Financial Forecasting 2019/20 – Empirical Project

- Prediction of the monthly miles traveled by vehicle in the United States of America -

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1. Introduction

The aim of our empirical project is to forecast the overall monthly miles traveled by vehicle in the United States of America. A prediction and a sound understanding of this specific time series is very important. For the state governments for example it makes sense to commission infrastructure - related construction work in months with low traffic volume in order to avoid big traffic jams and chaos. Furthermore, for regulating and fighting pollution it is quite important to have a sound future forecast in order to take actions in time. Our paper is structured in the following way. In Chapter 2, we describe the structure and content of our time series and test it for stationarity. In a next step we transform the data to make the underlying process stationary. Chapter 3 provides an overview of the according autocorrelation and partial autocorrelation functions to the reader. In Chapter 4 we present two possible ARMA models to forecast our out-of-sample data. After that, the forecast is presented in Chapter 5.

2. Times Series Description

The source of our time series is the Federal Reserve Bank of Saint Louis which provides a large range of macroeconomic and financial data.[1] The time series displays the number of million miles travelled by vehicle per month in the US from January 1970 until September 2019. Figure 1 plots the raw data. We use observations up to September 2017 to estimate our in-sample model. The last 24 months are then used to perform an out-of-sample forecast.

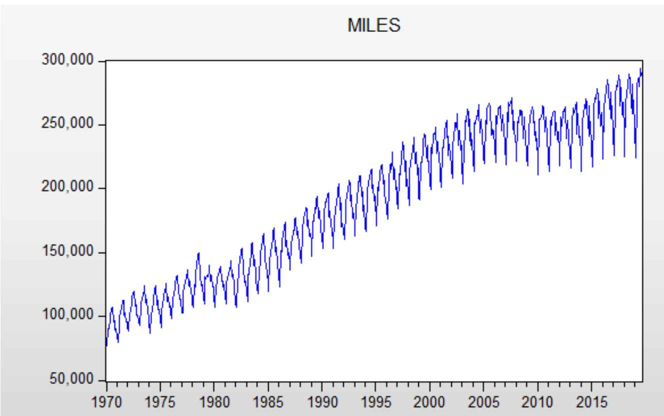


Figure 1: Million Miles travelled by vehicle per month in the US

[1] The timeseries can be found under the following link: https://fred.stlouisfed.org/series/TRFVOLUSM227NFWA

By looking at the data, the underlying process does not seem to be stationary. We observe a clear upwards sloping trend over time. This is in line with our economic expectations as traveling has become cheaper during the past years while at the same time the average wealth of the American citizens has increased as well as the population. The only clear extinction of the longtime upwards trend can be observed between the years 2008 and 2012. A possible explanation for this observation may be the world economic crisis in 2008 and its repercussions. In addition, we also observe an inconstant variance of the process. It seems to increase over time. These two observations (trend and time-varying unconditional variance) imply that the underlying series is not stationary. Applying the Augmented Dicky-Fuller test (ADF test) confirms our expectations.

Null Hypothesis: MILES has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 14 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.768344	0.9666
Test critical values:	1% level	-3.973901	
	5% level	-3.417559	
	10% level	-3.131200	

*MacKinnon (1996) one-sided p-values.

Figure 2: Augmented Dickey-Fuller test

Figure 2 displays the outcome of the test. Since we observe a very high p-value we cannot reject the Null Hypothesis which states that the underlying process is non-stationary.

Also, we discover a seasonality of 12 months with which we will deal with later in this chapter. This is also in line with our expectations since the overall traveled miles depends on factors like weather and holidays which again depends on the observed month. The highest peak is reached during the summer months.

2.1. Log Transformation

To stabilize the variance of the process throughout time we transform the underlying data using the natural log values of the monthly travelled miles. This leads to the graph presented in Figure 3.

