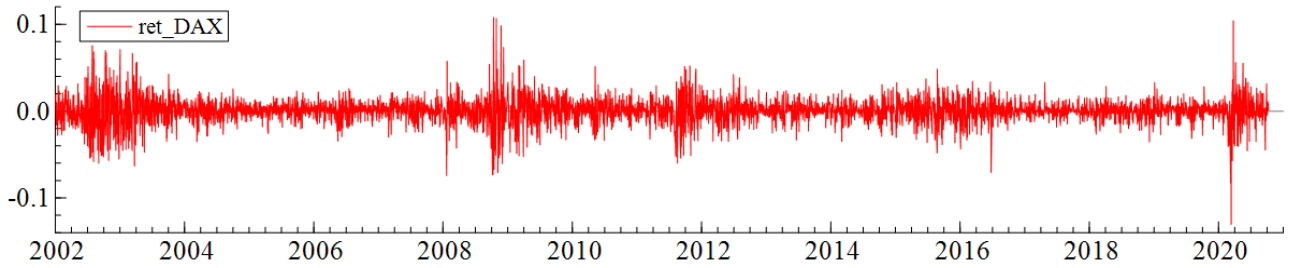
Stylized facts in DAX and BUX indices

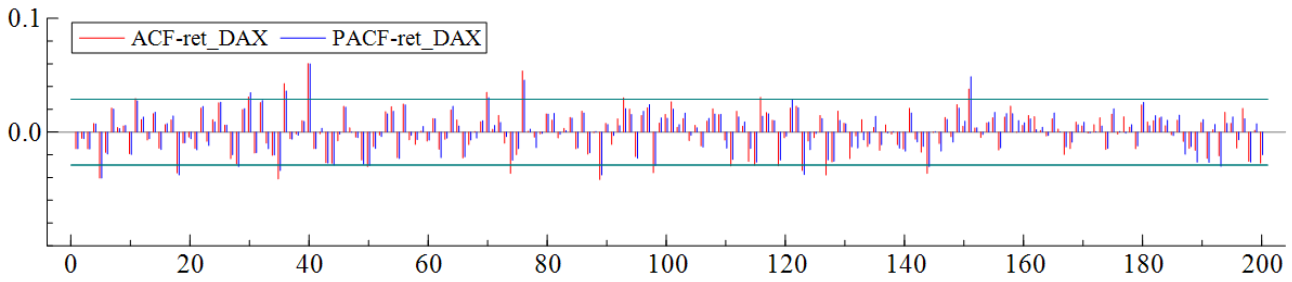
Daniele Melotti

I chose to work with DAX and BUX indices. An advantage of using these two indices is that the data is provided for the same range of time, between 2002-01-02 and 2020-10-09.
After uploading the data on OxMetrics, I calculated logarithmic returns for both indices, naming them ret_DAX and ret_BUX respectively.

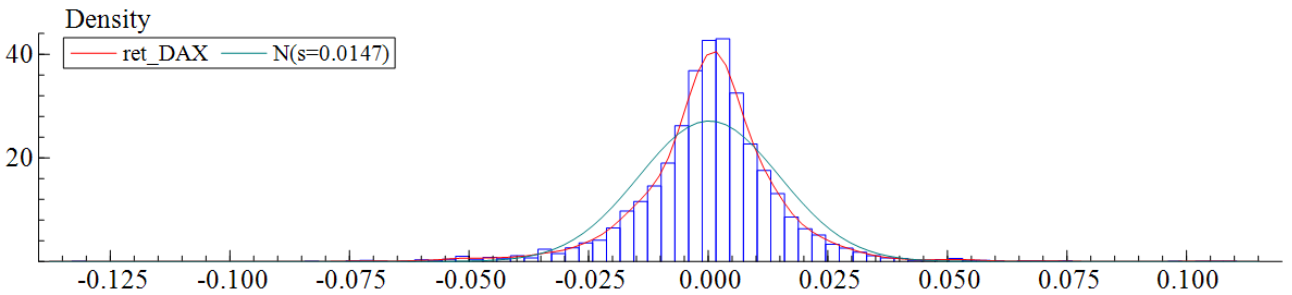
Starting with ret_DAX, I created an Actual series plot:



Some clusters of volatility are clearly visible: in the years between 2002 and 2004, probably due to the so-called “Early 2000s recessions”; around the year 2009, due to the global financial crisis; a little while before 2012, perhaps due to the European sovereign debt crisis; and at the beginning of 2020, due to the impact of the COVID-19 outbreak.
Then, I plotted an ACF-PACF graph with 200 lags displayed:



Here it possible to notice that in the initial lags there is not a lot of statistically significant autocorrelation. However, starting from the 30th lag, there is significant autocorrelation showing up to the 150th lag. The two most significant lags are the 40th and 76th/77th ones.
Finally, I plotted a distribution for ret_DAX:



This distribution is definitely non-normal. There is excess kurtosis and also negative skewness (left side).

