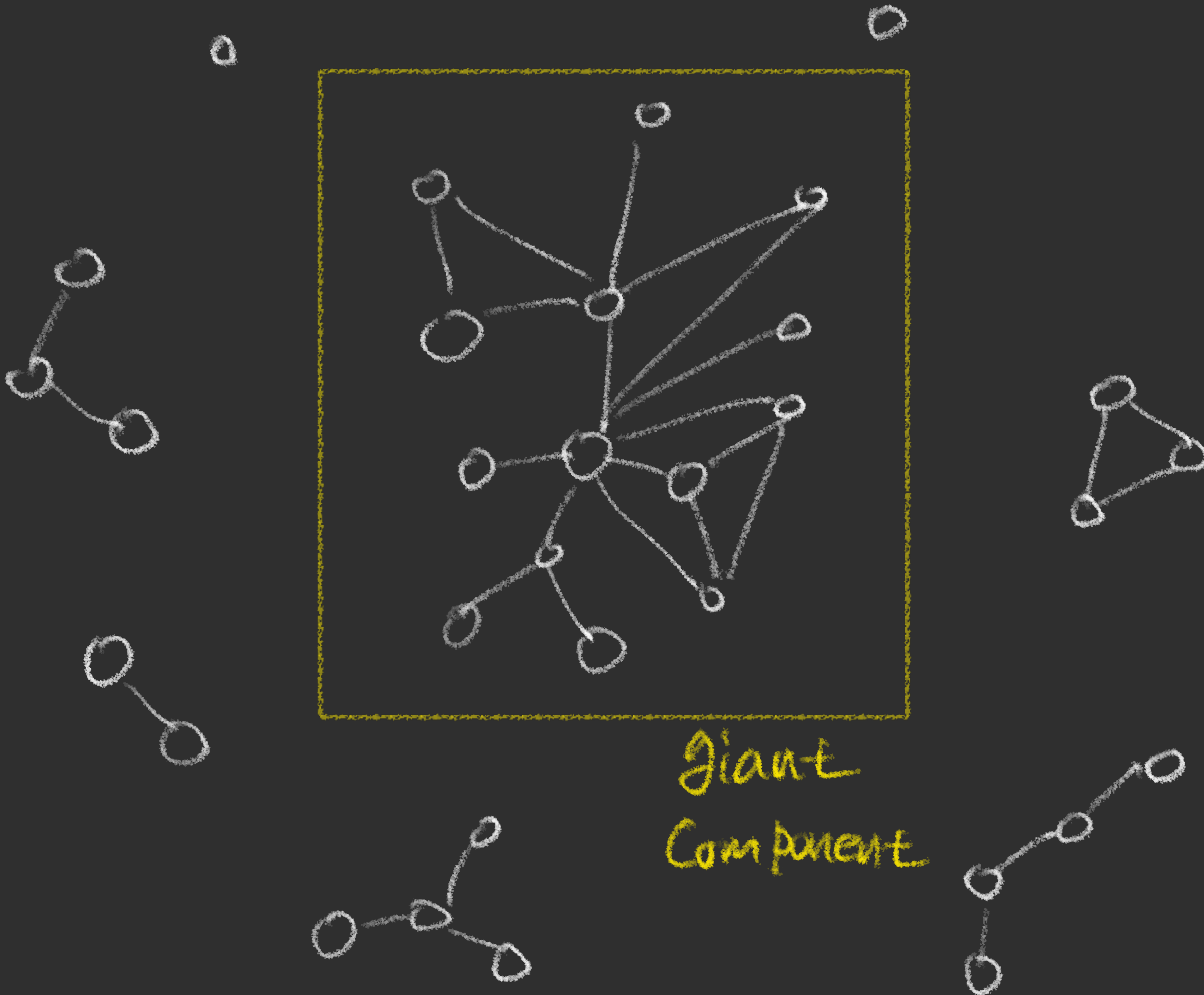
Message passing

percolation


Giant
Component

2) given a network of n nodes and a specific node within that network, does the node in question belong to the giant component?

