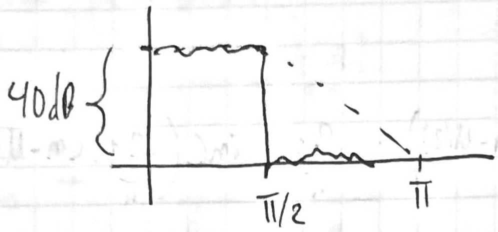
remoção do ruído

Kaiser 40 dB



$$E\{x_M^2\} \qquad E\{x^2[n]\} = \underbrace{\frac{1}{N} \cdot \underbrace{\sum_{n=0}^{N-1} x^2[n]}_{\text{Energia}}}_{\text{Potência do sinal}}$$

$$\underbrace{E\{x[n]\} = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x[n]}_{\text{média do sinal}} = w_N$$

w_N^2 = potência da componente D.C.

$$\underbrace{\sigma_N^2 = E\{(x[n] - w_N)^2\}}_{\text{Potência da componente A.C.}} = E\{x[n]^2 - 2 \cdot x[n] \cdot w_N + w_N^2\}$$

$$= E\{x[n]^2\} - 2 \cdot \underbrace{w_N \cdot E\{x[n]\}}_{w_N^2} + w_N^2$$

$$= \underbrace{E\{x^2[n]\}}_{P} - \underbrace{w_N^2}_{P(DC)}$$

$$\varphi_{xx}[m] = E\{x_n \cdot x^*_{n+m}\} = \begin{cases} E\{x_n^2\} = 1 & ; m = 0 \\ E\{x_n \cdot x_{n+m}\} = m_\mu^2 = 0 \end{cases}$$

$$\zeta = E\{x_n \cdot x_{n+m}\} = E\{x_n\} \cdot E\{x_{n+m}\} = w_N^2$$

$$\varphi_{xx}[n] = A \cdot \delta[n] \Rightarrow T.F.\{\varphi_{xx}[n]\} = \text{constante } (A)$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n].e^{-j\Omega n} \rightarrow \text{DTFT não dá para sinais aleatórios}$$

$$\gamma_{xy}[m] = E\{(x[n]-m_x).(y[n+x]-m_y)\}$$

$$= E\{(x[n]-m_x).(x[n+x]-m_x)^*\} = E\{x[n].x[n+m] - x[n].m_x - m_x.x^*[n+m] + m_x^2\}$$

$$= \underbrace{E\{x[n] * x^*[n-m]\}}_{\varphi_{xx}[m]} - m_x.\underbrace{E\{x[n]\}}_{m_x} - m_x.\underbrace{E\{x^*[n+m]\}}_{m_x} + m_x^2$$

$$= \varphi_{xx}[m] - m_x^2$$

$$= E\{(x[n]-m_x).(y[n+m]-m_y)^*\}$$

$$= E\{x[n].y[n+m] - x[n].m_y - y[n+m].m_x + m_x.m_y\}$$

$$= \underbrace{E\{x[n].y[n+m]\}}_{\varphi_{xy}[m]} - \underbrace{m_y.E\{x[n]\}}_{m_x.m_y} - \underbrace{m_x.E\{y[n+m]\}}_{m_y.m_x} + m_x.m_y$$

$$= \varphi_{xy}[m] - m_x.m_y$$

$$\varphi_{xx}[m] = E\{x[n].x[n+m]\} \Rightarrow \varphi_{xx}[0] = E\{x^2[n]\}$$

$$\gamma_{xx}[m] = E\{(x[m]-m_x).(x[n+m]-m_x)\} \xrightarrow{m=0} \gamma_{xx}[0] = E\{(x[n]-m_x)(x[n]-m_x)\}$$

$$= E\{x[n]-m_x\} = \sigma_x^2$$

$$\varphi_{xx}[m] = E\{x[n].x[n+m]\} = \frac{1}{N}.\sum_{n=0}^{N-1} x[n].x[n+m] = \frac{1}{N}.(x[0].x[m] + \dots + \dots)$$

Se $\Omega = n$

$$= \frac{1}{N}(x[m].x[0] + x[m+1] * x[1] + \dots) = E\{x[n+m].x[n]\}$$

$$= E\{x[\Omega].x[\Omega+m]\} = \varphi_{xx}[-m]$$

ex Slide

1. $x[n]$, $y[n]$ $\longrightarrow$ $z[n] = x[n] + y[n]$

$$m_z = E\{z[n]\} = \frac{1}{N} \cdot \sum_{n=0}^{N-1} z[n] = \frac{1}{N} \cdot \sum_{n=0}^{N-1} (x[n] + y[n])$$

$$= \underbrace{\frac{1}{N} \cdot \sum_{n=0}^{N-1} (x[n])}_{m_x} + \underbrace{\frac{1}{N} \cdot \sum_{n=0}^{N-1} (y[n])}_{m_y}$$

$$= m_x + m_y$$

$$\sigma_z^2 = E\{(z[n] - m_z)^2\} = E\{z[n]^2 - 2 \cdot z[n] \cdot m_z + m_z^2\}$$

$$= E\{z[n]^2\} - 2 \cdot m_z \cdot \underbrace{E\{z[n]\}}_{m_z} + m_z^2$$

$$= E\{z[n]^2\} - m_z^2$$

$$= E\{(x[n] + y[n])^2\} - (m_x + m_y)^2 =$$

$$= E\{x[n]^2 + 2\,x[n] \cdot y[n] + y[n]^2\} - m_x^2 - 2\,m_x \cdot m_y - m_y^2$$

$$= E\{x[n]^2\} + 2 \cdot \underbrace{E\{x[n] \cdot y[n]\}}_{} + E\{y[n]^2\} - m_x^2 - 2\,m_x \cdot m_y - m_y^2$$

$\hookrightarrow$ Como são não correlados

$$E\{x[n]\} \cdot E\{y[n]\} = m_x \cdot m_y$$

$$= E\{x[n]^2\} - m_x^2 + E\{y[n]^2\} - m_y^2 = \sigma_x^2 + \sigma_y^2$$

$$\sigma_z^2 = \begin{cases} \sigma_x^2 + \sigma_y^2 & \text{se } x \text{ e } y \text{ são não correlados} \\ \sigma_x^2 + \sigma_y^2 + \varphi_{xy}[n] - 2\,m_x \cdot m_y \end{cases}$$

2-

Se $e[n]$ é ruído branco

$\phi_{ee}[m] = E\{e[n] \cdot e[n+m]\} = A \cdot \delta[m] + m_e^2$ → assumindo que é zero

$\gamma_{ee}[m] = E\{(e[n] - m_e)(e[n+m] - m_e)\} = A \cdot \delta[m]$

$y[n] = s[n] \cdot e[n]$

$E\{y[n] \cdot y[n+m]\} = E\{\underline{s[n] \cdot e[n]} \cdot \underline{s[n+m] \cdot e[n+m]}\}$

