A GENERAL PARAMETRIC OPTIMAL POWER FLOW

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ABSTRACT: This paper is a generalization and extension of earlier research in parametric optimal power flow. Its principal features are: (i) the use of a full non-linear OPF model, (ii) a more general parameterization of equalities, inequalities and objective function and (iii) an algorithm that exactly tracks the OPF behaviour in terms of the continuation parameter. The parameterization allows us to distinguish between two phases. One serves to find the OPF solution to a static problem for a fixed load and network starting from an arbitrary initial condition. The second, finds the trajectories corresponding to varying loads. The optimal trajectories, in both phases, offer an excellent visualization of the complex nature of the OPF solutions, that is, the highly non-linear behaviour and the sensitivity of the solutions to parameter variations.

KEYWORDS: Optimal power flow; parametric optimization; relaxation of system load, inequality limits and objective function; optimal solution trajectories tracking; Newton method.

INTRODUCTION

Since the early work reported by Dillon [1] on the sensitivity of the optimal power flow (OPF) solution to parameter variations, several extensions of this idea have been proposed. Carpentier [2] presented some results on real power economic dispatch using parametric quadratic programming. Parametric optimization was also used by Vojdani and Galiana [3] where the continuation method and a varying load strategy were applied to solve the economic dispatch problem subject to the DC load flow and to transmission and generation inequalities. Later Galiana and Juman [4] proposed a parametric technique for the OPF based on the varying limits strategy. In Ponrajah's work [5], this varying limits strategy, together with the restart homotopy continuation algorithm, was applied to the economic dispatch where the losses are treated as a non-linear function of the control variables. More recently, Huneault and Galiana [6] suggested a successive linearization solution methodology for the full OPF problem where each linearized sub-problem is solved by a continuation method where the load and the limits are parameterized. Bacher and Van Meeteren [7] also reported results on the use of a parametric quadratic programming technique for real time generation control, while Gribik and Thomas [8] describe a parametric OPF formulation to perform sensitivity analysis of the optimum incremental losses with respect to the load. Recently, Ajjarapu and Christy [9] developed a continuation algorithm for load flow analysis parameterized by the system load.

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These parametric OPF approaches are limited by the specific nature of the models and algorithms used. Typically, they minimize a cost subject to linear constraints by parametric quadratic programming. Alternatively, they can be based on models consisting of non-linear constraints and costs that are linearized and solved through sequential quadratic programming. This last approach, in spite of using a general non-linear model, in some cases, can yield intermediate solutions "far" from the non-linear feasibility set so that it becomes difficult or impossible to restart the sequential process.

The present work is an extension and a generalization of the studies described in [3,4,5,6]. In the current research, we have examined a general parametric OPF model where the full non-linear load flow equalities and inequalities are enforced while minimizing an arbitrary objective function. Furthermore, in this work, the OPF problem can be independently parameterized by relaxing one or more of the following: (a) The objective function, (b) the inequality limits, (c) the equalities. This feature permits: (i) To systematically track and analyze the OPF behaviour in terms of general parameter variations (ii) To systematically solve the OPF from an arbitrary initial condition.

We distinguish between two main phases of the parametric OPF problem. Phase I finds the OPF solution for a given load level starting from an <u>arbitrary</u> initial condition. Phase II tracks the OPF solution as a function of the load level over a given interval starting from the Phase I solution.

In Phase I, the objective function, the equality and the inequality constraints are parameterized and relaxed by modifying this parameter in such a way that an arbitrary initial solution is forced to be optimal. The variation of the continuation parameter produces a sequence of non-linear optimization problems, with known active constraint set, whose solutions converge to the solution of the actual OPF problem. Since the active set is always known, the solution of intermediate optimization problems is reduced to simply solving the set of <u>equations</u> arising from the first order optimality conditions. Thus, starting from an arbitrary initial solution, the optimum is tracked through a strategy that basically consists of three steps: (i) The variation of the continuation parameter that brings the initial relaxed problem progressively closer to the actual one; (ii) The solution of the various intermediate optimization problems by Newton's method; and (iii) The actualization of the set of active inequality constraints.

The methodology described here varies the continuation parameter while monitoring changes in the set of active inequalities over the entire interval of variation. In this way, the challenge reported by Sun et al [10], of systematically identifying the set of active inequalities at the optimal solution, is successfully overcome.

