# Calibration of hyperon-nucleon interaction models using light hypernuclei

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# Outline

- **Introduction & motivation**
  - Strangeness, hypernuclei and hypernuclear interactions
- **Ab initio calculations of hypernuclei**
  - No-core shell model
  - Theoretical uncertainties
  - Hypertriton lifetime
  - Emulators
  - Global sensitivity analysis of hypernuclear spectra
  - Calibration of hyperon-nucleon interaction models
- **Summary & outlook**

# Many thanks to my collaborators

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# Introduction & motivation

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## Strangeness physics

- Deals with properties and interactions of strange particles, such as $\Lambda, \Sigma, \ldots$ hyperons and $K, \eta, \ldots$ mesons
- Interdisciplinary field connecting particle physics, nuclear physics, and astrophysics
- One of its major goals is to understand the elusive interaction of hyperons with nucleons and the nuclear medium

## Hyperon puzzle of neutron stars

- Neutron stars are compact objects with $M_{\mathrm{NS}} \approx M_{\mathrm{Sun}}$ and $R_{\mathrm{NS}} \approx 10\,\mathrm{km}$
- Their structure is governed by interplay of **gravity**, **strong interactions**, and QM Pauli **exclusion principle** under extreme conditions
- High density & strangeness-changing weak-interaction processes $\rightsquigarrow$ presence of hyperons energetically favorable
- Predictions of $M_{\mathrm{NS}} \lesssim 1.4\,M_{\mathrm{Sun}}$ in stark contrast with observations
- Different solutions, from modified gravity to **poor knowledge of hyperon interactions**

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## Constraining hyperon-nucleon interactions

- **YN scattering:** 'pure' but very difficult to realize $\rightsquigarrow$ sparse database with large uncertainties ($\sim$ 40 cross section measurements)
- **Final-state interactions** in hyperon photoproduction (CLAS Collaboration)
- Heavy-ion collisions: **production and decays** of light hypernuclei, correlation **femtoscopy** (ALICE, STAR Collaborations)
- **Lattice QCD:** interaction parameters, potentials (HAL QCD, NPLQCD Collaborations)
- **Exotic hyperatoms:** $\Sigma^-$, $\Xi^-$ (J-PARC)
- **Hypernuclei:** precise spectroscopy of hypernuclear energy levels
  - $\sim$ 40 species of $\Lambda$ hypernuclei observed: ${}^{3}_{\Lambda}\mathrm{H}$, ...${}^{208}_{\Lambda}\mathrm{Pb}$
  - few double-$\Lambda$ hypernuclei ${}^{6}_{\Lambda\Lambda}\mathrm{He}, \ldots, {}^{13}_{\Lambda\Lambda}\mathrm{Be}$
  - ${}^{4}_{\Sigma}\mathrm{He}$, ${}^{15}_{\Xi}\mathrm{C}$ candidate states
  - antihypernuclei ${}^{3}_{\bar{\Lambda}}\bar{\mathrm{H}}$, ${}^{4}_{\bar{\Lambda}}\bar{\mathrm{H}}$

# Introduction & motivation

## Theoretical analysis of hypernuclei

- Using **'effective'** *YN* interaction models & mean-field / shell-model approaches – successful but difficult to link with the underlying free-space *YN* interaction, limited predictive power
- Using **'realistic'** (free-space) *YN* interaction models
  - Combines modern developments of *YN* interactions based on $\chi$EFT ($\not\pi$EFT) and ab initio few- and many-body approaches
  - Computationally demanding
  - Can reveal deficiencies of existing *YN* interaction models

## Calibration of YN interaction models using hypernuclei requires

- Advanced ab initio **computational methods**
- Quantified **method uncertainties**, $\sigma_{\text{method}}$ – associated with the solution of the many-body problem
- Quantified **model uncertainties**, $\sigma_{\text{model}}$ – associated with the choice of the nuclear interaction
- Overcoming the **computational demands** – large number of evaluations needed

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# Ab initio calculations of light hypernuclei

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- Ab initio methods aim to solve the (hyper)nuclear many-body problem starting from realistic (free-space) interactions exactly or with controlled approximations

## Ab initio no-core shell model

- Quasi-exact method to solve the few- and many-body Schrödinger equation

$$\left(\sum \frac{\hat{\mathbf{p}}_i^2}{2m_i} + \sum \hat{V}_{NN;ij} + \sum \hat{V}_{NNN;ijk} + \sum \hat{V}_{YN;ij}\right)\Psi = E\Psi$$

[Navrátil et al., JPG 36, 083101 (2009); DG et al., FBS 55, 857 (2014); Wirth et al., PRL 113, 192502 (2014); Le et al., EPJA 56, 301 (2020)]

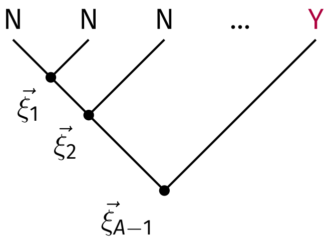
- Wave function is expanded and Hamiltonian is diagonalized in a *finite* $A$-particle harmonic oscillator (HO) basis

$$\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_A) = \sum_{N \leq N_{max}} \Phi^{HO}_{N,\omega}(\mathbf{r}_1, \ldots, \mathbf{r}_A)$$

