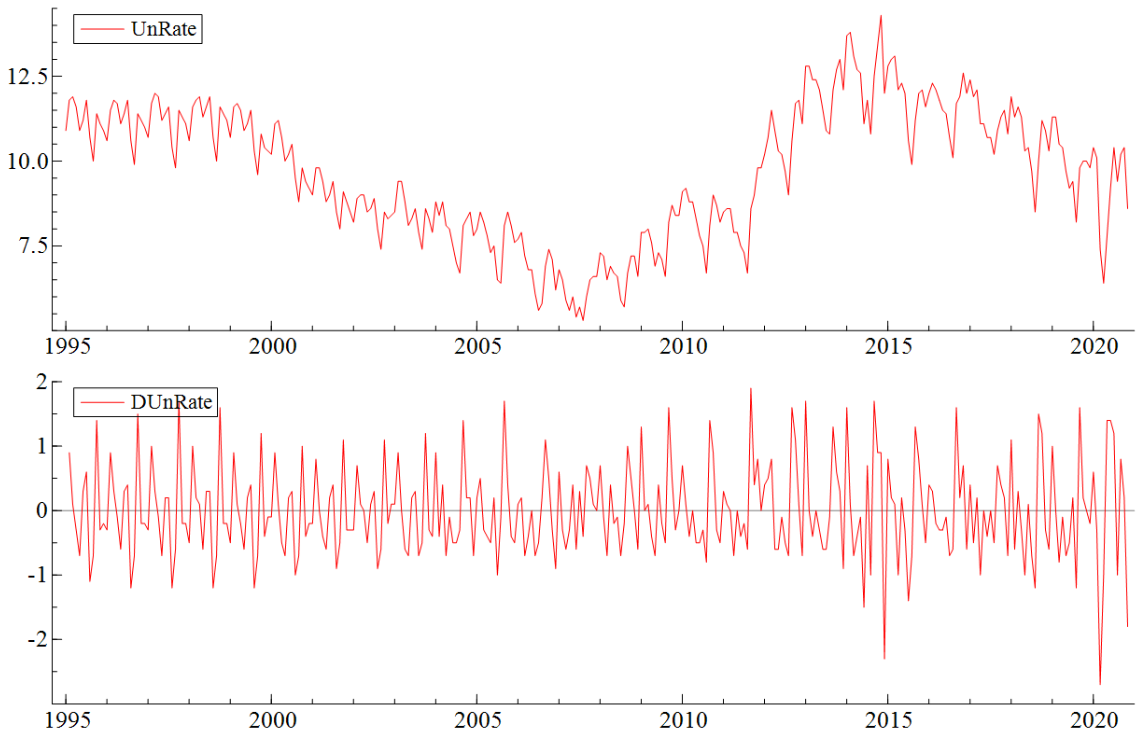
SARIMA and Unobserved Component Models applied on Unemployment Rate in Italy

Daniele Melotti

The time series that will be used in this project contains data related to the unemployment rates of all persons in Italy between 1995(1) and 2020(11), observed with a monthly frequency.
Such series is presented in the following graph:

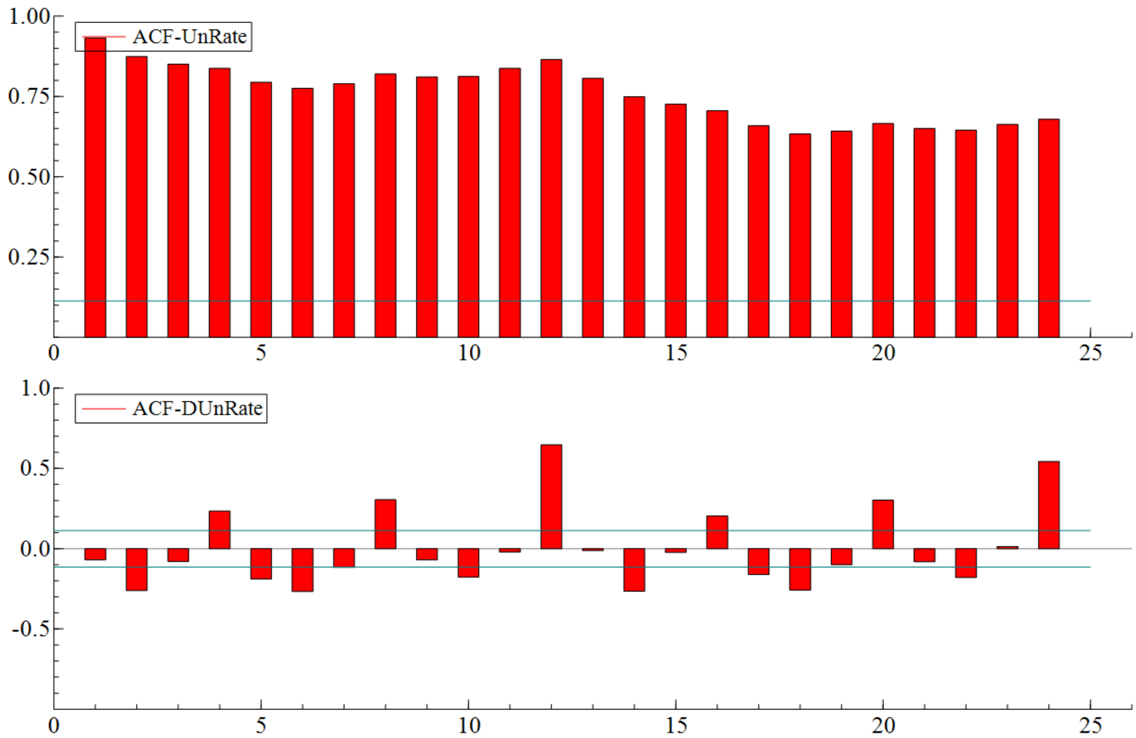


As we can see, seasonality seems to be present within the series. Tendentially, looking at the raw series, each year the lowest unemployment rates are recorded in the summer months, while the highest ones are found within the winter months. A general overlook of the plot indicates the presence of at least one cycle. Namely, in the beginning of the sample the rates are somewhat constant, then they start to decrease until 2007(8), that with a 5.3% represents the lowest rate of unemployment for the whole sample. Afterwards, the rates tend to increase until 2014(11), that with a 14.3% represents the highest rate of unemployment through the whole sample. Therefore, at first there is a downward trend, then an upward trend followed by another downward trend after the end of 2014. Interestingly, unemployment rates have decreased quite significantly within 2020, however, the number of inactive persons has increased during that same year.
On one hand, the raw series seems to be non-stationary, on the other hand, the first differences look stationary instead. An ADF test could confirm this:

```
ADF Test with 4 lags
No intercept and no time trend
H0: UnRate is I(1)

ADF Statistics: -0.724619
```

```
Asymptotic critical values, Davidson, R. and MacKinnon, J. (1993)

        1%        5%       10%
  -2.56572  -1.94093  -1.61663

---------------
H0: DUnRate is I(1)

ADF Statistics: -10.2223

Asymptotic critical values, Davidson, R. and MacKinnon, J. (1993)

        1%        5%       10%
  -2.56572  -1.94093  -1.61663

---------------
```

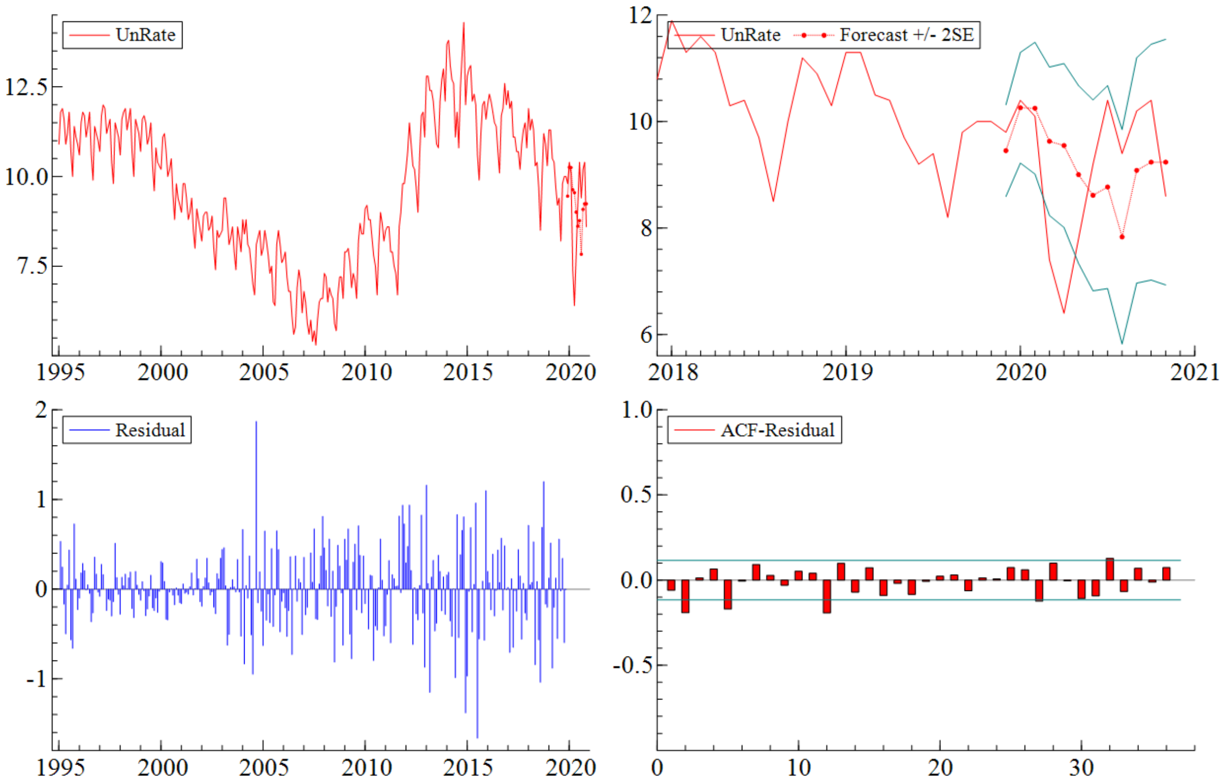
As we can see, the test confirms the supposition. Now, plotting the ACF for the raw series and first differences:



As we can see, the ACF for the raw series does not represent exactly a random walk process, as the autocorrelation does not decrease constantly and slowly throughout the whole sample. By looking at the first differences, it is visible that in the 12th and 24th lags we have the highest increases, while in the 2nd, 14th and perhaps 16th lags we have the largest decreases.

SARIMA

The **first model** will be a simple model where we assume only regular differencing. It will be with ARMA(1,0) both for the regular and the seasonal part, therefore our model will be an $ARIMA(1,\ 1,\ 0)\times(1,\ 0,\ 0)S$. The estimation will end 12 observations before, hence at 2019(11), and provide 12 forecasts. Here is the graphical output:



The main issue of this model is that the forecasts were not accurate. Apart from that, residuals didn't look IID, as there were several clusters of variance starting from 2003/2004. Also, there was statistically significant negative autocorrelation within the 2^{nd}, 5^{th} and 12^{th} lags. Such autocorrelations could be interpreted by saying that possibly what happened 2 months ago, 5 months ago and 12 months ago can have a negative impact on the next month's unemployment rate. According to the written output of the specification:

```
MODEL DEFINITION for UnRate
 Transformation:       No transformation
 ARIMA Model:  (1 1 0)(1 0 0)
 regARIMA Model Span: 1995.Jan to 2019.Nov

 MODEL ESTIMATION/EVALUATION
 Estimation converged in     5 ARMA iterations,    16 function evaluations.
 ARIMA Model:  (1 1 0)(1 0 0)
   Nonseasonal differences: 1
                                                  Standard
 Parameter                     Estimate             Errors
 -----------------------------------------------------
 Nonseasonal AR
   Lag  1                        -.3264             .05372

 Seasonal AR
   Lag 12                         .7805             .03541

 Variance                         .19342E+00
 SE of Var                        .15846E-01
 -----------------------------------------------------

 Likelihood Statistics
 ------------------------------------------------------------------
 Number of observations (nobs)                                  299
 Effective number of observations (nefobs)                      298
 Number of parameters estimated (np)                              3
 Log likelihood (L)                                       -183.7492
```

```
AIC                                                               373.4983
AICC (F-corrected-AIC)                                            373.5800
Hannan Quinn                                                      377.9381
BIC                                                               384.5896
------------------------------------------------------------------

Roots of ARIMA Model
 Root                  Real  Imaginary    Modulus  Frequency
 -----------------------------------------------------------
 Nonseasonal AR
  Root  1           -3.0639      .0000     3.0639      .5000
 Seasonal AR
  Root  1            1.2813      .0000     1.2813      .0000
 -----------------------------------------------------------


FORECASTING
 Origin  2019.Nov
 Number        12

 Confidence intervals with coverage probability ( .95000)
  ---------------------------------------
      Date       Lower  Forecast     Upper
  ---------------------------------------
  2019.Dec        8.59      9.46     10.32
  2020.Jan        9.22     10.26     11.30
  2020.Feb        9.01     10.25     11.49
  2020.Mar        8.24      9.63     11.03
  2020.Apr        8.01      9.55     11.09
  2020.May        7.33      9.01     10.68
  2020.Jun        6.82      8.62     10.41
  2020.Jul        6.86      8.77     10.68
  2020.Aug        5.82      7.84      9.85
  2020.Sep        6.97      9.08     11.20
  2020.Oct        7.03      9.24     11.45
  2020.Nov        6.93      9.24     11.55
  ---------------------------------------
```

The H_0 could be rejected for all parameters, therefore they were statistically significantly different from zero, which is good. The absolute values of the Roots of ARIMA Model were higher than 1, indicating that it could be considered to extend the part of differencing for seasonal too. The errors were computed on Excel:

Last 12 observed values		***ARIMA(1,1,0)x(1,0,0)S***	
Date	UnRate	Forecast	Error
2019(12)	9.8	9.46	-0.34
2020(1)	10.4	10.26	-0.14
2020(2)	10.1	10.25	0.15
2020(3)	7.4	9.63	2.23
2020(4)	6.4	9.55	3.15
2020(5)	7.8	9.01	1.21
2020(6)	9.2	8.62	-0.58
2020(7)	10.4	8.77	-1.63
2020(8)	9.4	7.84	-1.56
2020(9)	10.2	9.08	-1.12
2020(10)	10.4	9.24	-1.16
2020(11)	8.6	9.24	0.64

Errors for 12 periods	RMSE	1.4420
	RMSPE	1.9254
	MAE	1.1592
	MAPE	14.1097

The errors were calculated using the following formulas:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}$$

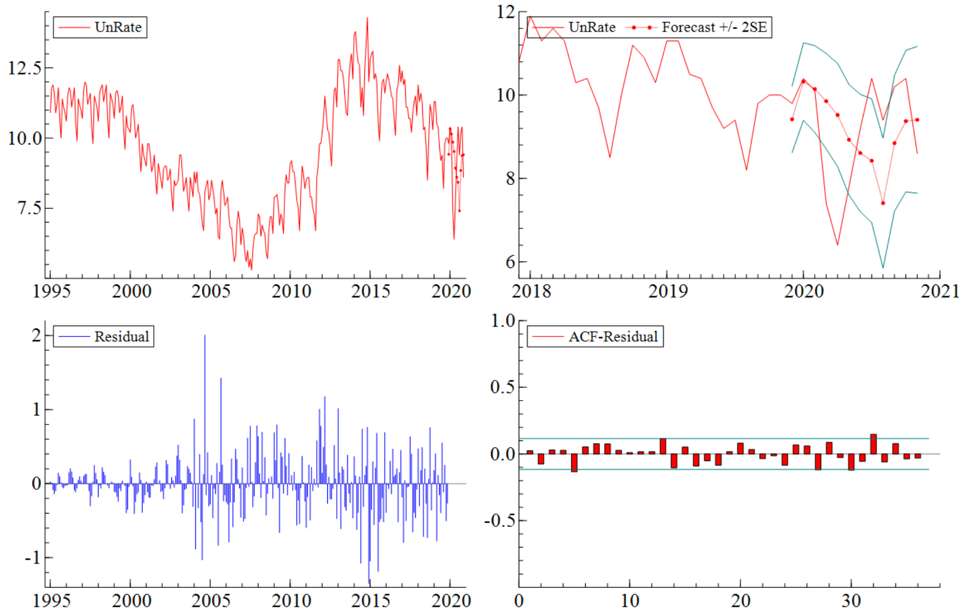
$$RMSPE = \sqrt{\frac{100\%}{n}\sum_{i=1}^{n}\left(\frac{y_i - \hat{y}_i}{y_i}\right)^2}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}|$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|y_i - \hat{y}_i|}{y_i}\cdot 100\%$$

In the **second model**, a first differencing was assumed both for the regular part and for the seasonal part, keeping an ARMA(1,0) for both. Therefore, the model was an $ARIMA(1,\ 1,\ 0)\times(1,\ 1,\ 0)S$. Despite an improvement in terms of log-likelihood and information criteria, the model still presented autocorrelation and had worse forecasts.

In the **third model**, again both differencing were assumed, this time using an ARMA(0,1). Therefore, the specification was of the so-called "airline" type: $ARIMA(0,\ 1,\ 1)\times(0,\ 1,\ 1)S$. Initially I choose this specification with the simple intention of trying how the airline model works, however, the same configuration was also chosen by OxMetrics when using the Automatic ARIMA setting. Here is the graphical output:



As we can see, the forecasts don't look satisfactory, actually they are a little worse than in the first model. However, this is mostly due to the fact that 2020 was a "special year" because of COVID-19. In fact, the forecasts related to 2020(1) and 2020(2) are quite accurate, nonetheless, we can notice how the actual unemployment rates go way below the lower error bar in 2020(3), which coincides with the first lockdown in Italy. The sharp decrease in unemployment rates was not led by an increase of occupied, but rather by an increase of inactives, hence the individuals who are unemployed and not searching for an occupation. On the other hand, the second major difference with the forecasts happened in 2020(6) and 2020(7), which are the first two months following the end of the lockdown: as more and more people started searching for a position, the unemployment rates initially grew up very quickly.
Looking at the ACF, we can see statistically significant autocorrelation (barely) only at lag 5. The residuals may look a little more IID than in the previous models, however there are still some clusters of variance. The written output:

```
MODEL DEFINITION for UnRate
 Transformation:      No transformation
 ARIMA Model:  (0 1 1)(0 1 1)
 regARIMA Model Span: 1995.Jan to 2019.Nov

 MODEL ESTIMATION/EVALUATION
 Estimation converged in    5 ARMA iterations,   16 function evaluations.
 ARIMA Model:  (0 1 1)(0 1 1)
   Nonseasonal differences: 1
   Seasonal differences:    1
                                              Standard
 Parameter                    Estimate          Errors
 -----------------------------------------------------
 Nonseasonal MA
   Lag  1                        .4060          .05374

 Seasonal MA
   Lag 12                        .5430          .05109

 Variance                        .16571E+00
 SE of Var                       .13857E-01
 -----------------------------------------------------

 Likelihood Statistics
 ------------------------------------------------------------------
 Number of observations (nobs)                                  299
 Effective number of observations (nefobs)                      286
 Number of parameters estimated (np)                              3
 Log likelihood (L)                                       -150.9583
 AIC                                                       307.9166
 AICC (F-corrected-AIC)                                    308.0017
 Hannan Quinn                                              312.3129
 BIC                                                       318.8846
 ------------------------------------------------------------------

 Roots of ARIMA Model
  Root                 Real  Imaginary    Modulus  Frequency
  ----------------------------------------------------------
  Nonseasonal MA
   Root  1           2.4633      .0000     2.4633      .0000
  Seasonal MA
   Root  1           1.8416      .0000     1.8416      .0000
  ----------------------------------------------------------


 FORECASTING
  Origin  2019.Nov
```

```
Number         12

Confidence intervals with coverage probability ( .95000)
 ---------------------------------------
     Date       Lower  Forecast     Upper
 ---------------------------------------
 2019.Dec        8.62      9.42     10.22
 2020.Jan        9.40     10.33     11.25
 2020.Feb        9.10     10.15     11.19
 2020.Mar        8.71      9.86     11.00
 2020.Apr        8.28      9.52     10.76
 2020.May        7.60      8.93     10.26
 2020.Jun        7.20      8.61     10.02
 2020.Jul        6.94      8.42      9.91
 2020.Aug        5.85      7.41      8.97
 2020.Sep        7.22      8.85     10.48
 2020.Oct        7.68      9.38     11.07
 2020.Nov        7.65      9.41     11.17
 ---------------------------------------
```

If we calculate the t-values for parameters, we will see that they are all statistically significant, the log-likelihood and information criteria (AIC, HQ, BIC) are the best ones so far. Therefore, this is the best specification in terms of diagnostics. Having the actual values and the forecasts, the errors can be calculated:

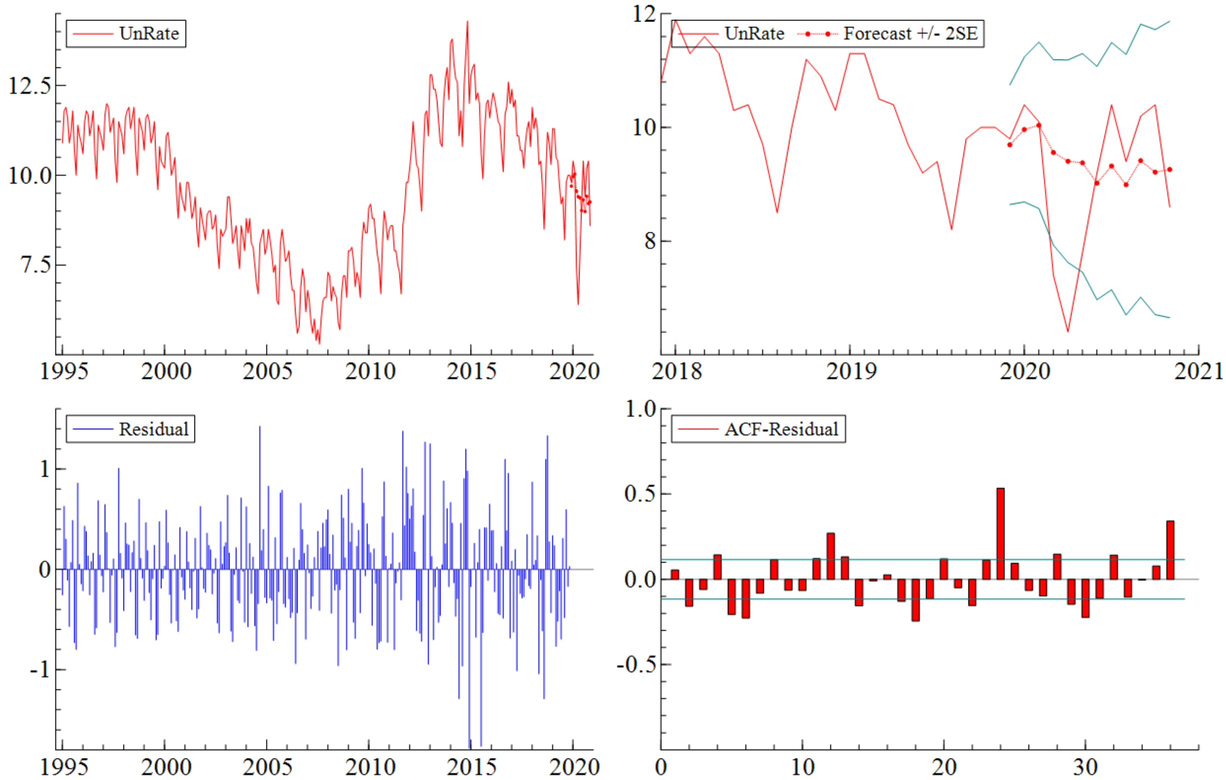
Last 12 observed values		***ARIMA(0,1,1)x(0,1,1)S***	
Date	UnRate	Forecast	Error
2019(12)	9.8	9.42	-0.38
2020(1)	10.4	10.33	-0.07
2020(2)	10.1	10.15	0.05
2020(3)	7.4	9.86	2.46
2020(4)	6.4	9.52	3.12
2020(5)	7.8	8.93	1.13
2020(6)	9.2	8.61	-0.59
2020(7)	10.4	8.42	-1.98
2020(8)	9.4	7.41	-1.99
2020(9)	10.2	8.85	-1.35
2020(10)	10.4	9.38	-1.02
2020(11)	8.6	9.41	0.81
Errors for 12 periods		RMSE	1.5536
		RMSPE	2.0247
		MAE	1.2458
		MAPE	15.0508

As we can see, unfortunately, all the errors have increased when compared to the first model. The graphical output already showed that the forecasting accuracy of this model seemed to be worse, the calculations confirm it.

In the **fourth model**, again both differencing were assumed, this time using an ARMA(0,2). Therefore, the model was an $ARIMA(0,\ 1,\ 2)\times(0,\ 1,\ 2)S$. In terms of diagnostics, the specification was nice as the parameters were all significant and autocorrelation was present only at

lag 5, however the forecasting accuracy was almost exactly the same as in the case of $ARIMA(0,\ 1,\ 1)\times(0,\ 1,\ 1)S$.
In the **fifth model**, again both differencing were assumed, this time using an ARMA(2,0). Therefore, the model was an $ARIMA(2,\ 1,\ 0)\times(2,\ 1,\ 0)S$. Unfortunately, the same issues were encountered as in the previous model. It looks like assuming both differencing (for the regular part and for the seasonal part) leads to a better specification in terms of diagnostics but worse in terms of forecasts.