μ_i : Prob. that node i does $\boxed{\text{not}}$ belong to the GC

$\Rightarrow \mu_i = 0$ or 1

$$\mu_i = \prod_{j \in N_i} \mu_j$$

N_i → neighbors of node i

$\Rightarrow$ node i is not in the GC iff none of its neighbors are in the GC

μ_i : Prob. that node i does $\boxed{\text{not}}$ belong to the GC

$\Rightarrow \mu_i = 0$ or 1

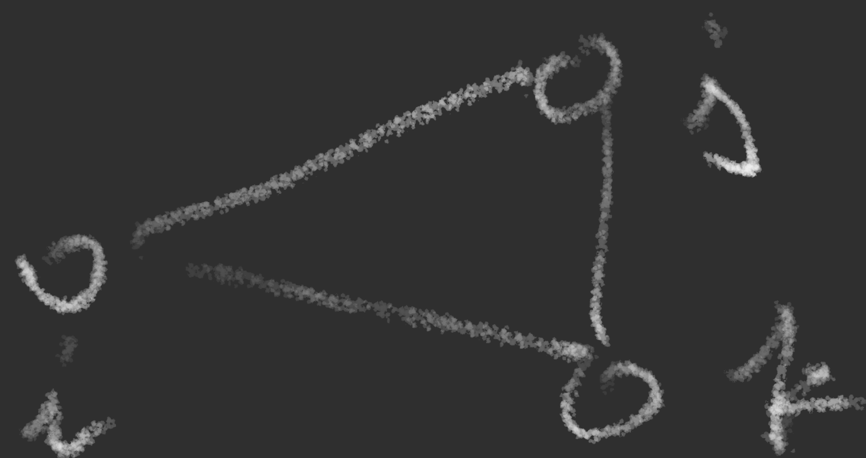
$$\cancel{\mu_i = \prod_{j \in N_i} \mu_j}$$

$\underbrace{j \in N_i}$ $\rightarrow$ neighbors of node i

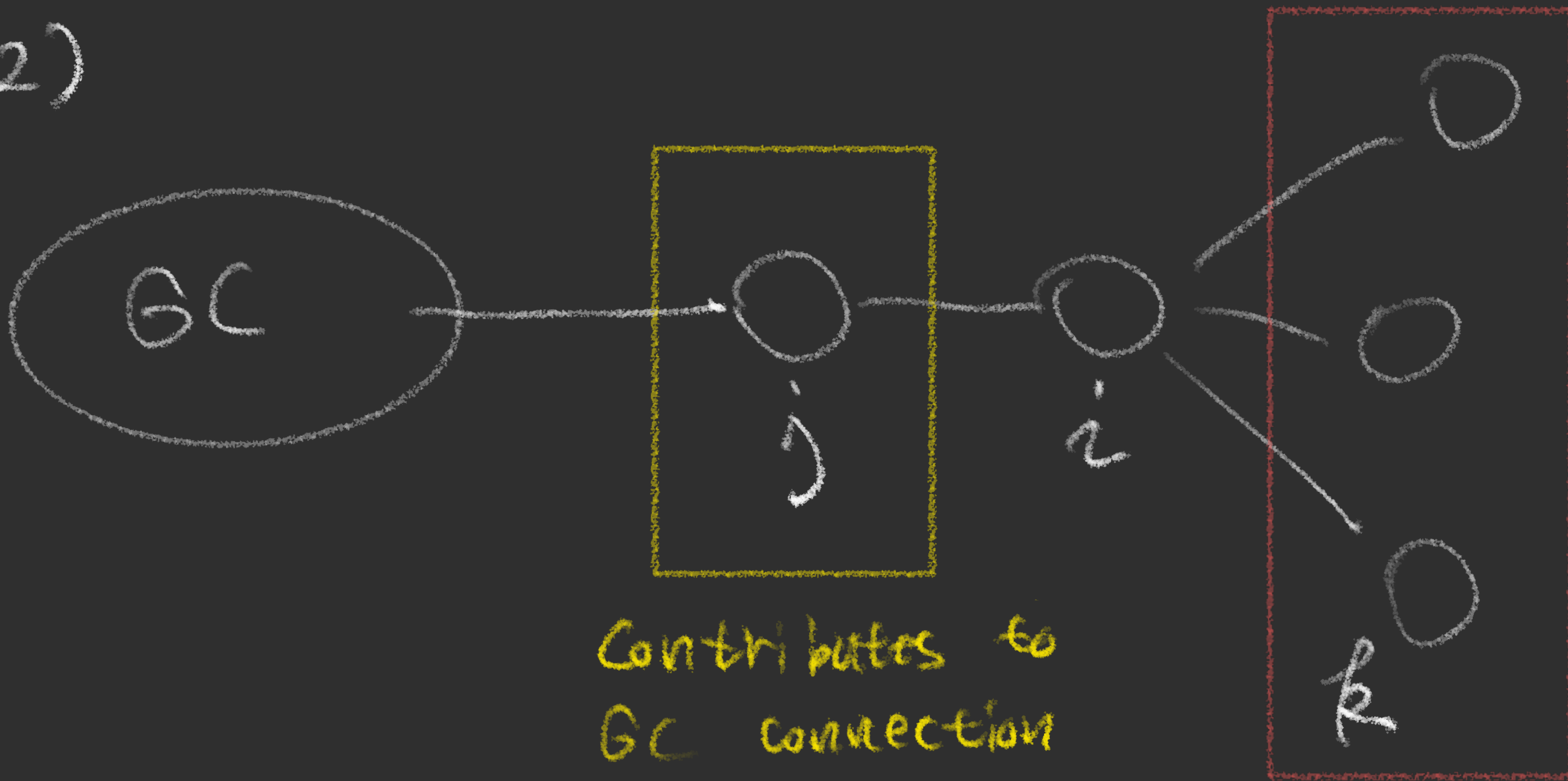
$\Rightarrow$ node i is not in the GC iff none of its neighbors are in the GC why?

