Option Greeks

Daniel Gutierrez Velez

Summary

This document outlines the theoretical approach to implementing various "Greeks" for assessing the sensitivity of options contracts. To validate these theoretical concepts, we use a real-world asset, International Consolidated Airlines, to examine the empirical values of the Greeks as provided by the Refinitiv software.

We then compare these empirical values to the ones calculated using our own VBA implementation and a set of predefined assumptions.

Greeks Motivation

The goal of this written is to understand Greeks from a theoretical perspective. The Greeks are metrics that indicate how sensitive derivative prices, e.g. options, are to changes in various underlying parameters. This analysis is based on the Black-Scholes model, which describes the price behavior of European Call and Put options

The types of Greeks are: Delta, Gamma, Vega, Theta, and Rho. Each represents a different partial derivative order, or sensitivity, such as price changes, volatility, time decay, and interest rate shifts.

Factors like transaction costs, market liquidity, and execution slippage aren't explicitly covered by theoretical models but impact Greeks' effectiveness. Black-Scholes has limitations (e.g., assumptions about market efficiency and constant volatility).

The Greeks are crucial for managing the risk of portfolios containing derivative instruments, allowing traders to isolate price sensitivity to various factors. By calculating these values daily, sophisticated options traders can assess changes that might impact their positions or outlook and decide if their portfolio needs rebalancing. This approach helps in hedging portfolios to achieve desired exposures, such as delta hedging, by addressing specific sensitivities while managing risk.

Greeks Details

The following are the greeks which are covered by this written (there are many other available greeks in the literature).

Delta Δ quantifies how much the theoretical value of an option changes in response to movements in the underlying asset's price. It is the first derivative of the option's value (V) with respect to the price of the underlying instrument (S)

$$Delta = \Delta = \frac{\partial V}{\partial S}$$

Theta Θ gauges how sensitive the value of a derivative is to the passage of time—often referred to as "time decay." This sensitivity reflects the diminishing value of the option as it approaches its expiration date.

$$\text{Theta} = \Theta = \frac{\partial V}{\partial \mathrm{T}}$$

Gamma $\boldsymbol{\Gamma}$ measures how quickly Delta changes in response to movements in the underlying asset's price. It represents the second derivative of the option's value with respect to the underlying price.

$$\text{Gamma} = \Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial\partial V}{\partial\partial S}$$

Vega $\boldsymbol{\nu}$ assesses the sensitivity of an option's value to changes in the volatility of the underlying asset. It represents the derivative of the option value with respect to the underlying asset's volatility.

$$\text{Vega} = \nu = \frac{\partial V}{\partial \sigma}$$

Rho $\boldsymbol{\rho}$ indicates how sensitive an option's value is to changes in interest rates. It represents the derivative of the option's value with respect to the risk-free interest rate for the relevant outstanding term.

$$\text{Rho} = \rho = \frac{\partial V}{\partial r}$$

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Part 1: Theorical Greeks on VBA

This section describes the implementation of the mentioned Greeks in Visual Basic for Excel (VBA). The functions for the four specified Greeks were programmed into a VBA module, allowing the attached Excel workbook to calculate the Greeks based on the following inputs:

Option type (as a string, either "Put" or "Call")
Spot price
Strike price
Time to maturity (in years)
Risk-free rate
Volatility
Dividend yield
For testing, the strike price was set to 100, with a risk-free rate of 1% and annualized volatility of 20%.

To evaluate different Greek values, we analyzed a range of prices from 60 to 140 with a step of 5, and time to maturity from 0.1 to 5 years. The time intervals varied: in the first year, the step was 0.1; after the first year, it was 1.

This analysis was performed for both call and put options. Additionally, we introduced a volatility shock of +50%, increasing the volatility to 70%.

The results are presented in the charts in section **1.1**, where you can observe that sensitivity is generally higher for longer times to maturity and higher spot prices.

1.1 Greeks usage in VBA

The following table shows how to use the coded greeks in VBA.

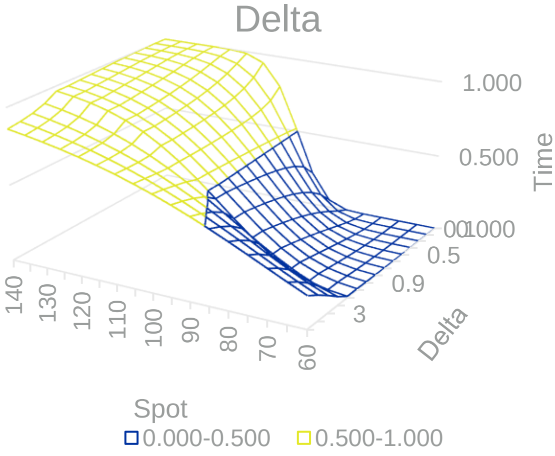
Formula
OptionDelta(OptionType, S, X, T, r, v, d)
OptionTheta(OptionType, S, X, T, r, v, d)
BSGamma(S, X, T, r, v, d)
Vega(S, X, T, r, v, d)
OptionRho(OptionType, S, X, T, r, v, d)

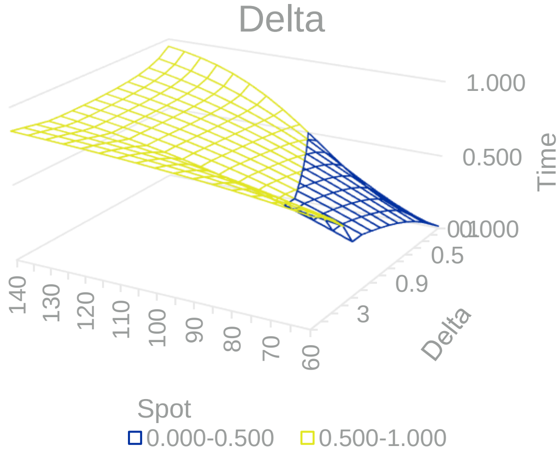
The inputs are almost the same for all formulas, except for Gamma and Vega, where we know that the call and put value are equal.

