Value at Risk (VaR)

Daniel Gutierrez Velez

Summary

This document introduces the Value at Risk (VaR) metric, which is calculated using five different approaches for an equibalanced portfolio and for the individual assets comprising it:

1. Parametric VaR
2. EWMA variance VaR
3. Monte Carlo VaR
4. Historical Simulation VaR
5. Historical Method VaR

We observe that the sum of the VaR metrics for the individual assets is always larger than the metric calculated for the equibalanced portfolio. Some methods, such as the second one, incorporate more recent information and, as a result, output higher VaR values given the recent market volatility.

Introduction

Value at Risk (VaR) is a widely used risk management tool that quantifies the potential loss in value of a risky asset or portfolio over a defined period, for a given confidence interval.
VaR represents the threshold value such that the probability that the loss on the portfolio over a specified period will not exceed this value. This threshold is based on a given probability, $\boldsymbol{\alpha}$. For example, if $\boldsymbol{\alpha}$ = 5%, then at a complementary 95% confidence level, it is expected that the loss will not exceed the VaR value.
The mathematical definition is as follows:

$$VaR_\alpha = \inf\{l : P(L > l) \leq 1 - \alpha\}$$

This makes the VaR the smallest value l for which the probability of the loss L exceeding l is no more than 1- α.
There are several approaches to calculate the VaR which will be covered in this written.

Portfolio Construction and Variance

A portfolio was built using two assets on Refinitiv: Ecopetrol (EC) and Bancolombia (CIB) which are the most traded Colombian equities listed in the NYSE. Using these assets an equibalanced portfolio was constructed for trading 126 days (6 months) and then the variance was calculated using different approaches, to later calculate the VaR of the portfolio and the single equities.

Company information

Ecopetrol is a Colombia-based multinational oil and gas company, which ranks as one of the largest in Latin America and is primarily owned by the Colombian government. The company is involved in the exploration, production, transportation, and sale of oil and natural gas.

Bancolombia is the largest bank in Colombia and has extensive operations across Central America. It offers a broad range of financial products and services, encompassing retail and commercial banking, wealth management, and investment banking. Bancolombia is also a key component of a significant Colombian conglomerate known as Grupo Empresarial Antioqueño (GEA). This group consists of major enterprises involved in diverse industries, including materials and construction, food, insurance, asset management, and banking.

Given their significant roles in the local economy, both stocks likely exhibit some degree of intrinsic correlation. This analysis will demonstrate how diversification can reduce the volatility of individual assets.

Portfolio Summary Figures

Daily returns	Portfolio	EC	CIB
Average	0.09%	-0.04%	0.21%
Variance	0.02%	0.04%	0.03%
Standard Deviation	1.41%	1.91%	1.65%
Annualized St. Dev.	22.43%	30.30%	26.15%

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Parametric VaR

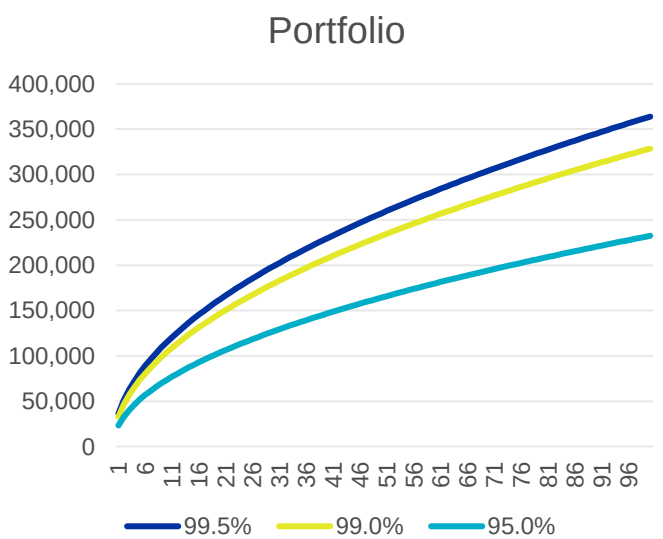
Parametric VaR, also known as the variance-covariance method, is one of the simplest methods to calculate VaR. This approach assumes that the returns of the portfolio are normally distributed.