$= \underline{E\{(s[n] \cdot s[n+m]) \cdot (e[n] \cdot e[n+m])\}}$

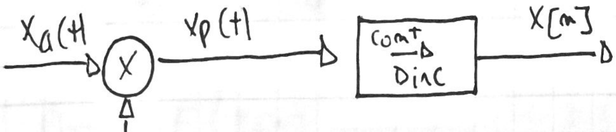
não correlados

$= E\{s[n] \cdot s[n+m]\} \cdot E\{e[n] \cdot e[n+m]\}$

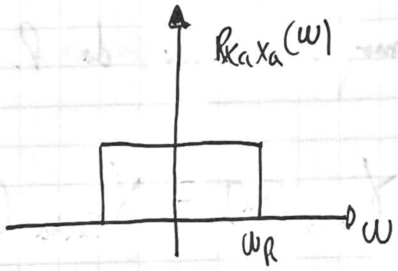
$= \phi_{ss}[m] \cdot A \cdot \delta[m]$

1	2	3	4	5	6
1	\	\	4	\	\

3-



$$p(t)=\sum_{k=-\infty}^{+\infty}\delta(t-kT)$$



$$x_p(t)=x_a(nT)=X[n]$$

$$P_{x_a x_a}(\omega)=T.F.\{\phi_{x_a x_a}(\tau)\}$$

$$=E\{x_a(t).\,x_a^*(t+\tau)\}$$

$$\gamma_{x_a x_a}(\tau)=\phi_{x_a x_a}(\tau)-m_{x_a}^2 \Rightarrow \text{Si } m_{x_a}=0 \Rightarrow \gamma_{x_a x_a}(\tau)=\phi_{x_a x_a}(\tau)$$

a) $\phi_{xx}[m]=E\{X[n].\,X^*[n+m]\}=E\{x_a(nT).\,x_a((n+m).T)\}$

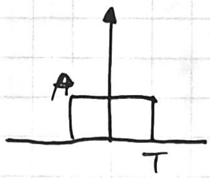
$$=\phi_{x_a x_a}(m.T)$$

Precisamos de calcular $\phi_{x_a x_a}(mT)$ $\rightarrow$ $\phi_{x_a x_a}(\tau)\xrightarrow{\tau=mT}\phi_{x_a x_a}(mT)\xrightarrow{m_a=0}\gamma_{x_a x_a}(\tau)$

$\phi_{x_a x_a}(\tau)=\frac{1}{2\pi}\cdot\int_{-\infty}^{+\infty}P_{x_a x_a}(\omega).\,e^{j\omega\tau}.\,d\tau$ ou usar a prop. de TF (dualidade)

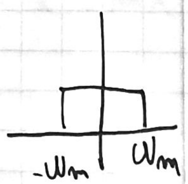
$x(t)\longrightarrow X(\omega)$

$X(t)\longrightarrow 2\pi.X(\omega)$



$\longleftrightarrow 2AT.\,\text{Sinc}\left(\frac{\omega T}{\pi}\right)$

$\frac{2.A.W_m.\,\text{sinc}\left(\frac{T.W_m}{\pi}\right)}{2\pi}\longleftrightarrow 2\pi.X$



$$\phi_{x_a x_a}(\tau)=\frac{W_m}{\pi}.\,\text{Sinc}\left(\frac{\tau.W_m}{\pi}\right) \Rightarrow \phi_{x_a x_a}(mT)=\frac{W_m}{\pi}.\,\text{Sinc}\left(\frac{W_m.\,mT}{\pi}\right)$$

Como $m_{x_a}=0 \Rightarrow \gamma_{x_a x_a}(mT)=\phi_{x_a x_a}(mT)$

Resp

Como a média do P.E é nula

$$\gamma_{x_0 x_0}[m] = \gamma_{x_a x_a}(mT) = \frac{W_m}{\pi} \cdot \text{Sinc}\left(\frac{W_m \cdot mT}{\pi}\right)$$

b) Para que $x[n]$ seja ruído branco ent $\gamma_{xx}[m] = \sigma_x^2 \cdot \delta[m]$

$$\text{Sinc}(x) = \frac{\sin(x\pi)}{x\pi} \Rightarrow \frac{W_m}{\pi} \cdot \text{Sinc}\left(\frac{W_m \cdot mT}{\pi}\right) = \frac{W_m}{\pi} \cdot \frac{\sin\left(\pi \cdot \frac{W_m \cdot mT}{\pi}\right)}{\pi \cdot \frac{W_m \cdot mT}{\pi}}$$

Se e só se $\frac{W_m \cdot T}{\pi} = 1$

$$\Rightarrow T = \frac{\pi}{W_m}$$

$$\phi_{zz}[m] = E\{z[n] \cdot z[n+m]\} = E\{(x[n]+y[n]) \cdot (x[n+m] + y[n+m])\}$$

$$= E\{x[n] \cdot x[n+m] + x[n] \cdot y[n+m] + y[n] \cdot x[n+m] + y[n] \cdot y[n+m]\}$$

$$= \phi_{xx}[m] + 2 \cdot \phi_{xy}[m] + \phi_{yy}[m] \xrightarrow[\text{ñ correlados}]{} \phi_{xy}[m] = m_x \cdot m_y$$

$E\{x[n] \cdot y[n+m]\} =$

$= E\{x[n]\} \cdot E\{y[n+m]\}$

$= m_x \cdot m_y$

$z[n] = x[n] \cdot y[n]$

$$\phi_{zz}[m] = E\{z[n] \cdot z[n+m]\} = E\{(x[n] \cdot y[n]) \cdot (x[n+m] \cdot y[n+m])\}$$

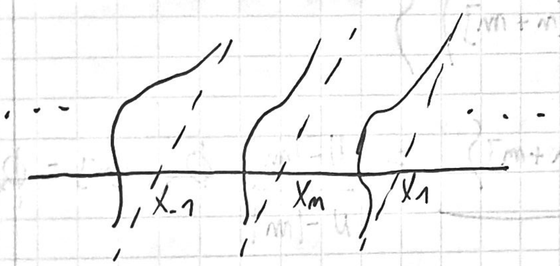
$$= E\{\underbrace{x[n] \cdot x[n+m]} \cdot y[n] \cdot y[n+m]\}$$

