Physics of Nuclear Reactors

Numerical Exercises

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Contents

Chapter 1

TP N° 1: Attenuation of neutrons from a 1D planar source

1.1 Goal 1 - Analytical solution

The analytical solution is:

$$\phi(x) = \frac{SL}{2D} \cdot \frac{sinh(\frac{x_0+d-x}{L})}{cosh(\frac{x_0+d}{L})} \tag{1.1}$$

The plot of the analytical solution:

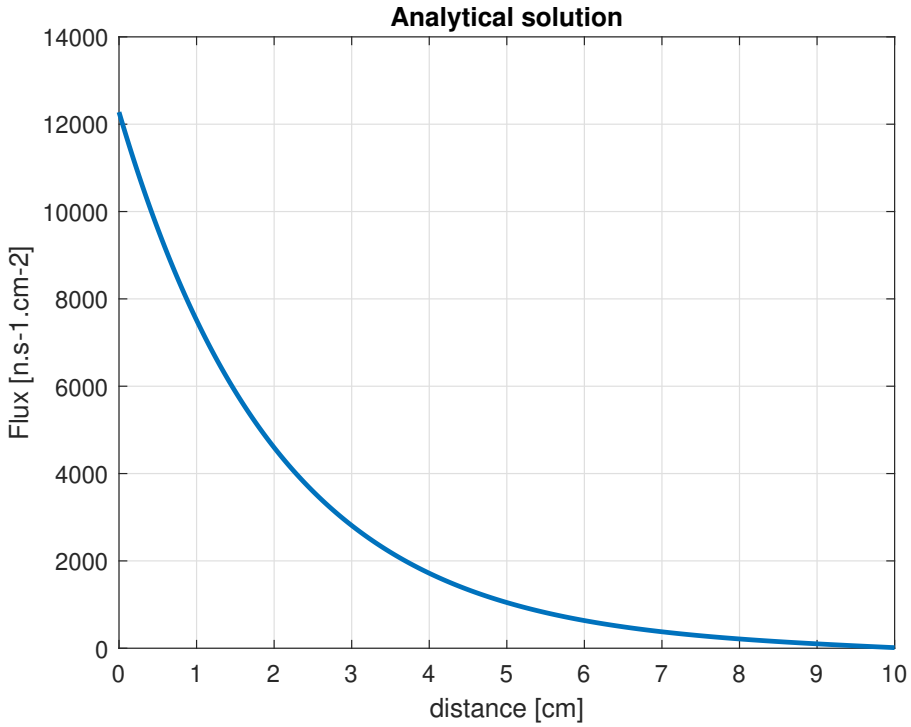


Figure 1.1: Analytical solution

Report **the values of the flux for the analytical solution**, in the following scientific format with 4 significant digits was adopted as indicated in the first printout:

$$\phi(0) = 1.2277e + 04 \tag{1.2}$$

$$\phi(0.05) = 1.1979e + 04 \tag{1.3}$$

$$\phi(1.05) = 7.3299e + 03 \tag{1.4}$$

$$\phi(x_0) = 1.3589e + 01 \tag{1.5}$$

1.2 Goal 2 and 3 - Discretization , boundary conditions and coefficients

The spatial discretization of the diffusion equation is described in detail here and is not repeated in the following reports, it yields:

$$\frac{D}{\Delta x^2} \cdot (\phi_{i+1} - 2\phi_i + \phi_{i-1}) - \Sigma_a \phi_i = 0 \tag{1.6}$$

Boundary conditions should be employed for the last and first node where the conditions on the **fixed net current near the source** and **net incoming current** at x=x0. The problematic solution is connected to the knowledge of ϕ_{i-1} on the first node and to the knowledge of ϕ_{i+1} in the last node. Hence we will use the most general form of the streaming term which yields:

$$\frac{2D}{\Delta x^2} \cdot (\phi_{i+1/2} - 2\phi_i + \phi_{i-1/2}) \tag{1.7}$$

Eventually on the first node we would be able to express $\phi_{i+1/2}$ thanks to current continuity in the inner material, while we can use the boundary condition to find a relation between ϕ_i and $\phi_{i-1/2}$. We will express the latter as a function of the former and find an expression depending only on ϕ_i and ϕ_{i+1}.

$$J_{i-1/2} = C = \frac{-2D}{\Delta x}(\phi_i - \phi_{i-1/2}) \tag{1.8}$$

$$\phi_{i-1/2} = \frac{+\Delta x C}{2D} + \phi_i \tag{1.9}$$

$$\phi_{i+1/2} = \frac{\phi_{i+1} + \phi_i}{2} \tag{1.10}$$

Using previous equations and diffusion equation the relation for **first node** of the mesh yields:

$$(-\frac{D}{\Delta x^2} - \Sigma_a)\phi_i + \frac{D}{\Delta x^2}\phi_{i+1} = \frac{-C}{\Delta x} \tag{1.11}$$

Hence a known term will appear as first element of the vector b of our system.

Similiarly on the last node we would be able to express $\phi_{i-1/2}$ thanks to current continuity in the inner material, while we can use the boundary condition to find a relation between ϕ_i and $\phi_{i+1/2}$. We will express the latter as a function of the former and find an expression depending only on ϕ_i and ϕ_{i-1}.

$$\phi_{i+1/2} = \frac{1}{\frac{1}{4} + \frac{D}{\Delta x}} \cdot \frac{D}{\Delta x}\phi_i \tag{1.12}$$

$$\phi_{i-1/2} = \frac{\phi_{i-1} + \phi_i}{2} \tag{1.13}$$

Using previous equations and diffusion equation the relation for **last node** of the mesh yields:

$$(\frac{2D}{\Delta x^2}(\frac{1}{\frac{\Delta x}{4D} + 1}) - \frac{3D}{\Delta x^2} - \Sigma_a)\phi_i + \frac{D}{\Delta x^2}\phi_{i-1} = 0 \tag{1.14}$$

The computed matrix will be a **tridiagonal matrix**. The values for **coefficients** in the upper and lower diagonal are verified to remain unchanged even when boundary conditions are set. The values for the main diagonal are instead everywhere equal apart from the first and the last term (i.e. A(1,1) and A(n,n)) due to the boundary conditions.

$$\text{main diagonal coefficients} = -\frac{2D}{\Delta x^2} - \Sigma_a \tag{1.15}$$

$$\text{A(1,1)} = -\frac{D}{\Delta x^2} - \Sigma_a \tag{1.16}$$

$$\text{A(n,n)} = \frac{2D}{\Delta x^2}(\frac{1}{\frac{\Delta x}{4D} + 1}) - \frac{3D}{\Delta x^2} - \Sigma_a \tag{1.17}$$

$$\text{upper and lower diagonal coefficients} = \frac{D}{\Delta x^2} \tag{1.18}$$

The values of the coefficients are reported hereafter:

$$\text{main diagonal coefficients} = -16.60 \tag{1.19}$$

$$\text{A(1,1)} = -8.312 \tag{1.20}$$

$$\text{A(n,n)} = -12.15 \tag{1.21}$$

$$\text{upper and lower diagonal coefficients} = 8.292 \tag{1.22}$$

In light of the future employment of the finite difference approximation for the following exercises, considering the indications in the TP intro slides and the request to **explicit the beta coefficients**, **the most general form of the 1-D discretised diffusion equation** is reported hereafter, together with the definition of the matrix coefficients according to the beta introduction. Beta coefficients for the given problem are simply equal to D, hence in the code they were defined in this way for the sake of simplicity, the complete implementation of the beta coefficients was carried out in TP3, using the following format:

$$-\Phi_i \left(\frac{\beta_{i+1} + \beta_{i-1}}{\Delta x^2} + \Sigma_{a,i,j} \right) + \frac{\beta_{i+1}}{\Delta x^2} \Phi_{i+1} + \frac{\beta_{i-1}}{\Delta x^2} \Phi_{i-1} = -Q_i \tag{1.23}$$

$$\beta_{i+1} = \frac{2D_i D_{i+1}}{D_{i+1} + D_i}; \quad \beta_{i-1} = \frac{2D_i D_{i-1}}{D_{i-1} + D_i}; \quad BC_i = \frac{1}{2\left(\frac{\Delta x}{4D_i} + 1\right)} \tag{1.24}$$

$$A(1,1) = -\frac{\beta_{i+1}}{\Delta x^2} - \Sigma_{a,i}; \quad A(n,n) = -\frac{BC_i \Delta x + \beta_{i-1}}{\Delta x^2} - \Sigma_{a,i} \tag{1.25}$$