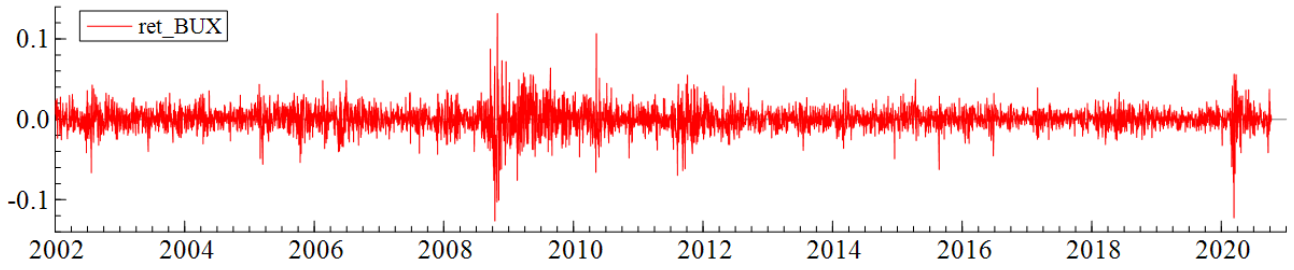
Then I switched to Descriptive statistics and prepared Basic stats, Normality test, ARCH test, the two Box-Pierce tests, ADF test, Geweke and Porter-Hudak test, Hurst-Mandelbrot and Lo R/S tests, and Runs test:

```
---- Database information ----
Sample:    2002-01-02 - 2020-10-09 (4767 observations)
Frequency: 1
Variables: 4

Variable               #obs  #miss    type          min         mean          max      std.dev
Date                   4767      0    date       2002-01-02            2020-10-09
ret_DAX                4766      1  double     -0.13055   0.00019438      0.10797     0.014702
Constant               4767      0  double            1            1            1            0
Trend                  4767      0  double            1         2384         4767       1376.1

Series #1/1: ret_DAX
---------

Normality Test

                   Statistic       t-Test      P-Value
Skewness            -0.14487       4.0842   4.4233e-05
Excess Kurtosis       6.4007       90.246      0.00000
Jarque-Bera           8152.4         .NaN      0.00000
---------------
ARCH 1-2 test:    F(2,4761) =   182.64 [0.0000]**
ARCH 1-5 test:    F(5,4755) =   160.62 [0.0000]**
ARCH 1-10 test:   F(10,4745)=   102.37 [0.0000]**
---------------
Box-Pierce Q-Statistics on Raw data
  Q(  5) =  10.2912   [0.0673932]
  Q( 10) =  15.9313   [0.1016153]
  Q( 20) =  30.6948   [0.0593512]
  Q( 50) =  111.880   [0.0000012]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
Box-Pierce Q-Statistics on Squared data
  Q(  5) =  1184.67   [0.0000000]**
  Q( 10) =  2250.22   [0.0000000]**
  Q( 20) =  3703.89   [0.0000000]**
  Q( 50) =  5640.12   [0.0000000]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hurst-Mandelbrot R/S test statistics: 1.11175
Lo R/S test statistics (q=1): 1.11998
Critical Values
90%: [0.861, 1.747]
95%: [0.809, 1.862]
99%: [0.721, 2.098]
H0: Hurst-Mandelbrot = no autocorrelation and Lo = no long-term dependence
---------------

RUNS TEST = 2.36040 [0.0182552]*
---------------
-----------------------------------------------

ADF Test with 3 lags
No intercept and no time trend
H0: ret_DAX is I(1)

ADF Statistics: -34.8293

Asymptotic critical values, Davidson, R. and MacKinnon, J. (1993)

        1%        5%       10%
```

