Time-varying Beta of Ferrari N.V. (RACE) stock

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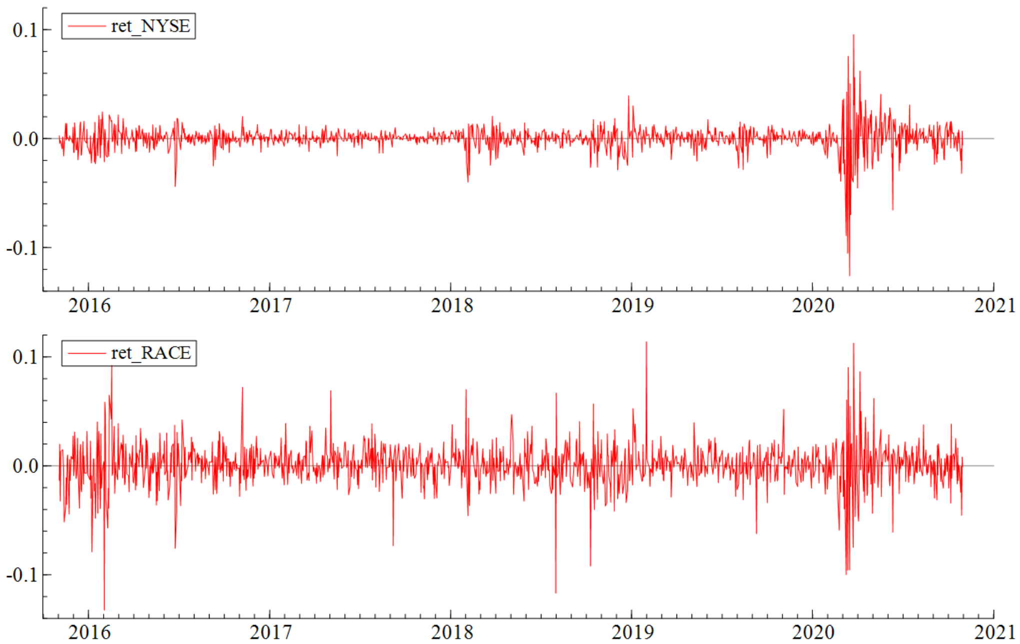
I decided to work with RACE stock from the NYSE Composite Index. This stock belongs to the Ferrari N.V. company, which is the society controlling Ferrari S.p.A, the worldwide known supercars manufacturer. It was a choice led by the passion for motorsport and the Italian brand with its headquarters in Maranello (Modena, Italy).

I downloaded historical data for both the stock and the index for the same time frame, namely the last 5 years (02.11.2015 – 30.10.2020). The data source is the "Yahoo! Finance" website, and the following are the links for Ferrari (RACE) and NYSE respectively:

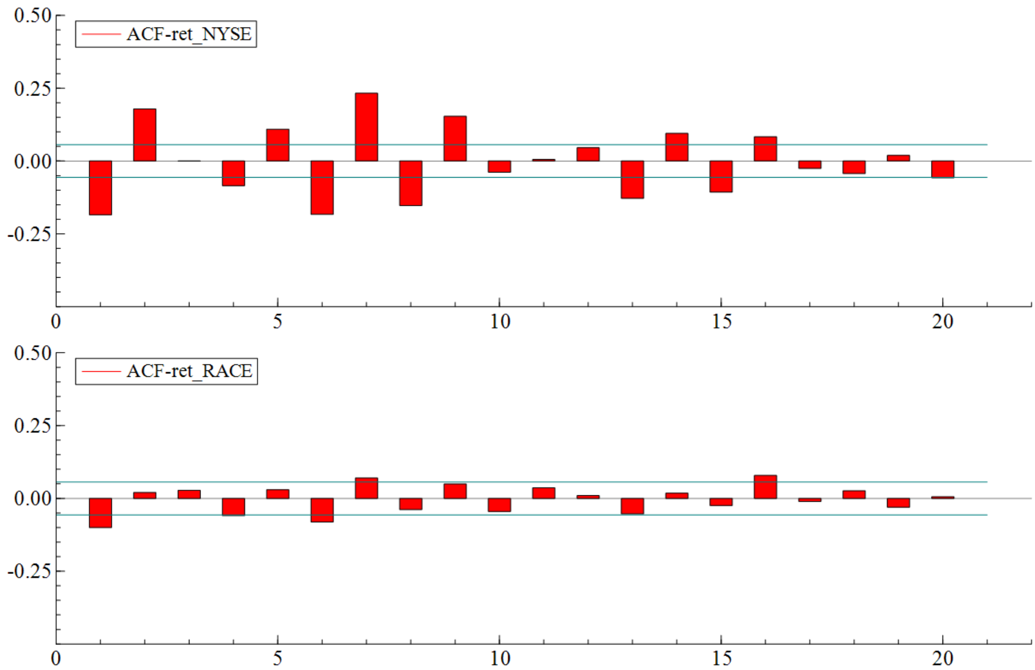
- https://finance.yahoo.com/quote/RACE/history/
- https://finance.yahoo.com/quote/%5Enya/history/

After uploading the closing prices onto OxMetrics, I calculated the logarithmic returns and plotted Actual series:

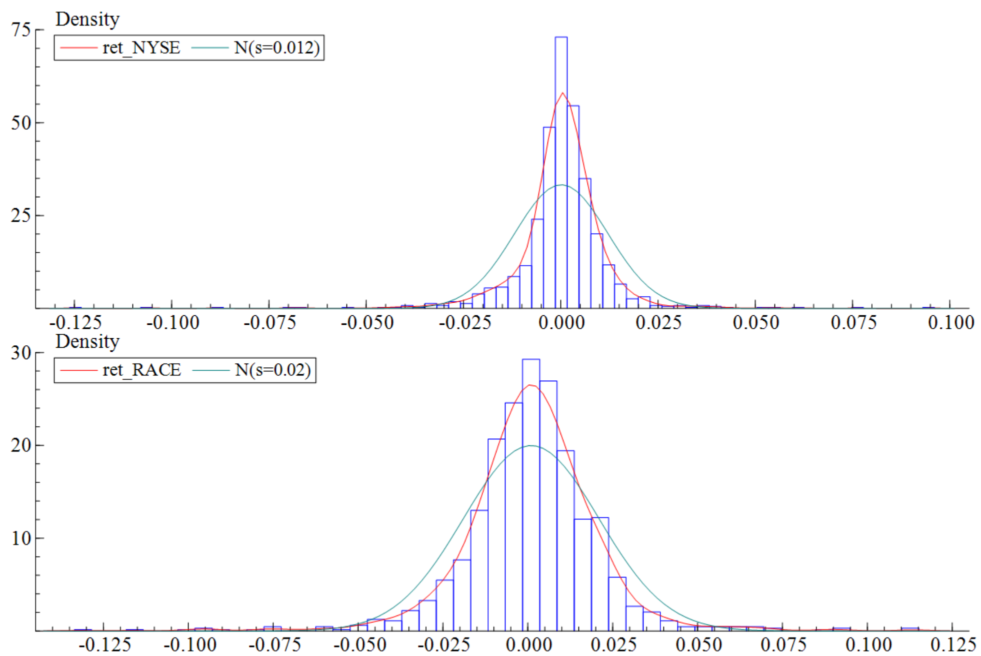


The two graphs are presented using the same scale, so it is easy to notice how the stock has been more volatile than the NYSE index for most of the considered time frame. Perhaps, the Beta will tend to be greater than 1 most of the time at the end. However, some similarities between Ferrari and NYSE are visible in the last period starting from March 2020, when the stock market literally crashed as a consequence of the COVID-19 outbreak.

Here are the ACF plots instead. They will be helpful in understanding which specifications to pick when creating Univariate models:



There is statistically significant autocorrelation that is quite elevate in the case of NYSE. This happens at the 1st, 2nd, 4th, 5th, 6th, 7th, 8th, 9th 13th, 14th, 15th and 16th lag. We could expect some long memory in the series, as the autocorrelation is prolonged so far in terms of lags.
The autocorrelation is less in Ferrari's stock, still the 1st, 4th (on the border), 6th, 7th, 13th (on the border) and 16th lags present statistically significant autocorrelation. As for NYSE, there could be long memory in this series.
Here are the distributions of the returns:



It seems that in the case of ret_NYSE there is a negative skewness. Both distributions show excess kurtosis; it is likely that none of them behaves as a normal distribution.