Hence, the **sixth model** sees to removal of seasonal differencing. The models is an $ARIMA(0,\ 1,\ 1)\times(0,\ 0,\ 1)S$, so it's like an airline model with the only difference of not having the seasonal differencing. The graphical output:



Notice how the forecasts vary quite significantly from the previous models: on one hand the 2020(3) remains extraordinary, however the values of 2020(6) and 2020(7) look much more fitted to what has really happened, which is a nice improvement. Also, the residuals look much more constant along the whole dataset, in fact there are some major spikes, like in 2015, but within the rest of the observations we see a similar pattern, therefore an almost constant variance. Unfortunately, the ACF displays several significant autocorrelations, starting from the 2nd, 4th, 5th and 6th lags. The written output:

```
MODEL DEFINITION for UnRate
 Transformation:       No transformation
 ARIMA Model:  (0 1 1)(0 0 1)
 regARIMA Model Span: 1995.Jan to 2019.Nov

 MODEL ESTIMATION/EVALUATION
 Estimation converged in   18 ARMA iterations,   55 function evaluations.
 ARIMA Model:  (0 1 1)(0 0 1)
   Nonseasonal differences: 1
                                              Standard
 Parameter                     Estimate         Errors
```

```
----------------------------------------------------
Nonseasonal MA
  Lag  1                    .3159          .05456

Seasonal MA
  Lag 12                   -.5661          .04929

Variance                    .28802E+00
SE of Var                   .23595E-01
----------------------------------------------------

Likelihood Statistics
-----------------------------------------------------------------
Number of observations (nobs)                                 299
Effective number of observations (nefobs)                     298
Number of parameters estimated (np)                             3
Log likelihood (L)                                      -239.7483
AIC                                                      485.4966
AICC (F-corrected-AIC)                                   485.5782
Hannan Quinn                                             489.9363
BIC                                                      496.5879
-----------------------------------------------------------------

Roots of ARIMA Model
 Root                   Real  Imaginary    Modulus  Frequency
 ----------------------------------------------------------
 Nonseasonal MA
  Root  1             3.1654      .0000     3.1654      .0000
 Seasonal MA
  Root  1            -1.7664      .0000     1.7664      .5000
 ----------------------------------------------------------


FORECASTING
 Origin  2019.Nov
 Number        12

 Confidence intervals with coverage probability ( .95000)
  --------------------------------------
      Date       Lower  Forecast     Upper
  --------------------------------------
  2019.Dec        8.64      9.70     10.75
  2020.Jan        8.69      9.96     11.24
  2020.Feb        8.57     10.04     11.50
  2020.Mar        7.93      9.56     11.19
  2020.Apr        7.62      9.41     11.19
  2020.May        7.46      9.38     11.30
  2020.Jun        6.97      9.02     11.07
  2020.Jul        7.15      9.32     11.50
  2020.Aug        6.70      8.99     11.28
  2020.Sep        7.02      9.42     11.82
  2020.Oct        6.71      9.21     11.72
  2020.Nov        6.65      9.26     11.87
  --------------------------------------
```

As we can see, the parameters are significant, which is a nice consolation of the fact that log-likelihood and information criteria have worsened heavily. If we look at the errors instead:

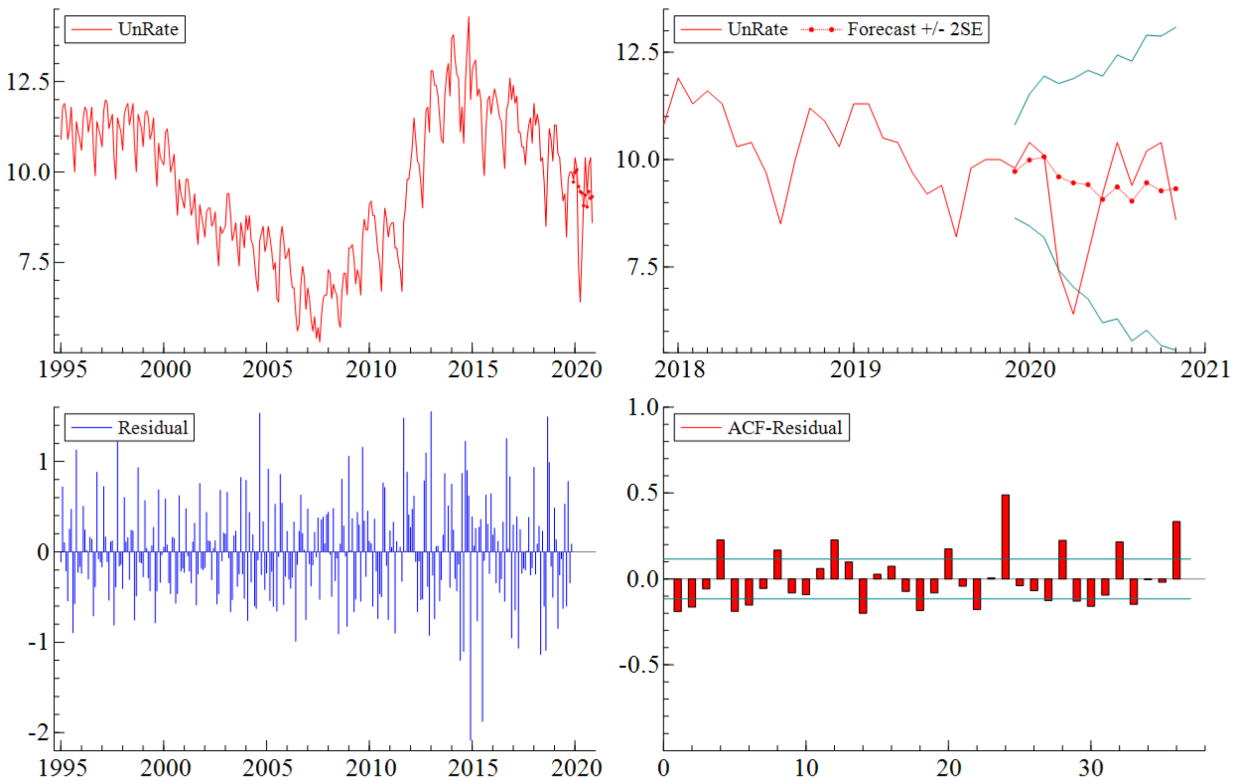
Last 12 observed values		***ARIMA(0,1,1)x(0,0,1)S***	
Date	UnRate	Forecast	Error
2019(12)	9.8	9.7	-0.10
2020(1)	10.4	9.96	-0.44
2020(2)	10.1	10.04	-0.06

2020(3)	7.4	9.56	2.16
2020(4)	6.4	9.41	3.01
2020(5)	7.8	9.38	1.58
2020(6)	9.2	9.02	-0.18
2020(7)	10.4	9.32	-1.08
2020(8)	9.4	8.99	-0.41
2020(9)	10.2	9.42	-0.78
2020(10)	10.4	9.21	-1.19
2020(11)	8.6	9.26	0.66
Errors for 12 periods		RMSE	1.2992
		RMSPE	1.7964
		MAE	0.9708
		MAPE	12.1491

We see that all the errors have decreased quite sensibly, even when compared to the first SARIMA specification. This is the confirmation of the fact that assuming both differencing (for the regular part and for the seasonal part) leads to a better specification in terms of diagnostics but worse in terms of forecasts, while assuming only first differencing for the regular part leads to a worse specification in terms of diagnostics but better in terms of forecasting accuracy.

The **seventh model**, an $ARIMA(0, 1, 2)x(0, 0, 2)S$ did not bring any improvement, apart from log-likelihood and information criteria.

Instead, the **eight model**, an $ARIMA(0, 1, 0)x(0, 0, 1)S$, did provide some interesting results:



It looks like the forecasts have gained a little more accuracy, as now 2020(3) is barely below the lower error bar. ACF presents quite a lot of significant autocorrelation unfortunately. The written output:

```
MODEL DEFINITION for UnRate
 Transformation:      No transformation
 ARIMA Model:  (0 1 0)(0 0 1)
 regARIMA Model Span: 1995.Jan to 2019.Nov

 MODEL ESTIMATION/EVALUATION
 Estimation converged in   12 ARMA iterations,   25 function evaluations.
 ARIMA Model:  (0 1 0)(0 0 1)
   Nonseasonal differences: 1
                                                     Standard
 Parameter                    Estimate                 Errors
 -----------------------------------------------------
 Seasonal MA
   Lag 12                       -.5487          .04976

 Variance                        .30704E+00
 SE of Var                       .25154E-01
 -----------------------------------------------------

 Likelihood Statistics
 ------------------------------------------------------------------
 Number of observations (nobs)                                  299
 Effective number of observations (nefobs)                      298
 Number of parameters estimated (np)                              2
 Log likelihood (L)                                       -249.0579
 AIC                                                       502.1158
 AICC (F-corrected-AIC)                                    502.1565
 Hannan Quinn                                              505.0756
 BIC                                                       509.5100
 ------------------------------------------------------------------

 Roots of ARIMA Model
  Root                  Real  Imaginary    Modulus  Frequency
  -----------------------------------------------------------
  Seasonal MA
   Root  1           -1.8226      .0000     1.8226      .5000
  -----------------------------------------------------------


 FORECASTING
  Origin  2019.Nov
  Number       12

  Confidence intervals with coverage probability ( .95000)
   ---------------------------------------
       Date      Lower  Forecast     Upper
   ---------------------------------------
   2019.Dec       8.64      9.72     10.81
   2020.Jan       8.45      9.99     11.53
   2020.Feb       8.18     10.06     11.95
   2020.Mar       7.43      9.60     11.77
   2020.Apr       7.03      9.46     11.89
   2020.May       6.76      9.42     12.08
   2020.Jun       6.20      9.07     11.95
   2020.Jul       6.29      9.36     12.44
   2020.Aug       5.78      9.03     12.29
   2020.Sep       6.03      9.46     12.90
   2020.Oct       5.67      9.27     12.88
   2020.Nov       5.56      9.32     13.08
   ---------------------------------------
```

If we don't focus on log-likelihood and information criteria, which are even worse than those of $ARIMA(0, 1, 1)\times(0, 0, 1)S$, the only parameter of the model is statistically significantly different from zero, and if we contemplate the errors:

Last 12 observed values		***ARIMA(0,1,0)x(0,0,1)S***	
Date	UnRate	Forecast	Error
2019(12)	9.8	9.72	-0.08
2020(1)	10.4	9.99	-0.41
2020(2)	10.1	10.06	-0.04
2020(3)	7.4	9.6	2.20
2020(4)	6.4	9.46	3.06
2020(5)	7.8	9.42	1.62
2020(6)	9.2	9.07	-0.13
2020(7)	10.4	9.36	-1.04
2020(8)	9.4	9.03	-0.37
2020(9)	10.2	9.46	-0.74
2020(10)	10.4	9.27	-1.13
2020(11)	8.6	9.32	0.72
Errors for 12 periods		RMSE	1.3097
		RMSPE	1.8202
		MAE	0.9617
		MAPE	12.1090

We can see that RMSE and RMSPE are just lightly higher than in $ARIMA(0, 1, 1)\times(0, 0, 1)S$, while MAE and MAPE are a little smaller.

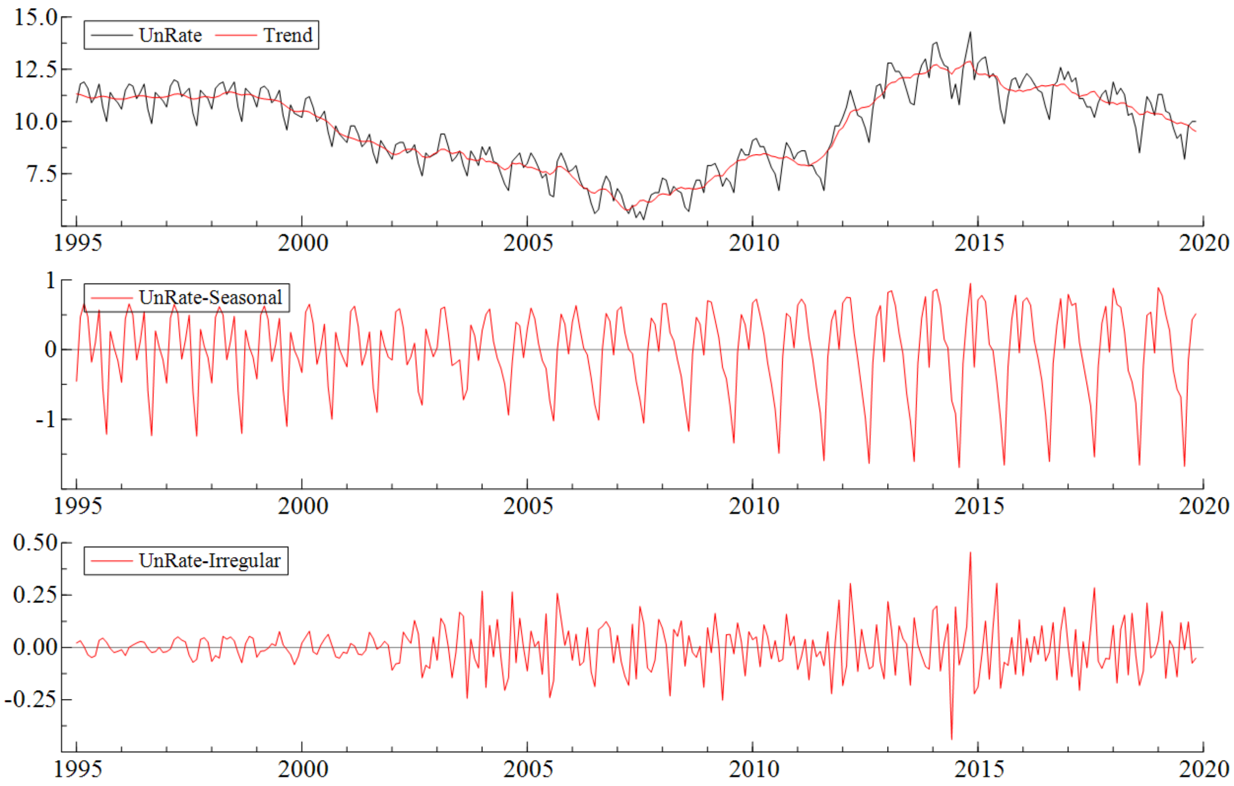
Afterwards, a few other specifications were tried, but none would lead to lower errors unfortunately. Summing up, the best models in terms of forecasts with relative errors are shown in the table below:

	ARIMA(1,1,0)x(1,0,0) S		**ARIMA(0,1,1)x(0,0,1) S**		**ARIMA(0,1,0)x(0,0,1) S**	
Date	Forecast	Error	Forecast	Error	Forecast	Error
2019(12)	9.46	-0.34	9.7	-0.1	9.72	-0.08
2020(1)	10.26	-0.14	9.96	-0.44	9.99	-0.41
2020(2)	10.25	0.15	10.04	-0.06	10.06	-0.04
2020(3)	9.63	2.23	9.56	2.16	9.6	2.2
2020(4)	9.55	3.15	9.41	3.01	9.46	3.06
2020(5)	9.01	1.21	9.38	1.58	9.42	1.62
2020(6)	8.62	-0.58	9.02	-0.18	9.07	-0.13
2020(7)	8.77	-1.63	9.32	-1.08	9.36	-1.04
2020(8)	7.84	-1.56	8.99	-0.41	9.03	-0.37
2020(9)	9.08	-1.12	9.42	-0.78	9.46	-0.74
2020(10)	9.24	-1.16	9.21	-1.19	9.27	-1.13
2020(11)	9.24	0.64	9.26	0.66	9.32	0.72

