The Completeness of Quantum Mechanics and the Locality Principle

This chapter presents Einstein's objections to the Copenhagen interpretation of wave mechanics and Bohr's response, followed by David Bohm's interesting interpretation of the implications of Einstein's argument. Einstein presented his objections in the form of various thought-experiments. The most probing of these appears in a paper "Can Quantum-Mechanical Description of Reality be Considered Complete?" by Einstein, Podolsky, and Rosen (referred to as EPR), which is still the object of debate among physicists and philosophers.

In discussions with Bohr and others over a period of years, Einstein had tried to devise a thought-experiment that would demonstrate the incompleteness of wave mechanics. Bohr offers the following summary of these conversations:

Niels Bohr: Einstein's Objections to Quantum Mechanics[1]

> To illustrate his attitude, Einstein referred at one of the sessions to the simple example, illustrated by Figure 1 of a particle (electron or photon) penetrating through a hole or a narrow slit in a diaphragm placed at some distance before a photographic plate. On account of the diffraction of the wave connected with the motion of the particle and indi-

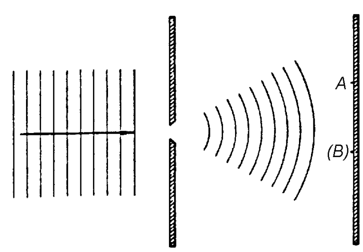

Figure 1

> cated in the figure by the thin lines, it is under such conditions not possible to predict with certainty at what point the electron will arrive at

[1]From "Discussion with Einstein on Epistemological Problems in Atomic Physics," in P.A. Schilpp, ed., *Albert Einstein: Philosopher-Scientist* (Evanston: The Library of Living Philosophers, 1949), 208-17. Reprinted in J. Wheeler and W. Zurek, eds., *Quantum Theory and Measurement* (Princeton University Press, 1983), 19-26. Hereafter the latter work will be referred to as *QTM*. Changes in notation will be made occasionally.

the photographic plate, but only to calculate the probability that, in an experiment, the electron will be found within any given region of the plate. The apparent difficulty, in this description, which Einstein felt so acutely, is the fact that, if in the experiment the electron is recorded at one point A of the plate, then it is out of the question of ever observing an effect of this electron at another point (B), although the laws of ordinary wave propagation offer no room for a correlation between two such events.

Einstein's attitude gave rise to ardent discussions within a small circle... The discussions, however, centered on the question of whether the quantum-mechanical description exhausted the possibilities of accounting for observable phenomena or, as Einstein maintained, the analysis could be carried further and, especially, of whether a fuller description of the phenomena could be obtained by bringing into consideration the detailed balance of energy and momentum in individual processes....

The problem raised by Einstein was now to what extent a control[2] of the momentum and energy transfer, involved in a location of the particle in space and time, can be used for a further specification of the state of the particle after passing through the hole. Here, it must be taken into consideration that the position and the motion of the diaphragm and the shutter have so far been assumed to be accurately coordinated with the space-time reference frame. This assumption implies, in the description of the state of these bodies, an essential latitude as to their momentum and energy which need not, of course, noticeably affect the velocities, if the diaphragm and the shutter are sufficiently heavy. However, as soon as we want to know the momentum and energy of these parts of the measuring arrangement with an accuracy sufficient to control the momentum and energy exchange with the particle under investigation, we shall, in accordance with the general indeterminacy relations, lose the possibility of their accurate location in space and time. We have, therefore, to examine how far this circumstance will affect the intended use of the whole arrangement and, as we shall see, this crucial point clearly brings out the complementary character of the phenomena.

Returning for a moment to the case of the simple arrangement indicated in Figure 1, it has so far not been specified to what use it is intended. In fact, it is only on the assumption that the diaphragm and the plate have well-defined positions in space that it is impossible, within

[2] ["a control": Here meaning *an ascertainment* or *a verification*. Compare Schrödinger's similar usage in Chapter IX, 162 above.]

the frame of the quantum-mechanical formalism, to make more detailed predictions as to the point of the photographic plate where the particle will be recorded. If, however, we admit a sufficiently large latitude in the knowledge of the position of the diaphragm it should, in principle, be possible to control the momentum transfer to the diaphragm and, thus, to make more detailed predictions as to the direction of the electron path from the hole to the recording point. As regards the quantum-mechanical description, we have to deal here with a two-body system consisting of the diaphragm as well as of the particle, and it is just with an explicit application of conservation laws to such a system that we are concerned in the Compton effect[3] where, for instance, the observation of the recoil of the electron by means of a cloud chamber allows us to predict in what direction the scattered photon will eventually be observed.

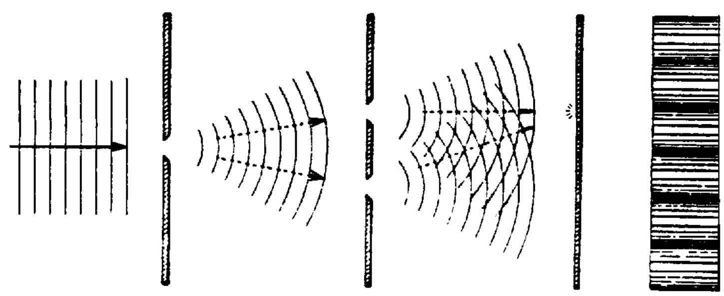

Figure 2

The importance of considerations of this kind was, in the course of the discussions, most interestingly illuminated by the examination of an arrangement where between the diaphragm with the slit and the photographic plate is inserted another diaphragm with two parallel slits, as is shown in Figure 2. If a parallel beam of electrons (or photons) falls from the left on the first diaphragm, we shall, under usual conditions, observe on the plate an interference pattern indicated by the shading of the photographic plate shown in front view to the right of the figure. With intense beams, this pattern is built up by the accumulation of a large number of individual processes, each giving rise to a small spot on the photographic plate, and the distribution of these spots follows a simple law derivable from the wave analysis. The same distribution

[3][See n. 15 in Chapter X.]