The inputs for the formulas are:

- **OptionType:** A string flag eigher “C” for call or “P” for put.
- **S:** For the spot price.
- **X:** For the strike price .
- **T:** For the time to maturity in years
- **R:** For the risk free rate.
- **v:** For the annualized volatility of the underlying
- **d:** For the dividend yield of the underlying, which for this written is always assumed as 0.

1.1 VBA Greeks Call + Shock





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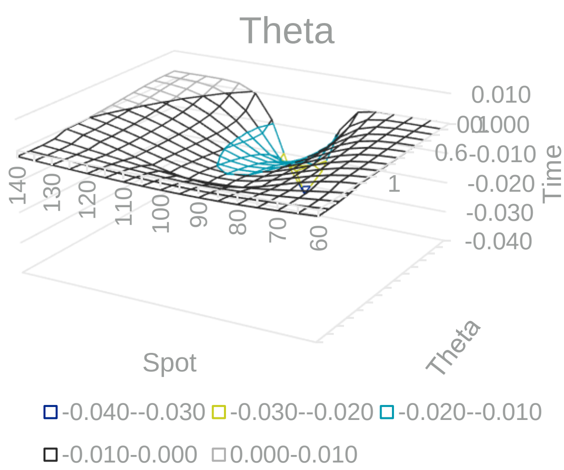
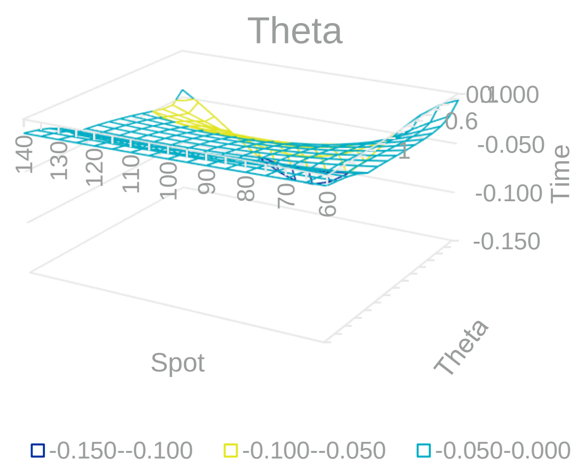
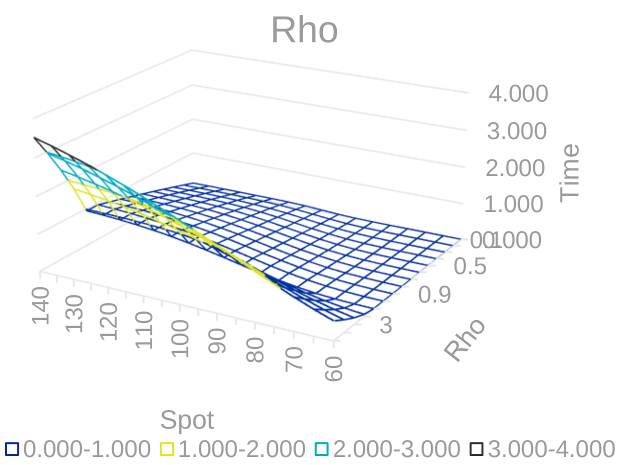
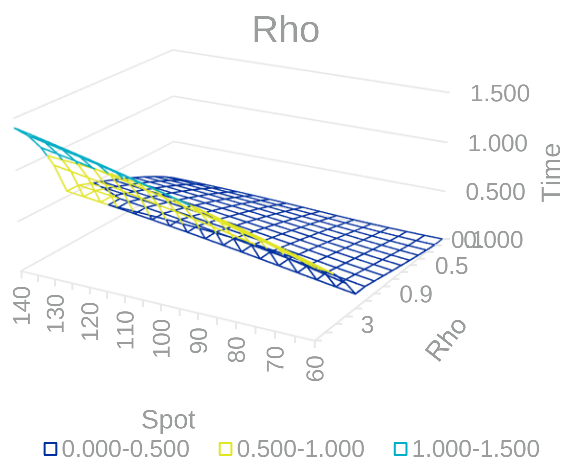
1.1 VBA Greeks Call + Shock