1.3 Goal 4 - Error

Flux values from **numerical solution** as requested:

$$\phi(0) = 1.2281e + 04 \tag{1.26}$$

$$\phi(0.05) = 1.1979e + 04 \tag{1.27}$$

$$\phi(1.05) = 7.3303e + 03 \tag{1.28}$$

$$\phi(x0) = 1.3624e + 01 \tag{1.29}$$

Error in sample point as requested:

$$\text{err}(1.05) = 3.7901e - 01 \tag{1.30}$$

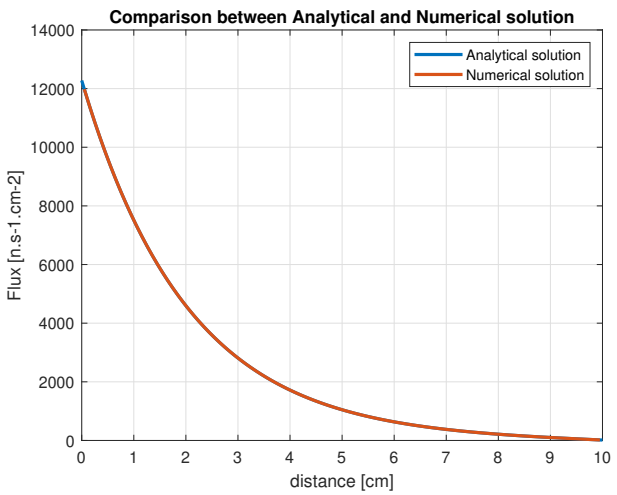


Figure 1.2: Numerical vs Analytical solution

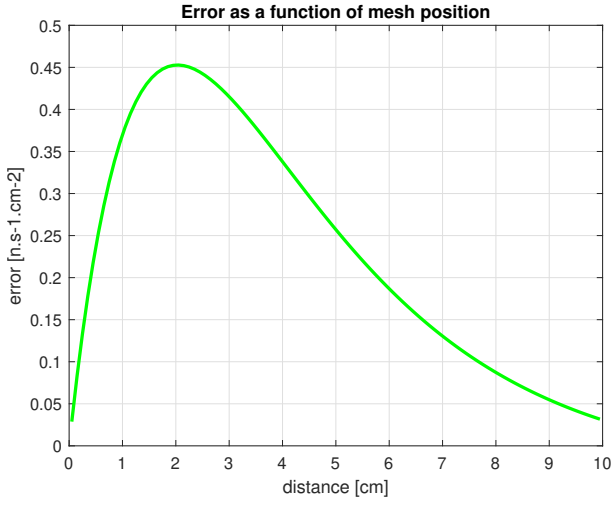


Figure 1.3: Error as a function of mesh position for mesh size h=0.1

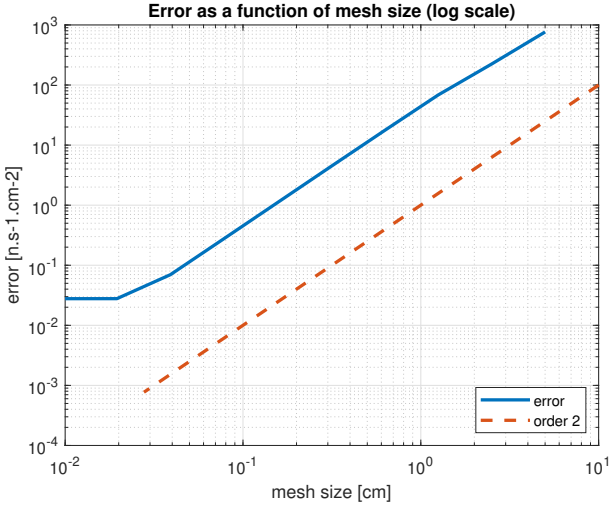


Figure 1.4: Infinite Norm Error as a function of mesh size, values at x0 and 0 were neglected

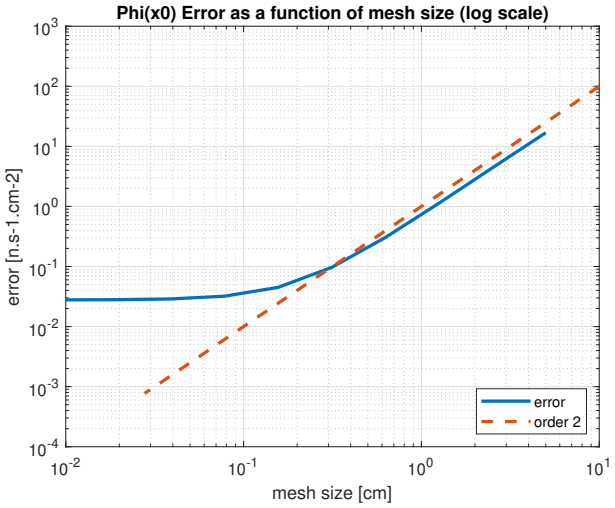


Figure 1.5: Error for flux at x0 as a function of mesh size

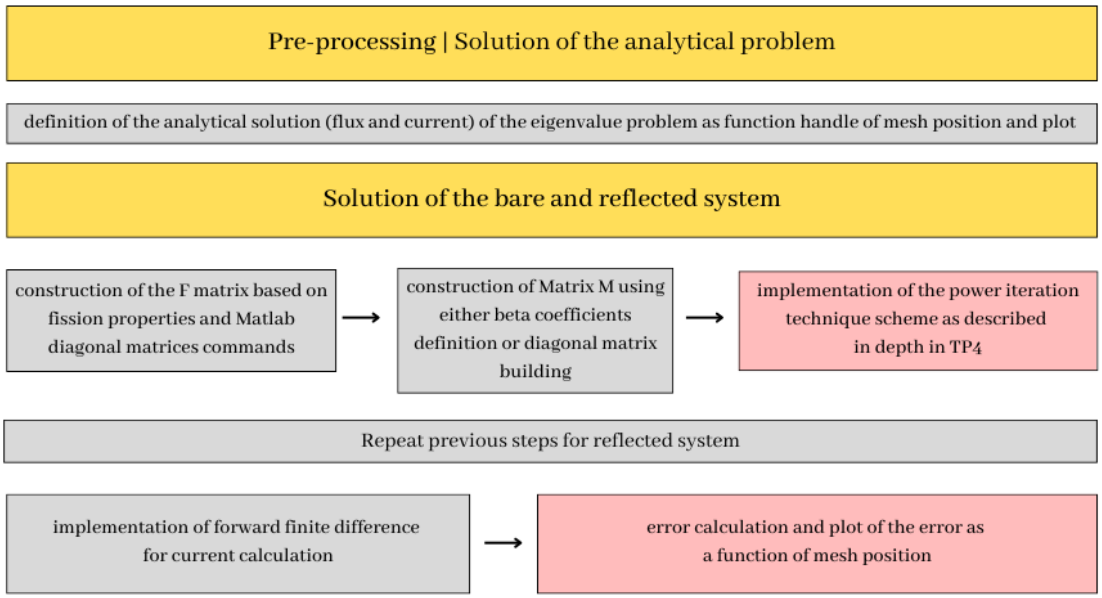
Comments The error computed as infinite norm of the numerical solution reduces as the mesh size diminishes. As expected, the numerical solution given with centered finite difference approximation gives a **convergence order equal to 2 asymptotically.** For very refined meshes the error reaches a "saturation" value. The same type of argument, description and comment can be made with regards to Figure 1.5, where the error on $\phi(x0)$ was plotted against the size of the mesh. Additionally, one could argue that this "saturation value" is due to a **systematic error** of our approximation procedure that cannot be eliminated: specifically, boundary conditions differ from the analytical solution ones because we never imposed the flux to be zero at the extrapolated boundary, we just asked to the incoming net current of neutrons at the boundary to be equal to zero. This important difference brings about an **inevitable saturation** of the error in $x0$.

Chapter 2

TP N° 2: Slab Reactor 1D-1G

2.1 Algorithm

Numerical problem from TP number 2 include a short visual scheme describing the main content of the script and the actions performed by the student. This style is adopted for all the following algorithm descriptions.



2.2 Pre processing - Analytical solution

The analytical solution for the flux is:

$$\phi(x) = A \cdot cos(Bx) \tag{2.1}$$

where $B = \pi/(2a + 2dc)$, with dc the extrapolation length. Hence the current can be found using Fick's law:

$$J(x) = D \cdot A \cdot B \cdot sin(Bx) \tag{2.2}$$

We set A=1 in order to normalise the flux (i.e. dividing all the value for the maximum one, which is at the center of the core). However, in order to be able to compare the analytical solution with the numerical one we actually normalise with the value of the analytical solution at x=0.05, hence $A = \frac{1}{cos(B \cdot 0.05)}$

2.3 Goal 2 - Numerical solver and iterative scheme

For the solution of the bare reactor, the matrix M was built analogously to the solution of the diffusion equation employed in TP1 (Matrix A). The problem was solved for half of the core, assuming **symmetry** (meaning current = 0 in x=0) and **net incoming current** at the boundary (x=a=20cm) equal to **zero** too.

Being in one group approximation, the scattering term only plays a role in the definition of the diffusion coefficient, hence matrix F for the solution of the eigenvalue problem is just a diagonal matrix yielding $\nu\Sigma_f$ on the diagonal.

Analogous reasoning can be done between reflector and core, the matrix will now be bigger (provided that we keep mesh size = 0.1), and from node (201,201) (for the main diagonal) it will yield the same kind of terms, bearing however the properties of the reflector. Similiarly, matrix F will still be a diagonal matrix, whose terms from (201,201) on will just be zero (no fission in the reflector).

Power iteration scheme was set and converged with **it=13** for the bare reactor, and **it2=14** for the reflected system.

2.4 Goal 3 - Flux, Current, Keff and error

Hereafter, the values requested as output from the Matlab code:

BARE REACTOR:
Keff for bare reactor: 9.73052e-01
Keff bare analytical: 9.73052e-01
error on keff: 2.50185e-08
Current at the core boundary for bare core NUMERICAL: 7.54358e-02
Current at the core boundary for bare core ANALYTICAL: 7.53677e-02
Error on boundary current = 6.80769e-05

REFLECTED SYSTEM:
Keff for reflected system: 1.12038e+0
Number of iterations for the reflected system: 14
Current at the core boundary for reflected core NUMERICAL: 5.44015e-02

2.5 Goal 4 - Plots and comments

Requested plots for **BARE REACTOR** and **REFLECTED SYSTEM** are included in the following page (Figure 2.1 to Figure 2.6).