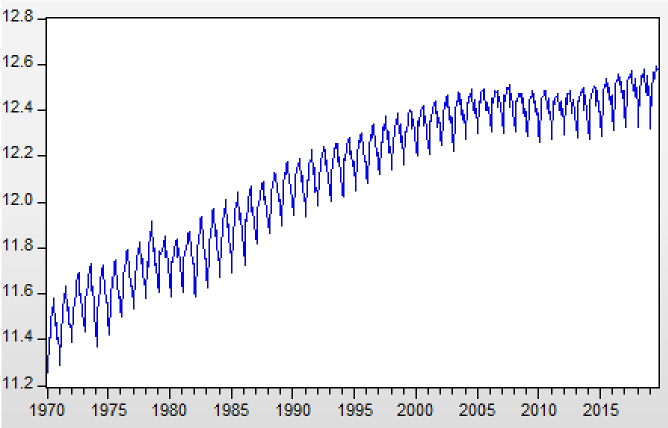


Figure 3: Log Transformation of the raw data

Accordingly, the variance seems to be more stable over time. However, we can still observe seasonality and an upwards sloping trend. In addition to that the ADF test in Figure 4 implies that we still have a non-stationary process:

Null Hypothesis: LOGMILES has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 13 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.826065	0.9615
Test critical values:	1% level	-3.973874	
	5% level	-3.417546	
	10% level	-3.131192	

*MacKinnon (1996) one-sided p-values.

Figure 4: Augmented Dickey-Fuller test oft he log-transformed data

2.2. Seasonal Differencing

In order to account for seasonality, we take in a second step the seasonal difference of the log-values with a seasonal period of 12. This leads to the output in Figure 5.

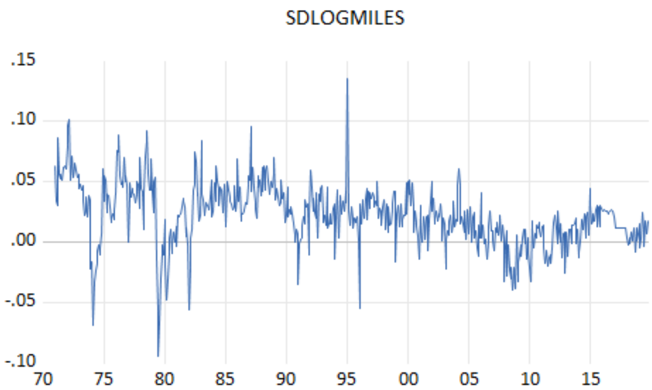


Figure 5: Seasonally differenced log-transformed data

At first sight there seems to be still a trend in the transformed time series, however this time it is a downwards sloping trend. The ADF test in Figure 6 though leads to the conclusion that we have a stationary process.

Null Hypothesis: SDLOGMILES has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 12 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.735544	0.0006
Test critical values:	1% level	-3.974180	
	5% level	-3.417695	
	10% level	-3.131280	

*MacKinnon (1996) one-sided p-values.

Figure 6: Augmented Dickey-Fuller test oft he seasonally differenced log-transformed data

Nevertheless, in this case we want to make sure that we are dealing with a stationary process. For this reason, we perform one last transformation.

2.3. Lag One Differencing

To get rid of the downwards sloping trend we have seen above we take the first difference of our log-seasonal transformed data. This leads to the output shown in Figure 7.

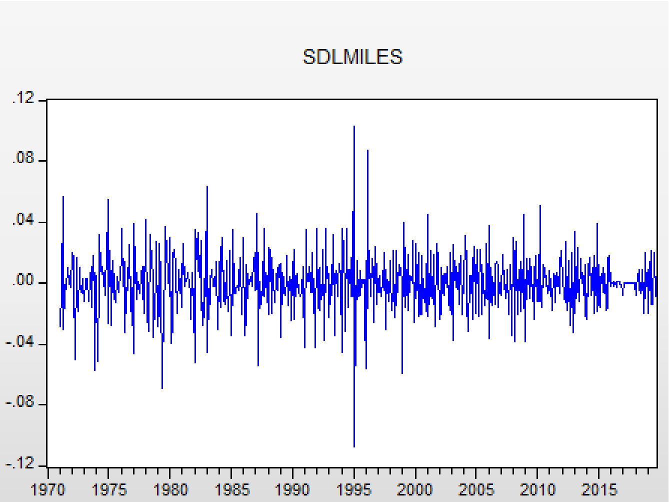


Figure 7: First Difference of the seasonally differenced, log-transformed data

By looking at the graph the transformed variable seems to be stationary now. The ADF test confirms our expectations (Figure 8). The p-value of zero allows us to reject the null hypothesis and therefore assume the time series to be stationary.