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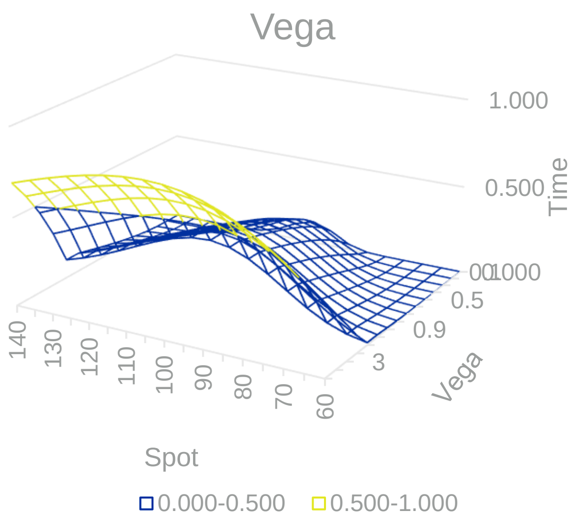
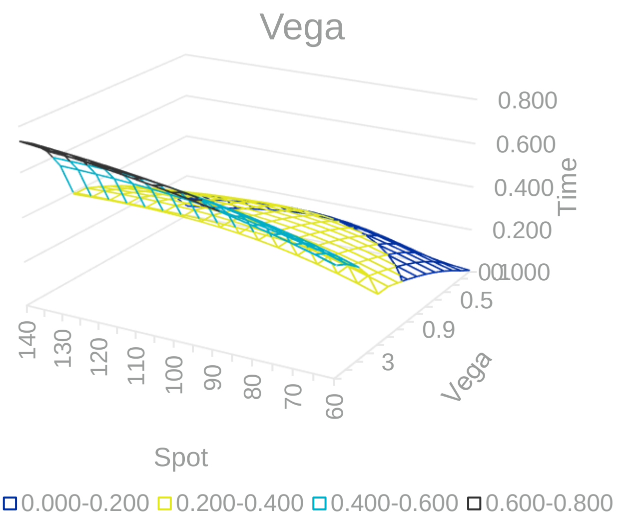
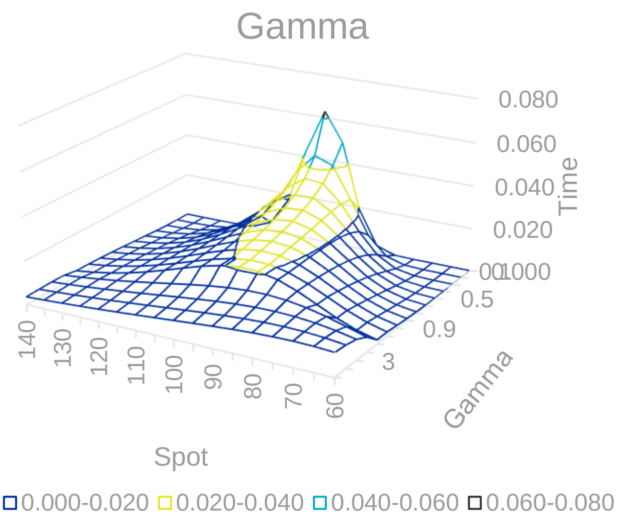
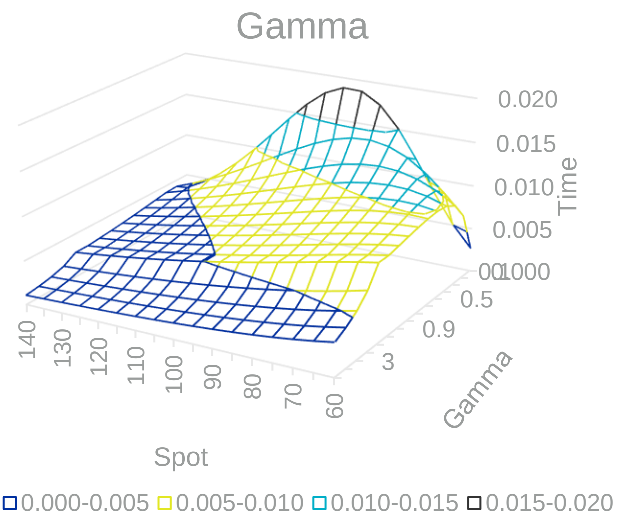
1.1 VBA Greeks (Call/Put) + Shock