Comments Error behaviour for flux along the mesh is reported in Figure 5 and requires some additional comments. Since the numerical boundary condition requires the incoming current to be equal to zero at the geometry boundary, while the analytical solution requires the flux itself to go to zero at the the extrapolated boundary, **the error** will peak closer to the latter.

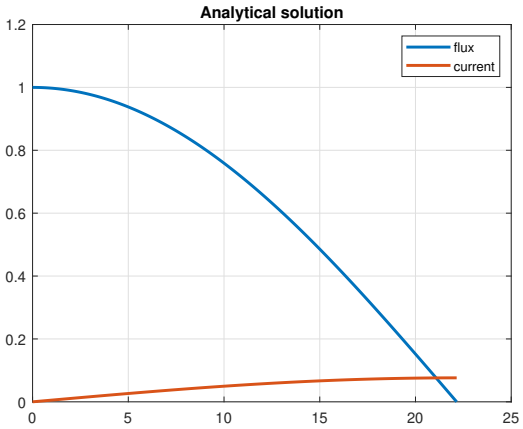


Figure 2.1: Analytical solution for FLUX and CURRENT - BARE REACTOR

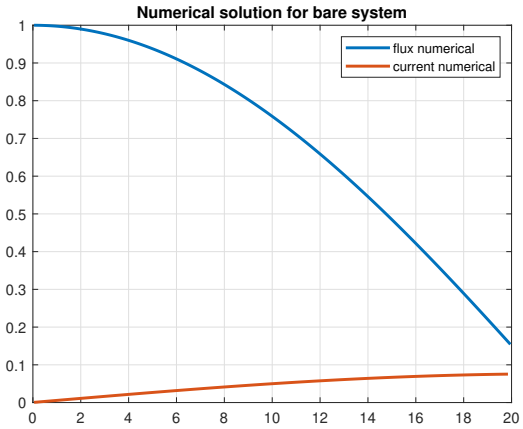


Figure 2.2: Numerical and analytical solution for FLUX - BARE REACTOR

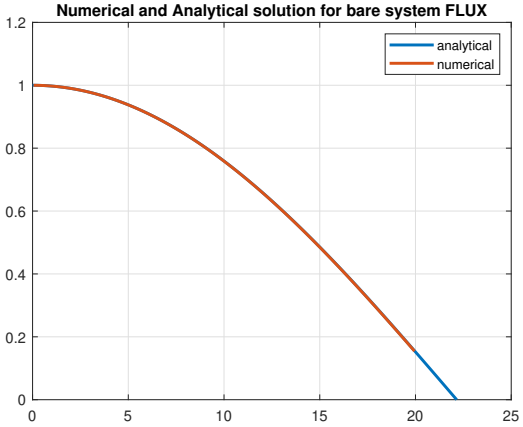


Figure 2.3: Numerical and Analytical solution for FLUX - BARE REACTOR

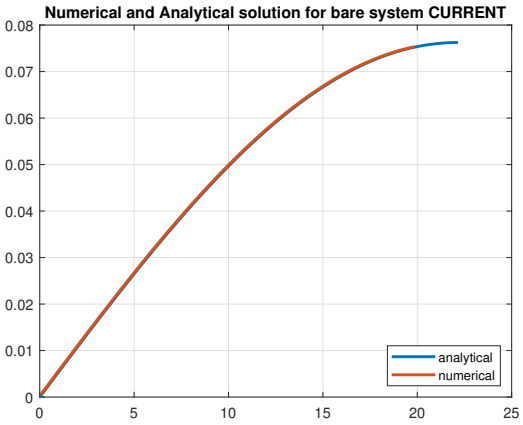


Figure 2.4: Numerical and analytical solution for CURRENT - BARE REACTOR

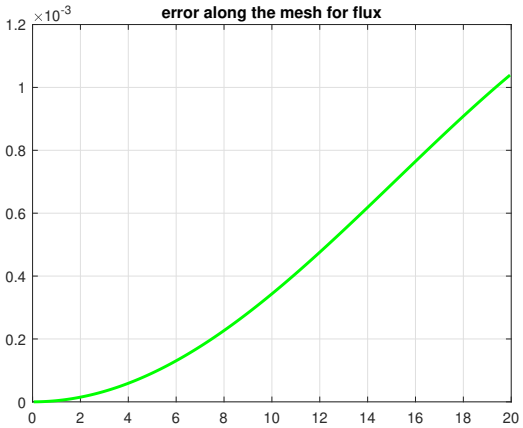


Figure 2.5: Error for flux along the mesh - BARE REACTOR

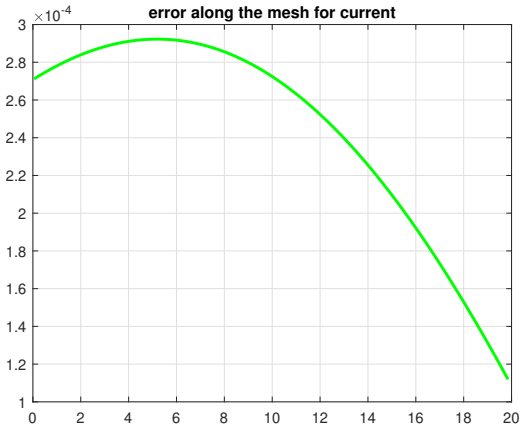


Figure 2.6: Error for current along the mesh - BARE REACTOR

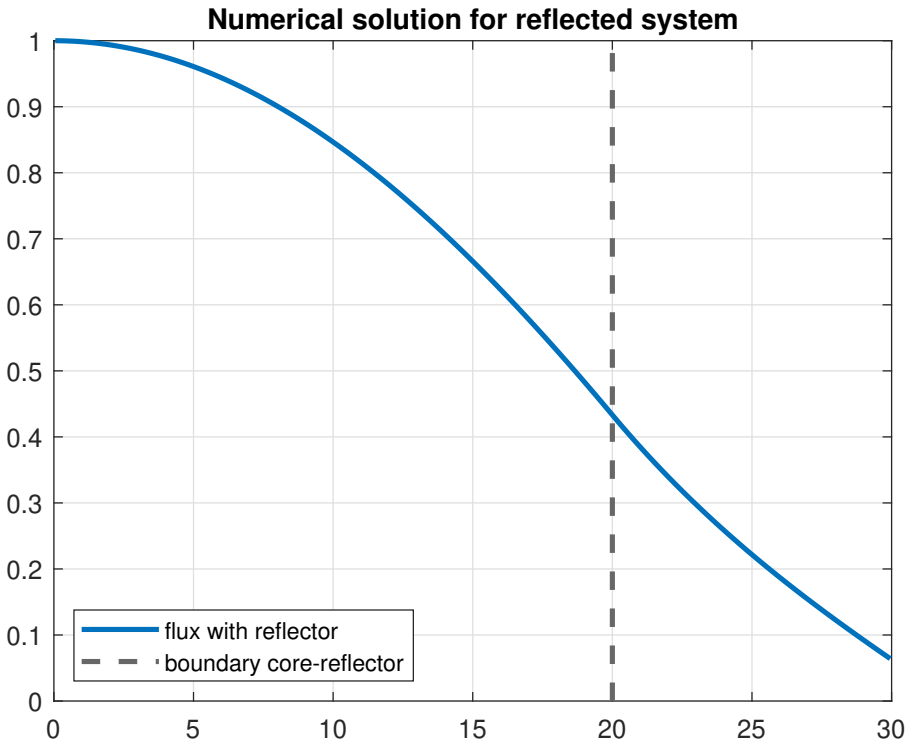
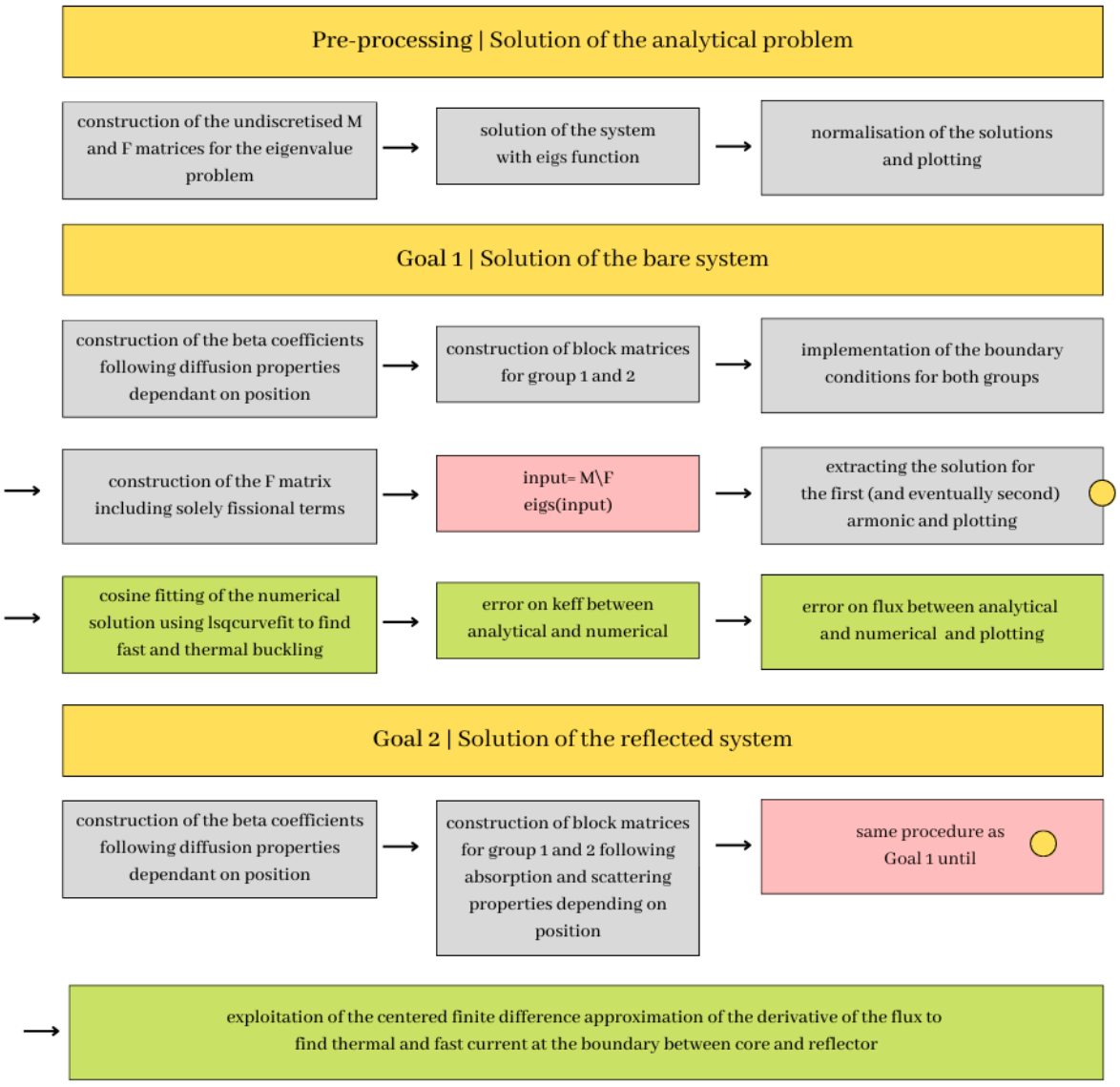


