CHAPTER 3

THE ELECTRON

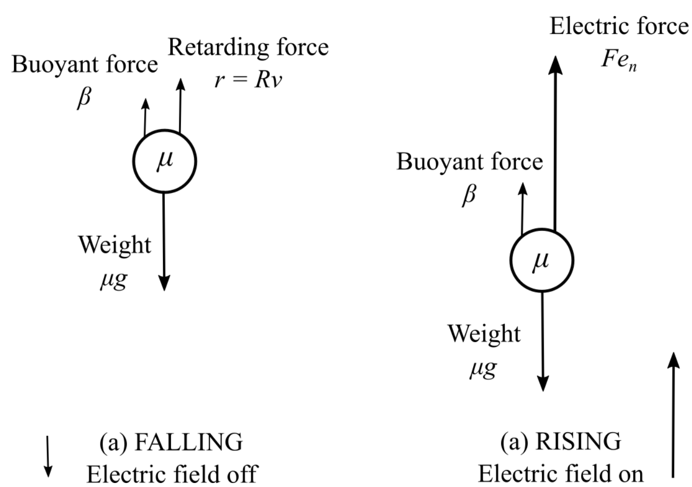
R. A. MILLIKAN

REMARKS

We turn now to the examination of "charge" itself, in particular to the question whether charge is discrete or continuous in its ultimate composition. That question was effectively answered with R. A. Millikan's discovery of a fundamental unit of charge—the so-called "electron."

The instrument Millikan employed for that delicate task of measurement is a tiny droplet of oil bearing a minute electrical charge, alternately hoisted up by an applied electric field, then allowed to fall back down under its own weight with the field turned off. From observations of the motion of such drops Millikan was able, first, to show that a fundamental unit of charge *exists*; second, to determine the magnitude of that unit.

In the selection that follows, Millikan takes as granted the equations governing the motion of the oil drop. Therefore the first phase in our reasoning must be to work out in a general way the forces that act on the drop and to derive the equation of its motion under those forces. It will be assumed at this point that the size and mass of the drop are known; actually their determination constitutes an important second phase of the analysis.



The forces which act on a falling drop in the absence of an electric field are sketched on the left side of the diagram, marked (a). The mass of the drop is μ and the weight is therefore μg. The drop falls through the air, which we shall regard as a very thin homogeneous fluid that both buoys up a body and resists motion through itself. We also assume, with Millikan, that this resisting force is *directly proportional to the velocity* of the moving drop. This proportionality characterizes the special type of friction called a "viscous" force, exerted by a "viscous" fluid on any body moving through it. The measure of a fluid's capacity for applying such a force is called its "viscosity." Millikan had good reason to believe that air exhibited this viscosity quite strictly.

With some constant of proportionality, R, then, the viscous force r resisting the droplet's fall will be

$$r = Rv \tag{3.1}$$

as shown in Fig. (a).

At the same time, the drop is buoyed up by a force equal to the weight of air which it displaces.[1] Thus a droplet of volume V will have mass $\mu = V\sigma$, but it will displace a mass $V\rho$ of air, where σ is the density of the oil and ρ is the density of the air. Hence the three forces acting on the falling drop are:

$$\begin{aligned} \text{Weight} \quad & W = V\sigma g \\ \text{Buoyancy} \quad & \beta = V\rho g \\ \text{Retarding force} \quad & r = Rv \end{aligned}$$

for a total downward force f of

$$\begin{aligned} f &= V\sigma g - V\rho g - Rv \\ &= V(\sigma - \rho)g - Rv \end{aligned}$$

On analogy with $V\sigma$, the actual mass of the oil drop, Millikan will call $V(\sigma - \rho)$ in the expression above the "effective" gravitational mass, m—as though the reduced downward tendency of the drop in the buoyant medium were due to a reduced gravitational mass $m = V(\sigma - \rho)$ rather than, as is actually the case, to the presence of an upward buoyant force $\beta = V\rho g$. The expression for the net downward force can then be written

$$f = mg - Rv \tag{3.2}$$

where $m = V(\sigma - \rho)$. There will be a downward acceleration—that is, a continuing increase in velocity v—so long as the resistance Rv of the medium is less than the effective weight mg.

However, as the velocity of the drop increases, so will the resistance; as Rv increases it will approach mg and the net force f will approach zero, as is clear from equation (3.2) above. The acceleration will become very small, and the droplet will continuously approach the *constant* velocity v_1,[2] characterized as the velocity at which

$$Rv = mg.$$

This gives

$$v_1 = \frac{mg}{R}. \tag{3.3}$$

[1] Archimedes, *On Floating Bodies*, I, Prop. 7.

[2] In practice, the drop almost immediatley accelerates to a velocity immeasurably close to its terminal velocity.

Suppose now that the droplet bears electric charge e_n. In the absence of an electric field, that charge will not affect the drop's motion. But when a field F tending to raise the drop is applied, an upward force Fe_n will act on the drop; and if it is strong enough to overbalance the effective weight of the drop, the drop will stop falling and begin to rise. Then while the drop is rising both the effective weight and the viscous force will be *downwards*, and there will act on the drop (see Fig. (b)) a net *upward* force equal to

$$Fe_n - mg - Rv.$$

Then by an argument similar to that for the drop when falling, the rising drop will shortly attain a constant upward velocity v_2, where

$$v_2 = \frac{Fe_n - mg}{R} \tag{3.4}$$

It is a striking characteristic of motion in a viscous medium that a body under the action of a constant force attains a *constant velocity proportional to that force*.

Equations (3.3) and (3.4) constitute all the theory that Millikan requires to establish from his observations that the electron *exists*; although calculation of its *quantity* involves, as we shall see, considerably more reasoning. Actually Millikan uses the *quotient* of equation (3.3) by equation (3.4), which is

$$\frac{v_1}{v_2} = \frac{mg}{Fe_n - mg} \quad \text{or} \quad e_n = \frac{mg}{Fv_1}(v_1 + v_2) \tag{1}$$

It will be called equation (9) in the selection that follows.