> should also be found in the statistical account of many experiments performed with beams so faint that in a single exposure only one electron (or photon) will arrive at the photographic plate at some spot shown in the figure as a small star. Since now as indicated by the broken arrows, the momentum transferred to the first diaphragm ought to be different if the electron was assumed to pass through the upper or the lower slit in the second diaphragm, Einstein suggested that a control of the momentum transfer would permit a closer analysis of the phenomenon and, in particular, to decide through which of the two slits the electron had passed before arriving at the plate.
>
> A closer examination showed, however, that the suggested control of the momentum transfer would involve a latitude in the knowledge of the position of the diaphragm which would exclude appearance of the interference phenomena in question.

* * *

In direct response to EPR, Bohr wrote the following article.

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?[4]

N. Bohr

[The] argumentation [of Einstein, Podolsky, and Rosen] would hardly seem suited to affect the soundness of quantum-mechanical description, which is based on a coherent mathematical formalism covering automatically any procedure of measurement like that indicated. The apparent contradiction in fact discloses only an essential inadequacy of the customary viewpoint of natural philosophy for a rational account of physical phenomena of the type with which we are concerned in quantum mechanics. Indeed the *finite interaction between object and measuring agencies* conditioned by the very existence of the quantum of action entails—because of the impossibility of controlling the reaction of the object on the measuring instruments if these are to serve their purpose—the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality. In fact, as we shall see, a criterion of reality like that proposed by the named authors contains—however cautious its formulation may appear—an essential ambiguity when it is applied to the actual problems with which we are here concerned. In order to make the argument to this end as clear as possible, I shall first consider in some detail a few simple examples of measuring arrangements.

[4][Bohr's reply to the EPR paper. *Physical Review* 48 (1935), 696-702, reprinted in *QTM*, 145-51.]

Let us begin with the simple case of a particle passing through a slit in a diaphragm, which may form part of some more or less complicated experimental arrangement. Even if the momentum of this particle is completely known before it impinges on the diaphragm, the diffraction by the slit of the plane wave giving the symbolic representation of its state will imply an uncertainty in the momentum of the particle, after it has passed the diaphragm, which is the greater the narrower the slit. Now the width of the slit, at any rate if it is still large compared with the wave-length, may be taken as the uncertainty Δx of the position of the particle relative to the diaphragm, in a direction perpendicular to the slit. Moreover, it is simply seen from de Broglie's relation between momentum and wave-length that the uncertainty Δp of the momentum of the particle in this direction is correlated to Δx by means of Heisenberg's general principle $\Delta p \Delta x \approx h$ .... Obviously the uncertainty Δp is inseparably connected with the possibility of an exchange of momentum between the particle and the diaphragm; and the question of principal interest for our discussion is now to what extent the momentum thus exchanged can be taken into account in the description of the phenomenon to be studied by the experimental arrangement concerned, of which the passing of the particle through the slit may be considered as the initial stage. Let us first assume that, corresponding to usual experiments on the remarkable phenomena of electron diffraction, the diaphragm, like the other parts of the apparatus—say a second diaphragm with several slits parallel to the first and a photographic plate—is rigidly fixed to a support which defines the space frame of reference. Then the momentum exchanged between the particle and the diaphragm will, together with the reaction of the particle on the other bodies, pass into this common support, and we have thus voluntarily cut ourselves off from any possibility of taking these reactions separately into account in predictions regarding the final result of the experiment—say the position of the spot produced by the particle on the photographic plate. The impossibility of a closer analysis of the reactions between the particle and the measuring instrument is indeed no peculiarity of the experimental procedure described, but is rather an essential property of any arrangement suited to the study of the phenomena of the type concerned, where we have to do with a feature of *individuality* completely foreign to classical physics. In fact, any possibility of taking into account the momentum exchanged between the particle and the separate parts of the apparatus would at once permit us to draw conclusions regarding the "course" of such phenomena—say through what particular slit of the second diaphragm the particle passes on its way to the photographic plate—which would be quite incompatible with the fact that the probability of the particle reaching a given element of area on this plate is determined not by the presence of any particular slit, but by the positions of all the slits of the second diaphragm within reach of the associated wave diffracted from the slit of the first diaphragm.

By another experimental arrangement, where the first diaphragm is not rigidly connected with the other parts of the apparatus, it would at least in principle be possible to measure its momentum with any desired accuracy before and after the passage of the particle, and thus to predict the momentum of the latter after it has passed through the slit. In fact, such measurements of momentum require only an unambiguous application of the classical law of conservation of momentum, applied for instance to a collision process between the diaphragm and some test body, the momentum of which is suitably controlled before and after the collision. It is true that such a control will essentially depend on an examination of the space-time course of some process to which the ideas of classical mechanics can be applied; if, however, all spatial dimensions and time intervals are taken sufficiently large, this involves clearly no limitation as regards the accurate control of the momentum of the test bodies, but only a renunciation as regards the accuracy of the control of their space-time coordination. This last circumstance is in fact quite analogous to the renunciation of the control of the momentum of the fixed diaphragm in the experimental arrangement discussed above, and depends in the last resort on the claim of a purely classical account of the measuring apparatus, which implies the necessity of allowing a latitude corresponding to the quantum-mechanical uncertainty relations in our description of their behavior.

