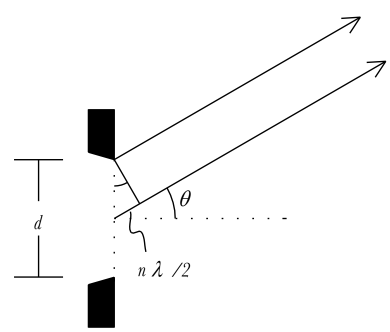
discontinuous change in the probability function does, to be sure, take place through the act of registering; for it is that discontinuous change of our knowledge at the moment of registering which is imaged in the discontinuous change of the probability function.

NOTE

SINGLE-SLIT DIFFRACTION AND THE LIMIT OF OPTICAL RESOLUTION

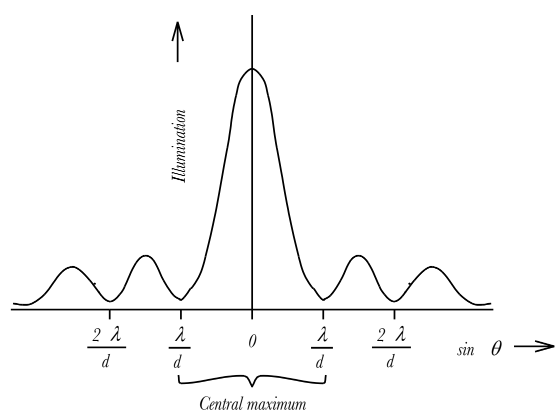
1. *Single-slit diffraction*. Let there be a narrow slit of width d, illuminated uniformly along the axis perpendicular to the plane of the slit. As in Huygens' treatment, consider the plane of the slit to be populated by infinitely many centers of wavelets, expanding in all forward directions. Thus the light beam will spread out as it leaves the slit.



There will be some angle, say θ, at which the wavelets that were emitted from *one edge* of the slit and from the *center* of the slit, respectively, will cover distances that differ by an integral number of half-wavelengths and arrive together at some distant point—where they will mutually cancel, being exactly out of phase with one another. The innumerable remaining wavelets may be similarly paired for mutual cancellation; so that the overall result will be a dark spot at angle θ from the perpendicular axis.

In the right triangle thus formed with hypotenuse $d/2$ and one leg an integral multiple of $\lambda/2$, the marked angle will be equal to θ and will have sine given by

