MATH 222 - Assignment 4

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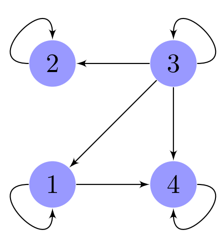
1. (a) In order for a relation to be both symmetric and antisymmetric, it must only contain reflexive elements. For each reflexive pair (x, x), there are 2 options: either the pair is in the relation, or it is not.
   $\therefore$ there are 2^n relations on A that fulfill the requirements

   (b) If R is a relation on A that is antisymmetric, then it may not contain any 2 pairs such that (x, y) and (y, x) are both in R unless $x = y$.
   $\therefore$ we can count the number of elements in the upper diagonal half of the corresponding relation matrix.
   This is equivalent to $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

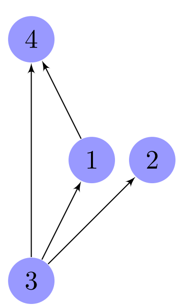
   (c) For each pair, either (x, y) or (y, x) may be in R, unless $x = y$.
   $\therefore$ there are $\frac{n(n+1)}{2} - n = \frac{n(n-1)}{2}$ pairs that may be swapped out for their inverse.
   As in part (a), we have 2 options for each of our pairs, so we have $2^{\frac{n(n-1)}{2}}$ relations

   (d) If R has domain A and is a function, then each element may map to only 1 other element in A, by the definition of a function.
   Since R is reflexive, for every $x \in A$ the pair (x, x) must be in R.
   Since every value in the domain is already mapped to itself, we know that no value can map to another due to our function restriction.
   $\therefore$ R is made up of all of the reflexive pairs (x, x) where $x \in A$

2. (a)



   (b)



(c) A total order must be antisymmetric, but must also contain at least one pair for every $x, y \in A$. There are currently 2 sets of pairs that are not included in R, these being $(2,1)/(1,2)$ and $(2,4)/(4,2)$.
For each of these 2 pairs there are 2 choices, with that choice being which will be in our total order.
$\therefore$ there are $2^2 = 4$ total orders containing our partial order