RMSE	1.4420	RMSE	1.2992	RMSE	1.3097
RMSPE	1.9254	RMSPE	1.7964	RMSPE	1.8202
MAE	1.1592	MAE	0.9708	MAE	0.9617
MAPE	14.1097	MAPE	12.1491	MAPE	12.1090

Unobserved Components Models

The **first model** is a simple default specification, as set by OxMetrics, with the estimation ending one year (12 observations) before the dataset's final value. Therefore, the model will be composed as Y = Trend + Seasonal + Irregular, with all these components being stochastic. The graphical output of the model is the following:



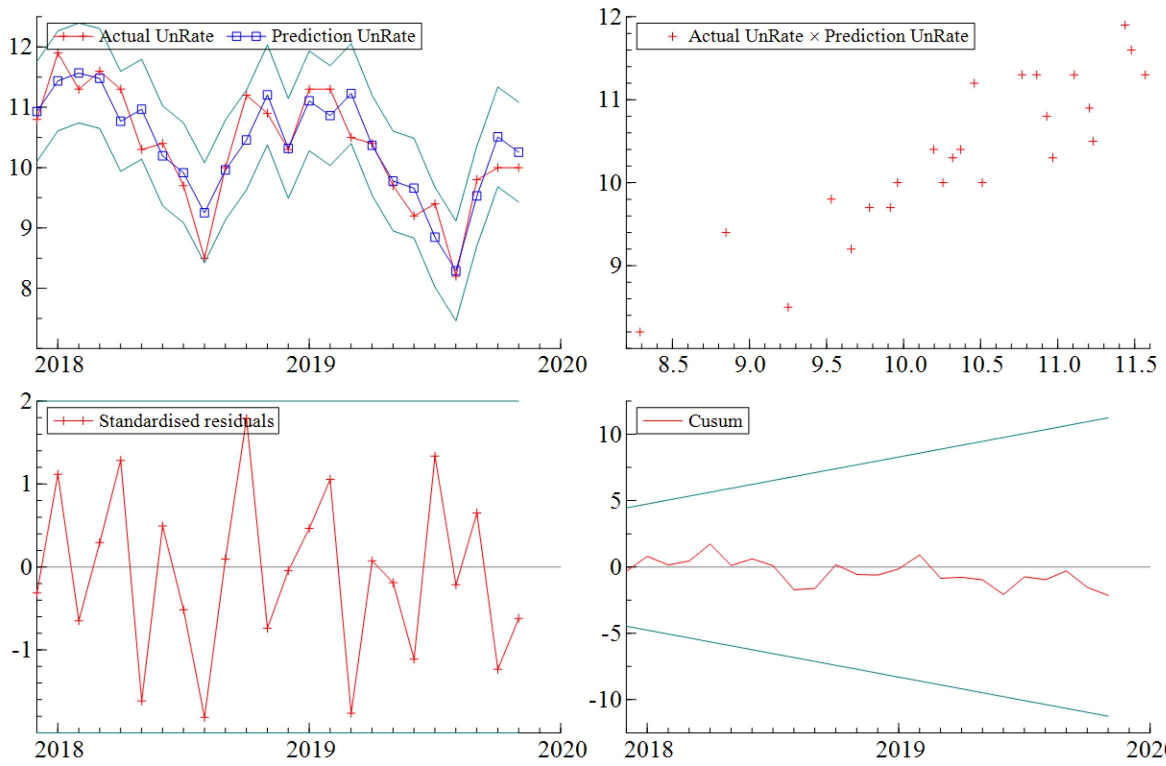
As we can see, a sort of trend is present and follows the actual series approximately. The Irregular part is also present, meaning that there are some values that could be taken into account as outliers. The written results are shown below:

```
UC( 1) Estimation done by Maximum Likelihood (exact score)
       The selection sample is: 1995(1) - 2019(11) (N = 1, T = 299)
       The dependent variable Y is: UnRate
       The model is: Y = Trend + Seasonal + Irregular

Profile Log-Likelihood:                   236.1567
Akaike Information Criterion (AIC):        -1.4659
Bayesian Information Criterion (BIC):      -1.2555
Prediction error variance:                  0.1639

Summary statistics:
                   UnRate
 T                    299
 Normality         40.486
 H(95)             6.8151
 DW                1.8218
```

```
 r(1)           0.088282
 q                    24
 p                     3
 r(q)          -0.060842
 Q(q,q-p)             38
 Rs^2            0.41663

Variances of disturbances:
                   Value    (q-ratio)
Level          0.0340922  (   0.9940)
Slope        0.000103798  ( 0.003026)
Seasonal     0.000332871  ( 0.009706)
Irregular      0.0342964  (    1.000)

State vector analysis at period 2019(11):
                            Value     Prob
Level                     9.53552 [0.00000]
Slope                    -0.05678 [0.21034]
Seasonal chi2 test      116.12440 [0.00000]

Seasonal effects:
                Period       Value     Prob
                     1     0.90437 [0.00006]
                     2     0.77108 [0.00047]
                     3     0.51249 [0.01694]
                     4     0.27169 [0.19421]
                     5    -0.29103 [0.15490]
                     6    -0.56726 [0.00470]
                     7    -0.67183 [0.00064]
                     8    -1.67433 [0.00000]
                     9    -0.14765 [0.42924]
                    10     0.43259 [0.01960]
                    11     0.51569 [0.00718]
                    12    -0.05581 [0.80543]
```

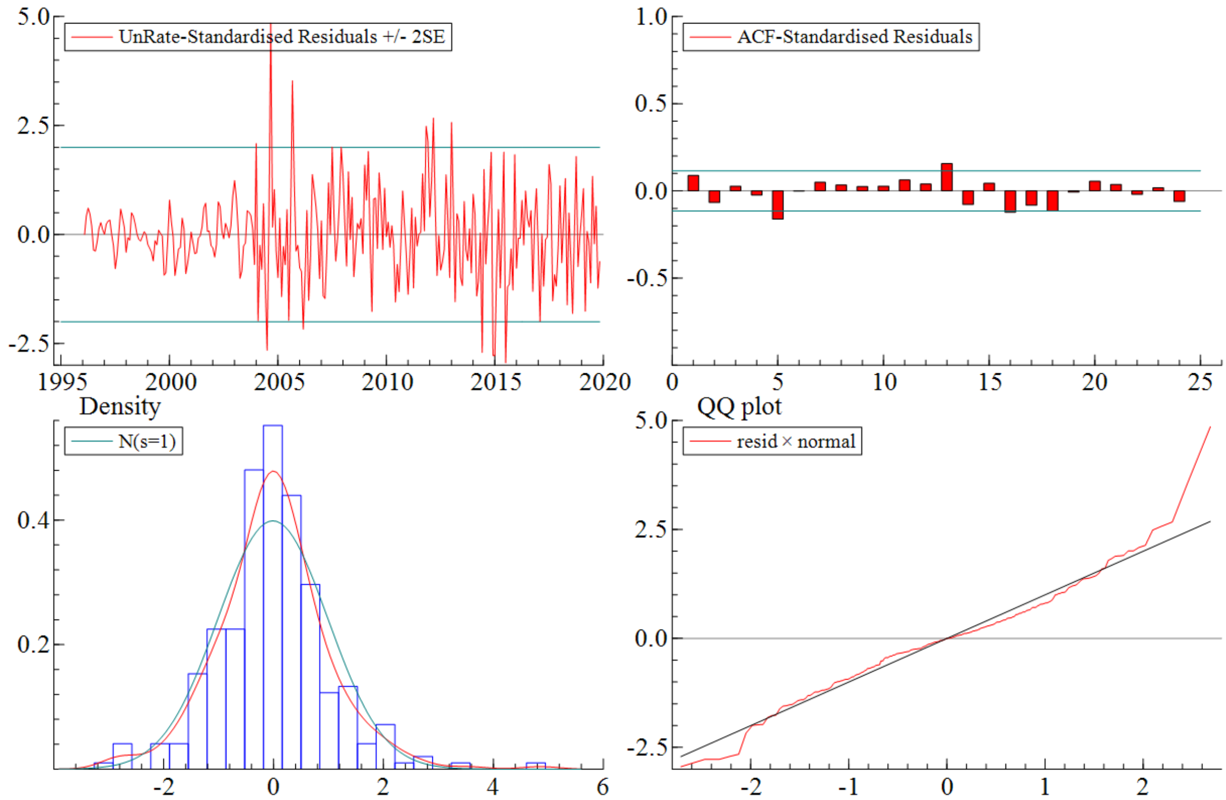
The residuals are non-normally distributed, as the value of Normality is way above the 5.99 value from the Chi-square distribution with 2 degrees of freedom. The value of DW is just lightly below 2, therefore we could say that there is no statistically significant autocorrelation of order 1. In fact, such autocorrelation has a very low value, equal to 0.088. The value of R^2 is about 41.66%, which is not too bad for the first specification. According to the Variances of disturbances section, the component with the highest variance for this model is the Irregular, closely followed by Level. All the components should remain stochastic for now, as none of them presents a q-ratio equal to zero. Looking at the seasonal effects section, we can see that the unemployment rate between May and September was lower than average, in the next two months it was higher, in December it was lower again, and finally between January and April it was above the average.

Some tests will be welcome, more precisely, outlier and break diagnostics, large absolute values, residuals graphics, prediction analysis and finally forecasts.
The graphical output for the prediction analysis:

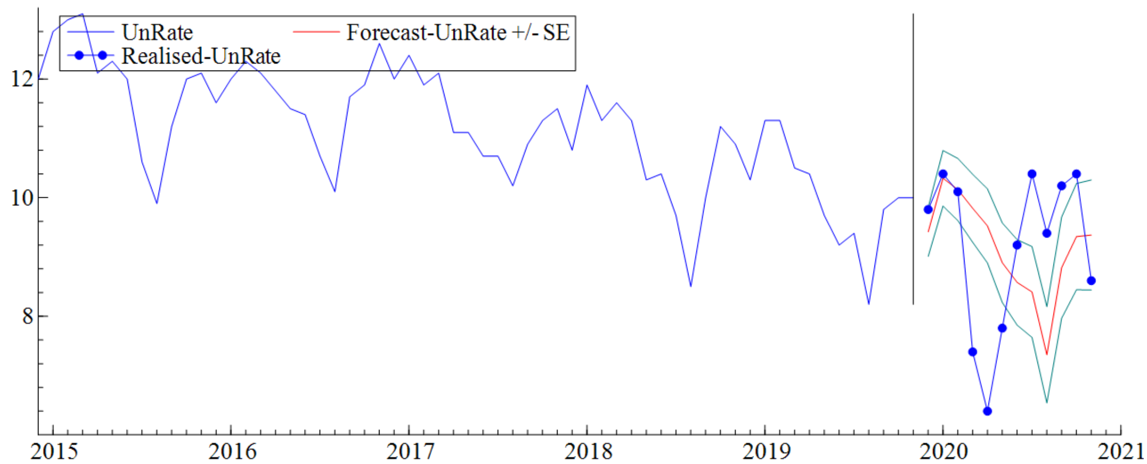


As we can see, the prediction values are not too far from the actual values. The actual vs prediction plot shows that the values follow a hypothetical line to some extent, which is good. The standardized residuals never go beyond the confidence interval, however they vary a lot across the whole graph. This is visible also in the Cusum plot, where the line at the end does not seem to go towards zero.

The output of residuals comes next:



At the beginning the residuals are very nice, but then they start to vary a lot, presenting also some outliers in 2004/2005 and 2014/2015. The ACF plot shows statistically significant autocorrelation only at lag 5, as it had already happened in some of the SARIMA specifications. The distribution presents some excess kurtosis and also a particular error, which seems to be due to the highest of the positive outliers from the standardized residuals' plot, that causes the distribution to be non-symmetric, and perhaps non-normal either. The QQplot presents outliers especially in the positive extreme. In general, after seeing the Irregular plot and this residuals graphics, we can be quite sure that we should try to adopt some interventions within the next model. Next the graphical forecasts for this model:



As we can see, the forecasts are not accurate at all; in several occasions the actual unemployment rates were beyond the error bars.
The whole written output is shown below:

```
Values larger than 3 for Irregular residual:
                    Value     Prob
2014(6)          -3.93033 [0.00005]
2014(11)          4.06260 [0.00003]

Normality test for Irregular residual:
                      Value
Sample size             299
Mean            -0.00024674
St.Dev              0.99394
Skewness            0.15783
Excess kurtosis        1.23
Minimum             -3.9303
Maximum              4.0626

                     Chi^2      Prob
Skewness            1.2414  [ 0.2652]
Kurtosis            18.849  [ 0.0000]
Bowman-Shenton       20.09  [ 0.0000]

Values larger than 3 for Level residual:
                    Value     Prob
2012(3)           3.09025 [0.00109]
2014(12)         -3.98418 [0.00004]
2015(7)          -3.33297 [0.00048]

Normality test for Level residual:
                      Value
Sample size             298
Mean             -0.0016433
St.Dev              0.99232
Skewness           0.019447
Excess kurtosis      1.2057
Minimum             -3.9842
Maximum              3.0902

                     Chi^2      Prob
Skewness          0.018783  [ 0.8910]
Kurtosis             18.05  [ 0.0000]
Bowman-Shenton      18.069  [ 0.0001]

Normality test for Slope residual:
                      Value
Sample size             297
Mean               -0.10059
St.Dev              0.97048
Skewness            0.14623
Excess kurtosis    0.081456
Minimum             -2.4342
Maximum              2.4663

                     Chi^2      Prob
Skewness            1.0584  [ 0.3036]
Kurtosis          0.082109  [ 0.7745]
Bowman-Shenton      1.1405  [ 0.5654]

Normality test for Residuals UnRate:
                      Value
Sample size             286
Mean              -0.013253
St.Dev              0.99991
Skewness            0.40407
Excess kurtosis       2.436
Minimum             -2.9437
Maximum              4.8593

                     Chi^2      Prob
```

```
Skewness            7.7826  [ 0.0053]
Kurtosis            70.716  [ 0.0000]
Bowman-Shenton      78.498  [ 0.0000]

Goodness-of-fit based on Residuals UnRate:
                                                              Value
Prediction error variance (p.e.v.)                          0.16388
Prediction error mean deviation (m.d.)                      0.11868
Ratio p.e.v. / m.d. in squares                               1.2137
Akaike Information Criterion (AIC) based on p.e.v.           -1.715
Bayesian Information Criterion (BIC) based on p.e.v.        -1.5417
Coefficient of determination R^2                            0.96032
... based on differences Rd^2                                0.6694
... based on diff around seas mean Rs^2                     0.41663

Serial correlation statistics for Residuals UnRate:
Durbin-Watson test is 1.82184
Asymptotic deviation for correlation is 0.0591312
   Lag       df     Ser.Corr    BoxLjung      Prob
     4        1    -0.023614      3.8886  [ 0.0486]
     5        2     -0.16223      11.603  [ 0.0030]
     6        3  -0.00058151      11.603  [ 0.0089]
     7        4     0.049599      12.329  [ 0.0151]
     8        5     0.034042      12.673  [ 0.0266]
    12        9     0.039403      14.725  [ 0.0988]
    24       21    -0.060842          38  [ 0.0129]
    36       33    -0.019062       63.83  [ 0.0010]

Forecasts with 68% confidence interval from period 2019(11) forwards:
          Forecast     Std.Err   Leftbound  Rightbound
1          9.42292     0.41391     9.00901     9.83684
2         10.32632     0.46754     9.85878    10.79387
3         10.13624     0.52401     9.61223    10.66026
4          9.82087     0.57250     9.24837    10.39337
5          9.52329     0.62458     8.89870    10.14787
6          8.90379     0.67095     8.23284     9.57474
7          8.57077     0.72048     7.85029     9.29125
8          8.40941     0.76524     7.64418     9.17465
9          7.35013     0.81201     6.53812     8.16214
10         8.82002     0.85379     7.96623     9.67381
11         9.34348     0.89498     8.44849    10.23846
12         9.36979     0.92853     8.44126    10.29833

Forecast accuracy measures from period 2019(11) forwards:
             Error        RMSE       RMSPE         MAE        MAPE
1         -0.37708     0.37708     0.38477     0.37708     3.84771
2         -0.07368     0.27167     0.27665     0.22538     2.27807
3          0.03624     0.22281     0.22683     0.16233     1.63833
4          2.42087     1.22572     1.64748     0.72697     9.40736
5          3.12329     1.77564     2.63334     1.20623    17.28616
6          1.10379     1.68240     2.47235     1.18916    16.76365
7         -0.62923     1.57565     2.30350     1.10917    15.34592
8         -1.99059     1.63329     2.25849     1.21935    15.82021
9         -2.04987     1.68467     2.24998     1.31163    16.48542
10        -1.37998     1.65673     2.17697     1.31846    16.18980
11        -1.05652     1.61143     2.09814     1.29465    15.64153
12         0.76979     1.55875     2.02537     1.25091    15.08400
```

As we can see, there is non-normality of residuals for all the components, apart from Slope. We can see that there are 2 values which are larger than 3 standard deviations from the Irregular component, namely 2014(6) and 2014(11), while there are 3 observations for the Level component, namely 2012(3), 2014(12) and 2015(7). Such observations could be considered for the interventions later. Since we considered the errors after 12 observations, we will do so even in Unobserved

Components Models. Hence, by looking at the errors, we can see that these forecasts are worse than those from any of the best 3 models obtained with SARIMA.

Let's set the interventions. Therefore, the **second model** takes into account the values larger than 3 standard deviations. It is still a Y = Trend + Seasonal + Irregular + Interventions model. The following interventions are applied:

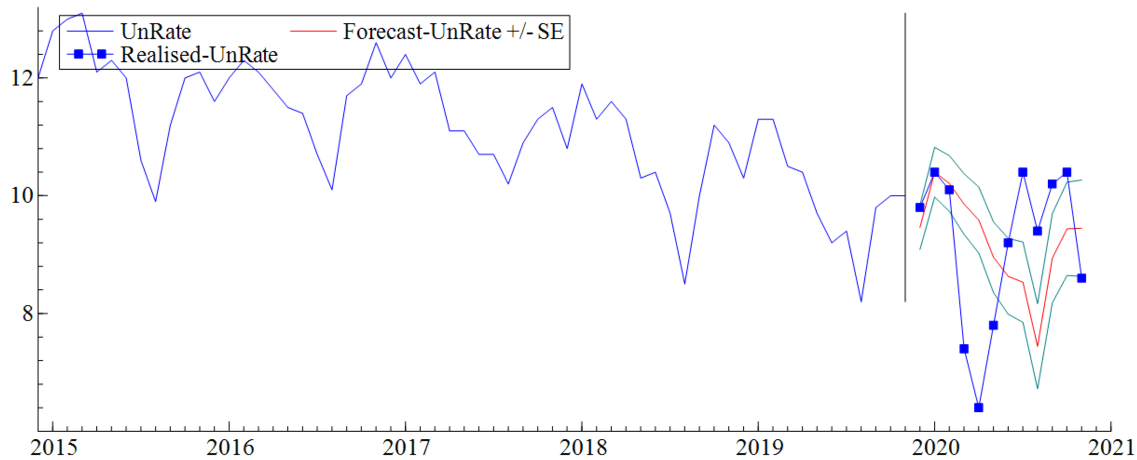
- 2014(6) as Irregular,
- 2014(11) as Irregular,
- 2012(3) as Level,
- 2014(12) as Level,
- 2015(7) as Level.

Here are the written outputs:

```
UC( 2) Estimation done by Maximum Likelihood (exact score)
       The selection sample is: 1995(1) - 2019(11) (N = 1, T = 299)
       The dependent variable Y is: UnRate
       The model is: Y = Trend + Seasonal + Irregular + Interventions

Profile Log-Likelihood:                 258.2240
Akaike Information Criterion (AIC):      -1.5801
Bayesian Information Criterion (BIC):    -1.3078
Prediction error variance:                0.1325

Summary statistics:
                   UnRate
 T                    299
 Normality         42.345
 H(93)             4.6586
 DW                1.8066
 r(1)            0.095396
 q                     24
 p                      3
 r(q)           -0.035555
 Q(q,q-p)          42.037
 Rs^2             0.53642

Variances of disturbances:
                   Value    (q-ratio)
Level          0.0273266  (    1.000)
Slope         7.44397e-05  ( 0.002724)
Seasonal      0.000339067  (  0.01241)
Irregular      0.0242147  (   0.8861)

State vector analysis at period 2019(11):
                             Value     Prob
Level                     10.37112 [0.00000]
Slope                     -0.04916 [0.21250]
Seasonal chi2 test       131.36878 [0.00000]

Seasonal effects:
                Period       Value     Prob
                     1     0.93671 [0.00001]
                     2     0.79440 [0.00016]
                     3     0.49367 [0.01568]
                     4     0.27297 [0.16945]
                     5    -0.31081 [0.10862]
                     6    -0.58570 [0.00200]
                     7    -0.63582 [0.00058]
                     8    -1.67777 [0.00000]
                     9    -0.13275 [0.44647]
                    10     0.41779 [0.01558]
```

```
                    11      0.47730 [0.00779]
                    12     -0.04999 [0.81790]

Regression effects in final state at time 2019(11):
                     Coefficient         RMSE     t-value      Prob
Outlier 2014(6)         -1.08665      0.27609    -3.93591 [0.00010]
Outlier 2014(11)         0.78126      0.32278     2.42044 [0.01614]
Level break 2012(3)      1.00970      0.29103     3.46943 [0.00060]
Level break 2014(12)    -0.88266      0.34445    -2.56253 [0.01091]
Level break 2015(7)     -0.93650      0.29466    -3.17826 [0.00165]
```

As we can see, the issue of normality remains, but one nice outcome is the significance of the proposed interventions. In terms of diagnostics, this specification is better than the first one, as now log-likelihood has become larger and information criteria have diminished. DW did not change significantly, while R^2 increased to 53.64%. Also, now the greatest variance lays in the Level component. However, when we look at the forecasts:

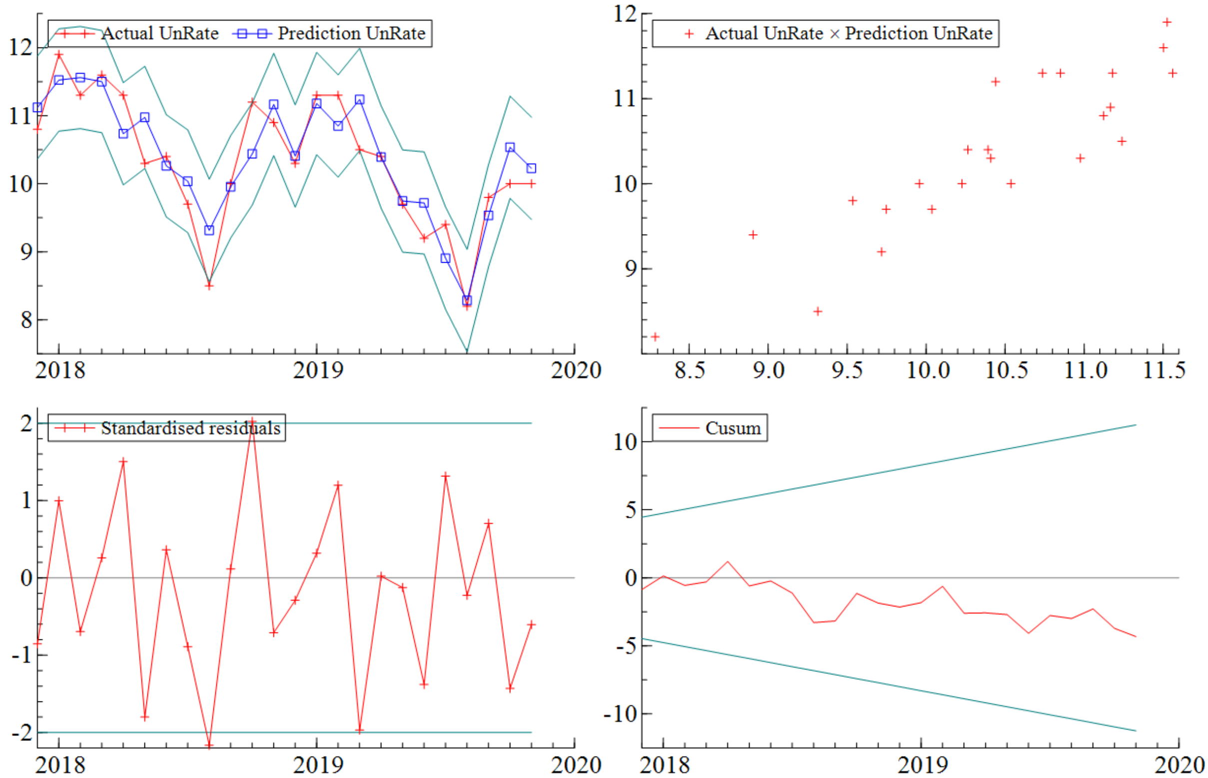


We can see that there has not been any major improvement. According to the written forecasts instead:

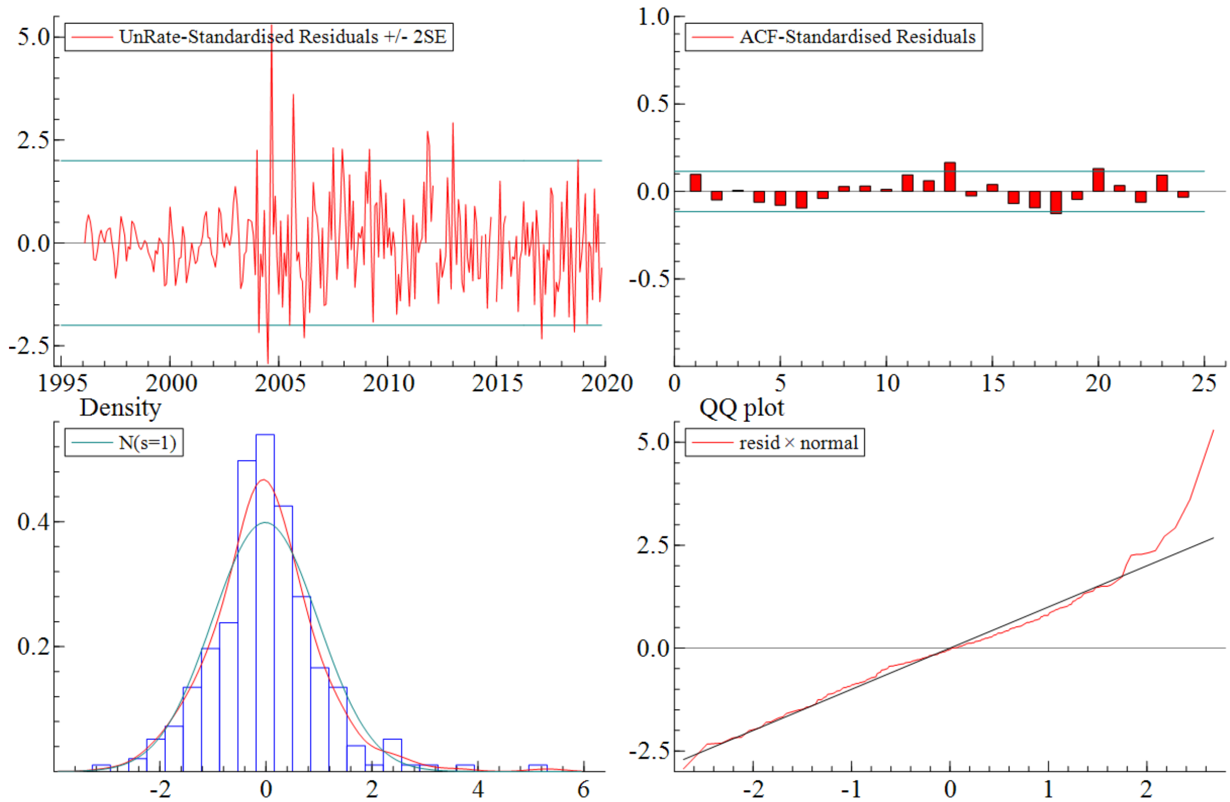
```
Forecasts with 68% confidence interval from period 2019(11) forwards:
          Forecast      Std.Err   Leftbound   Rightbound
1          9.46250      0.37570     9.08680      9.83821
2         10.40004      0.42294     9.97710     10.82298
3         10.20857      0.47317     9.73540     10.68174
4          9.85868      0.51527     9.34341     10.37395
5          9.58882      0.56096     9.02786     10.14979
6          8.95588      0.60087     8.35501      9.55675
7          8.63183      0.64386     7.98797      9.27569
8          8.53255      0.68207     7.85047      9.21462
9          7.44143      0.72217     6.71926      8.16361
10         8.93729      0.75735     8.17994      9.69464
11         9.43868      0.79179     8.64689     10.23047
12         9.44902      0.81886     8.63016     10.26789

Forecast accuracy measures from period 2019(11) forwards:
             Error         RMSE       RMSPE         MAE         MAPE
1         -0.33750      0.33750     0.34438     0.33750      3.44384
2          0.00004      0.23865     0.24352     0.16877      1.72213
3          0.10857      0.20469     0.20829     0.14870      1.50641
4          2.45868      1.24205     1.67103     0.72620      9.43615
5          3.18882      1.80773     2.68310     1.21872     17.51399
6          1.15588      1.71636     2.52293     1.20825     17.06482
7         -0.56817      1.60349     2.34741     1.11681     15.50924
8         -1.86745      1.63881     2.28574     1.21064     15.81512
```