A final remark: unlike Thomson (who had to deal with both electric and magnetic qualities in his calculations), Millikan, who deals with the electron only in its electrostatic relations, accordingly uses the *electrostatic system of units* (e.s.u.) in what we are about to read. Thus he will eventually state his fundamental unit of charge in *statcoulombs.* In order to relate Millikan's and Thomson's results, recall that the electromagnetic and the electrostatic systems of units are related by the constant *c* (the speed of light in centimeters per second), so that one abcoulomb equals 3.00×10^{10} statcoulombs.[3]

THE ELECTRON

R. A. MILLIKAN

General Proof of the Atomic Nature of Electricity[4]

Although the "balanced droplet method" just described[5] had eliminated the chief sources of uncertainty which inhered in preceding work on *e* and had made it possible to assert with much confidence that the unit charge was a real physical entity and not

[3]See Appendix (p. 203) for a fuller discussion of the relation between the "electrostatic" and "electromagnetic" system of units.

[4][Chapter IV of *The Electron*, University of Chicago Press (1924). Although we are reading his 1924 account, Millikan developed the methods here described from 1909 to 1913.]

[5][Millikan here refers to an earlier technique, omitted here.]

merely a "statistical mean," it was yet very far from an exact method of studying the properties of gaseous ions. The sources of error or uncertainty which still inhered in it arose from (1) the lack of stagnancy in the air through which the drop moved; (2) the lack of perfect uniformity of the electrical field used; (3) the gradual evaporation of the drops, rendering it impossible to hold a given drop under observation for more than a minute or to time a drop as it fell under gravity alone through a period of more than five or six seconds; and (4) the assumption of the validity of Stokes's Law.

The method which was devised to replace it was not only entirely free from all of these limitations, but it constituted an entirely new way of studying ionization and one which at once yielded important results in a considerable number of directions. This chapter deals with some of these by-products of the determination of *e* which are of even more fundamental interest and importance than the mere discovery of the exact size of the electron.

I. Isolation of Individual Ions and Measurement of Their Relative Charges

In order to compare the charges on different ions, the procedure adopted was to blow with an ordinary commercial atomizer an oil spray into the chamber C (Fig. 3). The air with which this spray was blown was first rendered dust-free by passage through a tube containing glass wool. The minute droplets of oil constituting the spray, most of them having a radius of the order of a one-thousandth of a millimeter, slowly fell in the chamber C, and occasionally one of them would find its way through the minute pinhole *p* in the middle of the circular brass plate M, 22 cm in diameter, which formed

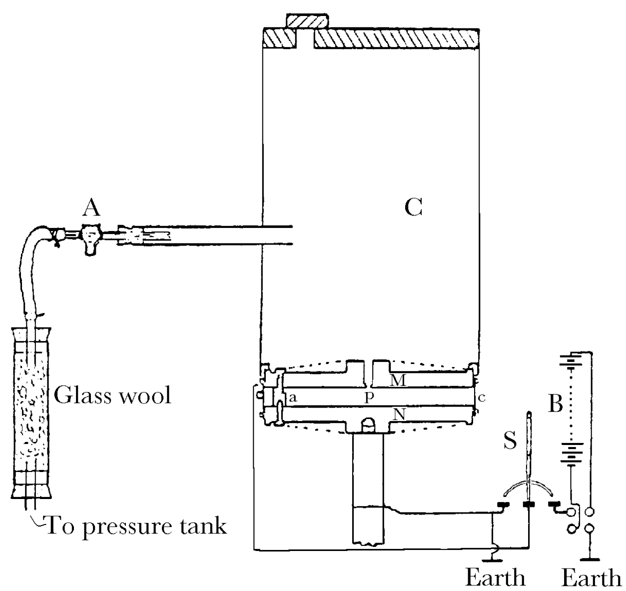


Figure 3

one of the plates of the air condenser. The other plate, N, was held 16 mm. beneath it by three ebonite posts *a*. By means of the switch S these plates could be charged, the one positively and the other negatively, by making them the terminals of a 10,000-volt storage battery B, while throwing the switch the other way (to the left) short-circuited them and reduced the field between them to zero. The oil droplets which entered at *p* were illuminated by a powerful beam of light which passed through diametrically opposite windows in the encircling ebonite strip *c*. As viewed through a third window in *c* on the side toward the reader, it appeared as a bright star on a black background. These droplets which entered *p* were found in general to have been strongly charged by the frictional process involved in blowing the spray, so that when the field was thrown on in the proper direction they would be pulled up toward M. Just before the drop under observation could strike M the plates would be short-circuited and the drop allowed to fall under gravity until it was close to N, when the direction of motion would be again reversed by throwing on the field. In this way the drop would be kept traveling back and forth between the plates. The first time the experiment was tried an ion[6] was caught within a few minutes, and the fact of its capture was signaled to the

TABLE IV

	t_g	t_F
	13.6	12.5
	13.8	12.4
	13.4	21.8
	13.4	34.8
	13.6	84.5
	13.6	85.5
	13.7	34.6
	13.5	34.8
	13.5	16.0
	13.8	34.8
	13.7	34.6
	13.8	21.9
	13.6	
	13.5	
	13.4	
	13.8	
	13.4	
Mean	13.595	

observer by the change in the speed with which [the oil drop] moved up when the field was on. The significance of the experiment can best be appreciated by examination of

[6]["Ion" was Faraday's term for the particles of matter which seemed, in electrolysis, to migrate from one region of the solution to another. Although Faraday himself distrusted the atomic view, researchers of a later generation collected much evidence that "ions" were actually *electrically-charged atoms* (or groups of atoms), which were not only found in electrolytes but could also be produced in gases by various causes including x-rays and the presence of radioactive substances. In an earlier chapter of *The Electron*, Millikan wrote: "In a word, then, the act of ionization in gases appears to consist in the detachment from a neutral atom of one or more negatively charged particles, called by Thomson *corpuscles*. The residuum of the atom is of course positively charged.... The detached corpuscle must soon attach itself, in a gas at ordinary pressure, to a neutral atom." Thus some of the air molecules in Millikan's chamber carry either a positive or a negative charge, and it is these charged molecules—ions—that periodically collide with the oil drop and yield to it whatever charges they bear.]

the complete record of one of the early experiments when the timing was done merely with a stop watch.