In Phase II, after obtaining an optimal solution for Phase I, the algorithm tracks the OPF solution trajectory as a function of the load

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over a given interval. In contrast to Phase I where, depending on the choice of the arbitrary initial solution, the parametric continuation process may involve numerous changes in the active set, in Phase II, small load variations usually lead to relatively few changes in the active set. The ability to track the OPF solution in terms of load has potential in an on-line environment by allowing system operators to pre-calculate optimal dispatch strategies based on a load forecast.

PARAMETRIC OPTIMAL POWER FLOW FORMULATION

Preliminary Remarks

The optimal power flow (OPF) can be formulated as a non-linear programming problem of the form,

$$\min_{x} \; c(x) \tag{1}$$

subject to

$$g(x) = 0 \qquad (\lambda) \tag{2}$$

$$h(x) \le 0 \qquad (\mu) \tag{3}$$

The scalar objective function, c(x), can measure economic and performance aspects of the system operation such as generation cost, transmission losses and voltage profile deviations from normal. The variable vector, x, can represent the voltage magnitudes and phase angles, transformer tap and phase-shifter settings and variable shunt admittances.

The equality constraints (2) represent the power balance equations at the load buses while the inequality constraints (3) typically depict both the functional inequalities, such as power flows, and the bounds on the variables x. The variables in brackets, λ and μ, respectively represent the Lagrange multipliers associated with the equality and inequality constraints.

A major difficulty encountered by OPF solution algorithms is to identify the correct set of active constraints. This is due to the large number of inequalities and to the fact that, out of these, relatively few are active at the optimal solution. The search for the optimum active set therefore becomes a combinatorial problem that cannot be solved by exhaustive evaluation. On the other hand, once the active inequalities are known, the solution of the OPF problem is reduced to that of a set of non-linear equations (corresponding to the first order optimality conditions) that can be solved by Newton method. Consequently, in order to systematically find the optimal active set, a parametric optimization approach is proposed here. This parametric approach has the additional positive feature of being able to continuously track an existing optimal solution as a function of a varying parameter such as the system load or a variable limit.

The Parametric OPF Problem

Many physical systems are operated under a finite number of equality and inequality constraints. A common feature of these systems is that they have two different sets of variables. One set is composed of the "parameters" of the system which normally cannot be directly controlled (e.g., the loads of a power system, the variable limits). When the parameters of a physical system are fixed, they partially or completely determine the behaviour of the decision variables (e.g., the voltage magnitudes). Parametric optimization deals with the characterization of the optimal decision variables for a range of parameters [11, 12]. The OPF problem (1)-(3) can be parameterized in the following general form:

Let x_0 and λ_0 be arbitrary initial guesses of x and λ respectively. By construction, we define the parametric optimal power flow (POPF) problem as,

$$\min_{x} \; c(x,\epsilon) \tag{4}$$

subject to

$$g(x,\epsilon) = g(x) - (1-\epsilon)\, g(x_0) = 0 \qquad (\lambda) \tag{5}$$

$$h(x,\epsilon) = h(x) - (1-\epsilon)\,\Delta h \le 0 \qquad (\mu) \tag{6}$$

where

$$c(x,\epsilon) = c(x) - (1-\epsilon)\, c_0^T x + \frac{1}{2}(1-\epsilon)\, W\, |x-x_0|^2 \tag{7}$$

$$c_0 = \frac{\partial c}{\partial x}(x_0) + \frac{\partial g^T}{\partial x}(x_0)\,\lambda_0 \tag{8}$$

$$\begin{aligned} \Delta h_i &= 0 \quad && \text{if} \quad h_i(x_0) \le 0 \\ \Delta h_i &> h_i(x_0) \quad && \text{if} \quad h_i(x_0) > 0 \end{aligned} \tag{9}$$

and where ϵ is a parameter that can assume values between 0 and 1. The above parameterization achieves four objectives:

(a) It relaxes the inequality limits according to (6) and (9) so that at $\epsilon = 0$ they are strictly inactive. This, therefore, implies that the corresponding Lagrange multiplier $\mu_0 = 0$. The parameter variation process then progressively tightens these limits back to their original values reached at $\epsilon = 1$ (See Figure 1).

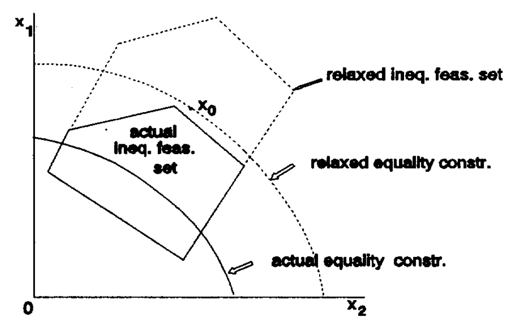

Fig. 1 - Relaxation of the feasible set.

