Herramientas Computacionales 2016661

Ajuste por mínimos cuadrados

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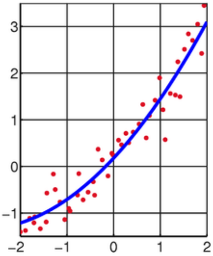
Problema a resolver



encontrar $\boldsymbol{a} = (a_0, a_1, a_2 \ldots)$ que minimice ϵ, donde:
$\epsilon = \sum_{i=1}^{n} \left(y_i - f\left(x_i, \boldsymbol{a}\right)\right)^2$

¿Que quiere decir esto?

Ajuste de n datos a combinación lineal de m funciones

$$f(x) = \sum_{i=0}^{m} a_i \phi_i(x) \; ; \; \epsilon = \sum_{i=1}^{n} (y_i - f(x_i, \mathbf{a}))^2$$

$$\begin{aligned}
\frac{\partial \epsilon}{\partial a_i} &= 0 \\
&= 2 \sum_{k=0}^{n} (f(x_k) - y_k)\, \phi_i(x_k) \\
&= 2 \left(\sum_{k=0}^{n} f(x_k)\, \phi_i(x_k) - \sum_{k=0}^{n} y_k \phi_i(x_k) \right)
\end{aligned}$$

lo que se puede escribir como:

$$\sum_{k=0}^{n} f(x_k)\, \phi_i(x_k) = \sum_{k=0}^{n} y_k \phi_i(x_k)$$

$$\sum_{k=0}^{n}\left(\sum_{i'=0}^{m} a_{i'}\phi_{i'}(x_k)\right)\phi_i(x_k) = \sum_{k=0}^{n} y_k\phi_i(x_k)$$

$$\sum_{i'=0}^{m} a_{i'}\sum_{k=0}^{n}\phi_{i'}(x_k)\phi_i(x_k) = \sum_{k=0}^{n} y_k\phi_i(x_k)$$

$$\begin{bmatrix} \phi_0(x_0) & \phi_0(x_1) & \cdots & \phi_0(x_n) \\ \phi_1(x_0) & \phi_1(x_1) & \cdots & \phi_1(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_m(x_0) & \phi_m(x_1) & \cdots & \phi_m(x_n) \end{bmatrix} \begin{bmatrix} \phi_0(x_0) & \phi_1(x_0) & \cdots & \phi_m(x_0) \\ \phi_0(x_1) & \phi_1(x_1) & \cdots & \phi_m(x_1) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_0(x_n) & \phi_1(x_n) & \cdots & \phi_m(x_n) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \cdots \\ a_m \end{bmatrix}$$

$$= \begin{bmatrix} \phi_0(x_0) & \phi_0(x_1) & \cdots & \phi_0(x_n) \\ \phi_1(x_0) & \phi_1(x_1) & \cdots & \phi_1(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_m(x_0) & \phi_m(x_1) & \cdots & \phi_m(x_n) \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$\boldsymbol{X}^T\boldsymbol{X}\boldsymbol{a} = \boldsymbol{X}^T\boldsymbol{y}$$

$$\boldsymbol{a} = \left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1}\boldsymbol{X}^T\boldsymbol{y}$$