Transformada de Fourier

11. Calcular a transformada de Fourier dos seguintes sistemas:

a) $\begin{cases} \frac{1}{3} & ; -1 \le n \le 1 \\ 0 & ; \quad - \end{cases}$

b) $h(n) = a^n u(n)$

c) $h(n) = r^n \cos(n\omega_0) u(n)$

12. Desenhe o módulo e a fase do seguinte sinal: $\delta(n-k) = \begin{cases} 1 & ; n = k \\ 0 & ; - \end{cases}$

13. Calcule a resposta em frequência e represente-a em módulo e fase para o seguinte sistema definido pela equação às diferenças: $y(n) = \dfrac{x(n) + 2x(n-1) + x(n-2)}{4}$

14. Calcule a resposta em frequência e represente-a graficamente, do sistema que tem a seguinte resposta impulsional: $h(n) = \begin{cases} \frac{1}{3} & ; 0 \le n \le 2 \\ 0 & ; \quad - \end{cases}$

15. Calcule a resposta impulsional do sistema que tem a seguinte resposta em frequência:

$$H(e^{j\omega}) = \begin{cases} 1 & ; |\omega| \le \omega_0 \\ 0 & ; \quad - \end{cases}$$

16. Calcular a resposta impulsional do filtro passa banda ideal. *(Sugestão: partir de um filtro passa baixo ideal).*

17. Desenhe o módulo e a fase do seguinte sistema: $H(e^{j\omega}) = \frac{1}{3}(1 + 2\cos\omega)$.

18. Represente graficamente o módulo e a fase da saída do sistema que tem como resposta impulsional: $h(n) = \begin{cases} a^n & ; n \geq 0 \\ 0 & ; - \end{cases}$

quando a entrada é: $x(n) = \begin{cases} \frac{1}{3} & ; -1 \leq n \leq 1 \\ 0 & ; - \end{cases}$

19. Calcule a resposta em frequência do seguinte sistema e represente-a graficamente:

$$h(n) = \begin{cases} -1 & ; n = -2 \\ 1 & ; n = 2 \\ 0 & ; - \end{cases}$$

20. Suponha que utiliza o sistema discreto $y(n) = \frac{1}{T}[x(n) - x(n-1)]$ para aproximar o diferenciador analógico $H_a(j\Omega) = j\Omega$.

a) Determine a resposta em frequência $H(e^{j\omega})$ do sistema discreto.

b) Represente graficamente (módulo e fase) $H(e^{j\omega})$ e $H_a(j\Omega)$ e indique em que condições a aproximação se pode considerar razoável.

c) Determine a resposta impulsional do sistema discreto constituído pela associação em série de dois sistemas idênticos ao dado.

21. A parte real da transformada de Fourier de um sinal causal é: $X_R(e^{j\omega}) = 1 + \cos\omega$. Determine a parte imaginária dessa mesma transformada.

22. Calcular a transformada inversa de $H(e^{j\omega}) = \frac{1}{3}(1 + 2\cos\omega)$.

23. Considere o sistema discreto: $y(n) = \frac{x(n) + x(n+3)}{2}$

a) Determine a sua resposta impulsional.

b) Determine a sua resposta em frequência, e represente-a graficamente.

Processamento Digital de Sinal

EXERCICIOS 2008 (7)

$\boxed{\text{TRANSFORMADA DE FOURRIER}}$

$\boxed{11}$ Calcular a transformada de fourier dos seguintes sistemas:

$\boxed{a)}$ $\begin{cases} 1/3 \ ; -1 \leq n \leq 1 \\ 0 \ ; \ \text{———} \end{cases}$

$$H(e^{jw}) = \sum_{n=-\infty}^{+\infty} h(n)\, e^{-jwn} = \frac{1}{3} e^{jw} + \frac{1}{3} e^{-jw0} + \frac{1}{3} e^{-jw} =$$

$$= \frac{1}{3}\left(\underbrace{e^{jw} + e^{-jw}} + 1\right) = 1/3\left(2\cos(w) + 1\right) = \frac{2}{3}\cos(w) + \frac{1}{3}$$

$\boxed{b)}$ $h(n) = a^n \cdot u(n) = \begin{cases} a^n \ ; \ n \geq 0 \\ \text{———} \end{cases}$

heavyside

$$H(e^{jw}) = \sum_{n=-\infty}^{+\infty} h(n) e^{-jwn} = \sum_{n=-\infty}^{+\infty} a^n e^{-jwn} = 1 \cdot \frac{1 - a^{\infty}}{1-a} e^{-jw} = \frac{1}{1-a} e^{-jw}$$