(b) It modifies the equality constraints (by essentially relaxing the load) according to (5) so that, at $\epsilon = 0$, the equalities are exactly satisfied for the arbitrary initial state, x_0. In a similar manner to (a) above, the parameter variation process progressively returns the loads back to their original values at $\epsilon = 1$ (See Figure 1). The Lagrange multipliers associated with the equalities are set to the arbitrary initial value λ_0.

(c) It translates the cost function by adding a parameterized linear term, so that the first order optimality conditions are satisfied at $\epsilon = 0$ for the arbitrary initial solution (x_0, λ_0, μ_0) (See Figure 2).

(d) It adds a quadratic term to the cost function, so that the second order optimality conditions are met near zero (See Figure 3). Note that at $\epsilon = 1$ this quadratic term disappears.

Note 1: Although we have parameterized the OPF problem with a single ϵ for reasons of clarity, it is possible to use four different ϵ's to independently parameterize the equalities, the inequalities, the first and second order optimality conditions. This gives us additional flexibility in the various phases of the tracking problem.

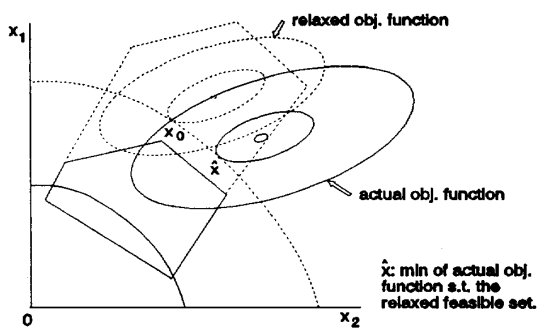

FIG. 2 - Relaxation of the objective function.

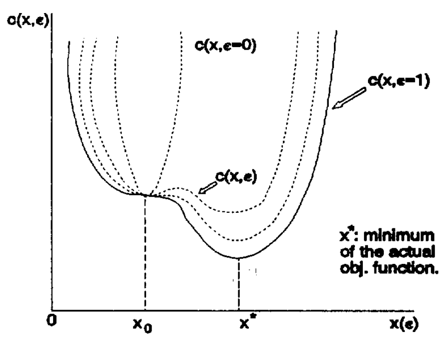

Fig. 3 - Modification of the objective function

Note 2: The above parameterization applies to Phase I. During the load tracking, the initial solution is already optimum, so the objective function does not need to be parameterized by the linear term.

PARAMETERIZED OPTIMALITY CONDITIONS

In order to express the optimality conditions as functions of ϵ, we first define A as the set of active inequalities for some value of ϵ, while I represents the corresponding set of inactive inequalities. Thus,

$$h_A(x,\epsilon) = 0 \tag{10}$$

$$h_I(x,\epsilon) < 0 \tag{11}$$

Note that at $\epsilon = 0$, because of (9), all inequalities are inactive, that is, the set A is empty.

In general, for an arbitrary ϵ, if we know A, then the problem Lagrangian is given by,

$$\mathcal{L}(x,\lambda,\mu,\epsilon) = c(x,\epsilon) + \lambda^T g(x,\epsilon) + \mu_A^T h_A(x,\epsilon) \tag{12}$$

The first order optimality conditions are then,

$$h_I(x,\epsilon) \le 0 \tag{13}$$

$$\mu_A \ge 0 \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\partial c(x,\epsilon)}{\partial x} + \frac{\partial g}{\partial x}^T(x,\epsilon)\lambda + \frac{\partial h_A}{\partial x}^T(x,\epsilon)\mu_A = 0 \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = g(x,\epsilon) = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_A} = h_A(x,\epsilon) = 0 \tag{17}$$

At $\epsilon = 0$, the above are, by construction, clearly satisfied by the initial solution $y_0 = (x_0, \lambda_0, \mu_0)$.

