Geometric Modeling

Assignment 2: Implicit Surface Reconstruction

Julian Panetta -- fjp234@nyu.edu

Acknowledgements: Olga Diamanti

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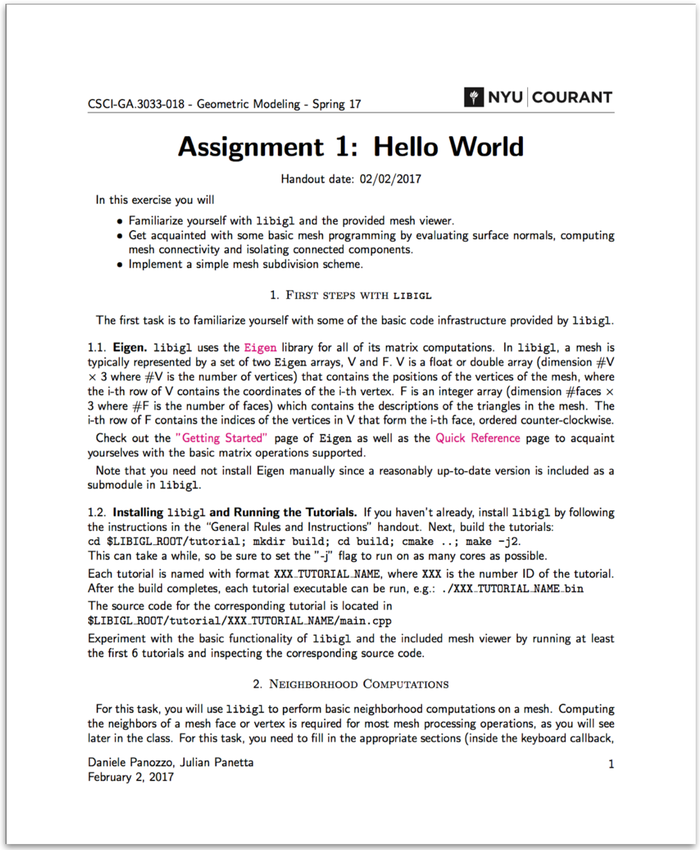
Administrative Items

- This is a "**Tutoring Session,**" not a "Recitation" (NYU Policy!)
  - Attendance isn't mandatory, but hopefully it's helpful…
- **HW 2 demo session:** March 22, 2017 2-3pm (Wednesday)
  - Schedule appointment if you can't make it to the session: fjp234@nyu.edu
- Assignments will overlap:
  - HW 3 out February 23, 2017 (optional)
  - HW 4 out March 9, 2017

HW 1 Questions?

- Please clone, build, and submit some code to test the process!
- Hit any issues?
  - Compilation
  - GitHub accounts
    (COMPLETE SURVEY IF YOU HAVEN'T)
  - Travis-CI



CSCI-GA.3033-018 - Geometric Modeling - Spring 17 NYU COURANT

Assignment 1: Hello World

Handout date: 02/02/2017

In this exercise you will

- Familiarize yourself with `libigl` and the provided mesh viewer.
- Get acquainted with some basic mesh programming by evaluating surface normals, computing mesh connectivity and isolating connected components.
- Implement a simple mesh subdivision scheme.

1. First steps with libigl

The first task is to familiarize yourself with some of the basic code infrastructure provided by `libigl`.

1.1. **Eigen.** `libigl` uses the Eigen library for all of its matrix computations. In `libigl`, a mesh is typically represented by a set of two `Eigen` arrays, V and F. V is a float or double array (dimension #V × 3 where #V is the number of vertices) that contains the positions of the vertices of the mesh, where the i-th row of V contains the coordinates of the i-th vertex. F is an integer array (dimension #faces × 3 where #F is the number of faces) which contains the descriptions of the triangles in the mesh. The i-th row of F contains the indices of the vertices in V that form the i-th face, ordered counter-clockwise.

Check out the "Getting Started" page of `Eigen` as well as the Quick Reference page to acquaint yourselves with the basic matrix operations supported.

Note that you need not install Eigen manually since a reasonably up-to-date version is included as a submodule in `libigl`.

1.2. **Installing `libigl` and Running the Tutorials.** If you haven't already, install `libigl` by following the instructions in the "General Rules and Instructions" handout. Next, build the tutorials:
`cd $LIBIGL_ROOT/tutorial; mkdir build; cd build; cmake ..; make -j2`.
This can take a while, so be sure to set the "`-j`" flag to run on as many cores as possible.

Each tutorial is named with format `XXX_TUTORIAL_NAME`, where `XXX` is the number ID of the tutorial. After the build completes, each tutorial executable can be run, e.g.: `./XXX_TUTORIAL_NAME.bin`

The source code for the corresponding tutorial is located in
`$LIBIGL_ROOT/tutorial/XXX_TUTORIAL_NAME/main.cpp`

Experiment with the basic functionality of `libigl` and the included mesh viewer by running at least the first 6 tutorials and inspecting the corresponding source code.

2. Neighborhood Computations

For this task, you will use `libigl` to perform basic neighborhood computations on a mesh. Computing the neighbors of a mesh face or vertex is required for most mesh processing operations, as you will see later in the class. For this task, you need to fill in the appropriate sections (inside the keyboard callback,

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February 2, 2017 1

Assignment 2: Implicit Surface Reconstruction

Handout date: 02/08/2017
Submission deadline: TBA
Demo date: TBA

In this exercise you will

- Compute an implicit MLS function approximating a 3D point cloud with given (but possibly unnormalized) normals.
- Sample the implicit function on a 3D volumetric grid.
- Apply the marching cubes algorithm to extract a triangle mesh of this zero level set.
- Experiment with various MLS reconstruction parameters.

Your main task is to construct an implicit function $f(\mathbf{x})$ defined on all $\mathbf{x} \in \mathbb{R}^3$ whose zero level set contains (or at least passes near) each input point. That is, for every point $\mathbf{p}_i$ in the point cloud, we want $f(\mathbf{p}_i) = 0$. Furthermore, ∇f (the isosurface normal) evaluated at each point cloud location should approximate the point's normal provided as input.

You will construct f by interpolating a set of target values, f_i, at "constraint locations," $\mathbf{c}_i$. The MLS interpolant is defined by minimization of the form $f(\mathbf{x}) = \operatorname{argmin}_\phi \sum_i \left(w(\mathbf{c}_i, \mathbf{x})[\phi(\mathbf{c}_i) - f_i]\right)^2$, where $\phi(\mathbf{x})$ lies in the space of admissible function (e.g., multivariate polynomials up to some degree) and w is a weight function that prioritizes each constraint equation depending on the evaluation point, $\mathbf{x}$.

1. Setting up the Constraints

Your first step is thus to build the set of constraint equations by choosing constraint locations and values. Naturally, each point $\mathbf{p}_i$ in the input point cloud should contribute a constraint with target value $f_i = 0$. But these constraints alone provide no information to distinguish the object's inside (where we want $f < 0$) from its outside (where we want $f > 0$). Even worse, the minimization is likely to find the trivial solution $f = 0$ (if it lies in the space of admissible functions). To address these problems, we introduce additional constraints incorporating information from the normals as follows:

- For each point $\mathbf{p}_i$ in the point cloud, add a constraint of the form $f(\mathbf{p}_i) = 0$.
- Fix an ε value, for instance $\varepsilon = 0.01 \times \text{bounding\_box\_diagonal}$.
- For each point $\mathbf{p}_i$ compute $\mathbf{p}_{i+N} = \mathbf{p}_i + \varepsilon \mathbf{n}_i$, where $\mathbf{n}_i$ is the normalized normal of $\mathbf{p}_i$. Check that $\mathbf{p}_i$ is the closest point to $\mathbf{p}_{i+N}$; if not, halve ε and recompute $\mathbf{p}_{i+N}$ until this is the case. Then, add another constraint equation: $f(\mathbf{p}_{i+N}) = \varepsilon$.
- Repeat the same process for $-\epsilon$, i.e., add equations of the form $f(\mathbf{p}_{i+2N}) = -\epsilon$. Do not forget to check each time that $\mathbf{p}_i$ is the closest point to $\mathbf{p}_{i+2N}$.

After these steps, you should have $3n$ equations for the implicit function $f(\mathbf{x})$.