Here is the Descriptive statistics using G@RCH for the two series, which can give a better insight for modeling:

```
---- Database information ----
Sample:    2015-11-02 - 2020-10-30 (1259 observations)
Frequency: 1
Variables: 5

Variable               #obs  #miss    type          min         mean          max      std.dev
Date                   1259      0    date       2015-11-02              2020-10-30
ret_NYSE               1258      1  double     -0.12595   0.00012807     0.095642     0.011978
ret_RACE               1258      1  double     -0.13235    0.0009637      0.11402     0.019963
Constant               1259      0  double            1            1            1            0
Trend                  1259      0  double            1          630         1259       363.44

Series #1/2: ret_NYSE
---------

Normality Test

                   Statistic       t-Test      P-Value
Skewness             -1.4862       21.546  5.8309e-103
Excess Kurtosis       24.123       174.99      0.00000
Jarque-Bera           30964.         .NaN      0.00000
---------------
ARCH 1-2 test:    F(2,1253) =   381.04 [0.0000]**
ARCH 1-5 test:    F(5,1247) =   160.59 [0.0000]**
ARCH 1-10 test:   F(10,1237)=   92.816 [0.0000]**
---------------
Box-Pierce Q-Statistics on Raw data
  Q(  5) =  107.478   [0.0000000]**
  Q( 10) =  280.220   [0.0000000]**
  Q( 20) =  346.405   [0.0000000]**
  Q( 50) =  388.671   [0.0000000]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
Box-Pierce Q-Statistics on Squared data
  Q(  5) =  1147.46   [0.0000000]**
  Q( 10) =  1729.82   [0.0000000]**
  Q( 20) =  1977.55   [0.0000000]**
  Q( 50) =  2023.60   [0.0000000]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hurst-Mandelbrot R/S test statistics: 1.19973
Lo R/S test statistics (q=1): 1.32867
Critical Values
90%: [0.861, 1.747]
95%: [0.809, 1.862]
99%: [0.721, 2.098]
H0: Hurst-Mandelbrot = no autocorrelation and Lo = no long-term dependence
---------------

Series #2/2: ret_RACE
---------

Normality Test

                   Statistic       t-Test      P-Value
Skewness            -0.31834       4.6151   3.9295e-06
Excess Kurtosis       7.0325       51.015      0.00000
Jarque-Bera           2613.5         .NaN      0.00000
---------------
ARCH 1-2 test:    F(2,1253) =   56.199 [0.0000]**
ARCH 1-5 test:    F(5,1247) =   28.754 [0.0000]**
ARCH 1-10 test:   F(10,1237)=   19.708 [0.0000]**
---------------
Box-Pierce Q-Statistics on Raw data
  Q(  5) =  19.7065   [0.0014185]**
```

```
  Q( 10) =  41.7908   [0.0000082]**
  Q( 20) =  58.6017   [0.0000117]**
  Q( 50) =  96.7788   [0.0000810]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
Box-Pierce Q-Statistics on Squared data
  Q(  5) =  200.411   [0.0000000]**
  Q( 10) =  388.955   [0.0000000]**
  Q( 20) =  476.272   [0.0000000]**
  Q( 50) =  492.132   [0.0000000]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hurst-Mandelbrot R/S test statistics: 1.37336
Lo R/S test statistics (q=1): 1.44778
Critical Values
90%: [0.861, 1.747]
95%: [0.809, 1.862]
99%: [0.721, 2.098]
H0: Hurst-Mandelbrot = no autocorrelation and Lo = no long-term dependence
---------------

Series #1/2: ret_NYSE

----  Log Periodogram Regression ----
d parameter         0.0809636 (0.0271128) [0.0028]
No of observations: 1258; no of periodogram points: 628

Series #2/2: ret_RACE

----  Log Periodogram Regression ----
d parameter       -0.00178909 (0.0270999) [0.9474]
No of observations: 1258; no of periodogram points: 629
```

From the **Basic stats**, the Mean of both returns is quite close to zero, while their Standard deviations are much bigger, indicating that perhaps the Mean is not statistically significantly different from zero.

The **Normality test** for NYSE's returns confirms that there is negative skewness and excess kurtosis, as both respective p-values are statistically significant. Jarque-Bera statistic is significant as well, hence the ret_NYSE series does not follow a normal distribution. According to the **ARCH test**, there is ARCH effect at all considered lags.
Both the tests of **Box-Pierce** show statistically significant autocorrelation at all the considered lags.
The **Hurst-Mandelbrot** and **Lo** statistics are included within the confidence intervals, hence there is no reason to reject H_0, and there is no autocorrelation and no long-term dependence.
Regarding returns from RACE, all the conducted tests' results lead to the same conclusions as in the case of NYSE.

Finally, looking at the output of the **Geweke and Porter-Hudak long-memory tests**, the d parameter is very small and statistically significant in the case of ret_NYSE, indicating perhaps a sort of medium memory. In the case of ret_RACE, the d parameter is negative but not significant, hence there is no long-term dependence within the series.

The first model will be created with OLS. It is a Cross-section regression using PcGive:

```
EQ( 1) Modelling ret_RACE by OLS (static model)
       The estimation sample is: 2015-11-03...2020-10-30

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant          0.000830842  0.0004409     1.88  0.0597   0.0028
```

```
ret_NYSE              1.03746    0.03680     28.2  0.0000   0.3875

sigma               0.0156361  RSS              0.307077111
R^2                  0.387487  F(1,1256) =   794.6 [0.000]**
Adj.R^2              0.386999  log-likelihood       3446.96
no. of observations      1258  no. of parameters          2
mean(ret_RACE)    0.000963704  se(ret_RACE)       0.0199709

Normality test:   Chi^2(2)  =   1010.4 [0.0000]**
Hetero test:      F(2,1255) =   3.0575 [0.0474]*
Hetero-X test:    F(2,1255) =   3.0575 [0.0474]*
RESET23 test:     F(2,1254) =   3.8856 [0.0208]*
```

The Beta is estimated as 1.03746. The tests indicate the non-normality of the distribution, and the presence of heteroskedastic residuals (despite the p-value is very close to the border). These information indicate that a GARCH specification could be used with a Student distribution.

Now, starting to model with “Univariate GARCH Models using G@RCH” on the ret_RACE series, the first model would be an ARMA(1,0) with GARCH(1,1) and Student distribution. The AR(1) lag is included because it is the biggest in the series and also the first one, hence it could be more relevant:

```
******************************
 ** G@RCH(1) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-11-03 - 2020-10-30
The dependent variable is: ret_RACE
Mean Equation:  ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 4.21051 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 3364
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.001426 0.00037350    3.819  0.0001
AR(1)               -0.056846   0.030811   -1.845  0.0653
Cst(V) x 10^4        0.203087   0.089437    2.271  0.0233
ARCH(Alpha1)         0.109347   0.033688    3.246  0.0012
GARCH(Beta1)         0.839597   0.049285    17.04  0.0000
Student(DF)          4.210511    0.53324    7.896  0.0000

No. Observations :      1258  No. Parameters  :         6
Mean (Y)         :   0.00096  Variance (Y)    :   0.00040
Skewness (Y)     :  -0.31834  Kurtosis (Y)    :  10.03245
Log Likelihood   :  3364.004  Alpha[1]+Beta[1]:   0.94894

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000397771
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is not observed.
The constraint equals 1.2652 and should be < 1.
  => See Ling & McAleer (2001) for details.
```

In this model the Constant in Mean is statistically significant, as well as the Constant in Variance equation. Alpha and Beta are positive and statistically significant, and their sum is lower than 1.

