Transformada de Z

24. Calcular a transformada z das seguintes sequências:

a) $h(n) = \begin{cases} \dfrac{1}{3} & ; -1 \le n \le 1 \\ 0 & ; \ - \end{cases}$

b) $h(n) = \begin{cases} a^n & ; n \ge 0 \\ 0 & ; \ - \end{cases}$

c) $h(n) = \begin{cases} r^n \cos(\omega_0 n) & ; n \ge 0 \\ 0 & ; \ - \end{cases}$

25. Calcule, aplicando as propriedades da transformada z, a saída do sistema.

a) $y(n) = \dfrac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]$

b) $y(n) = x(n) + a\,y(n-1)$

c) $y(n) = a_1 y(n-1) + a_2 y(n-2) + b_0 x(n) + b_1 x(n-1)$

26. Calcular a região de convergência e a transformada de z para as seguintes funções:

a) $\delta(n)$

b) $-\left(\dfrac{1}{2}\right)^n u(-n-1)$

c) $\left(\dfrac{1}{2}\right)^n u(-n)$

d) $\delta(n-1)$

e) $\delta(n+1)$

f) $\left(\dfrac{1}{2}\right)^n \left[u(n) - u(n-10)\right]$

27. Analise o seguinte sistema quanto à estabilidade:

$$y(n) = x(n) + a\,y(n-1)$$

28. Determine a região de convergência e a resposta impulsional do sistema discreto com função de transferência:

$$H(z) = \frac{1 - z^{-1}}{1 - 2z^{-1} + 0.75z^{-2}}$$

a) De modo que o sistema seja causal.

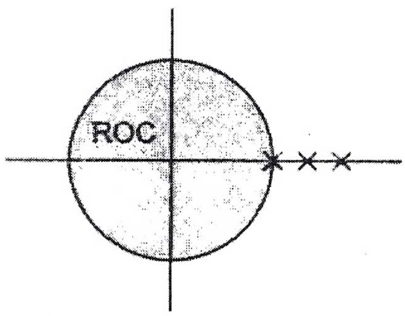
b) De modo que o sistema seja estável.

29. Calcular a transformada inversa de z da seguinte função limitada à esquerda:

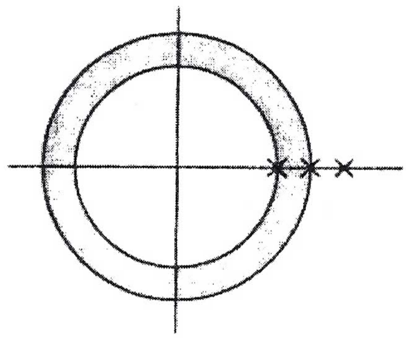
$$X(z) = \frac{-8z^{-1} + 18}{z^{-2} - 5z^{-1} + 6}$$

30. Sendo $X(z) = \dfrac{z}{z - \frac{1}{2}} + \dfrac{z}{z - \frac{1}{3}} + \dfrac{z}{z - \frac{1}{4}}$, calcule a transformada inversa de z nos quatro casos seguintes:

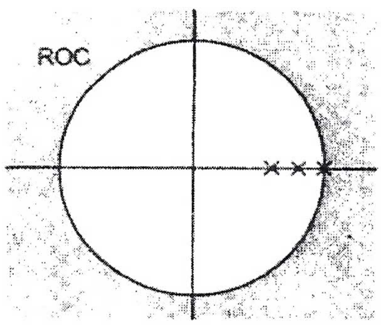
a)



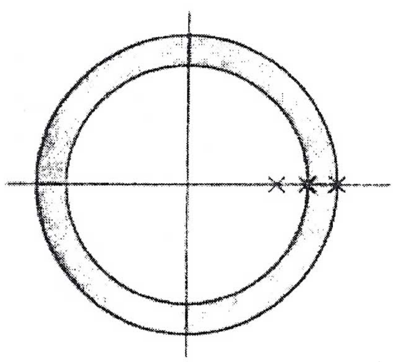
c)



b)



d)