• Se x e y são não correlados

$$\phi_{zz}[m] = E\{x[n] \cdot x[n+m]\} \cdot E\{y[n] \cdot y[n+m]\} = \phi_{xx}[m] \cdot \phi_{yy}[m]$$

x = randn(1, 10 000)

y = randn(1, 10 000)



$$\phi_{x_n x_n} = E\{x_n \cdot x^*_{n+m}\}$$

$$\{m_{x_n}, \tilde{\sigma}_{x_n}\} \xrightarrow{\text{estacionária}} (m_x \cdot \sigma_x^2) \xrightarrow{\text{ergódica}} m_x = \langle x \rangle$$

$$m_x = \langle x \rangle = \frac{1}{2N+1} \cdot \sum_{n=-N}^{N} x_n$$

$$\hat{m}_x = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x[n] \Rightarrow E\{\hat{m}_x\} = E\left\{\frac{1}{N} \cdot \sum_{n=0}^{N-1} x[n]\right\}$$

$$B = m_x - E\{\hat{m}_n\} \qquad = \frac{1}{N} \cdot \sum_{n=0}^{N-1} \underbrace{E\{x[n]\}}_{m_x} = \frac{1}{N} \cdot N \cdot m_x = m_x$$

$B \Rightarrow 0$

$$E\{m_x^2\} = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x[n] \cdot \frac{1}{N} \cdot \sum_{k=0}^{N-1} x[k]$$

$$= \frac{N(N-1)}{N} = \frac{N-1}{N}$$

$$\phi_{xx}[m] = E\{x[n] \cdot x[n+m]\}$$

$$\frac{1}{N-|m|}$$

$$\hat{\phi}_{xx}[0] = \frac{1}{4}\left(x^2[0] + x^2[1] + x^2[2] + x^2[3]\right)$$

$$\hat{\phi}_{xx}[1] = \frac{1}{3}\left(x[0] \cdot x[1] + x[1] \cdot x[2] + x[2] \cdot x[3]\right)$$

$$\hat{\phi}_{xx}[2] = \frac{1}{2}\left(x[0] \cdot x[2] + x[1] \cdot x[3]\right)$$

$$\hat{\phi}_{xx}[3] = \frac{1}{1}\left(x[0] \cdot x[3]\right)$$

$$E\{C'_{xx}[m]\} = E\left\{\frac{1}{N-|m|} \cdot \sum_{n=0}^{(N-|m|)-1} \left(x[n] \cdot x[n+m]\right)\right\}$$

$$= \frac{1}{N-|m|} \cdot \sum_{n=0}^{(N-|m|)-1} \underbrace{E\{x[n] \cdot x^*[n+m]\}}_{\phi_{xx}[m]} = \frac{N-|m|}{N-|m|} \phi_{xx}[m] = \phi_{xx}[m]$$

$$B_{C'_{xx}} = \phi_{xx}[m] - E\{C'_{xx}[m]\} = 0$$

$$E\{C_{xx}[m]\} = E\left\{\frac{1}{N} \cdot \sum_{n=0}^{(N-|m|)-1} x[n] \cdot x^*[n+m]\right\}$$

$$= \frac{1}{N} \cdot \sum_{n=0} E\{x[n] \cdot x^*[n+m]\} = \frac{N-|m|}{N} \cdot \phi_{xx}[m]$$

$$B_{C_{xx}} = \phi_{xx}[m] - E\{C_{xx}[m]\} = \phi_{xx}[m] - \frac{N-|m|}{N} \cdot \phi_{xx}[m] =$$

$$= \left[1 - \frac{N-|m|}{N}\right] \cdot \phi_{xx}[m] = +\frac{|m|}{N} \cdot \phi_{xx}[m]$$

$$\lim_{N \to +\infty} \frac{|m|}{N} \cdot \phi_{xx}[m] = 0 \rightarrow \text{consistente.}$$

$$C'_{xx}[m] = \frac{1}{N-|a|} \cdot \sum_{N=0}^{N-|w|-1} x[n] \cdot x[n+m]$$

$$C_{xx}[m] = \frac{1}{N} \sum_{M=0}^{N-|w|-1} x[n] \cdot x[n+m]$$

Sabemos que

$$P_{xx}(\Omega) = T.F\{\phi_{xx}[m]\} = \sum_{m=0}^{+\infty} \phi_{xx}[m] \cdot e^{-j\Omega m}$$

$$I_N(\Omega) = \hat{P}_{xx}(\Omega) = \sum_{m=-\infty}^{+\infty} C_{xx}[m] \cdot e^{-j\Omega m}$$

1-

$$I_N(\Omega) = \sum_{m=-\infty}^{+\infty} \underbrace{\left(\frac{1}{N} \cdot \sum_{n=0}^{N-m-1} x[n] \cdot x[n+m]\right)}_{C_{xx}[m]} \cdot e^{-j\Omega w}$$

$$= \frac{1}{N} \cdot \sum_{n=0}^{N-m-1} x[n] \cdot \sum_{m=-\infty}^{+\infty} x[n+m] \cdot e^{-j\Omega \cdot w}$$

$$= \frac{1}{N} \cdot \sum_{n=0}^{N-|m|-1} x[n] \cdot \underbrace{\sum_{m=-\infty}^{+\infty} x[n+m] \cdot e^{-j\Omega(n+m)} \cdot e^{j\Omega n}}_{X(\Omega)}$$

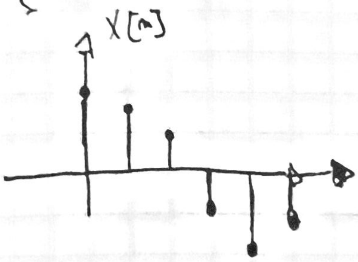
$$= \underbrace{\frac{1}{N} \cdot \sum_{n=0}^{N-|m|-1} \cdot x[n] \cdot e^{j\Omega n}}_{\frac{+\infty}{-\infty} = X(\Omega)} \cdot X(\Omega)$$

$$= \frac{1}{N} \cdot X(-\Omega) \cdot X(\Omega) = \frac{1}{N} \cdot X^*(\Omega) \cdot X(\Omega) = \frac{1}{N} |X(\Omega)|^2$$

$X^*(\Omega)$

~~$X^*(\Omega) = \frac{1}{N} \cdot X^*(\Omega) \cdot X(\Omega) = \frac{1}{N} ($~~

ξ



$C_{xx}[0] = \frac{1}{N} \cdot (x^2[0] + x^2[1] + \dots + x^2[5])$

$C_{xx}[1] = \frac{1}{N} (x[0] \cdot x[1] + x[1] \cdot x[2] + \dots + x[4] \cdot x[5])$

$C_{xx}[5] = \frac{1}{5} (x[0] \cdot x[5])$

$C_{xx}[6] = 0$ X

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Aula 7

Teorema da projeção

$E\left\{ \left(x_N - \sum_{i=1}^{N} a_i \cdot x_{N-i} \right) \cdot x_{N-k} \right\} = 0 \quad k = 1, \dots N$

$E\left\{ x_N \cdot x_{N-k} - \sum_{i=1}^{N} a_i \cdot x_{N-i} \cdot x_{N-k} \right\} = 0$

$\underbrace{E\left\{ x_N \cdot x_{N-k} \right\}}_{\phi_{xx}[k]} - \sum_{i=1}^{N} a_i \cdot E\left\{ x_{N-1} \cdot x_{N-k} \right\} = 0$

c.a.