1.1. **Creating the constraints.** For this task, you need to complete the appropriate sections (keyboard callback, keys '1' and '2') of `src/main.cpp`. Pressing key '1' displays the input point cloud (this part

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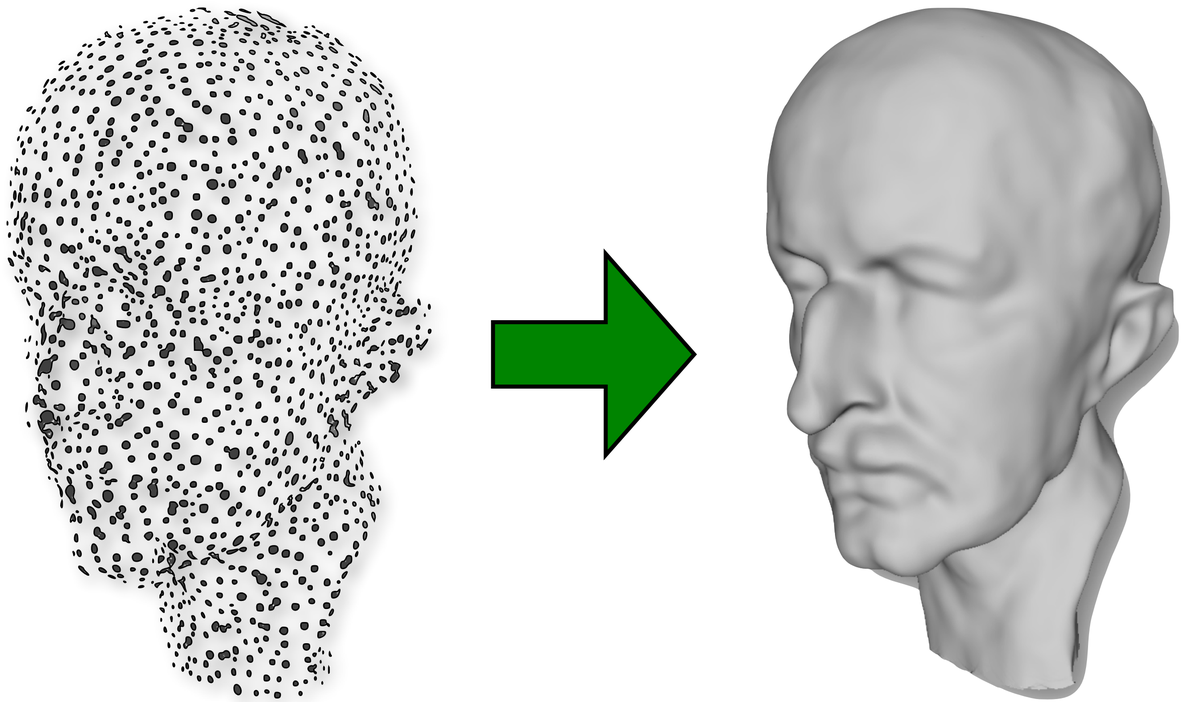
February 7, 2017

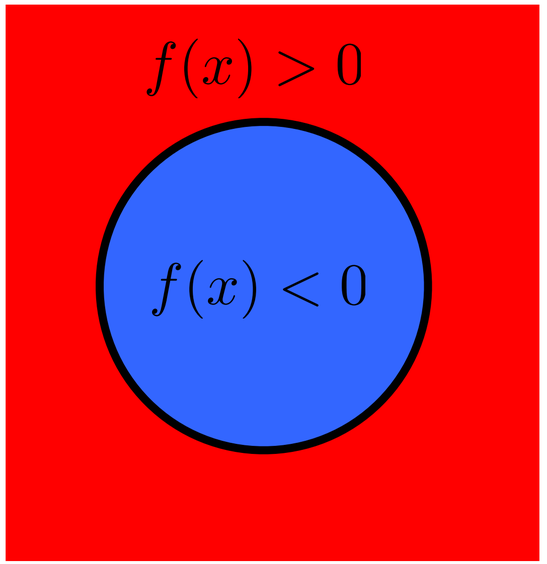
HW 2: Surface Reconstruction

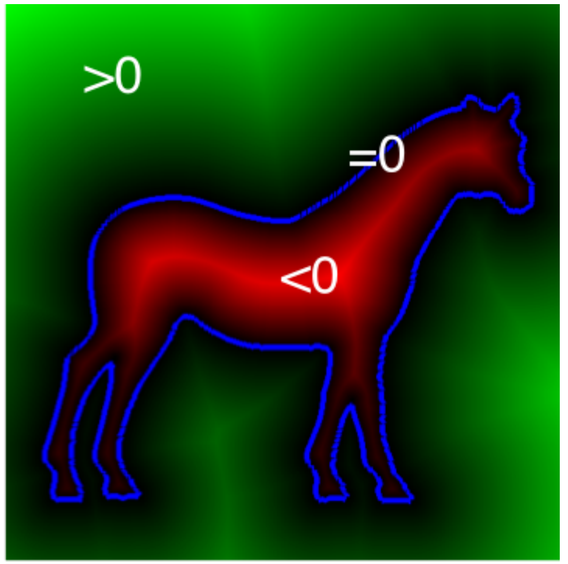


- **Input:** point cloud with normals
- **Output:** smooth surface mesh passing near each point
- **How?** Find out tomorrow from Daniele's lecture! (Or keep listening…)

Implicit Surface Reconstruction

- Remember: **surface representation matters!**
- Implicit representation bypasses many headaches an explicit approach would encounter.
- Guarantees by construction:
  - 2-Manifold
  - No holes (watertight)
- Robust to noisy point clouds



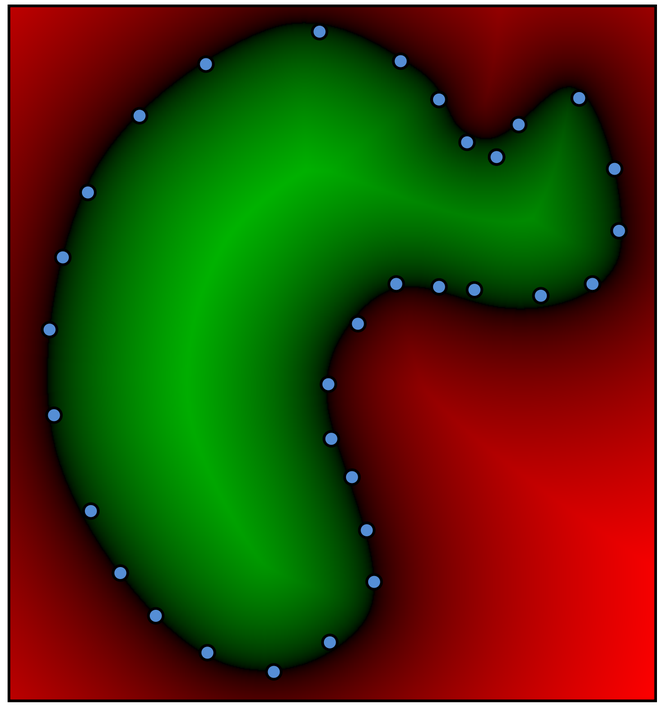


Simplifies the Problem

Surface interpolation

Scalar field interpolation

?

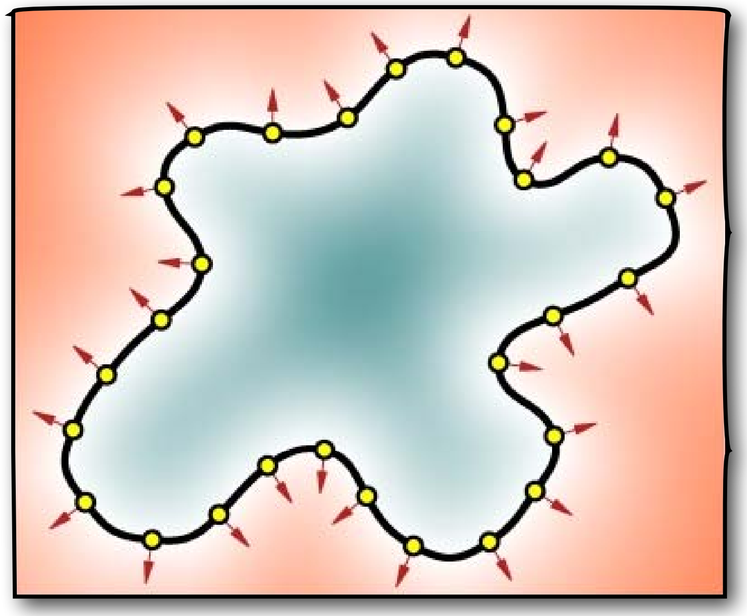


Constructing the Scalar Field

- Interpolate information from the input cloud:
  - **Points** tell us where the zero level set should go:

$$f(\mathbf{p}_i) = 0$$

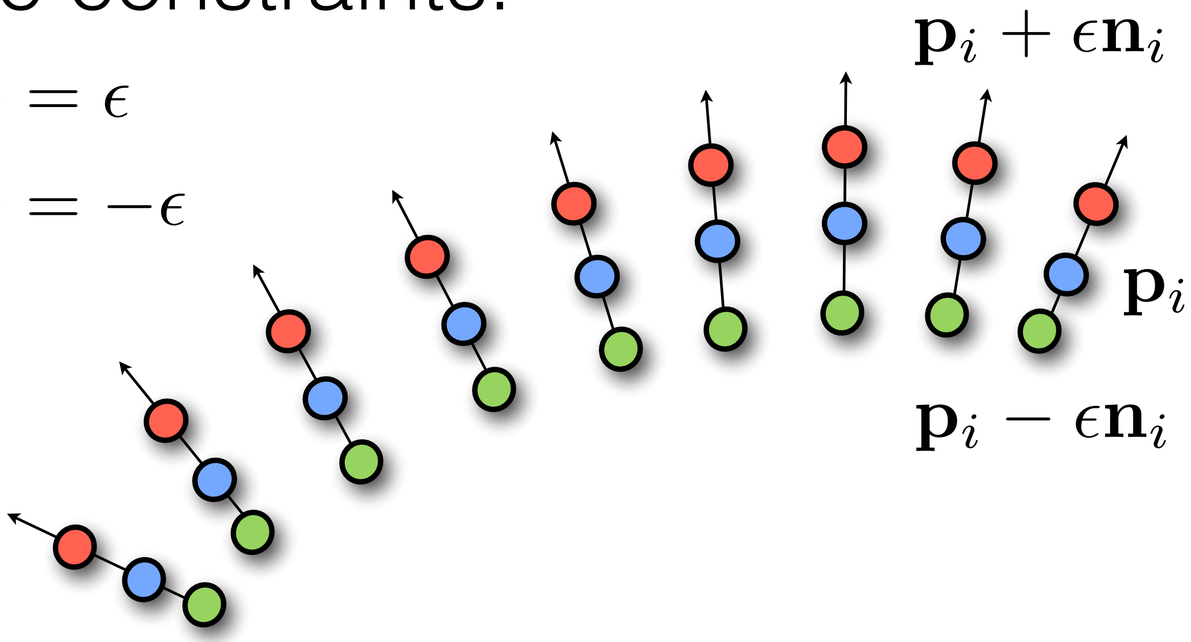
  - **Normals** define (locally) inside/outside

