

# Continuous Variables System

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# Introduction - Objectives

- Study Continuous Variables Quantum Key Distribution (CV-QKD) with 4 state discrete modulation.
- Both simulation and experimental results were obtained.
- Results were linked to theoretical expected values, not each other (missing detector information to compare simulation to experimental values).

# Introduction - Content in this presentation

- Simulation results:
  - Noise characterization.
  - Secret key generation rate in function of transmission for two levels of excess noise.
- Experimental results:
  - Phase drift compensation.
  - Noise characterization experiment.
  - Key distribution experiment with secret key generation rate estimation.

# Theoretical notes - Security of CV-QKD

In CV-QKD, the key for a One Time Pad (OTP) protocol is shared through a quantum channel. The security of this key is evaluated in terms of secret key generation rate (bits/symbol):

$$K = \beta I(A : B) - S(B : E). \quad (1)$$

The rate is positive if Alice and Bob manage to share more information,  $I(A : B)$ , than the information obtained by Eve on Bob's results,  $S(B : E)$ .  $\beta$  represents the efficiency of the employed error correction.

# Theoretical notes - Estimating information rates

The mutual classical information is estimated as:

$$I(A : B) = \log_2 \left( 1 + \frac{T \langle n \rangle}{1 + \frac{T}{2} \epsilon} \right). \quad (2)$$

This definition of  $I(A : B)$  assumes a Gaussian modulated signal, while we use discrete modulation. The effect of this inaccuracy on the mutual information is included in the error correction efficiency  $\beta$ .

# Theoretical notes - Estimating information rates

The quantum information Eve possesses,  $S(B : E)$ , is upper bounded by the quantum information between Alice and Bob,  $S(B : A)$ , which can be estimated by knowledge of the covariance matrices.

$$\gamma_{AB} = \begin{bmatrix} (1 + 2 \langle n \rangle) \mathbb{I}_2 & \sqrt{\frac{T}{2}} Z \sigma_Z \\ \sqrt{\frac{T}{2}} Z \sigma_Z & (T \langle n \rangle + 1 + \frac{T}{2} \epsilon) \mathbb{I}_2 \end{bmatrix} \quad (3)$$

The effect of the measurement on Alice's mode for the case of double-homodyne detection can be expressed as follows.

$$\gamma_{AB|B} = \left[ (1 + 2 \langle n \rangle) - \frac{\frac{T}{2} Z^2}{T \langle n \rangle + 2 + \frac{T}{2} \epsilon} \right] \mathbb{I}_2 \quad (4)$$

# Theoretical notes - Estimating information rates

The quantum information can then be calculated from the symplectic eigenvalues of the two previous covariance matrices through:

$$S(B : E) = \sum_{k=1}^2 \left[ (\bar{n}_k^{AB} + 1) \log_2(\bar{n}_k^{AB} + 1) - \bar{n}_k^{AB} \log_2 \bar{n}_k^{AB} \right] - (\bar{n}^{AB|B} + 1) \log_2(\bar{n}^{AB|B} + 1) - \bar{n}^{AB|B} \log_2 \bar{n}^{AB|B}, \quad (5)$$

where  $\bar{n} = (\mu - 1)/2$ , with  $\mu$  representing the symplectic eigenvalues.

# Theoretical notes - Security of practical CV-QKD

Taking into account the need to estimate the channel parameters, the covariance matrices are altered to:

$$\gamma_{\varepsilon} = \begin{bmatrix} (1 + 2 \langle n \rangle) \mathbb{I}_2 & t_{\min} Z \sigma_z \\ t_{\min} Z \sigma_z & (2t_{\min}^2 \langle n \rangle + \sigma_{\max}^2) \mathbb{I}_2 \end{bmatrix}, \quad (6)$$

and:

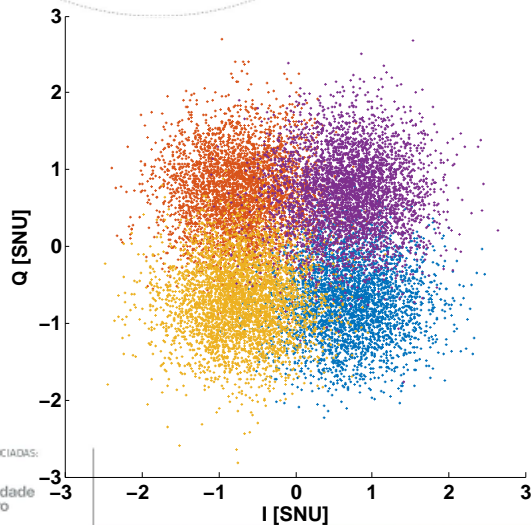
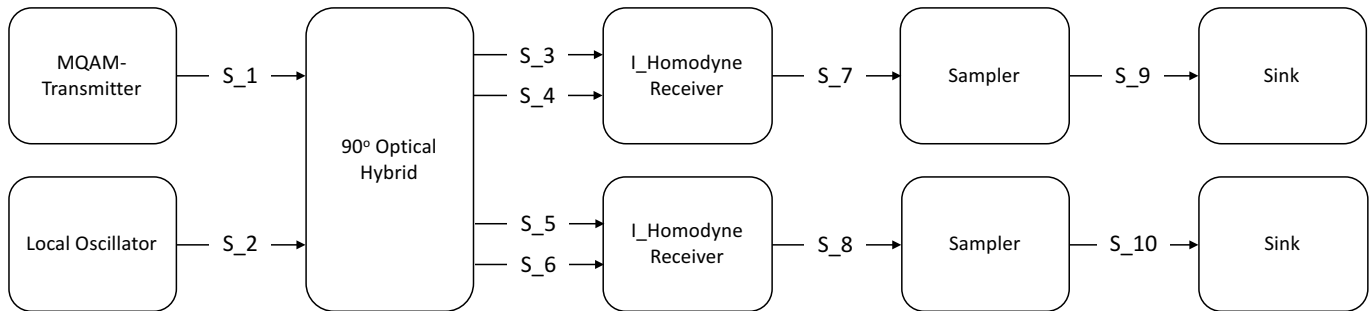
$$\gamma_{AB|B_{\varepsilon}} = \left( 1 + 2 \langle n \rangle - \frac{t_{\min}^2 Z^2}{2t_{\min}^2 \langle n \rangle + 1 + \sigma_{\max}^2} \right) \mathbb{I}_2. \quad (7)$$

$t_{\min}$  is the minimum value of  $t = \sqrt{\frac{T}{2}}$  except with a probability of  $\Delta/2$ .

$\sigma_{\max}^2$  is the maximum value of  $\sigma^2 = 1 + \frac{T}{2}\varepsilon$  except with a probability of  $\Delta/2$ .



# Simulation - Block diagram

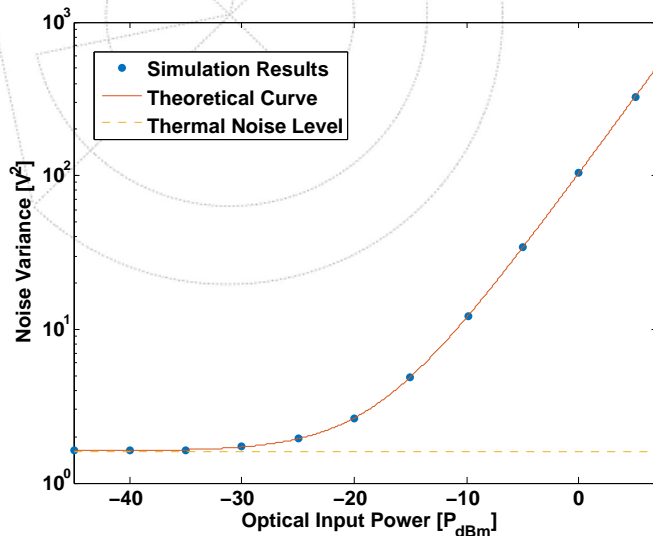


Two independent noise sources are considered:

- Thermal noise.
- Shot noise.

# Simulation - Detector noise characterization

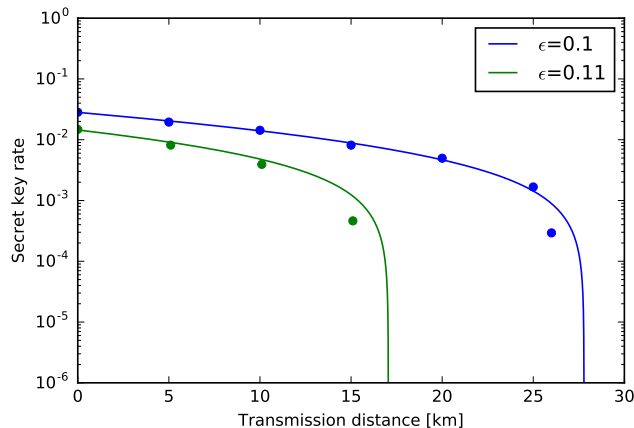
The first results presented here pertain to a noise variance characterization of the simulated homodyne receiver:



The simulation results closely follow the theoretical expectation values for all the studied power levels. The linear stage of the detector is seen to start at a Local Oscillator optical input power of -15 dBm.