Systematically improvable, converges to exact results for $N_{max} \to \infty$

# Ab initio no-core shell model

- NCSM formulated in single-particle Slater-determinat HO basis ($M$-scheme, heavier systems) or relative Jacobi-coordinate HO basis (few-body systems)




$$\vec{\xi}_0 \propto \text{center of mass}$$

$$\vec{\xi}_1 = \sqrt{\tfrac{1}{2}}\,(\vec{r}_1 - \vec{r}_2)$$

$$\vec{\xi}_2 = \sqrt{\frac{2mm_Y}{2m+m_Y}}\left[\frac{1}{2\sqrt{m}}(\vec{r}_1 + \vec{r}_2) - \frac{1}{\sqrt{m_Y}}\vec{r}_3\right]$$

$$\vdots$$

$$\vec{\xi}_{A-1} = \sqrt{\frac{(A-1)mm_Y}{(A-1)m+m_Y}}\left[\frac{1}{(A-1)\sqrt{m}}(\vec{r}_1 + \cdots \vec{r}_{A-1}) - \frac{1}{\sqrt{m_Y}}\vec{r}_A\right]$$

Basis states:

$$|\alpha, J^\pi T\rangle \equiv |\underbrace{N_{A-1} i_{A-1} J_1 T_1}_{\text{antisymmetric (A-1)N state}}, \overbrace{n_Y l_Y j_Y t_Y}^{\text{Y state}}, J^\pi T\rangle$$

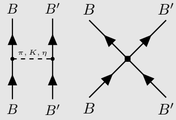
No spurious center-of-mass contributions

# Ab initio no-core shell model

## Input interactions derived from chiral EFT

- Effective theory of QCD at low energies
- Long-range part ($\pi, K, \eta$-exchange) predicted by $\chi$PT
- Short-range part parametrized by contact interactions, LECs fitted to experimental data

At LO:




## *NN*+*NNN* interaction

- N$^3$LO *NN* [Entem, Machleidt, PRC 68, 041001 (2003)]
  + N$^2$LO *NNN* potential [Navrátil, FBS 41, 14 (2007)], ...
- NNLO$_{sim}$ $NN + NNN$ potential family [Carlsson et al., PRX 6, 011019 (2016)]

## *YN* interaction

- Chiral LO potential [Polinder *et al.*, NPA 779, 244 (2006)], (available up to N$^2$LO)
- $\Lambda N - \Sigma N$ mixing explicitly taken into account:

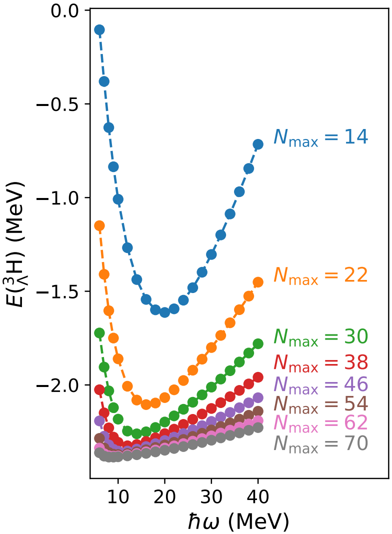
$$V_{YN} = \begin{pmatrix} V_{\Lambda N-\Lambda N} & V_{\Lambda N-\Sigma N} \\ V_{\Sigma N-\Lambda N} & V_{\Sigma N-\Sigma N} \end{pmatrix} + \Delta m$$

Coupled-channel Λ-hypernucleus – Σ-hypernucleus problem!

# Uncertainty quantification

# Ab initio calculations of light hypernuclei: method uncertainties

- Method uncertainties associated with convergence of the solution of the many-body problem




- NCSM-calculated energies typically exhibit undesired dependence on the HO basis frequency $\hbar\omega$ and truncation $N_{\max}$
- Convergence properties of observables calculated in finite HO bases are rather well understood [Wendt et al., PRC 91, 061391 (2015)]
  - NCSM model-space parameters $(N_{\max}, \hbar\omega)$ recast into infrared (IR) and ultraviolet (UV) scales $(L_{\mathrm{IR}}, \Lambda_{\mathrm{UV}})$
  - In a regime with negligible UV corrections, IR corrections are universal

$$E(L_{\mathrm{IR}}) = E_\infty + a_0 \exp(-2\kappa_\infty L_{\mathrm{IR}}) + \cdots$$

# Ab initio calculations of light hypernuclei: method uncertainties

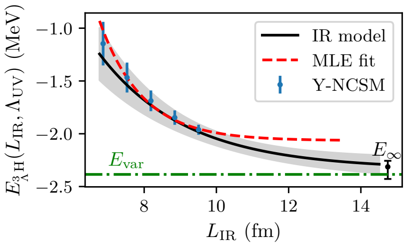
- **Infrared extrapolation formulated as a Bayesian inference problem**