1) it assumes independence of the neighbors

$\Rightarrow$ locally tree-like approximation



2)



$\Rightarrow$ node i's neighbor must be connected to the GC by some route other than via i itself.

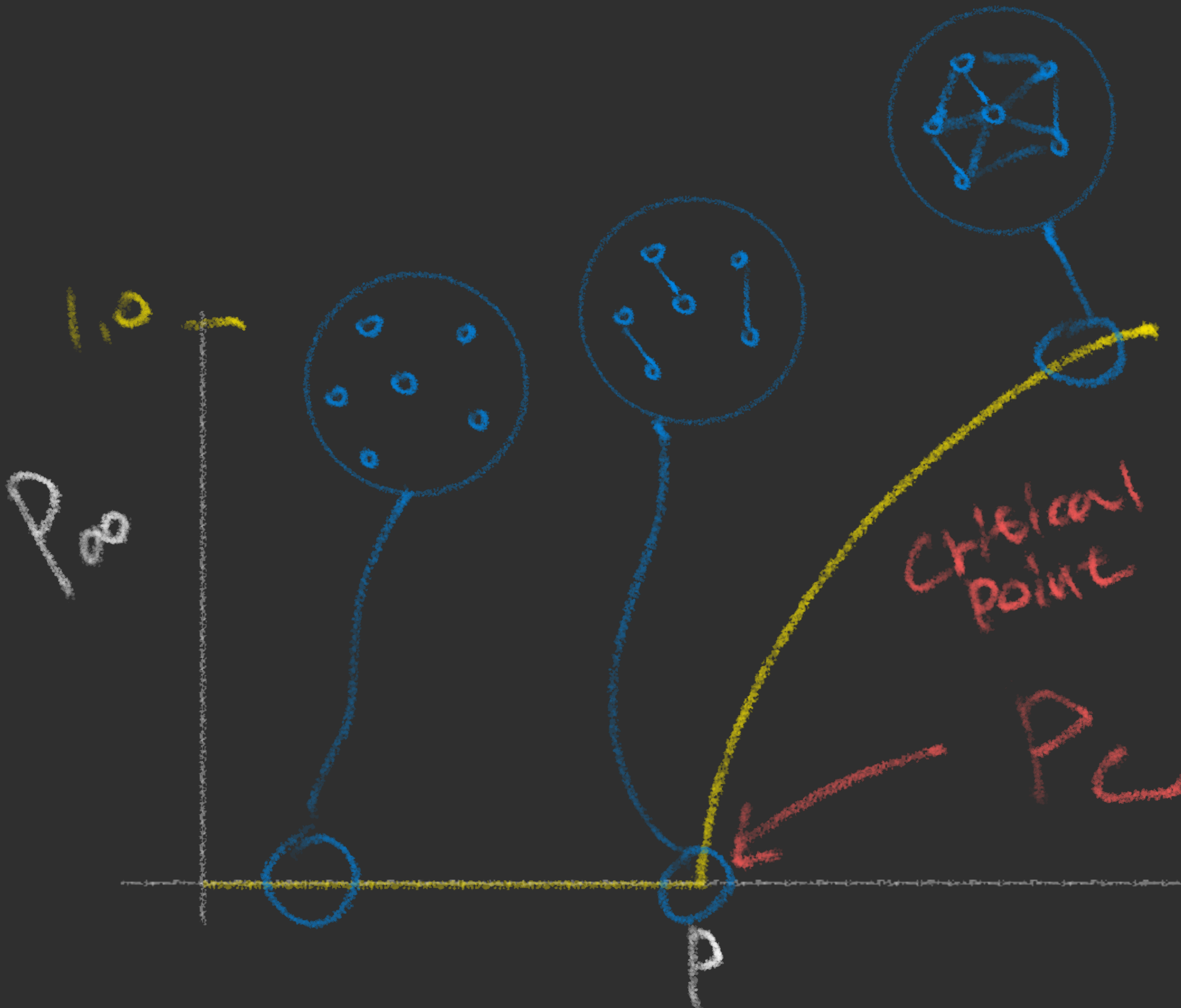
$$\mu_i = \prod_{j \in N_i} \underline{\mu_{i \leftarrow j}}$$