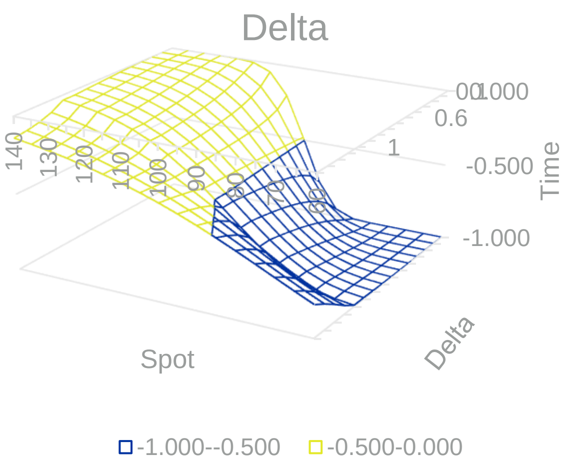


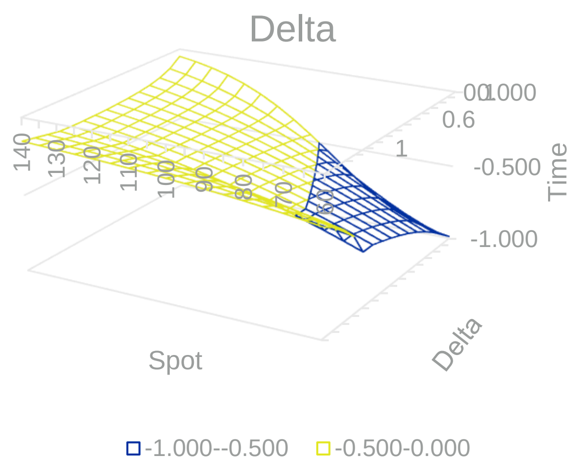


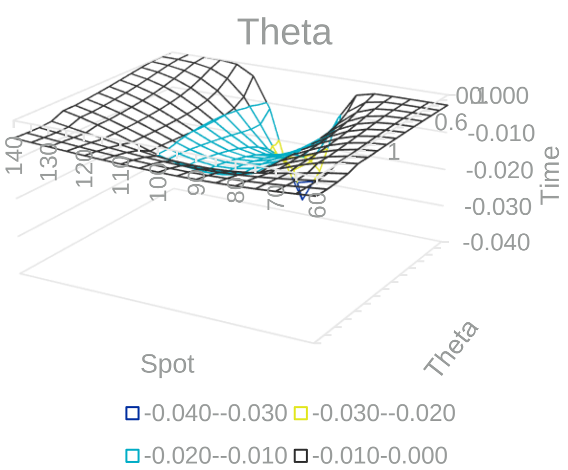


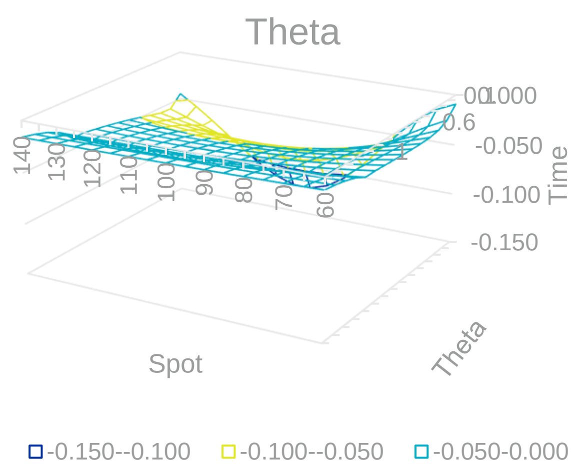


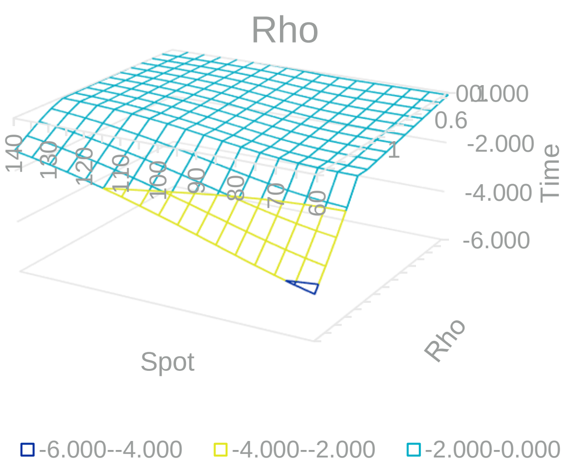
1.1 VBA Greeks Put + Shock

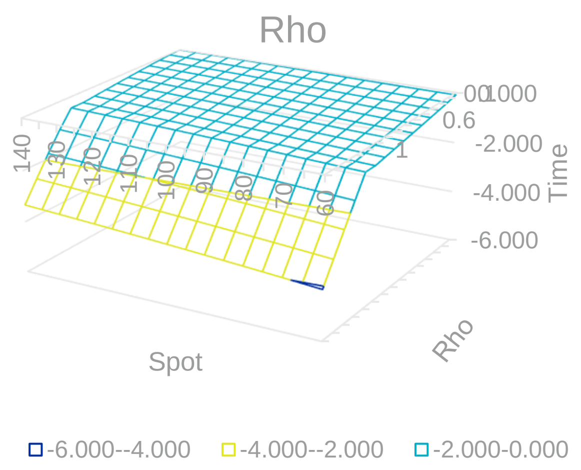












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Part 2: Observed Volatility Surface

For this section a non paying dividend stock was chosen for which there was a liquid enough book built in the market. The asset chosen was **International Consolidated Airlines (ICAG.MC)**. But a slight variation of the asset was chosen for accomplishing the goal of finding a valid book of European options, this asset is listed both in Spain and the UK, the Spanish version was chosen to access the book of European options.

International Consolidated Airlines Group S.A. is a United Kingdom-based airline company that holds interests of airline and ancillary operations. The Company's segments include British Airways, Iberia, Vueling, Aer Lingus and IAG Loyalty. The Company operates various aircraft fleet services.

The Company, through its subsidiaries, is engaged in providing airline marketing, airline operations, insurance, aircraft maintenance, storage and custody services, air freight operations, and cargo transport services. The Company's subsidiaries include AERL Holding Limited, British Airways Plc, IAG Cargo Limited, and IAG GBS Limited, among others.