The AR(1) lag is not significant, hence it is not necessary. The degrees of freedom are just a little more than 4, a little risky but acceptable.
Here are the tests for this model:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  --> P-values adjusted by 1 degree(s) of freedom
  Q(  5) =  1.83519   [0.7660387]
  Q( 10) =  3.30973   [0.9507401]
  Q( 20) =  9.13365   [0.9711930]
  Q( 50) =  50.7157   [0.4057313]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  1.24140  0.21446
Negative Size Bias t-Test         0.77611  0.43768
Positive Size Bias t-Test         0.30268  0.76214
Joint Test for the Three Effects  1.71069  0.63456

---------------
ARCH 1-2 test:    F(2,1251) =  0.15829 [0.8536]
ARCH 1-5 test:    F(5,1245) =  0.30243 [0.9116]
ARCH 1-10 test:   F(10,1235)=  0.25842 [0.9895]
---------------
```

According to these outcomes, there is no autocorrelation in standardized residuals and no ARCH effect. Moreover, in the Sign Bias test none of the p-values are showing any asymmetric relationship leading to Leverage effect.

A new model can be formulated, removing the AR(1) lag and keeping the other settings equal:

```
******************************
 ** G@RCH(2) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-11-03 - 2020-10-30
The dependent variable is: ret_RACE
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 4.31466 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 3361.59
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.001419 0.00039463    3.596  0.0003
Cst(V) x 10^4        0.208286   0.091584    2.274  0.0231
ARCH(Alpha1)         0.111074   0.034665    3.204  0.0014
GARCH(Beta1)         0.835374   0.051144    16.33  0.0000
Student(DF)          4.314658    0.54754    7.880  0.0000

No. Observations :      1258  No. Parameters  :         5
Mean (Y)         :   0.00096  Variance (Y)    :   0.00040
Skewness (Y)     :  -0.31834  Kurtosis (Y)    :  10.03245
Log Likelihood   :  3361.591  Alpha[1]+Beta[1]:   0.94645

The sample mean of squared residuals was used to start recursion.
```

```
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000388945
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is not observed.
The constraint equals 1.15569 and should be < 1.
  => See Ling & McAleer (2001) for details.
```

The Constant in Mean equation is still significant as well as the Constant in Variance equation. Alpha and Beta are still statistically significant and their sum is lower than 1.
The tests for this model follow:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  Q(  5) =  2.26309   [0.8116720]
  Q( 10) =  3.91644   [0.9510378]
  Q( 20) =  9.97612   [0.9686028]
  Q( 50) =  50.0718   [0.4705476]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                   Test  P-value
Sign Bias t-Test                 1.32373  0.18559
Negative Size Bias t-Test        0.84148  0.40008
Positive Size Bias t-Test        0.34052  0.73347
Joint Test for the Three Effects  1.94079  0.58479

---------------
ARCH 1-2 test:    F(2,1251) =  0.17006 [0.8436]
ARCH 1-5 test:    F(5,1245) =  0.29997 [0.9130]
ARCH 1-10 test:   F(10,1235)=  0.25927 [0.9894]
---------------
```

The outcome is positive as there is no autocorrelation, ARCH or Leverage effect.

However, since the ACF plots had indicated the possibility of having long memory in the series, I would try to embed the ARFIMA d parameter in the last specification:

```
******************************
 ** G@RCH(3) SPECIFICATIONS **
 ******************************
The estimation sample is:  2015-11-03 - 2020-10-30
The dependent variable is: ret_RACE
Mean Equation:  ARFIMA (0, d, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 4.22145 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 3363.77
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.001420 0.00028694    4.948  0.0000
d-Arfima            -0.046995   0.023428   -2.006  0.0451
Cst(V) x 10^4        0.201943   0.088195    2.290  0.0222
ARCH(Alpha1)         0.109109   0.033391    3.268  0.0011
GARCH(Beta1)         0.839788   0.048753    17.23  0.0000
```

```
Student(DF)          4.221455    0.53003    7.965  0.0000

No. Observations :      1258  No. Parameters  :         6
Mean (Y)         :   0.00096  Variance (Y)    :   0.00040
Skewness (Y)     :  -0.31834  Kurtosis (Y)    :  10.03245
Log Likelihood   :  3363.772  Alpha[1]+Beta[1]:   0.94890

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000395169
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is not observed.
The constraint equals 1.24676 and should be < 1.
  => See Ling & McAleer (2001) for details.
```