In order to guarantee second order optimality, we require that the second derivative of the Lagrangian with respect to x (the Hessian matrix),

$$\frac{\partial^2 \mathcal{L}}{\partial x^2} = \frac{\partial^2 c(x,\epsilon)}{\partial x^2} + \frac{\partial^2 (g^T(x,\epsilon)\lambda)}{\partial x^2} + \frac{\partial^2 (h_A^T(x,\epsilon)\mu_A)}{\partial x^2} \tag{18}$$

be positive definite on the null space of the equalities and active inequalities [13]. In other words, for all $d \bullet 0$ such that

$$\begin{bmatrix} \frac{\partial g}{\partial x}(x,\epsilon) \\ \frac{\partial h_A}{\partial x}(x,\epsilon) \end{bmatrix} d = 0 \tag{19}$$

we must have

$$d^T \frac{\partial^2 \mathcal{L}}{\partial x^2} d > 0 \tag{20}$$

At and near $\epsilon = 0$, the second order optimality conditions are met for a sufficiently large coefficient, W (see equation (7)), which makes the Hessian matrix diagonally positive dominant.

TRACKING THE OPTIMAL TRAJECTORY FOR $0 < \epsilon \le 1$

Conditions for Continuity of Trajectory

If at some value of ϵ, ϵ_0, we know the optimum (e.g. $\epsilon_0 = 0$), then for all ϵ in some neighbourhood around ϵ_0 there exists an optimum solution provided that,

(A) The Jacobian of the first order optimality conditions (15) to (17), that is,

$$H(y_0,\epsilon_0) = \begin{bmatrix} \frac{\partial^2 \mathcal{L}}{\partial x^2}(y_0,\epsilon_0) & \frac{\partial f}{\partial x}^T(y_0,\epsilon_0) \\ \frac{\partial f}{\partial x}(y_0,\epsilon_0) & 0 \end{bmatrix} \tag{21}$$

is non-singular, where,

$$f(y,\epsilon) \triangleq \begin{bmatrix} g(x,\epsilon) \\ h_A(x,\epsilon) \end{bmatrix} \tag{22}$$

and where $y = [x, \lambda, \mu_A]^T$. This ensures the continuity of the optimum solution $y(\epsilon) = [x(\epsilon), \lambda(\epsilon), \mu_A(\epsilon)]^T$ with respect to ϵ near ϵ_0 [11].

(B) The active inequality constraints remain active for all ϵ in some neighbourhood of ϵ_0, i.e.,

$$\mu_A(\epsilon) > 0 \tag{23}$$

(C) The inactive inequalities are not violated for any ϵ in the neighbourhood, i.e.,

$$h_I(x(\epsilon),\epsilon) < 0 \tag{24}$$

(D) Equation (20) is satisfied, ensuring that the second order optimality is met for all ϵ in the neighbourhood.

The basic idea of tracking the optimum trajectory is to start with a known optimum for some ϵ and find the maximum possible increase in ϵ which does not violate any of the above conditions. If conditions (B) or (C) establish such a maximum, the resulting point is called a break-point, and corresponds to either the release of an active inequality or to the binding of a previously inactive inequality. The tracking process consists of following the optimum path from one break-point to another until $\epsilon=1$. It is this procedure which enables this method to systematically identify the final active set. Furthermore, the active set throughout the path is also determined which has important implications in the optimum load tracking problem.

Singularities, Saddle Points, Degeneracies and Infeasibility

If, during the tracking process, condition (A) fails, then the path is not continuous, which may imply that a final solution cannot be found. Alternatively, if condition (D) fails, then the path being followed is not a minimum. The violation of one or both of these conditions does not however necessarily imply that the process fails to find a minimum at $\epsilon=1$. In some cases a saddle point can exist throughout part of the path, eventually changing to a minimum later on. When this change occurs, condition (A) is locally violated as illustrated in Figure 4. In this Figure, to the left of ϵ_0, the problem may correspond to a saddle point, that is, a problem where the projected Hessian has one negative eigenvalue. At ϵ_0, this eigenvalue crosses the zero axis to eventually become positive for values of ϵ beyond this level. At this point, the Jacobian is singular. Saddle points are difficult to detect and can result in unexpected behaviour of the algorithm. In order to help avoid saddle points near the initial part of the trajectory, we have added a quadratic term to the cost (equation (7)) which smooths the shape of the trajectory. The weighting factor associated with this term, W, is chosen by experimentation so that the initial solution satisfies the sufficient conditions for optimum, but not bigger than strictly necessary. An excessively large W would cause instability near $\epsilon=1$.

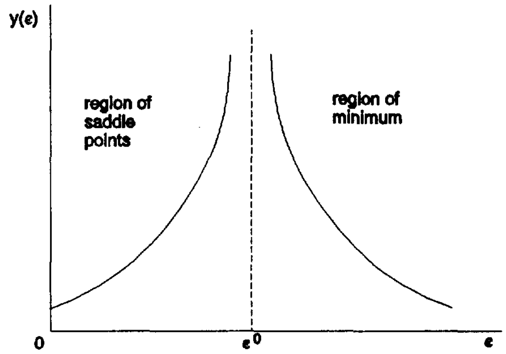

Fig. 4 - Singularity of the Jacobian.