The column headed t_g gives the successive times which the droplet required to fall [under the force of gravity, g] between two fixed cross-hairs in the observing telescope whose distance apart corresponded in this case to an actual distance of fall of .5222 cm. It will be seen that these numbers are all the same within the limits of error of a stop-watch measurement. The column marked t_F gives the successive times which the droplet required to rise under the influence of the electrical field [F] produced by applying in this case 5,051 volts of potential difference to the plates M and N. It will be seen that after the second trip up, the time changed from 12.4 to 21.8, indicating, since in this case the drop was positive, that a negative ion had been caught from the air. The next time recorded under t_F, namely, 34.8, indicates that another negative ion had been caught. The next time, 84.5, indicates the capture of still another negative ion. This charge was held for two trips, when the speed changed back again to 34.6, showing that a positive ion had now been caught which carried precisely the same charge as the negative ion which before caused the inverse change in time, i.e., that from 34.8 to 84.5.

In order to obtain some of the most important consequences of this and other similar experiments we need make no assumption further than this, that the velocity with which the drop moves is proportional to the force acting upon it and is independent of the electrical charge which it carries. Fortunately this assumption can be put to very delicate experimental test, as will presently be shown, but introducing it for the time being as a mere assumption, as Townsend, Thomson, and Wilson had done before, we get

$$\frac{v_1}{v_2} = \frac{mg}{Fe_n - mg} \quad \text{or} \quad e_n = \frac{mg}{Fv_1}(v_1 + v_2) \tag{9}$$

The negative sign is used in the denominator because v_2 will for convenience be taken as positive when the drop is going up in the direction of *F*, while v_1 will be taken as positive when it is going down in the direction of *g*. e_n denotes the charge on the drop, and must not be confused with the charge on an ion. If now by the capture of an ion the drop changes its charge from e_n to $e_{n'}$, then the value of the captured charge e_i is

$$e_i = e_{n'} - e_n = \frac{mg}{Fv_1}(v_2' - v_2) \tag{10}$$

and since mg/Fv_1 is a constant for this drop, any charge which it may capture will always be proportional to ($v_2' - v_2$), that is, to the change produced in the velocity in the field *F* by the captured ion. The successive values of v_2 and of ($v_2' - v_2$) are shown in Table V.

It will be seen from the last column that within the limits of error of a stop-watch measurement, all the charges captured have exactly the same value save in three cases. In all of these three the captured charges were just twice as large as those appearing in the other changes. Relationships of exactly this sort have been found to hold absolutely without exception, no matter in what gas the drops have been suspended or what sort of droplets were used upon which to catch the ions. In many cases a given drop has been held under observation for five or six hours at a time and has been seen to catch not eight or ten ions, as in the above experiment, but hundreds of them. Indeed, I have observed, all told, the capture of many thousands of ions in this way, and in no case have I ever found one the charge of which, when tested as above, did not have either

TABLE V[a]

v_2	$(v_2' - v_2)$
$\frac{.5222}{12.45} = .04196$	
	$.01806 \div 2 = .00903$
$\frac{.5222}{[21.85]} = .02390$	
	$.00885 \div 1 = .00885$
$\frac{.5222}{34.7} = .01505$	
	$.00891 \div 1 = .00891$
$\frac{.5222}{85.0} = .006144$	
	$.00891 \div 1 = .00891$
$\frac{.5222}{34.7} = .01505$	
	$.01759 \div 2 = .00880$
$\frac{.5222}{16.0} = .02364$	
	$.01759 \div 2 = .00880$
$\frac{.5222}{34.7} = .01505$	
	$[.00885 \div 1 = .00885]$
$\frac{.5222}{21.85} = .02390$	

[a] [The bracketed numbers are our corrections of errors in Millikan's original table.]

exactly the value of the smallest charge ever captured or else a very small multiple of that value. *Here, then, is direct, unimpeachable proof that the electron is not a "statistical mean," but that rather the electrical charges found on ions all have either exactly the same value or else small exact multiples of that value.*

II. Proof That All Static Charges Both on Conductors and Insulators Are Built Up of Electrons

The foregoing experiment leads, however, to results of much more fundamental importance than that mentioned in the preceding section. The charge which the droplet had when it first came under observation had been acquired, not by the capture of ions from the air, but by the ordinary frictional process involved in blowing the spray. If then ordinary static charges are built up of electrons, this charge should be found to be an exact multiple of the ionic charge which had been found from the most reliable measurement shown in Table V to be proportional to the velocity .00891. This initial charge e_n on the drop is seen from equations (9) and (10) to bear the same relation to (v_1 + v_2) which the ionic charge $e_{n'} - e_n$ bears to $(v_2' - v_2)$. Now, $v_1 = .5222 \div 13.595 = .03842$, hence $v_1 + v_2 = .03842 + .04196 = .08038$. Dividing this by 9 we obtain .008931, which is within about one-fifth of 1 per cent of the value found in the last column of Table V as the smallest charge carried by an ion. *Our experiment has then given us for the first time a means of comparing a frictional charge with the ionic charge, and the frictional charge has in this instance been found to contain exactly 9 electrons.* A more exact means of making this comparison will be given presently, but suffice it to say here that experiments like the foregoing have now been tried on thousands of drops in different media, some of the drops being made of non-conductors like oil, some of semi-conductors like glycerin, some of excellent metallic conductors like mercury. In every case, without a single exception, the initial charge placed upon the drop by the frictional process, and all of the dozen or more charges which have resulted from the capture by the drop of a larger or smaller number of ions, have been found to be exact multiples of the smallest charge caught from the air. Some of these drops have started with no charge at all, and one, two, three, four, five, and six elementary charges or electrons have been picked up. Others have started with seven or eight units, others with twenty, others with fifty, others with a hundred, others with a hundred and fifty elementary units, and have picked up in each case a dozen or two of elementary charges on either side of the starting point, so that in all, drops containing every possible number of electrons between one and one hundred and fifty have been observed and the number of electrons which each drop carried has been accurately counted by the method described. When the number is less than fifty there is not a whit more uncertainty about this count than there is in counting one's own fingers and toes. It is not found possible to determine with certainty the number of electrons in a charge containing more than one hundred or two hundred of them, for the simple reason that the method of measurement used fails to detect the difference between 200 and 201, that is, we cannot measure $v_2' - v_2$ with an accuracy greater than one-half of 1 per cent. But it is quite inconceivable that large charges such as are dealt with in commercial applications of electricity can be built up in an essentially different way from that in which the small charges whose electrons we can count are found to be. Furthermore, since it has definitely been proved that an electrical current is nothing but the motion of an electrical charge over or through a conductor, it is evident that the experiments under consideration furnish not only the most direct and convincing of evidence that all electrical charges are built up out of these very units which we have been dealing with as individuals in these experiments, but that all electrical currents consist merely in the transport of these electrons through the conducting bodies.