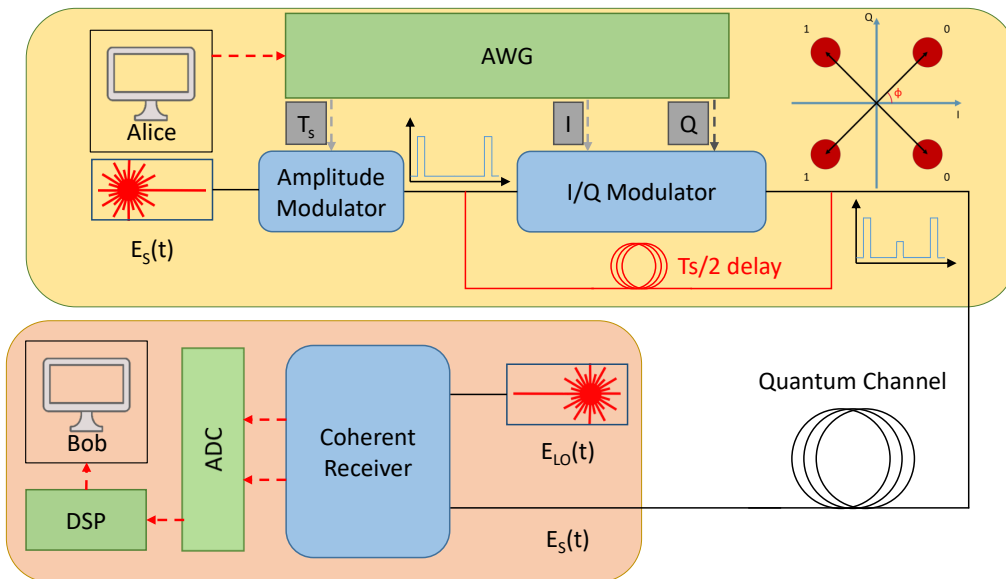
# Simulation - Secret key generation rate

Following the noise characterization, the secret key generation rate of the simulation was evaluated and compared to the theoretical expectation values:



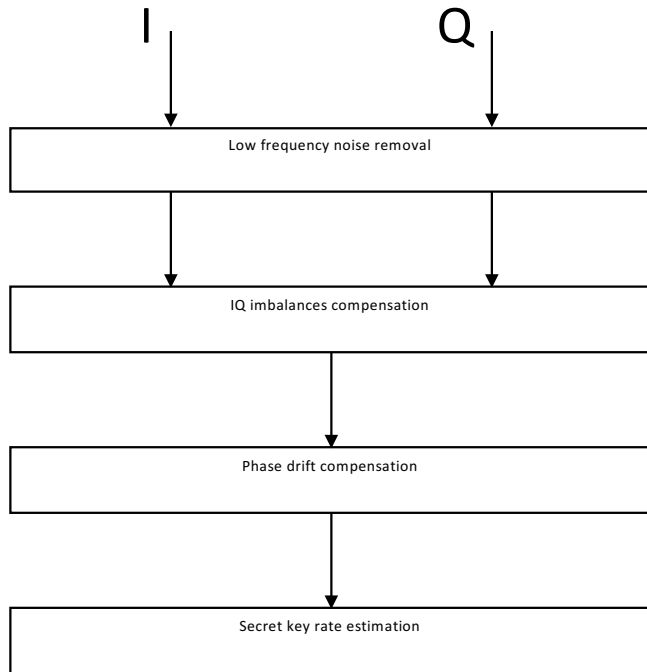
The simulation results closely follow the theoretical curve just until when the curve starts to quickly tend to 0, at which point they diverge.