Degeneracies can occasionally occur if two or more violations appear simultaneously. In some cases these can be resolved by trying out various combinations of the possible active sets. If a degeneracy cannot be resolved, then we have an infeasibility which means that the problem has no solution. Infeasibilities could also occur without degeneracies if the Jacobian matrix (21) becomes singular.

ALGORITHM

Basic POPF Algorithm

(1) Initialize $\epsilon=0$, and y(0) by $x(0)=x_0$, $\lambda(0)=\lambda_0$, $\mu(0)=0$.
(2) Increment ϵ.
(3) Solve (15)-(17) using a Newton approach for $y(\epsilon)$.
(4) Check inequalities (13) and (14):
    If there are no violations go to step (5).
    If there is a single violation in (13) go to step (6).
    If there is a single violation in (14) go to step (7).
    If there are multiple violations go to step (8).
(5) If $\epsilon=1$, stop. The solution of the OPF is y(1). Else go to step (2).
(6) Remove the violated inequality from h_I and add it to h_A. Go to step (3).
(7) Remove the active inequality with a violated Lagrange multiplier from h_A and add it to h_I. Go to step (3).
(8) Reduce ϵ. Go to step (3).

Note: In some cases, upon fixing one violation, other constraints are violated. In such cases, one must reduce ϵ and return to the original active set.

Initial Conditions

An important characteristic of the POPF algorithm is that it can be started from arbitrary initial conditions. The initial voltages are usually set to the flat voltage profile or to a solved load flow solution. If there are violations of inequalities, the algorithm relaxes the corresponding limits by an appropriate parameterization, so that initially there are no active inequalities. Consequently, the initial associated Lagrange multipliers are zero. The violated equalities in P and Q are also relaxed and parameterized, however the corresponding Lagrange multipliers are not necessarily zero. At present, the Q-equality multipliers are initialized by setting them all to one, while the P-equality multipliers are all set, initially, to the system incremental cost. Many tests were made with different initial lagrange multipliers for the equality constraints and it was observed that, contrarily to what occurs with other optimization methods, the value of the initial λ does not affect the convergence characteristics of the POPF. So, the choice of λ_0 is arbitrary. The POPF algorithm is capable of starting with other arbitrary initial conditions for both x and λ. We tested a number of cases, by starting Phase I from the two different initial conditions described above. In all examples, although the two Phase I optimum trajectories differed drastically, the final optimal solution was the same for both initial conditions.

Strategies for Incrementing and Reducing ϵ

This section describes in more detail steps (2) and (8) of the POPF algorithm. These are key steps in tracking the optimum solution trajectories as a function of the continuation parameter, ϵ. At present, we rely on a variation of a binary search to localize the next break-point. We have intentionally selected this conservative search so as to be able to follow the active set and the optimum trajectory precisely from one break-point to the next which is one of the objectives of this research. If one is interested in finding the optimum solution at $\epsilon=1$ only, then a more aggressive strategy for increasing ϵ may be followed. However, since the exact non-linear behaviour of the trajectory of each variable is not known à priori, large steps in ϵ can lead to many violations or to a singularity of the Jacobian. We are then faced with the combinatorial problem of deciding which violations to fix or which fixed variables to release. Before adopting an aggressive search strategy, we propose to analyze the behaviour of the exact trajectories to look for possible patterns that may eventually permit large search steps with greater confidence.

The binary search used at present, is illustrated in Figure 5 below showing three typical trajectories as a function of ϵ. These three voltages are from a 5-bus example [14], all of which have a maximum limit of 1.03 pu. The sequence of vertical lines denoted by ϵ_1 to ϵ_4 indicate the values of ϵ tried by the binary search to localize the first break-point. For each value of ϵ, the optimality conditions (15)-(17) are solved and the number of violations noted. Observe that it is not necessary to find the exact location of the break-point to stop the binary process, but simply to ensure that only one violation exists. In this example, the first break-point occurs near $\epsilon=0.4$ and requires that V_4 be fixed at its upper limit.