In order to show the beauty and precision with which these multiple relationships stand out in all experiments of this kind, a table corresponding to much more precise measurements than those given heretofore is here introduced (Table VI). The times of fall and rise shown in the first and second columns were taken with a Hipp chronoscope reading to one-thousandths of a second. The third column gives the reciprocals of these times. These are used in place of the velocities v_2 in the field, since distance of fall and rise is always the same. The fourth column gives the successive changes in speed due to the capture of ions. These also are expressed merely as time reciprocals.

For reasons which will be explained in the next section, each one of these changes may correspond to the capture of not merely one but of several distinct ions. The numbers in the fifth column represent simply the small integer by which it is found that the numbers in the fourth column must be divided in order to obtain the numbers in the sixth column. These [numbers in the sixth column] will be seen to be exactly alike within the limits of error of the experiment. The mean value at the bottom of the sixth column represents, then, the smallest charge ever caught from the air, that is, it is the elementary *ionic* charge. The seventh column gives the successive values of $v_1 + v_2$ expressed as reciprocal times. These numbers, then, represent the successive values of the *total* charge carried by the droplet. The eighth column gives the integers by which the numbers in the seventh column must be divided to obtain the numbers in the last column. These [numbers in the last column] also will be seen to be invariable. The mean at the bottom of the last column represents, then, *the electrical unit out of which the frictional charge on the droplet was built up, and it is seen to be identical with the ionic charge represented by the number at the bottom of the sixth column.*

TABLE VI[a]

t_g Sec.	t_F Sec.	$\frac{1}{t_F}$	$\frac{1}{t'_F} - \frac{1}{t_F}$	n'	$\frac{1}{n'}(\frac{1}{t'_F} - \frac{1}{t_F})$	$\frac{1}{t_g} + \frac{1}{t_F}$	n	$\frac{1}{n}(\frac{1}{t_g} + \frac{1}{t_F})$
11.848	80.708	.01236				.09655	18	.005366
11.890	22.366		.03234	6	.005390			
11.908	22.390	.04470				.12887	24	.005371
11.904	22.368		.03751	7	.005358			
11.882	140.565	.007192				.09138	17	.005375
			.005348	1	.005348			
11.906	79.600	.01254				.09673	18	.005374
11.838	34.748		.01616	3	.005387			
11.816	34.762	.02870				.11289	21	.005376
11.776	34.846							
11.840	29.286							
		.03414				.11833	22	.005379
11.904	29.236		.026872	5	.005375			
11.870	137.308	.007268				.09146	17	.005380
			.021572	4	.005393			
11.952	34.638	.02884				.11303	21	.005382
11.860			.01623	3	.005410			
11.846	22.104							
		.04507				.12926	24	.005386
11.912	22.268		.04307	8	.005384			
11.910	500.1	.002000				.08619	16	.005387
11.918	19.704		.04879	9	.005421			
		.05079				.13498	25	.005399
11.870	19.668							
			.03[79]4	7	.0054[20]			
11.888	77.630							
		.01285				.09704	18	.005390
11.894	77.806		.01079	2	.005395			
11.878	42.302	.02364				.10783	20	.005392
11.880			Means		.005386			.005384

Duration of exp.	= 45 min.	Pressure	= 75.62 cm.
Plate distance	= 16 mm.	Oil density	= .9199
Fall distance	= 10.21 mm.	Air viscosity	= $1,824 \times 10^{-7}$ [poise]
Initial volts	= 5,088.8	Radius (a)	= .[0]000276 cm.
Final volts	= 5,081.2	$\frac{l}{a}$ [mean free path $\div$ a]	= .034
Temperature	= 22.82°C.	Speed of fall	= .08584 cm./sec.

$e_i = 4.991 \times 10^{-10}$ [statcoulomb][b]

[a][The bracketed numbers are our corrections of errors in Millikan's original table.]
[b][The value presently accepted is 4.802×10^{-10} statcoulombs.]

It may be of interest to introduce one further table (Table VII) arranged in a slightly different way to show how infallibly the atomic structure of electricity follows from experiments like those under consideration.

TABLE VII

n	$4.917\times n$	Observed Charge	n	$4.917\times n$	Observed Charge
1	4.917	...	10	49.17	49.41
2	9.834	...	11	54.09	53.91
3	14.75	...	12	59.00	59.12
4	19.66	19.66	13	63.92	63.68
5	24.59	24.60	14	68.84	68.65
6	29.50	29.62	15	73.75	...
7	34.42	34.47	16	78.67	78.34
8	39.34	39.38	17	83.59	83.22
9	44.25	44.42	18	88.51	...