Null Hypothesis: SDLN_MILES has a unit root
Exogenous: Constant
Lag Length: 12 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-10.66723	0.0000
Test critical values:	1% level	-3.441573	
	5% level	-2.866383	
	10% level	-2.569409	

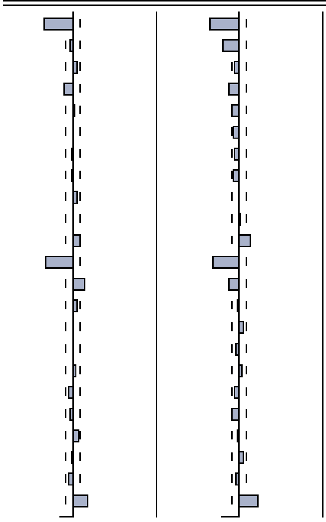
*MacKinnon (1996) one-sided p-values.

Figure 8: Augmented Dickey-Fuller test of the first-differenced seasonally differenced, log-transformed data

In the following chapters, if not stated otherwise, this transformed variable is used for estimating and evaluating the model.

3. Autocorrelation and Partial Autocorrelation Function

Autocorrelation Partial Correlation



	AC	PAC	Q-Stat	Prob
1	-0.356	-0.356	74.448	0.000
2	-0.044	-0.195	75.564	0.000
3	0.047	-0.049	76.849	0.000
4	-0.107	-0.136	83.568	0.000
5	0.010	-0.096	83.622	0.000
6	-0.001	-0.074	83.623	0.000
7	-0.012	-0.059	83.705	0.000
8	-0.009	-0.067	83.758	0.000
9	0.048	0.004	85.125	0.000
10	0.007	0.020	85.156	0.000
11	0.095	0.137	90.496	0.000
12	-0.350	-0.321	163.74	0.000
13	0.139	-0.131	175.27	0.000
14	0.056	-0.018	177.18	0.000
15	0.004	0.065	177.19	0.000
16	0.005	-0.038	177.21	0.000
17	0.037	0.025	178.03	0.000
18	-0.059	-0.051	180.15	0.000
19	-0.030	-0.084	180.68	0.000
20	0.069	-0.011	183.59	0.000
21	-0.017	0.065	183.76	0.000
22	-0.055	-0.034	185.62	0.000
23	0.179	0.227	205.06	0.000

Figure 9: Correlogram of the transformed time series

In order to find our forecast model, we look first at the corresponding autocorrelation and partial autocorrelation functions of the underlying process. EViews provides the graph depicted in Figure 9. It clearly shows that there is some structure in the data which we will try to capture best in the next chapter.

4. Model Selection

By considering the shape of the autocorrelation and partial autocorrelation functions we develop in a first step different appropriate MA, AR and ARMA Models. In a second step, we concentrate on the parameter´s p-values and the underlying unit roots and change our model-selection accordingly. In a third step we look at the Durbin-Watson statistic and plot the residual correlogram of each model to see whether the specific residuals are white noise or not. In the end we are left with the two models represented in Figure 10. In the following subchapters we will present and further investigate these models.

