## Making Time Count

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## Lecture

If you asked me the question "What is the time?" I could look at my watch—or these days, at my phone—and tell you. It's 8:05. But what if you changed your question, by leaving out the definite article? Suppose your question weren't "What is the time?", but more simply, "What is ... time?"

In that case, it's hard to know what my answer would be, but easy to suppose that it could not be as immediate, or exact, or conclusive as the time 8:05 was to your first question. And about a matter less suited to our phones than to some philosophy about the nature of time.

The Freshmen, in fact, have just finished reading one—Aristotle's; and my debts to it in this lecture should be clear before I'm done. But so will my debts to Euclid. And the question that guides me tonight will not be "What is time?"—at least if asking this question means turning away, either at once or for good, from our phones, our watches, our clocks; or more generally, from the numbers we use to tell time, and as if our telling of time had nothing to tell us—to teach us—about the nature of time. My question, then, is something less like "What is time?", and something more like: "What is it about time that allows it to be told?" What is it about time that allows us to ask about the time—definite article reintroduced—and to answer that question with all the immediacy, the exactness, and the conclusiveness of "it's 8:05"—or rather, "8:07." My conjecture for the next hour, then, will be that in telling you it's 8:07, or indeed in referring to "the next hour," and more generally in every ready-at-hand way we have through numbers of marking time, measuring time, reckoning time, there is something important to discover, even essential, about the nature of time. Something that accounts for time, in our counting of time.

But what could this be? After all, there is nothing essential in my calling the next hour "an hour." For I could just as well have called this hour "60 minutes." Or even "3,600 seconds"—if I've done my math right; and despite the fact that the number 3,600 is not the number 60, nor is 60 the number 1—if there is such a number. Our counting of time in this sense, the sense in

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which we use numbers to mark *intervals* of time, seems borrowed and arbitrary rather than rooted or necessary—a matter of convention rather than essence.

And there is even more convention, it would seem, in the counting of time that led me to say it was 8:05, or 8:07, or, right now, 8:08. This is the count where we use numbers to mark *moments* of time rather than intervals. For while it may be 8:08 in Annapolis on this side of King George Street, on the other side, it is 20:08—and in Santa Fe, only 6:08. I could also call it 6:08 in Flagstaff—if I were giving this lecture before Spring Break. But in giving it tonight, I will have to admit that it is only 5:08 in Flagstaff, since Daylight Saving Time goes unobserved in Arizona. And if I were giving this lecture before the Uniform Time Act of 1966, I wouldn't know what to say about the time elsewhere. For back then, cities in the United States could choose their own dates for Daylight Saving Time at will; and they did, in what Time Magazine called a "chaos of clocks." Not the clocks we find populating Einstein's theory of relativity, as if cities had discovered themselves in different frames of reference with no present moment in common; but rather the clocks we more innocently use in everyday life, where there is taken to be a present moment in common, but no like moment in number, it seems, no "now" that belongs to number to stop us from assigning the present moment different numbers at once—where "now" is 8:10 in Annapolis, 7:10 in Memphis, 6:10 in Santa Fe, or 5:10 in Flagstaff.

Giving content to this present moment, moreover, still gives us no sense of the number that might belong to it. The "now" that begins Friday Night lecture in Annapolis, for example, is officially 8, but it used to be 8:15. In Santa Fe it is 7:30, but used to be 8. Similarly with the "now" that begins seminar: for undergraduates in Annapolis, this is 8, but for GI students 7:30—and 7:30 in Santa Fe for both. —Except for the 4:30 seminars, one for each class, the year I was in Santa Fe. And if we include seminar-parties, where the start-time is assigned at will, well—there would seem no number excluded in principle: which is again as if there is no "now" in number. And if time itself exists only "now," in the present moment, since the past is no longer and the future not yet, then there is nothing of time in number.

But then we might have inferred this much—that time is one thing and number something else—even without such examples. For if we think of time as having any existence beyond the present moment, it is most likely when we think of this moment as ever-new, making time like a river, flowing ever onward. But our counting of time uses number to form a different kind of succession, which is never continuous, and usually recurring: going repeatedly back to 1 from day to day, week to week, month to month, and year to year, even as time itself rolls on—a fact soon enough written on our faces, even if we erase it from our clocks and calendars.