The principal difference between the two experimental arrangements under consideration is, however, that in the arrangement suited for the control of the momentum of the first diaphragm, this body can no longer be used as a measuring instrument for the same purpose as in the previous case, but must, as regards its position relative to the rest of the apparatus, be treated, like the particle traversing the slit, as an object of investigation, in the sense that the quantum-mechanical uncertainty relations regarding its position and momentum must be taken explicitly into account. In fact, even if we knew the position of the diaphragm relative to the space frame before the first measurement of its momentum, and even though its position after the last measurement can be accurately fixed, we lose, on account of the uncontrollable displacement of the diaphragm during each collision process with the test bodies, the knowledge of its position when the particle passed through the slit. The whole arrangement is therefore obviously unsuited to study the same kind of phenomena as in the previous case. In particular it may be shown that, if the momentum of the diaphragm is measured with an accuracy sufficient for allowing definite conclusions regarding the passage of the particle through some selected slit of the second diaphragm, then even the minimum uncertainty of the position of the first diaphragm compatible with such a knowledge will imply the total wiping out of any interference effect—regarding the zones of permitted impact of the particle on the photographic plate—to which the presence of more than one slit in the second diaphragm would give rise in case the positions of all apparatus are fixed relative to each other.

In an arrangement suited for measurements of the momentum of the first diaphragm, it is further clear that even if we have measured this momentum before the passage of the particle through the slit, we are after this passage still left with a *free choice* whether we wish to know the momentum of the particle or its initial position relative to the rest of the apparatus. In the first eventuality we need only to make a second determination of the momentum of the diaphragm, leaving unknown forever its exact position when the particle passed. In the second eventuality we need only to determine its position relative to the space frame with the inevitable loss of the knowledge of the momentum exchanged between the diaphragm and the particle. If the diaphragm is sufficiently massive in comparison with the particle, we may even arrange the procedure of measurements in such a way that the diaphragm after the first determination of its momentum will remain at rest in some unknown position relative to the other parts of the apparatus, and the subsequent fixation of this position may therefore simply consist in establishing a rigid connection between the diaphragm and the common support.

My main purpose in repeating these simple, and in substance well-known considerations, is to emphasize that in the phenomena concerned we are not dealing with an incomplete description characterized by the arbitrary picking out of different elements of physical reality at the cost of [sacrificing] other such elements, but with a rational discrimination between essentially different experimental arrangements and procedures which are suited either for an unambiguous use of the idea of space location, or for a legitimate application of the conservation theorem of momentum. Any remaining appearance of arbitrariness concerns merely our freedom of handling the measuring instruments, characteristic of the very idea of experiment. In fact, the renunciation in each experimental arrangement of the one or the other of two aspects of the description of physical phenomena—the combination of which characterizes the method of classical physics, and which therefore in this sense may be considered as complementary to one another—depends essentially on the impossibility, in the field of quantum theory, of accurately controlling the reaction of the object on the measuring instruments, i.e., the transfer of momentum in case of position measurements, and the displacement in case of momentum measurements. Just in this last respect any comparison between quantum mechanics and ordinary statistical mechanics, however useful it may be for the formal presentation of the theory, is essentially irrelevant. Indeed we have in each experimental arrangement suited for the study of proper quantum phenomena not merely to do with an ignorance of the value of certain physical quantities, but with the impossibility of defining these quantities in an unambiguous way.

The last remarks apply equally well to the special problem treated by Einstein, Podolsky and Rosen, . . . which does not actually involve any greater intricacies than the simple examples discussed above. The particular quantum-mechanical state of two free particles, for which they give an explicit mathematical expression, may be reproduced, at least in principle, by a simple experimental arrangement, comprising a rigid diaphragm with two parallel slits, which are very narrow compared with their separation, and through each of which one particle with given initial momentum passes independently of the other. If the momentum of this diaphragm is measured accurately before as well as after the passing of the particles, we shall in fact know the sum of the components perpendicular to the slits of the momenta of the two escaping particles, as well as the difference of their initial positional coordinates in the same direction; while of course the … difference of the components of their momenta, and the sum of their positional coordinates, are entirely unknown. In this arrangement, it is therefore clear that a subsequent single measurement either of the position or of the momentum of one of the particles will automatically determine the position or momentum, respectively, of the other particle with any desired accuracy; at least if the wave-length corresponding to the free motion of each particle is sufficiently short compared with the width of the slits. As pointed out by the named authors, we are therefore faced at this stage with a completely free choice whether we want to determine the one or the other of the latter quantities by a process which does not directly interfere with the particle concerned.

Like the above simple case of the choice between the experimental procedures suited for the prediction of the position or the momentum of a single particle which has passed through a slit in a diaphragm, we are, in the "freedom of choice" offered by the last arrangement, just concerned with a *discrimination between different experimental procedures which allow of the unambiguous use of complementary classical concepts.* In fact to measure the position of one of the particles can mean nothing else than to establish a correlation between its behavior and some instrument rigidly fixed to the support which defines the space frame of reference. Under the experimental conditions described such a measurement will therefore also provide us with the knowledge of the location, otherwise completely unknown, of the diaphragm with respect to this space frame when the particles passed through the slits. Indeed, only in this way we obtain a basis for conclusions about the initial position of the other particle relative to the rest of the apparatus. By allowing an essentially uncontrollable momentum to pass from the first particle into the mentioned support, however, we have by this procedure cut ourselves off from any future possibility of applying the law of conservation of momentum to the system consisting of the diaphragm and the two particles and therefore have lost our only basis for an unambiguous application of the idea of momentum in predictions regarding the behavior of the second particle. Conversely, if we choose to measure the momentum of one of the particles, we lose through the uncontrollable displacement inevitable in such a measurement any possibility of deducing from the behavior of this particle the position of the diaphragm relative to the rest of the apparatus, and have thus no basis whatever for predictions regarding the location of the other particle.