[DG, Htun, Forssén, PRC 106, 054001 (2022)]

$$E(L_{\mathrm{IR}}) = E_\infty + \Delta E_{\mathrm{IR}} \exp(-2\kappa_\infty \Delta L_{\mathrm{IR}}) \times \left(1 + \frac{\epsilon_{\mathrm{NLO}}}{\kappa_\infty (L_{\mathrm{IR,\,max}} + \Delta L_{\mathrm{IR}})}\right),$$

with data $\mathcal{D} = \{E(L_{\mathrm{IR},i})\}$ calculated in different model spaces and $\vec{\epsilon}_{\mathrm{NLO}} \sim N(0, \Sigma(\bar{\epsilon}, \rho))$ providing a stochastic model for the NLO energy correction

- Validation for ${}^{3}_{\Lambda}\mathrm{H}$




- **Method uncertainty** quantified by 68 % credible interval for the extrapolated energy $E_\infty$

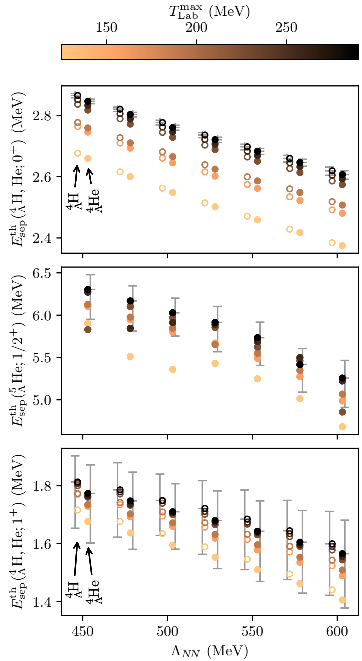
| | $B_\Lambda^{\mathrm{Exp}}$ (MeV) | $B_\Lambda^{\mathrm{th}}$ (MeV) median | 68 % $\mathrm{CI}_{\mathrm{method}}$ |
|---|---|---|---|
| ${}^{3}_{\Lambda}\mathrm{H}$ | 0.165(44) | 0.166 | $[-0.001, +0.001]$ |
| ${}^{4}_{\Lambda}\mathrm{H}$ | 2.157(77) | 2.78 | $[-0.01, +0.01]$ |
| ${}^{4}_{\Lambda}\mathrm{He}$ | 2.39(3) | 2.76 | $[-0.01, +0.01]$ |
| ${}^{5}_{\Lambda}\mathrm{He}$ | 3.12(2) | 6.03 | $[-0.28, +0.18]$ |
| ${}^{4}_{\Lambda}\mathrm{H}; 1^+$ | 1.067(80) | 1.75 | $[-0.12, +0.10]$ |
| ${}^{4}_{\Lambda}\mathrm{He}; 1^+$ | 0.984(50) | 1.71 | $[-0.13, +0.10]$ |

# Ab initio calculations of light hypernuclei: model uncertainties

- Energy levels of light hypernuclei are sensitive to details of the *YN* and *NN*+*NNN* interactions
- One can naively expect that calculated Λ separation energies should be insensitive to the choice of nuclear interaction
- We employed the family of 42 different $\mathrm{NNLO_{sim}}$ [Carlsson et al., PRX 6, 011019 (2016)] nuclear *NN*+*NNN* interactions to expose the magnitude of systematic model uncertainties in $B_\Lambda$
- Model uncertainty connected to the choice of nuclear Hamiltonian quantified by variance, $\sigma^2(\mathrm{NNLO_{sim}})$, of predictions for $B_\Lambda$

| | ${}^3_\Lambda\mathrm{H}$ | ${}^4_\Lambda\mathrm{H}$ | ${}^4_\Lambda\mathrm{He}$ | ${}^5_\Lambda\mathrm{He}$ | ${}^4_\Lambda\mathrm{H}; 1^+$ | ${}^4_\Lambda\mathrm{He}; 1^+$ |
|---|---|---|---|---|---|---|
| $\sigma_{\mathrm{model}}$ (MeV) | 0.02 | 0.08 | 0.08 | 0.36 | 0.07 | 0.07 |

[DG, Htun, Forssén, PRC 106, 054001 (2022)]



# Theoretical uncertainties in hypertriton lifetime

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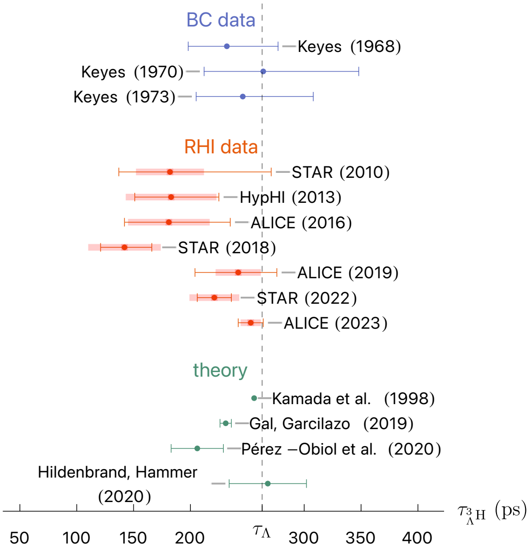
## Hypertriton