# Experimental results - Experimental system



# Experimental results - Output data processing

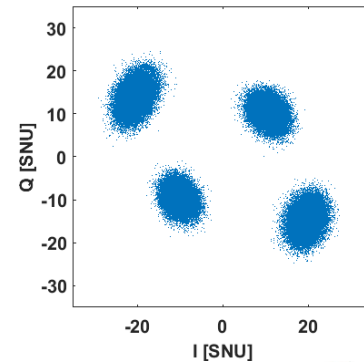
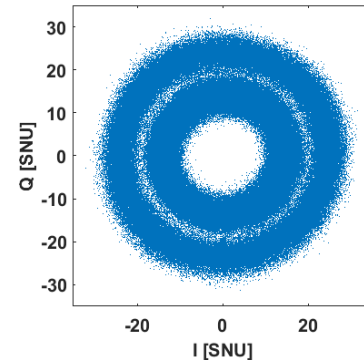
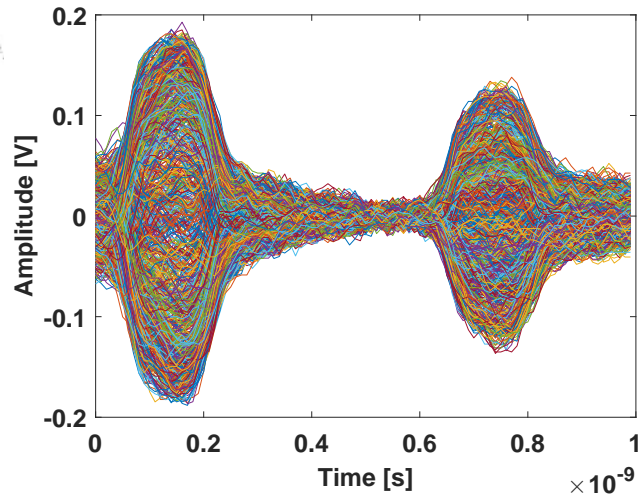
The following digital signal processing was employed:



- The IQ imbalances are removed by applying a Gram-Schmidt orthogonalization process.
- The phase drift is compensated by measuring the relative phase between the two lasers and removing that value from the decoded results.
- The secret key rate is estimated through the finite size analysis method presented above.

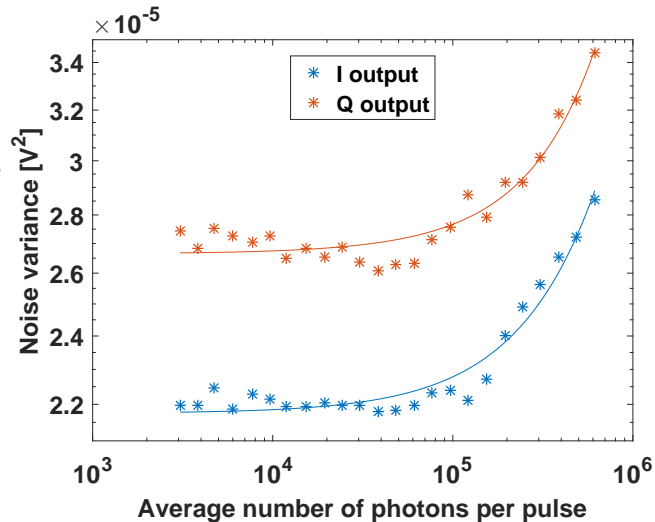
# Experimental results - Phase drift compensation

The phase drift compensation scheme was tested.



# Experimental results - Detector noise characterization

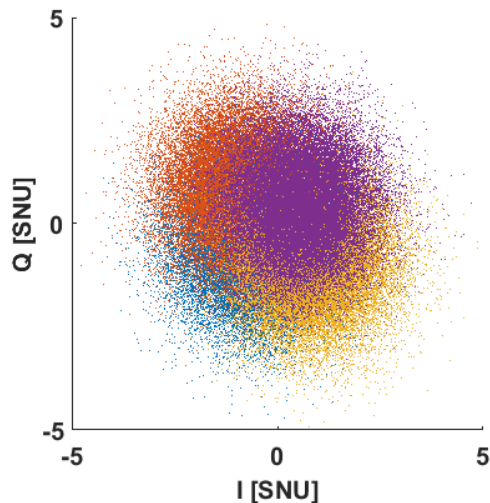
A shot noise characterization of the detector was performed



- The linear was stage was found to be below  $\langle n \rangle \sim 4 \times 10^6$ .
- Our detector has a very short linear stage, not ideal for this application.
- Shot Noise Units (SNU) conversion factor was obtained.