[5]From our point of view we now see that the wording of the above-mentioned

[5][This paragraph has been moved from its place in the original, which was marred by a typesetting error.]

criterion of physical reality proposed by Einstein, Podolsky and Rosen contains an ambiguity as regards the meaning of the expression "without in any way disturbing a system." Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of an *influence on the very conditions which define the possible types of predictions regarding the future behavior of the system*. Since these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete. On the contrary this description, as appears from the preceding discussion, may be characterized as a rational utilization of all possibilities of unambiguous interpretation of measurements, compatible with the finite and uncontrollable interaction between the objects and the measuring instruments in the field of quantum theory. In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, the coexistence of which might at first sight appear irreconcilable with the basic principles of science. It is just this entirely new situation as regards the description of physical phenomena, that the notion of *complementarity* aims at characterizing.

* * *

Th[e] necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are to be treated as measuring instruments and those which constitute the objects under investigation may indeed be said to form a *principal distinction between classical and quantum-mechanical description of physical phenomena.* It is true that the place within each measuring procedure where this discrimination is made is in both cases largely a matter of convenience. While, however, in classical physics the distinction between object and measuring agencies does not entail any difference in the character of the description of the phenomena concerned, its fundamental importance in quantum theory, as we have seen, has its root in the indispensable use of classical concepts in the interpretation of all proper measurements, even though the classical theories do not suffice in accounting for the new types of regularities with which we are concerned in atomic physics.

* * *

Another Interpretation of the EPR Argument

Bohr's response did not induce Einstein to change his interpretation of quantum mechanics. He continued to think that, while it might give the correct statistical laws, it involved an inadequate conception of individual elementary processes.

Thus, he saw it as a provisional theory that would eventually be superseded because it was based on the wrong concepts, that is, on ones that did not correspond to the real physical states. He may have been thinking that concepts such as potential energy and material point that quantum mechanics inherited from classical mechanics had to be replaced before physics could go beyond the statistical laws of quantum theory. Then Einstein would have been using the EPR argument to try to induce physicists to join him in the search for radically new physical concepts.

However, Einstein did agree with at least one aspect of Bohr's reply, namely, his emphasis on the central role of the expression "without in any way disturbing a system" in the EPR argument. This phrase refers to the assumption that "no real change can take place in the second system in consequence of anything that may be done to the first system." This assumption is one formulation of what has become known as the *locality principle*. In EPR's statement of the argument, it is not emphasized that this is the major assumption from which the conclusion follows. This lack of clarity in the presentation of the argument may have been due to Podolsky, who actually wrote the paper. For Einstein himself believed that the truth of the locality principle was what forced the conclusion that quantum mechanics was incomplete.

According to Einstein the "orthodox" quantum theoreticians argue as follows:[6]

> If the partial systems I and II [for example, in the thought-experiment of the EPR paper] form a total system which is described by its ψ-function. . . . , there is no reason why any mutually independent existence (state of reality) should be ascribed to the partial systems I and II viewed separately, *not even if the partial systems are spatially separated from each other at the particular time under consideration*. The assertion that, in this latter case, the real situation of II could not be (directly) influenced by any measurement taken on I is, therefore, within the framework of quantum theory, unfounded and (as the paradox [that is, the EPR argument] shows) unacceptable.
>
> By this way of looking at the matter it becomes evident that the paradox forces us to relinquish one of the following two assertions:
>
> (1) the description by means of the ψ-function is *complete*,
>
> (2) the real states of spatially separated objects are independent of each other.
>
> On the other hand, it is possible to adhere to (2), if one regards the ψ-function as the description of a (statistical) ensemble of systems (and therefore relinquishes (1)). However, this view blasts the framework of the "orthodox quantum theory."

[6] P. Schilpp, ed., *Albert Einstein: Philosopher-Scientist* (Open Court, 1970), 681-82. Einstein's symbols have been changed.

Now, Einstein believed that the locality principle was true. For it seemed to him that if one were to say that the locality principle is false, one would have to accept either a kind of causal influence (from system I to system II) unknown to current physical theories or a signal (from system I to system II) that traveled faster than the speed of light. However, in his special theory of relativity, Einstein had shown that the speed of light was the fastest speed with which a signal could be transmitted. Thus, Einstein relinquished assertion (1).

Bohr, on the other hand, in his response argued that the real physical state of system II was not independent of the experimental arrangement that was set up to measure some quantity of system I. Thus, he was not forced to abandon the completeness of quantum mechanics.

However, given the argument as Einstein stated it above, there is another possibility. For one might relinquish both of the assertions and hold both that quantum mechanics is incomplete *and* that all things are interconnected. This position was developed by David Bohm.

As opposed to Einstein, Bohm accepted the dynamical concepts with which quantum mechanics operates, but he supplemented them with new variables standing for aspects of reality not taken into account by quantum mechanics. For this reason such a development of quantum theory is known as a *hidden variables* theory. Bohm claimed that the non-locality involved neither conflicts with special relativity nor is attributable to an influence of the experimental apparatus "on the very conditions which define the possible types of predictions regarding the future behavior of the system."