$a^{\infty} \to 0$

com $|a| < 1$ infinitésimo

$\boxed{c)}$ $h(n) = r^n \cos(n w_0)\, u(n) = \begin{cases} r^n \cos(n w_0) \ ; \ n \geq 0 \\ 0 \ ; \ \text{———} \end{cases}$

$$H(e^{jw}) = \sum_{n=-\infty}^{+\infty} r^n \cos(n w_0)\, e^{-jwn} = \cos(w_0)\, \frac{1 - r^{\infty}}{1 - r} e^{-jw}$$

$r^{\infty} \to 0$

$|r| < 1$ infinitésimo

$$= \frac{\cos w_0}{1-r} e^{-jw} \quad \text{com } |r| < 1$$

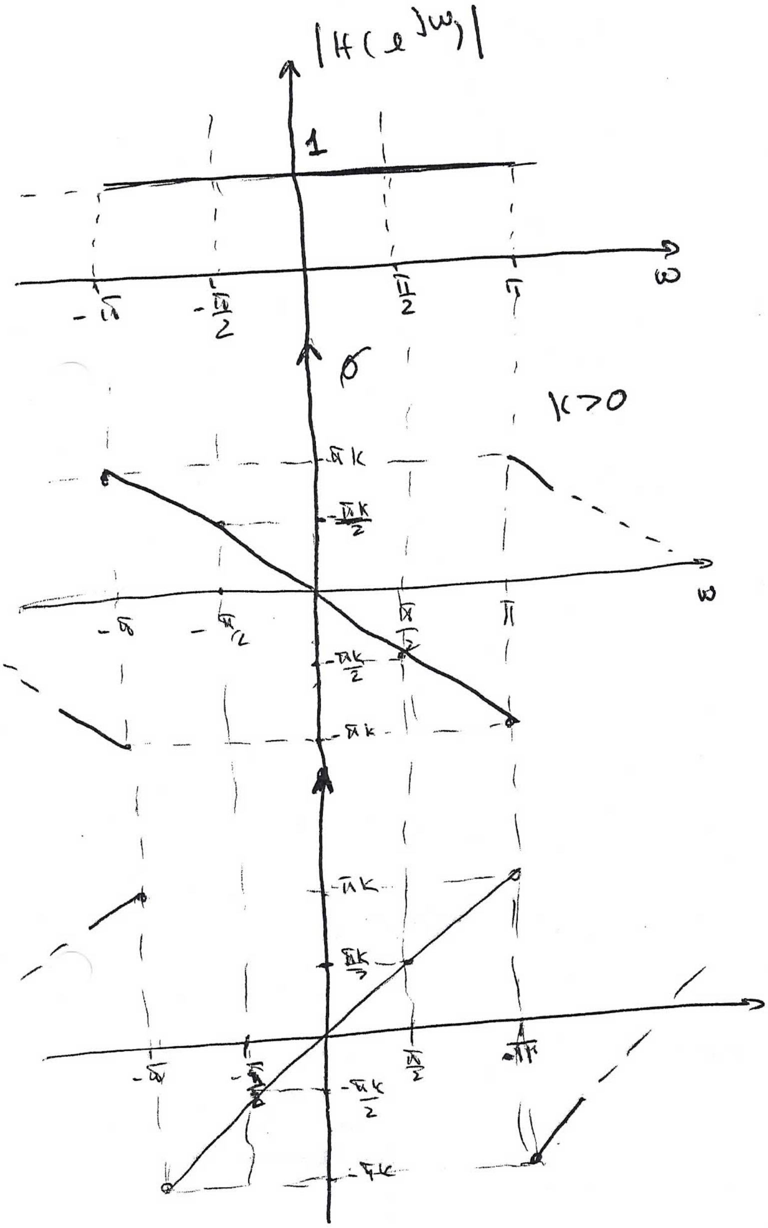
$\boxed{12}$ Desenhe o modulo e a fase do seguinte sinal $\delta(n-k) = \begin{cases} 1 ; & n=k \\ 0 ; & — \end{cases}$

1) Módulo e fase de $\delta(n-k) = \begin{cases} 1 ; & n = k \\ 0 ; & \text{---} \end{cases}$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} \delta(n-k)\, e^{-j\omega n} = 1 \cdot e^{-j\omega k} = e^{-j\omega k}$$

$|H(e^{j\omega})| = 1$ (Módulo)