$N - k - (N-1) =$

$= -k + i$

$\phi_{xx}[k] = \sum_{i=1}^{N} a_i \cdot \phi_{xx}[-k+i]$

$\underline{-k+i} \to -(k-i)$

Par

$N \to N=5$

$\phi_{xx}[2] = \sum_{i=1}^{N} a_i \cdot \phi_{xx}[-2+i]$

$i = 1 \to -2+1 = -1$

$i = 2 \to -2+2 = 0$

$\vdots$

$i = 5 \to -2+5 = 3$

$\phi_{xx}[6] = \sum_{i=1}^{5} a_i \cdot \phi_{xx}(6-i)$

$= a_1 \cdot \phi_{xx}[5] + a_2 \cdot \phi_{xx}[4] + a_5 \cdot \phi_{xx}[1]$

$$\phi_{xx}[k] = \sum_{i=1}^{N} \cancel{\phi}\, a_1 \cdot \phi_{xx}[k-i]$$

$$\phi_{xx}[0] = a_1 \cdot \phi_{xx}[-1] + a_2 \cdot \phi_{xx}[-2] + \dots + a_N \cdot \phi_{xx}[-N]$$

$$\phi_{xx}[1] = a_1 \cdot \phi_{xx}[0] + a_2 \cdot \phi_{xx}[-1] + \dots + a_N \cdot \phi_{xx}[1-N]$$

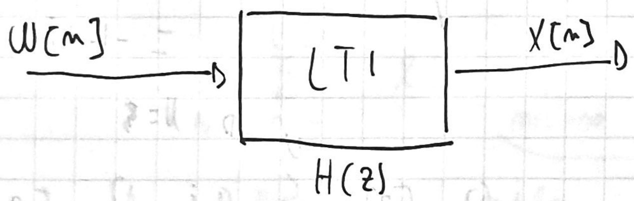
$$\phi_{xx}[2] = a_1 \cdot \phi_{xx}[1] + a_2 \cdot \phi_{xx}[0] + \dots + a_N \cdot \phi_{xx}[2-N]$$

$$\phi_{xx}[3] = a_1 \cdot \phi_{xx}[2] + a_2 \cdot \phi_{xx}[1] + \dots$$

$$\cancel{E\{(x_N - \hat{x}_N)^2\}} =$$

$$E\{x_N(x_N - \hat{x}_N)\} = E\{x_N^2\} - E\{x_N \cdot \sum_{i=1}^{N} a_i \cdot x_{N-i}\} =$$

$$= \phi_{xx}[N] - \sum_{i=1}^{N} a_i \cdot E\{x_N \cdot x_{N-i}\} = \phi_{xx}[0] - \sum_{i=1}^{N} a_i \cdot \phi_{xx}[i]$$



$$H(z) = \frac{A}{1 - B z^{-1}} \Rightarrow$$

$$\Rightarrow \frac{X(z)}{W(z)} = H(z)$$

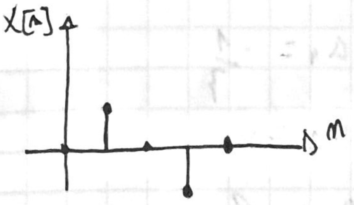
$$\Rightarrow X(z) \cdot [1 - Bz^{-1}] = A W(z)$$

inv Z $\Rightarrow x[n] = B \cdot x[n-1] + A w[n]$

$$\hat{x}[n] = B \cdot x[n-1]$$

Erro ~~análise~~ $= (x[n] - \hat{x}[n]) = A w[n] \Rightarrow$ é ruído branco

Exemplo 76



$N = 5$

a) $Cxx[m] = \frac{1}{N} \sum_{n=0}^{N-|m|-1} \left(x[n] \cdot x[n+m] \right)$

$|m| < 4$

$Cxx[m] = Cxx[-m]$

$Cxx[0] = \frac{1}{5}\left(x[0]^2 + x[1]^2 + x[2]^2 + x[3]^2 + x[4]^2 + x[5]^2 \right) = \frac{2}{5}$

$Cxx[1] = Cxx[-1] = \frac{1}{5}\left(x[0] \cdot x[1] + x[1] \cdot x[2] + x[2] \cdot x[3] + x[3] \cdot x[4] \right) = 0$

$Cxx[2] = Cxx[-2] = \frac{1}{5}\left(x[0] \cdot x[2] + x[1] \cdot x[3] + x[2]\, x[4] \right) = -\frac{1}{5}$

b) $\phi_{xx}(m) = \sum_{i=1}^{M} a_i \cdot \phi_{xx}(m-i) \qquad m = M+1, \ldots, +\infty$

$\phi_{xx}[1] = a_1 \cdot \phi_{xx}(\underset{1-1}{m-i}) + a_2 \cdot \underset{\phi_1}{\phi_{xx}(1-2)} + a_3 \cdot \phi_{xx}[-2] + a_4 \cdot \phi_{xx}[-3]$

$\phi_{xx}[2] = a_1\, \phi_{xx}[1] + a_2 \cdot \phi_{xx}[0] + a_3\, \phi_{xx}[-1] + a_4 \cdot \phi_{xx}[-2]$

$$\begin{bmatrix} \phi_{xx}[0] & \phi_{xx}[1] & \phi_{xx}[2] & \phi_{xx}[3] \\ \phi_{xx}[1] & \phi_{xx}[0] & \phi_{xx}[1] & \phi_{xx}[2] \\ \phi_{xx}[2] & \phi_{xx}[1] & \phi_{xx}[0] & \phi_{xx}[1] \\ \phi_{xx}[3] & \phi_{xx}[2] & \phi_{xx}[1] & \phi_{xx}[0] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \phi_{xx}[1] \\ \phi_{xx}[2] \\ \phi_{xx}[3] \\ \phi_{xx}[4] \end{bmatrix}$$

$\phi_{xx}[3] = a_1\, \phi_{xx}[2] + a_2\, \phi_{xx}[1] + a_3 \cdot \phi_{xx}[0] + a_4 \cdot \phi_{xx}[1]$

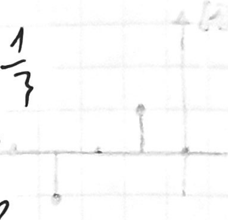
$\phi_{xx}[4] = a_1\, \phi_{xx}[3] + a_2\, \phi_{xx}[2] + a_3 \cdot \phi_{xx}[1] + a_4 \cdot \phi[0]$

$$2\times\begin{cases} 2\cdot a_1 - a_3 = 0 \\ 2\cdot a_2 - a_4 = -1 \\ -a_1 + 2\cdot a_3 = 0 \\ -a_2 + 2a_4 = 0 \end{cases} \iff \begin{cases} 0 \quad - 3\cdot a_3 = 0 \\ 2a_2 - a_4 = -1 \\ -a_1 + 2\cdot a_3 = 0 \\ 2\times \quad -a_2 + 2\cdot a_4 = 0 \end{cases}$$