It seems to have been a good idea, as the d parameter is negative and statistically significant (barely), indicating a **short memory**. The constants are still statistically significant, as well as Alpha and Beta, and their sum is lesser than 1. Here are the tests:

```
TESTS :
---------

Q-Statistics on Standardized Residuals
  Q(  5) =  3.83021   [0.5741116]
  Q( 10) =  5.62605   [0.8456417]
  Q( 20) =  11.8133   [0.9223525]
  Q( 50) =  53.4556   [0.3430120]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Diagnostic test based on the news impact curve (EGARCH vs. GARCH)
                                     Test  P-value
Sign Bias t-Test                  1.10268  0.27017
Negative Size Bias t-Test         0.68625  0.49256
Positive Size Bias t-Test         0.26552  0.79061
Joint Test for the Three Effects  1.34698  0.71801

---------------
ARCH 1-2 test:    F(2,1251) =  0.14542 [0.8647]
ARCH 1-5 test:    F(5,1245) =  0.27537 [0.9267]
ARCH 1-10 test:   F(10,1235)=  0.25029 [0.9908]
---------------
```

One more time, the tests don't show any sign of autocorrelation, ARCH or Leverage effect. So, this model seems to be the accurate for ret_RACE series.

Now that GARCH model and OLS Beta have been estimated, it is time to introduce Multi-GARCH processes, which will allow to get Conditional Variance and Conditional Covariance so as to calculate Beta.
The model will be built assuming Dynamic correlation, hence a DCC (Engle), with ARMA(0,0), GARCH(1,1) with the inclusion of ARFIMA and Student distribution:

```
  ********************
 **   FIRST STEP   **
*******************

--------------------------Estimating the univariate GARCH model for
ret_RACE---------------------------


 ********************
```

```
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-11-03 - 2020-10-30
The dependent variable is: ret_RACE
Mean Equation:  ARFIMA (0, d, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3256.09
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.001451 0.00040879    3.550  0.0004
d-Arfima            -0.019954   0.028030  -0.7119  0.4767
Cst(V) x 10^4        0.248364    0.13270    1.872  0.0615
ARCH(Alpha1)         0.094505   0.032627    2.897  0.0038
GARCH(Beta1)         0.840396   0.061320    13.71  0.0000

No. Observations :      1258  No. Parameters  :         5
Mean (Y)         :   0.00096  Variance (Y)    :   0.00040
Skewness (Y)     :  -0.31834  Kurtosis (Y)    :  10.03245
Log Likelihood   :  3256.088  Alpha[1]+Beta[1]:   0.93490

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000381521
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.891904 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.001451;-0.019954; 0.248364; 0.094505; 0.840396
Elapsed Time : 0.447 seconds (or 0.00745 minutes).

---------------------------Estimating the univariate GARCH model for
ret_NYSE---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-11-03 - 2020-10-30
The dependent variable is: ret_NYSE
Mean Equation:  ARFIMA (0, d, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 4323.23
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000634 0.00013064    4.855  0.0000
d-Arfima            -0.053738   0.031597   -1.701  0.0892
Cst(V) x 10^4        0.030770   0.010043    3.064  0.0022
ARCH(Alpha1)         0.243991   0.052572    4.641  0.0000
GARCH(Beta1)         0.744677   0.039461    18.87  0.0000
```

```
No. Observations :      1258  No. Parameters  :         5
Mean (Y)         :   0.00013  Variance (Y)    :   0.00014
Skewness (Y)     :  -1.48619  Kurtosis (Y)    :  27.12264
Log Likelihood   :  4323.226  Alpha[1]+Beta[1]:   0.98867

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000271536
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is not observed.
The constraint equals 1.09653 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000634;-0.053738; 0.030770; 0.243991; 0.744677
Elapsed Time : 0.459 seconds (or 0.00765 minutes).


  ********************
 **  SECOND STEP   **
*******************


  *************
 ** SERIES  **
*************
#1: ret_RACE
#2: ret_NYSE

The estimation sample is:  2015-11-03 - 2020-10-30

 ******************************
 ** MG@RCH(1) SPECIFICATIONS **
 ******************************
Conditional Variance : Dynamic Correlation Model (Engle)
Multivariate Student distribution, with 4.74031 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 7984.07
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                 Coefficient  Std.Error  t-value  t-prob
rho_21              0.543299   0.028051    19.37  0.0000
alpha               0.072993   0.021152    3.451  0.0006
beta                0.737429   0.067726    10.89  0.0000
df                  4.740309    0.32352    14.65  0.0000
No. Observations :      1258  No. Parameters  :        14
No. Series       :         2  Log Likelihood  :  7984.073
Elapsed Time : 0.03 seconds (or 0.0005 minutes).
```

With this approach, the d parameter is not statistically significant neither in ret_RACE nor in ret_NYSE. Perhaps, this is due to the fact that the Multivariate approach automatically assumes a Normal distribution in the First step instead of a Student distribution as specified in the model settings. It would be better to remove ARFIMA and reformulate the model as ARMA(0,0) with GARCH(1,1) and Student distribution, which in the Univariate approach was a good model too:

```
  ********************
 **   FIRST STEP   **
*******************
```