Processamento Digital de Sinal

EXERCICIOS 2008/2009

TRANSFORMADA DE Z

24 Calcular a transformada de Z das seguintes sequências:

a) $h(n) = \begin{cases} 1/3 \; ; & -1 \leq n \leq 1 \\ 0 \; ; & \text{---} \end{cases}$ $= \frac{1}{3}\left(z + 1 + z^{-1}\right) \;, \; z \neq 0$

$$H(z) = \sum_{-\infty}^{+\infty} \frac{1}{3} z^{-n} = \frac{1}{3} z^{-1} + \frac{1}{3} z^{0} + \frac{1}{3} z^{1} \;; \; z \neq 0$$

b) $h[n] = \begin{cases} a^n \; ; & n \geq 0 \\ 0 \; ; & n < 0 \end{cases}$

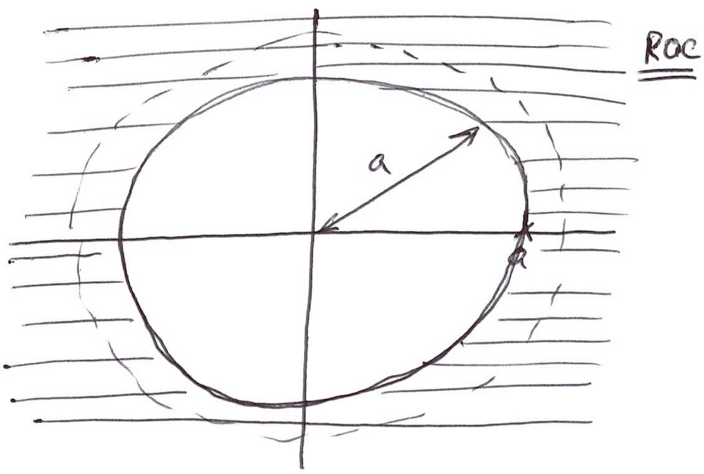
$$H(z) = \sum_{n=-\infty}^{+\infty} h(n)\, z^{-n}$$

$$\sum_{n=N_0}^{N} r^n = \frac{r^{N_0} - r^{N+1}}{1 - r}$$

$|r| < 1$

$$H(z) = \sum_{n=0}^{+\infty} a^n \cdot z^{-n} = \sum_{n=0}^{+\infty} \left(a z^{-1}\right)^n =$$

$$= \frac{\cancel{(a z^{-1})^{0}} - \cancel{(a z^{-1})^{\infty}}}{1 - a z^{-1}} = \frac{1}{1 - a z^{-1}} = \frac{z}{z - a}$$



causal

$|a z^{-1}| < 1 \iff |z| > |a|$

Para $|a| < 1$ o sistema é estável.

$\boxed{e)}$ $h[n] = \begin{cases} r^n \cos(w_0 n) & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$

$$\cos(w_0 n) = \frac{e^{jw_0 n} + e^{-jw_0 n}}{2}$$

$$H(z) = \sum_{n=0}^{+\infty} r^n \cos(w_0 n) z^{-n} = \sum_{0}^{+\infty} (r z^{-1})^n \cos(w_0 n) =$$

$$= \sum_{0}^{+\infty} \frac{1}{2}(r z^{-1})^n \left(e^{jw_0 n} + e^{-jw_0 n}\right) = \frac{1}{2}\sum_{0} (r z^{-1}) e^{jw_0 n} + \frac{1}{2}\sum_{0}^{+\infty} (r z^{-1}) e^{-jw_0 n} =$$

