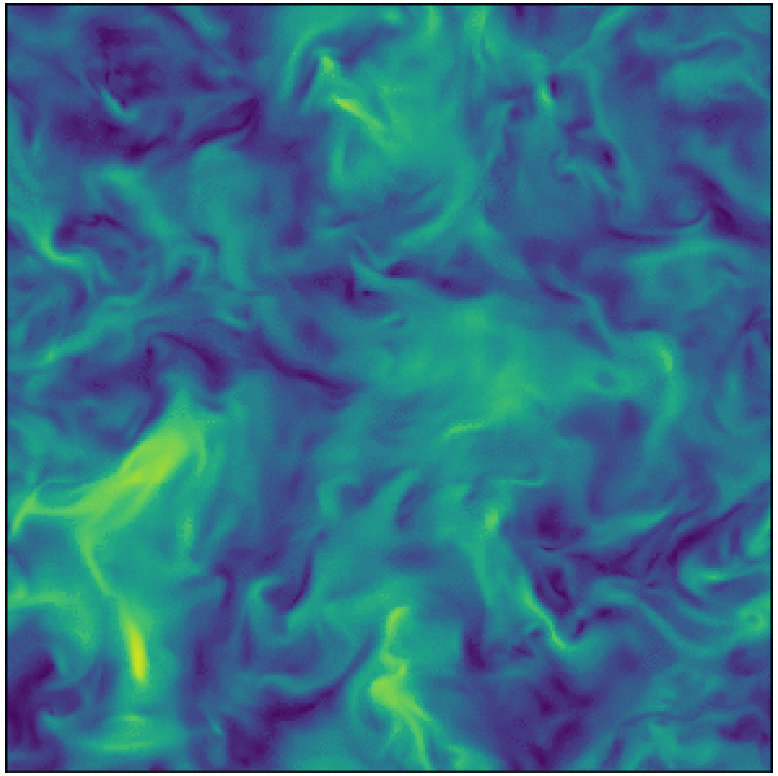
Studies of turbulent diffusion through Direct Numerical Simulation

- Daniel Ørnes Halvorsen

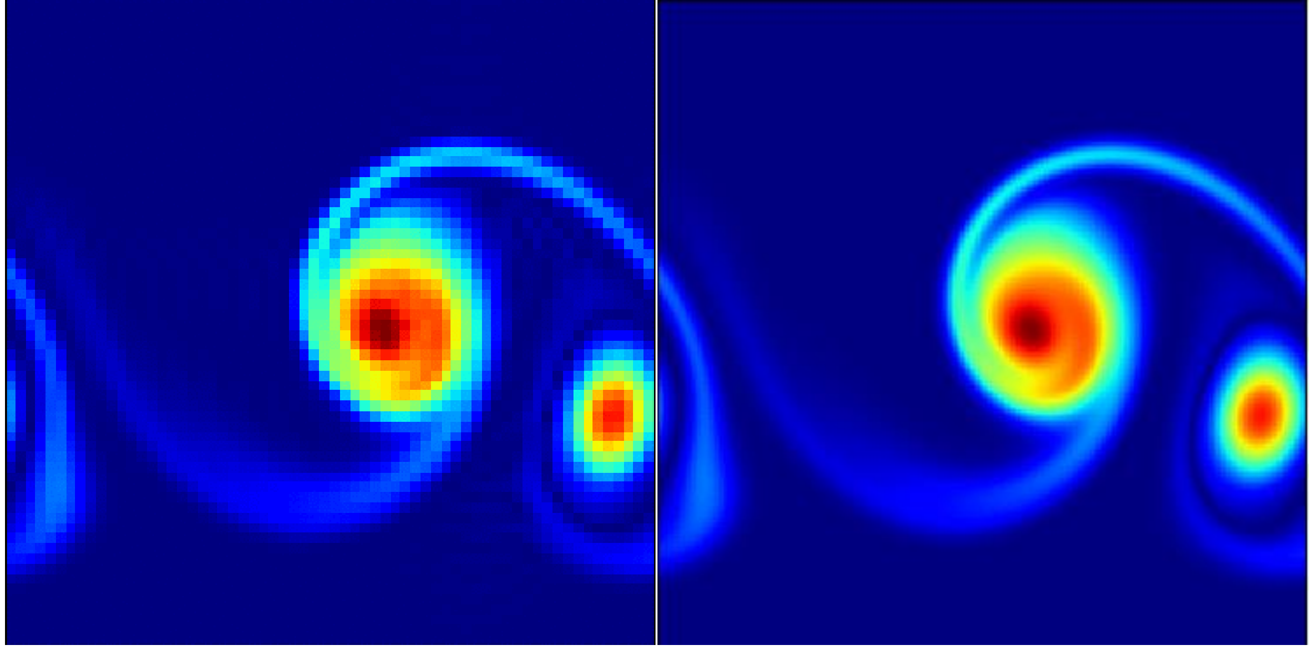
Supervisor: Tor Nordam



What is turbulence?

