MATH 222 Assignment #4
Due: Tuesday, March 21, 2017

Assignments are due in class **before** the start of lecture. Late assignments will be accepted up to 24 hours late, but will be deducted a 20% late penalty.

All answers must include full explanations. For full marks, your work must be stapled, neatly written, and contain enough detail that it is clear how you arrived at your solutions.

1. Let $A = \{1, 2, 3, \ldots, n\}$.

   (a) How many relations on A are both symmetric and antisymmetric?

   (b) If R is a relation on A that is antisymmetric, what is the maximum number of ordered pairs that can be in R?

   (c) How many antisymmetric relations on A have the maximum size that you determined in part (b)?

   (d) If R is an equivalence relation on A and is also a function with domain A, describe R.

2. Consider the relation on $A = \{1, 2, 3, 4\}$ with relation matrix:

$$M(\mathcal{R}) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   Assume that the rows and columns of the matrix refer to the elements of A in the order $1, 2, 3, 4$.

   (a) Draw the digraph for the given partial order.

   (b) Draw the Hasse Diagram for the partial order.

   (c) How many total orders contain the given partial order as a subset?

3. Let S be the set of all nonzero real numbers. Consider the relation R on S given by xRy iff $xy > 0$.

   (a) Prove that R is an equivalence relation on S, and give an example of equivalence classes of R that give a partition of S.

(b) Why is the relation R_2 on S given by xR_2y iff $xy < 0$ NOT an equivalence relation?

4. Let $X = \{1, 2, 3, 4, 5, 6, 7\}$ and let R be a partial order on X. For $Y \subset X$ (i.e. Y a proper subset of X), let R_Y be the relation on Y given by $(a, b) \in R_Y$ iff $a, b \in Y$ and $(a, b) \in R$.

   (a) Prove that R_Y is a partial order on Y.

   (b) Prove or disprove: If R is a total order on X, then R_Y is a total order on Y.

5. Determine the sequence generated by each of the following generating functions.

   (a) $f(x) = \dfrac{x^3}{1 - x^2}$

   (b) $f(x) = (4x - 1)^3$

   (c) $f(x) = \dfrac{1 - x}{1 + x}$

6. Find a generating function for each of the given sequences. When possible, give your answer in closed form rather than as a series.

   (a) $0, 1, -2, 4, -8, 16, \ldots$

   (b) $0, 1, 0, -1, 0, 1, 0, -1, \ldots$

   (c) the sequence $a_0, a_1, a_2, \ldots$, where a_n is the number of ways to give a player \$$n$ using only \$5 red poker chips, \$10 blue poker chips, \$25 green poker chips, and \$100 black poker chips.