# Experimental results - System performance

The system's performance was evaluated for 4 different setups, utilizing a single/double laser setup coupled with either a direct connection or a 10 km transmission channel. The secret key rate was estimated through the finite size analysis presented before.

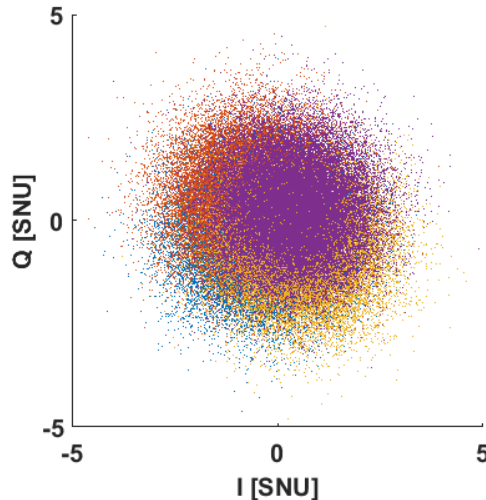


|               |                    |                     |
|---------------|--------------------|---------------------|
| $\varepsilon$ | 0.074              |                     |
|               | <b>Theoretical</b> | <b>Experimental</b> |
| $T$           | Efficiency         | 0.9519              |
| $K$           | 0.0288             | 0.0213              |



# Experimental results - System performance

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|               |                    |                     |
|---------------|--------------------|---------------------|
| $\varepsilon$ | 0.0156             |                     |
|               | <b>Theoretical</b> | <b>Experimental</b> |
| $T$           | 0.4547             | 0.4317              |
| $K$           | 0.0132             | 0.0094              |

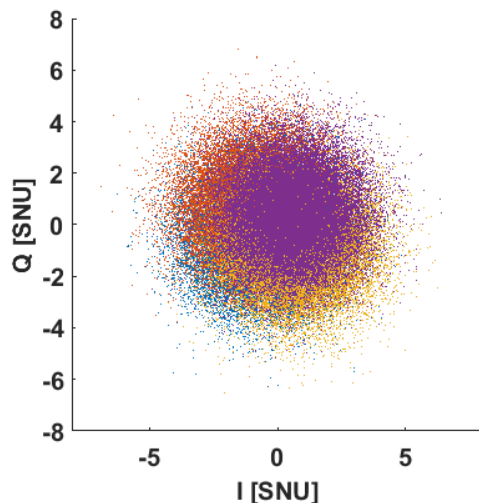
Single laser, 10 km recovered  
constellation

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# Experimental results - System performance

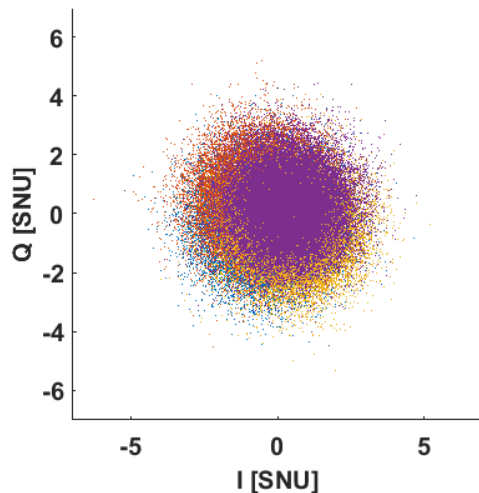
The system's performance was evaluated for 4 different setups, utilizing a single/double laser setup coupled with either a direct connection or a 10 km transmission channel. The secret key rate was estimated through the finite size analysis presented before.



|               |                    |                     |
|---------------|--------------------|---------------------|
| $\varepsilon$ | 2.9915             |                     |
|               | <b>Theoretical</b> | <b>Experimental</b> |
| $T$           | $\sim 0.95$        | 0.9557              |
| $K$           | Negative           | -0.8011             |

# Experimental results - System performance

The system's performance was evaluated for 4 different setups, utilizing a single/double laser setup coupled with either a direct connection or a 10 km transmission channel. The secret key rate was estimated through the finite size analysis presented before.



|               |                    |                     |
|---------------|--------------------|---------------------|
| $\varepsilon$ | 2.8196             |                     |
|               | <b>Theoretical</b> | <b>Experimental</b> |
| $T$           | 0.4547             | 0.4282              |
| $K$           | Negative           | -0.4769             |

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double laser, 10 km recovered  
constellation