9	-1.95857	1.67735	2.26417	1.29374	16.37298
10	-1.26271	1.64061	2.18336	1.29064	15.97363
11	-0.96132	1.59089	2.10032	1.26070	15.36180
12	0.84902	1.54275	2.03100	1.22639	14.90434

The accuracy has slightly increased as all the errors, apart from RMSE, have decreased. Looking at the prediction analysis:



We can see that the patterns remained similar, apart from the Cusum, which is more negative, while in the case of the residuals graphics:



It is visible that now there is no more significant autocorrelation, at least within the first 12 lags. Also, the QQplot shows more fitted values in the negative extreme, but still the issue in the positive extreme is visible.
Finally, we may say that this specification is definitely better in terms of diagnostics and autocorrelation, and the forecasts have also improved, even if by little.

In the **third model** I have tried to add two more interventions related to 2004(5) and 2004(7), since the residuals for those observations are very large. However, the interventions proved to be insignificant and lead to worse forecasts.

In the **fourth model**, I removed the interventions for 2004(5) and 2004(7), and added 2007(8) as a Slope; in fact, that is the point in time when the slope of the actual series changes from downward to upward. Such addition did not change the model much in terms of diagnostics, and neither in terms of forecasts.

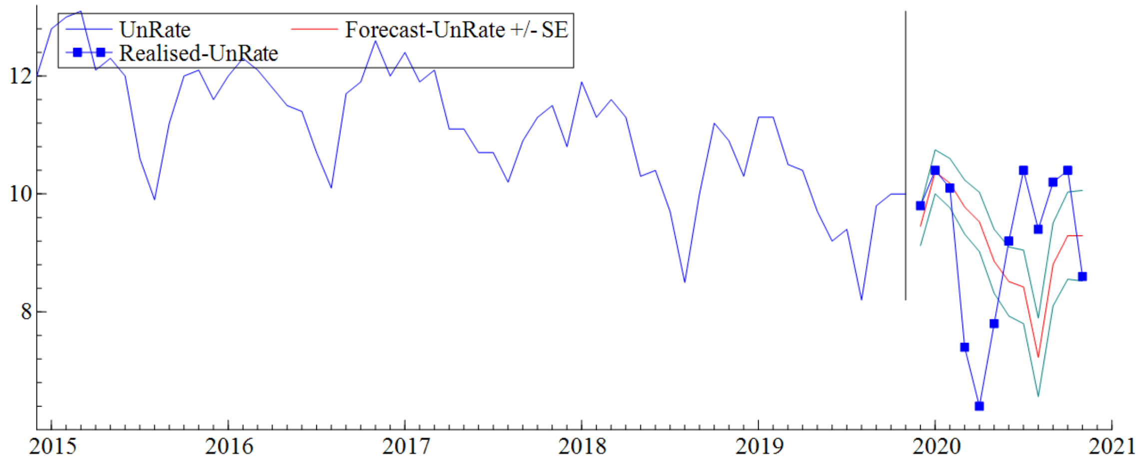
In the **fifth model**, I assumed the presence of a short cycle, however it did not prove helpful, as both the forecasts and the diagnostics became slightly worse.

In the **sixth model**, I tried to use the automatic intervention setting. Therefore, the model which was created follows:

```
UC( 9) Estimation done by Maximum Likelihood (exact score)
       The selection sample is: 1995(1) - 2019(11) (N = 1, T = 299)
       The dependent variable Y is: UnRate
       The model is: Y = Trend + Seasonal + Irregular + Interventions

Profile Log-Likelihood:                 275.9159
Akaike Information Criterion (AIC):      -1.6583
Bayesian Information Criterion (BIC):    -1.3118
Prediction error variance:                0.1080
```

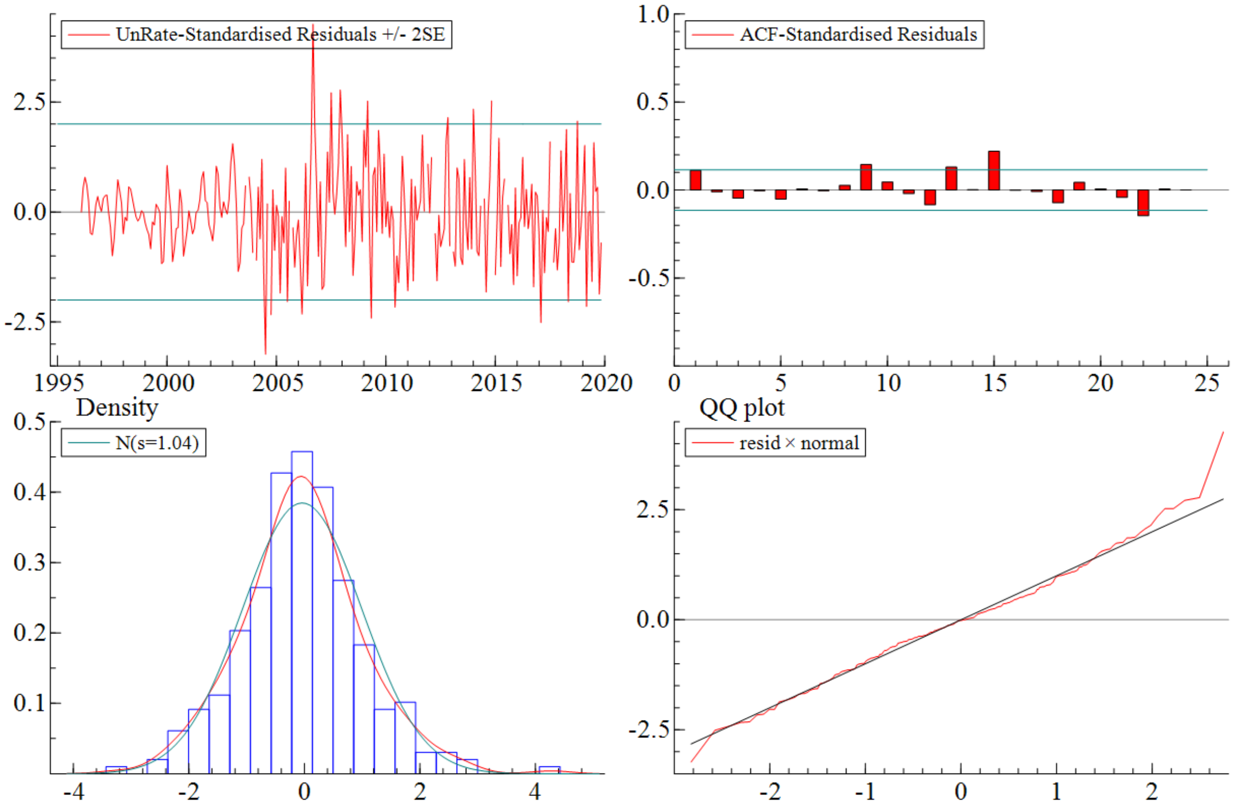
```
Summary statistics:
                  UnRate
 T                   299
 Normality        11.556
 H(91)            3.8686
 DW               1.7805
 r(1)            0.10723
 q                    24
 p                     3
 r(q)         -0.0025935
 Q(q,q-p)         43.452
 Rs^2            0.63027

Variances of disturbances:
                   Value    (q-ratio)
Level          0.0187365  (    1.000)
Slope        0.000116414  ( 0.006213)
Seasonal     0.000303968  (  0.01622)
Irregular      0.0167671  (   0.8949)

State vector analysis at period 2019(11):
                            Value     Prob
Level                     6.92131 [0.00000]
Slope                    -0.06198 [0.13083]
Seasonal chi2 test      175.83042 [0.00000]

Seasonal effects:
                Period       Value     Prob
                     1     0.95296 [0.00000]
                     2     0.82101 [0.00002]
                     3     0.47826 [0.00963]
                     4     0.28942 [0.10632]
                     5    -0.31731 [0.06850]
                     6    -0.59752 [0.00045]
                     7    -0.62851 [0.00015]
                     8    -1.76087 [0.00000]
                     9    -0.12259 [0.42809]
                    10     0.42829 [0.00524]
                    11     0.48846 [0.00229]
                    12    -0.03161 [0.87276]

Regression effects in final state at time 2019(11):
                     Coefficient        RMSE     t-value     Prob
Outlier 2014(6)         -1.09573     0.23868    -4.59085 [0.00001]
Level break 2012(3)      1.01854     0.25169     4.04676 [0.00007]
Level break 2014(12)    -1.27582     0.25219    -5.05898 [0.00000]
Level break 2015(7)     -0.92692     0.25525    -3.63137 [0.00034]
Outlier 2017(8)          0.80494     0.23685     3.39857 [0.00078]
Level break 2011(11)     0.94535     0.25164     3.75672 [0.00021]
Outlier 2003(9)         -0.45340     0.23997    -1.88943 [0.05989]
Outlier 2004(1)          0.72371     0.23575     3.06978 [0.00236]
Level break 2004(9)      0.99468     0.26459     3.75932 [0.00021]
Level break 2005(9)      1.10033     0.26193     4.20078 [0.00004]
Level break 2013(1)      0.76946     0.25213     3.05188 [0.00250]
```

The specification improved significantly in terms of normality, log-likelihood, information criteria and R^2, however, it did so by implementing an important number of interventions, 11 in total! The Level component presents the highest disturbances in this model.
The forecasts are the following:

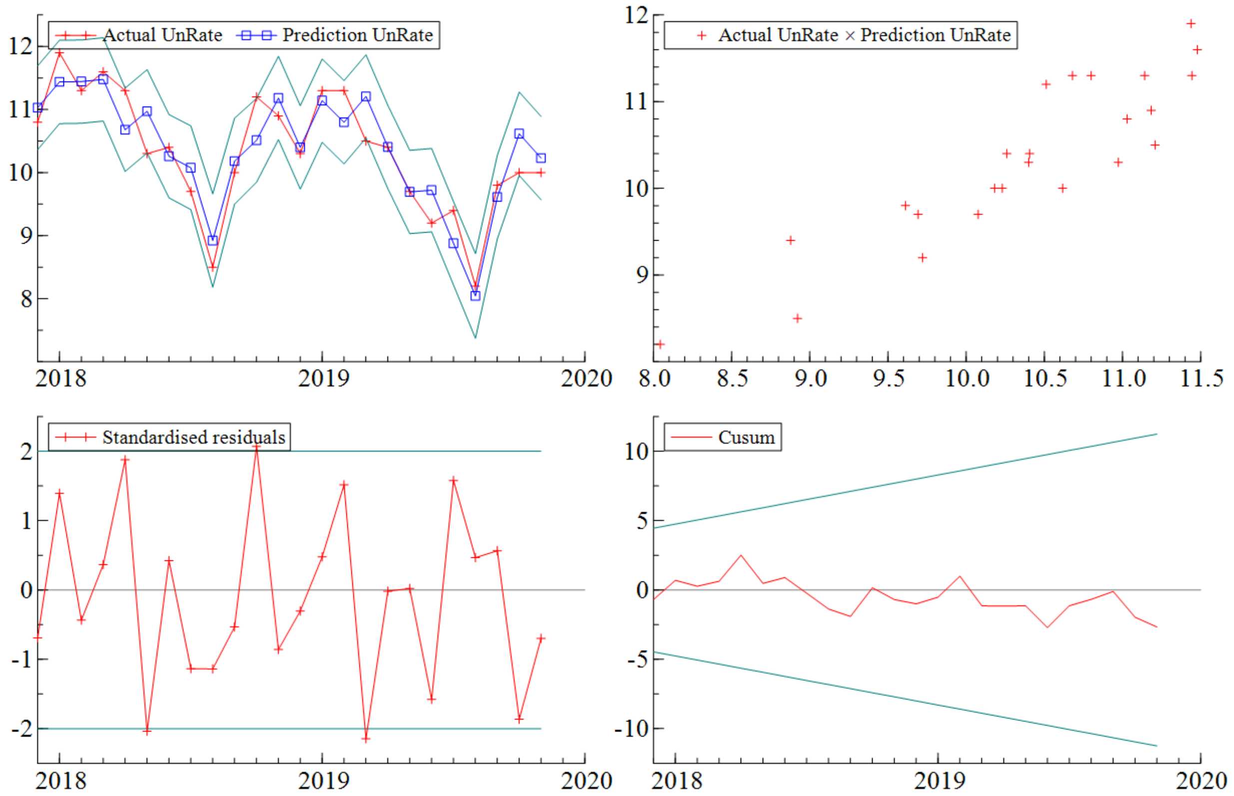



```
Forecasts with 68% confidence interval from period 2019(11) forwards:
          Forecast     Std.Err   Leftbound  Rightbound
1          9.45335     0.33043     9.12292     9.78379
2         10.37595     0.37216    10.00379    10.74811
3         10.18202     0.41866     9.76336    10.60067
4          9.77729     0.45833     9.31896    10.23562
5          9.52648     0.50266     9.02383    10.02914
6          8.85778     0.54199     8.31578     9.39977
7          8.51559     0.58512     7.93047     9.10071
8          8.42262     0.62403     7.79859     9.04664
9          7.22829     0.66570     6.56259     7.89399
10         8.80459     0.70226     8.10233     9.50685
11         9.29349     0.73880     8.55469    10.03230
12         9.29169     0.76851     8.52318    10.06020

Forecast accuracy measures from period 2019(11) forwards:
             Error        RMSE       RMSPE         MAE        MAPE
1         -0.34665     0.34665     0.35372     0.34665     3.53720
2         -0.02405     0.24570     0.25065     0.18535     1.88422
3          0.08202     0.20613     0.20996     0.15090     1.52683
4          2.37729     1.20198     1.61654     0.70750     9.17652
5          3.12648     1.76374     2.61982     1.19130    17.11147
6          1.05778     1.66697     2.45480     1.16904    16.51976
7         -0.68441     1.56484     2.29003     1.09981    15.22254
8         -1.97738     1.62216     2.24513     1.20951    15.69638
9         -2.17171     1.69206     2.25247     1.31642    16.51938
10        -1.39541     1.66477     2.18023     1.32432    16.23548
11        -1.10651     1.62198     2.10337     1.30452    15.72676
12         0.69169     1.56571     2.02717     1.25345    15.08644
```

