TEK4050 Sotchastic Systems – Spring 2025
COMPULSORY ASSIGNMENT 1

Deadline: **Friday, April 25, 2025**
Submit in the form of a short report in an email with "TEK4050" in the subject field to **kjetil-bergh.anonsen@ffi.no**. Include code listings.

Task 1

The random variable X has the following probability density function:

$$f_X(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0. \end{cases} \tag{1}$$

a) Find the cumulative distribution function of X and compute $P\{X \geq 2\}$ and $P\{1 \leq X \leq 2\}$.

b) Find the expectation and variance of X.

Task 2

A scalar 1. order continuous Gauss-Markov process is described as

$$\dot{x} = -\frac{1}{T}x + v, \tag{2}$$

where $T > 0$ is a time constant and v is Gaussian white noise, with $E[v(t)] = 0$ and $E[v(t)v(\tau)] = \tilde{q}\delta(t-\tau)$.

a) Show that the discretization of (2), with constant time step Δt, is given as

$$x_{k+1} = e^{-\frac{1}{T}\Delta t}x_k + v_k, \tag{3}$$

where v_k is a white noise sequence with $E[v_k] = 0$ and $E[v_k v_l] = q\delta_{kl}$, $k = 1, 2, \ldots$ and δ_{kl} is the Kronecker delta, defined as:

$$\delta_{kl} = \begin{cases} 1, & k = l \\ 0, & k \neq l. \end{cases}$$

What is the correspondence between q and $\tilde{q}$?

b) Simulate the discrete system in a) on the interval $t_0 = 0$ s to $t_{\text{final}} = 100$ s in a suitable programming language (e.g. Matlab or Python). Use the time step $\Delta t = 0.01$ s. Let $x_0 \sim \mathcal{N}(\bar{x}_0, p_0)$, where $\bar{x}_0 = 10$ and the variance $p_0 = 2$, $q = 1$ and the time constant $T = 10$ s. Make a plot that shows one realization of the process.

c) Is the process you have simulated stationary?

d) Experiments with different values for the time consntant T. How does this affect the process? Show some realizations with high and low values for T. What happens to the process when we let $T \to 0$ and $T \to \infty$, respectively?