1. A triangular region R in the $xy-$ plane has its edges defined by the horizontal line $y=-2$, the vertical line $x=1$, and the line $y=-\dfrac{1}{2}+\dfrac{3}{2}x$. Which of the following is an integral of $f(x,y)$ over that region?

(a) $I=\displaystyle\int_{-1}^{0}\int_{x=-1}^{\frac{1}{3}+\frac{2}{3}y} f(x,y)\,dx\,dy\,,$ (b) $I=\displaystyle\int_{-2}^{1}\int_{x=\frac{2}{3}y+\frac{1}{3}}^{1} f(x,y)\,dx\,dy\,,$

(c) $I=\displaystyle\int_{0}^{1}\int_{y=\frac{3}{2}x-\frac{1}{2}}^{1} f(x,y)\,dy\,dx\,,$ (d) $I=\displaystyle\int_{-1}^{1}\int_{y=-1}^{-\frac{1}{2}+\frac{3}{2}x} f(x,y)\,dy\,dx$

(e) none of the above.

2. Integrating the function $f(x,y)=4x+y$ over the region R from the previous question, we obtain

(a) $I=-2$, (b) $I=2$, (c) $I=-1$, (d) $I=1$,

(e) none of the above.

3. If $x=\rho\cos\phi\,,y=\rho\sin\phi$ and $f(x,y)=\frac{1}{2}(x^2+y^2)\,$, then $\left[\dfrac{\partial f}{\partial\rho},\dfrac{\partial f}{\partial\phi}\right]=$

(a) $[\rho\cos\phi\,,\sin\phi]$ (b) $[\rho\,,0]$ (c) $[0\,,\rho]$ (d) $[\rho\,,\phi]$

(e) none of the above.

4. A function is given by

$$f(x)=\begin{cases}2-x & \text{for}\quad 0\le x<1\\ x-1 & \text{for}\quad 1\le x<2\end{cases}$$

and $f(x+2)=f(x)$ for all x. The Fourier series is calculated for this function (you do not need to do this!) and converges to the following values

(a) $\dfrac{1}{2}$ at $x=2\,$, and 1 at $x=0$ (b) $-\dfrac{1}{2}$ at $x=0\,$, and 0 at $x=-1$

(c) $\dfrac{1}{2}$ at $x=1\,$, and $\dfrac{3}{2}$ at $x=-2$ (d) $\dfrac{3}{2}$ at $x=0\,$, and $-\dfrac{1}{2}$ at $x=2$

(e) none of the above.

The next two questions refer to $f(x)$, the period-2, even extension of a function which is equal to $2 - x$, on the interval $0 < x \leq 1$.

5. Determine which of the following is a correct Fourier series for $f(x)$ (Formulae at end)

a) $\displaystyle \frac{3}{2} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos[(2n-1)\pi x]$, b) $\displaystyle \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cos(nx)$,

c) $\displaystyle \frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(nx)$, d) $\displaystyle \frac{1}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos(n\pi x)$,

(e) none of the above.

6. The complex Fourier coefficients c_n when $n \neq 0$, for $f(x)$ are

(a) $\displaystyle \frac{2}{(n\pi)^2} \cos(n\pi)$, (b) $\displaystyle \frac{(-1)^{n+1} i}{n}$, (c) $\displaystyle \frac{i}{n^2}[1 - \cos(n\pi)]$, (d) $\displaystyle \frac{1}{\pi n^2} \sin(n\pi/2)$,

(e) none of the above

7. Consider the following five statements about $F(\omega)$, the Fourier transform of $f(t)$:

(I) If $f(t) = e^{-|t|}$, then $F(\omega)$ is an odd function of ω ,

(II) The Fourier transform of $\dfrac{df}{dt}$ is equal to $-i\omega F(\omega)$,

(III) If $|F(\omega)|$ is an even function, then $f(t)$ is real-valued ,

(IV) If $f(t) = \cos(\pi t)$, then $F(\omega)$ will involve the Dirac delta function ,

(V) If $f(t)$ is an odd function, the imaginary part of $F(\omega)$ is equal to zero ,

Then

(a) (I) and (IV) are both false , (b) (V) is true and (II) is false ,

(c) (II) is false and (III) is true , (d) (III) and (V) are both true ,

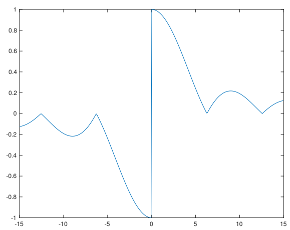
(e) none of the above.

8. A switch function is given by $f(t) = -H(t) + H(t-1)$, where $H(t)$ is the Heaviside function. The Fourier transform of $f(t)$ is $F(\omega) =$

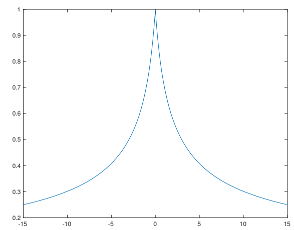
(a) $\displaystyle \frac{i}{\omega}\left(e^{i\omega} - 1\right)$ (b) $\displaystyle \frac{1}{i\omega}\left(e^{-i\omega} + 1\right)$ (c) $\displaystyle -\frac{i}{\omega}\left(1 - e^{i\omega}\right)$ (d) $\displaystyle -\frac{i}{\omega}\left(e^{-i\omega} - 1\right)$

(e) none of the above.

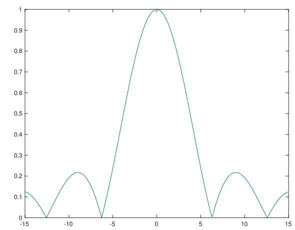
9. From the previous question, which of the following shows the amplitude spectrum of $F(\omega)$?



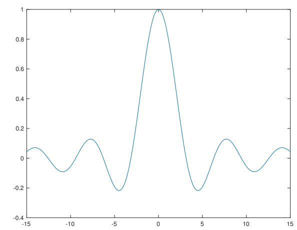
(a)



(b)



(c)



(d)

The next two questions concern the function of two variables $f(x,y) = 6x - y - x^3 + y^3 - 23$.

10. Which of the following points are stationary points for $f(x,y)$?

(a) $\left(\frac{1}{2}, \sqrt{3}\right)$ and $\left(-\frac{1}{2}, -\sqrt{3}\right)$ (b) $\left(\frac{1}{\sqrt{2}}, \frac{1}{3}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \frac{1}{3}\right)$

(c) $\left(\sqrt{2}, -\frac{1}{\sqrt{3}}\right)$ and $\left(\sqrt{2}, \frac{1}{\sqrt{3}}\right)$ (d) $(\sqrt{2}, \sqrt{3})$ and $(-\sqrt{2}, \sqrt{3})$

(e) none of the above.

11. The stationary points of $f(x,y)$, found in the previous question, are

(a) a maximum and a saddle (b) a minimum and a saddle

(c) two saddles (d) a maximum and a minimum

(e) none of the above.

Fourier Transform:

$$G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-i\omega t}\, dt\,, \quad \Longleftrightarrow \quad g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{i\omega t}\, d\omega\,,$$

Fourier Series:

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2n\pi x}{T}\right) + b_n \sin\left(\frac{2n\pi x}{T}\right)\right]$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\ \cos\left(\frac{2n\pi x}{T}\right)\, dx\,,\ b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\ \sin\left(\frac{2n\pi x}{T}\right)\, dx\,,$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i2\pi nx/T}\,, \qquad c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(x)e^{-i2n\pi x/T}\, dx\,.$$