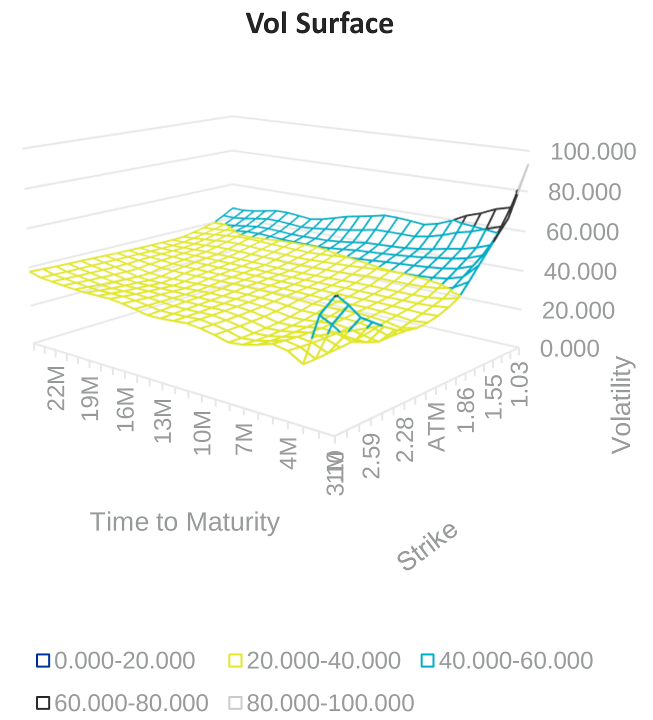
2.1 Volatility Surface

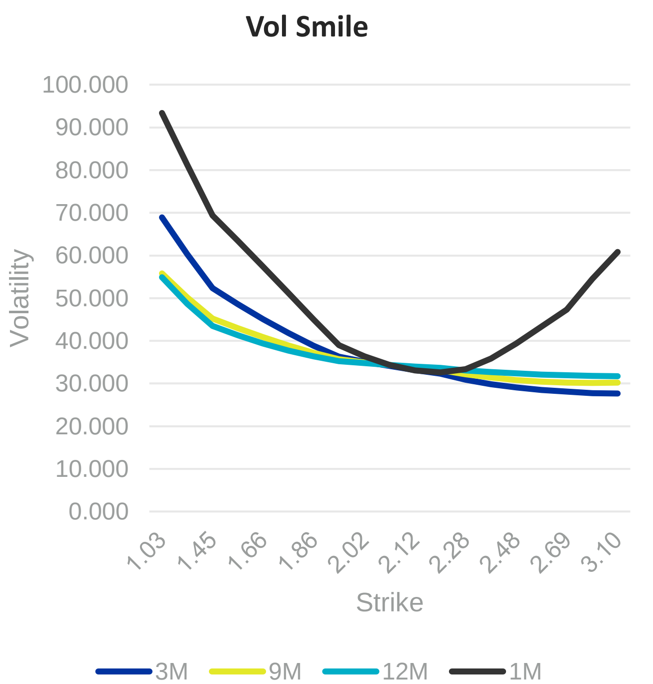
The first aspect we examine in sensitivity of the options is the volatility surface. The plot represents the strike prices, the time to maturity, and the implied volatility corresponding to these factors.

The data from Refinitiv shows that the volatility surface for call options is quite smooth when the maturity period is longer and when the strike price is higher. This suggests that as the strike price and time to maturity increase, volatility tends to be more stable.

However, as we look toward the lower-right corners of the plot—where time to maturity is short—we see that volatility increases, particularly for lower strike prices. This makes sense, as the price would need to change drastically in a short time to reach those lower strike levels.

2.1.1 Call Surface and Smile





As for the smile, in the plot we see that there is a skewness for the lower strike levels, and the skewness is more evident in the shorter times to maturity (1 and 3 months vs the longer ones like 12 months)

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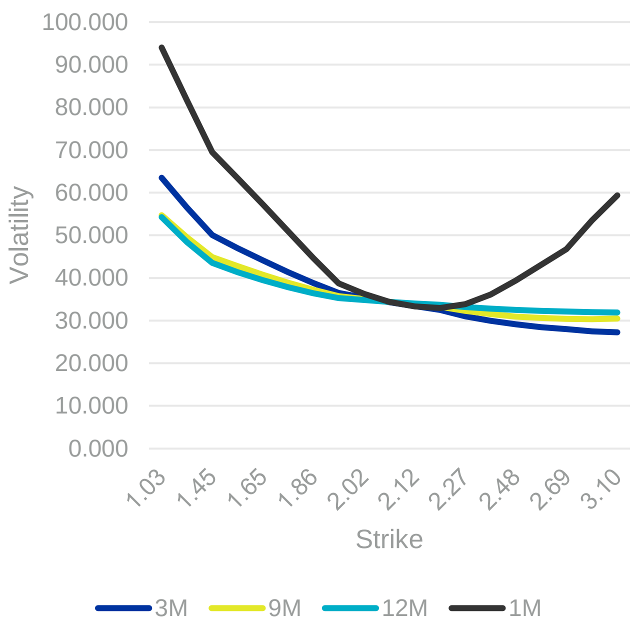
2..1.2 Put Surface and Smile

