Group Theory Assignment 1:

\# A **binary operatⁿ** or **law of compositⁿ** on a set G is a functⁿ $G \times G \to G$ that assigns to each pair $(a, b) \in G \times G$ a unique element $a \circ b$ or ab in G, called the compositⁿ of ~~G~~ a and b.

\# A ~~grog~~ **group** $(G, \circ)$ is a set G together w̄ a law of compositⁿ $(a, b) \longmapsto a \circ b$ that satisfies the following axioms: associativity, existence of identity element, and existence of inverse element.

1a. $S = \mathbb{R}$, $a * b = \dfrac{a}{a^2 + b^2}$

Let $a = b = 0$, $a * b = 0 * 0 = \dfrac{0}{0 + 0} = \dfrac{0}{0} \notin \mathbb{R}$

$\therefore$ $*$ is <u>not</u> a binary operatⁿ on S.

1b. $S = \mathbb{R}$, $a * b = a$

As long as $a \in S = \mathbb{R}$, $*$ is a binary operatⁿ.

1c. $S = \{1, 6, 3, 2, 18\} = \{1, 2, 3, 6, 18\}$, $a * b = ab$

Let $a = 2$, $b = 6$, $a * b = 2 * 6 = 12 \notin S$:

$\therefore$ $*$ is <u>not</u> a binary operatⁿ on S.

1e. $S = \{1, -2, 3, 2, -4\}$, $a * b = |b|$

Let $a = 1$, $b = -4$, $a * b = 1 * -4 = |-4| = 4 \notin S$.

$\therefore$ $*$ is <u>not</u> a binary operatⁿ on S.