$$\sin\theta = \frac{n\lambda/2}{d/2} = \frac{n\lambda}{d}.$$

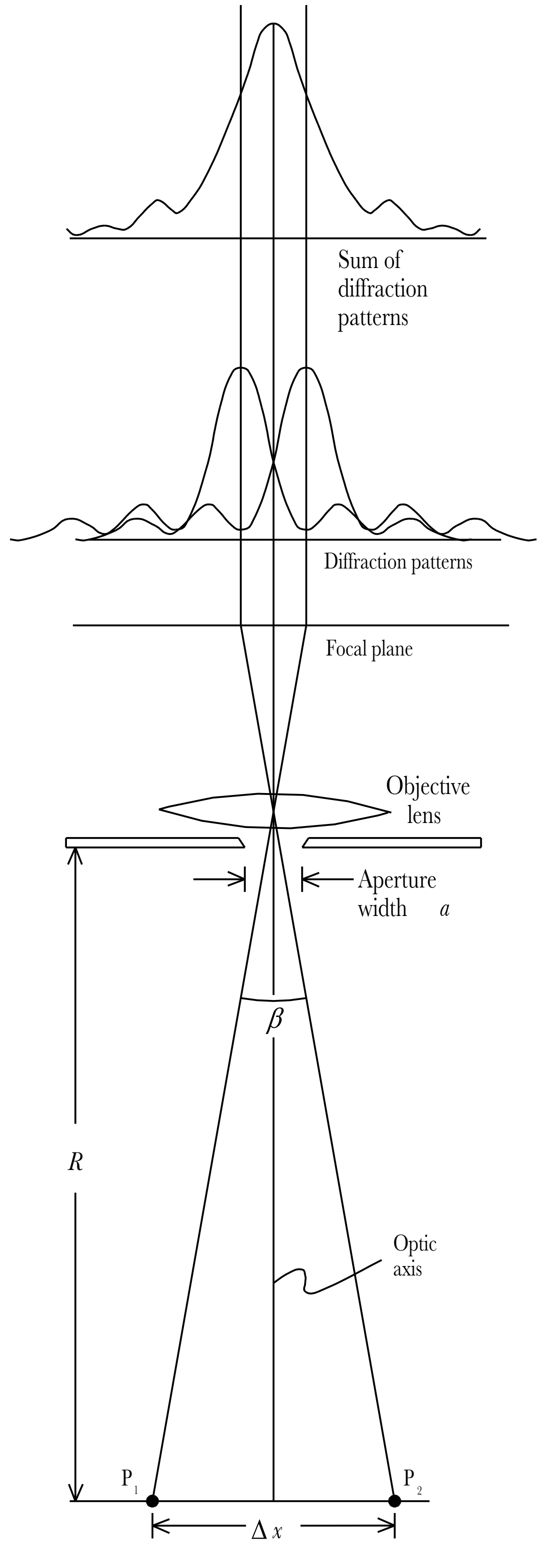


Since n is any integer, there will be alternating bands of illumination and darkness for all angles up to 90° on either side of the perpendicular axis. But in practice the greatest illumination is found within the "central maximum," that region bounded by the two minima for which $n = 1$ (called "first-order" minima). Suppose these minima appear at angles θ_1 on either side of the axis. Then θ_1 will be the *angular half-width* of the central maximum, and will be given by

$$\sin\theta_1 = \frac{\lambda}{d}.$$

2. *Optical resolution*.[28] Consider points P_1 and P_2 on the surface of some object. Let their angular separation be β, and let it be supposed that both points emit or reflect light of wavelength λ. For the sake of simplicity, we will let the aperture through which light is admitted to the instrument be a slit of width a (a circular aperture can be analogously treated, but numerous complications arise for that case). Light from each point forms its own independent diffraction pattern with a central maximum flanked by pairs of minima. As shown in the previous section 1, each central maximum will have an angular half-width equal to θ_1 such that

$$\sin \theta_1 = \lambda / a.$$



Now the central maxima of the two patterns lie in the focal plane of the instrument lens and (since rays passing through the center of a lens are not bent) must be separated from one another by the angle β. But as aperture *a* is made smaller, the angular half-width of each central maximum increases, until eventually $\theta_1 = \beta$—that is, the central maximum of each pattern coincides with a first-order minimum of the other pattern. The central bright maxima will then be immediately adjacent to one another. Inspection of the sum of the two intensity graphs under these conditions shows that instead of forming two distinct bands, the central maxima will merge into a *single band*.

Someone viewing such an image through the instrument would be quite unable to distinguish either of these maxima from the other; and thus the points P1 and P2 would become *indistinguishable*. No increase of magnifying power, so long as the same aperture width is retained, can remedy this limitation. However, if the aperture width a is increased even the slightest amount, so that the separation between the two patterns increases by any degree at all, there will be found a dip between the two maxima when the sum of the intensity graphs is plotted as before. Thus, that the central maximum of each pattern

[28] After Curtis Wilson, c. 1980.

shall coincide with the first-order minimum of the other is a *limiting condition* for the resolution of the images of two points; it is called *Rayleigh's criterion*.

Heisenberg's equation (16), page 140 above, derives directly from Rayleigh's criterion. For suppose $\theta_1 = \beta$ as described above; that is, let

$$\sin\beta = \lambda/a. \tag{4}$$

Let the object distance be R, and suppose also that angle β is very small. Then Δx is the chord of a circle with center at the vertex of β, and so it very nearly equals the arc which it subtends. This small arc equals $R\beta$ (radians); while in its turn a very small angle β (in radians) nearly equals $\sin\beta$. This gives $\Delta x \approx R \times \sin\beta$, so that

$$\sin\beta \approx \Delta x/R. \tag{5}$$

From equations (1) and (2) it follows that

$$\frac{\Delta x}{R} = \frac{\lambda}{a}. \tag{6}$$

Similarly, the aperture of width a located at distance R from a point will subtend an angle ε such that, if ε is very small,

$$\sin\varepsilon \approx a/R,$$

from which

$$a \approx R\sin\varepsilon. \tag{7}$$

Substitution of equation (4) into equation (3) above yields

$$\frac{\Delta x}{R} \approx \frac{\lambda}{R\sin\varepsilon}$$

which becomes Heisenberg's equation (16), when R is canceled from both sides. Q.E.D.