Bohm agreed with the position on non-locality which Heisenberg had formulated several years before the EPR paper. Heisenberg thought that quantum mechanics involved a propagation at a speed greater than the speed of light but that this did not conflict with special relativity. Consider an experiment like a double slit experiment where, however, only one particle (in this case a photon) is directed at the slits. If there is no photographic plate, the wave function ψ has two parts, one corresponding to each slit; and they are each non-zero. But if a photographic plate is introduced near one slit and registers the photon, then the part of the wave function corresponding to the other slit

> immediately becomes zero. The experiment at the position of the [first slit] thus exerts a kind of action [reduction of the wave packet] at the distant [region near the second slit], and one sees that this action is propagated with a velocity greater than that of light. However, it is also obvious that this kind of action can never be utilized for the transmission of signals so that it is not in conflict with the postulates of the theory of relativity.[7]

[7][From W. Heisenberg, *The Physical Principles of the Quantum Theory* (University of Chicago Press,

Bohm based his interpretation on his 1952 article in which he had refuted objections made in 1927 to de Broglie's pilot wave theory. Bohm had devoted the major portion of that article to the development of a consistent non-local theory of hidden variables which was able to reproduce the predictions of quantum mechanics. He later formulated his interpretation in less technical terms as follows:

> 2. General Considerations on the Sub-Quantum Mechanical Level[8]
>
> We note, first of all, that if one adopts the hypothesis of a sub-quantum mechanical level containing hidden variables, then, ... we are led to regard the statistical character of the current quantum theory as originating in random fluctuations of new kinds of entities, existing in the lower level. If we consider only those entities which can be defined at the quantum-mechanical level alone, these will be subjected to a genuine indeterminacy in their motions, because determining factors that are important (i. e. the hidden variables) simply cannot be defined in this level. Hence, as in the usual interpretation of the quantum theory, we regard the indeterminacy implied by Heisenberg's principle as an objective necessity and not just as a consequence of a simple lack of knowledge on our part concerning some hypothetical "true" states of the quantum-mechanical variables. Thus, it is not the existence of indetermination and the need for a statistical theory that distinguishes our point of view from the usual one. For these features are common to both points of view. The key difference is that we regard this particular kind of indeterminacy and the need for this particular kind of statistical treatment as something that exists only within the context of the quantum-mechanical level, so that by broadening the context we may diminish the indeter-minacy below the limits set by Heisenberg's principle....
>
> To illustrate in more detail what the indeterminacy principle would mean in terms of a sub-quantum mechanical level, it will be helpful to [consider] the analogy of Brownian movement....
>
> [T]he motion of a smoke particle is subject to random fluctuations, originating in collisions with the atoms which exist at a lower level. As a result of these collisions, its motions cannot be completely determined by any variables (e.g. the position and velocity of the particle) existing at the level of the Brownian motion itself. Indeed, the lack of determination is not only qualitatively analogous to that obtained in the

1930), 39.]

[8] [From D. Bohm, *Causality and Chance in Modern Physics* (Philadelphia: University of Pennsylvania Press, 1957).]

quantum theory, but as has been shown by Furth, the analogy even extends to the quantitative form of the indeterminacy relations. Thus if we observe a moving smoke particle throughout some short interval of time, Δt, we will find random fluctuations of magnitude Δx in the mean position, and of magnitude, Δp, in its mean momentum, which satisfy the relationship

$\Delta p \Delta x \approx C$

Here C is a constant, which depends on the temperature of the gas, as well as on other properties such as its viscosity.... [T]he form of this relationship is just the same as that of Heisenberg, except that the Planck's constant, h, has been replaced by the constant, C, which depends on the state of the gas.

There is, however, an important respect in which the analogy between the Brownian motion and the quantum theory is not complete. This difference arises essentially in the fact that C is not a universal constant whereas h is. As a result, in principle at least, one is able by changing conditions suitably to make C arbitrarily small (e. g. by lowering the temperature) and thus reduce the indeterminacy below any desired value. On the other hand, the constant, h, does not depend on conditions in any known way, so that Heisenberg's relations imply, as far as we have been able to tell, an indeterminacy that is universal, at least within the quantum-mechanical domain....

Finally, the analogy of Brownian motion also serves to bring out two different limiting modes in which the indeterminacy originating in random sub-quantum mechanical fluctuations may manifest itself. For let us consider, not the Brownian motion of smoke particles, but rather that of very fine droplets of mist. It is evident that there is a certain indeterminacy in the motion of these droplets that could be removed only by going to a broader context, including the air molecules with which these droplets are continually being struck. It remains true, however, that in their irregular Brownian motions the droplets retain their characteristic mode of existence as very small bodies of water. On the other hand, as we approach the critical temperature and pressure of the gas[9] a new behaviour appears; for the fine droplets begin to become unstable. The substance then enters a phase in which the droplets are always

[9]The critical temperature and pressure define a point at which the distinction between gas and liquid disappears. Above this point there is no sharp qualitative transition between liquid and gas, while below it such a transformation can take place. If we heat a liquid confined in a strong container past its critical point, the meniscus separating gaseous and liquid phases disappears, showing that there is now only one phase, which may be thought of as a very dense gas.

forming and dissolving and, as a result, the substance becomes opalescent.

Here we have a new kind of fluctuation, which leads to an indeterminacy in the very mode of existence of the substance (i. e. between existence in the form of droplets and existence in the form of a homogeneous gas).