In this table 4.917 is merely a number obtained precisely as above from the change in speed due to the capture of ions and one which is proportional in this experiment to the ionic charge. The column headed 4.917 × *n* contains simply the whole series of exact multiples of this number from 1 to 18. The column headed "Observed Charge" gives the successive observed values of $(v_1 + v_2)$. It will be seen that during the time of observation, about four hours, this drop carried all possible multiples of the elementary charge from 4 to 18, save only 15. *No more exact or more consistent multiple relationship is found in the data which chemists have amassed on the combining powers of the elements and on which the atomic theory of matter rests than is found in the foregoing numbers.*

Such tables as these—and scores of them could be given—place beyond all question the view that an electrical charge wherever it is found, whether on an insulator or a conductor, whether in electrolytes or in metals, has a definite granular structure, that it consists of an exact number of specks of electricity (electrons) all exactly alike, which in static phenomena are scattered over the surface of the charged body and in current phenomena are drifting along the conductor. Instead of giving up, as Maxwell thought we should some day do, the "provisional hypothesis of molecular charges," we find ourselves obliged to make all our interpretations of electrical phenomena, *metallic as well as electrolytic*, in terms of it.

III. Mechanism of Change of Charge of a Drop

All of the changes of charge shown in Table IV were spontaneous changes, and it has been assumed that all of these changes were produced by the capture of ions from the air. When a negative drop suddenly increases its speed in the field, that is, takes on a larger charge of its own kind than it has been carrying, there seems to be no other conceivable way in which the change can be produced. But when the charge suddenly *decreases* there is no a priori reason for thinking that the change may not be due as well to the direct loss of a portion of the charge as to the neutralization of this same amount of electricity by the capture of a charge of opposite sign. That, however, the changes do actually occur, when no X-rays or radioactive rays are passing between the plates,[7] only by the capture of ions from the air, was rendered probable by the fact that drops not too heavily charged showed the same tendency on the whole to increase as to decrease in charge. This should not have been the case if there were two causes tending to decrease the charge, namely, direct loss and the capture of opposite ions, as against one tending to increase it, namely, capture of like ions. The matter was very convincingly settled, however, by making observations when the gas pressures were as low as 2 or 3 mm. of mercury. Since the number of ions present in a gas is in general proportional to the pressure,[8] spontaneous changes in charge should almost never occur at these low pressures; in fact, it was found that drops could be held for hours at a time without changing. The frequency with which the changes occur decreases regularly with the pressure, as it should if the changes are due to the capture of ions. For the number of ions formed by a given ionizing agent must vary directly as the pressure.

Again, the changes do not, in general, occur when the electrical field is on, for then the ions are driven instantly to the plates as soon as formed, at a speed of, say, 10,000 cm per second, and so do not have any opportunity to accumulate in the space between them. When the field is off, however, they do so accumulate until, in ordinary air, they reach the number of, say, 20,000 per cubic centimeter. These ions, being endowed with the kinetic energy of agitation characteristic of the temperature, wander rapidly through the gas and become a part of the drop as soon as they impinge upon it. It was thus that all the changes recorded in Table IV took place.

* * *

COMMENT

"WEIGHING" THE OIL DROP

As Millikan shows in his Table V, the two equations

$$\frac{v_1}{v_2} = \frac{mg}{Fe_n - mg} \quad \text{or} \quad e_n = \frac{mg}{Fv_1}(v_1 + v_2) \tag{9}$$

and

$$e_i = e_{n'} - e_n = \frac{mg}{Fv_1}(v_2' - v_2) \tag{10}$$

suffice to determine *relative* values for e_n (total charge on an oil drop) and e_i (charge of a captured ion), using nothing more than stopwatch measurements on the motion of the drop. The equations will determine *exact* values for those charges as well; but only if he can evaluate *mg*, the "effective" weight of the drop in a buoyant medium, which

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[7][As he elsewhere explains, Millikan repeatedly directed x-rays through the chamber or brought radioactive material into proximity with it. These measures would greatly accelerate the formation of ions in the air and so increase the chances of capturing one. But since they might also, by the same token, directly affect the charge carried by a droplet, Millikan considers only those changes that occur in the absence of such external influences.]

[8][That the number of *molecules* present in a gas is strictly proportional to the pressure, is an element in Avogadro's hypothesis. A relatively constant *fraction* of these molecules will become ionized under given conditions independent of pressure; hence the proportionality cited.]

we have already expressed above as[9]

$$mg = V(\sigma - \rho)g$$

or, assuming the drop to be a sphere of radius *a*,

$$mg = \frac{4\pi a^3}{3}(\sigma - \rho)g. \tag{3.6}$$

Millikan has no way to measure this quantity directly; but reasoning from the viscous (resistive) properties of the medium, air, he is able to devise an indirect measurement.

We have already derived in equation (3.3) an expression for the limiting velocity attained by a body that falls through a viscous medium:

$$v_1 = \frac{mg}{R} \tag{3.3}$$

where R is a so far unspecified proportionality constant. This constant is different for every different-sized oil drop. However, Millikan was able to relate it to a general, measurable quantity. The *viscosity* of a fluid is its resistance to being passed through, as we mentioned, or to flowing past something. This property can be more precisely defined and measured by various procedures.[10] Accurate tables of the viscosity of air were available to Millikan. In addition, *Stokes's Law* (derived by George Stokes in 1845) applies this property to the case of a smooth sphere moving through a fluid. Millikan assumed that his oil drops were spheres; with air viscosity η and drop radius a, Stokes's Law gives $R = 6\pi\eta a$.[11] Substituting into Equation (3.3) gave Millikan

$$v_1 = \frac{mg}{6\pi\eta a}. \tag{3.7}$$

He then eliminated *a* between equations (3.6) and (3.7) to obtain

$$m = \frac{4\pi}{3}(\sigma - \rho)\left(\frac{mg}{6\pi\eta v_1}\right)^3,$$

from which

$$\left(\frac{mg}{v_1}\right)^2 = \frac{3 \cdot 6^3\pi^2\eta^3 v_1}{4g(\sigma - \rho)}.$$

Hence for the coefficient mg/Fv_1 in Millikan's equations (9) and (10) we have

$$\frac{mg}{Fv_1} = \left(\frac{9\pi}{F}\sqrt{\frac{2\eta^3}{g(\sigma - \rho)}}\right) \cdot \sqrt{v_1} \tag{3.8}$$

which is sufficient to evaluate e_n and e_i.