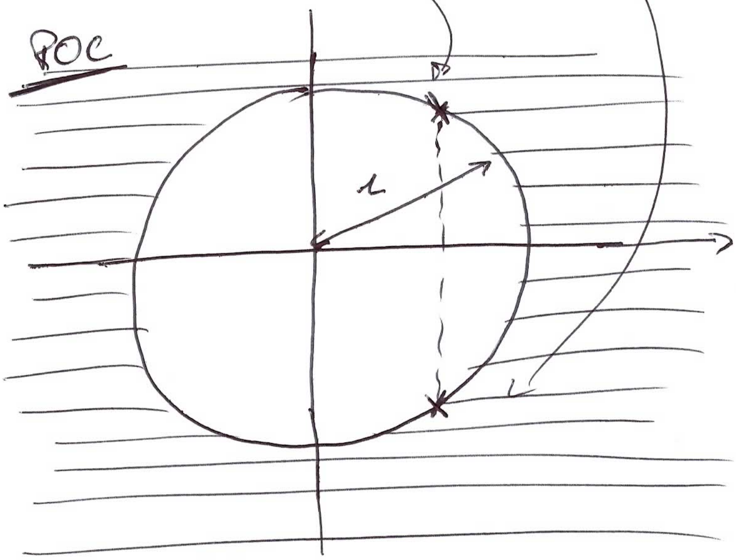
$$= \frac{1}{2}\sum_{0}^{+\infty}(r z^{-1} e^{jw_0})^n + \frac{1}{2}\sum (r z^{-1} e^{-jw_0})^n =$$

$$= \frac{1}{2}\frac{1}{1 - r z^{-1} e^{jw_0}} + \frac{1}{2}\frac{1}{1 - r z^{-1} e^{-jw_0}} = \underbrace{\frac{1/2\, z}{z - r e^{jw_0}}}_{} + \underbrace{\frac{\frac{1}{2} z}{z - r e^{-jw_0}}}_{}$$

$z - re^{jw_0} = 0 \ \wedge \ z - re^{-jw_0} = 0$

$\Leftrightarrow z = re^{jw_0} \ \wedge \ z = re^{-jw_0}$

$|z| > |r|$



25 Calcule, aplicando as propriedades da Transformada de Z, a saída do sist.

a) $y[n] = \frac{1}{3}\left[x[n-1] + x[n] + x[n+1]\right]$

Propriedades úteis

$x[n] \longleftrightarrow X(z)$

$a x_1[n] + b x_2[n] \longleftrightarrow a X_1(z) + b X_2(z)$ LINEARIDADE

$x[n-n_0] \longleftrightarrow X(z)\cdot z^{-n_0}$ ATRASO NO TEMPO

$$Y(z) = \frac{1}{3}X(z)z^{1} + \frac{1}{3}X(z)z^{0} + \frac{1}{3}X(z)z^{-1}$$

$$= \frac{1}{3}X(z)\left(z + 1 + z^{-1}\right) \Longrightarrow \frac{Y(z)}{X(z)} = \frac{1}{3}\left(z + 1 + z^{-1}\right)$$

b) $y[n] = x[n] + a\cdot y[n-1]$

$$\updownarrow$$

$$Y(z) = X(z) + a\, Y(z) z^{-1} \Leftrightarrow$$

$$\Leftrightarrow Y(z) - a\, Y(z) z^{-1} = X(z) \Leftrightarrow$$

$$\Leftrightarrow Y(z)\left(1 - a z^{-1}\right) = X(z) \Leftrightarrow$$

$$\Leftrightarrow \frac{Y(z)}{X(z)} = \frac{1}{1 - a z^{-1}} \Leftrightarrow \boxed{\frac{Y(z)}{X(z)} = \frac{z}{z-a}}$$

$\boxed{c)}$ $y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n] + b_1 x[n-1]$

$$Y(z) = a_1 Y(z) z^{-1} + a_2 Y(z) z^{-2} + b_0 X(z) + b_1 z^{-1} X(z) \Leftrightarrow$$

$$\Leftrightarrow Y(z) - a_1 Y(z) z^{-1} - a_2 Y(z) z^{-2} = b_0 X(z) + b_1 z^{-1} X(z) \Leftrightarrow$$

$$\Leftrightarrow Y(z)\left(1 - a_1 z^{-1} - a_2 z^{-2}\right) = X(z)\left(b_0 + b_1 z^{-1}\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