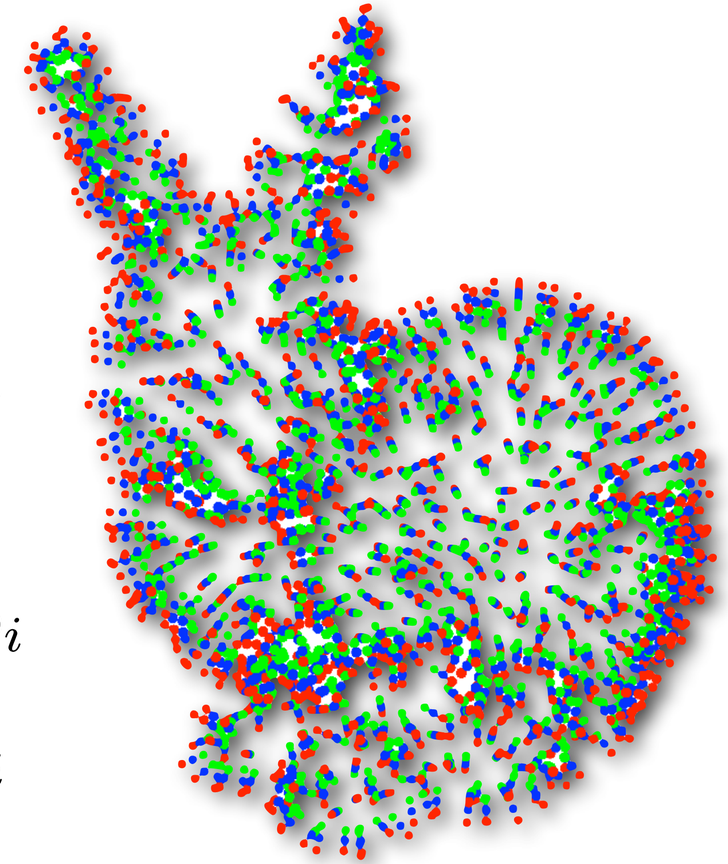


Step 1: Build the Constraint Set

- Point constraints $f(\mathbf{p}_i) = 0$ are insufficient (trivial solution f = 0)
- Incorporate normal info with additional off-surface constraints:

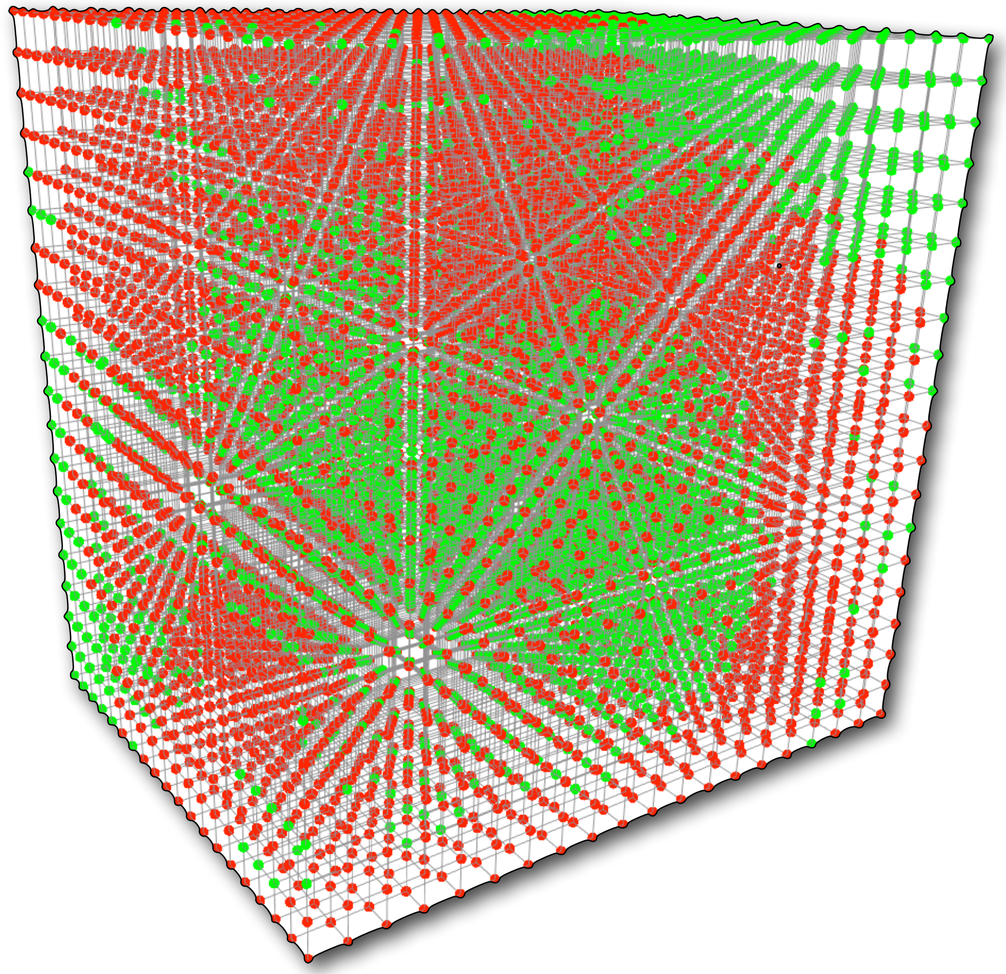
$$f(\mathbf{p}_i + \epsilon\mathbf{n}_i) = \epsilon$$
$$f(\mathbf{p}_i - \epsilon\mathbf{n}_i) = -\epsilon$$





Step 2: Construct Interpolant

- Construct regular grid
- Compute nodal scalar field satisfying constraints (approximately).
- Method: **MLS (Moving Least Squares)**



Interpolation Problem

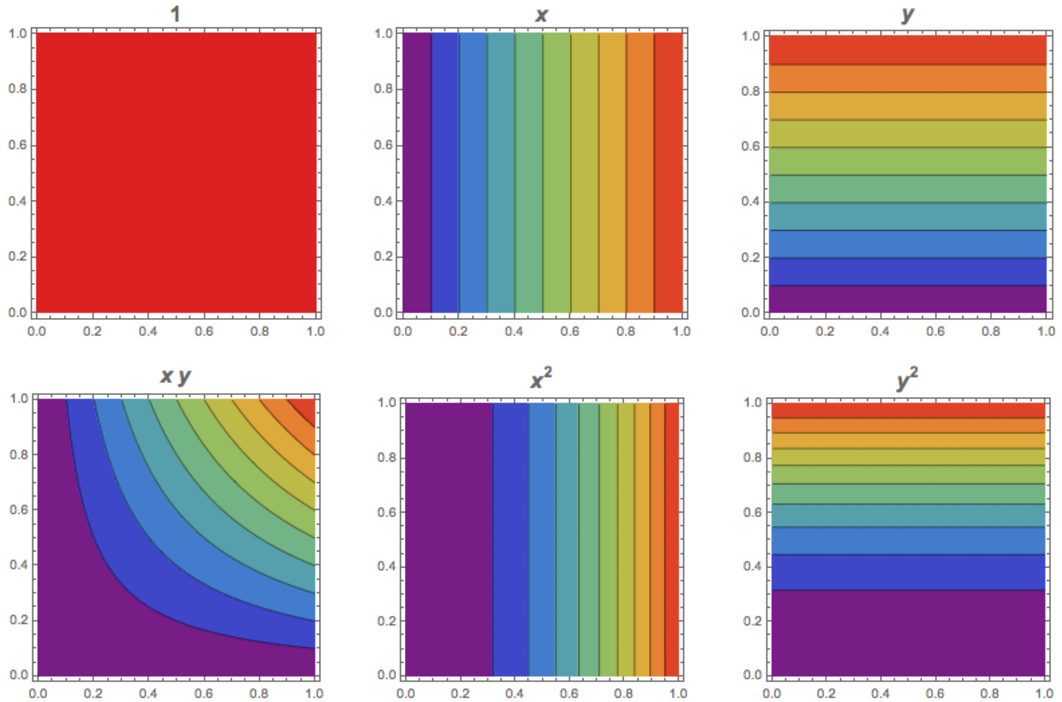
- List of 3N constraint locations, $\mathbf{c}_i$ (e.g. $\mathbf{p}_0$, $\mathbf{p}_0+\varepsilon\mathbf{n}_0$, …)
- List of 3N values, d_i
- Together, they describe 3N constraints of the form $f(\mathbf{c}_i) = d_i$
- **Goal:** find the "best" f in the span of chosen basis functions b($\mathbf{x}$):

$$f(\mathbf{x}) = \sum_j b_j(\mathbf{x}) a_j$$

(By tuning weights a_j to best approximate constraints)