Our counting of time in these respects even suggests that the count is not really of time, but rather of place, in following motions that, unlike time, continue and finally complete themselves with respect to place. We count to 365, then, in following the yearly trip of the earth around the sun; as we count to 30—or close enough—in following the monthly trip of the moon around the

earth; or to 24, in following the daily trip of the earth around itself. And dividing this 24 into its units, accordingly divides the earth into twenty-four places—each a so-called time zone, explaining the difference in time between Annapolis and Santa Fe as really a difference in place. Even our weekly count to 7 in this sense, which otherwise has no source in the completed motion of any celestial body, might still be said to follow one—the very motion that, on one account at least, created every celestial body, along with everything else, over six days of work finished by a seventh of rest. Nor need we look so far above us for seeming proof that our telling of time is really of place. For we can find this proof writ small on the face of any watch, where every number has its place, and every time in turn, its position. Yet if time itself has no place or position—and we might even consider it a negation of any place or position—then our every clock, it seems, tells a bald-faced lie.

But this is to argue against what I earlier called my conjecture for the next hour—now my conjecture for the next forty-five minutes—which is that our every clock tells the truth. And perhaps it is time—whatever that may be—to make the case for clocks. Consider again our sense of time itself, when we think of it like a river. In certain respects—decisive respects, I think—the simile falls short. For the motion of a river, we can see and say, is made by the river, as it moves along its banks, now more swiftly, now more slowly. But in the motion of time, there is nothing like the river or its banks to see, nothing visibly moving relative to anything else. And no surprise, for there would only be anything visible, it seems, if the motion of time were a motion in space. This motion is already peculiar, then, in belonging to time and time alone. And it becomes still more peculiar when we consider its progress. We sometimes fancy that, again like a river moving swifter or slower, time itself can fly or drag. But if time could really do this, how would we know? When anything else changes its speed, after all, we can use some unit of time to confirm it, as a change in the distance traveled, for example, per minute. But we are deprived of this unit to confirm any change of speed in time itself; for the unit is now in question: is this minute, when compared to the last, or the next, faster or slower, longer or shorter? And even before this question we must ask another, since to say that time can change its speed assumes that time has a speed. But what could we mean by time's speed? In what sense could time itself travel any distance, if we are then deprived of any measure in space to account for it? And how long would it take time to travel this distance, if we are then deprived of any measure in time to account for it?

It begins to look as if our attributing any motion to time involves a category mistake, claiming for time itself, and apart from space, what can only happen in time and space together. And this correction suggests that time itself is not a motion, but rather a medium for motion—a qualification implicit in our sense that when something moves from place to place, it does so in, or even through, some corresponding length of time. But to admit even this much risks another conflation of time and space; for what could we mean by a "length" of time? Indeed, the only conception of time so far that promises to escape any such conflation with space, is one that reduces the length of time to "now," the

present moment, since the present moment can be conceived to have no length. It makes sense to think that this moment lasts only, well, a moment, or instant, before a new such moment takes its place, so to speak. Or, to borrow an earlier formulation, it makes sense to think of the present moment as ever-new. But this is also what tempts us to compare time to a river. And if that analogy is false in being spatial, then what is time really like in being temporal?

It is just here, I think, that our counting of time deserves some credit. For let us suppose that the present moment indeed has no length. If it has no length, then it is no larger, we could say, than a geometric point. But saying this risks another spatial analogy. For even though the point has no part according to Euclid's definition, its integrity in this sense, being spatial, makes the point a position, or location. The point is where a line begins and ends, for example; or where any two lines intersect. But the integrity of the present moment, being temporal, is opposed to this; and we might even characterize it as if it negated the integrity of the point, by saying that the present moment can occupy every location in space, not just the one location occupied by a point. But saying this still relies on an appeal to space—all of space in fact. So what should we say instead? A more promising characterization, I think, is one I just mentioned a moment ago: namely, that the present moment is ever-new. For as the present moment begins, it ends, in having no parts; and as it ends, it is replaced, in having no position. This is why the integrity of the present moment is temporal rather than spatial; for in having neither part nor position, this moment establishes time as a dimension of succession rather than simultaneity. But this gives us a way to distinguish absolutely the present moment from the geometric point. For if the present moment is indeed evernew in lacking parts and position, then one such moment will be replaced by the next immediately, without any space between the two. And the lack of space would be another mark of the moment's temporal integrity. But space is precisely what remains as a mark of spatial integrity, as the position without parts possessed by the point, allows it to occupy space without consuming it. Thus, there is no way to encounter points in succession, from one to the next; between any two, no matter how close, there will be space enough for a third. But the present moment, then, is not only not a point, it is not even like a point. And if we have any hope of finding something that the present moment is truly like, it will have to be what has neither parts nor position.