- The lightest bound hypernucleus with spin-parity $J^\pi = \frac{1}{2}^+$
- A '$\Lambda pn$' bound state with tiny $\Lambda$ hyperon separation energy $B_\Lambda = 164(43)$ keV, implying a $\Lambda{-}^2$H mean distance $\approx 10$ fm
- Is expected to have lifetime within few % of the free $\Lambda$ lifetime $\tau_\Lambda$ governed to 99.7% by nonleptonic $\Lambda \to N\pi$ weak decay

## Hypertriton lifetime puzzle

- World average of measured $\tau({}^3_\Lambda\mathrm{H})$ is $\sim 20\%$ shorter than $\tau_\Lambda$=263(2) ps!



# Theoretical uncertainties in hypertriton lifetime

## Hypertriton decay channels

- Mesonic modes due to $\Lambda \to N\pi$ (not Pauli blocked as in heavier hypernuclei)

  ${}^{3}_{\Lambda}\mathrm{H} \to {}^{3}\mathrm{He}+\pi^-$ ${}^{3}_{\Lambda}\mathrm{H} \to {}^{3}\mathrm{H}+\pi^0$

  ${}^{3}_{\Lambda}\mathrm{H} \to d+p+\pi^-$ ${}^{3}_{\Lambda}\mathrm{H} \to d+n+\pi^0$

  ${}^{3}_{\Lambda}\mathrm{H} \to p+p+n+\pi^-$ ${}^{3}_{\Lambda}\mathrm{H} \to p+n+n+\pi^0$

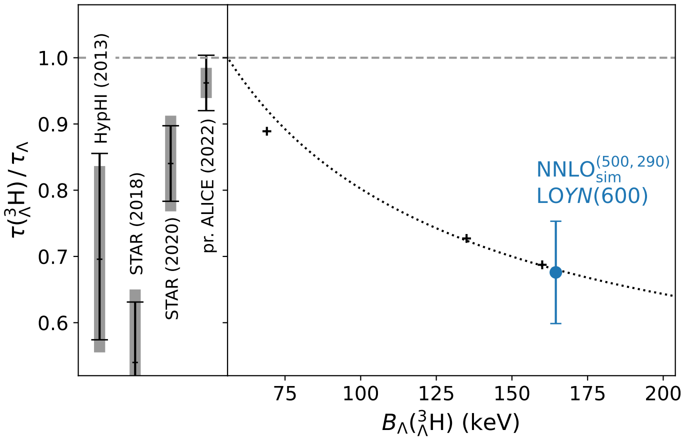
- Rare non-mesonic modes due to $\Lambda N \to NN$

  ${}^{3}_{\Lambda}\mathrm{H} \to d+n$ ${}^{3}_{\Lambda}\mathrm{H} \to n+n+p$

- Employed ab initio NCSM ${}^{3}_{\Lambda}\mathrm{H}$, ${}^{3}\mathrm{He}$ wave functions to compute the ${}^{3}_{\Lambda}\mathrm{H}$ 2-body $\pi^-$ decay rate $\Gamma_{{}^{3}_{\Lambda}\mathrm{H}\to{}^{3}\mathrm{He}+\pi^-}$
- Deduced the ${}^{3}_{\Lambda}\mathrm{H}$ lifetime $\tau({}^{3}_{\Lambda}\mathrm{H})$ by using the measured branching ratio $R_3 = \Gamma_{{}^{3}_{\Lambda}\mathrm{H}\to{}^{3}\mathrm{He}+\pi^-}/\Gamma_{\pi^-} = 0.35(4)$ to obtain the inclusive $\pi^-$ ${}^{3}_{\Lambda}\mathrm{H}$ decay rate and employing the $\Delta T = 1/2$ rule to include all $\pi^0$ decay modes
- Accounted for significant but opposing contributions of pionic FSI and $\Sigma \to N\pi$ due to $\Sigma NN$ admixtures in ${}^{3}_{\Lambda}\mathrm{H}$
- Quantified theoretical uncertainties due to hypernuclear interactions

[Pérez-Obiol, DG, Friedman, Gal, PLB 811, 135916 (2020);
DG, Pérez-Obiol, Friedman, Gal, PRC 109, 024001 (2024)]

# Theoretical uncertainties in hypertriton lifetime




- $B_\Lambda({}^3_\Lambda\mathrm{H})$ poorly known experimentally and suffers from large theoretical uncertainties
- None of the conflicting RHI measured $\tau({}^3_\Lambda\mathrm{H})$ can be excluded but rather associated with its own value of $B_\Lambda$