Unfortunately, there was no improvement in terms of forecast in this model.
Instead, according to residuals graphics, the ACF presents some statistically significant autocorrelation at the 9th lag, and a quite nice Cusum, moreover the distribution is closer to normality:



For what concerns the prediction analysis:

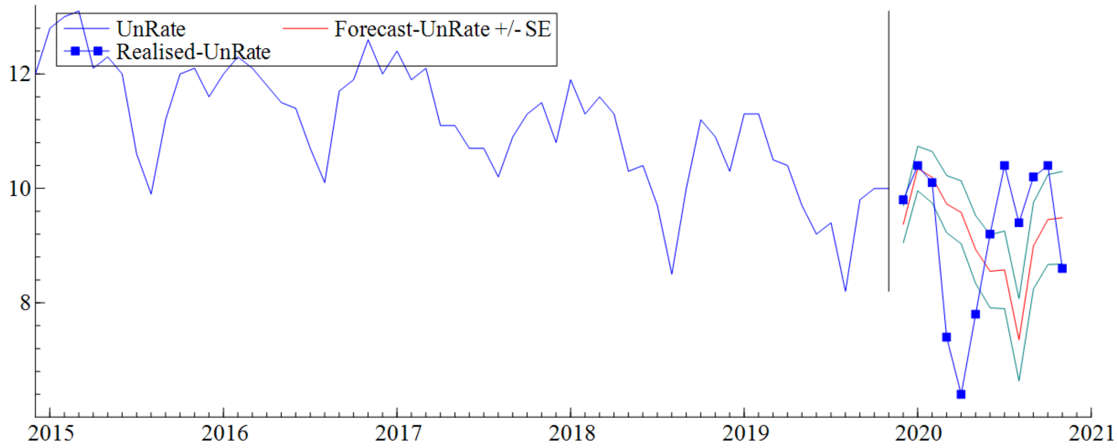


We can see that the theoretical values still follow approximately the empirical ones. Within the other plots, there seems to be no major improvement.

In the **seventh model** I used the automatic interventions again, this time however I unchecked the Irregular part. The model follows:

```
UC(15) Estimation done by Maximum Likelihood (exact score)
       The selection sample is: 1995(1) - 2019(11) (N = 1, T = 299)
       The dependent variable Y is: UnRate
       The model is: Y = Trend + Seasonal + Irregular + Interventions

Profile Log-Likelihood:                 259.1455
Akaike Information Criterion (AIC):      -1.5662
Bayesian Information Criterion (BIC):    -1.2568
Prediction error variance:                0.1252

Summary statistics:
                   UnRate
 T                    299
 Normality          5.667
 H(92)             6.2713
 DW                2.1173
 r(1)           -0.060072
 q                     24
 p                      3
 r(q)            0.013342
 Q(q,q-p)          61.255
 Rs^2             0.56828

Variances of disturbances:
                    Value    (q-ratio)
Level           0.0351563  (    1.000)
Slope         4.28965e-05  ( 0.001220)
Seasonal      0.000361468  (  0.01028)
Irregular        0.000000  (   0.0000)

State vector analysis at period 2019(11):
                             Value     Prob
Level                      6.32873 [0.00000]
Slope                     -0.04279 [0.23312]
Seasonal chi2 test       163.62790 [0.00000]

Seasonal effects:
                Period       Value     Prob
                     1     0.94021 [0.00000]
                     2     0.83062 [0.00003]
                     3     0.40529 [0.03297]
                     4     0.30222 [0.10090]
                     5    -0.30394 [0.08826]
                     6    -0.64156 [0.00022]
                     7    -0.57371 [0.00061]
                     8    -1.75420 [0.00000]
                     9    -0.06855 [0.65758]
                    10     0.43369 [0.00456]
                    11     0.50748 [0.00240]
                    12    -0.07755 [0.70442]

Regression effects in final state at time 2019(11):
                     Coefficient        RMSE     t-value     Prob
Outlier 2014(6)         -1.08483     0.21565    -5.03051 [0.00000]
Level break 2012(3)      1.07958     0.25906     4.16723 [0.00004]
Level break 2015(7)     -0.89128     0.26593    -3.35163 [0.00092]
Outlier 2004(9)          0.90176     0.21560     4.18251 [0.00004]
Outlier 2009(5)         -0.66274     0.21030    -3.15136 [0.00180]
Outlier 2017(8)          0.74975     0.21085     3.55589 [0.00044]
Level break 2005(9)      1.13828     0.26551     4.28722 [0.00003]
Level break 2011(11)     0.95525     0.25909     3.68694 [0.00027]
Level break 2013(1)      0.88196     0.25913     3.40356 [0.00076]
```

First of all, the Normality value is now below 5.99, indicating that residuals follow a normal distribution, for the first time! Log-likelihood and information criteria's variation is not great. DW has become larger than 2, but we can still assume that there is no autocorrelation. We can see that again OxMetrics has implemented several interventions. If we look at the forecasts:



We can see that again the forecasts are not very precise. If we look at the written results however:

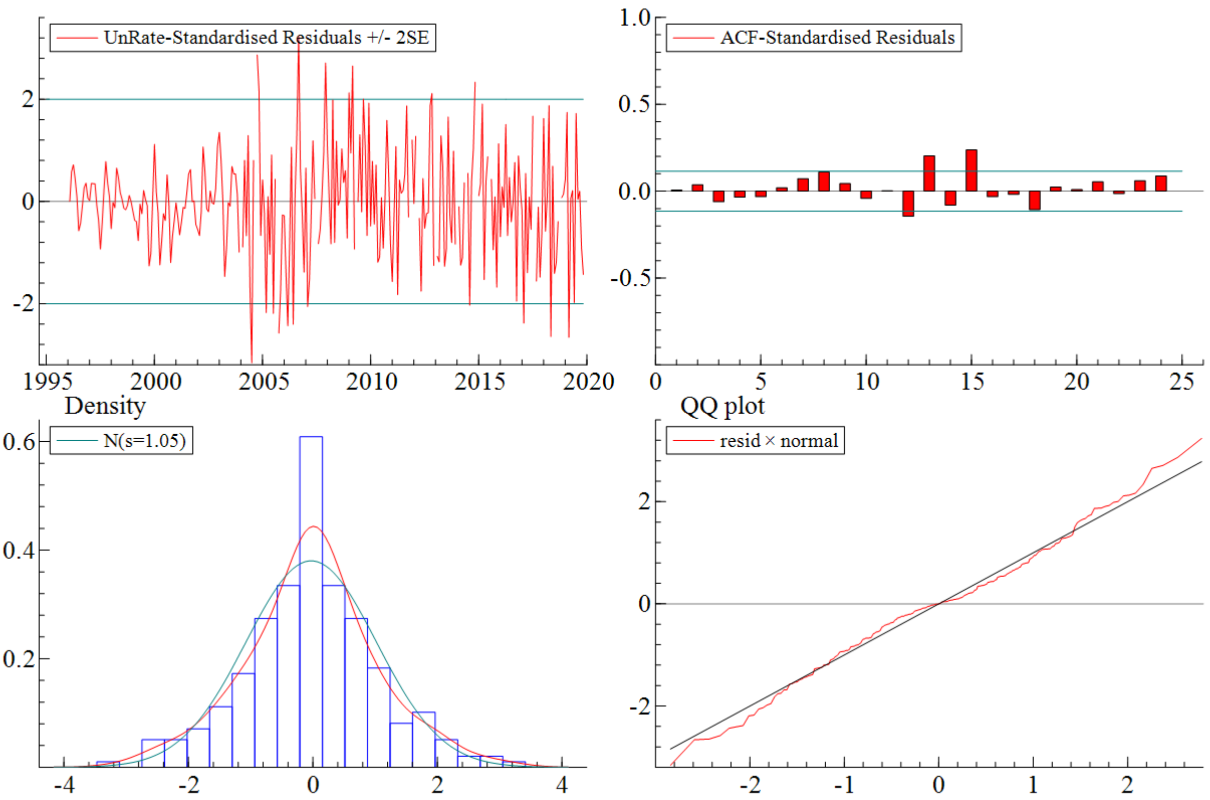
Forecasts with 68% confidence interval from period 2019(11) forwards:

	Forecast	Std.Err	Leftbound	Rightbound
1	9.37219	0.32358	9.04861	9.69577
2	10.34716	0.38828	9.95888	10.73544
3	10.19479	0.45063	9.74416	10.64541
4	9.72667	0.49935	9.22732	10.22602
5	9.58081	0.55044	9.03037	10.13125
6	8.93186	0.59349	8.33838	9.52535
7	8.55145	0.63883	7.91262	9.19029
8	8.57652	0.67847	7.89805	9.25498
9	7.35324	0.71931	6.63393	8.07255
10	8.99610	0.75446	8.24165	9.75056
11	9.45555	0.78713	8.66842	10.24268
12	9.48655	0.80884	8.67771	10.29540

Forecast accuracy measures from period 2019(11) forwards:

	Error	RMSE	RMSPE	MAE	MAPE
1	-0.42781	0.42781	0.43654	0.42781	4.36543
2	-0.05284	0.30481	0.31077	0.24033	2.43677
3	0.09479	0.25482	0.25946	0.19181	1.93734
4	2.32667	1.18408	1.58805	0.72553	9.31337
5	3.18081	1.77346	2.63775	1.21658	17.39073
6	1.13186	1.68359	2.47973	1.20246	16.91078
7	-0.64855	1.57786	2.31119	1.12333	15.50202
8	-1.82348	1.61061	2.24904	1.21085	15.75595
9	-2.04676	1.66472	2.24120	1.30373	16.42463
10	-1.20390	1.62454	2.15870	1.29375	15.96246
11	-0.94445	1.57489	2.07637	1.26199	15.33689
12	0.88655	1.52941	2.01012	1.23071	14.91788

We can see that this model is the best one in terms of forecasting accuracy, as RMSE and RMSPE have decreased, while MAE and MAPE have increased very lightly when compared to the second model that was built. Further, we can check the residuals' graphics:



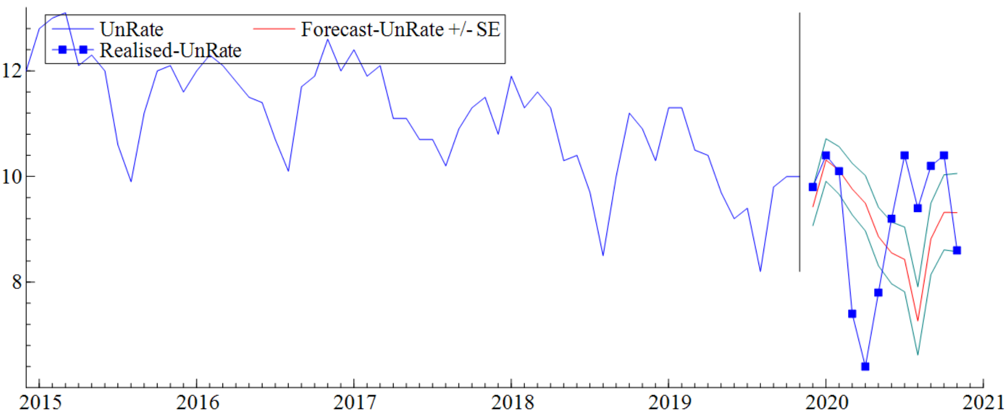
We can notice how within the first 12 lags there is statistically significant autocorrelation only at lag 12. The QQplot is the most fitted one among all other specifications. Therefore, both for matters of diagnostics and forecasts, this seems to be the best model obtained so far within UC Models.