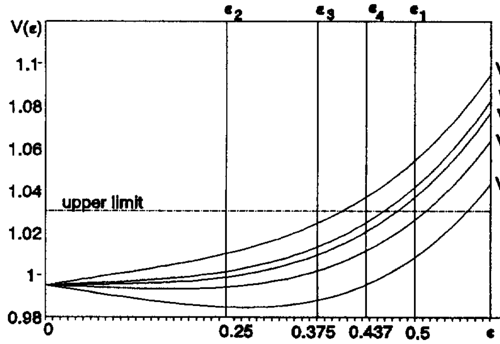

Fig. 5 - Binary search mechanism.

Figure 6 illustrates, for the same example, the optimum trajectories (dotted lines) based on the initial active set (A_0), and how these drastically change when one variable is fixed and the active set is modified (solid lines). This figure also shows the complete optimum trajectory with all three break-points included. One can observe, for example, that if the initial active set is maintained until $\epsilon=1$, then several voltages would have violated the upper limit of 1.03 pu. An aggressive but non-systematic approach could have fixed all violations at $\epsilon=1$, however from the figure, we see that this would not correspond to the optimum solution. In fact, the optimum requires that only three of the voltages be fixed at a limit at $\epsilon=1$. It is important to point out that this parametric approach gives us an optimum trajectory rather than just one optimum point.

During the load tracking, Phase II, the parameter ϵ represents the system load. The interval of ϵ between 0 and 1, therefore, represents a change in system load between two levels which we can specify. If this load change is sufficiently small, then a more aggressive strategy can be followed where ϵ is incremented from 0 directly to 1. This approach allows us to follow the optimal load tracking trajectory closely and, at the same time, reduce the number of trial iterations.

Comparison of POPF with Conventional OPF

The objectives of the proposed POPF are quite distinct from those of a common OPF. The POPF has been developed with two main objectives: (i) To continuously track and analyze the behaviour of the optimal solution in terms of general parameter variations such as load, inequality and objective function; (ii) To systematically solve a

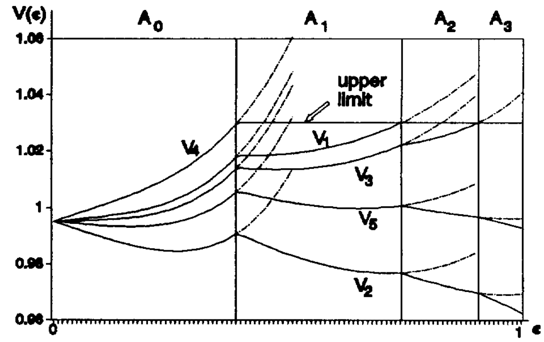

Fig. 6 - Optimal trajectories of voltage magnitudes.

given OPF from arbitrary initial conditions by the use of a "smart" parameter relaxation technique and a subsequent progressive tightening of the relaxed parameters. On the other hand, the OPF is only interested in the optimum solution for one set of parameters. The Newton approach, in combination with powerful heuristics to find the optimal active set, has been successfully used to solve the OPF and, for this purpose, the OPF is much more efficient than the POPF. The same Newton method, together with an algorithm for finding the active set, is also used by the POPF; however, this is done over an entire range of parameter variations rather than for a single parameter.

To increase its efficiency, the OPF tries to identify as many active constraints as possible at each iteration, whereas the POPF, in trying to track the complete optimal trajectory, follows the path from one change in the active set to another. In this way, if the tracking cannot continue due to the failure of the Newton method, one can analyze the previous optimum point in the path to determine the cause of the failure, which can be due to loss of optimality or infeasibility. The efficiency of the POPF would considerably increase if similar heuristics to those of the OPF for identifying the active set were used. However, this would be at the expense of losing the capability of continuously tracking the optimal solution path in terms of a varying parameter. Our experience shows that the tracking of the optimal trajectory from one break-point to another would be possible even for larger systems, since the occurrence of several violations of the inequality limits at the same ϵ is very rare, although changes can occur very close to each other.

RESULTS

Preliminary Remarks

The results presented here emphasize the behaviour of the optimum trajectories in phase I (finding an optimum from an arbitrary initial condition) and phase II (tracking the optimum as the load varies).

The systems tested and described here are: The 14 and 30-bus IEEE networks, and a 34-bus network with 64 lines characterized by high levels of reactive power and voltage instability. The minimization criteria examined were of three types: Real transmission losses, generation cost and deviations from a flat voltage profile. Linear combinations of these three were also considered. In the load tracking phase, the load was uniformly varied with time according to a given curve.

In the table below the average cpu time of one ϵ iteration is presented for some examples tested on a Sun Sparc 10. In the table, "trial iteration" means a tentative increase in ϵ that cannot be performed because it leads to multiple violations of inequality limits. A "good iteration" describes a successful increment in ϵ with no violations. This prototype version of the POPF was programmed in Matlab without an efficient sparsity exploitation. Newer versions further exploiting sparsity and more efficient strategies for tracking ϵ are being developed.