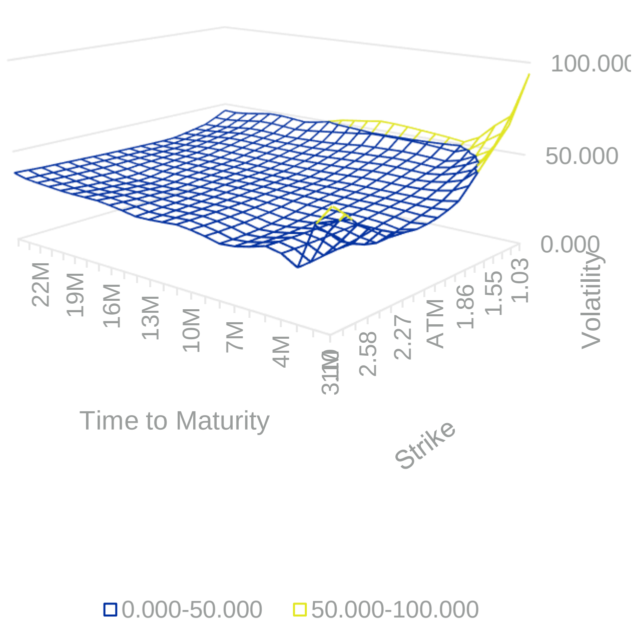
In this section, we focus on put contracts, examining their volatility surface and volatility smiles. As with call options, the volatility surface for puts is smooth on the left side of the plot, indicating longer times to maturity. Conversely, the right side of the chart shows an increase in implied volatility, especially at lower strikes. This suggests that the market anticipates the asset's price to continue rising in both the short and long term, as indicated by the blue areas on the volatility surface. Across all maturities, the implied volatility is higher at lower strike prices.

The volatility smile confirms this pattern, showing higher implied volatility at lower strikes across different times to maturity.

Vol Smile



Vol Surface



2.2 Empirical Greeks

The Refinitiv platform does not offer Greek surfaces for equity options. It allows users to access historical Greeks based on daily price changes or build a portfolio to track real-time Greek movements according to price fluctuations. Therefore, in the following section, we see the Greeks for a given price change (the last observed) and we compare it with the computed Greeks with VBA given a set of assumptions.

We can for example determine the Greeks for a given price and extract the delta for each maturity and strike pair.

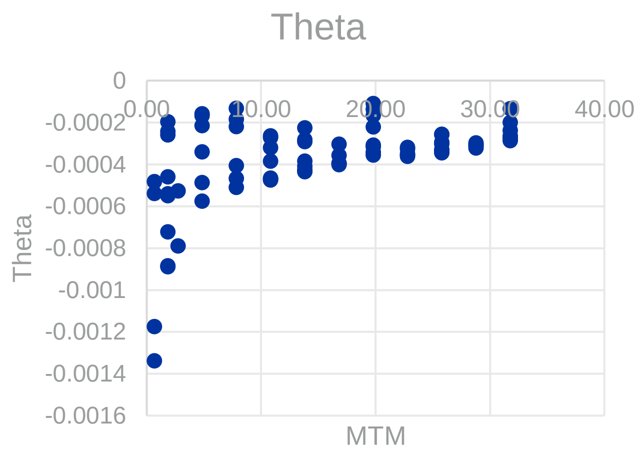
Due to insufficient data points for creating a complete surface, the Greeks will be presented as a scatterplot, showing the relationship between the Greeks and maturity. Each combination of time to maturity and Greek contains multiple data points, representing various strike values for the option.

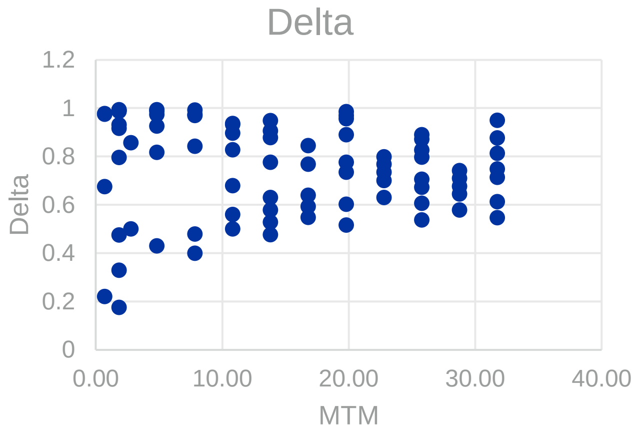
2..2.1 Empirical Greeks

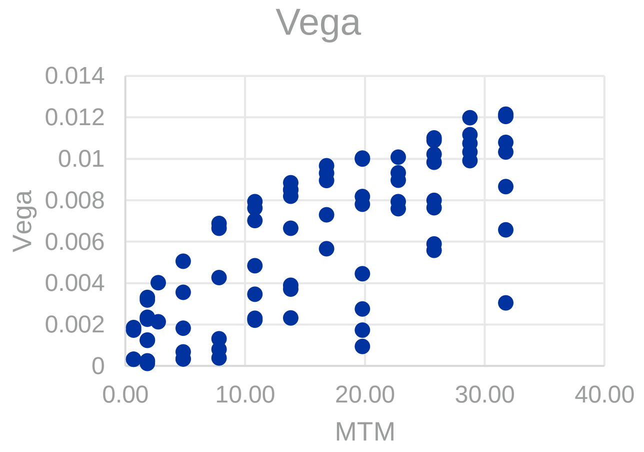
The following set of plots shows the greeks obtained for the book of European call options of the underlying **ICAG.MC.** The book contains a range from strikes of 1 to 2.3 with a tick of 0.1, so in total 14 strikes. And for the maturity we limited the book for up to 32 months, so we have a range from less than one month up to 32 monts (13 different maturities).