$\angle H(e^{j\omega}) = -\omega k$ (fase) $= \phi$



Módulo:

ω	$\lvert H(e^{j\omega})\rvert$
$-\pi$	1
$-\frac{\pi}{2}$	1
0	1
$\frac{\pi}{2}$	1
π	1

fase: $k>0$

ω	$\angle H(e^{j\omega}) = -\omega k$
$-\pi$	$+\pi k$
$-\frac{\pi}{2}$	$+\frac{\pi k}{2}$
0	0
$\frac{\pi}{2}$	$-\frac{\pi k}{2}$
π	$-\pi k$

$k < 0$

ω	$\phi = -\omega k$
$-\pi$	$-\pi k$
$-\frac{\pi}{2}$	$-\frac{\pi k}{2}$
0	0
$\frac{\pi}{2}$	$\frac{\pi k}{2}$
π	πk

transformada de fourier

11) transformada de fourier?

a) $h[n] = \begin{cases} \frac{1}{3} ; & -1 \leq n \leq 1 \\ 0 ; & \text{———} \end{cases}$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n]\, e^{-j\omega n} = \frac{1}{3} e^{j\omega} + \frac{1}{3} e^{-j\omega 0} + \frac{1}{3} e^{-j\omega \cdot 1}$$

$$= \frac{1}{3} \left(\underbrace{e^{j\omega} + e^{-j\omega}}_{\cos \omega = \frac{e^{j\omega} + e^{-j\omega}}{2}} + 1 \right) = \frac{1}{3} (2 \cos(\omega) + 1)$$

$$= \frac{2}{3} \cos(\omega) + \frac{1}{3}$$

b) $h[n] = a^n u[n]$

$h[n] = \begin{cases} a^n ; & n \geq 0 \\ 0 , & \text{———} \end{cases}$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n] \cdot e^{-j\omega n} = \underbrace{\sum_{n=0}^{+\infty} a^n \cdot e^{-j\omega n}}_{\text{↳ prog. Geométrica de razão } a}$$

$$= 1 \cdot \frac{1 - a^{\infty \to 0}}{1 - a} \cdot e^{-j\omega} \quad , |a| < 1 \text{ (infinitésimo)}$$

$$= \frac{1}{1-a} e^{-j\omega} \quad , \text{com } |a| < 1$$

c) $h[n] = a^n \cos(n\omega_0)\, u[n]$

$h[n] = \begin{cases} a^n \cos(n\omega_0) , & n \geq 0 \\ 0 , & \text{———} \end{cases}$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n] \cdot e^{-j\omega n}$$

$$= \sum_{n=0}^{+\infty} a^n \cdot \cos(n\omega_0) \cdot e^{-j\omega n} = \cos(\omega_0) \cdot \frac{1 - a^{\infty \to 0}}{1 - a} e^{-j\omega}$$

$|a| < 1$ (infinitésimo)

$$= \frac{\cos(\omega_0)}{1-a} \cdot e^{-j\omega} , \quad \text{com } |a| < 1$$

13) $y[n] = \dfrac{x[n] + 2x[n-1] + x[n-2]}{4}$

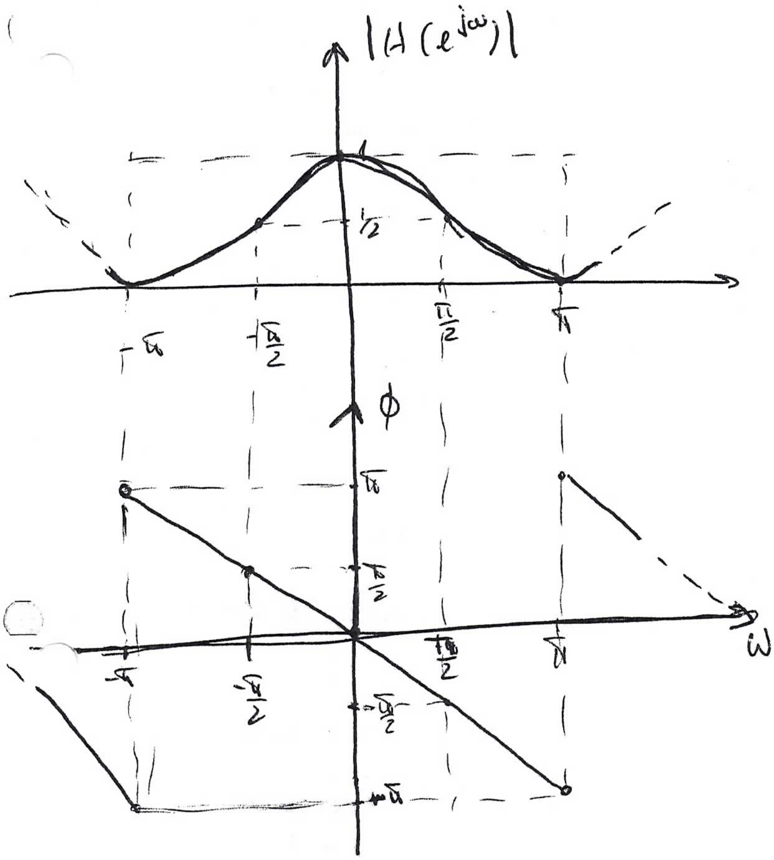
$$h[n] = \frac{1}{4}\,\delta[n] + \frac{1}{2}\,\delta[n-1] + \frac{1}{4}\,\delta[n-2]$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n]\, e^{-j\omega n}$$