But saying "what has neither parts nor position," as it happens, is one way to define the unit, out of which, at least according to classical arithmetic, every number is made. I say "classical," since we are tempted, by later developments in the theory of number, to think of the unit itself as a number. I even mentioned the number 1 near the beginning of my lecture, before catching myself. And if we do think of the unit as a number, then it is easy to suppose it has both position and parts, being the first of the so-called counting numbers, which we can always divide by a larger such number to yield some fraction of itself. But it is precisely those suppositions that the classical understanding of number calls into question. And it is the unit on that understanding, I now propose, that not only makes most sense of how we count when counting time; but is

also what time is most like when we erase any hint of space from our sense of it. Or to put this proposal another way: we are all Euclideans when we tell the time—and time itself is Euclidean most of all.

To make sense of this proposal, let me first offer a simple sketch of the likeness between number and time that I have in mind. I just concluded that the present moment, in lacking parts and position, is ever-new. But then we could also say this of the unit, since it too lacks parts and position. Or to put this another way, since the unit lacks parts and position, we can think of it as a recurring whole, which corresponds to the ever-new present moment. And if we then assign the numeral "1" to the unit, we can conceive its recurrence as producing a count of the ever-new present moment, like so: 1, 1, 1, and so on. But if the unit remains whole in its recurrence, then it would seem as if the unit must somehow encompass its recurrence. That is, the unit must somehow encompass the fact that it is being taken once, twice, thrice ..., in the count 1, 1, 1.... So the count in question is better conceived as going 1, 2, 3, and so on, proceeding through every number in turn. And as the evernew present moment establishes the succession we call time, this count of the present moment can be understood as belonging to time, as being a counting of time.

In light of this sketch, which all but identifies time with number, my earlier reasons for denying any connection can be reconsidered. The first of these reasons was that there seems to be nothing essential in my calling the next hour "an hour." For I could just as well call this hour 60 minutes, or even 3,600 seconds. If my sketch above is correct, however, I think we will have to say that this is mistaken. For counting to 3,600 is different from counting to 60, while counting to 1 isn't even possible, since 1 defines the start rather than the end of any count. But why, then, would we think that the different numbers in this case are equal, and equal in turn to the unit? I think it's because we are once again conflating time and space. We refer to the next hour and divide it at once into minutes, as if the hour we tell time by had position and parts. But when we give the unit position and parts, we are no longer telling time by it; that is, we are no longer using the unit for counting. We are rather using the unit for measuring. Or to put it another way, the unit that can indeed possess position and parts makes for the numbers we read off a ruler, not the numbers we read off a clock.

What I mean by this is that the succession of numbers on a ruler, say from 1 to 12, is meant to carry us through space rather than time, in an act of measurement rather than counting. For when we apply the ruler to a length, the length, being spatial, will carry us through the successive numbers on the ruler at once. And if the length should end somewhere between two such numbers—say between 8 and 9—there is nothing to stop us from dividing the unit between them into as many parts as needed, again at once, to give a more accurate reckoning of the length. And this reckoning, so-called, will thus be a matter of inspection rather than calculation. But on a clock, at least if we are using it for a reckoning of time, numbers work radically differently, even if they look the same. And this is why a clock tells the truth. As an example, take