Figure 2.7: Flux NUMERICAL - REFLECTED SYSTEM

Chapter 3

TP N° 3: Slab Reactor 1D-2G

3.1 Summary of the algorithm used



3.2 Goal 1 - Analytical solution

The analytical solution for the flux is given by the solution of the eigenvalue problem from reactor equation:

$$\nabla^2\phi(x) + B^2 \cdot \phi(x) = 0 \tag{3.1}$$

where $B = pi/2a$, following indications from the professor, the extrapolated distance, which would have been different between the two groups, was neglected. The diffusion equation for the two groups yields:

$$\begin{cases} D_1 \cdot B^2 \cdot \phi_1 + (\Sigma_{a1} + \Sigma_{s12}) \cdot \phi_1 = \frac{1}{k} \cdot (\nu\Sigma_{f1}\phi_1 + \nu\Sigma_{f2}\phi_2) \\ D_2 \cdot B^2 \cdot \phi_2 + \Sigma_{a2} \cdot \phi_2 = \Sigma_{s12} \cdot \phi_1 \end{cases}$$

After bringing the scattering term to the LHS of the second equation, the above mentioned set of equations can be written in **matrix form** and solved thanks to Matlab `eigs` function. The eigenvector will yield two values ϕ_1 and ϕ_2, which were normalised accordingly to the bigger one (fast flux ϕ_1). From this two values, we can now express the **analytical solution as function handle**. Code implementation of the described procedure is included in SECTION 2 of the handed script.

3.3 Goal 2 - Numerical solver and iterative scheme

Algorithm for the solution of the problem is discussed in this section. Following from Exercise 1, **discretization** of the diffusion equation yields for N spatial mesh points and G energy groups, there will be a set of $N \times G$ simultaneous algebraic equations.

$$a_i^g\Phi_{i,g} + a_{i+1}^g\Phi_{i+1,g} + a_{i-1}^g\Phi_{i-1,g} - \sum_{g' \neq g}^{G} \Sigma_{s_i}^{g' \to g}\Phi_{i,g'} = \frac{\chi^g}{k}\sum_{g'=1}^{G} v_{g'}\Sigma_{f_i}^{g'}\Phi_{i,g'} \tag{3.2}$$

For the sake of matrix building when material properties change within the mesh, **we must define the β coefficients**, which are dependant on the **material composition** and hence on the position inside the mesh when we are provided with different values for the **diffusion coefficient**. The beta coefficients are computed accordingly to the position along the mesh using the provided data for bare and reflected core. Their construction is implemented in SECTION 3 of the script. As requested in the printouts, the relationship between ϕ_i, ϕ_{i-1} and ϕ_{i+1} is explicited hereafter, discussing the terms of the built matrices.

The building of the block matrices for group 1 and group 2 subsequently yields the following terms:

- construction of the **block matrix M1** by means of diagonal matrices construction. Scattering outside group 1 shall be included in the matrix main diagonal coefficients, making a total cross-section Σ_t that accounts for both phenomena.

$$\text{diagonal terms} = \frac{(\beta_+ + \beta_-)}{h^2} + \Sigma_{t,core} \tag{3.3}$$

$$\text{upper diagonal terms} = \frac{-\beta_+}{h^2} \tag{3.4}$$

$$\text{lower diagonal terms} = \frac{-\beta_-}{h^2} \tag{3.5}$$

- implementation of boundary conditions for half the core requires changing **the first and last value of the main diagonal of the tridiagonal Matrix M1** considering **current simmetry in 0** and **zero incoming current at the boundary**.
- **the same procedure, including BCs, is repeated for M2**. Please mind that the diagonal terms in block matrix M2 won't be featuring the upscattering term in the overall "removal" term, but solely the absorption term.

$$\text{diagonal terms} = \frac{(\beta_+ + \beta_-)}{h^2} + \Sigma_{a,core} \tag{3.6}$$

$$\text{upper diagonal terms} = \frac{-\beta_+}{h^2} \tag{3.7}$$

$$\text{lower diagonal terms} = \frac{-\beta_-}{h^2} \tag{3.8}$$

- the construction of matrix M is carried out by concatenation of block matrix M1, two `zeros` matrices and again block matrix M2.
- scattering term is added by imposing diagonal of order $-nc$, where nc are the mesh number points for the core (i.e. a/h), to be equal to negative down-scattering cross section.
- Matrix M is now twice the size of the mesh (2nc, 2nc).

Construction of **matrix F** required the definition of two diagonal terms: **main diagonal** is a vector of **length** `2nc`, with fast fission cross section (and ν factor) on the first `nc` terms and zero afterwards; **upper diagonal** is a vector of length `nc` with the thermal fission cross section (and ν factor). The latter will take the place of the diagonal of order `nc` of the final F matrix `2nc,2nc`.
Expliciting the overall discretisation for the two groups:

$$\text{GROUP 1: } (\frac{(\beta_+ + \beta_-)}{h^2} + \Sigma_{t,core}) \cdot \phi_{1,i} + \frac{-\beta_+}{h^2} \cdot \phi_{1,i+1} + \frac{-\beta_-}{h^2} \cdot \phi_{1,i-1} = \frac{1}{k} \cdot (\nu\Sigma_{f1} \cdot \phi_{1,i} + \nu\Sigma_{f2} \cdot \phi_{2,i}) \tag{3.9}$$

$$\text{GROUP 2: } (\frac{(\beta_+ + \beta_-)}{h^2} + \Sigma_{a,core}) \cdot \phi_{2,i} + \frac{-\beta_+}{h^2} \cdot \phi_{2,i+1} + \frac{-\beta_-}{h^2} \cdot \phi_{2,i-1} = \Sigma_{s12} \cdot \phi_{2,i} \tag{3.10}$$

The same procedure is **repeated for the reflective system**, where the β coefficients are redefined taking into account the spatial change in the diffusion coefficient between core and reflector. The same block matrices construction, differing only for the dimension of the matrix and for the **consistent use of the reactor region properties when building the matrix**, is carried out.

3.4 Goal 3 - Flux, Current, Keff and error

Hereafter, the values requested as output from the Matlab code:

BARE REACTOR:
Keff analytical of the bare system: 1.08440e+0
Keff numerical of the bare system: 1.08677e+0
Buckling of the fast neutron flux: 3.02284e-02
Buckling of the thermal neutron flux: 3.03546e-02
Error on keff in pcm: 240 pcm
REFLECTED SYSTEM:
Keff numerical of the reflected system: 1.09199e+0
Keff associated to second harmonic: 9.19226e-01
Current at the boundary of reflected system FAST: 3.95116e-02
Current at the boundary of reflected system THERMAL: -5.87471e-03

Comments The requested plots and numerical results have been included in the previous pages, the most significant comments are made hereafter:

- the choice not to include the extrapolation distance in the definition of the analytical solution for the bare reactor leads to **enourmous percentage error** nearby the boundary. This is due to the analytical assumption of the flux going to zero at the reactor boundary not matching the numerical boundary condition imposed, which only accounts for the incoming current being zero. In Figure 4, the error is plotted for mesh positions further away from the boundary, before ramping up due to the above mentioned reasons.
- **reflected system** shows the wanted properties (i.e. reduction of the thermal current at the boundary, hence reduction of thermal leakage). This is shown by thermal current being negative at the boundary and represents the advantage of the reflected configuration with respect to the bare one: reflected configuration allows for smaller critical size geometries thanks to reduced leakage.