[9] See sentence with equation (3.2), p. 26.
[10] The details are beyond our scope.
[11] Again, we cannot follow the details.

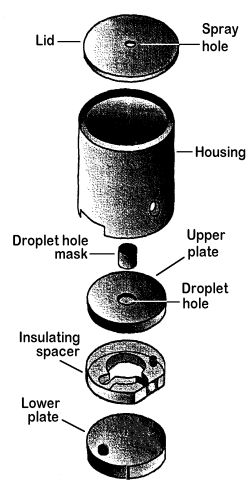
EXPERIMENT: MEASUREMENT OF THE "ATOM OF CHARGE"

We use the Pasco oil-drop apparatus, which is similar to Millikan's, only far smaller. In addition, the viewing telescope and associated illumination are integrally mounted. Make sure that the apparatus is level, the telescope illumination *on*, and the high voltage *off*. If the following preliminary adjustments have not already been made by the laboratory assistant, perform them now:

Focus the telescope: Remove the chamber housing, lid, and droplet hole mask, as illustrated in the drawing. Unscrew the focusing wire from its storage position and carefully insert it into the droplet hole in the center of the upper plate. Adjust the Reticle Focusing Ring on the telescope to bring the viewing reticle into focus. Adjust the Droplet Focusing Ring to bring the wire image into sharp focus. When the telescope is focused, each minor reticle division corresponds to a droplet fall distance of .01 cm; each major division to .05 cm.

Focus the lamp: Turn the lamp's Horizontal Adjustment to bring the right edge of the wire into highest contrast compared to the center of the wire. Turn the Vertical Adjustment to direct the brightest light onto the part of the wire that is within the reticle. Remove and return the focusing wire to its storage position and reassemble the chamber.

Adjust the plate voltage: Connect the high-voltage supply. Turn the cable-mounted plate switch to the GROUNDED position. Adjust the supply to deliver between 400 and 500 volts, and record the voltage. The apparatus is now ready for use.



Begin by introducing oil droplets into the chamber. Move the vent lever to the Spray Droplets position; this will permit air to escape from the chamber as the oil spray enters. The atomizer nozzle should be turned vertically downward. After one or two test squeezes to make sure oil sprays correctly, point the nozzle into the spray hole in the lid of the chamber.

While looking through the telescope, give the atomizer *one quick squeeze*, followed immediately by *one slow squeeze* to create a vertical downdraft. When you see a shower of drops through the telescope, move the vent lever to the OFF position.[12]

With a number of drops in view, observe that different drops fall with noticeably different speeds. This illustrates the relation expressed in equation (3.3)—heavier drops fall proportionately faster than lighter ones. Pay attention to those droplets that fall *slowly*—about 10–25 seconds to travel the distance between major reticle lines (.05 cm).

Energizing the plates will reveal which drops bear a charge, and whether the applied voltage is in the right direction to *reverse* a drop's falling motion (which is desired)

[12]Avoid immoderate use of the atomizer. The object is to introduce a small number of drops, from which a single drop can be chosen. If a large, bright cloud appears in the viewing area, spraying has been excessive. You can try waiting a few minutes to see if the drops settle out of view, but most likely the chamber will have to be disassembled and cleaned.

or not. Repeatedly move the cable-mounted plate switch to its + or – positions. When a droplet is found that can be driven back towards the upper plate by applying the voltage, try to keep it in capture by alternately applying and removing that voltage.

Measurements on a single drop

Execution of the experiment will require two people, one to observe the drop and operate the stopwatch, the other to read the watch and record the timed intervals. It is usually best to time successive rising and falling transits of the droplet over any *major* (.05 cm) division on the telescope's viewing reticle—ten to twenty measurements in each direction are desirable. Do not raise or lower the plate voltage adjustment once you begin timing a drop. Record the fall times and rise times in separate columns. A series of measurements of these times, *all performed on the same drop*, will enable you to construct a table like Millikan's Table VI. See Table A (figures are from 1997).

Table A[a]

	d = 0.05cm		d = 0.05cm	Charge on ion			Frictional charge		
t_g	$v_1(=d/t_g)$ (cm/sec)	t_F	$v_2(=d/t_F)$ (cm/sec)	$(v_2'-v_2)$	n'	$\frac{v_2'-v_2}{n'}$	v_1+v_2	n	$\frac{v_1+v_2}{n}$
18.2	$\overline{.00286}$	3.8	.01316				.01602	3	.00534
18.6				.00470	1	.00470			
19.2		2.8	.01786						
18.0				.01561	3	.00520			
17.2		22.2	.00225						
15.4				.00544	1	.00544			
16.7		6.5	.00769						
18.0				.00541	1	.00541			
15.4		21.9	.00228						
17.3				.01123	2	.00562			
18.4		3.7	.01351						
$\overline{17.5}$						$\overline{.00527}$			$\overline{.00534}$

[a]Overlined numbers are averages.

Measurements on Multiple Drops

Although it would be convenient to make all measurements on a single droplet, it is seldom possible to do so; for it proves extremely difficult to hold any one drop in play for long periods of time. But droplets of the same material differ from one another only in mass. Therefore—see Equation (3.8)—if the electric field intensity F is not changed, the coefficient in Millikan's equations (9) and (10) will vary, from drop to drop, only in proportion to $\sqrt{v_1}$. You may therefore combine data from different drops by tabulating and comparing

$$\sqrt{v_1}(v_1+v_2) \quad \text{and} \quad \sqrt{v_1}(v_2'-v_2)$$

instead of $(v_1 + v_2)$ and $(v'_2 - v_2)$ directly. See Table B (the figures are from 1983).

Table B

Obsvd. t_g	v_1 (i.e., $1/t_g$)	Obsvd. t_F	v_2 (i.e., $1/t_F$)	$(v_2' - v_2)$	$\sqrt{v_1}(v_2' - v_2)$
7.2 sec	0.1389	2.1 sec	0.4762	0.1984	0.0739
		3.6	0.2778	0.1978	0.0737
		12.5	0.0800		
5.8 sec	0.1724	1.9	0.5263	0.3792	0.1575
		6.8	0.1471	0.8529	0.3541
		1.0	1.000	0.7059	0.2931
		3.4	0.2941		

The table entries reflect measurements on two different droplets. In the sixth column the observed changes in rise speed are multiplied by $\sqrt{v_1}$ as explained above. The resulting quantities are very nearly integral multiples of about .076. This trial, therefore, would indicate the existence of a fundamental ionic charge, with a relative value of about .076.