$$= \frac{1}{4}\cdot e^{-j\omega 0} + \frac{1}{2}\, e^{-j\omega 1} + \frac{1}{4}\, e^{-j\omega 2}$$

$$= \frac{1}{4}\, e^{-j\omega}\left(e^{j\omega} + 2 + e^{-j\omega}\right) \qquad \cos\omega = \frac{e^{j\omega} + e^{-j\omega}}{2}$$

$$= \frac{1}{4}\, e^{-j\omega}\left(2\cos(\omega) + 2\right) = \underbrace{\frac{1}{2}\left(1 + \cos(\omega)\right)}_{\text{Módulo}} \cdot e^{-j\omega} \rightarrow \text{fase}$$



Módulo:

ω	$\lvert H(e^{j\omega})\rvert$
$-\pi$	0
$-\frac{\pi}{2}$	$\frac{1}{2}$
0	1
$\frac{\pi}{2}$	$\frac{1}{2}$
π	0

Fase:

ω	$\angle H(e^{j\omega})$ $= -\omega$
$-\pi$	π
$-\frac{\pi}{2}$	$\frac{\pi}{2}$
0	0
$\frac{\pi}{2}$	$-\frac{\pi}{2}$
π	$-\pi$

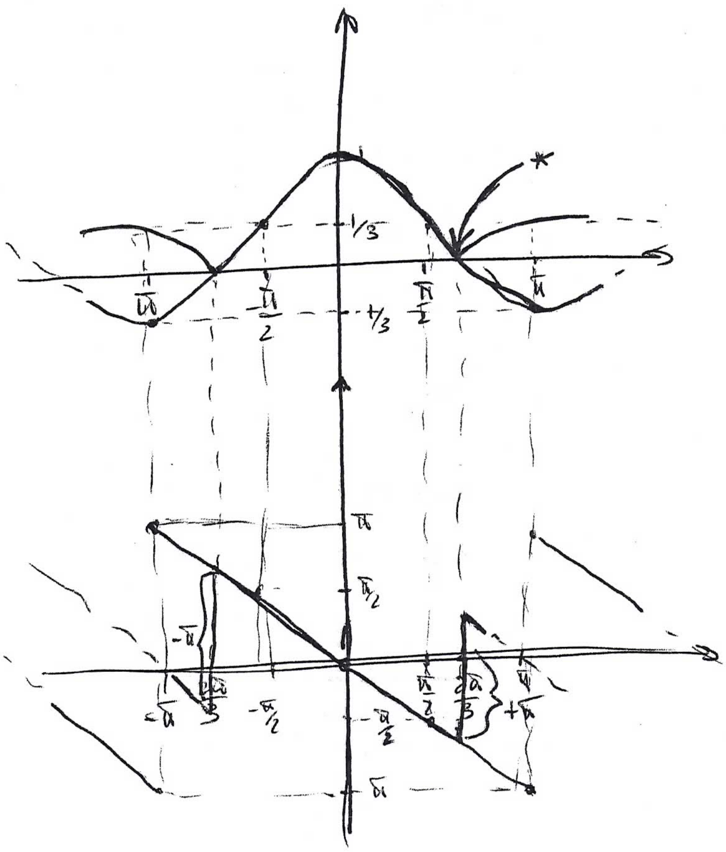
4)

$$h[n] = \begin{cases} 1/3 & , 0 \leq n \leq 2 \\ 0 & , \text{———} \end{cases}$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n] \cdot e^{-j\omega n} = \frac{1}{3} e^{-j\omega 0} + \frac{1}{3} e^{-j\omega 1} + \frac{1}{3} e^{-j\omega 2}$$

$$= \frac{1}{3} e^{-j\omega} \left(\underbrace{e^{j\omega} + 1 + e^{-j\omega}}_{\cos(\omega) = \frac{e^{j\omega} + e^{-j\omega}}{2}} \right) = \frac{1}{3} \cdot (1 + 2\cos(\omega)) \, e^{-j\omega}$$

$$= \underbrace{\left(\frac{1}{3} + \frac{2}{3} \cos(\omega) \right)}_{\text{Módulo}} \cdot e^{-j\omega} \to \text{fase}$$



