ADF tests (T=35, Constant+Trend; 5%=-3.54 1%=-4.24)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob       AIC  F-prob
  0     0.9058        1.0323    8.736                        4.417

DFDEFX: ADF tests (T=35, Constant+Trend; 5%=-3.54 1%=-4.24)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob       AIC  F-prob
  0     -5.950**   -0.045389    8.838                        4.440

DDFEDFX: ADF tests (T=35, Constant+Trend; 5%=-3.54 1%=-4.24)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob       AIC  F-prob
  0     -11.47**    -0.59476    10.23                        4.733
```

The results obtained in Oxmetrics are divided in FDEFX, DFDEFX, DDFDEFX. For each series, we need to observe the empirical value (*t-adf*) and compare it with the critical value for the selected significance interval (1%). Namely, if:

$$(t - adf) > critical\ value \rightarrow Accept\ H_0$$

$$(t - adf) < critical\ value \rightarrow Reject\ H_0$$

Therefore:

- FDEFX's series' empirical value (0.9058) is greater than the critical value (-4.24), so there is no reason to reject the null hypothesis. There is unit root in this series, which

is a non-stationary process. Moreover, since Beta value is greater than 1, there is AR in this series.

- For DFDEFX series the empirical value (-5.950) is greater than the critical value (-4.24), so there is no reason to reject the null hypothesis. There is unit root in this series, which is a non-stationary process. Moreover, since Beta value is smaller than 1, there is no AR in this series.
- For DDFDEFX series the empirical value (-11.47) is greater than the critical value (-4.24), so there is no reason to reject the null hypothesis. There is unit root in this series, which is a non-stationary process. Moreover, since Beta value is smaller than 1, there is no AR in this series.

We can now perform an Augmented Dickey-Fuller test, which means that we increase the number of lags. For example, let's try with 3 lags.

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Unit-root tests
The sample is: 2011(2) - 2019(4) (39 observations and 3 variables)

FDEFX: ADF tests (T=35, Constant+Trend; 5%=-3.54 1%=-4.24)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob       AIC  F-prob
  3    -0.5867       0.97215    8.330      2.238  0.0331     4.395
  2     0.5618        1.0247    8.869     0.8075  0.4258     4.497  0.0331
  1      1.066        1.0412    8.820    -0.6310  0.5326     4.461  0.0727
  0     0.9058        1.0323    8.736                        4.417  0.1268

DFDEFX: ADF tests (T=35, Constant+Trend; 5%=-3.54 1%=-4.24)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob       AIC  F-prob
  3     -2.372       0.39668    8.378    -0.1126  0.9111     4.406
  2     -2.449       0.39286    8.239     -2.266  0.0308     4.349  0.9111
  1     -3.541       0.15413    8.771     -1.222  0.2310     4.450  0.1006
  0     -5.950**   -0.045389    8.838                        4.440  0.1089

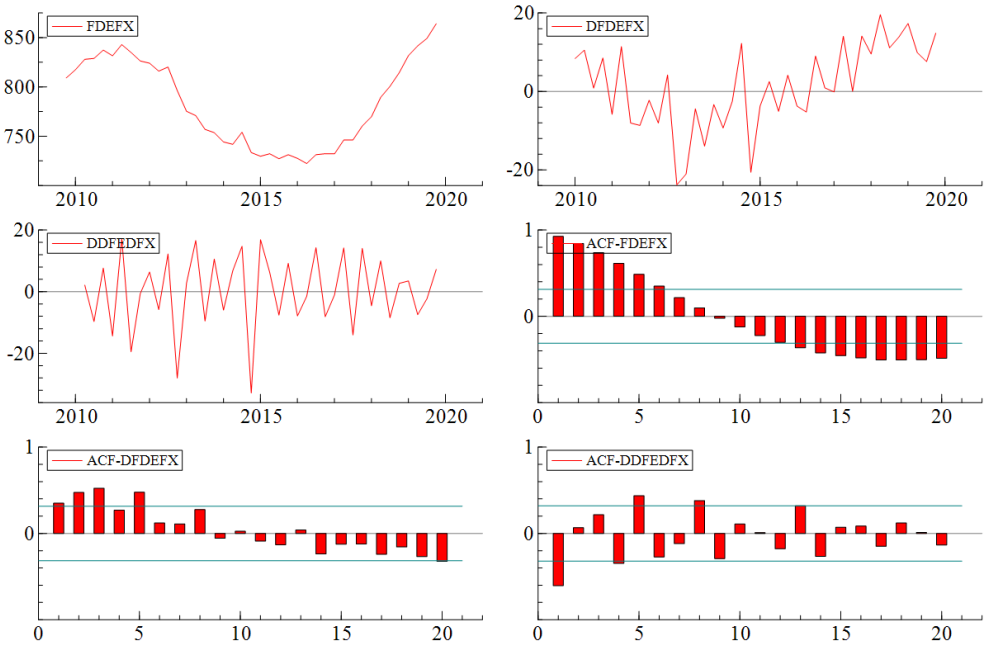
DDFEDFX: ADF tests (T=35, Constant+Trend; 5%=-3.54 1%=-4.24)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob       AIC  F-prob
  3     -5.283**     -2.5876    8.525      2.108  0.0438     4.441
  2     -5.014**     -1.5983    9.001     0.4026  0.6901     4.526  0.0438
  1     -8.899**     -1.4217    8.878      3.390  0.0019     4.474  0.1170
  0     -11.47**    -0.59476    10.23                        4.733  0.0034
```