The VaR is given by:

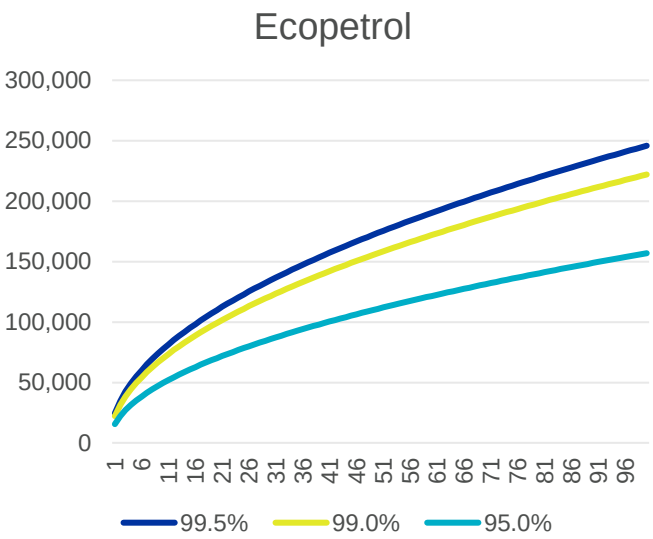
$$VaR_{\alpha} = (z_{\alpha}\,\sigma) * Port\_value$$

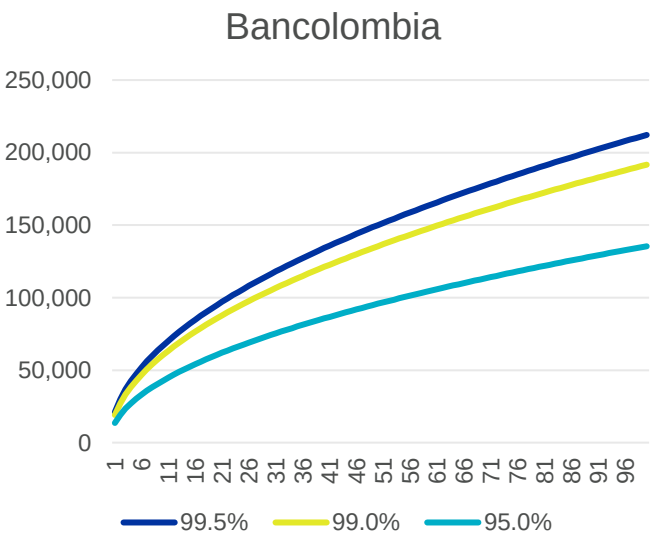
Where σ is the standard deviation of the asset (or portfolio), z_{α} is the z-score corresponding to the desired confidence level α. For all cases we chose the confidence levels 95%, 99% and 99.5%.
It is important to remark that in order to calculate the σ we simply calculate the standard deviation of the last six months (126 trading days) of the portfolio and the individual assets, and then to transform the VaR to the desired period we multiply $\sqrt{t}\sigma$ where t is the number of days to evaluate.

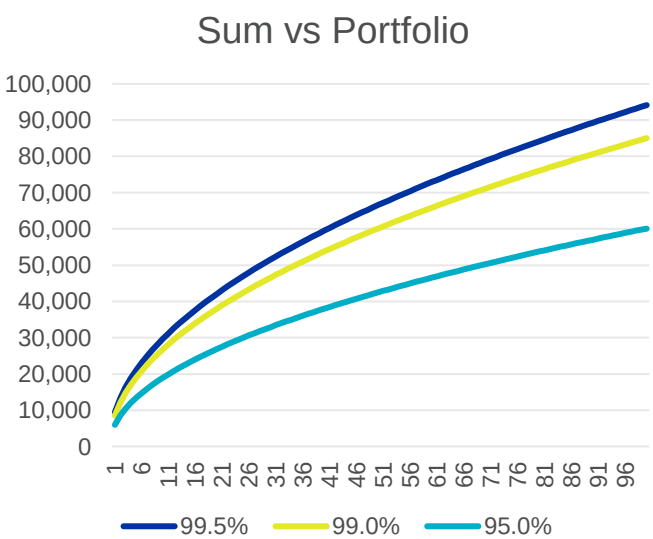
VaR results for 100 days



VaR results for 100 days (cont)