Módulo:

ω	
$-\pi$	$-1/3$
$-\frac{\pi}{2}$	$1/3$
0	1
$\frac{\pi}{2}$	$1/3$
π	$-1/3$

fase

ω	$\phi = -\omega$
$-\pi$	π
$-\frac{\pi}{2}$	$\frac{\pi}{2}$
0	0
$\frac{\pi}{2}$	$-\frac{\pi}{2}$
π	$-\pi$

$* \ \frac{1}{3} + \frac{2}{3} \cos(\omega) = 0$

$\Leftrightarrow \frac{2}{3} \cos(\omega) = -\frac{1}{3}$

$\Leftrightarrow \cos(\omega) = -\frac{3}{6} = -\frac{1}{2}$

$\Leftrightarrow \omega = \cos^{-1} -\frac{1}{2} \Leftrightarrow \omega = \frac{2\pi}{3} \pm k\pi, k \in \mathbb{Z}$

15) $H(e^{j\omega}) = \begin{cases} 1 \;, |\omega| \leq \omega_0 \\ 0 \;, \text{———} \end{cases}$ $\quad h[n] = ?$

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2} \frac{1}{jn} \int_{-\omega_0}^{\omega_0} jn \cdot 1 \cdot e^{j\omega n}\, d\omega = \frac{1}{2\pi jn} \left[e^{j\omega n} \right]_{-\omega_0}^{\omega_0}$$

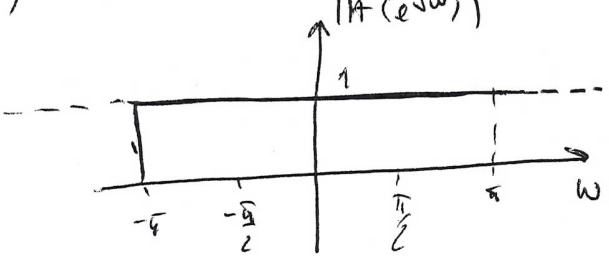
$$= \frac{1}{2\pi jn} \underbrace{\left[e^{j\omega_0 n} - e^{-j\omega_0 n} \right]}_{\sin(\omega_0 n) = \frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}}$$

$$= \frac{1}{2\pi jn} \times 2j \sin(\omega_0 n) = \frac{1\,\omega_0 n}{\pi n} \times \frac{\sin(\omega_0 n)}{\omega_0 n}$$

Como $\operatorname{sinc}(\omega_0 n) = \dfrac{\sin(\omega_0 n)}{\omega_0 n}$ então

$$h[n] = \frac{\omega_0}{\pi} \operatorname{sinc}(\omega_0 n)$$

16)



← filtro passa-banda ideal

$|H(e^{j\omega})| = 1$

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} 1 \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \frac{1}{jn} \int_{-\pi}^{\pi} jn\, e^{j\omega n}\, d\omega = \frac{1}{2\pi jn} \left[e^{j\omega n} \right]_{-\pi}^{\pi}$$