$a_3 = 0$

$a_4 = -\frac{1}{3}$

$a_1 = 0$



$a_2 = -\frac{2}{3}$

$a_3 = 0$

$3 \cdot a_4 = -1$

$$\hat{x}[n] = \sum_{i=1}^{N} a_i x[n-i]$$

$$\hat{x}[5] = a_1 \cdot x[4] + a_2 \cdot x[3] + a_3 x[2] + a_4 x[1]$$

$$= 0 + -\frac{2}{3}\underbrace{x[4]}_{-1} + 0 + -\frac{1}{3}\cdot(1) = 1/3$$

$$\hat{x}[6] = a_1 \cancel{\hat{x}[5]} + a_2 \cdot x[4] + a_3 \cdot \cancel{x[3]} + a_4 \cdot x[2]$$

$$= -\frac{2}{3}(0) + \left(-\frac{1}{3}\right)\cdot 0 = 0$$

$$\hat{x}[7] = a_1 \cancel{\hat{x}[6]} + a_2 \cdot \hat{x}[5] + a_3 \cdot \cancel{x[4]} + a_4 \cdot x[3]$$

$$= -\frac{2}{3}\cdot\frac{1}{3} + -\frac{1}{3}(-1) = \frac{1}{9}$$

$$\hat{x}[8] = a_1 \cdot \cancel{\hat{x}[7]} + a_2 \cdot \cancel{\hat{x}[6]} + a_3 \cancel{\cdot \hat{x}[5]} + a_4 \cdot \cancel{\hat{x}[4]}$$

$$= 0$$

Erro do Preditor

$$MMSE = \phi_{xx}[0] - \sum_{k=1}^{M} a_k \cdot \phi_{xx}[k]$$

$$= \frac{2}{5} - \underbrace{\sum_{i=1}^{4} a_i \cdot \phi_{xx}(i)}_{a_2 \cdot X(-1/5) + (-1/3) \cdot 0} = \cancel{2/5}\ \frac{2}{5} - \frac{2}{15} = \frac{4}{15}$$

$a_2 \cdot X(-1/5) + (-1/3) \cdot 0$

$-2/3$

---

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4)

Se um processo é autoregressivo de ordem m ent

$$\hat{x}[n] = \sum_{i=1}^{k} a_i \cdot x[n-i] \qquad \phi_{xx}[m] = E\{x[n] \cdot x[n+m]\}$$

$$E\{x[n+m] x[n]\} = E\{\sum_{k=1}^{M} a_k \cdot X[n+k] \cdot K[n+m]\}$$

$n + m - (n - k)$

$m - k$

$$\phi_{xx}[m] = \sum_{k=1}^{M} a_k \cdot \underbrace{E\{x[n-k] \cdot x[n+m]\}}_{\phi_{xx}[m+k]}$$

$$\phi_{xx}[m] = \sum_{k=1}^{M} a_k \cdot \phi_{xx}[m+k] \qquad X$$

1º $E\{x[n-m] \cdot x[n]\} = E\{\sum_{k=1}^{M} a_k \cdot x[n-k] \cdot x[n-m]\}$

Par

$$\phi_{xx}[m] = \sum_{k=1}^{M} a_k \cdot E\{x[n-k] \cdot x[n-m]\}$$

$\cancel{n} - m - (\cancel{n} - k)$

$k - m \Rightarrow m - k$

Seq de autocor é par

$$\phi_{xx}[m] = \sum_{k=1}^{M} a_k \cdot \phi_{xx}(m-k)$$

$\phi(0) \cdot a_1 = \phi(1)$

$2 \cdot a_1 = 1 \Rightarrow \boxed{a1 = 1/2}$

b) $\phi[2] = a_1 \cdot \phi[1] = \frac{1}{2} \cdot 1 = 1/2$

c) $\phi[0] = 2$, $\phi(1) = 1$, $\phi(2) = \frac{1}{2}$

Preditor linear de ordem 2

$$\begin{bmatrix} \phi_{xx}[0] \cdot \phi_{xx}[1] \\ \phi_{xx}[1] \cdot \phi_{xx}[0] \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \phi_{xx}[1] \\ \phi_{xx}[2] \end{bmatrix} \Rightarrow \begin{cases} 2a_1 + a_2 = 1 \\ 1 \cdot a_1 + 2 \cdot a_2 = \frac{1}{2} \end{cases}$$

$-2\times$

$0 \cdot a_1 - 3 \cdot a_2 = 0 \Rightarrow a_2 = 0$

$a_1 + 0 = \frac{1}{2} \Rightarrow a_1 = \frac{1}{2}$

$$C_{xx}[m] = \frac{1}{N} \cdot \sum_{n=0}^{N-|m|-1} x[n] \cdot x[n+m]$$

$$E\{C_{xx}[m]\} = \frac{1}{N} \cdot \sum_{n=0}^{N-|m|-1} \phi_{nn}[m]$$

$$= \frac{1}{N} \cdot (N - |m|) \cdot \phi_{nn}[m]$$

$$= \frac{N-|m|}{N} \cdot \phi_{nn}[m]$$

$$C'_{nn}[m] = \frac{1}{N-|m|} \cdot \sum x[n] x[n+m]$$

$$E\{C'_{nn}[m]\} = \phi_{xx}[m]$$

Diana predictions canada

Considere um sinal discreto $x[n]$ de comprimento N, estacionário de média nula com características de ruído branco, ou seja:

$$E\{x[n].x[m]\} = \sigma_n^2 \delta[n-m]$$

a) Mostre que a média da DTFT de $x[n]$ é nula.

b) Determine a correlação conjugada entre os valores da DTFT de $x[n]$.

a) $$X(k) = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x[n] \cdot e^{-jk\frac{2\pi}{N} \cdot n} \quad k = 0, 1, \dots, N-1$$

$$E\{X(k)\} = \frac{1}{N} \sum_{n=0}^{N-1} \underbrace{E\{x[n]\}}_{\text{média}=0} \cdot e^{-jk\frac{2\pi}{N} \cdot n} = 0$$

b) $$E\{X_k \cdot X_n^*\} \rightarrow E\{X(k) \cdot X^*(n)\} = E\left\{\frac{1}{N} \cdot \sum_{n_1=0}^{N-1} x[n_1] \cdot e^{-jk\frac{2\pi}{N} n_1} \cdot \frac{1}{N} \cdot \sum_{n_2=0}^{N-1} x[n_2] \cdot e^{-jn\frac{2\pi}{N} n_2}\right\}$$

$$= \frac{1}{N^2} \cdot \sum_{n} \sum_{n_2} \underbrace{E\{x[n_1] \cdot x[n_2]\}}_{\sigma_x^2 \cdot \delta[n_2-n_1]} e^{-j\frac{2\pi}{N} k n_1} \cdot e^{-j\frac{2\pi}{N} \cdot n \cdot n_2}$$

