

I rounded to the second digit

Formula for K =2

$$P(X) = 0.47 * \text{norm}(3.19, 1.7^{**}0.5) + 0.53 * \text{norm}(11.0, 9.56^{**}0.5)$$

K=3

$$P(X) = 0.25 * \text{norm}(2.03, 0.34^{**}0.5) + 0.24 * \text{norm}(4.47, 0.08^{**}0.5) + 0.51 * \text{norm}(11.32, 7.74^{**}0.5)$$

K=4

$$P(X) = 0.25 * \text{norm}(2.03, 0.34^{**}0.5) + 0.25 * \text{norm}(4.47, 0.08^{**}0.5) + 0.25 * \text{norm}(8.89, 0.35^{**}0.5) + 0.25 * \text{norm}(13.9, 1.52^{**}0.5)$$

K=5

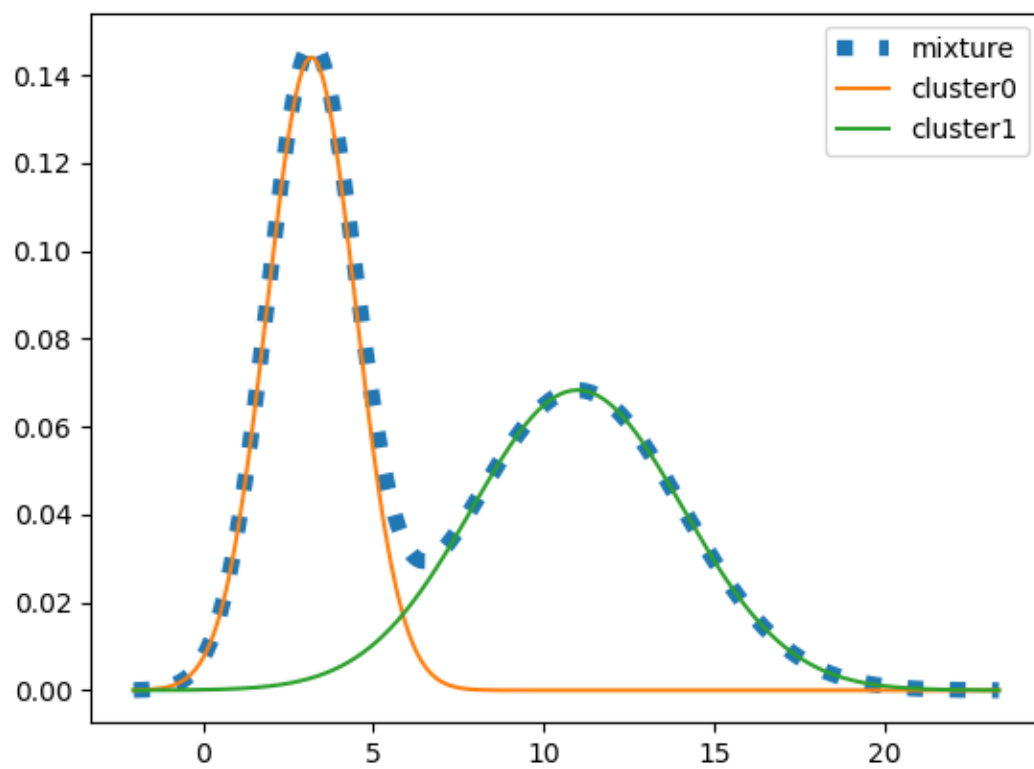
$$P(X) = 0.25 * \text{norm}(2.03, 0.35^{**}0.5) + 0.25 * \text{norm}(4.47, 0.08^{**}0.5) + 0.06 * \text{norm}(8.73, 0.3^{**}0.5) + 0.19 * \text{norm}(8.94, 0.36^{**}0.5) + 0.25 * \text{norm}(13.9, 1.52^{**}0.5)$$