Basis Functions

- For this assignment, we'll use **polynomial basis functions**:



(but in 3D)

Constraints in the Basis

- We can express our constraints in this basis:

$$f(\mathbf{c}_i) = \sum_j b_j(\mathbf{c}_i) a_j = d_i$$

In matrix form:

$$B\mathbf{a} = \mathbf{d}$$

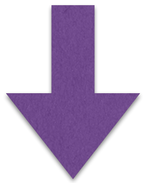
- Where matrix $B_{ij} := b_j(c_i)$
  (columns hold basis function's value at every constraint location).

$$B = \begin{bmatrix} 1 & x_1 & y_1 & \dots \\ 1 & x_2 & y_2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Overconstrained Linear System

- We'll have many more constraints than basis functions…
- Least-squares solution?

$$\min_f \sum_i (f(\mathbf{c}_i) - d_i)^2$$

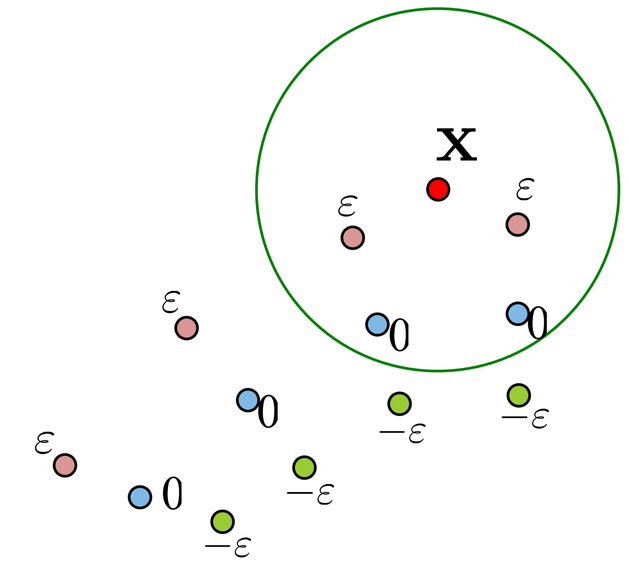


$$\min_{\mathbf{a}} \|B\mathbf{a} - \mathbf{d}\|^2$$

- What's bad about this?

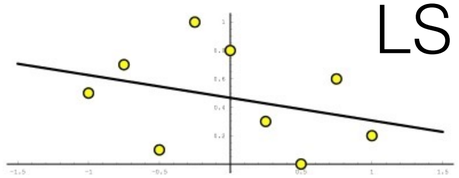
Problems with Least-squares

- **Global problem:** large matrices (even if basis functions are local)
- **Need many, high-degree basis functions**
  - Evaluating interpolant becomes expensive
- Better idea:
  - Construct **low degree**, **local interpolants** and stitch them together

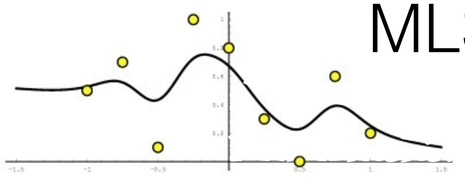


Moving Least Squares (MLS)

LS



MLS



- MLS builds distinct local interpolant **around every eval pt!**
- But final stitched function is still guaranteed smooth.
- Idea: weight the constraints based on distance to eval pt **x:**

$$f_{\mathbf{x}} := \underset{f}{\operatorname{argmin}} \sum_i w\left(\|\mathbf{x} - \mathbf{c_i}\|\right)\left(f(\mathbf{c}_i) - d_i\right)^2$$

- **Constraints with zero weight disappear!**
  (Choose weight function so few kept ==> **small linear system**)

MLS in Matrix Form

$$\min_f \sum_i w\left(\|\mathbf{x}-\mathbf{c}_i\|\right)\left(f(\mathbf{c}_i)-d_i\right)^2$$

$$\min_a \|B\mathbf{a}-\mathbf{d}\|^2_{W(\mathbf{x})}$$

Note: some papers call this $W(\mathbf{x})^2$

$$\|B\mathbf{a}-\mathbf{d}\|^2_{W(\mathbf{x})} := \left(B\mathbf{a}-\mathbf{d}\right)^T W(\mathbf{x})\left(B\mathbf{a}-\mathbf{d}\right)$$

$$W(\mathbf{x}) = \begin{bmatrix} w(\|\mathbf{x}-\mathbf{c}_1\|) & & \\ & \ddots & \\ & & w(\|\mathbf{x}-\mathbf{c}_{3N}\|) \end{bmatrix}$$

MLS Coefficients, Closed Form

- MLS objective function is quadratic in coefficients **a**; find optimum by differentiating and solving a linear system:

$$\begin{aligned} 0 =& \nabla_{\mathbf{a}} \left(\left(B\mathbf{a} - \mathbf{d} \right)^T W(\mathbf{x}) \left(B\mathbf{a} - \mathbf{d} \right) \right) \\ =& \quad 2B^T W(\mathbf{x}) B\mathbf{a} - 2B^T W(\mathbf{x})\mathbf{d} \end{aligned}$$

- Thus the coefficients **for point x** are given by solving the system:

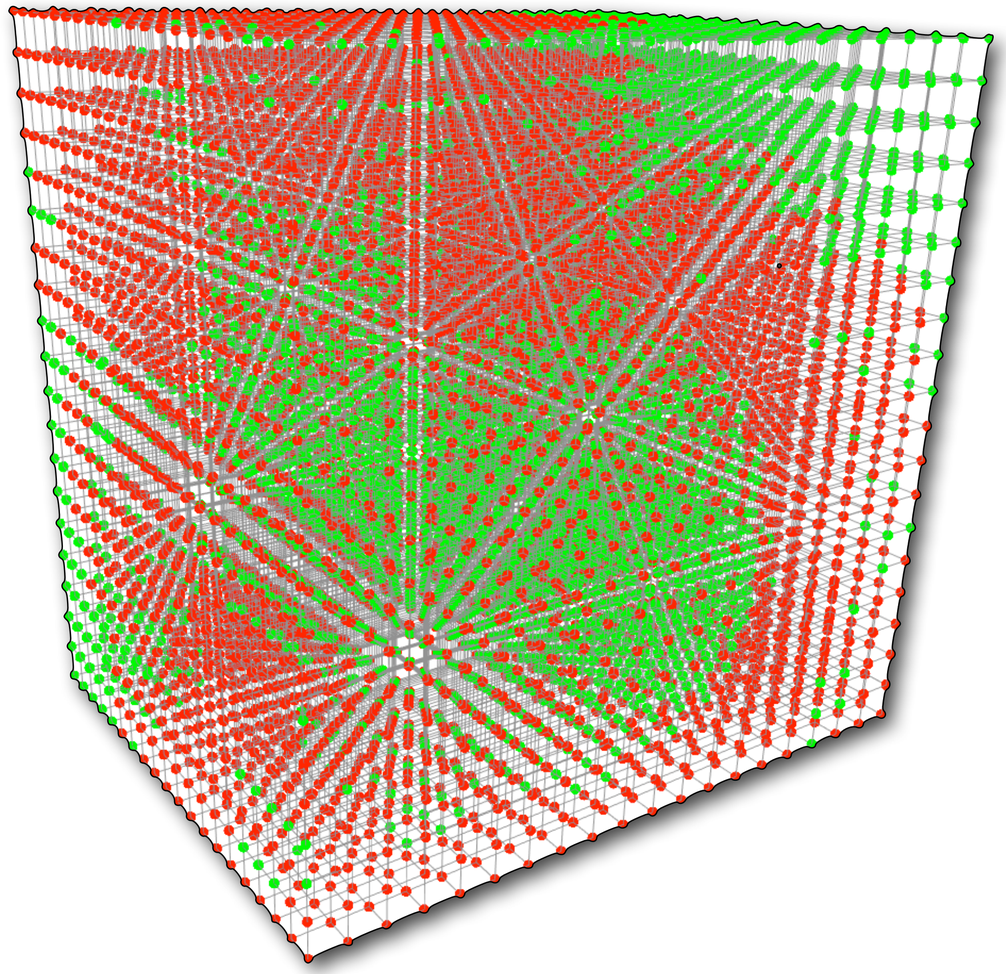
$$\left(B^T W(\mathbf{x}) B \right) \mathbf{a}(\mathbf{x}) = B^T W(\mathbf{x}) \mathbf{d}$$

for **a(x)**.

Step 2: Construct Interpolant

- Finally, fill in the grid!
- Evaluate local MLS interpolant at each grid point **x**.