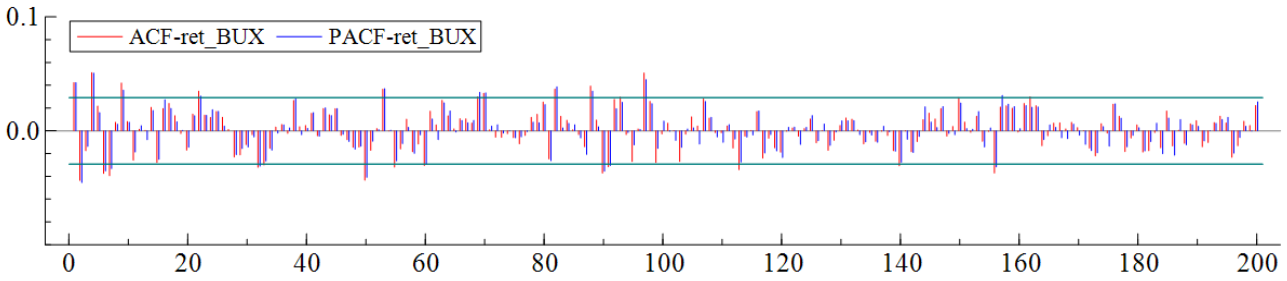
```
  -2.56572  -1.94093  -1.61663

OLS Results
                  Coefficient    t-value
y_1                 -1.027299    -34.829
dy_1                 0.012755    0.50216
dy_2                 0.007279    0.35255
dy_3                -0.007360   -0.50784
RSS                  1.029106
OBS                 4762.000000
Information Criteria (to be minimized)
Akaike         -5.600175  Shibata        -5.600176
Schwarz         -5.594742  Hannan-Quinn    -5.598266
---------------

----  Log Periodogram Regression ----
d parameter      -0.000259742 (0.0136717) [0.9848]
No of observations: 4766; no of periodogram points: 2382
```

From the **Basic stats**, the Mean is quite close to zero, while Standard deviation is much bigger, indicating that perhaps the Mean is not statistically significantly different from zero.
The **Normality test** confirms that there is negative skewness and excess kurtosis, as both respective p-values are statistically significant. Jarque-Bera statistic is significant as well, hence the returns from DAX don't follow a normal distribution.
Looking at the **ARCH test**, there is a lot of ARCH effect in every lag. Regarding the **Box-Pierce** on standardized residuals, there is autocorrelation only at 50 lags, while for the squared standardized residuals autocorrelation is present massively.
The **Hurst-Mandelbrot** and **Lo** statistics lay in the middle of the confidence intervals, hence they are signifying no autocorrelation and no long-term dependence.
According to the **Runs test** the null hypothesis should be rejected, namely there is some series (patterns) in ret_DAX. The **ADF** statistic is much smaller than any of the critical values (1%, 5% or 10%), therefore, the returns are stationary.
Looking at the d parameter generated by **Geweke and Porter-Hudak** test, it is negative but not statistically significant. So, according to this test, there is no long-term dependence within the series.

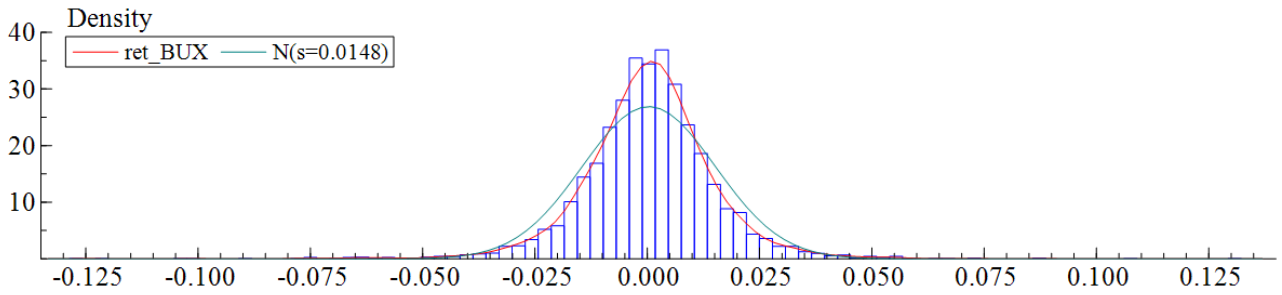
Switching to ret_BUX, I created the respective Actual series plot:



Here the volatility seems to be lesser compared to the one seen in ret_DAX. Peaks of volatility are recorded again around the year 2009, due to the global financial crisis; and at the beginning of 2020, due to Coronavirus outbreak.
The ACF-PACF graph:



It is visible that here there is significant autocorrelation in the first lags, especially the 1st, 2nd and 4th ones; later it tends to decrease, and then it re-appears before the 100th lag.
And here is the distribution:



In this case there is excess kurtosis and there seems to be no skewness. This series seems to be non-normally distributed but it will be checked with a Normality test.