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EWMA VaR

The Exponentially Weighted Moving Average (EWMA) VaR approach is similar to the previous one (parametric VaR) but one transformation is made to reflect the fact that recent volatility plays a more important role than older volatility in the assets.

The parametric VaR assumes that the variance of the assets can be calculated as:

$$\sigma^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-1}^2$$

Where:

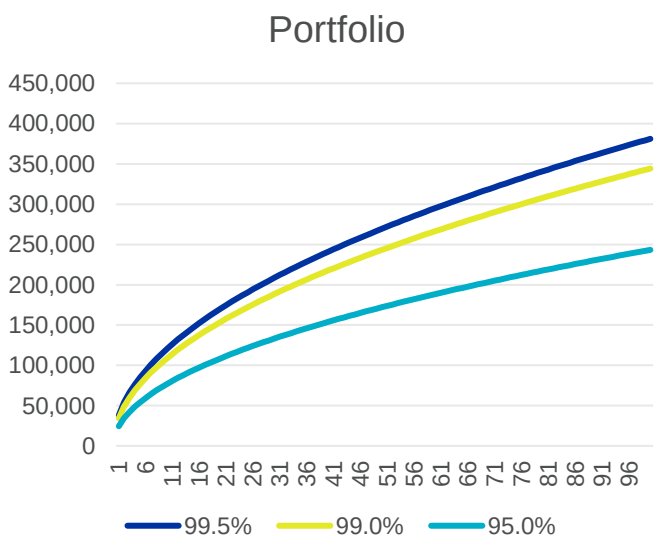
- m = Number of days measured
- n = Day i
- u = Difference between return i from average return

One can interpret that the variance is equally weighted by m. The EWMA approach weights the recent days of the sample with a higher value as:

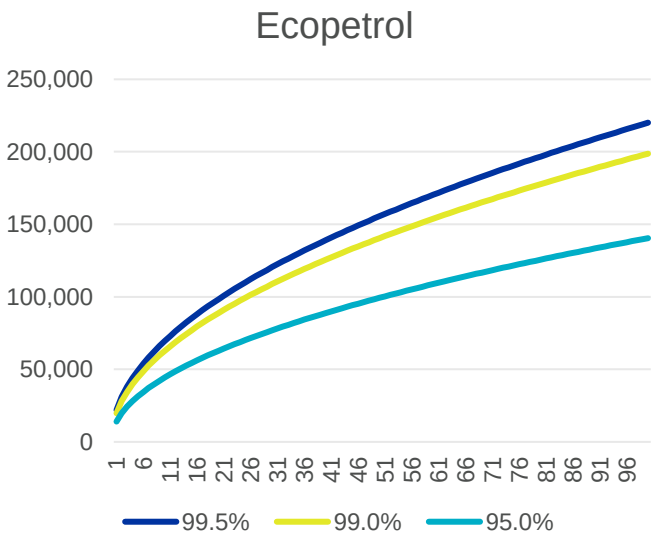
$$\sigma_n^2(EWMA) = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$$

