# Lsg Vorschlag A+N Ü009 Maximilian Maag

## Aufgabe A

$sinh(x) = \frac{1}{2} * (e^x - e^{-x})$

$sinh(x) = \frac{1}{2}(e^x + e^{-x})$

$g(x)\ ax^5 + bx^4 + cx^3 + dx^2 + ix + f$ ~~...~~

Ableitungen $g(x)$

$g'(x)\ 5ax^4 + 4bx^3 + 3cx^2 + 2dx + i$

$g''(x)\ 20ax^3 + 12bx^2 + 6cx + 2d$

$g'''(x)\ 60ax^2 + 24bx + 6c$

$g''''(x)\ 120ax + 24b$

$g'''''(x)\ 120a$

Ableitungen $sinh(x)$

$sinh(x)\ \frac{1}{2}(e^x - e^{-x})$

$sinh'(x)\ \frac{1}{2}(e^x + e^{-x})$

$sinh''(x)\ \frac{1}{2}(e^x - e^{-x})$ u.s.w.

Bilde LGS:

A1: $f = \frac{1}{2}(e^0 - e^0) = 0$

B1: $i = \frac{1}{2}(e^0 + e^0) = 1$

C1: $2d = 0$

D1: $6c = \frac{1}{2}(e^0 + e^0)$

D2: $6c = 1 \quad c = \frac{1}{6}$

E1: $24b = 0$

F1: $120a = \frac{1}{2}(e^0 + e^0)$

F2: $12a = 1 \quad a = \frac{1}{120}$

$a = \frac{1}{120}$
$b = 0$
$c = \frac{1}{6}$
$d = 0$
$i = 1$
$f = 0$

$a, b, c, d, i, f$ in $g(x)$

$g(x) = \frac{1}{120}x^5 + \frac{1}{6}x^3 + x$

$g(x) = \frac{1}{120}x^5 + \frac{1}{6}x^3 + x$

## Aufgabe B

Ansatz nach Newton:
$f(x) = a + b(x-0) + c(x-0)(x-1) + d(x-0)(x-1)(x-2)$

A: $a = 1$
B: $a + b = 2$
C: $a + 2b + 2c = 9$

a = 1; b = 1; c = 3
in f(x)
$f(x) = 1 + x + 3(x - 0)(x - 1)$
$f(x) = 1 + x + 3x(x - 1)$
$f(x) = 1 + x + 3x^2 - 3x$
$f(x) = 3x^2 - 2x + 1$

# Aufgabe 1

## a)

$f(x) = 3e^{-\frac{1}{2}x}$
Ansatz nach Taylor:
$g(x) = ax^2 + bx + c$
soll die Funktion f approximieren.

Bedingungen:
$f(0) = g(0)$
$f'(0) = g'(0)$
$f''(0) = g''(0)$

Ableitungen g(x):
$g'(x) = 2ax + b$
$g''(x) = 2a$

Ableitungen f(x):
$f'(x) = 3 * (-\frac{1}{2}) * e^{-\frac{1}{2}x}$
$f'(x) = -\frac{3}{2} * e^{-\frac{1}{2}x}$
$f''(x) = \frac{3}{4} * e^{-\frac{1}{2}x}$

Aus Bedingungen resultiert LGS
A: $a * 0^2 + b * 0 + c = 3e^{-\frac{1}{2}*0}$
B: $2a * 0 + b = -\frac{3}{2} * e^{-\frac{1}{2}*0}$
C: $2a = \frac{3}{4} * e^{-\frac{1}{2}*0}$

A1: $c = 3e^0$
B1: $b = -\frac{3}{2} * e^0$
C1: $2a = \frac{3}{4} * e^0$

Aus $e^0 = 1$ ergibt sich:
A2: $c = 3$
B2: $b = -\frac{3}{2}$
C2: $2a = \frac{3}{4}$

C3: $a = \frac{3}{8}$
a, b und c in g(x) ergibt: $g(x) = \frac{3}{8}x^2 - \frac{3}{2}x + 3$ an der Entwicklungsstelle 0.

