CS 161: Homework # 5

Daniel Geng
ID: 504588536

Fall 2016

1)

P	Q	$P \implies \neg Q$	$Q \implies \neg P$
0	0	**1**	**1**
0	1	**1**	**1**
1	0	**1**	**1**
1	1	**0**	**0**

$P \implies \neg Q$ and $Q \implies \neg P$ are equivalent.

P	Q	$P \iff \neg Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
0	0	**0**	**0**
0	1	**1**	**1**
1	0	**1**	**1**
1	1	**0**	**0**

$P \iff \neg Q$ and $((P \wedge \neg Q) \vee (\neg P \wedge Q))$ are equivalent.

2)

		$[\alpha]$	$[\beta]$	
S	F	$S \implies F$	$\neg S \implies \neg F$	$\alpha \implies \beta$
0	0	**1**	**1**	**1**
0	1	**1**	**0**	**0**
1	0	**0**	**1**	**1**
1	1	**1**	**1**	**1**

Invalid, but satisfiable.

			$[\alpha]$	$[\beta]$	
S	F	H	$S \implies F$	$\neg S \implies \neg F$	$\alpha \implies \beta$
0	0	0	**1**	**1**	**1**
0	0	1	**1**	**0**	**0**
0	1	0	**1**	**1**	**1**
0	1	1	**1**	**1**	**1**
1	0	0	**0**	**0**	**1**
1	0	1	**0**	**0**	**1**
1	1	0	**1**	**1**	**1**
1	1	1	**1**	**1**	**1**

Invalid, but satisfiable.

S	F	H	$[\alpha]$ $(S \vee H) \implies F$	$[\beta]$ $(S \implies F) \vee (H \implies F)$	$\alpha \iff \beta$
0	0	0	**1**	**1**	**1**
0	0	1	**1**	**1**	**1**
0	1	0	**1**	**1**	**1**
0	1	1	**1**	**1**	**1**
1	0	0	**1**	**1**	**1**
1	0	1	**0**	**0**	**1**
1	1	0	**1**	**1**	**1**
1	1	1	**1**	**1**	**1**

Valid (and satisfiable).

3) Knowledge base: Mythical (Y), Immortal (I), Mammal (M), Horned (H), Magical (G)

- $Y \implies I$
- $\neg Y \implies (\neg I \wedge M)$
- $(I \vee M) \implies H$
- $H \implies G$

CNF

$Y \implies I$

$= (\neg Y \vee I)$

$\neg Y \implies (\neg I \wedge M)$

$= Y \vee (\neg I \wedge M)$

$= (Y \vee \neg I) \wedge (Y \vee M)$

$(I \vee M) \implies H$

$= \neg(I \vee M) \vee H$

$= (\neg I \wedge \neg M) \vee H$

$= (H \vee \neg I) \wedge (H \vee \neg M)$

$H \implies G$

$= (\neg H \vee G)$

CNF: $(\neg Y \vee I) \wedge (Y \vee \neg I) \wedge (Y \vee M) \wedge (H \vee \neg I) \wedge (H \vee \neg M) \wedge (\neg H \vee G)$

1. $(\neg Y \vee I)$
2. $(Y \vee \neg I)$
3. $(Y \vee M)$
4. $(H \vee \neg I)$
5. $(H \vee \neg M)$
6. $(\neg H \vee G)$

Prove Mythical (Y):

7. $\neg Y$ (Try to find contradiction)
8. $\neg I$ (2, 7)
9. M (3, 7)
10. H (5, 9)
11. G (6, 10)

No contradictions found, cannot prove that the unicorn is mythical with this knowledge base.

Prove Magical (G):

7. $\neg G$ (Try to find contradiction)
8. $\neg H$ (6, 7)
9. $\neg I$ (4, 8)
10. $\neg M$ (5, 8)
11. Y (3, 10)
12. I (1, 11)

Statements 9 and 12 contradict each other, so the unicorn must be magical.

Prove Horned (H):

7. $\neg H$ (Try to find contradiction)
8. $\neg I$ (6, 7)
9. $\neg M$ (5, 8)
10. Y (3, 9)
11. I (1, 10)

Statements 8 and 11 contradict each other, so the unicorn must be horned.