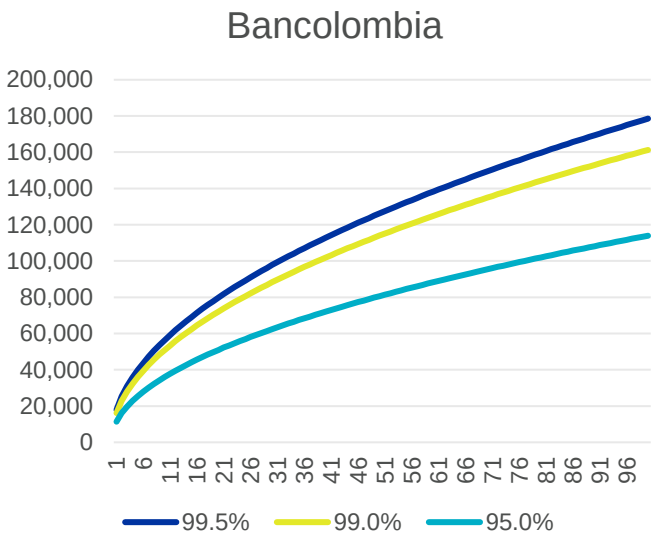
Here the variance today depends on the variance of yesterday and the square return of yesterday, both adjusted by a weighting factor λ, in our calculations equal to 0.94 as suggested by RiskMetrics method.

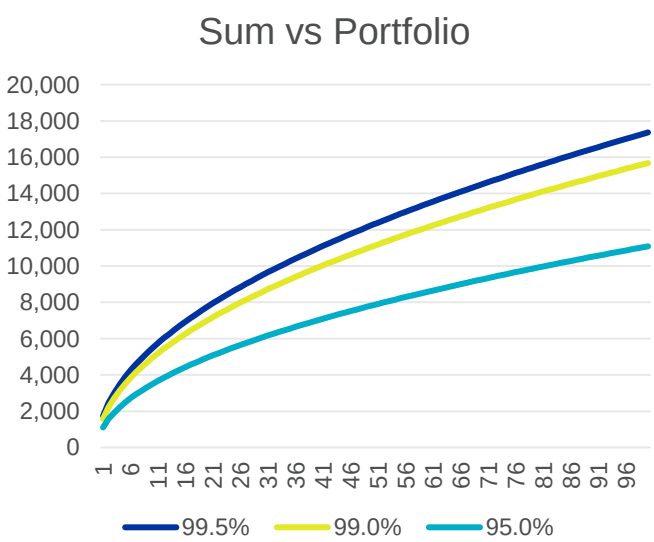
EWMA VaR results for 100 days



EWMA VaR results for 100 days (cont)







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Monte Carlo VaR

The approach of using Monte Carlo simulations to estimate the VaR assumes that the assets follow a Geometric Brownian Motion path. N prices are simulated for a given time target t. In this particular case N = 1000 (so one thousand simulations) for t = 10 days. We will find how much is the maximum loss given a confidence level for the portfolio in 10 trading days.

The stock prices for the simulations are given by:

$$S_T = S_0 \exp\{\left(r + \frac{1}{2}\sigma^2\right)T + \sigma\sqrt{T}\,z$$

Where:

- S_T is the stock price at the end of the period
- S_0 is the initial stock price at time = 0
- r is the risk-free rate
- σ is the annualized standard deviation of the asset (or portfolio). This is the same used in the parametric approach.
- T is the time to maturity in years
- z is a standard normal random variable realization (that is with mean 0 and variance 1)

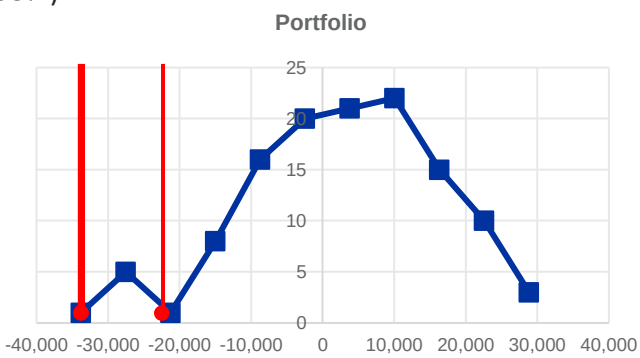
The VaR is given at the end by observing the worst cases the desired confidence level. That is, comparing the loss of the portfolio between t = 0 and t = T and getting the i-th percentile. The following table summarizes the results for our approach:

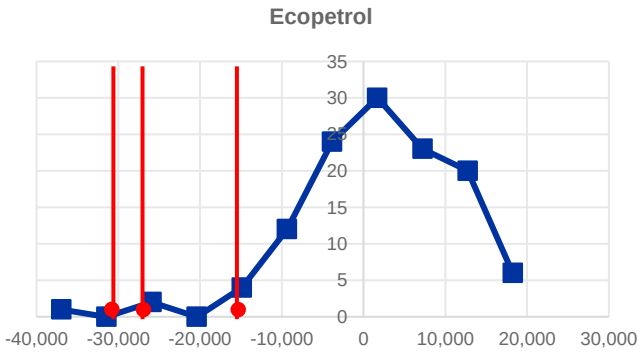
VaR %	99.5%	99.0%	95.0%
Portfolio	10.7%	9.6%	7.0%
EC	16.2%	14.5%	10.2%
CIB	13.3%	12.9%	9.4%

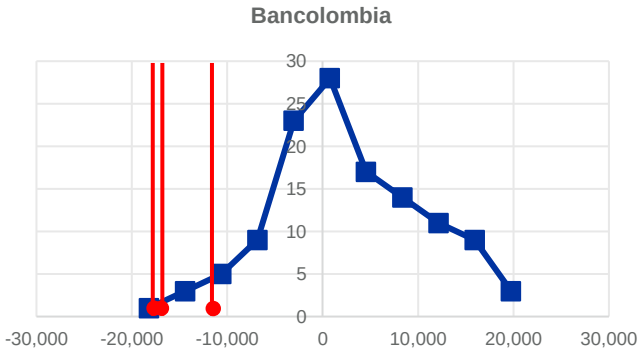
VaR $	99.5%	99.0%	95.0%
Portfolio	107,308	96,318	69,882
EC	80,771	72,434	50,750
CIB	66,715	64,713	46,813
Sum vs Portfolio	40,179	40,829	27,681

Historical Simulation

The historical simulation approach is one of the simplest. Here using historical data of the behavior of the portfolio and the individual assets we estimate the losses of the current portfolio as if assets will follow the same distribution. In simple terms, we observe the past losses and estimate our current maximum losses based on the i-th percentile of the losses given in the observed period.
The following plots present the 1-day loss distribution, and the vertical red lines represent the VaR at the 3 confidence levels (99.5%, 99% and 95%)







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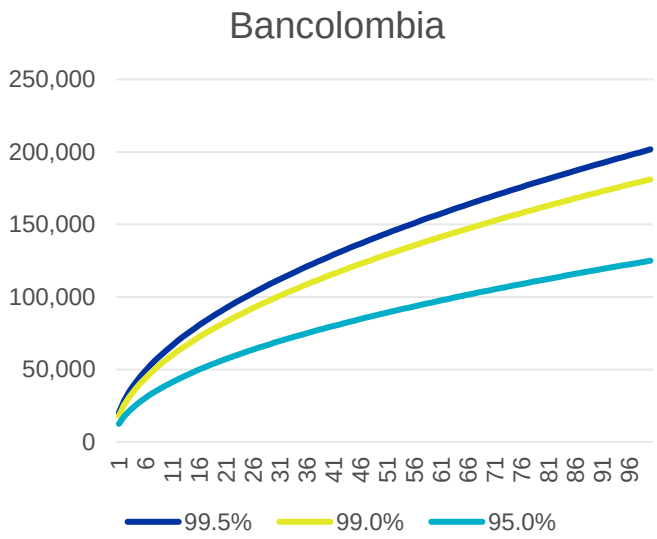
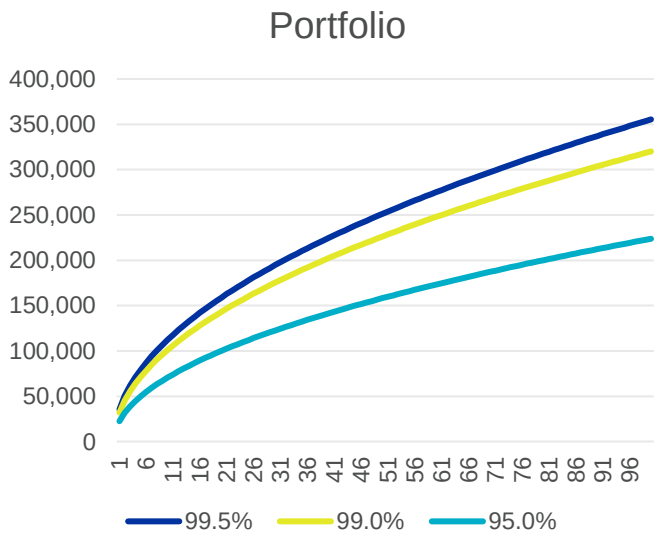
Historical Method VaR