Similarly, it is possible that the very mode of existence of the electron will eventually be found to be indeterminate, when we have understood the detailed character of quantum fluctuations. Indeed, the fact that the electron shows a characteristic wave particle duality in its behaviour would suggest that perhaps this second kind of indeterminacy will turn out to be the relevant one; for if such an indeterminacy exists, it would lead to a concept of the electron as an entity that was continually fluctuating from wave-like to particle-like character, and thus capable of demonstrating both modes of behaviour, each of which would, however, be emphasized differently in the different kinds of environment supplied, for example, by different arrangements of laboratory apparatus.

Of course, we have no way at present to decide which of these interpretations of the indeterminacy principle is the correct one. Such a decision will be possible only when we shall have found an adequate theory that goes below the level of the quantum theory. Meanwhile, however, it is important to keep both possibilities in mind.

* * *

4. A Specific Example of an Alternative Interpretation of the Quantum Theory

In this section, we shall sketch in a qualitative way a specific example of an alternative interpretation of the quantum theory. This example is not the original one proposed by the author, but already contains a number of modifications and new features, which are aimed at removing some of the unsatisfactory aspects of the earlier proposals.

We begin by recalling that in the quantum-mechanical domain matter is able, under different conditions, to show either wave-like or particle-like behaviour, so that it is evident that the wave concept and the particle concept are each, *by themselves*, incapable of dealing with the full richness of properties demonstrated by matter in this domain. Now, the first and simplest idea to suggest itself in the face of this problem is that perhaps the difficulty arises out of the fact that in previously existing theories only two possibilities were considered, namely, that of the pure wave and that of the pure particle, these two possibilities being regarded as mutually exclusive. On the other hand, it is evidently possible that in any given process, both wave and particle could be present *together* in some kind of interconnection. Of course, this proposal does not constitute a very great enrichment of the concepts that were hitherto used, but, as we shall see, it is already able to represent the essential properties of matter in the quantum domain.

We now formulate this point of view in more detail. We first postulate that connected with each of the "fundamental" particles of physics (e.g. an electron) is a body existing in a small region of space. The probable size of this region we shall discuss later, but for the present we assume only that it is smaller than the size of an atom, and indeed so small that in most applications at the atomic level the body can be approximated as a mathematical point (just as in the earliest forms of the atomic theory one was able for many purposes to approximate atoms as points).

The next step is to assume that associated with this body there is a wave, without which the body is never found. This wave will be assumed to be an oscillation in a new kind of field, which is represented mathematically by the ψ field.... In other words, we no longer suppose that the ... wave function is nothing more than a mathematical symbol convenient to manipulate in order to calculate certain probabilities, but, instead, represents an objectively real field, somewhat like the gravitational and the electromagnetic, but having some new characteristics of its own. Instead of satisfying Maxwell's equations or the equations of the gravitational field, this field satisfies Schrödinger's equation, which provides, however, as in the case of the other fields, a partial differential equation determining the future changes of the field in terms of its value at all points in space at a given instant of time.

We now assume that the ψ field and the body are interconnected in the sense that the ψ field exerts a new kind of "quantum-mechanical" force on the body [associated with which there is a "quantum potential"], a force that first begins to manifest itself strongly in the atomic domain, so that we can understand why it has not previously turned up in the study of the large-scale domain. We also suppose that the body may exert a reciprocal influence on the ψ field, but that this reciprocal influence is small enough to be neglected in the quantum-mechanical domain, even though it is, as we shall see later, likely to be significant in the sub-quantum mechanical domain....

All that is important for the present is to suppose that the force is such as to produce a tendency to pull the body into regions where $|\psi|^2$ is largest.

If the above tendency were all that were present, the body would eventually find itself at the place where the ψ field had the highest intensity. We now further assume that this tendency is resisted by random motions undergone by the body, motions which are analogous to the Brownian movement. These random motions clearly could have many sources. They could, for example, come from random fluctuations in the ψ field itself. Indeed, it has been characteristic of all other fields known thus far that typical solutions to the field equations represent in general only some kind of average motion. For example, real electromagnetic fields do not oscillate in some simple and regular way, but in general they have complicated and irregular fluctuations (e.g. those representing the thermal radiation coming from atoms in the walls of containers, etc.) Similarly, hydrodynamic fields, representing the velocity and density distributions of real fluids, generally show turbulent fluctuations, about an average satisfying certain kinds of simplified hydrodynamical equations. Hence, it is not unreasonable to suppose that the ψ field is undergoing random fluctuation about an average that satisfies Schrödinger's equation and that these fluctuations communicate themselves to the body. The details of these fluctuations would then represent properties of the field associated with a sub-quantum mechanical level, since the quantum-mechanical level is treated in terms of the mean part, which satisfies Schrödinger's equation. On the other hand, the bodies could obtain a random motion from a sub-quantum mechanical level in other ways, for example, as in ordinary Brownian motion, by direct interaction with new kinds of entities existing in this lower level. Indeed, at the present stage of the theory, it is not relevant where such fluctuations come from. All that is important here is to assume that they exist and to see their effects. The question of their origin can then appropriately be raised only in a study of the sub-quantum mechanical level.

Once admitting the existence of these fluctuations, we then see that they will produce a tendency for the body to wander in a more or less random way over the whole space accessible to it. But this tendency is opposed by the "quantum-force" which pulls the body into the places where the ψ field is most intense. The net result will be to produce a mean distribution in a statistical ensemble of bodies, which favours the regions where the ψ field is most intense, but which still leaves some chance for a typical body to spend some time in the places where the ψ field is relatively weak. Indeed, a rather similar behaviour is obtained in classical Brownian motion of a particle in a gravitational field, where the random motion which tends to carry the particle into all parts of the containers is opposed by the gravitational field, which tends to pull it towards the bottom.... In the quantum-mechanical problem, one can show by means of a treatment that is given elsewhere that with physically reasonable assumptions concerning the quantum force and the random motions coming from the sub-quantum mechanical level, we obtain Born's probability distribution, $P = |\psi|^2$.

