# Lsg Vorschlag D+S Ü010 Maximilian Maag

## Aufgabe A

Nummer zwei stimmt.

## Aufgabe B

zu zeigen: $1 * 2 + 2 * 2^2 .... n * 2^n = (n-1) * 2^{n+1} + 2$
Induktionsbasis:
setze ein n = 1
$n * 2^n = (n-1) * 2^{n+1} + 2$
$1 * 2^1 = (1-1) * 2^{1+1} + 2$
$2 = 2$
Induktionsschritt:
$1 * 2 + 2 * 2^2 .... n * 2^n = (n-1) * 2^{n+1} + 2$
$n \rightarrow n+1$
$\rightarrow (n-1) * 2^{n+1} + (n+1) * 2^{n+1} + 2$
$(n-1) * 2^{n+1} + 2 = (n-1+n+1)2^{n+1} + 2$
$= (n-1+n+1)2^{n+1} + 2$
$= 2n2^{n+1} + 2$
$= n2^{n+2} + 2$
q.e.d

## Aufgabe C

zu zeigen: $1 + f_1 + f_2 + f_3 ... + f_n = f_{n+2}$
Basis: $setzen = 1$
$f_1 + f_1 = f_2$ $1 + 1 = 2$
Indikationsschritt: $n \rightarrow n+1$
$f_{n+1} + f_{n+2} = f_{n+3}$
$f_2 + f_3 = f_4$
$2 + 3 = 5$
q.e.d

## Aufgabe 1

zu zeigen: $1 + 2 + 3 ... n^2 = \frac{n*(n+1)*(2n+1)}{6}$
Basis: setze n = 1
$1^2 = \frac{1*(1+1)*(2+1)}{6}$
$1^2 = \frac{2*3}{6}$
$1^2 = \frac{6}{6}$
$1 = 1$

Induktionsschritt: $n \to n+1$
$1+2+3...n^2+(n+1)^2 = \frac{(n+1)*((n+1)+1)*(2(n+1)+1)}{6}$
$\frac{(n+1)*((n+1)+1)*(2(n+1)+1)}{6} = \frac{n*(n+1)*(2n+1)}{6} + (n+1)^2$
$\frac{(n+1)*((n+1)+1)*(2(n+1)+1)}{6} = \frac{n*(2n^2+n+2n+1)}{6} + \frac{(n+1)^2}{1}$
$\frac{(n+1)*((n+1)+1)*((2n+2)+1)}{6} = \frac{(2n^3+3n^2+n)}{6} + \frac{n^2+2n+1}{1}$
$\frac{(n+1)*((n+2))*((2n+2)+1)}{6} = \frac{(2n^3+3n^2+n)}{6} + \frac{6n^2+12n+6}{6}$
$\frac{(n+1)*((n+2)*((2n+2)+1)}{6} = \frac{(2n^3+3n^2+n+6n^2+12n+6)}{6}$
$\frac{(n+1)*(2n^2+2n+n+4n+4+2)}{6} = \frac{(2n^3+9n^2+13n+6)}{6}$
$\frac{2n^3+2n^2+n^2+4n^2+4n+2n+2n^2+2n+n+4n+4+2}{6} = \frac{(2n^3+9n^2+13n+6)}{6}$
$\frac{2n^3+9n^2+13n+6}{6} = \frac{2n^3+9n^2+13n+6}{6}$
q.e.d

# Aufgabe 2

Zu zeigen: $1+q+q^2+q^3...+q^n = \frac{1-q^{n+1}}{1-q}$
Induktionsbasis: setze n = 0
$1+q+q^2+q^3...+q^1 = \frac{1-q^{n+1}}{1-q}$
$q^0 = \frac{1-q^{0+1}}{1-q}$
$1 = \frac{1-q}{1-q}$
$1 = 1$
Induktionsschritt: $n \to n+1$
$1+q^1+q^2+q^n+q^{n+1} = \frac{1-q^{n+1+1}}{1-q}$
$\frac{1-q^{n+1}}{1-q} + q^{n+1} = \frac{1-q^{n+2}}{1-q}$
$1-q^{n+1}+(1-q)q^{n+1} = 1-q^{n+2}$
$1-q^{n+1}+q^{n+1}-q^{n+2} = 1-q^{n+2}$
$1-q^{n+2} = 1-q^{n+2}$
q.e.d

# Aufgabe 3

zu zeigen: $1+2^3+3^3+...n^3 = (1+2+2...+n)^2$
Tipp: $(1+2+2...+n)^2 \equiv (\frac{n(n+1)}{2})^2$

Induktionsbasis: n = 1
$1^3 = 1^2$

Induktionsschritt: $n \to n+1$
$1+2^3+3^3...+n^3+(n+1)^3 = (\frac{(n+1)(n+2)}{2})^2$
$(\frac{n*(n+1)}{2})^2+(n+1)^3 = (\frac{(n+1)(n+2)}{2})^2$

$\frac{n^2*(n+1)^2}{4}+(n+1)^3=\frac{(n+1)^2(n+2)^2}{4}$
$\frac{n^2*(n+1)^2}{4}+n^3+3n^2+1=\frac{(n+1)^2(n+2)^2}{4}$
$n^2*(n+1)^2+4n^3+12n^2+12n+4=(n+1)^2(n+2)^2$
$n^2*(n^2+2n+1)+4n^3+12n^2+12n+4=(n+1)^2(n+2)^2$
$n^4+2n^3+n^2+4n^3+12n^2+12n+4=(n+1)^2(n+2)^2$
$n^4+6n^3+13n^2+12n+4=(n+1)^2(n+2)^2$
$n^4+6n^3+13n^2+12n+4=(n^2+2n+1)(n^2+4n+4)$
$n^4+6n^3+13n^2+12n+4=n^4+4n^3+4n^2+2n^3+8n^2+8n+n^2+4n+4$
$n^4+6n^3+13n^2+12n+4=n^4+6n^3+13n^2+12n+4$
q.e.d