3.5 Goal 4 - Plots

Requested plots for **BARE REACTOR** and **REFLECTED SYSTEM** are included hereafter:

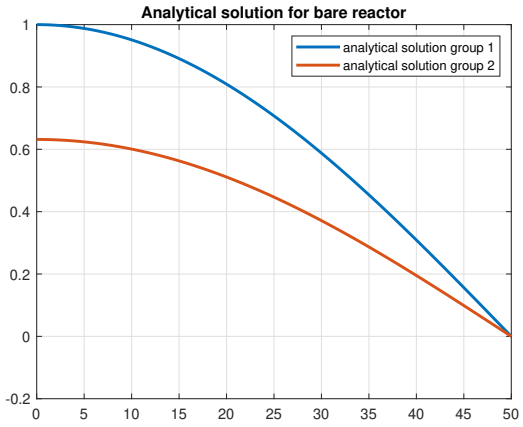


Figure 3.1: Analytical solution for FLUX thermal and fast - BARE REACTOR

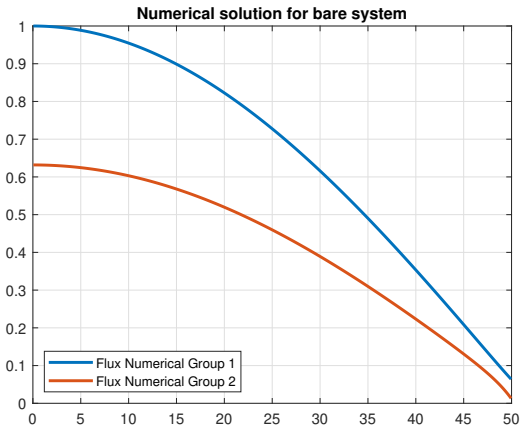


Figure 3.2: Numerical solution for FLUX thermal and fast - BARE REACTOR

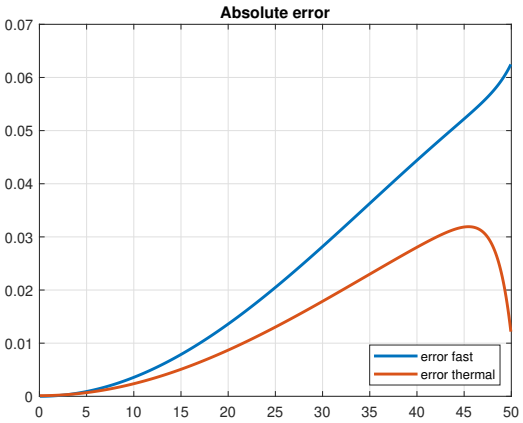


Figure 3.3: Absolute Error for flux along the mesh - BARE REACTOR

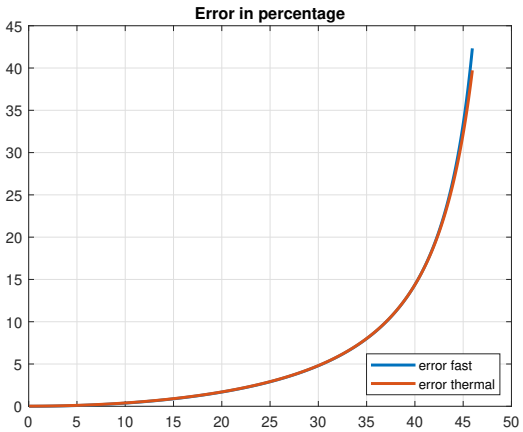


Figure 3.4: Relative error for flux along the mesh - further away from the boundary - BARE REACTOR

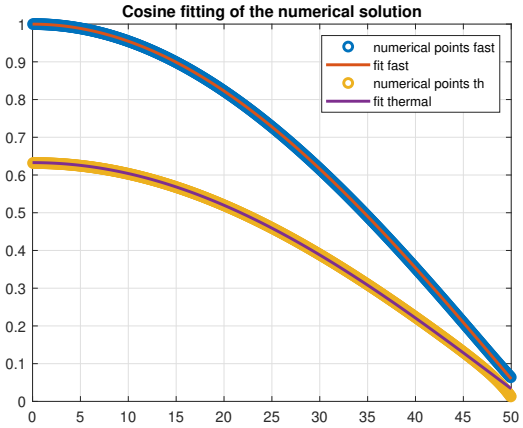


Figure 3.5: Cosine fitting of the bare reactor in order to obtain the buckling

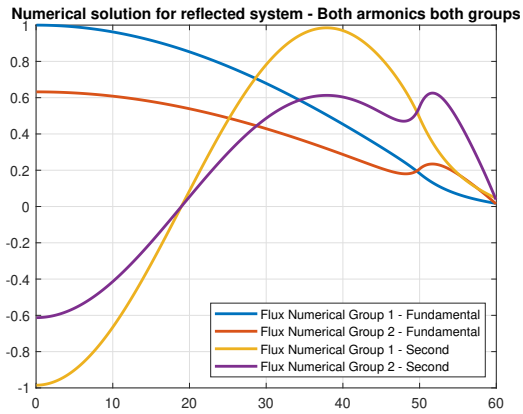
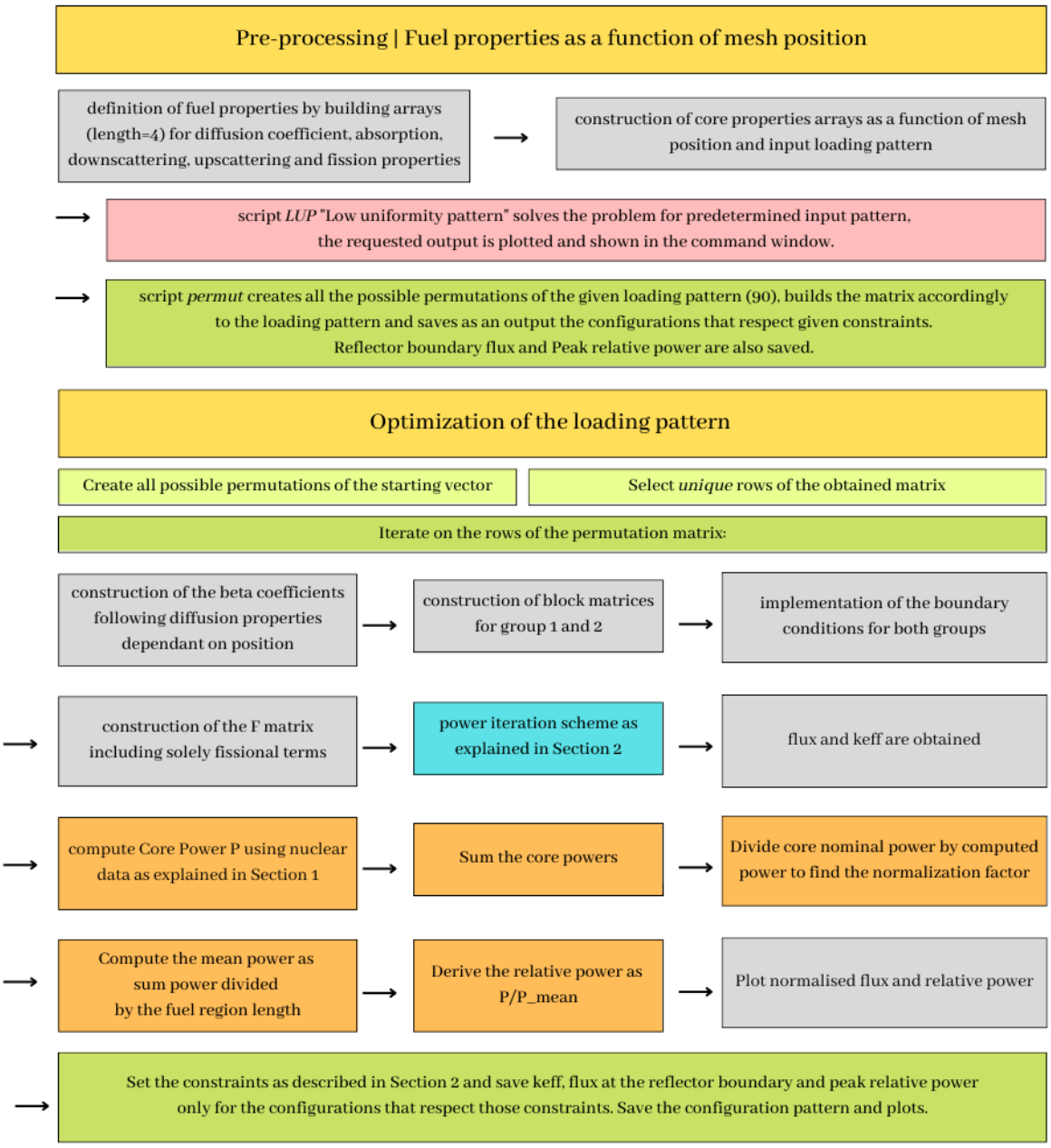


Figure 3.6: Numerical solution for FLUX - REFLECTED SYSTEM (armonics)

Chapter 4

TP N° 4: Optimization of loading pattern

4.1 Summary of the algorithm



4.2 Goal 1 - Solution of the basis configuration

Following from Exercise 1, **discretization** of the diffusion equation yields for N spatial mesh points and G energy groups, there will be a set of $N \times G$ simultaneous algebraic equations.

$$a_i^g \Phi_{i,g} + a_{i+1}^g \Phi_{i+1,g} + a_{i-1}^g \Phi_{i-1,g} - \sum_{g' \neq g}^{G} \Sigma_{s_i}^{g' \to g} \Phi_{i,g'} = \frac{\chi^g}{k} \sum_{g'=1}^{G} v_{g'} \Sigma_{f_i}^{g'} \Phi_{i,g'} \tag{4.1}$$

Matrices were built in accordance to what explained in the algorithm section. The solution of the eigenvalue problem is carried out by means of a **power iteration scheme** with convergence criteria set only for the k_{eff} (`epsilon` $= 10^{-7}$). Details of the power iteration scheme are included in the algorithm description requested at the beginning of the handed scripts.