What is the meaning of this result? It means that instead of starting from Born's probability distribution as an absolute and final and unexplainable property of matter, we have shown how this property could come out of random motions originating in a sub-quantum mechanical level.

A more detailed treatment[10] ... shows that the above result is sufficient to lead to an interpretation that is consistent with all the essential results of the quantum theory. Here, however, we shall illustrate only one way in which this happens, namely, the explanation of the wave-particle duality. To do this, we consider an experiment in which electrons are sent separately and independently with perpendicular incidence into a system containing two slits, illustrated in Fig. 3. Every electron is assumed to have initially the same momentum, and therefore the same wave function,[11] which in fact takes the form of a plane wave incident perpendicularly on the slit system. These waves will be diffracted through the slit system as shown in the figure, and a pattern of high and low intensity will be obtained at the detecting screens, just as in the case of light quanta....

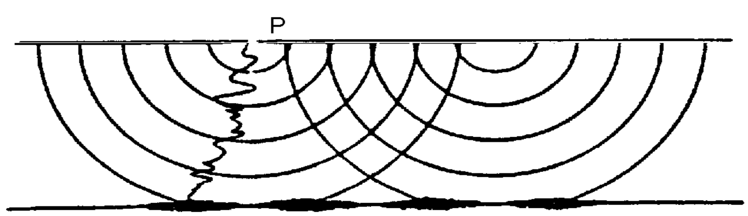

Figure 3

The small body connected with the electron undergoes, however, a random motion. Thus, it follows an irregular path starting out from point

[10] [D. Bohm, *Phys. Rev.* 85 (1952), 166ff. and 180ff.]

[11] This follows from the de Broglie relationship, $p = h/\lambda$.... Actually of course, free-electron waves must consist of packets; but in this case the packet is so much bigger than the slits that we can approximate it as an infinite plane wave.

P, as indicated in Fig. 3. Each electron then arrives at the screen at a certain point. After a large number of electrons have passed through the slit system, we will obtain a statistical pattern of such points, in which the density of electrons is proportional to the field intensity, $|\psi|^2$, at the screen. The statistical tendency to appear where $|\psi|^2$ is greatest is due to the effects of the "quantum-force" while the random motions explain why the precise points at which the various particles appear fluctuate in an irregular way.

Now suppose that we close slit B. The wave pattern will now, as shown in Fig. 4, cease to have strong and weak fringes. Thus, a new pattern of electrons is obtained at the screen. Hence, the closing of slit B influences even those particles that pass through slit A, because it influences the "quantum-force" felt by the particle as it moves between the slit system and the screen.

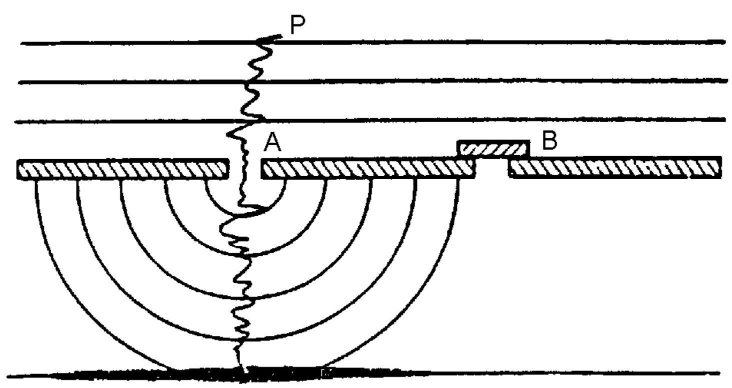

Figure 4

In this way, we can understand how the wave-particle duality originates. On the other hand, in the usual interpretation, no such an understanding is possible. All that we can do is to accept without further discussion the fact that electrons enter the slit system, and appear at the screen with an interference pattern. As to how this came about, such a question cannot even be raised within the framework of the usual interpretation.

In two later articles[12] Bohm presented his interpretation and applied it to the

[12]D. Bohm and B. Hiley, "The de Broglie Pilot Wave Theory and the Further Development of New

EPR argument as follows:

> Originally, Schrödinger regarded the wave intensity ... as the actual density of electric charge, and this gave an image of the electron as a unique and independent reality. However ... this assumed charge density would spread out without limit in a short time; and yet, the particle is always found in a small region of space. To resolve this difficulty, Born proposed that the wave intensity is the probability density of finding a particle. However, this left open the question of whether the localized particle exists independently, or whether it is in some sense produced or at least localized in the act of observation (as is indeed implied in Heisenberg's analysis of the measurement of position and momentum).
>
> It is well-known that in the usual interpretation of the quantum theory, the latter point of view is adopted.. The most consistent version of this interpretation is that given by Bohr, in which no meaning can be ascribed to precisely defined particle properties beyond the limits specified by the Heisenberg uncertainty principle. This lack of meaning goes beyond regarding these properties as independently existent, but uncertain to us because of limits in our ability to obtain knowledge of them through measurements. Rather, in Bohr's view, the universe is basically an unanalyzable whole, in which the notion of the separateness of particle and environment is an abstraction that has no content, except as an approximation that may be applied within the limit of Heisenberg's principle.
>
> Most physicists have adopted views similar in key ways to those described above, though the details vary considerably. However, de Broglie, Einstein, Schrödinger, and others disagreed with this approach, because they felt that there is a uniquely defined reality, which can be grasped in thought and is yet independent of thought. Without considering this reality, science is reduced to a set of formulas and recipes for predicting the results of experiments. Indeed, a large number of modern physicists have since then, at least tacitly, come to adopt such a point of view, perhaps because it is part of the pragmatic spirit of the age. However, in our view, this pragmatic approach is not the only possible one, nor is it even the best. For one can see that concepts that do not give immediate new experimental predictions may still be valuable, in that they permit new insight and understanding (from which

Insights Arising Out of It," *Foundations of Physics* 12 (1982), 1001-10; D. Bohm, "Hidden Variables and the Implicate Order" in *Quantum Implications*, B.J. Hiley and F.D. Peat, Edd. (London: Routledge & Kegan Paul, 1987), 37-39.

new predictions may ultimately emerge). An approach that discourages this kind of insight will thus tend to prevent creative new perceptions, such as those of Einstein, de Broglie, etc.