$\boxed{26}$ Calcular a região de convergência e a transformada de z para as seguintes funções

$\boxed{a)}$ $\delta(n)$

$$\Delta(z) = \sum_{-\infty}^{+\infty} \delta(n) z^{-n} = 1 \qquad \delta(n) = \begin{cases} 1 ; & n=0 \\ 0 ; & \text{—} \end{cases}$$

$ROC \Rightarrow \forall z$

$\boxed{d)}$ $\delta(n-1)$

$$\Delta(z) = \sum_{-\infty}^{+\infty} \delta(n-1) z^{-n} = z^{-1} = \frac{1}{z} \qquad \delta(n-1) = \begin{cases} 1 ; & n=1 \\ 0 ; & \text{—} \end{cases}$$

$ROC \Rightarrow z \neq 0$

$\boxed{e)}$ $\delta(n+1)$

$$\Delta(z) = \sum_{-\infty}^{+\infty} \delta(n+1) z^{-n} = z \qquad \delta(n+1) = \begin{cases} 1 ; & n=-1 \\ 0 ; & \text{—} \end{cases}$$

$ROC \Rightarrow |z| < \infty$

$\boxed{27}$ Analise o sistema quanto à estabilidade

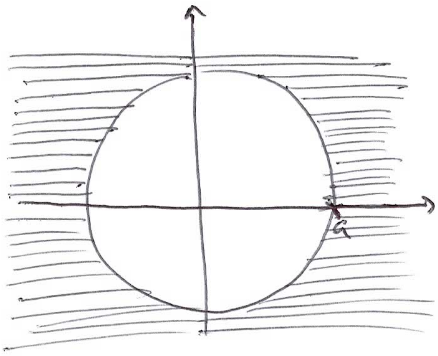
$$y[n] = x[n] + a\, y[n-1]$$

$$Y(z) = X(z) + a\,Y(z)\,z^{-1} \Leftrightarrow Y(z) - a\,Y(z)\,z^{-1} = X(z) \Leftrightarrow$$

$$\Leftrightarrow \frac{Y(z)}{X(z)} = \frac{1}{1 - a z^{-1}} = \frac{z}{z-a}$$

$\Rightarrow$ Se for causal

$|z| > |a|$

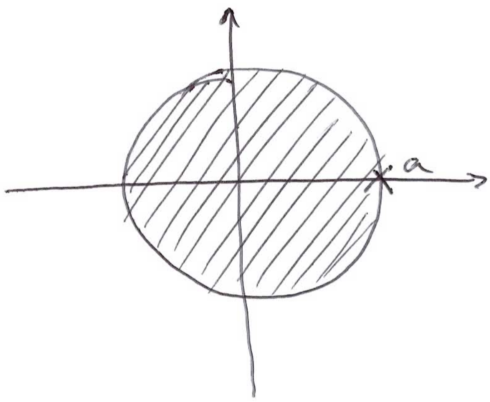


$a < 1$ estável

$a > 1$ instável

$\Rightarrow$ Se for não causal.

$|z| < |a|$



$a > 1$: estável

$a < 1$: instável

$\boxed{28}$ Determine a região de convergência e a resposta impulsional do sistema discreto com função de transferência

$$H(z) = \frac{1 - z^{-1}}{1 - 2z^{-1} + 0,75z^{-2}}$$

a) De modo que o sistema seja causal.

b) De modo que o sistema seja estável.

