Topology - X400416

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Contents

These notes are based on Topology (2^{nd} edition) by James R. Munkres.

1 Topological Spaces and Continuous Functions

A metric on a set X is a map $d : X \times X \to [0, \infty)$ that satisfies

1. $d(x, y) = d(y, x)$
2. $d(x, x) = 0$
3. $d(x, y) > 0, x \neq y$
4. $d(x, y) \leq d(x, z) + d(z, y)$

A set $U \subset X$ is open if for all $x \in U$ and some $r > 0$

$$B(x, r) = \{y \in X \mid d(x, y) < r\}$$

s.t. $B(x, r) \subset U$. Union (finite, countable or uncountable) of open sets is open and finite intersections of open sets is open (infinite intersections need not be open)

A function between metric spaces is continuous if and only if a preimage of an open set is open.

Let X be a set. Then a topology on X is a set $\mathcal{T} \subset \mathcal{P}(x)$ s.t.

1. $\varnothing \in \mathcal{J}, X \in \mathcal{J}$
2. For $U_\alpha \in \mathcal{J}, \bigcup_\alpha U_\alpha \in \mathcal{J}$
3. For $(U_i)_{0 \leq i \leq n} \subset \mathcal{J}, \bigcap_{i=0}^n U_i \in \mathcal{J}$

A topological space is the pair $(X, \mathcal{J})$

If X is a set, the a *basis* of a topology is a collection $\mathcal{B}$ of subsets of X s.t.

1. $\forall x \in X, \exists B \in \mathcal{B}$ s.t. $x \in B$
2. If $x \in B_1 \cap B_2$ with $B_1, B_2 \in \mathcal{B}$ then there exists $B_3 \in \mathcal{B}$ with $x \in B_3 \subset B_1 \cap B_2$.