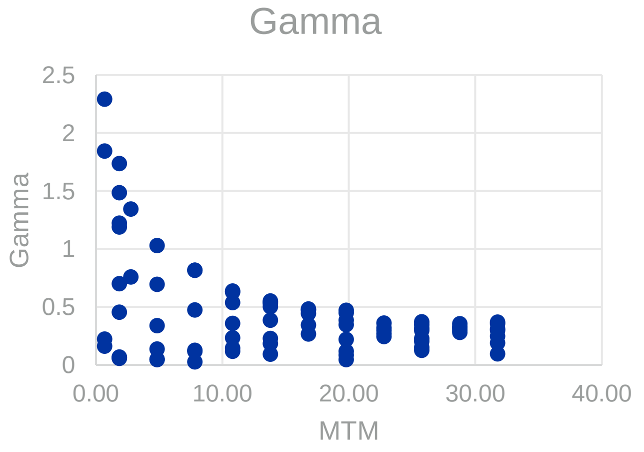
For all almost all greeks we see how the value of the Greek is more concentrated in a single region (smoother) as time to maturity increases. Vega an Rho are the exceptions, where in Vega we see a spreadout in longer times to maturity, while in Rho we see the same concentration along the graph.

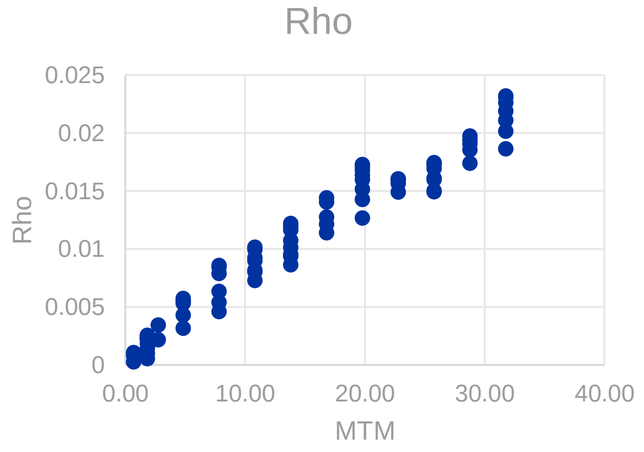
2.2.1 Empirical Greeks











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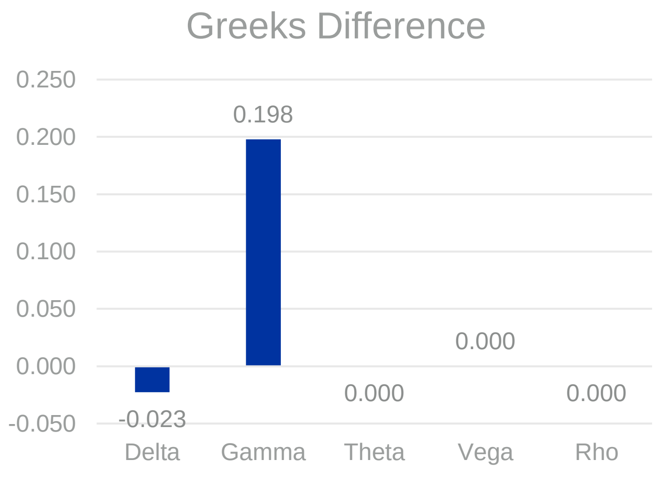
3.1 Theorical vs Empirical Greeks

In this section, we calculate our own version of the Greeks and compare them to those provided by Refinitiv. We focus on an at-the-money (ATM) 3-month call option. To calculate the theoretical Greeks, we use the asset's historical volatility based on data from the last three years. For the risk-free rate, we refer to the OIS-EUR curve and take the 3-month point.

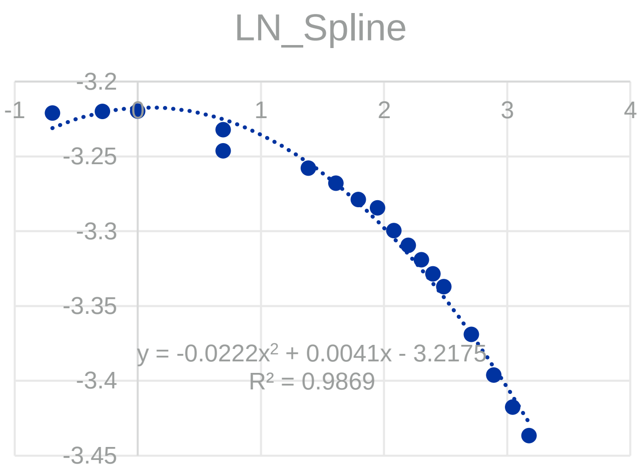
Below is a table outlining the assumptions used for calculating the Greeks:

Input	Theorical	Empirical
Spot	2.07	2.07
Strike	2.10	2.10
Months to Maturity	2.76	2.76
Risk Free rate	3.95%	Not given
Volatility	34.91%	32.52%
Delta	0.524	0.501
Gamma	1.147	1.345
Theta	-0.001	-0.001
Vega	0.004	0.004
Rho	0.002	0.002



3.2 Greeks Surface

For plotting the Greeks surfaces we did a series of assumptions. The first, is that we used the implied volatility for every pair of points (strike, time to maturity) given by the volatility surface of Refinitiv. This way we can calculate the Greeks with their respective volatility and not use just the static historic volatility. A second assumption is that we used the OIS-EUR curve for calculating each Greek risk free rate at each point in time. For the points with no value in the curve, we fitted a quadratic spline on the log-log values of the rate vs maturity to interpolate the curve as follows:



We made a significant assumption: the spot prices for each point on the plot are derived from the range of strike prices in the options book. This means the volatility used corresponds to the original range of strikes. For this analysis, we kept the strike constant at 2.06.