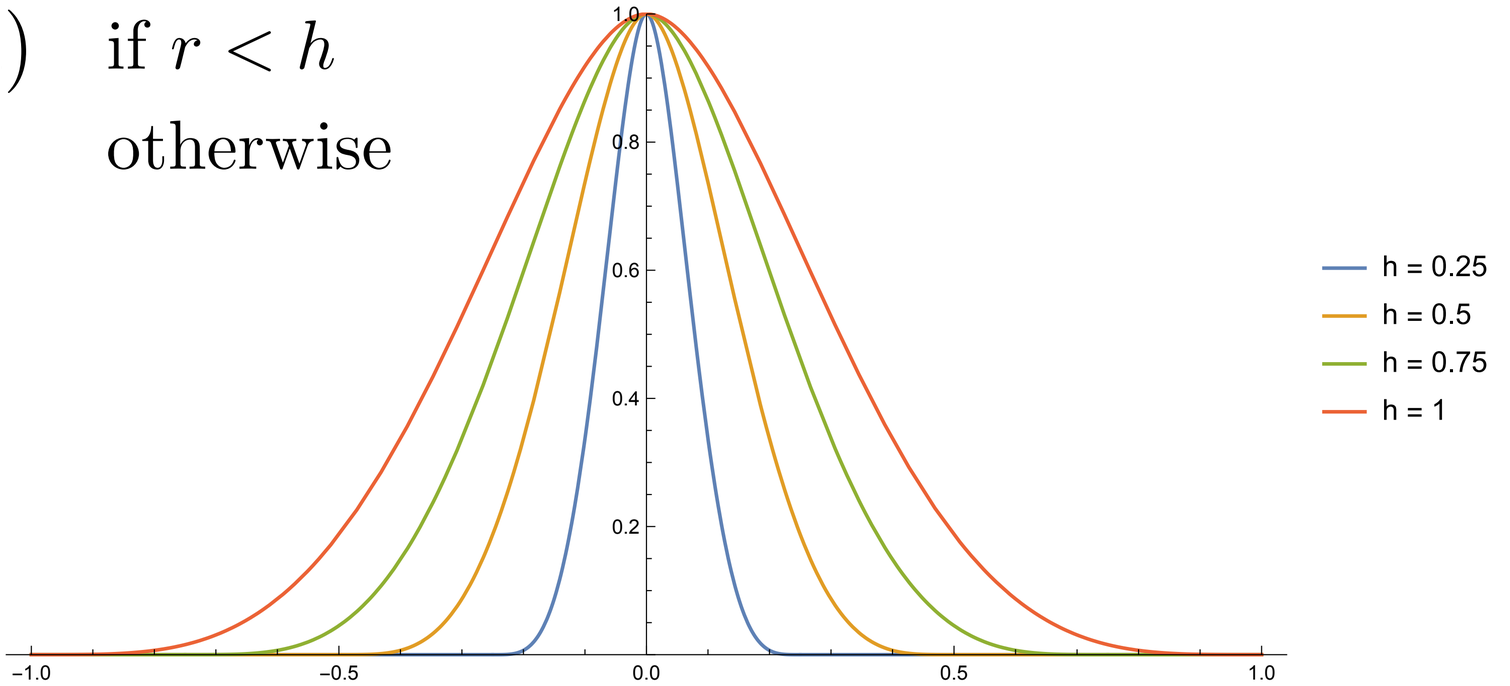
$$f_{\mathbf{x}}(\mathbf{x}) = \sum_j b_j(\mathbf{x}) a_j(\mathbf{x})$$



Wendland Weights

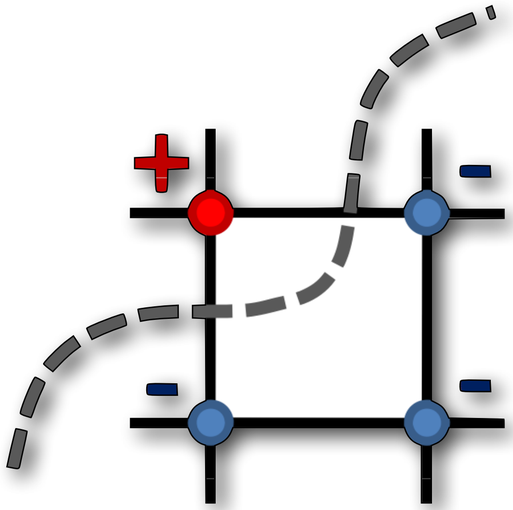
- You'll use **Wendland weights** for w in this assignment
- Vanish at dist "h" from eval pt (most constraints disappear)

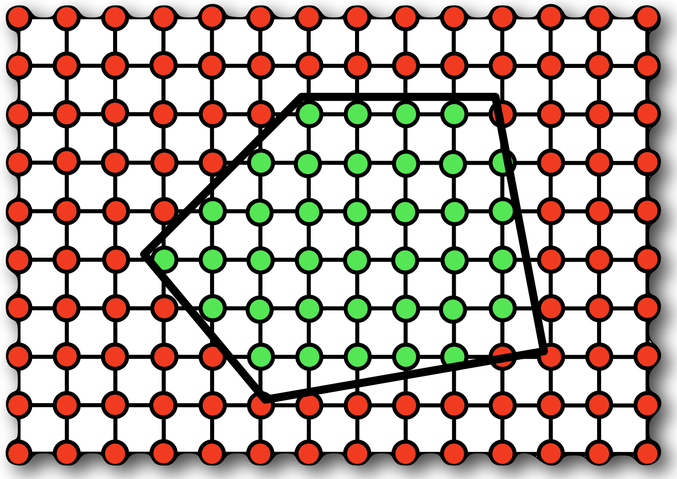
$$w(r) := \begin{cases} \left(1 - \frac{r}{h}\right)^4 \left(4\frac{r}{h} + 1\right) & \text{if } r < h \\ 0 & \text{otherwise} \end{cases}$$

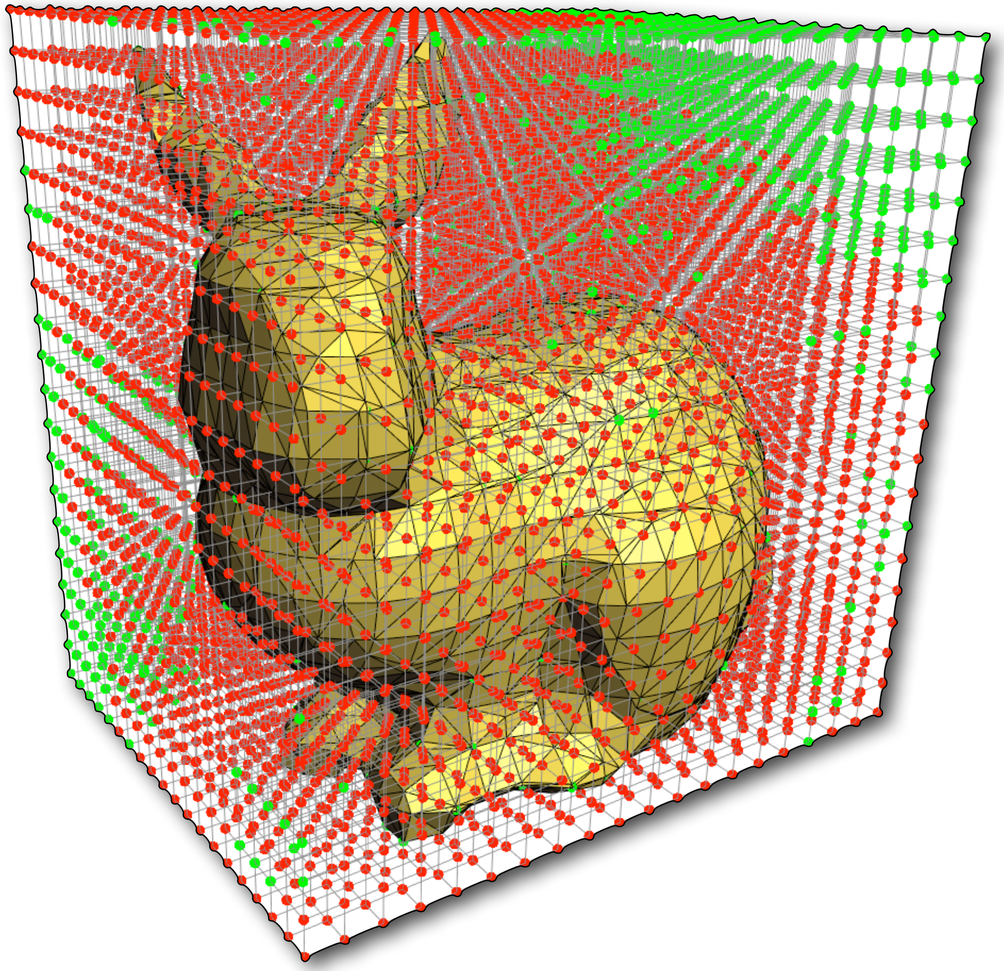


Step 3: Extract Zero Level Set

- Use the **marching cubes** algorithm to extract the grid function's zero isosurface
- Just call igl::copyleft::marching_cubes