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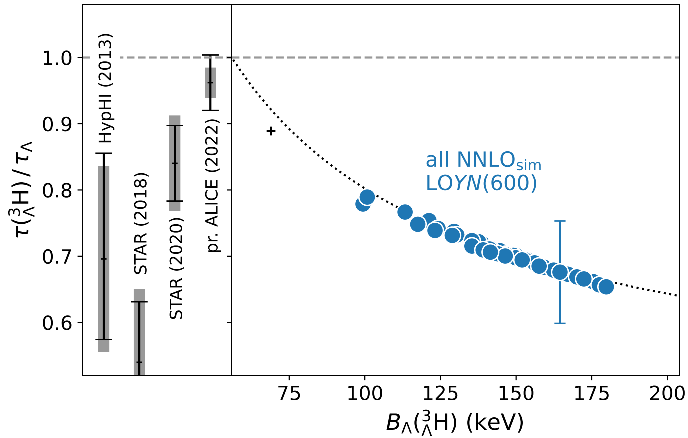


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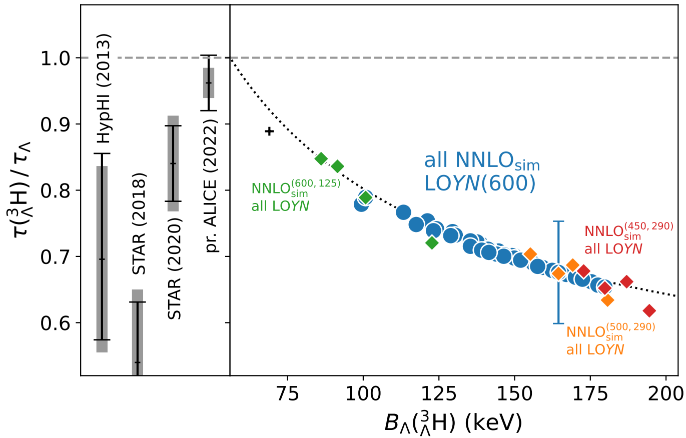


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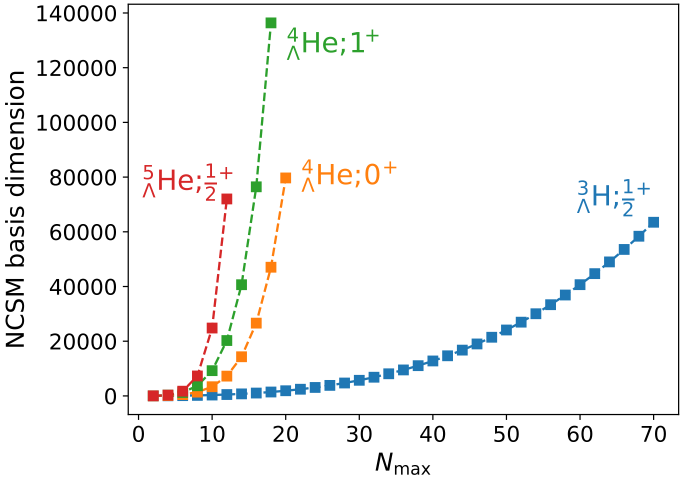
# Calibration of hyperon-nucleon interaction models using light hypernuclei

# Ab initio calculations of hypernuclei: the curse of dimensionality

- Ab initio methods provide a reliable link between the properties of hypernuclei and the underlying hyperon–nucleon interactions
- Is it possible to directly incorporate them in optimization of hyperon-nucleon forces which require a large number of model evaluations?




- This is not feasible given their computational cost

# Emulating ab initio NCSM calculations: eigenvector continuation

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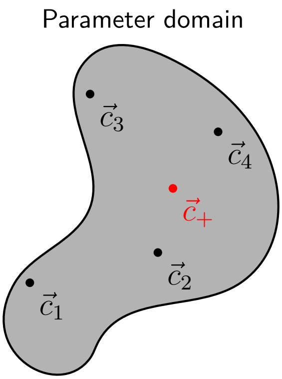
- **Eigenvector continuation** is based on the fact that when a Hamiltonian depends smoothly on some real-valued control parameter(s), any eigenvector is a smooth function of that parameter(s) and its trajectory is confined to a very low-dimensional subspace

  [Frame et al., PRL 121, 032501 (2018); König et al., PLB 810, 135814 (2020)]



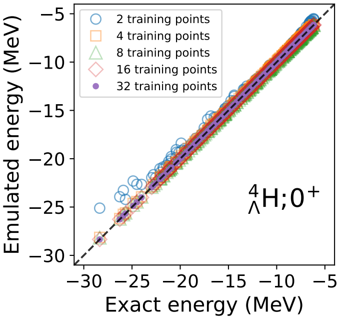

- Write the Hamiltonian in a linearized form $H(\vec{c}) = H_0 + \sum c_i H_i$
- Select 'training' points $\{\vec{c}_i\}$ and solve the exact problem $H(\vec{c}_i)\,|\psi_i\rangle = E_i\,|\psi_i\rangle$
- Project the Hamiltonian onto the subspace of training eigenvectors $\{|\psi_i\rangle\}$ and diagonalize the generalized eigenvalue problem