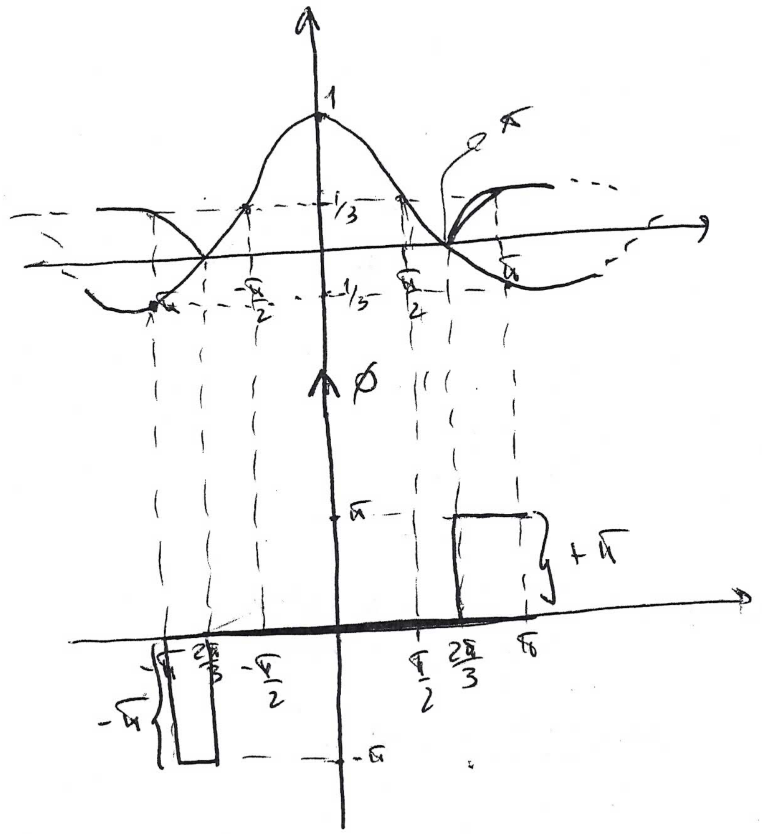
$$= \frac{1}{2\pi} \left[e^{j\omega\pi} - e^{-j\omega\pi} \right] = \frac{1}{2\pi jn} 2j \sin(\pi n)$$

$$= \frac{1}{\pi n} \sin(\pi n) = \operatorname{sinc}(\pi n)$$

7) $H(e^{j\omega}) = \frac{1}{3}(1+2\cos\omega) = \left(\frac{1}{3}+\frac{2}{3}\cos\omega\right)e^{-j\omega}$

Módulo: $|H(e^{j\omega})| = \frac{1}{3} + \frac{2}{3}\cos\omega$

fase: $\phi = \angle H(e^{j\omega}) = 0$



Módulo

ω	$\lvert H(e^{j\omega})\rvert$
$-\pi$	$-1/3$
$-\frac{\pi}{2}$	$1/3$
0	1
$\frac{\pi}{2}$	$1/3$
π	$-1/3$

$* \ \frac{1}{3} + \frac{2}{3}\cos\omega = 0$

$\Leftrightarrow \frac{2}{3}\cos\omega = -\frac{1}{3}$

$\Leftrightarrow \cos\omega = -\frac{1}{2}$

$\Leftrightarrow \omega = \frac{2\pi}{3} \pm k\pi, k \in \mathbb{Z}$

18) $h[n] = \begin{cases} a^n & ; n \geq 0 \\ 0 & ; \text{---} \end{cases}$; $x[n] = \begin{cases} 1/3 & ; -1 \leq n \leq 1 \\ 0 & ; \text{---} \end{cases}$

$$y[n] = x[n] * h[n] = \sum_{n=-\infty}^{+\infty} x[n] \cdot h[n-k] = \sum_{n=-\infty}^{+\infty} h[n] \cdot x[n-k]$$

$$= \sum_{n=0}^{+\infty} a^n \cdot \left[\frac{1}{3}e^{j\omega} + \frac{1}{3}e^{-j\omega} + \frac{1}{3}e^{-j\omega}\right]$$

$$= \sum_{n=0}^{+\infty} a^n \left[\frac{1}{3}\left(e^{j\omega} + e^{-j\omega} + 1\right)\right] = \sum_{n=0}^{+\infty} a^n \frac{1}{3}(2\cos\omega + 1)$$

$$= \sum_{n=0}^{+\infty} a^n \left[\frac{2}{3}\cos\omega + \frac{1}{3}\right] = \sum_{n=0}^{+\infty} \frac{2a^n}{3}$$

19) Resposta em frequência ?