K=6

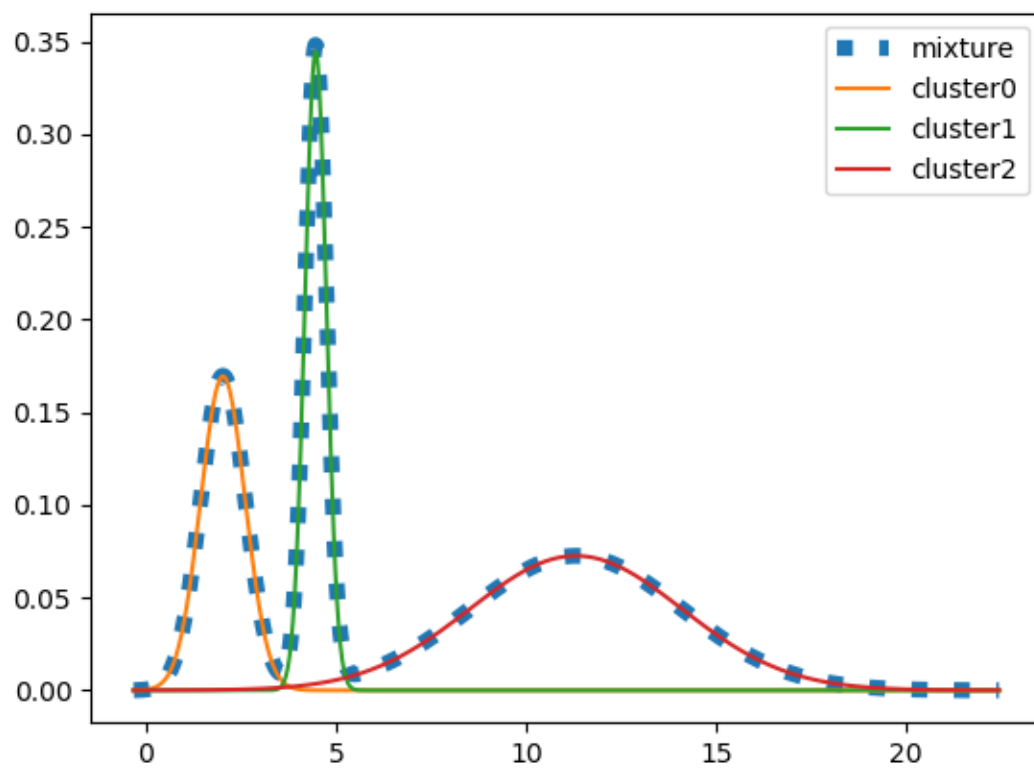
$$P(X) = 0.22 * \text{norm}(1.99, 0.33^{**}0.5) + 0.03 * \text{norm}(2.61, 0.78^{**}0.5) + 0.25 * \text{norm}(4.47, 0.08^{**}0.5) + 0.25 * \text{norm}(8.89, 0.35^{**}0.5) + 0.01 * \text{norm}(13.29, 1.49^{**}0.5) + 0.24 * \text{norm}(13.93, 1.51^{**}0.5)$$

3. The log likelihood increased from -1010 to -910 after k is 3 and the log likelihood just stay around there until k=6.

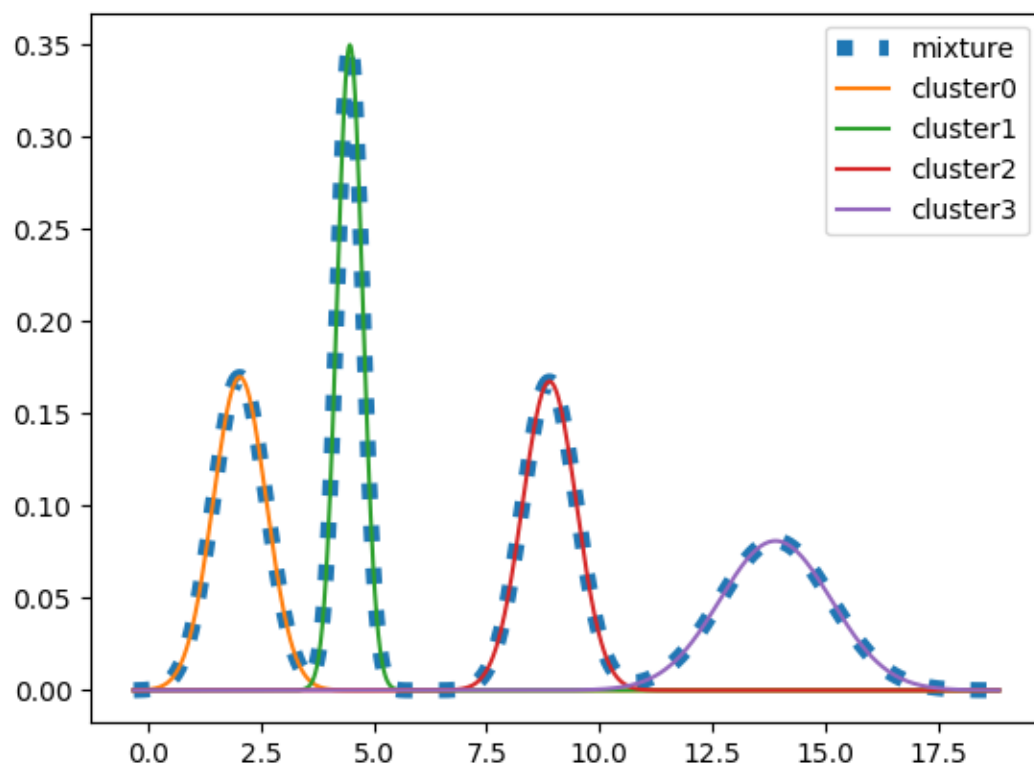
4. There are 4 clusters in this dataset, because when the k is 6. There are two clusters are almost identity to the other 4 and does not have enough data.



