CS161: Homework #7

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1) Generalized product rule: Prove $P(A,B|K) = P(A|B,K)P(B|K)$

$$P(A,B|K) = \frac{P(A,B,K)}{P(K)} = \frac{P(A|B,K)P(B|K)P(K)}{P(K)} = P(A|B,K)P(B|K)$$

Generalized Bayes' rule: Prove $P(A|B,K) = P(B|A,K)P(A|K)/P(B|K)$

$$P(A|B,K) = \frac{P(B,K|A)P(A)}{P(B,K)} = \frac{P(B|K,A)P(K|A)P(A)}{P(B,K)} = \frac{P(B|K,A)P(A|K)P(K)}{P(B,K)}$$

$$= \frac{P(B|A,K)P(A|K)P(K)}{P(B|K)P(K)} = \frac{P(B|A,K)P(A|K)}{P(B|K)}$$

2) $P(oil) = 0.5$

$P(natural) = 0.2$

$P(neither) = 0.3$

$P(positive|oil) = 0.9$

$P(positive|natural) = 0.3$

$P(positive|neither) = 0.1$

$$P(oil|positive) = \frac{P(positive|oil)P(oil)}{P(positive)}$$

$$P(positive) = P(positive|oil)P(oil)+P(positive|natural)P(natural)+P(positive|neither)P(neither)$$

$$P(positive) = (0.9 * 0.5) + (0.3 * 0.2) + (0.1 * 0.3) = 0.54$$

$$P(oil|positive) = \frac{0.9 * 0.5}{0.54} = 0.8\bar{3}$$

3)

World	Black	Square	One	$P()$
1	0	0	0	1/13
2	0	0	1	1/13
3	0	1	0	1/13
4	0	1	1	1/13
5	1	0	0	2/13
6	1	0	1	1/13
7	1	1	0	4/13
8	1	1	1	2/13

$P(\alpha_1) = P(\omega_5) + P(\omega_6) + P(\omega_7) + P(\omega_8) = 9/13$

$P(\alpha_2) = P(\omega_3) + P(\omega_4) + P(\omega_7) + P(\omega_8) = 8/13$

$P(\alpha_3) = P(square|one \vee black) = P(square, (one \vee black))/P(one \vee black)$

$P(\alpha_3) = \frac{P(\omega_4)+P(\omega_7)+P(\omega_8)}{P(\omega_2)+P(\omega_4)+P(\omega_5)+P(\omega_6)+P(\omega_7)+P(\omega_8)} = \frac{7/13}{11/13} = 7/11$

Find sentences that show α is independent of β given γ: $P(\alpha|\gamma) = P(\alpha|\beta, \gamma)$

$\alpha = one, \beta = square, \gamma = \neg black$: $P(one|\neg black) = 2/4$, $P(one|\neg black, square) = 1/2$

$\alpha = one, \beta = square, \gamma = black$: $P(one|black) = 3/9$, $P(one|black, square) = 2/6$

4) a) $I(A, \varnothing, \{B, E\})$

$I(B, \varnothing, \{A, C\})$

$I(C, A, \{B, D, E\})$

$I(D, \{A, B\}, \{C, E\})$

$I(E, B, \{A, C, D, F, G\})$

$I(F, \{C, D\}, \{A, B, E\})$

$I(G, F, \{A, B, C, D, E, H\})$

$I(H, \{E, F\}, \{A, B, C, D, G\})$

b) $d\_sep(A, BH, E)$ is false because path $ACFHE$ is unblocked.

$d\_sep(G, D, E)$ is false because path $GFCADBE$ is unblocked.

$d\_sep(AB, F, GH)$ is false because path BEH is unblocked.

c) $P(a, b, c, d, e, f, g, h) =$

$P(a|b, c, d, e, f, g, h)P(b|c, d, e, f, g, h)P(c|d, e, f, g, h)P(d|e, f, g, h)P(e|f, g, h)P(f|g, h)P(g|h)P(h)$

d) $P(A = 0, B = 0) = P(A = 0)P(B = 0) = 0.8 * 0.3 = 0.24$

E and A are independent, so just find $P(E = 1)$.

$P(E = 1) = P(E = 1|B = 0)P(B = 0) + P(E = 1|B = 1)P(B = 1) = 0.9 * 0.3 + 0.1 * 0.7 = 0.34$