For the first time during the numerical exercises, the students were given information on the **power of the reactor core** (i.e. 90MW full core), together with nuclear data referring to the **energy produced by fission events** ($\kappa\Sigma_{f,i}^{g'}$) in the different assemblies and groups. Such information was used in order to derive the normalisation factor F_{norm}, allowing for the plot of the normalised flux, and the *Relative Power*.

$$\boldsymbol{P}_{fiss} = \sum_i \boldsymbol{h} \Delta \boldsymbol{x}_i \sum_{g'}^{G} \kappa \Sigma_{f,i}^{g'} \Phi_i^{g'} \tag{4.2}$$

$$F_{\text{norm}} = \frac{P_{\text{core}}^{\text{nom}}}{P_{\text{fiss}}} \tag{4.3}$$

$$\Phi_{i,\,\text{norm}}^{g} = F_{\text{norm}}\, \Phi_i^g \tag{4.4}$$

$$\overline{\boldsymbol{P}_{fiss}} = \frac{\boldsymbol{P}_{fiss}}{\boldsymbol{N}} \tag{4.5}$$

$$\boldsymbol{P}_{i,fiss}^{\text{rel}} = \frac{\boldsymbol{P}_{i,fiss}}{\overline{\boldsymbol{P}_{\text{fiss}}}} \tag{4.6}$$

where h is the height of the fuel assemblies, Δx_i is the mesh size (i.e. 1cm for the attached code), N is the number of mesh points in the fuel region and $\overline{\boldsymbol{P}_{fiss}}$ is the average fission power.

For the sake of clarity, the handed scripts include: **a general script** solving any input pattern, **a *permut* script** computing the solutions that respect the specified parameters, **a Low Non-Uniformity pattern and a Low-Leakage pattern script** solving the initial configuration and the optimised one, printing the requested outputs.

RESULTS FOR THE LOW-NON UNIFORMITY PATTERN:
Number of iterations: 72
The flux at the reflector boundary: 7.22601e+13
Keff: 1.21882e+0
Peak power relative to the average: 2.44e+0

Comments on relative power profile: the relative power profile as expected peaks in the fresh fuel assemblies (type 1), as the greatest amount of fissions take place here, due to the relatively higher macroscopic cross-section of fission.

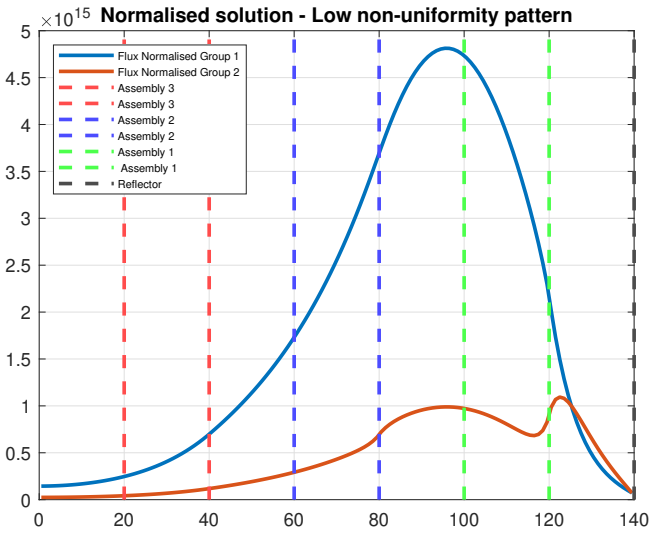


Figure 4.1: Low non-uniformity pattern - Low flux

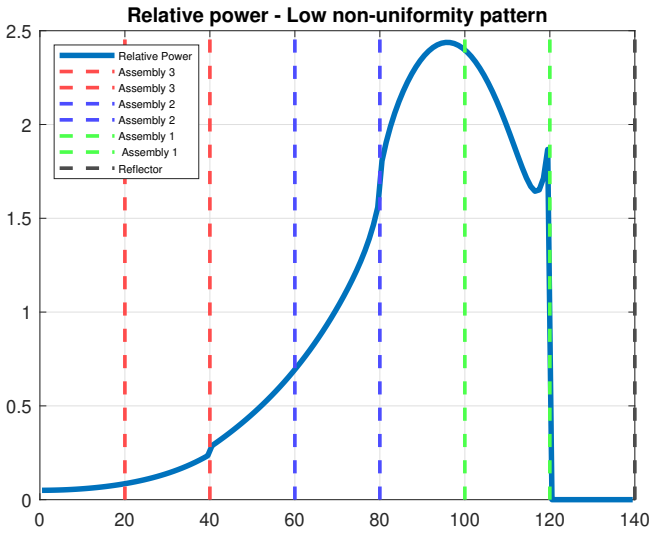


Figure 4.2: Low non-uniformity pattern - Relative Power

4.3 Goal 2 - Optimization of the loading pattern

The only difference with the previous point is the necessity to solve the numerical problem for different assembly loading patterns. As explained in the introductory algorithm description, this is done considering all the possible permutations without repetitions of the starting array **[3 3 2 2 1 1 4]**. The results are stored if two operational constraints are verified:

- flux at the last mesh point (i.e. flux at the reactor pressure vessel boundary) is $< 10^{13}$ so to limit RPV embrittlment;
- relative power peak is < 3 so to limit radial disuniformities in power distribution, hence differential thermal stresses for the fuel;

19 configurations among the 90 possible permutations without repetitions were found to be consistent with the previous constraints. All the results are included hereafter.

Pattern	Flux RB	Peak RP	Pattern	Flux RB	Peak RP
[1 2 1 2 3 3 4]	1.54E+12	2.99	[2 2 1 1 3 3 4]	6.24E+12	2.15
[2 1 1 2 3 3 4]	2.42E+12	2.16	[2 3 1 1 2 3 4]	9.75E+12	2.22
[2 1 1 3 2 3 4]	2.59E+12	2.24	[3 1 1 2 2 3 4]	4.24E+12	2.22
[2 1 1 3 3 2 4]	2.61E+12	2.28	[3 1 1 2 3 2 4]	4.28E+12	2.28
[2 1 2 1 3 3 4]	4.63E+12	1.95	[3 1 1 3 2 2 4]	4.87E+12	2.37
[2 1 2 3 1 3 4]	6.61E+12	2.31	[3 1 2 1 2 3 4]	9.28E+12	1.73
[2 1 2 3 3 1 4]	6.34E+12	2.55	[3 1 2 1 3 2 4]	9.36E+12	1.81
[2 1 3 1 2 3 4]	7.43E+12	1.92	[3 2 1 1 2 3 4]	8.90E+12	2.17
[2 1 3 1 3 2 4]	6.90E+12	2.13	[3 2 1 1 3 2 4]	9.43E+12	2.25
[2 1 3 2 3 1 4]	8.74E+12	2.58			

Table 4.1: Flux RB is the fast flux at the reflector boundary, Peak RP is the rel. power peak.

Pattern	Keff
[1 2 1 2 3 3 4]	1.26019
[2 1 1 2 3 3 4]	1.25325
[2 1 1 3 2 3 4]	1.24519
[2 1 1 3 3 2 4]	1.24443
[2 1 2 1 3 3 4]	1.21362
[2 1 2 3 1 3 4]	1.19399
[2 1 2 3 3 1 4]	1.19127
[2 1 3 1 2 3 4]	1.19253
[2 1 3 1 3 2 4]	1.18802
[2 1 3 2 3 1 4]	1.17597
[2 2 1 1 3 3 4]	1.23376
[2 3 1 1 2 3 4]	1.23012
[3 1 1 2 2 3 4]	1.23671
[3 1 1 2 3 2 4]	1.23471
[3 1 1 3 2 2 4]	1.22423
[3 1 2 1 2 3 4]	1.20157
[3 1 2 1 3 2 4]	1.19308
[3 2 1 1 2 3 4]	1.23970
[3 2 1 1 3 2 4]	1.23002

Table 4.2: Keff for the selected configurations

Comments The two-pronged optimization problem requires a compromise between the objectives set in the beginning. The **final optimised pattern** can be chosen accordingly to a safety oriented reasoning, opting for the so called **low-leakage pattern**:

- the objective of a lower vessel flux can be considered priority as it implies successful continuous operation of the reactor, it can be full-filled by choosing the configuration with **the lowest vessel flux**;
- configuration with the lowest vessel flux corresponds to a peak relative power below the requested threshold, specifically **the highest peak relative power** among the ones permitted.

