1 The Model - DRAFT

1.1 Initial model - without retirement

The initial model that is taken into consideration is the basic heterogeneous agents model a la Aiyagari. Where the households are ex ante homogeneous but face different idiosyncratic earnings risk from shocks on productivity.

The problem that Households face is as follows:

$$
\begin{aligned}
&\max_{c_t, a_{t+1}} && \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \\
&\text{subject to} && a_{t+1} + c_t \leq y(s'|s)lw + (1-r)a_t \\
& && c_t \geq 0 \\
& && a_t \geq -b
\end{aligned}
$$

Where $y(s'|s)$ captures the idiosyncratic earning risk through two states of productivity and follows a finite state Markov chain process, b is the borrowing limit and β the utility discount factor.

Thus, the value function for this problem, normalising labor demand $l = 1$, reads:

$$V(a_t) = \max_{a_{t+1} \geq 0} \left[u((1-r)a_t + y(s'|s)w_t - a_{t+1}) + \sum_{t=0}^{\infty} \beta^t [u((1+r)a_{t+1}+) + y(s'|s)w_{t+1} - a_{t+2}] \right]$$

From the optimality conditions we know

$$
\begin{aligned}
U'(c_j(a,s)) &= \sum_{j=0}^{\infty} [U'(c_{j+1}(a',s'))\pi(s'|s)]\beta(1+r) \\
c_j(a,s)^{-1} &= \sum_{j=0}^{\infty} [c_{j+1}(a',s'))\pi(s'|s)]^{-1}\beta(1+r) \\
a' &= (1+r)a + y_j(s) - c_j(a,s)
\end{aligned}
$$

1.2 Initial model - with retirement

We denote a households age by $j \in W, R$. Young households face a constant probability of retiring of $1-\theta \in [0,1]$ and old households face a constant probability of dying of $1-\nu \in [0,1]$, if the agent/household dies, it is replaced by a new young household.

The dynamic programming problem of retired households is characterized as follows:

$$V_R(a) = \max_{c,a'}\{u(c) + \nu\beta V_R(a')\}$$

s. t.

$$c + a' = b_R + (1+r)a$$

Where b_R denotes the retirement benefits.
For young hoseholds the programming problem is given by:

$$V_W(a) = \max_{c,a'}\{u(c) + \beta \sum_{s\in S} \pi(s'|s)[\theta V_W(a') + (1-\theta)V_R(a')]\}$$

s. t.

$$c + a' = (1-\tau)wy(s) + (1+r)a$$