Table I - Computational effort - Phase I.

test system	# trials ϵ iterations	# good ϵ iterations	cpu time per iter. (sec.)
14-bus	0	12	2.24
30-bus	10	14	3.20
34-bus	75	50	3.38

The sample trajectories of the 14-bus and 30-bus systems shown below are normalized so that all system variables and Lagrange multipliers range between 0 and 100%, corresponding to their minimum and maximum levels. This makes it easier to visualize and compare the behaviour of the trajectories.

Phase I Results

The graphs presented here show some selected optimal trajectories for the 14 and 34-bus systems when minimizing generation cost plus voltage deviations from the flat profile.

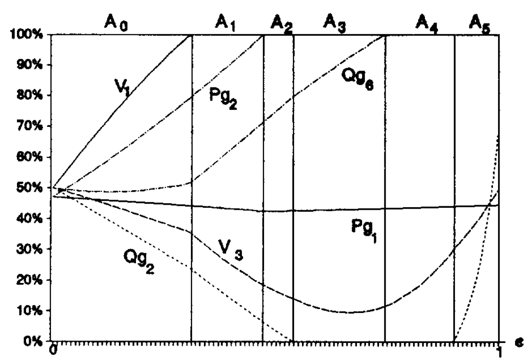

Fig. 7a - Results for the 14-bus system - Phase I

The results of the 14-bus system are presented in figures 7a and 7b. Phase I, in this example, displays five break-points in the interval of ϵ from 0 to 1. The change in the optimal trajectories caused by each break-point are clearly observable. Note, for example, that after the first break-point, when V_1 is fixed to its upper bound, the trajectory of the real generation, P_{g2}, is not affected much, while the trajectories of the remaining free voltage and reactive generations are more drastically modified. Alternatively, when P_{g2} reaches its upper limit, the trajectories of the voltage and reactive generations are not much affected. This behaviour is expected in a general sense. The highly non-linear behaviour of the POPF problem is more easily observable in figure 7b where the Lagrange multipliers are depicted.

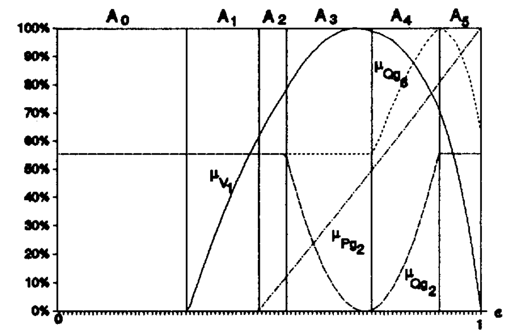

Fig. 7b - Results for the 14-bus system - Phase I.

Lagrange multipliers can be thought of as the tendency of a variable to stay at a limit. It can be seen that this tendency varies in a very non-linear way. For example, when Q_{g2} is released from its lower limit at $\epsilon=0.9$, the tendency for Q_{g6} to stay at its upper bound begins to decrease very rapidly, whereas before the break-point it tended to increase.

Figure 8a represents the optimal voltage magnitude trajectories for the 34-bus system in Phase I. This is a system with a high degree of voltage instability, very sensitive to parameter changes. In this example, 34 break-points were observed when the initial solution is a converged load flow. The extreme non-linear behaviour of the optimal trajectories can also be observed in figure 8b showing the corresponding Lagrange multipliers. In particular, the last break-point results in a very sharp increase of the Lagrange multiplier corresponding to V_{17} fixed at its upper limit. We also examined the same case starting from the flat voltage profile. The voltage results, shown in figure 8c, clearly show totally different trajectories, however the final solutions at $\epsilon=1$ are identical. There were 84 break-points in this case which starts "farther" from the optimum compared to a solved load flow.

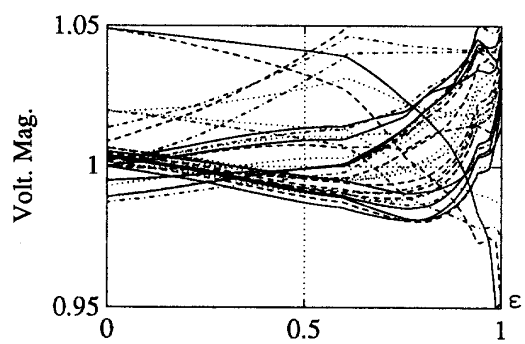

Fig. 8a - Optimal trajectories.