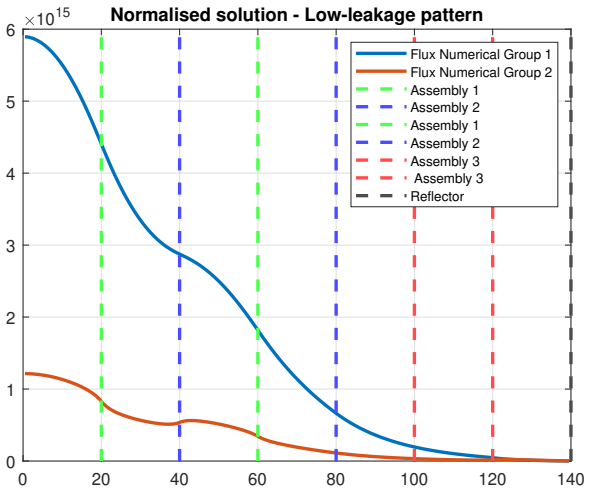


Figure 4.3: Chosen pattern: Low-leakage pattern - Flux

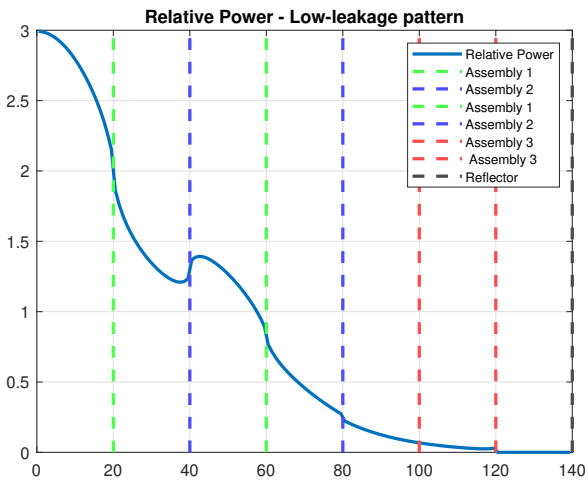
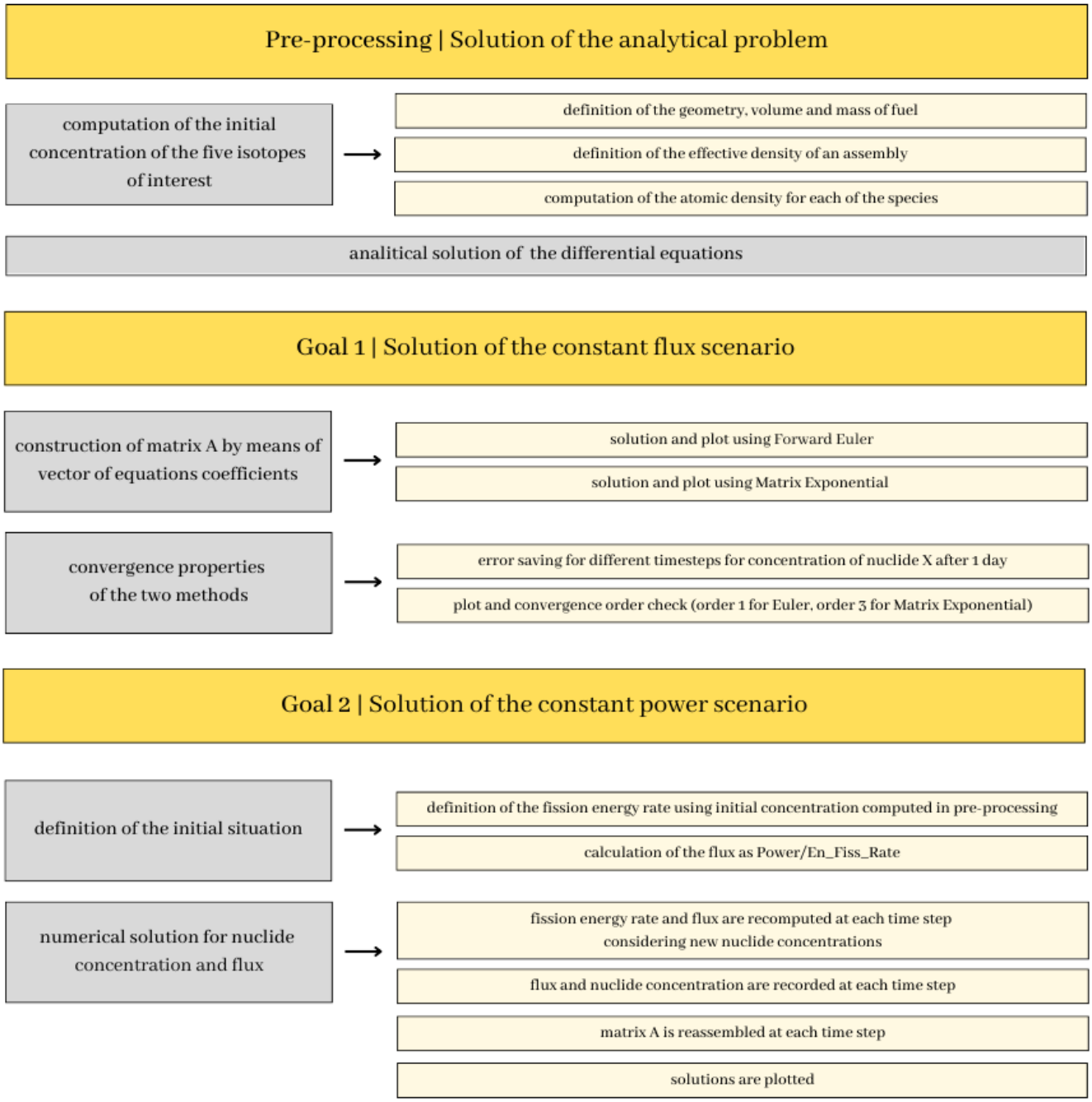


Figure 4.4: Chosen pattern: Low-leakage pattern - Relative Power

Chapter 5

TP N° 5: Fuel evolution with exposure

5.1 Summary of the algorithm



5.2 Pre-processing - Analytical solution of the differential equations

The following set of differential equations describes the evolution of the five nuclides of interest.

$$\begin{aligned}
\frac{dN_{U-235}}{dt} &= -\Phi\sigma_{c,U-235}N_{v-235} - \Phi\sigma_{f,U-235}N_{U-235} \\
\frac{dN_{U-238}}{dt} &= -\Phi\sigma_{c,U-238}N_{U-238} - \Phi\sigma_{f,U-238}N_{U-238} \\
\frac{dN_{Pu-239}}{dt} &= \Phi\sigma_{c,U-238}N_{U-238} - \Phi\sigma_{c_{Pu-239}}N_{Pu-239} - \Phi\sigma_{f,Pu-239}N_{Pu-239} \\
\frac{dN_x}{dt} &= \Phi\gamma_{i\to j}N_{U-235}\sigma_{f,U-235} - \Phi\sigma_{c,x}N_x - \lambda_x N_x \\
\frac{dN_y}{dt} &= \Phi\gamma_{i\to j}N_{U-235}\sigma_{f,U-235} - \Phi\sigma_{c,y}N_y
\end{aligned} \tag{5.1}$$

In the previous set of equations it was assumed that the fission products only derive from the fission of U-235. With this assumption each differential equation can be solved accordingly to the general formula.

$$\begin{aligned}
&\frac{dy}{dx} + p(x)y = q(x) \\
&y(x) = \frac{1}{\alpha(x)}\left(\int \alpha(x)q(x)dx + c\right) \\
&\alpha(x) = e^{\int p(x)dx}
\end{aligned} \tag{5.2}$$

The following solutions describe the evolution of the five nuclides:

$$\boxed{N_8(t) = N_8(0)\cdot\exp\left(-\sigma_a^8\Phi t\right)} \tag{5.3}$$

$$\boxed{N_5(t) = N_5(0)\cdot\exp\left(-\sigma_a^5\Phi t\right)} \tag{5.4}$$

$$\boxed{N_{Pu-239}(t) = K\left(e^{-\Phi\sigma_{a,238}\cdot t} - e^{-\Phi\sigma_{c,239}\cdot t}\right) \text{ with } K = \frac{N_{238}\sigma_{c,8}}{\sigma_{a,9} - \sigma_{a,8}}} \tag{5.5}$$

$$\boxed{N_x(t) = K\left(e^{-\sigma_{a,5}t} - e^{-\left(\bar{\Phi}\sigma_c + \lambda x\right)t}\right) \text{ with } K = \frac{\Phi\cdot FYX\cdot\sigma_{f,5}\cdot N_5(0)}{\Phi\sigma_{c,x} + \lambda_x - \sigma_{a,5}\Phi}} \tag{5.6}$$

$$\boxed{N_y(t) = K\left(e^{-\Phi\sigma_{a,5}t} - e^{-\Phi\sigma_{c,y}t}\right) \text{ with } K = \frac{N_5(0)\cdot\sigma_{f,5}\cdot FYY}{\sigma_{c,y} - \sigma_{a,5}}} \tag{5.7}$$

In the following, only the most significant graphs have been included. For the sake of conciseness **solution for 1 year** was plotted for Forward Euler Method, while a **1 day snippet** was plotted for the analytical solution. All the plots for both Euler Method, Matrix exponential and Analytical solution are included in the script. Figure 5.3 and 5.4 display the order of convergence of the two methods, confirming the theoretical expected behaviour. Finally, Figure 5.5 shows the relative difference between numerical and analytical solution for the given nuclides.