The pilot wave theory of de Broglie was indeed a significant and fruitful example of imaginative concepts that helped lead to new insights. . . .

In essence, de Broglie assumed that there is a physically real wave satisfying Schrödinger's equation, at least as a linear approximation, along with a particle following a well defined trajectory [and] being "guided" by the background wave, and for this reason, de Broglie called the latter a "pilot wave." (One may here consider the analogy of an airplane guided by radar waves, which carry information about the whole environment.)

* * *

[Quantum mechanics implies] that each particle will be acted on, not only by the classical potential, V, but also by an additional quantum potential Q. . . . In this interpretation, the new features of quantum mechanics are seen to arise basically from Q.

The first main difference [between classical and quantum mechanics] can be seen by noting that the quantum potential, Q . . . does not fall to zero at long distances, where the wave intensity becomes negligible. However, the classical notion of analyzability of a system into independent parts depends critically on the assumption that whenever the parts are sufficiently far removed from each other, they do not significantly interact. This means that the quantum theory implies a new kind of wholeness, in which the behavior of a particle may depend significantly on distant features of the over-all environment. This dependence produces consequences similar to those implied by Bohr's notion of unanalyzable wholeness, but different in that the universe can be understood as a unique and in principle well defined reality.

To illustrate in more detail what is meant here, we consider an interference experiment, in which a beam of electrons of definite momentum is sent through a two slit system. In Fig. 5 we show the results of a computation of the quantum potential. . . .

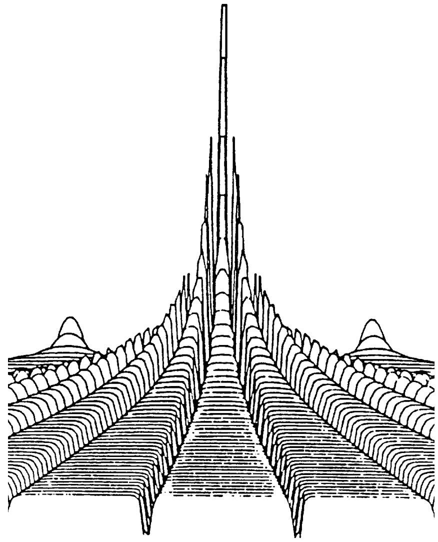

Figure 5

What is especially significant in Fig. 5 is that the quantum potential remains large at long distances from the slits, taking the form of a set of valleys and high ridges, which latter gradually flatten out into broad plateau.... The fact that the quantum potential does not in general fall off with the distance is thus what explains inter-ference and diffraction patterns, and this is clearly also what implies the kind of wholeness of particle and environment to which we have referred above.

One may return here to the analogy of the airplane guided by radar waves. Evidently, it is not a case of mechanical pressure of these waves on the airplane, but rather, information concerning the whole environment is enfolded by the waves, and carried into each region of space. The airplane thus responds actively to the *form* of the waves, and this form is not altered as the intensity falls off with the distance. A similar response to the *form* of the quantum potential is seen to be characteristic of the behavior of the electron. This means that in the microworld, the concept of active information is relevant.

* * *

Thus, one could at least in principle have a strong and direct (non-local) connection between particles that are quite distant from each other. This sort of non-locality would, for example, give a simple and direct explanation of the paradox of Einstein, Podolsky and Rosen, because in measuring some property of one of a pair of particles with correlated wave functions, one will alter the "non-local" quantum potential so that the other particle responds in a corresponding way.

* * *

[W]hen the properties of the first particle are measured, the quantum potential brings about a corresponding disturbance of the second particle. And from this, it can be shown that in a statistical ensemble of similar measurements, Heisenberg's uncer-tainty solutions . . . will still be obtained.

It follows then that with the aid of the quantum potential, something like Heisenberg's original explanation of the uncertainty principle can be maintained. The uncertainties in the properties of particles are indeed now seen to follow from disturbances produced by the quantum potential, whose effects are moreover unpredictable and uncontrollable, because the current form of the theory is only able to provide information concerning statistical distributions over the initial conditions of all the particles concerned. (So that we cannot in practice make observations of individual particles going beyond Heisenberg's principle.)

* * *

[C]lassically, the whole is merely the result of the parts and their preassigned interactions, so that the primary reality is the set of parts while the behavior of the whole is derived entirely from those parts and their interactions. With the quantum potential, however, the whole has an independent and prior significance such that, indeed, the whole may be said to organize the activities of the parts. For example, in a superconducting state it may be seen that electrons are not scattered because, through the action of the quantum potential, the whole system is undergoing a coordinated movement more like a ballet dance than like a crowd of unorganized people. Clearly, such quantum wholeness of activity is closer to the organized unity of functioning of the parts of a living being than it is to the kind of unity that is obtained by putting together the parts of a machine.