$\boxed{a)}$

$x = z^{-1}$

$1 - 2x + 0,75x^2 = 0 \Leftrightarrow x = 2 \quad \vee \quad x = 2/3$

$$\frac{4/3(1-x)}{(x-2)(x-2/3)} = \frac{A}{(x-2)} + \frac{B}{(x-2/3)}$$

$\Leftrightarrow A\left(x - \frac{2}{3}\right) + B(x-2) = (1-x)\frac{4}{3} \Leftrightarrow$

$\Leftrightarrow Ax - A\frac{2}{3} + Bx - B2 = \frac{4}{3} - \frac{4}{3}x \Leftrightarrow$

$$\Leftrightarrow \begin{cases} A + B = -4/3 \\ -\frac{2A}{3} - 2B = 4/3 \end{cases} \Leftrightarrow \begin{array}{l} 2A + 2B = -8/3 \\ -\frac{2A}{3} - 2B = 4/3 \quad (+) \\ \hline \frac{4}{3}A = -\frac{4}{3} \Leftrightarrow \boxed{A = -1} \end{array}$$

$-\frac{2}{3}\cdot(-1) - 2B = 4/3 \Leftrightarrow -2B = \frac{2}{3} \Leftrightarrow \boxed{B = -1/3}$

então:

$$H(z) = -\frac{1}{z^{-1}-2} - \frac{\frac{1}{3}}{z^{-1}-2/3} = \cancel{-}\frac{1}{\cancel{-}2\left(1-\frac{z^{-1}}{2}\right)} - \frac{1/3}{-\frac{2}{3}\left(1-\frac{3}{2}z^{-1}\right)} =$$

$$= \frac{1/2}{1-\frac{z^{-1}}{2}} + \frac{1/2}{1-\frac{3}{2}z^{-1}}$$

<u>Causal:</u>

$$x[n] = \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(\frac{3}{2}\right)^n u[n] = \frac{1}{2}u[n]\left[\left(\frac{1}{2}\right)^n + \left(\frac{3}{2}\right)^n\right]$$

Processamento Digital de Sinal

EXERCICIOS 2008/2009

29 Calcular a transformada inversa de z da seguinte função limitada à esquerda.

$$X(z) = \frac{-8z^{-1}+18}{z^{-2}-5z^{-1}+6}$$

Considere-se o sistema causal

$x = z^{-1}$

$x^2 - 5x + 6 = 0 \Leftrightarrow x = 3 \vee x = 2$

$$\frac{-8x+18}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

	$x[n]$	$X(z)$	ROC
CAUSAL	$a^n u[n]$	$\frac{1}{1-az^{-1}}$ $\frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
$\overline{\text{CAUSAL}}$	$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$

$A(x-2) + B(x-3) = -8x + 18 \Leftrightarrow$

$\Leftrightarrow Ax - 2A + Bx - 3B = -8x + 18 \Leftrightarrow \begin{cases} A + B = -8 \\ -2A - 3B = 18 \end{cases} \Leftrightarrow \begin{cases} 2A + 2B = -16 \\ -2A - 3B = 18 \end{cases} +$

$-1B = -2 \Leftrightarrow$

$\Leftrightarrow \boxed{B = 2}$

$2A + 2B = -16 \Leftrightarrow \boxed{A = -6}$

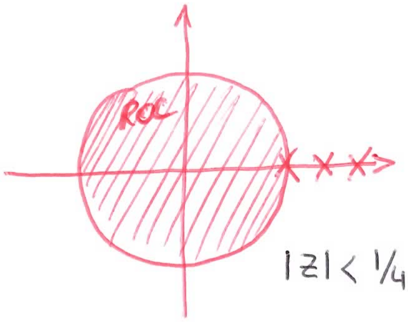
$$X(z) = -\frac{6}{z^{-1}-3} - \frac{2}{z^{-1}-2} = -\frac{6}{-3\left(1-\frac{z^{-1}}{3}\right)} - \frac{2}{-2\left(1-\frac{z^{-1}}{2}\right)} =$$

$$= \frac{2}{1-z^{-1}/3} + \frac{1}{1-z^{-1}/2}$$

$$\boxed{x[n] = 2\left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^n u[n]}$$

$\boxed{30}$ Sendo $X(z) = \dfrac{z}{z-1/2} + \dfrac{z}{z-1/3} + \dfrac{z}{z-1/4}$, calcule a transformada inversa de Z nos seguintes casos:

a)