Enhancing Ionization

It is hoped that the droplet will acquire an ion during one of its falls, as is indeed illustrated in the foregoing chart. Capture of an ion does not change the drop's falling velocity, but it will change the drop's rising velocity when the plate voltage is next applied. However, should you find that your droplet has not caught an ion after 10–20 successive rises, move the vent lever to the IONIZATION ON position for a few seconds during the next fall period. That will expose the air in the chamber to a low-level radioactive thorium source, thus increasing the number of ionized particles in the air and multiplying the chances for the falling drop to encounter an ion. Repeat the exposure if again the drop fails to capture an ion after 10-20 measurements, and continue for as long as the droplet can be held in play. It is desirable to measure as many different charges on a single drop as possible.

Frictional Charge and Ionic Charge

As Millikan explains, the *initial* charge on a droplet, prior to the capture of an ion, must have been acquired by friction during the spraying process. In the foregoing chart it is assumed that the first measurement made on the drop reflects this initial frictional charge. While that may be a reasonable assumption provided the first measurement was made *soon* after spraying, and the observed frequency of ion capture by a drop is not high, remember that it is *only* an assumption—the drop may have already picked up an ion by the time we first measure it. That is especially to be suspected when, as in this example, the apparent number n of unit charges on the drop is only *3*—a number so small it could as easily reflect ion capture as frictional electrification.

If it is desired to measure *frictional charges specifically*, do not use the ionization source—we want to minimize the chances of capturing an ion. As soon after spraying as possible, try to select a drop whose rising velocity is noticeably *faster* than others. Rapid rising indicates a high charge, which is more likely to have been frictionally acquired. As before, time the drop for 10–20 transits, or until it appears to have encountered an ion. Let the drop escape; spray again, and repeat the measurements on another drop.

Calculation of the Fundamental Charge

The stopwatch measurements illustrated in the previous chart suffice to determine values of and which, as Millikan's equations (9) and (10) make clear, are proportional to the *initial frictional charge on a drop* and the *charge on a captured ion*, respectively. But in order to calculate exact values of these charges, not just relative ones, we must evaluate the coefficient that appears in those equations. By our equation (3.8) above, the coefficient can be calculated from the following quantities:

(a) the velocity v_1 of the falling drop. This you have already measured; just remember to cite the correct value for each different drop. (Remember to average and to take the square root.)

(b) the electric field intensity F between the plates. To compare our results numerically with Millikan's we will calculate this in e.s.u., so we must express the potential difference between the plates in e.s.u., i.e. in *statvolts*. Our meters give volts and 1 volt = 1/300 statvolts, so *the voltmeter reading must be divided by 300*. The plate separation is 0.76 cm, unless a different value is labeled on the side of your apparatus. Then the voltage in statvolts divided by the plate separation in cm will give electric field intensity in e.s.u.[13]

(c) the density of the oil, σ. We use Squibb #5597 non-volatile mineral oil, for which $\sigma = .886$ g/cm^3.

(d) the density of the air, ρ, obtained from Graph B below. You will have to note the temperature of the air in the chamber and the barometric pressure in the room. Typical values for ρ are about $.0012$ g/cm^3.

(e) the viscosity of the air, η, obtained from Graph A below. Typical values for η are around 1825×10^{-7} poise (gm/cm-sec).

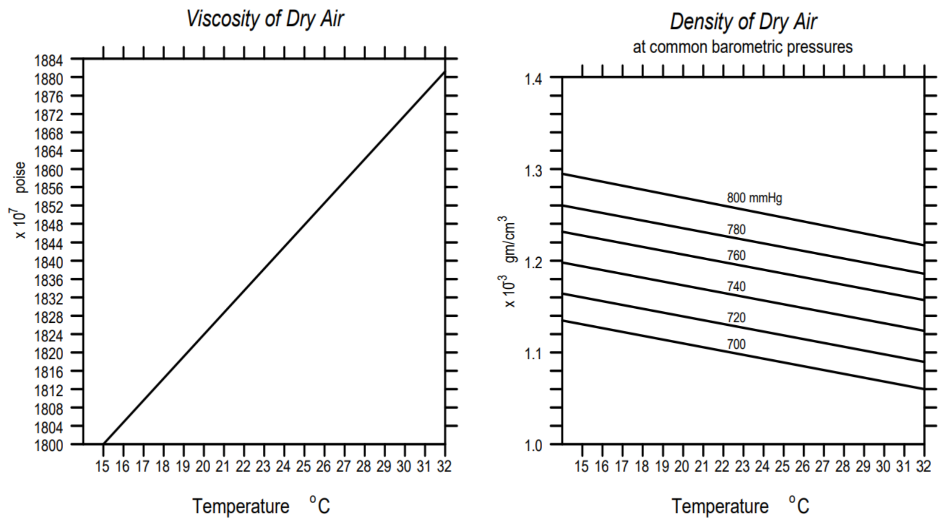
(f) the acceleration of gravity, $g = 980$ cm/sec^2.

Remarks on Avogadro's Number

As Millikan himself pointed out, mere *measurement* of the fundamental charge is not necessarily, in itself, the most important fruit of this experiment. Of far greater consequence, perhaps, is the demonstration that the same "electron" lies at the root of both *static electricity* (the frictional charge on the drop) and *chemical activity* (the ionic charge). This is strong, even conclusive evidence, for the interpretation of chemical power as being fundamentally electrical, for Millikan's results indicate that the *fundamental unit of charge* corresponds also to the *fundamental unit of chemical combining power*. If the electron has indeed this dual significance, then Millikan's measurement of the electron charge will at last permit accurate calculation of that theoretically pivotal but experimentally elusive quantity, *Avogadro's number*—the number of atoms in a gram-atomic weight of any element.[14]

[13] See Appendix (p. 203) for an explanation of the units and for the relation between F and the potential difference.

