Introduction to Homology and Cohomology

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Abstract

In this paper we introduce the reader to the theory of homology. In Part 1 we begin by covering the simple case of simplicial homology. We then seek generalisations and variants, and eventually formulate a more general notion of homology through category theory in Part 2. Finally, we use this more general notion to sample other forms of homology that arise in topics such as physics, algebraic geometry, and data science.

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Part I

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