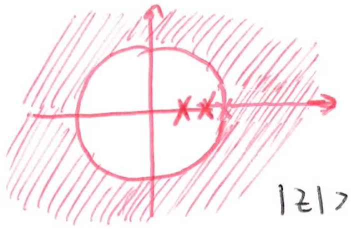
$|z| < 1/4$

sistema não causal.

$$x[n] = -\left(\tfrac{1}{2}\right)^n u(-n-1) - \left(\tfrac{1}{3}\right)^n u(-n-1) - \left(\tfrac{1}{4}\right)^n u[-n-1]$$

$$\Leftrightarrow x[n] = -\left[\left(\tfrac{1}{2}\right)^n + \left(\tfrac{1}{3}\right)^n + \left(\tfrac{1}{4}\right)^n\right] u[-n-1]$$

b)



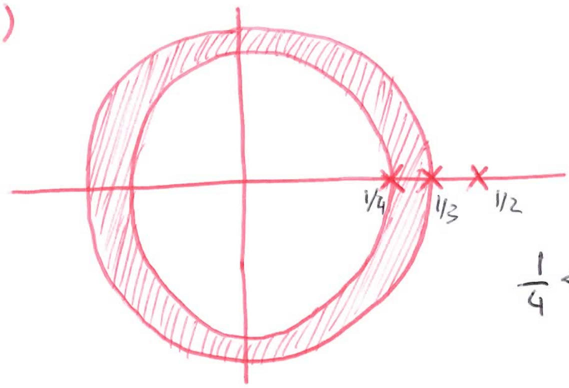
$|z| > 1/2$

sistema causal.

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n] + \left(\tfrac{1}{3}\right)^n u[n] + \left(\tfrac{1}{4}\right)^n u[n] \Leftrightarrow$$

$$\Leftrightarrow x[n] = \left[\left(\tfrac{1}{2}\right)^n + \left(\tfrac{1}{3}\right)^n + \left(\tfrac{1}{4}\right)^n\right] u[n]$$

c)



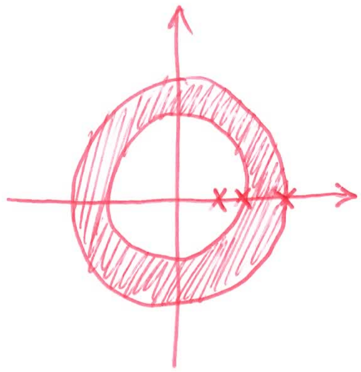
$\frac{1}{4} < |z| < \frac{1}{3}$

$x[n]$	$X(z)$	ROC
$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$ $\dfrac{z}{z-a}$	$\|z\| > \|a\|$ causal
$-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$ $\dfrac{z}{z-a}$	$\|z\| < \|a\|$ ñ causal

$$X(z) = \underbrace{\frac{z}{z-1/2} + \frac{z}{z-1/3}}_{\text{ñ causal.}} + \underbrace{\frac{z}{z-1/4}}_{\text{causal}}$$

$$x[n] = -\left(\tfrac{1}{2}\right)^n u[-n-1] - \left(\tfrac{1}{3}\right)^n u[-n-1] + \left(\tfrac{1}{4}\right)^n u[n] = \left(\tfrac{1}{4}\right)^n u[n] - \left[\left(\tfrac{1}{2}\right)^n + \left(\tfrac{1}{3}\right)^n\right] u[-n-1].$$

$\boxed{d)}$



$\frac{1}{3} < |z| < \frac{1}{2}$

$$X(z) = \underbrace{\frac{z}{z-1/2}}_{\bar{n}\text{ causal}} + \underbrace{\frac{z}{z-1/3} + \frac{z}{z-1/4}}_{\text{causal.}}$$

$$x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] + \left[\left(\frac{1}{3}\right) + \left(\frac{1}{4}\right)^n\right] u[n]$$