Model	Durbin - Watson	Adjusted R^2	Akaike	Schwarz
AR(1) AR(2) AR(4) AR(5) MA(6) MA(12)	2.02	0.42	-5.49	-5.43
AR(1) AR(2) AR(4) AR(5) AR(6) AR(7) MA(12)	2.05	0.42	-5.49	-5.43

Figure 10: Selected models

4.1. AR(1) AR(2) AR(4) AR(5) MA(6) MA(12)

Figure 11 shows the estimation output for this specific model. As we already mentioned in the previous paragraph all included variables (except of the intercept) are significant at a level of 5%. Also all unit roots are outside the unit circle. A Durbin-Watson statistic close to 2 is a first indicator that the residuals of the model are white noise. Since it only concideres the residuals of lag one, we also plot the correlogram of the residuals in Figure 12. We observe significant spikes at lag 6, 14 and 23. However these are only barely significant and are therefore neglected. These results amplifies our assumption that the residuals are white noise.

Dependent Variable: DDS_LN_MILES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/26/19 Time: 15:43
Sample: 1971M02 2017M09
Included observations: 560
Convergence achieved after 32 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-5.83E-05	6.80E-05	-0.857591	0.3915
AR(1)	-0.409037	0.031648	-12.92457	0.0000
AR(2)	-0.173585	0.033479	-5.184832	0.0000
AR(4)	-0.151044	0.043148	-3.500604	0.0005
AR(5)	-0.115023	0.049075	-2.343800	0.0194
MA(6)	-0.121783	0.036235	-3.360915	0.0008
MA(12)	-0.714251	0.033173	-21.53104	0.0000
SIGMASQ	0.000231	1.02E-05	22.69726	0.0000

R-squared	0.423123	Mean dependent var	-9.00E-05
Adjusted R-squared	0.415808	S.D. dependent var	0.020043
S.E. of regression	0.015319	Akaike info criterion	-5.487393
Sum squared resid	0.129542	Schwarz criterion	-5.425566
Log likelihood	1544.470	Hannan-Quinn criter.	-5.463251
F-statistic	57.83949	Durbin-Watson stat	2.022012
Prob(F-statistic)	0.000000		

Inverted AR Roots	.45+.47i	.45-.47i	-.39+.60i	-.39-.60i
	-.53			
Inverted MA Roots	.98	.83-.48i	.83+.48i	.49-.85i
	.49+.85i	.00-.96i	-.00+.96i	-.49+.85i
	-.49-.85i	-.83-.48i	-.83+.48i	-.98

Date: 11/26/19 Time: 15:48
Sample: 1970M01 2017M09
Included observations: 560
Q-statistic probabilities adjusted for 6 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.013	-0.013	0.0993	
		2	-0.034	-0.035	0.7651	
		3	-0.077	-0.078	4.1542	
		4	-0.013	-0.016	4.2446	
		5	-0.017	-0.024	4.4180	
		6	-0.074	-0.082	7.5228	
		7	-0.058	-0.065	9.4125	0.002
		8	-0.053	-0.067	11.032	0.004
		9	0.054	0.034	12.726	0.005
		10	-0.005	-0.022	12.739	0.013
		11	0.038	0.025	13.550	0.019
		12	0.065	0.063	16.008	0.014
		13	0.027	0.021	16.429	0.021
		14	0.081	0.085	20.194	0.010
		15	-0.004	0.015	20.203	0.017
		16	-0.015	0.002	20.336	0.026
		17	-0.017	0.011	20.497	0.039
		18	-0.004	0.011	20.505	0.058
		19	-0.022	-0.003	20.792	0.077
		20	0.030	0.050	21.321	0.094
		21	0.026	0.036	21.719	0.115
		22	-0.053	-0.045	23.361	0.104
		23	0.129	0.128	33.155	0.011

Figure11: Estimation Output of Model 1

Figure 12: Correlogram of the Residuals of Model 1

For futher evalution we look how much struture of the uderlying process is explained by our model. Figure 13 shows the actual and theoretical correlogram. We observe a reasonably good fit.