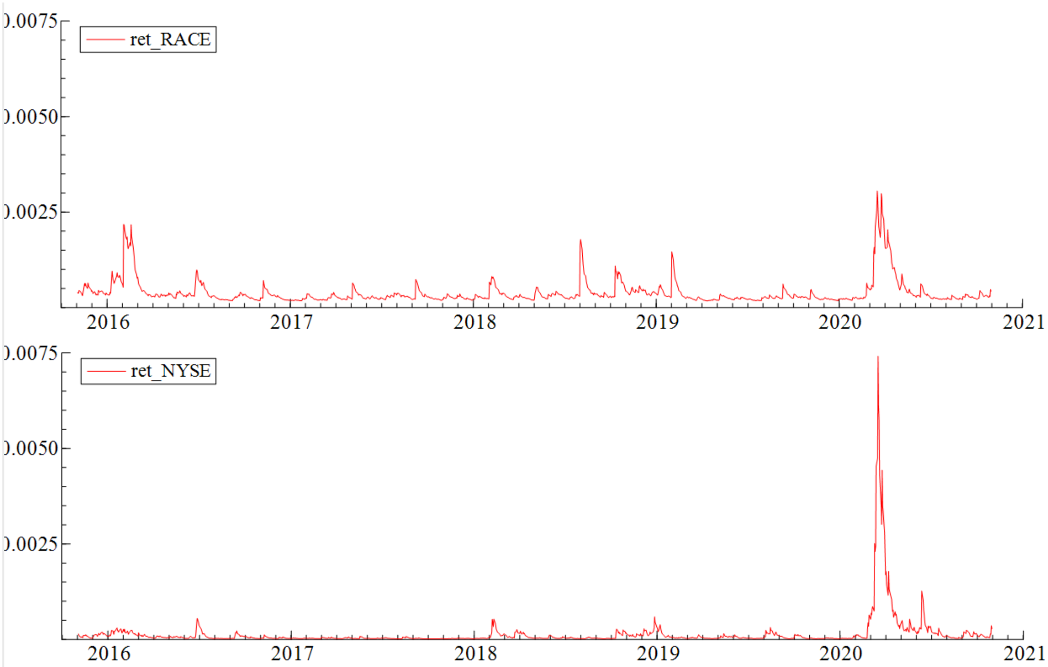
```
---------------------------Estimating the univariate GARCH model for
ret_RACE---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-11-03 - 2020-10-30
The dependent variable is: ret_RACE
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 3255.8
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.001448 0.00046441    3.117  0.0019
Cst(V) x 10^4        0.252571    0.13426    1.881  0.0602
ARCH(Alpha1)         0.095462   0.032582    2.930  0.0035
GARCH(Beta1)         0.838296   0.061764    13.57  0.0000

No. Observations :      1258  No. Parameters  :         4
Mean (Y)         :   0.00096  Variance (Y)    :   0.00040
Skewness (Y)     :  -0.31834  Kurtosis (Y)    :  10.03245
Log Likelihood   :  3255.805  Alpha[1]+Beta[1]:   0.93376

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000381286
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is observed.
The constraint equals 0.89013 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.001448; 0.252571; 0.095462; 0.838296
Elapsed Time : 0.045 seconds (or 0.00075 minutes).

---------------------------Estimating the univariate GARCH model for
ret_NYSE---------------------------


 ********************
 ** SPECIFICATIONS **
 ********************
The estimation sample is:  2015-11-03 - 2020-10-30
The dependent variable is: ret_NYSE
Mean Equation:  ARMA (0, 0) model.
No regressor in the conditional mean
Variance Equation:  GARCH (1, 1) model.
No regressor in the conditional variance
Normal distribution.

Strong convergence using numerical derivatives
Log-likelihood = 4321.32
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000664 0.00018540    3.580  0.0004
```

```
Cst(V) x 10^4        0.031707   0.010221    3.102  0.0020
ARCH(Alpha1)         0.242603   0.050534    4.801  0.0000
GARCH(Beta1)         0.743600   0.038625    19.25  0.0000

No. Observations :      1258  No. Parameters  :         4
Mean (Y)         :   0.00013  Variance (Y)    :   0.00014
Skewness (Y)     :  -1.48619  Kurtosis (Y)    :  27.12264
Log Likelihood   :  4321.318  Alpha[1]+Beta[1]:   0.98620

The sample mean of squared residuals was used to start recursion.
The positivity constraint for the GARCH (1,1) is observed.
This constraint is alpha[L]/[1 - beta(L)] >= 0.
The unconditional variance is 0.000229824
The conditions are alpha[0] > 0, alpha[L] + beta[L] < 1 and alpha[i] + beta[i] >= 0.
  => See Doornik & Ooms (2001) for more details.
The condition for existence of the fourth moment of the GARCH is not observed.
The constraint equals 1.09031 and should be < 1.
  => See Ling & McAleer (2001) for details.

Estimated Parameters Vector :
 0.000664; 0.031707; 0.242603; 0.743600
Elapsed Time : 0.074 seconds (or 0.00123333 minutes).


  ********************
 **  SECOND STEP   **
*******************


  *************
 ** SERIES  **
*************
#1: ret_RACE
#2: ret_NYSE

The estimation sample is:  2015-11-03 - 2020-10-30

 *******************************
 ** MG@RCH(2) SPECIFICATIONS **
 *******************************
Conditional Variance : Dynamic Correlation Model (Engle)
Multivariate Student distribution, with 4.78431 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 7983.64
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
rho_21               0.547847   0.027941    19.61  0.0000
alpha                0.073159   0.021561    3.393  0.0007
beta                 0.738712   0.070344    10.50  0.0000
df                   4.784306    0.33044    14.48  0.0000
No. Observations :      1258  No. Parameters  :        12
No. Series       :         2  Log Likelihood  :  7983.639
Elapsed Time : 0.035 seconds (or 0.000583333 minutes).
```