We used a range of spot prices from 1.03 to 3.10 (19 values) and a range of dates from 1 month to 24 months (24 values).

The resulting Greek surfaces are presented on the following page.

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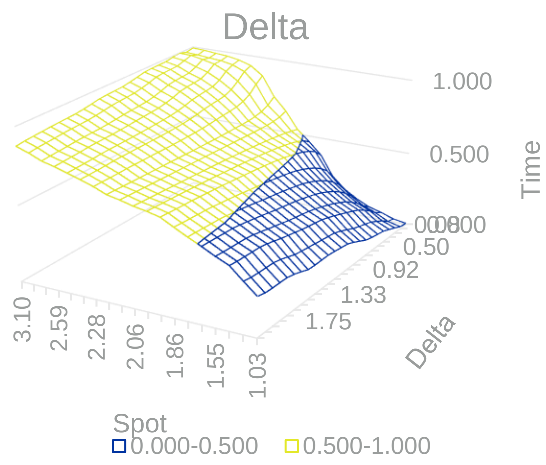
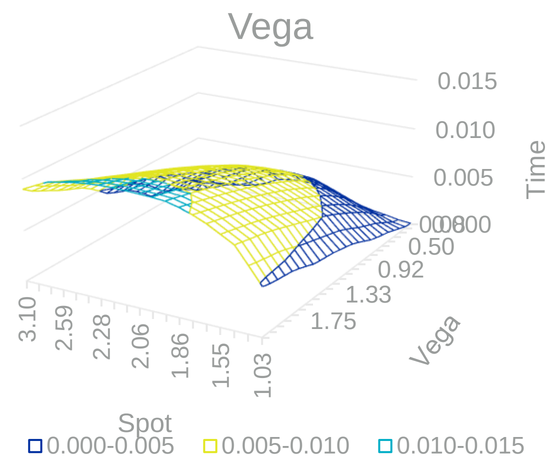
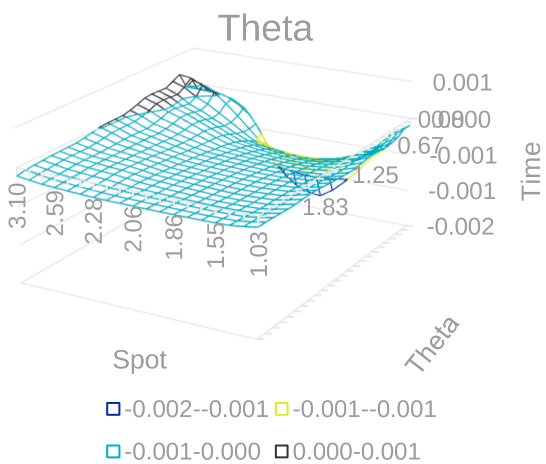
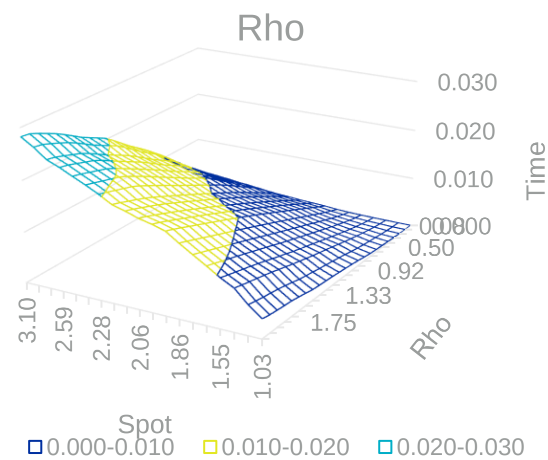
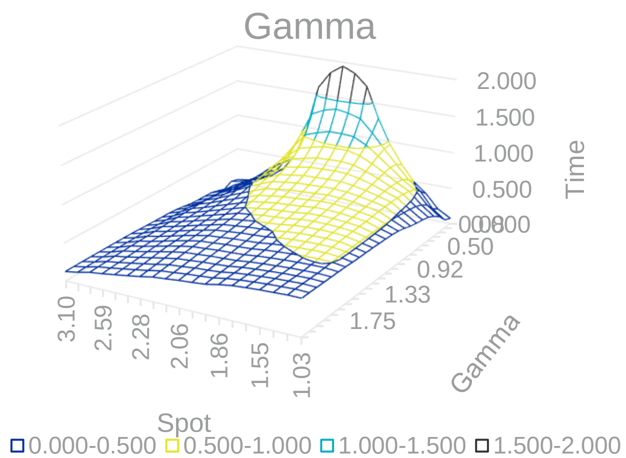
3.2 Greeks Surface (Call)











Conclusions

The Greeks we calculated exhibit a similar pattern in terms of shape to the first theoretical Greeks we presented earlier. While the surface's orientation might be slightly different or the highest points could be in another area, the overall behavior closely aligns with the original calculations. This suggests that our VBA implementation of the Greeks is functioning correctly.

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