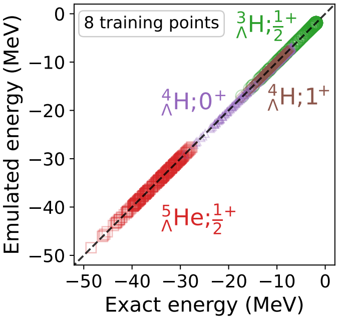
$$\tilde{H}(\vec{c}_+)\,|\tilde{\psi}\rangle = \tilde{E}_+\tilde{N}\,|\tilde{\psi}\rangle\,,$$

where $\tilde{H}_{ij} = \langle\psi_i|H(\vec{c}_+)|\psi_j\rangle$, $\tilde{N}_{ij} = \langle\psi_i|\psi_j\rangle$, and $\tilde{E}_+$ approximates $E_+$

# Emulating ab initio NCSM calculations: eigenvector continuation

- Hypernuclear Hamiltonian with LO *YN* interactions can be linearized,
$$H(\vec{C}) = H_0 + C_{27}V_{27} + C_{10^*}V_{10^*} + C_{10}V_{10} + C_{8a}V_{8a} + C_{8s}V_{8s},$$
where $C_i$s are the 5 independent $\mathrm{SU}_f(3)$ LECs
- **Cross validation**
  - Select 2, 4, 8, 16, 32 points in the 5-dimensional space of LO *YN* LECs using the Latin hypercube space-filling design in a $\pm 40\,\%$ domain around the nominal values to train the emulators
  - Select randomly 256 exact NCSM calculations within the same domain




- We can achieve relative accuracy of $|\delta_{\mathrm{rel}}| < 1, 0.1, 0.002\,\%$ using 8, 16, 32 training points

- Accurate and lighting-fast emulation of ab initio NCSM calculations

# Application: global sensitivity analysis of hypernuclear spectra

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## Global sensitivity analysis

- Addresses the question of how variance of the output of a model can be apportioned to variances of the model inputs [Saltelli et al., CPC 181, 259 (2010)]
- Allows to identify the most influential LECs of $\chi$EFT YN interactions which determine the hypernuclear energy spectra

- For an output $Y = f(\vec{\alpha})$ of a model $f$, we decompose the total variance as

$$\mathrm{Var}\,[Y] = \sum_{i=1}^{d} V_i + \sum_{i<j=1}^{d} V_{ij} + \cdots ,$$

where

$$V_i = \mathrm{Var}\,[E_{\vec{\alpha}\sim(\alpha_i)}[Y|\alpha_i]],$$

$$V_{ij} = \mathrm{Var}\,[E_{\vec{\alpha}\sim(\alpha_i,\alpha_j)}[Y|\alpha_i, \alpha_j]] - V_i - V_j,$$

are variances of conditional expectation of $Y$

- The variance integrals are computed by using quasi-MC sampling, including 95 % confidence intervals
- The first-, second-, and higher-order (Sobol') sensitivity indices

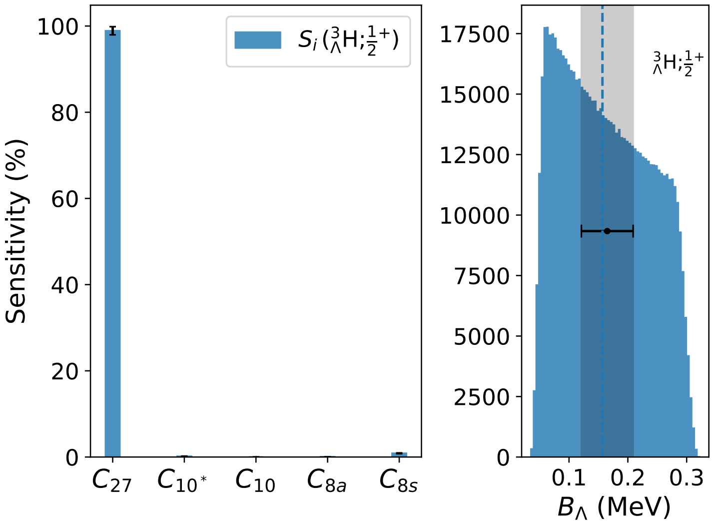
$$S_i = \frac{V_i}{\mathrm{Var}\,[Y]}, \quad S_{ij} = \frac{V_{ij}}{\mathrm{Var}\,[Y]}, \quad \cdots$$

- Total effect

$$S_{Ti} = S_i + S_{ij} + \cdots$$

# Application: global sensitivity analysis of hypernuclear spectra

- **Identify the most influential LECs:**
  $Y =$ Λ separation energies of ${}^{3}_{\Lambda}\mathrm{H}_{\frac{1}{2}^+}$, ${}^{4}_{\Lambda}\mathrm{H}_{0^+}$, ${}^{4}_{\Lambda}\mathrm{He}_{0^+}$, ${}^{4}_{\Lambda}\mathrm{H}_{1^+}$, ${}^{4}_{\Lambda}\mathrm{He}_{1^+}$, ${}^{5}_{\Lambda}\mathrm{He}_{\frac{1}{2}^+}$,
  $\vec{\alpha} =$ the 5 SU(3) LECs of the LO $YN$ interaction; independent and uniformly distributed within $\pm 2\,\%$ ($\pm 20\,\%$) variation around the nominal values of LO$YN$($\Lambda_{YN}$=600 MeV) for ${}^{3}_{\Lambda}\mathrm{H}$ ($A = 4, 5$)