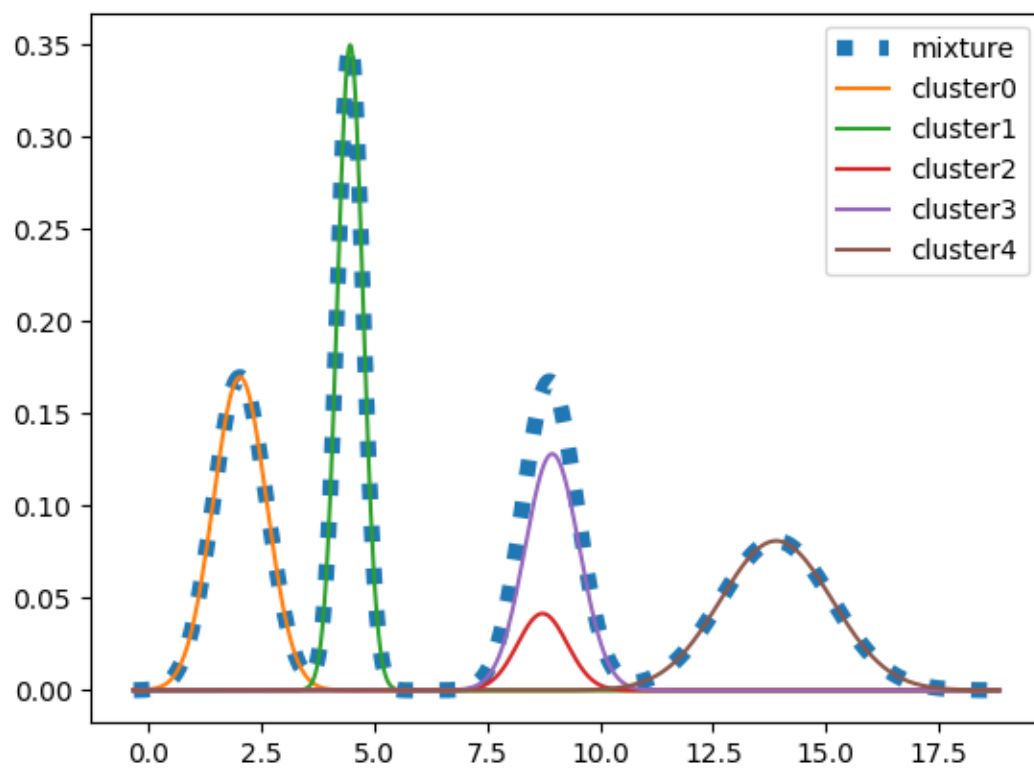
K=2



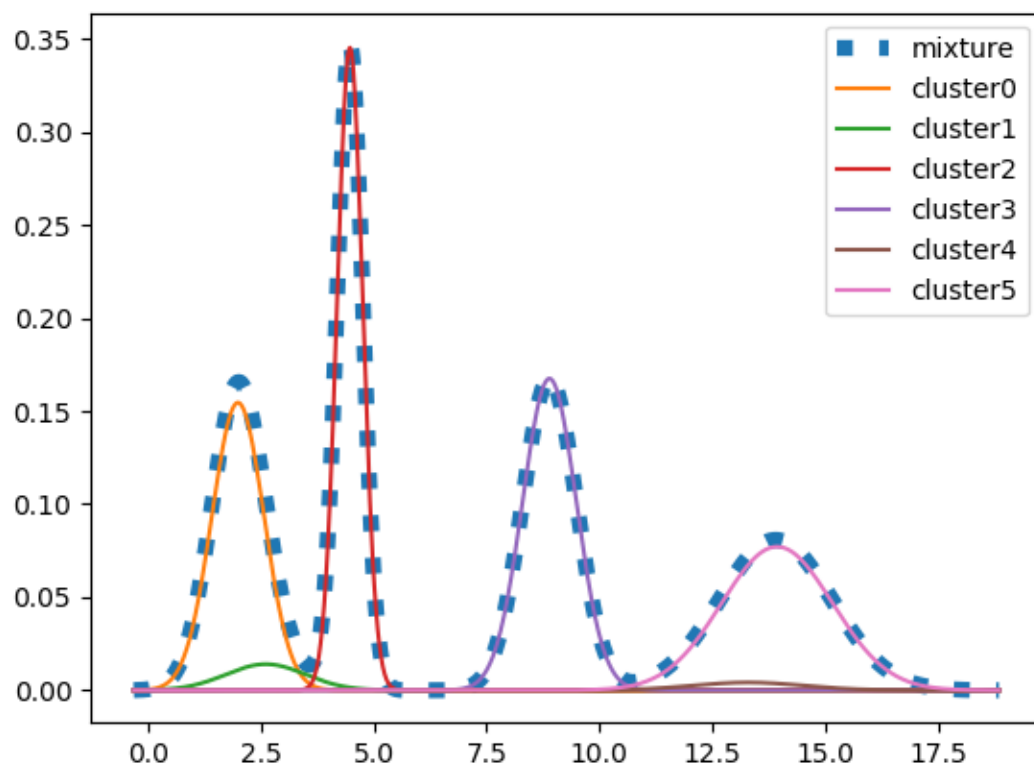
K=3



K=4



K=5



K=6