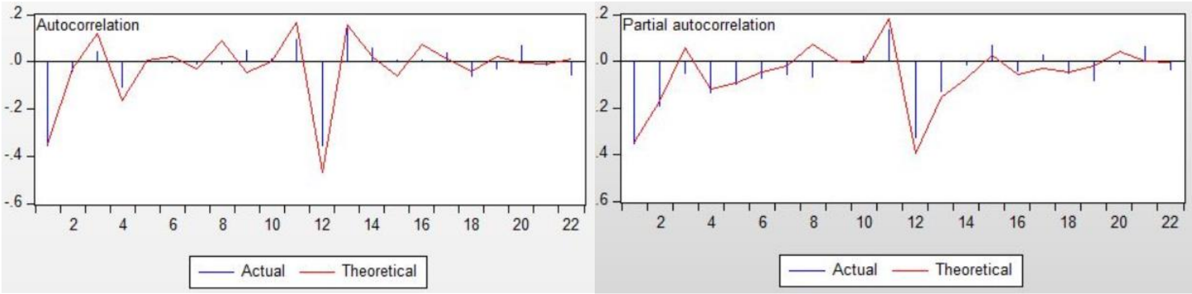


Figure 13: Actual and theoretical Correlogram of Model 1

4.2. AR(1) AR(2) AR(4) AR(5) AR(6) AR(7) MA(12)

The estimation output of the model is presented in Figure 14. Again all variables (excluding the intercept) are significant at the 5% level. All unit roots are outside the unit circle and the Durbin-Watson statistic is again close to 2. Looking at the residual correlogram in Figure 15, we observe significant spikes at lag 8, 12, 14 and 23. However as before, the spikes are barely significant and therfore again neglected. Hence we assume the resudials again to be white noise.

Dependent Variable: DDS_LN_MILES
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/26/19 Time: 15:49
Sample: 1971M02 2017M09
Included observations: 560
Convergence achieved after 28 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-5.79E-05	7.91E-05	-0.731474	0.4648
AR(1)	-0.400939	0.032211	-12.44741	0.0000
AR(2)	-0.174240	0.034168	-5.099445	0.0000
AR(4)	-0.144135	0.043471	-3.315703	0.0010
AR(5)	-0.153123	0.052822	-2.898864	0.0039
AR(6)	-0.174194	0.053726	-3.242258	0.0013
AR(7)	-0.119277	0.050333	-2.369787	0.0181
MA(12)	-0.763127	0.030949	-24.65720	0.0000
SIGMASQ	0.000229	1.00E-05	22.81341	0.0000

R-squared	0.429544	Mean dependent var	-9.00E-05
Adjusted R-squared	0.421262	S.D. dependent var	0.020043
S.E. of regression	0.015248	Akaike info criterion	-5.493474
Sum squared resid	0.128100	Schwarz criterion	-5.423918
Log likelihood	1547.173	Hannan-Quinn criter.	-5.466315
F-statistic	51.86184	Durbin-Watson stat	2.045427
Prob(F-statistic)	0.000000		

Inverted AR Roots	.66+.46i	.66-.46i	-.02-.71i	-.02+.71i
	-.53-.54i	-.53+.54i	-.64	
Inverted MA Roots	.98	.85+.49i	.85-.49i	.49-.85i
	.49+.85i	.00+.98i	-.00-.98i	-.49-.85i
	-.49+.85i	-.85-.49i	-.85+.49i	-.98

Figure 14: Estimation Output of Model 2

Date: 11/26/19 Time: 15:49
Date: 11/26/19 Time: 15:49
Sample: 1970M01 2017M09
Included observations: 560
Q-statistic probabilities adjusted for 7 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.025	-0.025	0.3508	
		2	-0.038	-0.038	1.1510	
		3	-0.078	-0.080	4.5737	
		4	-0.020	-0.026	4.8064	
		5	0.013	0.005	4.8962	
		6	-0.016	-0.024	5.0417	
		7	-0.015	-0.020	5.1748	
		8	-0.082	-0.085	9.0133	0.003
		9	0.062	0.054	11.225	0.004
		10	-0.032	-0.040	11.823	0.008
		11	0.027	0.016	12.232	0.016
		12	0.080	0.084	15.878	0.007
		13	0.019	0.023	16.081	0.013
		14	0.094	0.103	21.199	0.003
		15	0.006	0.029	21.218	0.007
		16	0.008	0.020	21.251	0.012
		17	-0.014	0.015	21.366	0.019
		18	-0.074	-0.074	24.531	0.011
		19	-0.039	-0.031	25.413	0.013
		20	0.042	0.048	26.453	0.015
		21	0.035	0.022	27.185	0.018
		22	-0.069	-0.055	29.931	0.012
		23	0.137	0.137	40.948	0.001