prob. that
j is not in GC
when i is removed

$$\mu_{i \leftarrow j} = \prod_{k \in N_j / i} \boxed{\mu_{j \leftarrow k}}$$

message?
belief?

$\Longleftrightarrow$ solving $2m$ equations w.
$2m$ variables $\mu_{i \leftarrow j}$



1.0
P_∞
critical point
P_c
p

$$\mu_i = \prod_{j \in N_i} \mu_{i \leftarrow j}$$

$$\mu_{i \leftarrow j} = \prod_{k \in N_j / i} \mu_{j \leftarrow k}$$

$p \in [0, 1)$

$$\mu_i = \prod_{j \in N_i} (1 - p + p\,\mu_{i \leftarrow j})$$

$$\mu_{i \leftarrow j} = \prod_{k \in N_j / i} (1 - p + p\,\mu_{j \leftarrow k})$$

$p = 1$

Message passing VS Simulation

$\downarrow$ single calculation

$\downarrow$ multiple realizations

average GC size ?

$$P_\infty = \sum_{\{s_i\}} \left[\frac{1}{n}\sum_i s_i\right]\left[\prod_i (1-\mu_i)^{s_i}\mu_i^{1-s_i}\right]$$

$$= \frac{1}{n}\sum_i \sum_{s_i=0,1} s_i (1-\mu_i)^{s_i}\mu_i^{1-s_i}$$

$$= 1 - \frac{1}{n}\sum_{i=1}^{n}\mu_i$$

Critical point?

$$\mu_{i \leftarrow j} = \prod_{k \in N_j / i} (1 - p + p\,\mu_{j \leftarrow k})$$

trivial solution: $\mu_{i \leftarrow j} = 1$

$\Longleftrightarrow$ No GC

$\Rightarrow$ given some perturbation, let's see how stable this solution is.

$$\mu_{i \leftarrow j} = 1 - \varepsilon_{i \leftarrow j}$$

$$1 - \varepsilon_{i \leftarrow j} = \prod_{k \in N_j / i} \left(1 - p\, \varepsilon_{j \leftarrow k} \right)$$

$$= 1 - p \sum_{k \in N_j / i} \varepsilon_{j \leftarrow k} + O(\varepsilon^2)$$

$$\Longleftrightarrow \varepsilon_{i \leftarrow j} = p \sum_{k \in N_j / i} \varepsilon_{j \leftarrow k}$$

$$\varepsilon_{i \leftarrow j} = p \sum_{k \in N_j / i} \varepsilon_{j \leftarrow k}$$

$$\Longleftrightarrow \quad \varepsilon = p \boxed{B} \varepsilon$$

2m -element vector

2m x 2m

Hashimoto edge-incidence matrix

non-backtracking matrix

unstable trivial sol

$\iff$ ε grows under

$$\varepsilon = p B \varepsilon$$

$$\iff p \boxed{\lambda} > 1$$

leading eigenvalue of B

$$p_c = \frac{1}{\lambda}$$