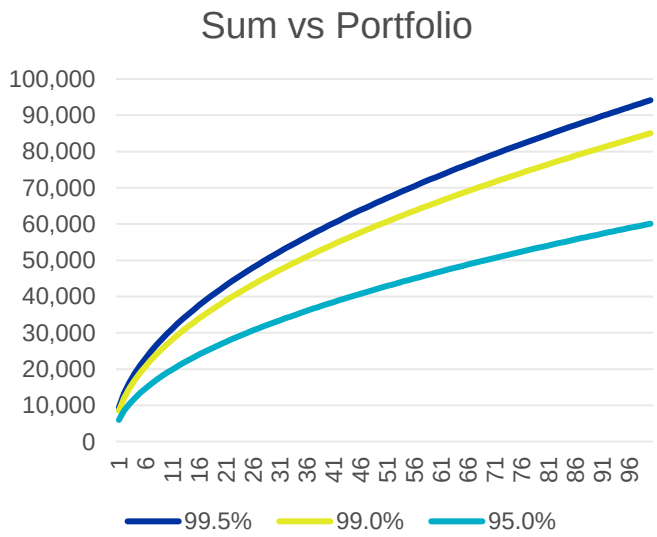
The historical method is very similar to the parametric VaR. In this case the variation lies in the estimation of the random variable *z.*
Recall the formula for the VaR at a given period and confidence level:

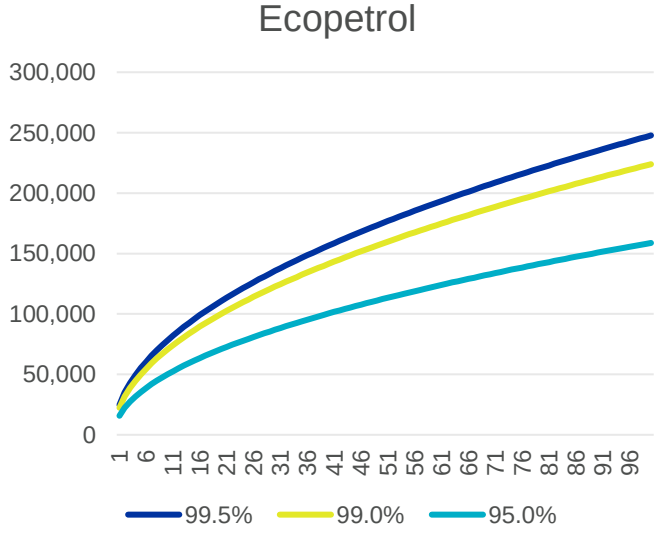
$$VaR_{\alpha} = (z_{\alpha}\ \sigma) * Port\_value$$

In this case z is not a standard normal random variable, but a standard random variable that follows a distribution $N(\mu, \sigma^2)$. Where μ is the average return of the asset and σ^2 is the variance of the asset.
The results of the historical method are presented bellow:









Conclusions

In all cases we observe that the Value at Risk of the portfolio is lower than the sum of the separate parts, which is shown in the plots Sum vs Portfolio (which shows the excess loss if one calculates the separate VaRs and sums them). We can see that even given two assets that are correlated the VaR is lower for an equibalanced portfolio. Overall, the methods output similar results, but the one that seems to incorporate more realistically the recent information to this risk metric is the EWMA method.

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