- 3D
- Unsteady
- Vorticity
- Diffusive
- Dissipative
- High Reynolds number
- Continuum phenomenon

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} - \boldsymbol{u}\times\boldsymbol{\omega}\right) = -\nabla\left(p + \boldsymbol{u}\cdot\frac{\boldsymbol{u}}{2}\right) + \mu\nabla^2\boldsymbol{u} + \rho\boldsymbol{f}$$



N=64 N=256

Kolmogorov microscale:

$$\eta = l\left(\frac{ul}{\nu}\right)^{\frac{-3}{4}}$$

Spectral method

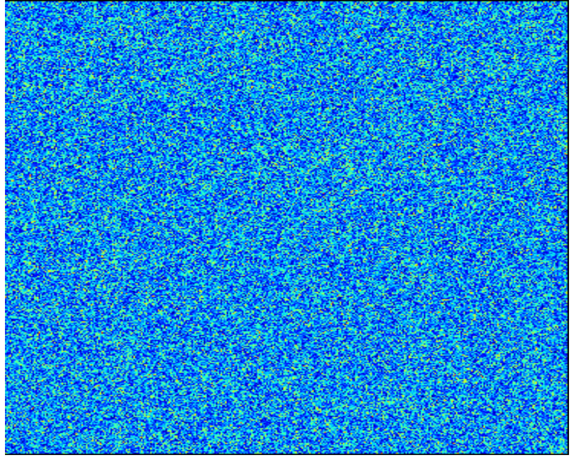
$$\boldsymbol{u}(\boldsymbol{x},t) = \frac{1}{N^3}\sum_{\boldsymbol{k}} \widehat{\boldsymbol{u}}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\widehat{\boldsymbol{u}}_{\boldsymbol{k}}(t) = \sum_{\boldsymbol{x}} \boldsymbol{u}(\boldsymbol{x},t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

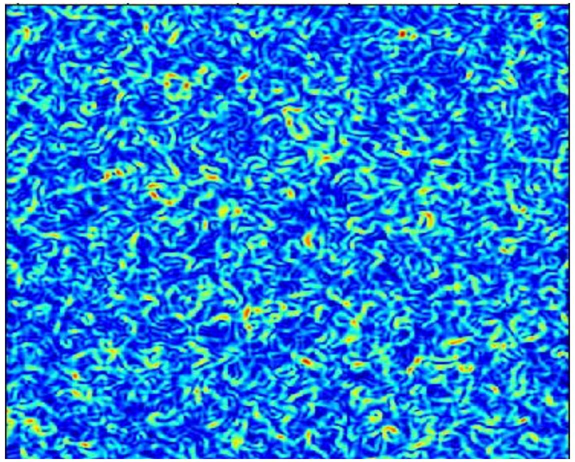
$$\frac{d\widehat{\boldsymbol{u}_{\boldsymbol{k}}}}{dt} = (\widehat{\boldsymbol{u}\times\boldsymbol{\omega}}) - \nu|\boldsymbol{k}|^2\widehat{\boldsymbol{u}_{\boldsymbol{k}}} - \boldsymbol{k}\frac{\boldsymbol{k}\cdot(\widehat{\boldsymbol{u}\times\boldsymbol{\omega}})_{\boldsymbol{k}}}{|\boldsymbol{k}|^2}$$

Temporal term solved using your favorite ODE solver!

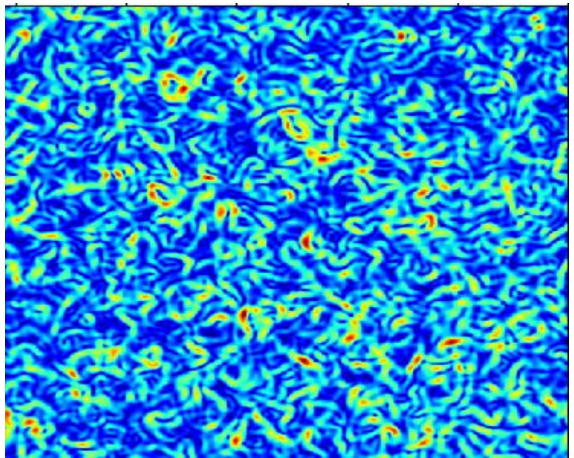
Exponential convergence: $error \propto \left(\frac{L}{N}\right)^N$



N=512
$t_0 = 0$



N=512
$t_1 > t\_0$



N=512
$t_2 > t_1$

Summary

- Turbulence is hard
- DNS - all spatial scales resolved
- Spectral methods used for spatial discretization
- Use the turbulent flow field as input in an advection-diffusion equation to simulate transportation of some species.