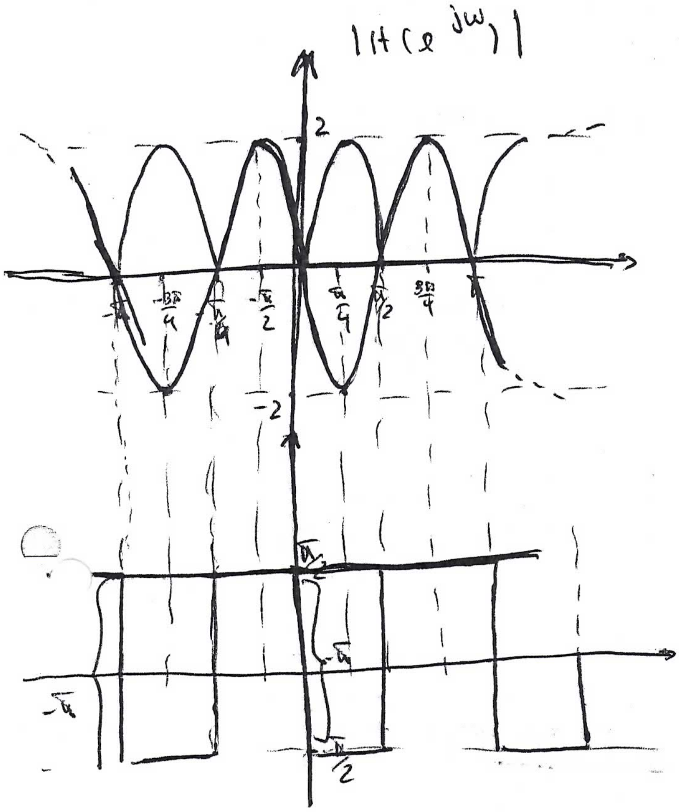
$H(e^{j\omega}) = ?$

$$h[n] = \begin{cases} -1 & ; n=-2 \\ 1 & ; n=2 \\ 0 & ; \text{———} \end{cases}$$

$$h[n] = -\delta[n+2] + \delta[n-2]$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n] e^{-j\omega n} = -1 e^{j\omega 2} + 1 e^{-j\omega 2}$$

$$\Leftrightarrow H(e^{j\omega}) = e^{-j\omega 2} - e^{j\omega 2} = -2j \sin(\omega 2) = -2 \sin(2\omega)\, e^{-j\frac{\pi}{2}}$$



Módulo

ω	$H(e^{j\omega})$
$-\pi$	0
$-\frac{\pi}{2}$	0
$-\frac{\pi}{4}$	2
0	0
$\frac{\pi}{4}$	-2
$\frac{\pi}{2}$	0
π	0

fase

ω	$\angle H(e^{j\omega})$
$-\pi$	$\frac{\pi}{2}$
$-\frac{\pi}{2}$	.
0	.
$\frac{\pi}{2}$	.
π	$\frac{\pi}{2}$

22) $H(e^{jw}) = \frac{1}{3}(1+2\cos w)$

$h[n] = ?$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{jw}) \cdot e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{3}(1+2\cos w)e^{jwn}\, dw = \frac{1}{6\pi}\int_{-\pi}^{\pi}\left(1+\cancel{2}\left(\frac{e^{jw}+e^{-jw}}{\cancel{2}}\right)\right)\cdot e^{jwn}\, dw$$

$$= \frac{1}{6\pi}\int_{-\pi}^{\pi} e^{jwn} + e^{jw(n+1)} + e^{jw(n-1)}\, dw$$

$$= \frac{1}{6\pi}\left[\frac{e^{jwn}}{jn} + \frac{e^{jw(n+1)}}{j(n+1)} + \frac{e^{jw(n-1)}}{j(n-1)}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{6\pi jn}\left[e^{j\pi n} - e^{-j\pi n}\right] + \frac{1}{6\pi j(n+1)}\left[e^{j\pi(n+1)} - e^{-j\pi(n+1)}\right] + \frac{1}{6\pi j(n-1)}\left[e^{j\pi(n-1)} + e^{-j\pi(n}\right.$$

$$= \frac{1}{6\pi jn} \times 2j\sin(\pi n) + \frac{1}{6\pi \cancel{j}(n+1)} 2\cancel{j}\sin(\pi(n+1)) + \frac{1}{6\pi\cancel{j}(n-1)} 2\cancel{j}\sin(\pi(n-1))$$

$$= \frac{1}{3}\operatorname{sinc}(\pi n) + \frac{1}{3}\operatorname{sinc}(\pi(n+1)) + \frac{1}{3}\operatorname{sinc}(\pi(n-1))$$

23) $y[n] = \dfrac{x[n] + x[n+3]}{2}$

a) $h[n] = ?$ $\qquad h[n] = \dfrac{\delta[n] + \delta[n+3]}{2}$

$n=0$: $h[0] = \frac{1}{2}\delta\overset{=1}{[0]} + \frac{1}{2}\delta\overset{=0}{[3]} = \frac{1}{2}$