Phase II Results

The algorithm was used to generate optimal trajectories that follow a pre-specified load curve (see Figure 9a) for the three test networks

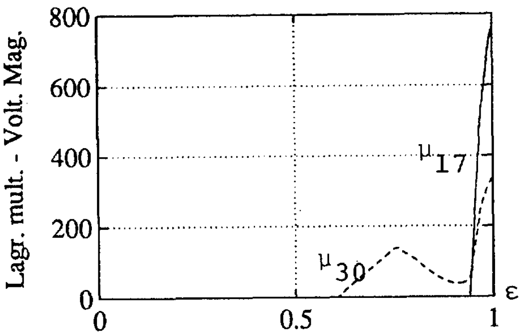

Fig. 8b - Lagrange multipliers

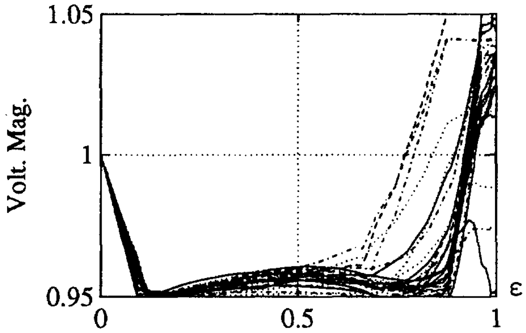

Fig. 8c - Optimal trajectories, x_0= flat

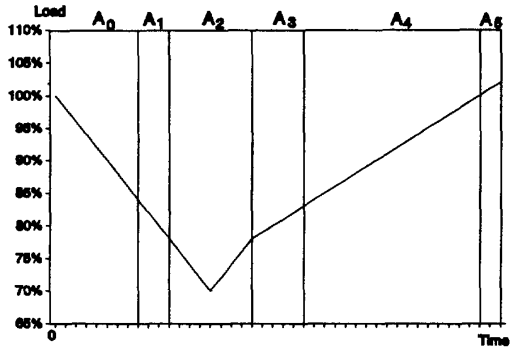

Fig. 9a - Load curve.

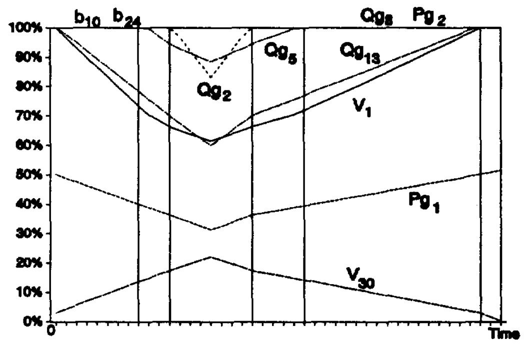

Fig. 9b - Optimal trajectories.

under different objective functions. In all cases, the initial conditions for Phase II correspond to those found at the end of Phase I. In the 30-bus example, the pre-specified load curve versus time is shown in figure 9a depicting a decrease of 30% followed by an increase of 32%. Figure 9b shows the corresponding behaviour of selected optimum trajectories as a function of time. We observe that during the load tracking there are relatively few changes in the active set and that the trajectories are approximately linear for small load changes. This suggests that a linear prediction of the next break-point could be used efficiently by the algorithm in the load tracking phase. In this example, the maximum feasible load turns out to be 102%. Beyond this level, the Jacobian matrix becomes singular.

CONCLUSIONS

The POPF method was applied to three different transmission systems and in all cases, both for phase I and II, it was able to generate the optimal trajectories of the solutions and to show the high non-linear nature of the OPF problem. The parametric approach, besides providing a systematic way of tracking the changes in the active set, permits an excellent visualization of the behaviour of all variables during the solution. The optimal trajectories show very clearly the difficulties encountered while solving an OPF problem, that is, the sensitivity of the optimal solution and the active set to parameter changes. For example, as the parameter varies, numerous modifications of the active set and sudden changes in the tendency of a variable to stay at a limit, expressed by the abrupt increase (or decrease) of the corresponding Lagrange multiplier are observed, giving an idea of the high sensitivity of the optimal solution in some examples. This, in the end, can significantly affect the convergence of methods that use trial iterations or linear predictions to find the feasible set.

The parametric approach also shows that, contrarily to what occurs in Phase I, during Phase II the optimal trajectories created while tracking the system load are approximately linear and the changes in the active set are comparably few. This result justifies the use of a parametric methodology to optimally track the load variation.

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