- $S_i \approx S_{Ti} \rightsquigarrow$ energies are additive in all LECs
- $C_{27}$ is responsible for most of the variation in energy

$$C^{\Lambda\Lambda}_{{}^1S_0} = \frac{1}{10}(9C_{27} + C_{8s})$$
$$C^{\Lambda\Lambda}_{{}^3S_1} = \frac{1}{2}(C_{10^*} + C_{8a})$$
$$C^{\Sigma\Sigma}_{{}^3S_1} = C_{10}$$

# Application: global sensitivity analysis of hypernuclear spectra

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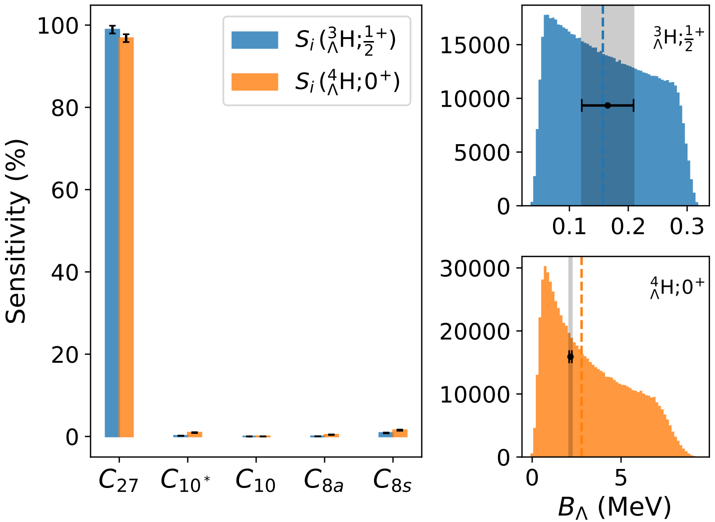


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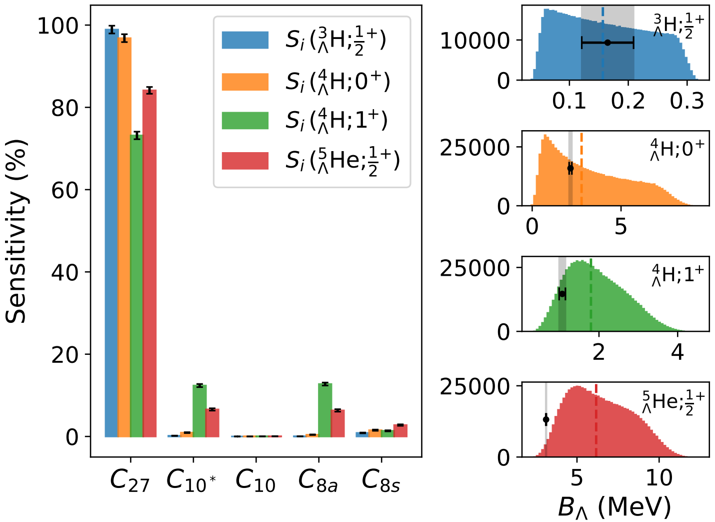


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- $C_{27}$ is responsible for most of the variation in energy

$$C^{\Lambda\Lambda}_{^1S_0} = \frac{1}{10}(9C_{27} + C_{8s})$$
$$C^{\Lambda\Lambda}_{^3S_1} = \frac{1}{2}(C_{10^*} + C_{8a})$$
$$C^{\Sigma\Sigma}_{^3S_1} = C_{10}$$

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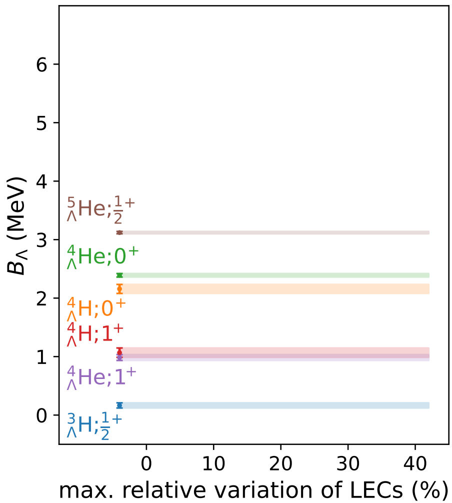
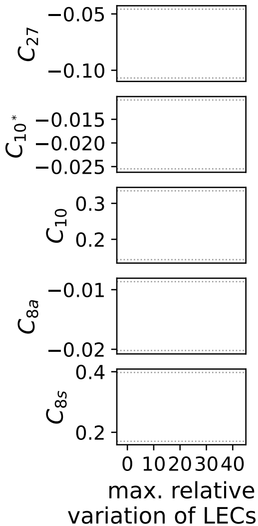
- Simultaneous fitting of *bound-state and scattering* observables is inevitable
- Can we improve the description of $\Lambda$ separation energies in light hypernuclei with a small variation of LO *YN* LECs?




