MATH 222 Assignment #3

Due: Friday, March 3, 2017

Assignments are due in class **before** the start of lecture. Late assignments will be accepted up to 24 hours late, but will be deducted a 20% late penalty.

All answers must include full explanations. For full marks, your work must be stapled, neatly written, and contain enough detail that it is clear how you arrived at your solutions.

1. Consider all compositions of the number 16. Recall that a composition of n refers to a way to write n as an ordered sum of positive integers. So $2 + 10 + 4$ and $10 + 4 + 2$ are different compositions of 16.

   (a) How many compositions of 16 are there?

   (b) How many compositions of 16 have all even summands?

2. When players A and B play chess against each other they keep track of who wins each game. A *run* of wins for a player refers to any number of games in a row that the player wins. For example, if the list of winners for the last 15 games played was $ABBBAABABBBBAAA$, then A had a run of length 1, followed by B having a run of length 3, then A a run of length 2, etc. The example shown has 7 runs, in which A won 7 games and B won 8.

   (a) How many different outcomes are there for 15 consecutive games that result in exactly 7 runs where A wins 7 games and B wins 8?

   (b) How many different outcomes are there for 15 consecutive games that result in exactly 7 runs? In this case, each player may win any number of the 15 games.

3. How many integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 22$ satisfy $0 \le x_1 \le 6$, $0 \le x_2 \le 7$, $3 \le x_3 \le 10$, and $4 \le x_4 \le 10$?

4. Consider all permutations of the digits $1, 2, 3, 4, 5, 6, 7, 8, 9$. How many of the permutations satisfy that for all $1 \le i < 9$, i is not immediately followed by $i + 1$?

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5. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Consider the functions $f : A \to A$ that are onto. How many of these functions satisfy that for all even x, $f(x) \neq x$?

6. Show that among any 13 integers, there must be 2 whose difference is divisible by 12.

7. While learning about compositions of integers, a MATH 222 student is thinking about all of the different compositions of 20 that have exactly 14 summands. There are 27132 of them, so the student certainly can't write them all out, but does notice that no matter how he tries there is always a collection of consecutive summands which sum to 5. Prove that this must always be the case.