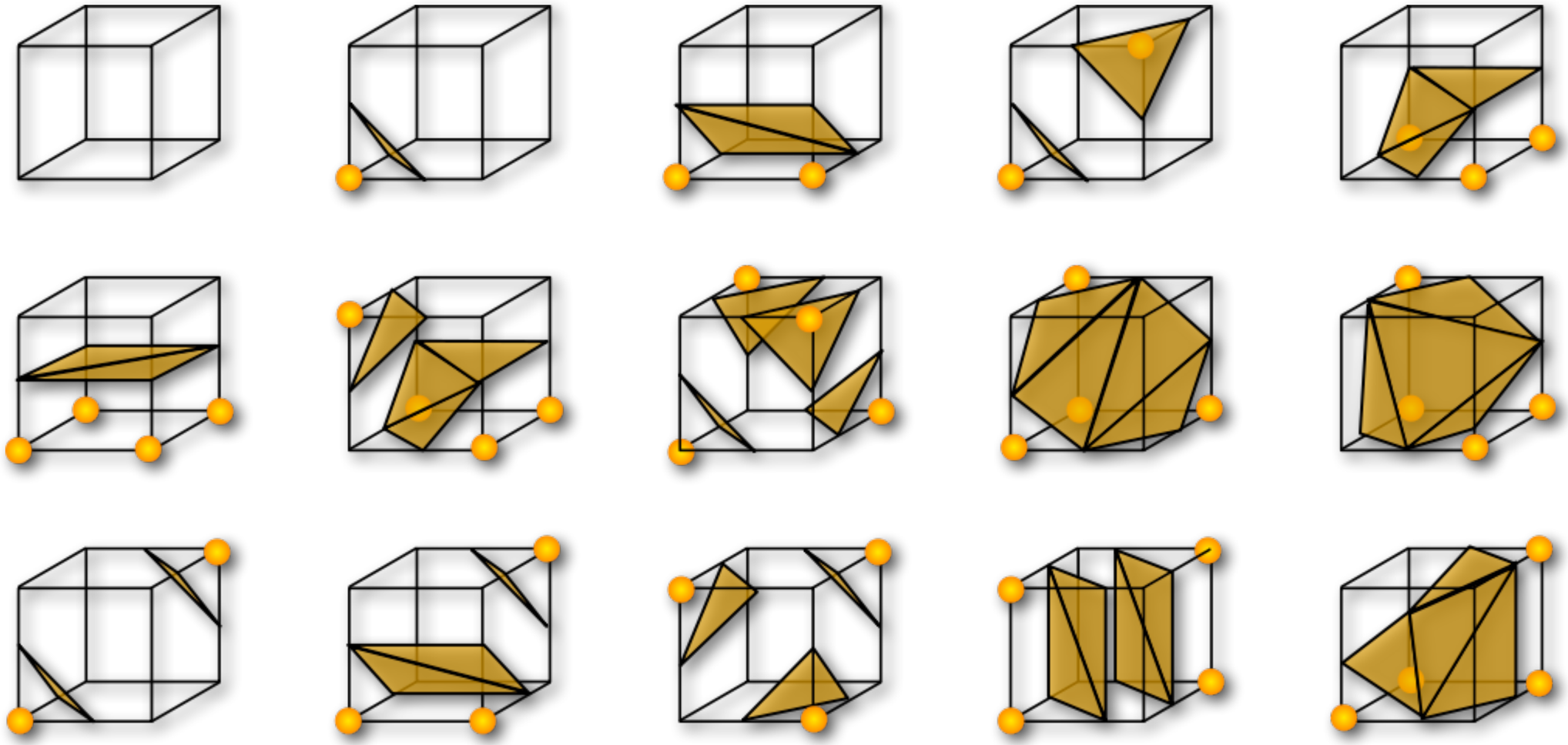




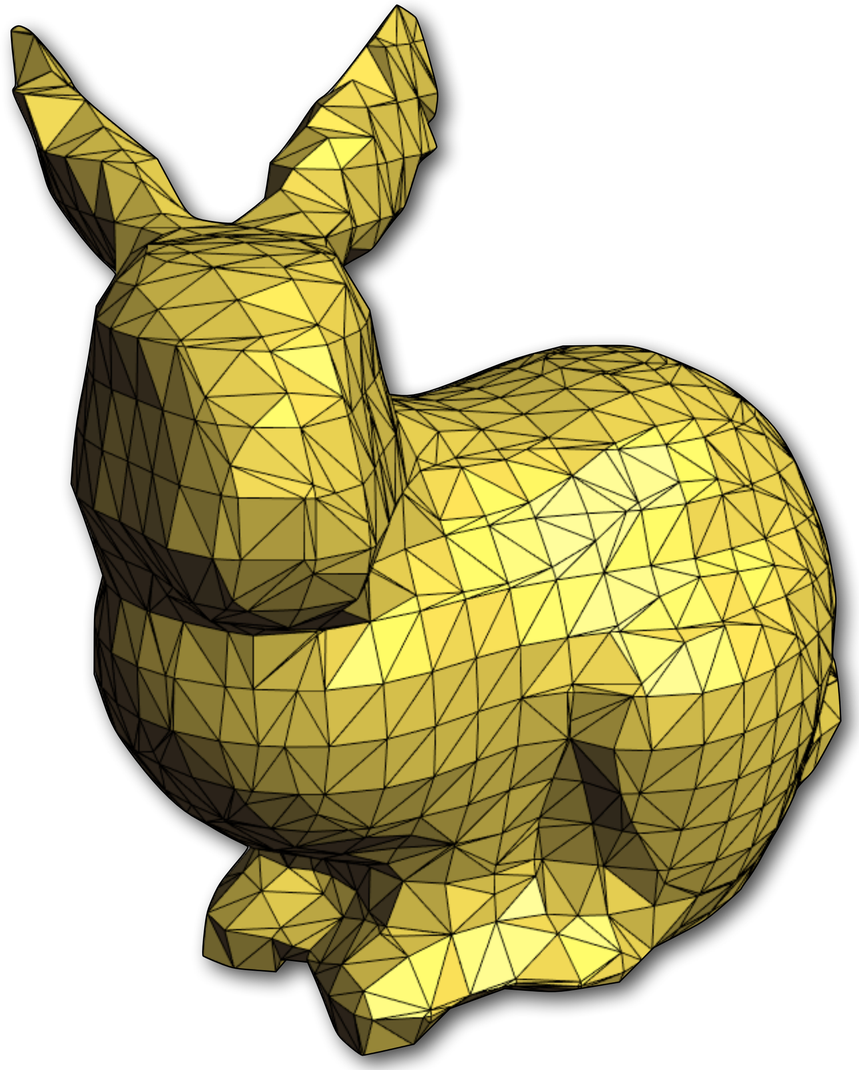


Marching Cubes: General Idea

- Look up triangles to create in each grid cell based on corner values:

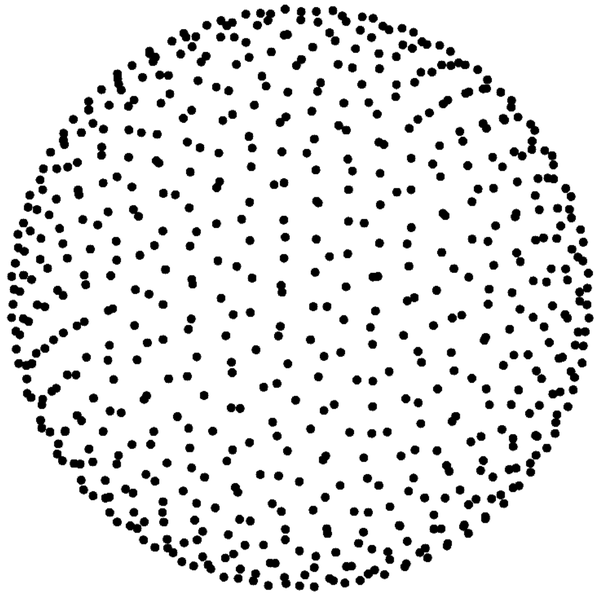


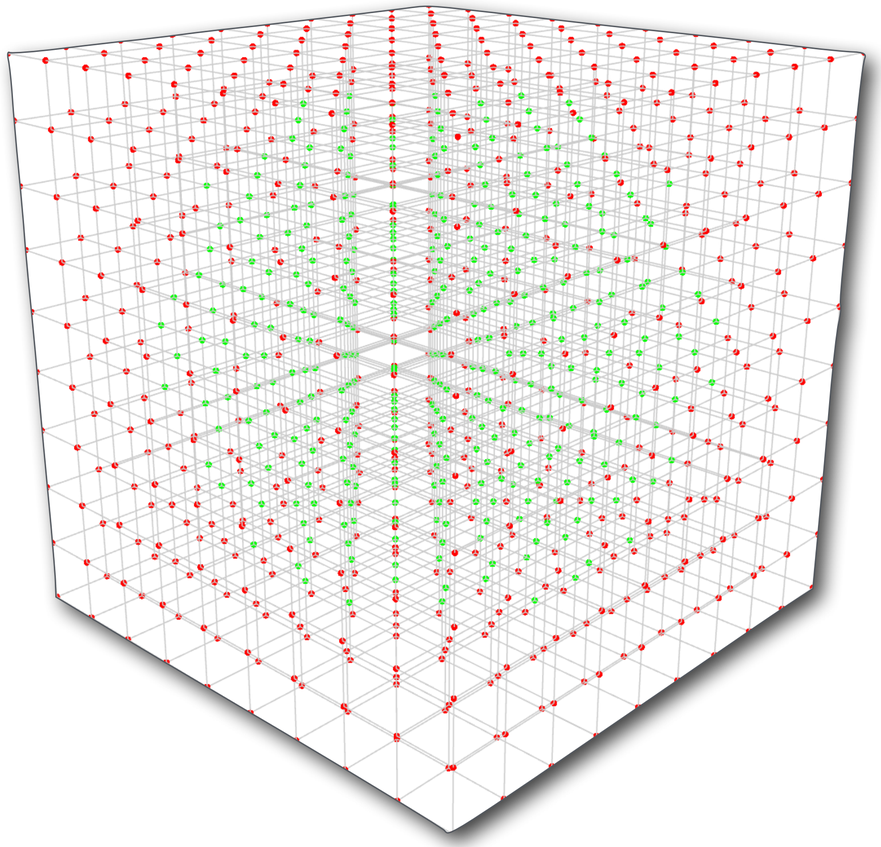
Final Result from Marching Cubes

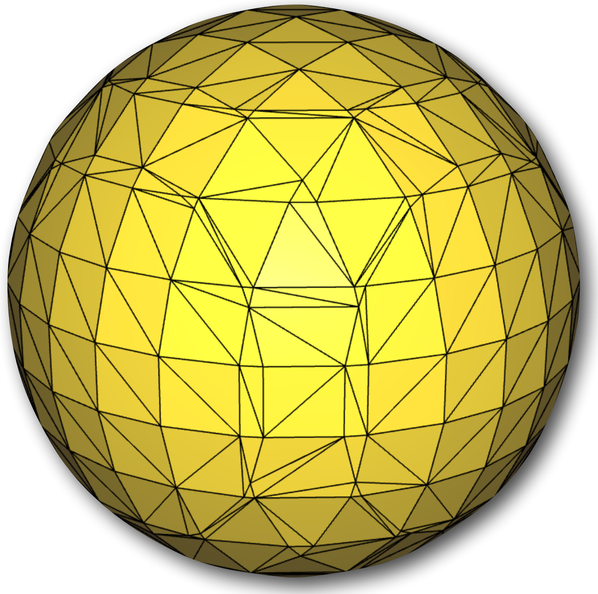


Provided Code

- Implements pipeline but uses analytic signed distance fn for sphere in place of MLS

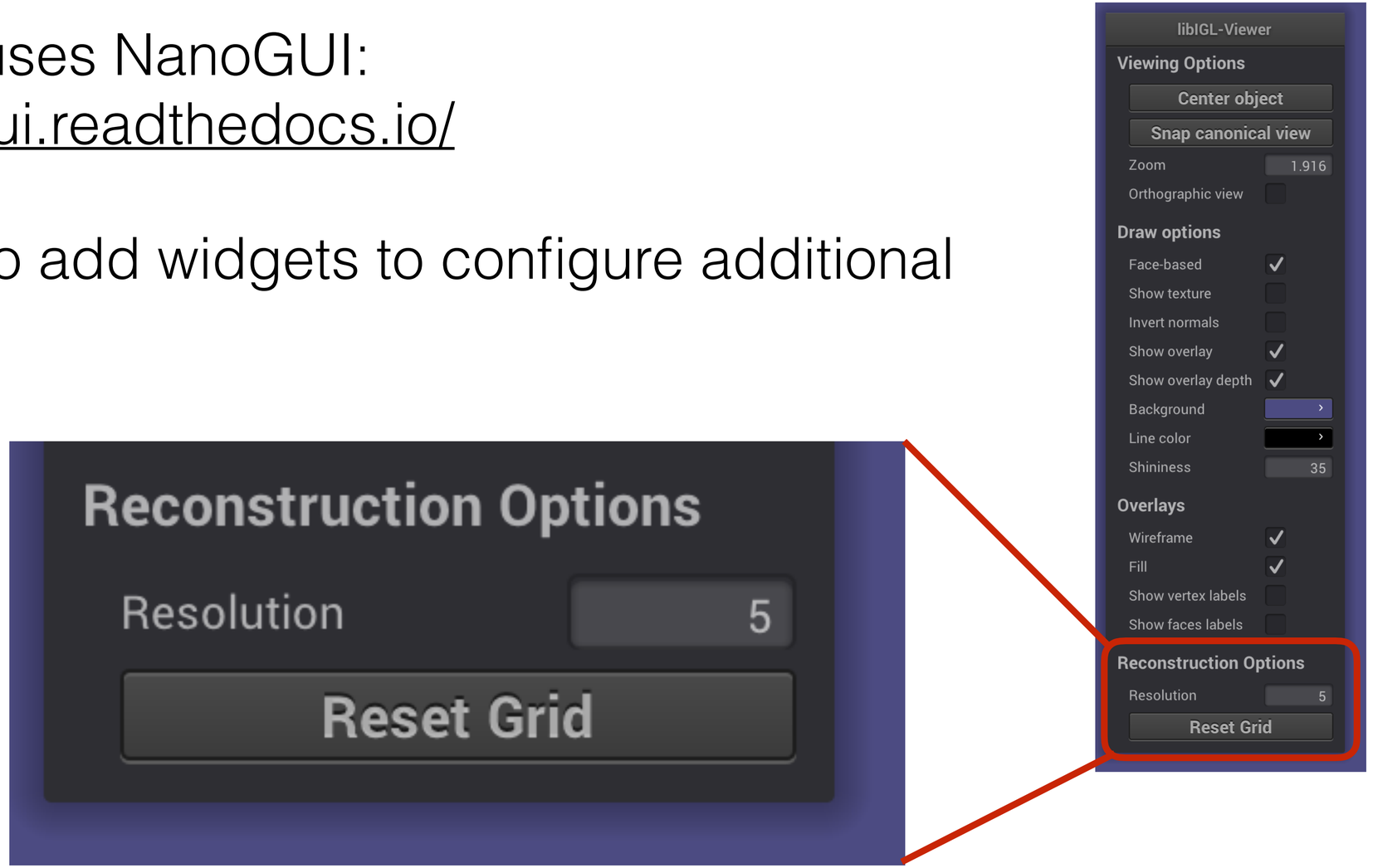






NanoGUI

- IGL Viewer uses NanoGUI: http://nanogui.readthedocs.io/
- You'll need to add widgets to configure additional variables.



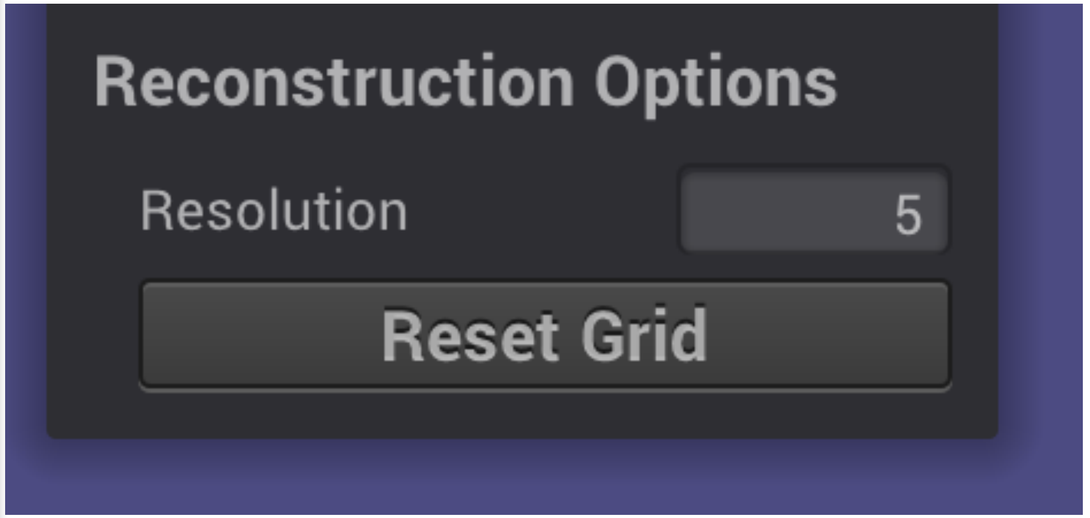
NanoGUI: Adding Settings

- Thankfully, this is really easy:

```
viewer.callback_init = [&](Viewer &v) {
    // Add widgets to the sidebar.
    v.ngui->addGroup   ("Reconstruction Options");
    v.ngui->addVariable("Resolution", resolution);
    v.ngui->addButton  ("Reset Grid", [&](){
        // Recreate the grid
        createGrid();
        // Switch view to show the grid
        callback_key_down(v, '3', 0);
    });

    // Add more parameters to tweak here...

    v.screen->performLayout();
    return false;
};
```



- (C++ lambda expressions)

Provided Example: Implicit Sphere

- Step 1: Compute an axis-aligned bounding box

```
        /******** createGrid() ********/
// Grid bounds: axis-aligned bounding box
Eigen::RowVector3d bb_min, bb_max;
bb_min = P.colwise().minCoeff();
bb_max = P.colwise().maxCoeff();

// Bounding box dimensions
Eigen::RowVector3d dim = bb_max - bb_min;
```