$\sigma_x^2 \cdot \delta[n_2 - n_1]$ se $n_1 = n_2$

0 se $n_1 != n_2$

$\delta[0] = 1$

$\delta[1] = 0$

$\delta[2] = 0$

$$= \frac{1}{N^2} \cdot \underbrace{\sum_{n} \sigma_x^2 \cdot e^{-j\frac{2\pi}{N} n(k-n)}}_{\sum_{n=0}^{N-1} 1 = N}$$

$\rightarrow$ se $k - n != 0 \Rightarrow$

$\Rightarrow E\{X(k) \cdot X^*(n)\} = 0 \rightarrow$ não existe correlação

$\rightarrow$ se $k - n = 0 \Rightarrow$

$=$

$$\Rightarrow E\{X(k) \cdot X^*(n)\} = \frac{\sigma_x^2}{N^2} \cdot N$$

$$= \frac{\sigma_x^2}{N}$$

Extra

$m_x \neq 0$

$$E\{X(0)\} = \frac{1}{N} \cdot \sum E\{x[n]\} \cdot \underbrace{e^{0}}_{1}$$

$$= m \cdot X$$

Considere um sinal discreto ~~[illegible]~~

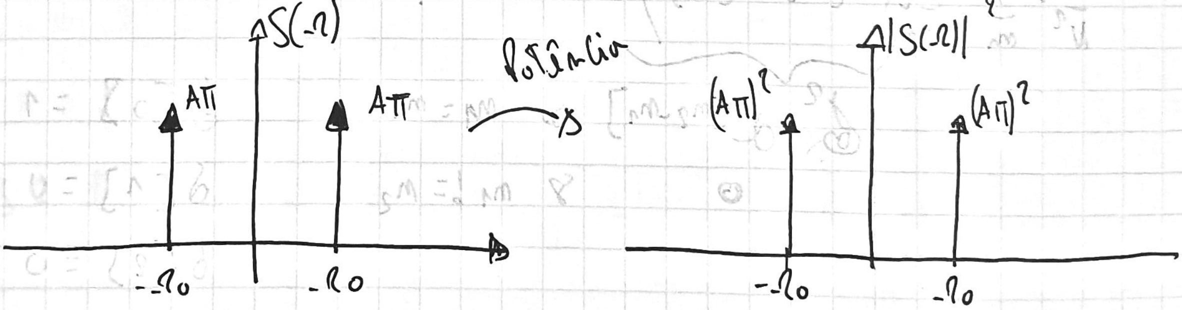
$$s[n] = A \cdot \cos(\Omega_0 n + \varphi)$$

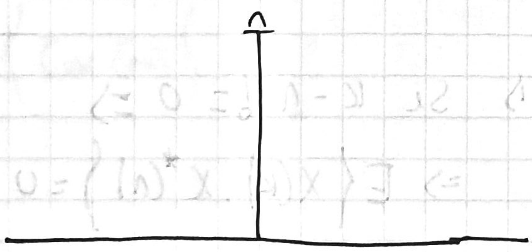
Com φ como uma variável aleatória com distribuição uniforme no intervalo $[0, 2\pi]$

mostre que

$$P_{ss}(\Omega) = \frac{A^2}{2} \cdot \pi\, \delta(\Omega - \Omega_0)$$

$$A \cdot \cos(\Omega_0 n + \varphi) \leftrightarrow A\pi\left[\delta[\Omega - \Omega_0] \cdot e^{j\varphi} + \delta[\Omega - \Omega_0] \cdot e^{-j\omega}\right]$$





$$P_{xx}(\Omega) = \frac{1}{N} \cdot \left(X(\Omega)\right)^2$$

$$P_{ss}(\Omega) = \frac{1}{2\pi} \cdot |S(\Omega)|^2 = \frac{(A\pi)^2}{2\pi} \cdot \frac{A^2}{2} \cdot \pi \cdot \delta(\Omega - \Omega_0)$$

$$\phi_{ss}[m] = E\{ s[n] \cdot s[n+m] \} = E\{ A \cdot \cos(\Omega_0 n + \varphi) \cdot A \cdot \cos(\Omega_0 (n+m) + \varphi) \}$$

$$= A^2 \cdot E\{ \underbrace{\cos(\Omega_0 n + \varphi)}_{a} \cdot \underbrace{\cos(\Omega_0 (n+m) + \varphi)}_{b} \}$$

$\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$

$\cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$

$\frac{\cos(a+b) + \cos(a-b)}{2} = \cos(a) \cdot \cos(b)$

$$= \frac{A^2}{2} \cdot E\{ \cos((\Omega_0 n + \varphi) + (\Omega_0 (n+m) + \varphi)) + \cos((\Omega_0 n + \varphi) - (\Omega_0 (n+m) + \varphi)) \}$$

$$= \frac{A^2}{2} \cdot E\{ \cos(2\Omega_0 n + \Omega_0 m + 2\varphi) + \cos(-\Omega_0 m \cancel{+2\varphi}) \}$$

$$= \frac{A^2}{2} \cdot \left(E\{ \cos(2\Omega_0 n + \Omega_0 m + 2\varphi) \} + E\{ \cos(\Omega_0 m) \} \right) = \frac{A^2}{2} \cdot \cos(\Omega_0 m)$$

$$P_{ss}(\Omega) = T.F\{ \phi_{xx}[m] \} = \frac{A^2}{2} \cdot \pi \cdot \delta[\Omega - \Omega_0]$$

Filtragem Adaptativa (Wiener)

$x[n] = s[n] + v[n]$ => Sinal e o ruído não são correlados

$w_x = w_s + w_v$ <= sempre

$$\phi_{xx}[m] = E\{x[n] . x[n+m]\} = E\{(s[n] + v[n]).(s[n+m] + v[n+m])\}$$

$$= E\{s[n].s[n+m] + s[n].v[n+m] + v[n].s[n+m] + v[n].v[n+m]\}$$

Como a média da soma é igual à soma das médias

$$\phi_{xx}[m] = \phi_{ss}[m] + \underbrace{\phi_{sv}[m] + \phi_{vs}[m]}_{2.\phi_{sv}[m]} + \phi_{vv}[m]$$

Como o sinal e ruído não são correlados => $\phi_{sv}[m] = m_s . \cancel{m_v}^{=0}$