Figure 15: Correlogram of the residuals of Model 2

For the evaluation of our model we compare again the actual with the theoretical correlogram. Figure 16 indicates a good fit of our model.

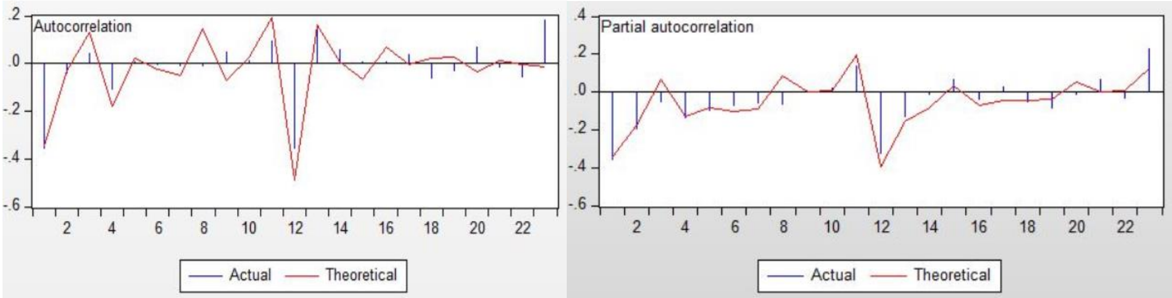


Figure 16: Actual and Theoretical Correlogram of Model 2

4.3. Model Comparison

Looking at the Akaike and Schwarz criterion we cannot rank the two models. Both estimation models have an Akaike criterion of - 5.49 and a Schwarz criterion of ca. – 5.43. Using these criteria, we cannot conclude that one model is better than the other.

5. Forecast

Since we now selected our models using the in-sample, we want to test their prediction power by forecasting our out-of-sample data. Figure 17 shows the forecast of the out-of-sample using model 1 and Figure 18 when using model 2, respectively.

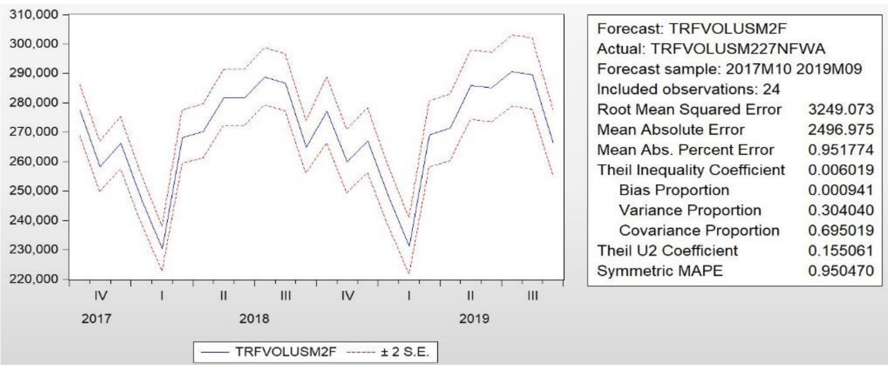


Figure 17: 24-month forecast Model 1 (out-of-sample)