$n=-3$: $h[-3] = \frac{1}{2}\delta\overset{=0}{[-3]} + \frac{1}{2}\delta\overset{=1}{[0]} = \frac{1}{2}$

$$h[n] = \begin{cases} \frac{1}{2} & ; n = 0, -3 \\ 0 & , \end{cases}$$

c) $H(e^{j\omega}) = ?$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n] \cdot e^{-j\omega n} = \frac{1}{2} e^{-j\omega 0} + \frac{1}{2} e^{+j\omega 3}$$

$$= \frac{1}{2} e^{j\omega\frac{3}{2}} \left(\underline{e^{-j\omega\frac{3}{2}} + e^{j\omega\frac{3}{2}}} \right)$$

$$\cos\left(\frac{\omega 3}{2}\right) = \frac{e^{j\omega\frac{3}{2}} + e^{-j\omega\frac{3}{2}}}{2}$$

$$= \frac{1}{\cancel{2}} e^{j\omega\frac{3}{2}} \cdot \cancel{2} \cos\left(\frac{\omega 3}{2}\right) = \underbrace{\cos\left(\frac{\omega 3}{2}\right)}_{\text{Módulo}} e^{\underbrace{j\omega\frac{3}{2}}_{\text{fase}}}$$

C.A

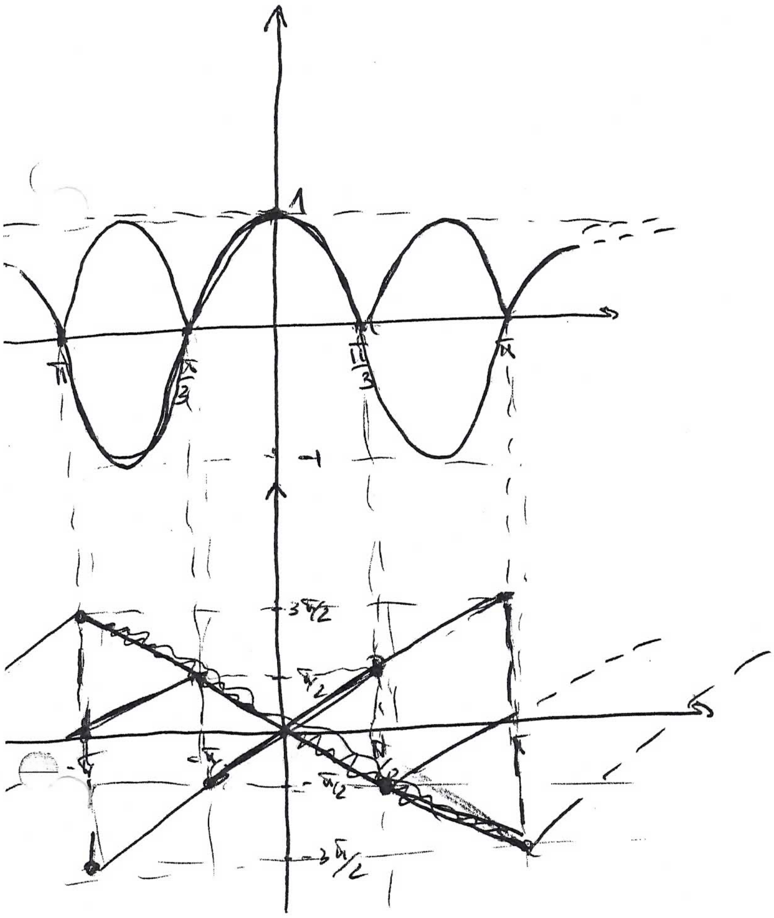
$\rightarrow e^{j\omega\frac{3}{2}} \times e^{-j\omega\frac{3}{2}}$

$= e^{j\omega\frac{3}{2} - j\omega\frac{3}{2}} = e^{0}$

$\rightarrow e^{j\omega\frac{3}{2}} \times e^{j\omega\frac{3}{2}}$

$= e^{j\omega\frac{3}{2} + j\omega\frac{3}{2}}$

$= \cancel{2} j\omega\frac{3}{\cancel{2}} = e^{j\omega 3}$



Módulo

ω	$\lvert H(e^{j\omega})\rvert$
$-\pi$	0
$-\frac{\pi}{3}$	0
0	1
$\frac{\pi}{3}$	0
π	0

Fase

ω	$\angle H(e^{j\omega})$
$-\pi$	$-\frac{3}{2}\pi$
$-\frac{\pi}{3}$	$-\frac{\pi}{2}$
0	0
$\frac{\pi}{3}$	$\frac{\pi}{2}$
π	$\frac{3}{2}\pi$