Initial atomic concentrations of the isotopes of interest:
U-235: 2.31e+20
U-238: 7.47e+21
Pu-239, X, Y: 0

5.3 Goal 1 - Solution of the constant flux scenario

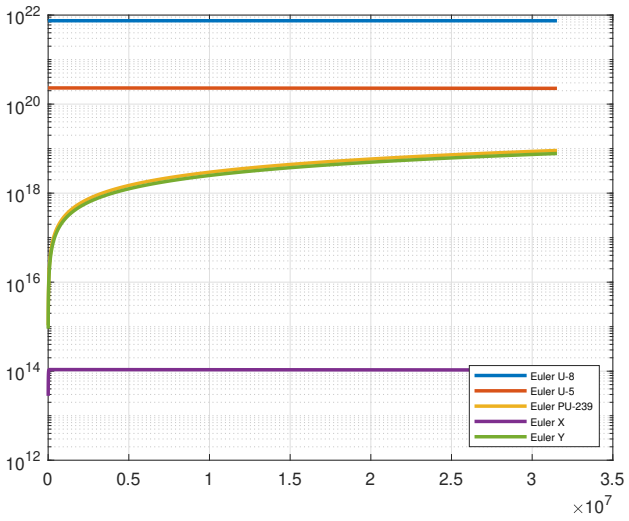


Figure 5.1: Example of numerical solution (Euler) for 1 year depletion problem

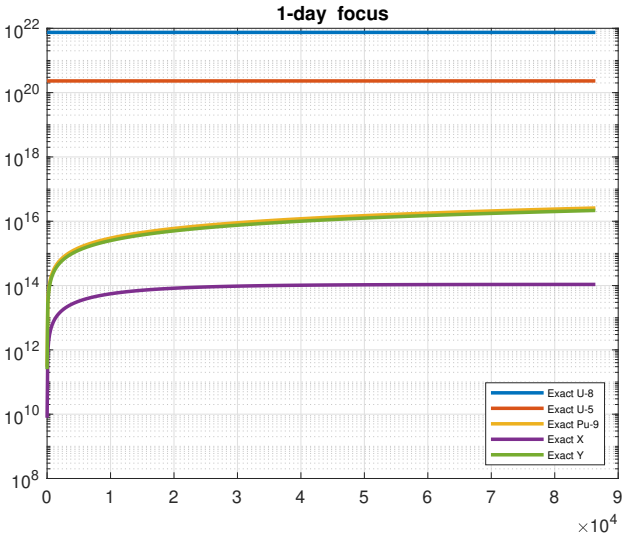


Figure 5.2: 1 day focus for depletion problem - analytical solution

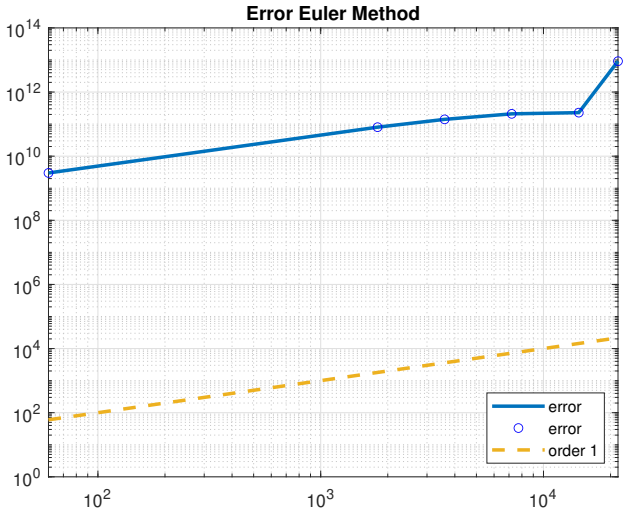


Figure 5.3: Convergence of Forward Euler Method

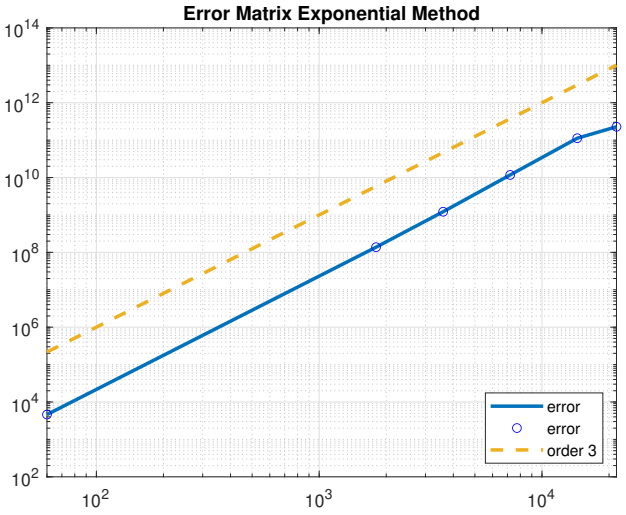


Figure 5.4: Convergence of Matrix Exponential (order 3)

The imposed flux is **not sufficient to effectively burn U-235**, meaning that its concentration, together with that of U-238 remains constant. In the 1 day snippet we can notice how the fission product X has already almost reached its **equilibrium concentration**, while Plutonium and fission product Y keep slowly building up until the end of the year. With respect to **convergence order**, it was evaluated considering nuclide X concentration after 1 day: as expected Forward Euler shows an asymptotic error evolution of order 1, while Matrix Exponential is much faster, proving order 3 in accordance to the Taylor expansion order adopted.

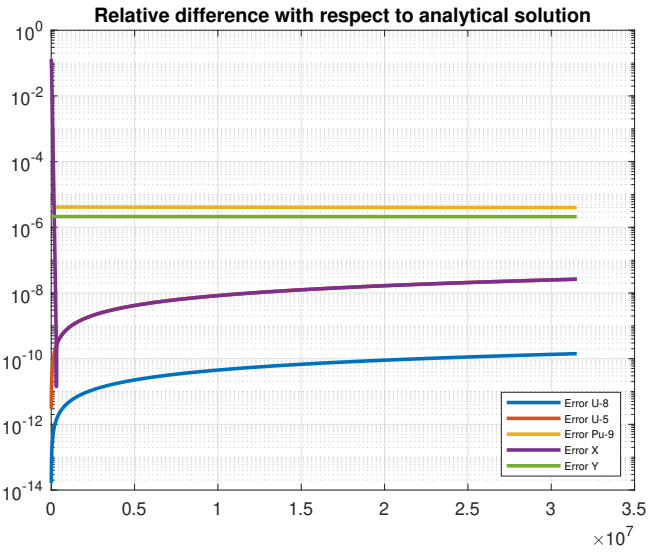


Figure 5.5: Relative difference between analytical and numerical solution

5.4 Goal 2 - Solution of the constant power scenario

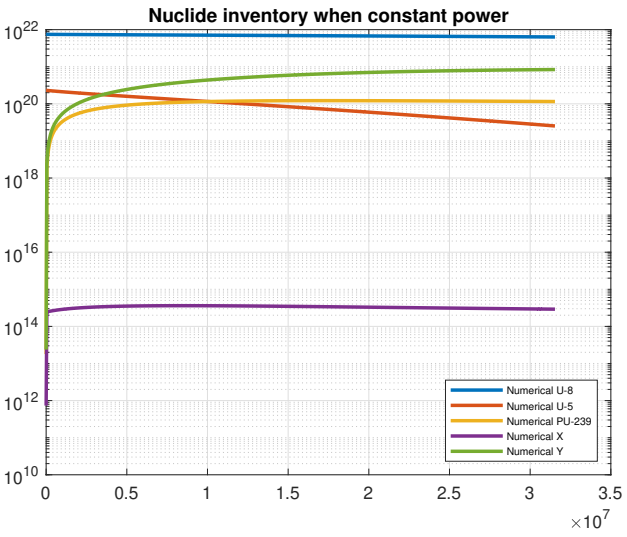


Figure 5.6: Nuclides' concentrations for constant power scenario

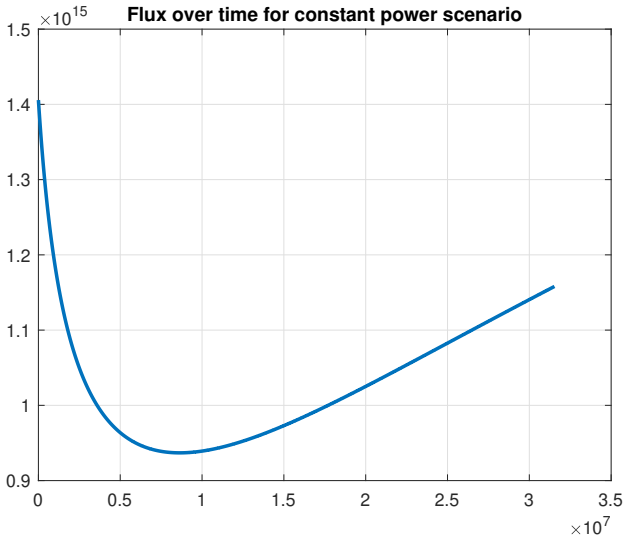


Figure 5.7: Flux evolution for constant power scenario over 1 year

Comments In the constant power scenario, the flux level is such that we **burn U-235**, in log-scale its concentration decreases linearly. Subsequently, the **flux** will tend to **decrease as Pu-239 is built-up** and contributes to fission inside the assembly, up to 116 days. After that, **U-235 concentration falls below that of Pu-239** and flux is bound to **increase in order to keep constant power** (i.e. in order to cause the same amounts of fission reactions despite low U-235 concentration).