Se a média do ruído é zero => $\phi_{xx}[m] = \boxed{\phi_{ss}[m] + \phi_{vv}[m]}$

$$P_{xx}(\Omega) = T.F\{\phi_{xx}[m]\} = \sum_{m=-\infty}^{+\infty} \phi_{xx}[m].e^{-j\Omega m}$$

$$P_{xx}(\Omega) = P_{ss}(\Omega) + P_{vv}(\Omega)$$

$$P_{xx}(\Omega) \longrightarrow \boxed{H(\Omega)} \longrightarrow P_{ss}(\Omega)$$

$$H(\Omega) = \frac{P_{ss}(\Omega)}{P_{ss} + P_{vv}(\Omega)}$$ = Constante <= Estacionaridade X

$$\int_{-\pi}^{\pi} P_{xx}(\Omega).d\Omega = \sigma_x^2 + w_x^2 = E\{x^2[x]\}$$

1º hipòtesi ⇒ m_s i $m_v = 0$

$$H(\Omega) = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_v^2} \Rightarrow h[n] = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_v^2} \cdot \delta[n]$$

$$s[n] = x[n] * h[n]$$

$$s[n] = m_x + \left[(x[n] - m_x) \right] * \frac{\sigma_s^2}{\sigma_s^2 + \sigma_v^2} \cdot \delta[n]$$

$$\boxed{s[n] = m_x + (x[n] - m_x) \cdot \frac{\sigma_s^2}{\sigma_s^2 + \sigma_v^2}}$$

Implementació → σ_s^2 ? ⇒ $\sigma_s^2 = \sigma_x^2 - \sigma_v^2$ → Variància de soroll

hem d'estimar (?)

Estimem un inici de array on encara no comencen a parlar

Teste 2019 - 2020

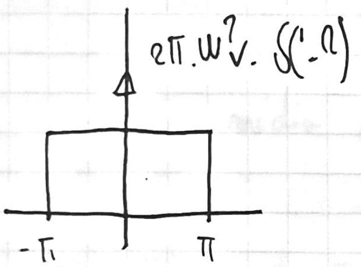
1-a)

Família de variáveis aleatórias não correlacionadas - Ruído branco

Transformada de Fourier constante

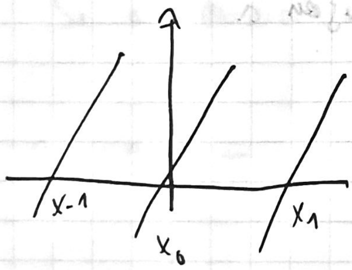
$$\phi_{vv}[m] = \sigma_x^2 \cdot \delta[m] + w_v^2$$

$$P_{xx}(\Omega) = T.F\{\phi_{xx}[m]\}$$



Estacionaridade → parâmetros estatísticos (m, σ_v^2) constantes e $\phi_{vv}[m]$ só depende de m.

Ergodicidade → A estatística de conjunto é igual à estatística temporal



A previsibilidade prática é nenhuma.

$$\gamma_{xx}[m] = E\{(x[n] - m_x).(x[n+m] - m_x)\}$$

$$= E\{x[n].x[n+m] - x[n].m_x - m_x.x[n+m] + m_x^2\}$$

a média da soma = soma das médias

$$\gamma_{xx}[m] = \phi_{xx}[m] - w_x^2 \quad c/ \quad \phi_{xx}[m] = \sigma_x^2 \cdot \delta[m] + m_x^2$$

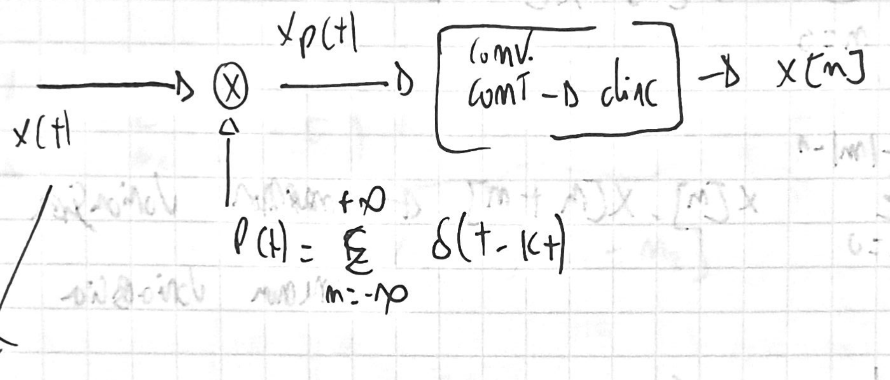
$$\gamma_{xx}[m] = \sigma_x^2 \cdot \delta[m]$$

como a média é nula então

$$\gamma_{xx}[m] = \phi_{xx}[m]$$

---

b)



$m_x = 0$

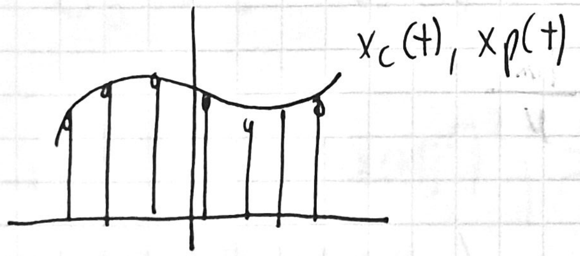
$$\gamma_{xx}(\tau) = E\{x_c(t) \cdot x_c(t-\tau)\} \Rightarrow \phi_{xx}[m] = E\{x[n] \cdot x[n+m]\}$$

$$\gamma_{x_c x_c}(n \cdot T) = E\{x_c(nT) \cdot x_c((n+m) \cdot T)\}$$

$$= \gamma_{xx}[m]$$

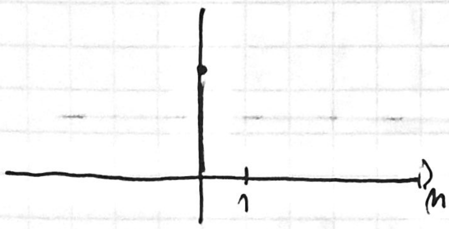
devida à amostragem temos

$$x_p(t) = x_c(n \cdot T)$$



$$x[n] = x_c(n \cdot T)$$

$$\gamma_{xx}[m] = \gamma_{x_c x_c}(m \cdot T)$$



c) Tomam W.N de $m_x = 0$

$$\Rightarrow \gamma_{xx}[m] = \phi_{xx}[m] = \sigma_x^2 \cdot \delta[m]$$

Estudo-mos e estimativa da representação da autocorrelação

$$c'_{xx}[m] = \frac{1}{N-|m|} \cdot \sum_{n=0}^{N-|m|-1} x[n] \cdot x[n+m]$$

$$c_{xx}[m] = \frac{1}{N} \cdot \sum_{n=0}^{N-|m|-1} x[n] \cdot x[n+m]$$ ← menor variância

menor variância

$$B_{cxx} = \phi_{xx}[m] - E\{c_{xx}[m]\}$$

$$E\{c_{xx}[m]\} = \frac{1}{N} \cdot \sum_{n=0}^{N-|m|-1} \underbrace{E\{x[n] \cdot x[n+m]\}}_{\phi_{xx}[m]} = \phi_{xx}[m] \cdot \frac{N-|m|}{N}$$

$$B_{cxx} = \phi_{xx}[m] \cdot \left[1 - \frac{N-|m|}{N}\right] = \phi_{xx}[m] \cdot \frac{|m|}{N}$$