### b)

## Aufgabe 2

$cosh(x) = \frac{1}{2}(e^x + e^{-x})$

### a)

Ansatz nach Taylor für Taylorpolynom 6. Ordnung
$g(x) = ax^6 + bx^5 + cx^4 + dx^3 + jx^2 + fx + g$

$sinh(0) = g(x)$
$sinh'(0) = g'(0)$
$sinh''(0) = g''(0)$
$sinh'''(0) = g'''(0)$
$sinh''''(0) = g''''(0)$
$sinh'''''(0) = g'''''(0)$
$sinh''''''(0) = g''''''(0)$

Ableitungen g(x)
$g(x) = ax^6 + bx^5 + cx^4 + dx^3 + jx^2 + fx + g$
$g'(x) = 6ax^5 + 5bx^4 + 4cx^3 + 3dx^2 + 2jx + f$
$g''(x) = 30ax^4 + 20bx^3 + 12cx^2 + 6dx + 2j$
$g'''(x) = 120ax^3 + 60bx^2 + 24cx + 6d$
$g''''(x) = 360ax^2 + 120bx + 24c$
$g'''''(x) = 720ax + 120b$
$g''''''(x) = 720a$

Ableitungen cosh(x)
$cosh(x) = \frac{1}{2}(e^x + e^{-x})$
$cosh(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$
$cosh'(x) = \frac{1}{2}e^x * 1 + \frac{1}{2}e^{-x} * -1$
$cosh'(x) = \frac{1}{2}(e^x - e^{-x})$
$cosh''(x) = \frac{1}{2}(e^x + e^{-x})$
$cosh'''(x) = \frac{1}{2}(e^x - e^{-x})$
$cosh''''(x) = \frac{1}{2}(e^x + e^{-x})$
$cosh'''''(x) = \frac{1}{2}(e^x - e^{-x})$
$cosh''''''(x) = \frac{1}{2}(e^x + e^{-x})$

Bilde LGS mit Tayloransatz:

A1: $g = \frac{1}{2}(e^0 + e^0) = 1$
B1: $f = \frac{1}{2}(e^0 - e^0) = 0$
C1: $2j = \frac{1}{2}(e^0 + e^0)$
C2: $j = \frac{1}{2}$
D1: $6d = \frac{1}{2}(e^0 - e^0)$
D2: $d = 0$
E1: $24c = \frac{1}{2}(e^0 + e^0)$
E2: $c = \frac{1}{24}$
F1: $120b = \frac{1}{2}(e^0 - e^0)$
F2: $b = 0$
G1: $720a = \frac{1}{2}(e^0 + e^0)$
G2: $720a = 1$
G3: $a = \frac{1}{720}$
$a = \frac{1}{720}$; $b = 0$; $c = \frac{1}{24}$; $d = 0$; $j = \frac{1}{2}$; $f = 0$; $g = 1$

in g(x)
$g(x) = \frac{1}{720}x^6 + \frac{1}{24}x^4 + \frac{1}{2}x^2 + 1$

### b)

### c)

## Aufgabe 3

### a)

Ansatz für Stützpolynom nach Newton:
(1,1); (3,4); (5,9)
$f(x) = a + b(x-1) + c(x-1)(x-3) + d(x-1)(x-3)(x-5)$

LGS bilden:
A: $a = 1$
B: $a + 2b = 4$
C: $a + 4b + 6c = 9$
$a = 1$, $b = \frac{3}{2}$, $c = \frac{1}{3}$

in f(x)
$f(x) = 1 + \frac{3}{2}(x-1) + \frac{1}{3}(x-1)(x-3)$

### b)

(1,1); (3,4); (5,9); (4,6)
$f(x) = a + b(x-1) + c(x-1)(x-3) + d(x-1)(x-3)(x-5)$

LGS bilden:

A: $a = 1$
B: $a + 2b = 4$
C: $a + 4b + 6c = 9$
D: $a + 3b + 4c + 3d = 6$
B2: $1 + 2b = 4$
B3: $b = \frac{3}{2}$
C2: $1 + 4 * \frac{3}{2} + 6c = 9$
C3: $7 + 6c = 9$
C3: $c = \frac{1}{3}$
D2: $1 + \frac{9}{2} + \frac{4}{3} + 3d = 6$
D3: $1 + \frac{54}{12} + \frac{16}{12} + 3d = 6$
D4: $3d = 6 - \frac{54}{12} - \frac{16}{12} - \frac{12}{12}$
D5: $3d = \frac{72}{12} - \frac{54}{12} - \frac{16}{12} - \frac{12}{12}$
D6: $3d = -\frac{10}{12}$
D6: $d = -\frac{10}{36}$

$f(x) = 1 + \frac{3}{2}(x-1) + \frac{1}{3}(x-1)(x-3) - \frac{10}{36}(x-1)(x-3)(x-5)$