In the **eight model**, I assumed the same interventions which were implemented within the previous model, with the addition of a short cycle and the reintroduction of the Irregular component. Here is the specification:

```
UC(17) Estimation done by Maximum Likelihood (exact score)
       The selection sample is: 1995(1) - 2019(11) (N = 1, T = 299)
       The dependent variable Y is: UnRate
       The model is: Y = Trend + Seasonal + Irregular + Cycle 1 + Interventions

Profile Log-Likelihood:                    266.6186
Akaike Information Criterion (AIC):         -1.5894
Bayesian Information Criterion (BIC):       -1.2305
Prediction error variance:                   0.1184

Summary statistics:
                   UnRate
 T                    299
 Normality         8.9201
 H(92)             4.6536
 DW                1.8063
 r(1)            0.093902
 q                     24
 p                      6
 r(q)            0.025021
 Q(q,q-p)          35.231
 Rs^2             0.59195

Variances of disturbances:
                    Value    (q-ratio)
Level           0.0165724  (   0.6568)
```

```
Slope         8.93486e-05  ( 0.003541)
Seasonal      0.000286480  (  0.01135)
Cycle          0.00133084  (  0.05274)
Irregular       0.0252326  (    1.000)

Cycle other parameters:
                      Value
Variance            0.01436
Period             11.20418
Period in years     0.93368
Frequency           0.56079
Damping factor      0.95252
Order               1.00000

State vector analysis at period 2019(11):
                             Value     Prob
Level                      6.82481 [0.00000]
Slope                     -0.06145 [0.10238]
Seasonal chi2 test       125.81999 [0.00000]
Cycle 1 amplitude          0.00226 [   .NaN]

Seasonal effects:
                Period       Value     Prob
                     1     0.89750 [0.00002]
                     2     0.76116 [0.00022]
                     3     0.46818 [0.01925]
                     4     0.26100 [0.18093]
                     5    -0.30908 [0.10600]
                     6    -0.55732 [0.00300]
                     7    -0.61979 [0.00075]
                     8    -1.72264 [0.00000]
                     9    -0.10536 [0.54470]
                    10     0.45920 [0.00759]
                    11     0.51648 [0.00332]
                    12    -0.04932 [0.81516]

Regression effects in final state at time 2019(11):
                     Coefficient        RMSE     t-value     Prob
Outlier 2014(6)         -1.02651     0.26465    -3.87882 [0.00013]
Level break 2012(3)      1.03818     0.27348     3.79618 [0.00018]
Level break 2015(7)     -0.96221     0.27528    -3.49541 [0.00055]
Outlier 2004(9)          0.84000     0.26299     3.19402 [0.00157]
Outlier 2009(5)         -0.72763     0.26103    -2.78756 [0.00568]
Outlier 2017(8)          0.76272     0.26286     2.90158 [0.00401]
Level break 2005(9)      0.80774     0.27320     2.95653 [0.00338]
Level break 2011(11)     0.98480     0.27215     3.61865 [0.00035]
Level break 2013(1)      0.84542     0.27282     3.09878 [0.00214]
```

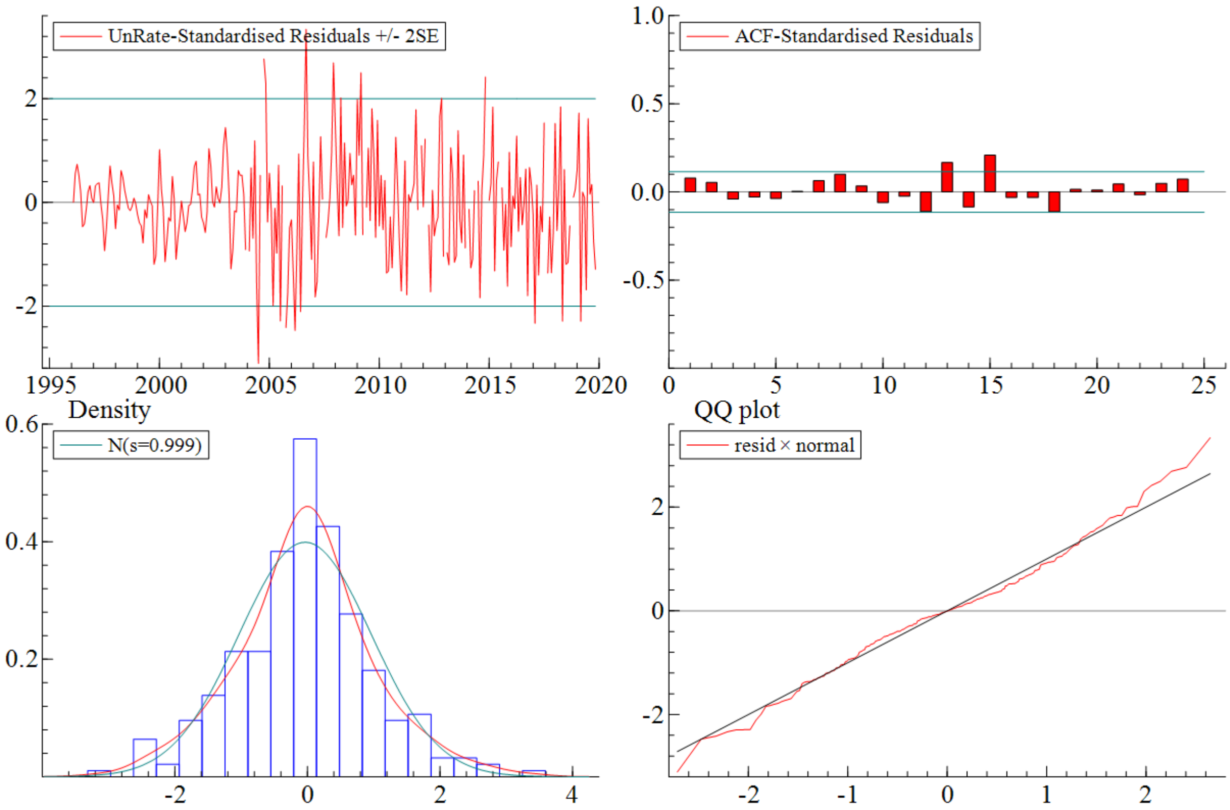
The biggest change that we can notice when introducing the cycle is that the Irregular part gains the highest q-ratio. Also, log-likelihood and information criteria are even better than those of the previous model, as AIC and prediction error variance have decreased. Unfortunately, we lose the normality of the residuals. If we look at forecasts:



We see that again there are several values which were beyond the error bars. The written forecasts:

```
Forecasts with 68% confidence interval from period 2019(11) forwards:
          Forecast     Std.Err    Leftbound   Rightbound
1          9.42576     0.35745      9.06831      9.78321
2         10.31181     0.40277      9.90903     10.71458
3         10.11509     0.45040      9.66469     10.56549
4          9.76175     0.48764      9.27412     10.24939
5          9.49393     0.52446      8.96947     10.01840
6          8.86272     0.55390      8.30881      9.41662
7          8.55280     0.58512      7.96768      9.13792
8          8.42821     0.61338      7.81483      9.04159
9          7.26307     0.64639      6.61668      7.90945
10         8.81812     0.67716      8.14096      9.49529
11         9.32075     0.71080      8.60995     10.03155
12         9.31652     0.73955      8.57697     10.05607

Forecast accuracy measures from period 2019(11) forwards:
             Error        RMSE        RMSPE         MAE         MAPE
1         -0.37424     0.37424      0.38188     0.37424      3.81877
2         -0.08819     0.27188      0.27661     0.23122      2.33338
3          0.01509     0.22216      0.22601     0.15917      1.60539
4          2.36175     1.19645      1.60774     0.70982      9.18294
5          3.09393     1.74919      2.59652     1.18664     17.01489
6          1.06272     1.65468      2.43467     1.16599     16.44984
7         -0.64720     1.55134      2.26970     1.09188     15.10483
8         -1.97179     1.60991      2.22641     1.20186     15.58667
9         -2.13693     1.67667      2.23167     1.30576     16.38074
10        -1.38188     1.64956      2.16006     1.31337     16.09745
11        -1.07925     1.60611      2.08317     1.29209     15.57744
12         0.71652     1.55158      2.00893     1.24412     14.97363
```

We can see that despite other errors have increased, the value of RMSPE has decreased. If we check the residuals' graphics:



An interesting success for this model is the removal of the autocorrelation at the 12th lag, even though it's on the edge now. This model could be interpreted as better or worse than the seventh one depending on whether one is interested in diagnostics or forecasts; it is a little better in terms of diagnostics and a little worse in terms of forecasts.

Unfortunately, it was not possible to improve the forecasting accuracy significantly by any other means. It seems that it is harder for Unobserved Components Models to cope with the unexpected decrease in unemployment rates in the spring of 2020 and with the unexpected rise in the summer of that same year. Here is a short summary of the best UC models in terms of forecasting:

2nd model (5 interventions)		7th model (automatic interventions, Irregular unchecked)		8th model (same interventions as 7th model, short cycle)	
RMSE	1.5428	RMSE	1.5294	RMSE	1.5516
RMSPE	2.0310	RMSPE	2.0101	RMSPE	2.0089
MAE	1.2264	MAE	1.2307	MAE	1.2441
MAPE	14.9043	MAPE	14.9179	MAPE	14.9736

Summary

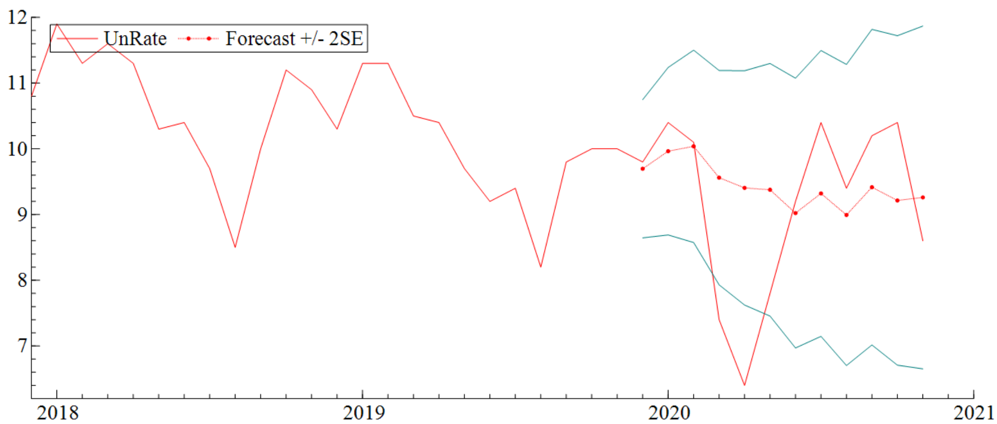
Now, if we compare the best three models which were achieved with SARIMA and UCM:

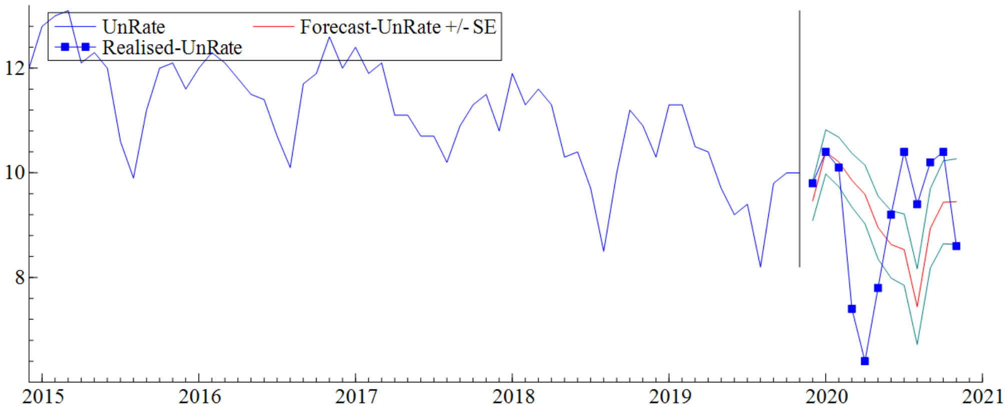
ARIMA(1,1,0)x(1,0,0) S		ARIMA(0,1,1)x(0,0,1) S		ARIMA(0,1,0)x(0,0,1) S	
RMSE	1.4420	RMSE	1.2992	RMSE	1.3097

RMSPE	1.9254	RMSPE	**1.7964**	RMSPE	1.8202
MAE	1.1592	MAE	0.9708	MAE	**0.9617**
MAPE	14.1097	MAPE	12.1491	MAPE	**12.1090**

2nd model (5 interventions)		7th model (automatic interventions, Irregular unchecked)		8th model (same interventions as 7th model, short cycle)	
RMSE	1.5428	RMSE	1.5294	RMSE	1.5516
RMSPE	2.0310	RMSPE	2.0101	RMSPE	2.0089
MAE	1.2264	MAE	1.2307	MAE	1.2441
MAPE	14.9043	MAPE	14.9179	MAPE	14.9736

We can already notice quickly how the forecasting accuracy of SARIMA was better in general. Especially, $ARIMA(0, 1, 1)x(0, 0, 1)S$ is the best model in terms of RMSE and RMSPE, while $ARIMA(0, 1, 0)x(0, 0, 1)S$ is the best one in terms of MAE and MAPE.
If we compare the forecasts of $ARIMA(0, 1, 1)x(0, 0, 1)S$, which beats $ARIMA(0, 1, 0)x(0, 0, 1)S$ in terms of diagnostics too, with the best UCM model in terms of forecasts, so the second one:





We can notice how SARIMA managed to limit the overestimation at 2020(3), while it also managed to limit the overestimation at 2020(6) and 2020(7), which are all features that UCM never managed to do this well.