A estimativa é consistente porque $\lim_{N \to +\infty} B_{cxx} = \emptyset$

---

d) $I_N(\Omega) = T.F.\{c_{xx}[m]\} = |X(\Omega)|^2$

$$E\{|X(\Omega)|^2\} = E\{X(k) \cdot X^*(k)\}^N$$ c/ $\boxed{X(k) = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x[n] \cdot e^{-jk\frac{2\pi}{N} \cdot n}}$

DFT

(1) → $X(k) = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x[n] \cdot e^{-jk\frac{2\pi}{N} \cdot n}$

$$x[n] = \sum_{k=0}^{N-1} X(k) \cdot e^{jk\frac{2\pi}{N} \cdot n}$$

$$\text{DFT (2)} \to X(k) = \sum_{m=0}^{N-1} x[m] \cdot e^{-jk\frac{2\pi}{N}\cdot m}$$

$$x[m] = \frac{1}{N} \cdot \sum_{k=0}^{N-1} X(k) \cdot e^{jk\cdot\frac{2\pi}{N}\cdot m}$$

$$E\{|X(k)|^2\} = E\left\{\frac{1}{N} \cdot \sum_{m_1=0}^{N-1} x[m_1] \cdot e^{-jk\frac{2\pi}{N}\cdot m_1} \cdot \frac{1}{N} \cdot \sum_{m_2=0} x[m_2] \cdot e^{jk\cdot\frac{2\pi}{N}\cdot m_2}\right.$$

$$= \frac{1}{N^2} \sum_{m_1}\sum_{m_2} E\left\{\underbrace{x[m_1] \cdot x[m_2]}_{\sigma_x^2 \cdot \delta[m_1 - m_2]} \cdot e^{-jk\frac{2\pi}{N}(m_1 - m_2)}\right\}$$

$$= \frac{\sigma_x^2}{N^2} \cdot \underbrace{\sum_m (1)}_{N} = \frac{\sigma_x^2}{N}$$

---

e)

$$y = |X(k)|^2$$

$$\sigma_y^2 = ? \Rightarrow \sigma_y^2 = E\{y^2\} - m_y^2 \Rightarrow \boxed{m_y = \frac{\sigma_x^2}{N}}$$

$$E\{y^2\} = E\{(|X(k)|^2)^2\} = E\{|X(k)|^4\}$$

$$= E\left\{\left(\frac{1}{N} \cdot \sum_{m_1=0}^{N-1} x_1[m_1] \cdot e^{-jk\cdot\frac{2\pi}{N}\cdot m_1}\right) \cdot \left(\frac{1}{N} \cdot \sum_{m_2=0} x[m_2] \cdot e^{jk\frac{2\pi}{N}\cdot m_2}\right) \cdot\right.$$

$$\cdot \left(\frac{1}{N} \cdot \sum_{m_3=0}^{N-1} x[m_3] \cdot e^{-jk\cdot\frac{2\pi}{N}\cdot m_3}\right) \cdot \left(\frac{1}{N} \cdot \sum_{m_4=0}^{N-1} x[m_4] \cdot e^{jk\frac{2\pi}{N}\cdot m_4}\right)$$

$$= \frac{1}{N^4} \cdot \sum_{m_1}\sum_{m_2} \cdot E\left\{\underbrace{x[m_1] x[m_2]}_{\sigma_x^2 \cdot \delta[m_1 - m_2]} \cdot e^{-jk\frac{2\pi}{N}(m_1 - m_2)}\right\} \cdot \sum_{m_3}\sum_{m_4} \cdot E\left\{\underbrace{x[m_3] \cdot x[m_4]}_{\sigma_x^2 \cdot \delta[m_3 - m_4]} \cdot e^{-jk\frac{2\pi}{N}(m_3 - m_4)}\right\}.$$

$$= \frac{\sigma_x^4}{N^4} \cdot \left[\sum_{m_1}\sum_{m_2} 1 + \sum_{m_3}\sum_{m_4} 1\right] = \frac{2 \cdot \sigma_x^4 \cdot N^2}{N^4} = \frac{2\sigma_x^4}{N^2}$$

$$\sigma_y^2 = E\{y^2\} - m_y^2 = \frac{2 \cdot \sigma_x^4}{N^2} - \left(\frac{\sigma_x^2}{N}\right)^2 = \frac{\sigma_x^4}{N^2}$$

---

e.)

Autocorrelação do $|X(k)|^2$

$$E\{|X(k)|^2 \cdot |X(n)|^2\}$$

$$E\left\{\left(\frac{1}{N} \cdot \sum_{m_1=0}^{N-1} x[m_1] \cdot e^{-jk\frac{2\pi}{N} \cdot m_1}\right) \cdot \left(\frac{1}{N} \cdot \sum_{m_2=0}^{N-1} x[m_2] \cdot e^{jk\frac{2\pi}{N} \cdot m_2}\right)\right.$$

$$\left. \cdot \left(\frac{1}{N} \cdot \sum_{m_3=0}^{N-1} x[m_3] \cdot e^{-jn\frac{2\pi}{N} m_3}\right) \cdot \left(\frac{1}{N} \cdot \sum_{m_4=0}^{N-1} x[m_4] \cdot e^{jn\frac{2\pi}{N} \cdot m_4}\right)\right\}$$

$$= \frac{1}{N^4} \cdot \sum_{m_1} \sum_{m_2} \sum_{m_3} \sum_{m_4} \cdot \underbrace{E\{x[m_1] \cdot x[m_2] \cdot x[m_3] \cdot x[m_4]\}}_{\sigma_x^4 [\delta[m_1 - m_2] \cdot \delta[m_3 - m_4]]} \cdot e^{-jk\frac{2\pi}{N}(m_1 - m_2)} \cdot e^{-jn\frac{2\pi}{N} \cdot (m_3 - m_4)}$$

$$= 2 \cdot \frac{\sigma_x^4}{N^4} \cdot \sum_{m_1} \sum_{m_2} \sum_{m_3} \sum_{m_4} \cdot e^{-j\left(\frac{2\pi}{N}(m_1 - m_3)(k + n)\right)} \cdot e^{-j\frac{2\pi}{N}(m_2 - m_4) \cdot (k + n)}$$

$$= 2 \cdot \frac{\sigma_x^4}{N^4} \underbrace{\sum_{m_1} \cdot \sum_{m_4} e^{-j(k-n) \cdot (m_1 + m_4) \cdot \frac{2\pi}{N}} \cdot \sum_{m_2} \sum_{m_3} e^{-j(k-n) \cdot (m_2 + m_3) \cdot \frac{2\pi}{N}}}$$

$\hookrightarrow k = n \Rightarrow 1$

$k \neq n \Rightarrow \emptyset$

$$= \frac{2\sigma_x^4}{N^4} \cdot \delta(k - n)$$