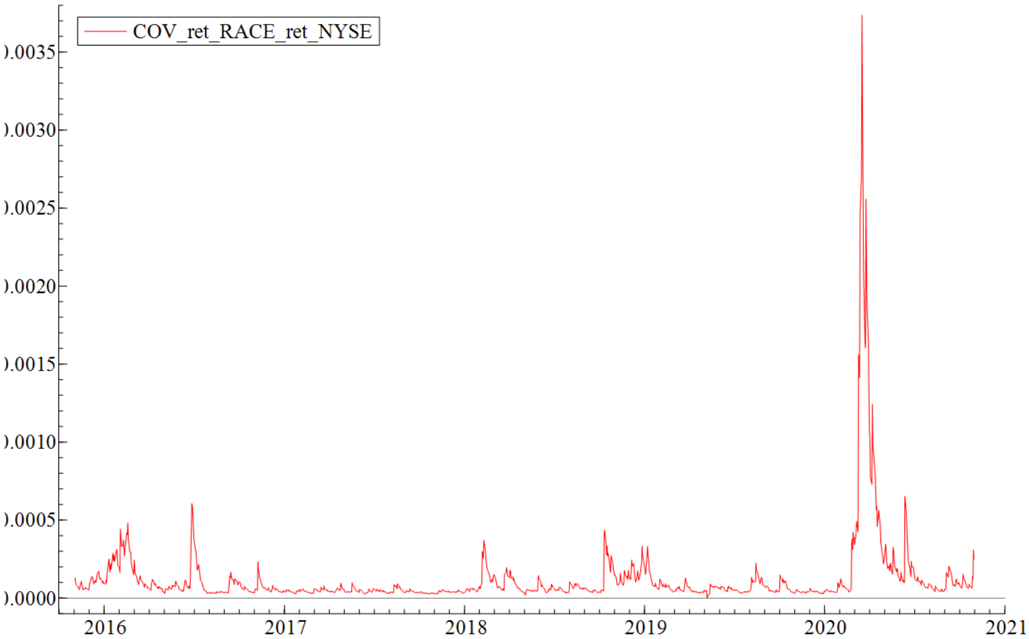
In the First step, the parameters of ret_RACE are all significant apart from the Constant in the Variance equation, however, the unconditional variance exists and is positive. Regarding ret_NYSE, all the parameters are statistically significant, while Beta is a little smaller than usually and Alpha is a little greater.
In the Second step, the Correlation coefficient rho is obtained, and it is equal to 0.547847, which is approximately half of the Beta estimate obtained with OLS in the beginning.
Here are the tests for this specification:

```
  ***********
 ** TESTS **
***********
Q-Statistics on Standardized Residuals

 Series: ret_RACE
  Q(  5) =  2.03938   [0.8436694]
  Q( 10) =  4.28249   [0.9337046]
  Q( 20) =  12.1343   [0.9113771]
  Q( 50) =  56.0025   [0.2598288]

 Series: ret_NYSE
  Q(  5) =  6.81535   [0.2347394]
  Q( 10) =  10.1251   [0.4295857]
  Q( 20) =  16.8376   [0.6634945]
  Q( 50) =  46.6899   [0.6069992]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Q-Statistics on Squared Standardized Residuals

 Series: ret_RACE
  Q(  5) =  1.06493   [0.9571606]
  Q( 10) =  1.82692   [0.9975028]
  Q( 20) =  8.90585   [0.9839723]
  Q( 50) =  27.9960   [0.9949887]

 Series: ret_NYSE
  Q(  5) =  4.32685   [0.5033794]
  Q( 10) =  12.5518   [0.2498189]
  Q( 20) =  16.0284   [0.7148577]
  Q( 50) =  31.6997   [0.9797624]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hosking's Multivariate Portmanteau Statistics on Standardized Residuals
  Hosking(  5) =  20.7031   [0.4147880]
  Hosking( 10) =  33.3510   [0.7621965]
  Hosking( 20) =  63.3507   [0.9141611]
  Hosking( 50) =  198.541   [0.5158574]
---------------

Hosking's Multivariate Portmanteau Statistics on Squared Standardized Residuals
  Hosking(  5) =  15.2439   [0.6451555]
  Hosking( 10) =  27.9676   [0.8835381]
  Hosking( 20) =  45.3782   [0.9988463]
  Hosking( 50) =  132.590   [0.9998946]
Warning: P-values have been corrected by 2 degrees of freedom
---------------
```

