

CS161: Homework #7

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1) Generalized product rule: Prove $P(A, B|K) = P(A|B, K)P(B|K)$

$$P(A, B|K) = \frac{P(A, B, K)}{P(K)} = \frac{P(A|B, K)P(B|K)P(K)}{P(K)} = P(A|B, K)P(B|K)$$

Generalized Bayes' rule: Prove $P(A|B, K) = P(B|A, K)P(A|K)/P(B|K)$

$$\begin{aligned} P(A|B, K) &= \frac{P(B, K|A)P(A)}{P(B, K)} = \frac{P(B|K, A)P(K|A)P(A)}{P(B, K)} = \frac{P(B|K, A)P(A|K)P(K)}{P(B, K)} \\ &= \frac{P(B|A, K)P(A|K)P(K)}{P(B|K)P(K)} = \frac{P(B|A, K)P(A|K)}{P(B|K)} \end{aligned}$$

2) $P(oil) = 0.5$

$P(natural) = 0.2$

$P(neither) = 0.3$

$P(positive|oil) = 0.9$

$P(positive|natural) = 0.3$

$P(positive|neither) = 0.1$

$$P(oil|positive) = \frac{P(positive|oil)P(oil)}{P(positive)}$$

$$P(positive) = P(positive|oil)P(oil) + P(positive|natural)P(natural) + P(positive|neither)P(neither)$$

$$P(positive) = (0.9 * 0.5) + (0.3 * 0.2) + (0.1 * 0.3) = 0.54$$

$$P(oil|positive) = \frac{0.9 * 0.5}{0.54} = 0.8\bar{3}$$

3)

| World | Black | Square | One | $P()$ |
|-------|-------|--------|-----|-------|
| 1 | 0 | 0 | 0 | 1/13 |
| 2 | 0 | 0 | 1 | 1/13 |
| 3 | 0 | 1 | 0 | 1/13 |
| 4 | 0 | 1 | 1 | 1/13 |
| 5 | 1 | 0 | 0 | 2/13 |
| 6 | 1 | 0 | 1 | 1/13 |
| 7 | 1 | 1 | 0 | 4/13 |
| 8 | 1 | 1 | 1 | 2/13 |

$$P(\alpha_1) = P(\omega_5) + P(\omega_6) + P(\omega_7) + P(\omega_8) = 9/13$$

$$P(\alpha_2) = P(\omega_3) + P(\omega_4) + P(\omega_7) + P(\omega_8) = 8/13$$

$$P(\alpha_3) = P(\text{square} | \text{one} \vee \text{black}) = P(\text{square}, (\text{one} \vee \text{black})) / P(\text{one} \vee \text{black})$$

$$P(\alpha_3) = \frac{P(\omega_4) + P(\omega_7) + P(\omega_8)}{P(\omega_2) + P(\omega_4) + P(\omega_5) + P(\omega_6) + P(\omega_7) + P(\omega_8)} = \frac{7/13}{11/13} = 7/11$$

Find sentences that show α is independent of β given γ : $P(\alpha|\gamma) = P(\alpha|\beta, \gamma)$

$$\alpha = \text{one}, \beta = \text{square}, \gamma = \neg \text{black}: P(\text{one} | \neg \text{black}) = 2/4, P(\text{one} | \neg \text{black}, \text{square}) = 1/2$$

$$\alpha = \text{one}, \beta = \text{square}, \gamma = \text{black}: P(\text{one} | \text{black}) = 3/9, P(\text{one} | \text{black}, \text{square}) = 2/6$$

4) a) $I(A, \emptyset, \{B, E\})$

$$I(B, \emptyset, \{A, C\})$$

$$I(C, A, \{B, D, E\})$$

$$I(D, \{A, B\}, \{C, E\})$$

$$I(E, B, \{A, C, D, F, G\})$$

$$I(F, \{C, D\}, \{A, B, E\})$$

$$I(G, F, \{A, B, C, D, E, H\})$$

$$I(H, \{E, F\}, \{A, B, C, D, G\})$$

b) $d_sep(A, BH, E)$ is false because path $ACFHE$ is unblocked.

$d_sep(G, D, E)$ is false because path $GFCADBE$ is unblocked.

$d_sep(AB, F, GH)$ is false because path BEH is unblocked.

c) $P(a, b, c, d, e, f, g, h) =$

$$P(a|b, c, d, e, f, g, h)P(b|c, d, e, f, g, h)P(c|d, e, f, g, h)P(d|e, f, g, h)P(e|f, g, h)P(f|g, h)P(g|h)P(h)$$

d) $P(A = 0, B = 0) = P(A = 0)P(B = 0) = 0.8 * 0.3 = 0.24$

E and A are independent, so just find $P(E = 1)$.

$$P(E = 1) = P(E = 1 | B = 0)P(B = 0) + P(E = 1 | B = 1)P(B = 1) = 0.9 * 0.3 + 0.1 * 0.7 = 0.34$$