Observing the results of the test, we will compare again the empirical and critical values, following the same criteria as for the Dickey-Fuller test, hence:

- In FDEFX series the empirical values are all greater than the critical value (-4.24), regardless of the number of lags, so there is no reason to reject the null hypothesis. There is unit root in this series, which is a non-stationary process.
- In DFDEFX series the empirical values for 1,2 and 3 lags are greater than the critical value (-4.24) and the series is non-stationary. However, for 0 lags the empirical value is smaller than the critical value, and there the process is stationary.
- In DDFDEFX series the empirical values for all lags are smaller than the critical value (-4.24), so we should reject the null hypothesis. There is no unit root in this series, which is a stationary process.

One way to understand how many lags should be used would be by looking at the Akaike Information Criteria (AIC).
For FDEFX we can see how the smallest AIC is when we use 0 lags, for DFDEFX the smallest AIC is when we use 2 lags, however the only stationary process was found to happen when we use 0 lags, so we would rather select to have 0 lags. Finally, for DDFDEFX, the AIC is the smallest for 3 lags, and since the process was found to be stationary for all the lags we can clearly choose 3 lags.

Another way would be preparing ACF graphs and interpreting them.



As we can see, in the ACF graph for FDEFX there is non-stationarity, as the graph decreases very slowly. The same can be observed for DFDEFX while if for DDFDEFX we focus on the first lags only, we can see how the first one is quite statistically significant. Hence, an ADF of order 1 can be executed. We can notice that, somehow, the interpretation of these graphs leads to different conclusions than those indicated by the AICs.

G@RCH

We can try to use G@RCH model class and implement an ADF test, again with 3 lags. We should compare the empirical value (ADF statistics) and the critical value, with the same criteria as for PcGive's DF test. Also, if a series is integrated at least with order one, then the process is non-stationary, otherwise the process is stationary.

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ADF Test with 3 lags
No intercept and no time trend
H0: FDEFX is I(1)

ADF Statistics: 0.778528

Asymptotic critical values, Davidson, R. and MacKinnon, J. (1993)

        1%         5%        10%
  -2.56572  -1.94093  -1.61663

OLS Results
                  Coefficient    t-value
y_1                  0.001620    0.77853
dy_1                 0.062557    0.36487
dy_2                 0.302284     1.8664
dy_3                 0.479765     2.7991
RSS               2119.486744
OBS                 31.000000
Information Criteria (to be minimized)
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Akaike          7.330002  Shibata         7.301512
Schwarz         7.515032  Hannan-Quinn    7.390317
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```

For FDEFX we can see that the ADF Statistics (0.77853) is greater than the critical value (-2.56572), hence we have no reason to reject the null hypothesis and we can conclude that FDEFX is integrated at least with order 1. There is unit root in this series, which is a non-stationary process.

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H0: DFDEFX is I(1)

ADF Statistics: -0.834992

Asymptotic critical values, Davidson, R. and MacKinnon, J. (1993)

        1%        5%       10%
  -2.56572  -1.94093  -1.61663

OLS Results
                  Coefficient    t-value
y_1                 -0.159966   -0.83499
dy_1                -0.749055    -3.0203
dy_2                -0.435664    -1.7202
dy_3                 0.035855    0.19043
RSS               2164.159029
OBS                 31.000000
Information Criteria (to be minimized)
Akaike          7.350860  Shibata         7.322370
Schwarz         7.535890  Hannan-Quinn    7.411175
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```

For DFDEFX we can see that the ADF Statistics (-0.83499) is greater than the critical value (-2.56572), hence we have no reason to reject the null hypothesis and we can conclude that FDEFX is integrated at least with order 1. There is unit root in this series, which is a non-stationary process.

```
H0: DDFEDFX is I(1)

ADF Statistics: -4.97239

Asymptotic critical values, Davidson, R. and MacKinnon, J. (1993)

        1%        5%       10%
  -2.56572  -1.94093  -1.61663

OLS Results
                  Coefficient    t-value
y_1                 -3.245360    -4.9724
dy_1                 1.360499     2.4605
dy_2                 0.647538     1.7573
dy_3                 0.335594     1.9553
RSS               1944.680109
OBS                 31.000000
Information Criteria (to be minimized)
Akaike          7.243925  Shibata         7.215435
Schwarz         7.428956  Hannan-Quinn    7.304241
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For DDFDEFX we can see that the ADF Statistics (-4.97239) is greater than the critical value (-2.56572), hence we should reject the null hypothesis and we can conclude that FDEFX is not integrated at least with order 1. There is no unit root in this series, which is a stationary process.