MATH 222 Assignment #1
Due: Tuesday, January 24, 2017

Assignments are due in class **before** the start of lecture. Late assignments will be accepted up to 24 hours late, but will be deducted a 20% late penalty.

All answers must include full explanations. For full marks, your work must be stapled, neatly written, and contain enough detail that it is clear how you arrived at your solutions.

1. Let $G = (V, E)$ be a graph with $|E| = 29$ and $deg(v) \geq 4$ for all $v \in V$. What is the maximum number of vertices that G can have?

2. Prove that if every vertex of a simple graph G has degree at least 2, then G contains a cycle.

3. Consider the graph K_6 with vertices labelled $1, 2, 3, 4, 5, 6$. How many different subgraphs are isomorphic to a path of length 3?

4. Consider a graph G with 20 vertices that has the property that $deg(x) + deg(y) \geq 19$ for all $x, y \in V$. Prove that G is connected.

5. You and a friend meet three other couples at a party and several handshakes take place. Nobody shakes hands with himself/herself, there are no handshakes between a person and their partner, and no one shakes hands with the same person more than once. At the end of the party you ask each of the other people how many handshakes they had and get 7 different answers! How many hands did you shake? How many did your friend shake? (A graph will be helpful!)

6. Let G be the graph with 64 vertices representing the squares on a chessboard. Two vertices are adjacent iff you can move legally between the corresponding squares with a single move of a knight. Note that the moves of a knight are L-shaped and consist of moving two squares in one direction (vertically or horizontally) and then one square in the perpendicular direction.

   (a) Explain why the graph G is bipartite. If you are stuck on this, you may find it helpful to actually look at a chess board and a few legal moves of a knight.

   (b) What are the degrees of the vertices? Note that they will not all be the same!

7. Let $G = (V, E)$ be a graph. Recall from class that the complement of G, denoted $\overline{G}$, is the graph that has vertex set V in which two vertices are adjacent if and only if they are not adjacent in G. i.e. $E(\overline{G}) = \{uv \mid uv \notin E(G),\ u, v \in V\}$. Also recall that G is called self-complementary if G is isomorphic to $\overline{G}$.

   (a) Is there a graph G on 5 vertices such that G and $\overline{G}$ are both bipartite? If not, explain why not. If so, give an example of such a graph G.

   (b) Prove that if G is self-complementary and $|V(G)| = n$, then $n = 4k$ or $n = 4k + 1$ for some positive integer k. Hint: We calculated $|E(G)|$ for self-complementary graphs in class.

8. Prove or disprove each of the following statements:

   (a) If $K_{m,n}$ has both an Euler circuit and a Hamilton cycle, then $m = n$ and n is even.

   (b) If G has an Euler circuit, then G has a Hamilton cycle.

   (c) If G has a Hamilton cycle, then G has an Euler circuit.

   (d) There exist two nonisomorphic graphs with 5 vertices and 6 edges.