[14] There had been earlier efforts to estimate Avogadro's number, predating Millikan's oil-drop measurements; Einstein cites one of his own in his paper that appears in Chapter 6 below. But the most reliable



Graphs A and B

Recall that the "equivalent weight" of an element is that much of it, measured on the scale of atomic weights with Hydrogen = 1, which exercises *one unit* of chemical combining power. But the combining power of one hydrogen atom is taken as the unit for chemical combining power generally; thus for hydrogen and all atoms equal to hydrogen in their combining power, the atomic weight and the equivalent weight are the same.[15]

Now if one unit of chemical combining power actually means *one electron of electric charge*, then one gram-atomic weight of hydrogen will exercise the cumulative charge of Avogadro's number of electrons, which is

$$N_0 \times e,$$

where N_0 denotes Avogadro's number and *e* is the electron charge. Now the above is also the total charge associated with a gram-equivalent weight of hydrogen, since for it the gram-atomic weight is equal to the gram-equivalent weight. But we found in electrolysis that the total charge associated with liberation or transfer of one gram-equivalent weight of *any* element is 96,500 coulombs, which equals 2.895×10^{14} stat-coulombs; hence we may write

$$2.895 \times 10^{14} \text{ statC} = N_0 \times e$$

While Millikan has shown that (modern value):

$$e = 4.802 \times 10^{-10} \text{ statC}$$

Thus we have

$$N_0 = \text{Avogrado's number} = \frac{2.895 \times 10^{14}}{4.802 \times 10^{-10}} = 6.028 \times 10^{23}.$$

(Other, more recent, determinations yield 6.023×10^{23}.)

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determinations of Avogadro's number and many other constants all depend on accurate knowledge of the electron charge (cf. *The Electron*, Chapter X).

[15]Chemical combining power is commonly expressed in units of "valence"; hydrogen and all atoms equivalent to it are said to be *univalent* atoms.

Weighing and Sizing Elementary Particles

Determination of Avogadro's number puts us in a position to "weigh all the atoms"; for if there are N_0 atoms in a gram-atomic weight of an element, then each atom must weigh $1/N_0$ of the gram-atomic weight. For example, a gram-atomic weight of hydrogen weighs 1.008 grams. Then a single hydrogen atom, for example, must weigh

$$1.008 \div (6.023 \times 10^{23}) = 1.674 \times 10^{-24}\ \text{g}$$

and similarly for all the other atoms.

In Chapter II of *The Electron* Millikan pointed out that the term "electron" had been first introduced in 1891 as a name for the supposed "natural unit of electricity," namely, that quantity of electricity exhibited by a hydrogen ion or any other univalent ion. Thus it denoted simply a *definite quantity of electricity* without reference to any mass or inertia which might be associated with it. It is clear that Millikan's oil-drop experiment has proved the existence of "electron" in this original sense.

But measurements by Thomson and others of the charge-to-mass ratio of ions in both gases and liquids strongly implicated Thomson's *cathode-ray corpuscle* as the essential constituent whereby atoms acquire either an excess or deficiency of negative charge to become ions.[16] This implies that *the charge borne by each cathode ray corpuscle must be the very quantity of charge that had been termed "electron."* As a result, Millikan explains, the word "electron" gradually changed its meaning to become synonymous with the cathode-ray corpuscle: a particle having definite mass as well as charge, a constituent not only of cathode rays but of atoms themselves.[17] Since the electron (in the new sense of *particle*) has the charge-to-mass ratio that was determined for cathode-rays, and has, on the other hand, the quantity of charge that is disclosed in Millikan's experiment, we are able to calculate its mass, to "weigh" the electron itself.

We have, from the cathode rays,

$$m/e = .5685 \times 10^{-7}\text{g/abC} = 1.895 \times 10^{-18}\ \text{g/statC}.$$

Then, since the electron charge is

$$e = 4.802 \times 10^{-10}\ \text{statC},$$

we may calculate

$$m = 1.895 \times 10^{-18} \times 4.802 \times 10^{-10} = .9099 \times 10^{-27}\ \text{g}.$$

Hence the electron will be almost 1840 times lighter than the hydrogen atom. That an electron weighs such a slight fraction of even the lightest atom will prove important in formulating a conception of the atom's structure, as discussed in Rutherford's paper which follows.

[16] See footnote 6 above; Millikan devotes the whole of his Chapter II to this very interesting story.

[17] Millikan views this alteration of usage with real distress: "It is unfortunate that modern writers have not been more careful to retain the original significance of ['electron'], for it is obvious that a word is needed which denotes merely the elementary unit of electricity and has no implication as to where that unit is found, to what it is attached, with what inertia it is associated, or whether it is positive or negative in sign."

"Sizing" an Atom

Another magnitude interesting in itself and also important in Rutherford's paper is the approximate *size* of an atom. Using Avogadro's Number this is easy to calculate for elemental solids.

Take gold, which is what Rutherford will be dealing with. The density of gold is 19.3 g/cm^3, while its gram-atomic weight is 197. Therefore there are .098 gram equivalents in a cubic centimeter of gold. But since there are N_0 atoms in a gram-atomic weight of an element, we can now say that there are $.098 \times (6.023 \times 10^{23})$, or 5.90×10^{22}, atoms in a cubic centimeter of gold. Each atom thus occupies a volume of $1/(5.90 \times 10^{22}) = 1.69 \times 10^{-23}$ cubic centimeters. The cube root of this, 2.57×10^{-8} cm, will give the side of a cube of this volume. Let us assume that gold atoms are spherical and that in solid gold they fit together in a more or less cubical pattern. Then half of 2.57×10^{-8} cm, i.e.

$$1.285 \times 10^{-8} \text{ cm},$$

is a reasonable rough estimate for the radius of a gold atom. Rutherford implicitly adopts this size estimate as he analyses how particles that are *much smaller* than this interact with gold atoms.