Next step concerns the analysis of the Descriptive statistics for ret_BUX. The chosen tests are the same as for ret_DAX:

```
---- Database information ----
Sample:    2002-01-02 - 2020-10-09 (4689 observations)
Frequency: 1
Variables: 4

Variable               #obs  #miss    type          min          mean          max      std.dev
Date                   4689      0    date   2002-01-02                2020-10-09
ret_BUX                4688      1  double     -0.12649   0.00033126      0.13177     0.014825
Constant               4689      0  double            1            1            1            0
Trend                  4689      0  double            1         2345         4689       1353.6

Series #1/1: ret_BUX
---------

Normality Test

                   Statistic       t-Test      P-Value
Skewness            -0.28172       7.8773   3.3440e-15
Excess Kurtosis       7.6691       107.24      0.00000
Jarque-Bera           11551.         .NaN      0.00000
---------------
ARCH 1-2 test:    F(2,4683) =   350.18 [0.0000]**
ARCH 1-5 test:    F(5,4677) =   167.02 [0.0000]**
ARCH 1-10 test:   F(10,4667)=   129.03 [0.0000]**
---------------
Box-Pierce Q-Statistics on Raw data
  Q(  5) =  33.2413   [0.0000034]**
  Q( 10) =  55.9852   [0.0000000]**
  Q( 20) =  71.5845   [0.0000001]**
  Q( 50) =  118.331   [0.0000002]**
```

```
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
Box-Pierce Q-Statistics on Squared data
  Q(  5) =  1366.29   [0.0000000]**
  Q( 10) =  2885.51   [0.0000000]**
  Q( 20) =  4004.87   [0.0000000]**
  Q( 50) =  4826.68   [0.0000000]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------

Hurst-Mandelbrot R/S test statistics: 1.27311
Lo R/S test statistics (q=1): 1.24695
Critical Values
90%: [0.861, 1.747]
95%: [0.809, 1.862]
99%: [0.721, 2.098]
H0: Hurst-Mandelbrot = no autocorrelation and Lo = no long-term dependence
---------------

RUNS TEST =-0.828551 [0.4073586]
---------------
-----------------------------------------------

ADF Test with 3 lags
No intercept and no time trend
H0: ret_BUX is I(1)

ADF Statistics: -33.1993

Asymptotic critical values, Davidson, R. and MacKinnon, J. (1993)

        1%        5%       10%
  -2.56572  -1.94093  -1.61663

OLS Results
                  Coefficient    t-value
y_1                 -0.961613    -33.199
dy_1                 0.006377    0.25607
dy_2                -0.035739    -1.7703
dy_3                -0.051156    -3.5051
RSS                  1.023474
OBS                 4684.000000
Information Criteria (to be minimized)
Akaike         -5.589120  Shibata        -5.589121
Schwarz         -5.583610  Hannan-Quinn    -5.587182
---------------

----  Log Periodogram Regression ----
d parameter         -0.018561 (0.0137715) [0.1777]
No of observations: 4688; no of periodogram points: 2344
```

Firstly, **Basic stats** show again a very small Mean and bigger Standard deviation, indicating that probably the Mean is not statistically significantly different from zero.
The **Normality test** actually shows that there is statistically significant negative skewness and excess kurtosis. So, there is no normality in this distribution.
In the **ARCH test** a lot of ARCH effect is underlined, while both **Box-Pierce tests** show statistically significant autocorrelation at any considered lag, both in standardized residuals and in squared standardized residuals.
The **Hurst-Mandelbrot** and **Lo** test statistics are higher than those for ret_DAX, however, still included in all 3 confidence intervals, indicating no autocorrelation and no long-term dependence.
Looking at the **Runs test** there is no series embedded in these returns in opposition to what was observed in the returns of DAX. Like for ret_DAX, the **ADF** statistic is much smaller than any of the critical values, indicating the stationarity of the returns.

The d parameter from **Geweke and Porter-Hudak** test is negative, but not statistically significant. Hence, there is no long-term dependence within the series.

Here's a table summing up the results for ret_DAX and ret_BUX:

	DAX	**BUX**
Basic stats	Very small Mean value and bigger Standard deviation	Very small Mean value and bigger Standard deviation
Normality test	Non-normal distribution	Non-normal distribution
ARCH test	ARCH effect at all considered lags	ARCH effect at all considered lags
Box-Pierce on Standardized residuals	Autocorrelation present only at the 50^{th} lag	Autocorrelation present at all considered lags
Box-Pierce on Squared standardized residuals	Autocorrelation present at all considered lags	Autocorrelation present at all considered lags
Hurst-Mandelbrot test	Test statistic included within all intervals, indicating no autocorrelation	Test statistic included within all intervals, indicating no autocorrelation
Lo R/S test	Test statistic included within all intervals, indicating no long-term dependence	Test statistic included within all intervals, indicating no long-term dependence
Runs test	Series present within returns	Series not present within returns
ADF test	Stationary	Stationary
Geweke and Porter-Hudak test	No long memory	No long memory

The tests for the two series appear to have similar outcomes, apart from small differences in Runs test and Box-Pierce on Standardized residuals. This is an interesting outcome because it shows many similarities between two indices that are actually quite different, especially in terms of development.