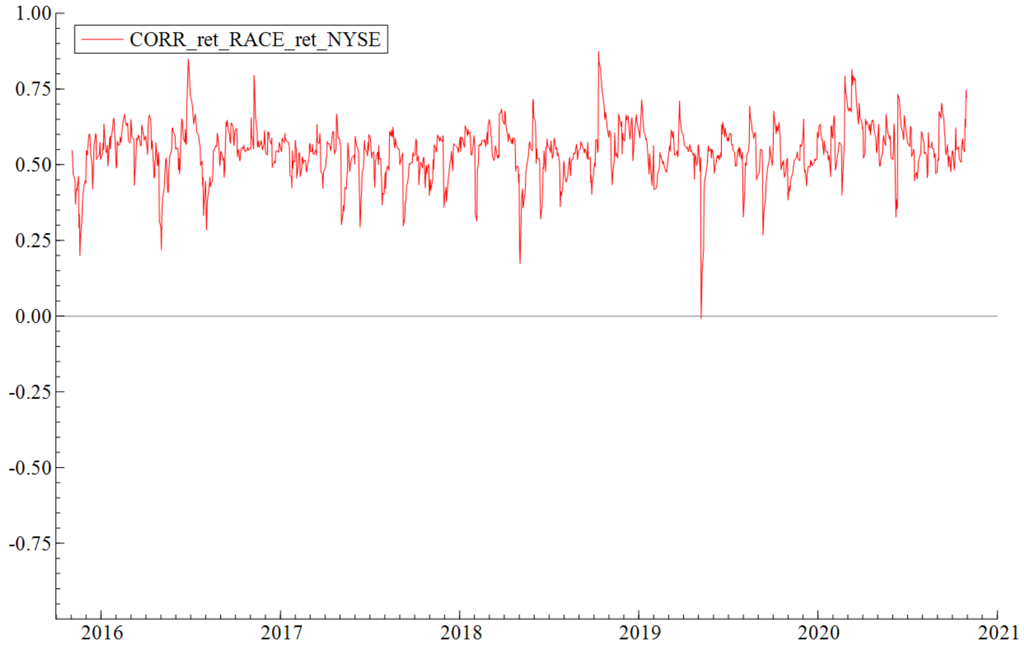
All the conducted tests show that there is no autocorrelation left, which is good.
Conditional Variances, Conditional Covariance and Conditional Correlation can be displayed thanks to the Graphic analysis tool.
Starting with the Conditional Variances:



The plots have quite a different behavior, similarly to what was initially seen in the Actual series to some extent, in the sense that Variance is generally greater in RACE. The highest Variance is present at the same point in time for both series, however, the Variance of NYSE is approximately three times greater than the one of RACE.
The Covariance plot is below:



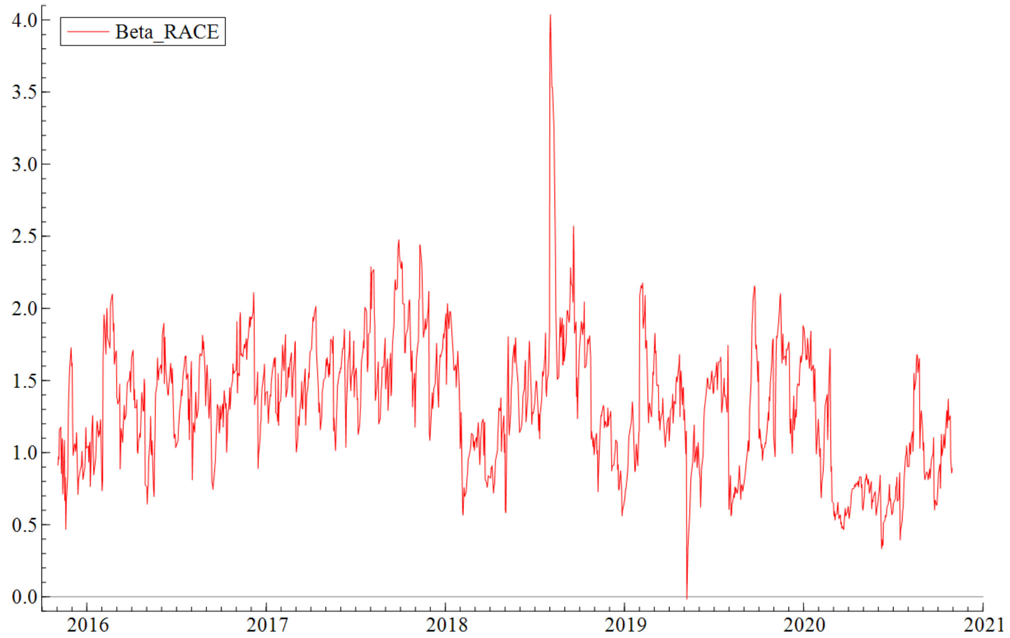
And the Correlation plot:



The Correlation does not look very constant. The lowest point was reached around April 2019, when the value was equal to zero.
Now, Conditional Variances and Conditional Covariance can be stored. Taking into account the formula:

$$\beta_i = \frac{cov(R_i, R_M)}{var(R_M)}$$

The Beta can be estimated using the Calculator tool. And after doing this, here is the plot of the estimated Beta:



As expected in the beginning of this project, the value of Beta is higher than 1 in many days, indicating higher volatility than NYSE Composite. Also, it does not look to be very stable. The highest value of Beta was approximately 4, reached in July 2018, which indicates that at that time the stock was 4 times more volatile than the market! In April 2019, instead, the value was lightly negative, indicating an opposite movement of the stock in relation with the movement of the NYSE Index.