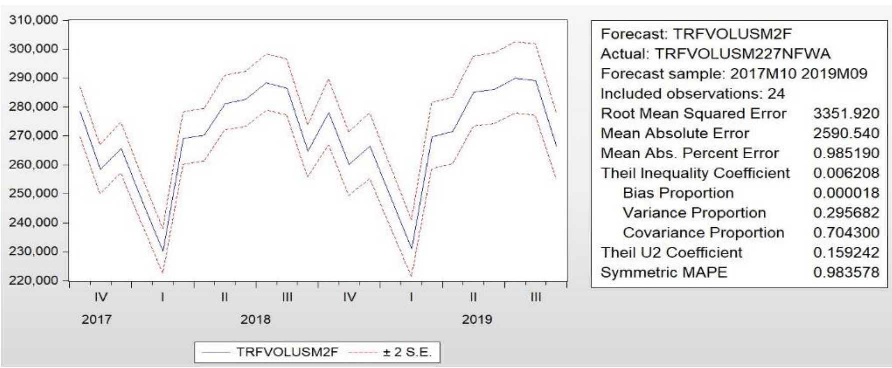


Figure 18: 24-month forecast of Model 2 (out-of-sample)

We can see that both forecasted series continue the real pattern quite well. Looking at Figure 17 and 18 we see indeed that the difference between the actual and the forecast is quite small in both models. When comparing the error measures (either RMSE, MAE, MAPE and symmetric MAPE) it's clear model 1 dominates model 2 in terms of prediction accuracy.

In a last step we want to forecast to the monthly miles traveled by vehicle in the US for the next 12 months (and so for the period from October 2019 until September 2020). Figure 19 shows the forecast using model 1 and Figure 20 shows the forecast using model 2, respectively. Here we use data up to September 2019 to estimate our two ARMA models. The results regarding the outputs and residuals represented in the chapters before are not significantly different.

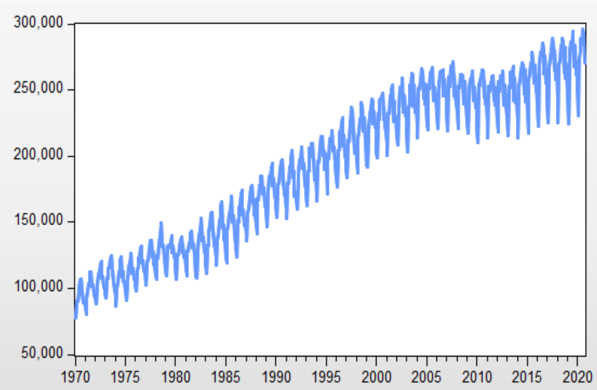


Figure 19: Forecast for the next 12 month Model 1

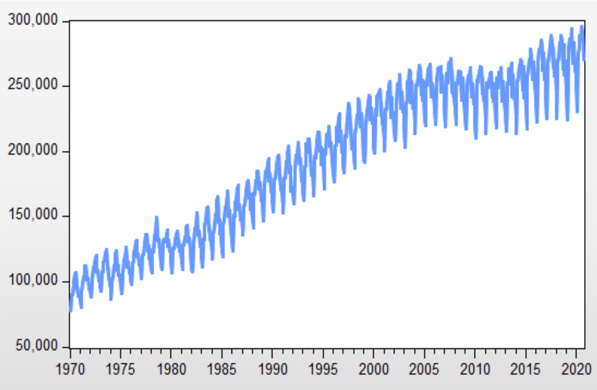


Figure 20: Forecast for the next 12 month Model2

6. Conclusions

In this paper we model a time series representing millions of miles travelled by vehicle in each month from January 1970 to September 2017. We recognized in the series weak non-stationarity, both in mean and in variance, and seasonality of period 12. Therefore, we take the log values to stabilize the variance, the 12th lag difference of these log values to account for the seasonality and the first difference of this in order to stabilize the mean.

We then analyse the structure of the autocorrelation and partial autocorrelation functions to come up with two selected models: an ARMA(6,12) and an ARMA(7,12). Finally, we conclude both our selected models have good prediction power. Indeed, they adequately capture the structure in the data (residuals are white noise) and the ex-post forecasts are very similar to the actual observations. One can criticize the fact that our two models are nearly identical when it comes to prediction. However, after trying many of possible specifications, those two are by far the ones with the best R^2 and Akaike-and Schwarz criterion among the models where residuals seem to be white noise.