Provided Example: Implicit Sphere

- Step 2: construct a grid over the bounding box

```
                                /******** createGrid() ********/
// Grid spacing
const double dx = dim[0] / (double)(resolution - 1);
const double dy = dim[1] / (double)(resolution - 1);
const double dz = dim[2] / (double)(resolution - 1);
// 3D positions of the grid points -- see slides or marching_cubes.h for ordering
grid_points.resize(resolution * resolution * resolution, 3);
// Create each gridpoint
for (unsigned int x = 0; x < resolution; ++x) {
    for (unsigned int y = 0; y < resolution; ++y) {
        for (unsigned int z = 0; z < resolution; ++z) {
            // Linear index of the point at (x,y,z)
            int index = x + resolution * (y + resolution * z);
            // 3D point at (x,y,z)
            grid_points.row(index) = bb_min + Eigen::RowVector3d(x * dx, y * dy, z * dz);
        }
    }
}
```

Provided Example: Implicit Sphere

- Step 3: Fill grid with the values of the implicit function

```
                    /******** evaluateImplicitFunc() ********/
// Scalar values of the grid points (the implicit function values)
grid_values.resize(resolution * resolution * resolution);

// Evaluate sphere's signed distance function at each gridpoint.
for (unsigned int x = 0; x < resolution; ++x) {
    for (unsigned int y = 0; y < resolution; ++y) {
        for (unsigned int z = 0; z < resolution; ++z) {
            // Linear index of the point at (x,y,z)
            int index = x + resolution * (y + resolution * z);
            // Value at (x,y,z) = implicit function for the sphere
            grid_values[index] = (grid_points.row(index) - center).norm() - radius;
        }
    }
}
```

$$f(\mathbf{x}) = \|\mathbf{x} - \mathbf{c}\| - r$$

Provided Example: Implicit Sphere

- Step 4: run marching cubes

```
igl::copyleft::marching_cubes(grid_values, grid_points, resolution, resolution, resolution, V, F);
```

input: implicit function values at grid points

Provided Example: Implicit Sphere

- Step 4: run marching cubes

```
igl::copyleft::marching_cubes(grid_values, grid_points, resolution, resolution, resolution, V, F);
```

input: grid point positions

Provided Example: Implicit Sphere

- Step 4: run marching cubes

```
igl::copyleft::marching_cubes(grid_values, grid_points, resolution, resolution, resolution, V, F);
```

input: grid size (x, y, z)

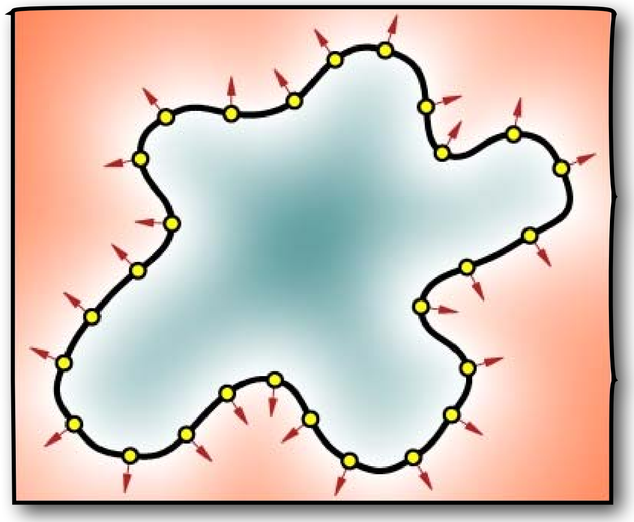
Provided Example: Implicit Sphere

- Step 4: run marching cubes

```
igl::copyleft::marching_cubes(grid_values, grid_points, resolution, resolution, resolution, V, F);
```

output: vertices and faces

Bonus: Better Normal Constraints



- Our method implemented only point constraints
- Normals “constrained” using inward- and outward-offset value constraints
    - Leads to undesirable surface oscillation
- Solution: use the normal to define a linear function at each sample point; interpolate these **functions** with MLS.

- Chen Shen, James F. O'Brien, and Jonathan R. Shewchuk. "**Interpolating and Approximating Implicit Surfaces from Polygon Soup**". In *Proceedings of ACM SIGGRAPH 2004*, pages 896–904. ACM Press, August 2004. **(Section 3.3)**

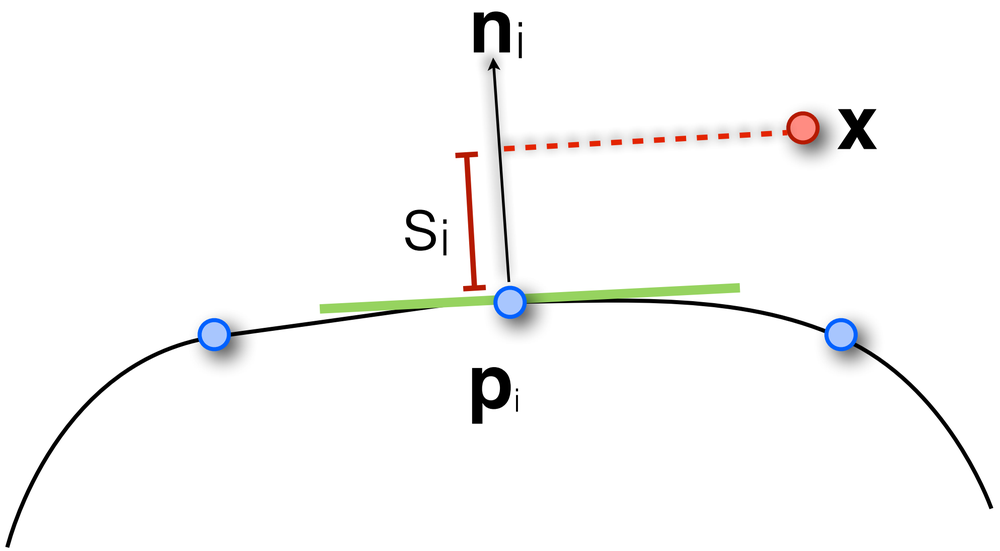
Bonus: Better Normal Constraints

- Recall, we computed our interpolant by solving:

$$\min_a \|B\mathbf{a} - \mathbf{d}\|^2_{W(\mathbf{x})}$$

  with constraint value d_i for the 3N constraint locations.

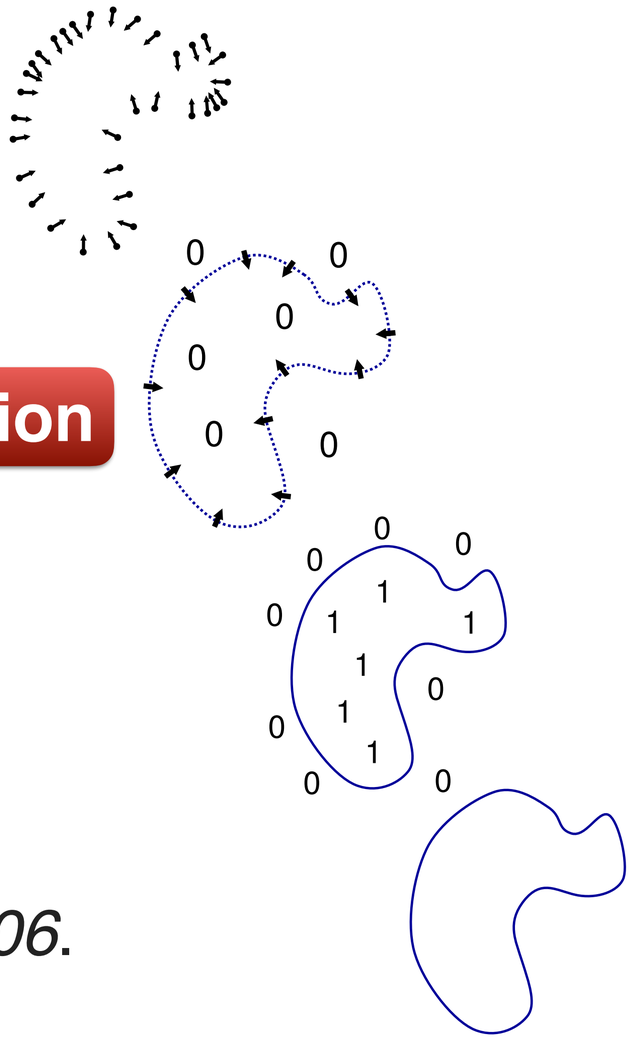
- **New scheme:** use just one constraint per sample pt
- Replace d_i with: $s_i(\mathbf{x}) = (\mathbf{x} - \mathbf{p}_i) \cdot \mathbf{n}_i$
- s_i is the linear function computing signed distance to $\mathbf{p}_i$'s tangent plane
- Note: $\nabla_{\mathbf{x}} s_i = \mathbf{n}_i$



Bonus: Poisson Reconstruction

- Explicitly a fit scalar function's gradient to the normals.
  - Smooth out sampled normals to create a global vector field $\vec{V}$
  - Find scalar function χ whose gradient best approximates this vector field: $min_\chi \|\nabla\chi - \vec{V}\|$
- Advantages:
  - No spurious sheets far from the surface!
  - Robust to noise
- Michael Kazhdan,Matthew Bolitho, Hugues Hoppe. "**Poisson Surface Reconstruction.**" In *Eurographics Symposium on Geometry Processing, 2006*.

Approximate indicator function



Questions?