- Proof of principle, simple least-squares optimization
- LECs restricted up to $\pm 40\,\%$ variation around the nominal values of LO*YN*($\Lambda_{YN}$=600 MeV)
- Theoretical precision $\sigma^2_{\text{th}} = \sigma^2_{\text{method}} + \sigma^2_{\text{model}}$

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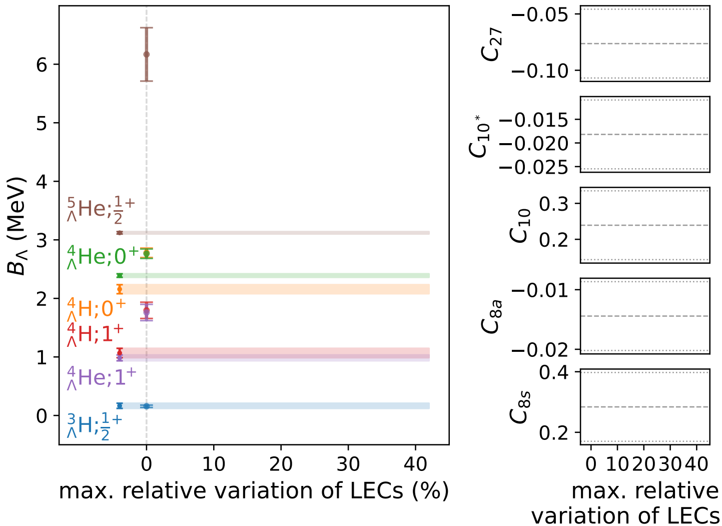


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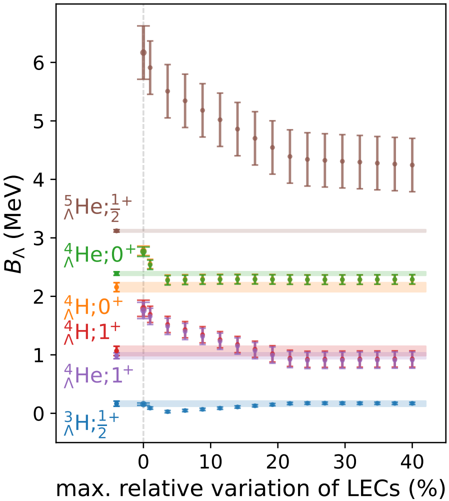


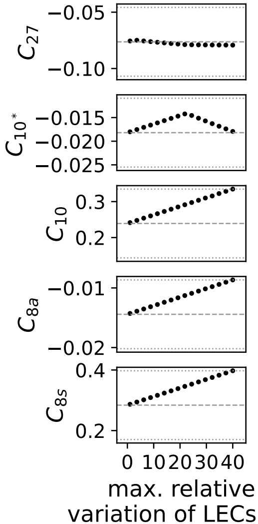


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# Summary & outlook

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## Ab initio calculations of light hypernuclei

- Hypernuclear observables, such as **Λ separation energies** in light hypernuclei, suffer from **sizable theoretical uncertainties** associated with the choice of nuclear interaction

## Emulating ab initio NCSM

- **Eigenvector continuation** provides **fast and accurate** emulation of ab initio calculations of light hypernuclei
- Global sensitivity analysis identifies **the most influential LECs** of $\chi$EFT YN interaction which **determine the energy spectra** of light hypernuclei
- A significantly better description of energy levels of light hypernuclei can be achieved with a relatively small variation of the LO$YN$($\Lambda_{YN}$=600 MeV) LECs

## Outlook

- **Simultaneous optimization** of YN interactions using bound-state and scattering observables with accompanying **uncertainty quantification**

**Thank you!**

# Backup slides

## The NNLO$_{sim}$ family of *NN*+*NNN* potentials

- Parameters fitted to reproduce simultaneously $\pi N$, *NN*, and *NNN* low-energy observables
- family of 42 Hamiltonians where the experimental uncertainties propagate into LECs

$$\left.\begin{array}{ll} T_{NN}^{\text{lab,max}} & \leq 125, \ldots, 290 \text{ MeV} \\ \Lambda_{\text{EFT}} & \leq 450, \ldots, 600 \text{ MeV} \end{array}\right\} 42\ V_{NN}+V_{NNN} \text{ potentials}$$

- All Hamiltonians give equally good description of the fit data
- Note that $\Delta E^{(^3\text{He}/^3\text{H})} \approx 0$ (fitted) while $\Delta E_{g.s.}^{(^4\text{He})} \approx 1.5$ MeV

[Carlsson et al., PRX 6, 011019 (2016)]