the current time on my phone: 8:29. At first glance, this time may look like the number in my ruler example: a number between 8 and 9, which is taken by dividing the unit between 8 and 9 into parts—in this case, sixty parts, or minutes. But one sign this first glance is deceptive is the punctuation mark I see on my phone—a colon—that separates the 8 from the 29 And if I confirmed the time on my phone by looking at a watch, my having to read the 8 from the position of one hand and the 29 from the position of another hand would reflect the truth. The time 8:29 is really composed of two numbers, neither of which is reached at once, but rather in separate counts. The one number is reached in a count to 8; while the other number is reached in a count to 29, preceded by at least 8 counts to 60. And we are stopped, then, from dividing the unit between 8 and 9 into parts in this case—our so-called "next hour"—since there is no unit to divide: the count that reaches 8, in being a count, hasn't yet reached 9; while the count that reaches 29, in being a count, started at 1, in a reflection of the unit's integrity rather than divisibility. So while different numbers may make no difference to a measurement of time; they make all the difference to a reckoning of time.

In this analysis of the numbers on a clock, however, there is a strange implication for the present moment, which I will only mention here before returning to it later. If we are making two counts when we tell the time, in this case having reached 8 by the one count and 29 by the other, then it looks as if there are two present moments: the one that belongs to 8, and that we call "the hour"; and then the one that belongs to 29, and that we call "the minute." It would be easy to say that the minute is contained in the hour, reducing the two moments to one, if we could say that the hour had parts. But I just argued against this. Yet how could the hour, then, be one present moment and the minute be yet another present moment? Especially if the present moment, in having no parts, has no length, and ends as soon as it begins? Doesn't this imply that there can be merely one such moment at once, and indeed that our reckoning of it, whether by hours or by minutes, is rough? Inexact?

As I said, I will return to this matter later, but I think I must accept the strange implication. If time and number are identified in the way I sketched above, then I think we will have to admit more than one present moment at once—indeed, as many such moments as are reached in different counts. And when I declare the time to be 8:29, or rather 8:34, I am counting to different present moments at once, the one that still belongs to 8, and the one that now belongs to 34. We will also have to admit, I think, that there is nothing rough or inexact in either count, since the present moment, in having no parts, is no bigger or smaller than the unit in either count, which also has no parts.

But this brings me to the second reason I gave earlier for thinking that time is one thing and number something else. For if it is now 8:34 in Annapolis, then it is now 6:34 in Santa Fe. Before, I took this to mean that while Santa Fe and Annapolis share the same present moment, they assign it different numbers, and that this reflected a difference in place rather than time. But if my sketch above is correct, I think we will have to say that Santa Fe and Annapolis do not share the same present moment, and for reasons unrelated to any theory

of relativity. For Annapolis has reached the number 8 in a count by hour that Santa Fe has only made to 6, which in turn means that our respective counts by minute differ by factors of 8 and 6. Thus, Santa Fe's present moment is indeed different from ours, and in fact a present moment that lies in our past, in just the way that 6 is different from 8, and in the count to 8, already passed. So because Santa Fe is at a lesser number in its counting of time than Annapolis, Santa Fe is, right now, at an earlier moment in time than Annapolis, right now, apart from any difference in place. And again, in a matter I will have to return to, it looks as if identifying time with number means admitting that there can be more than one present moment at once; and in this case we have at least four: the two that belong to the counts of hour and minute in Annapolis; and the two that belong to the counts of hour and minute in Santa Fe.

This multiplication of present moments, moreover, is implicit in the way I would now want to correct my earlier conclusion about the difference between time and number. Then, I said there was no "now" belonging to number to stop us from assigning the present moment different numbers at once. But in identifying time with number, I think I will have to say just the opposite: for the "now" belongs to every number in turn; which means there is indeed something stopping us from assigning the same present moment different numbers at once. For if the numbers are different as the result of different counts, then the present moments are different as well.

This brings me to my last reason, which was more general, for denying any connection between time and number. I said that the present moment, being ever-new, makes time like a river, flowing ever onward; while our counting of time uses number to form a different kind of succession, which is never continuous and usually recurring. But since then, I think I've given us better reasons for denying that time is like a river. And on the matter of continuity, I think we can say that temporal succession is no different from numerical succession, at least if we conceive the numerical in this case, again, along classical lines. For then there will be no gaps in our count, as every number is composed by the unit, and reached by the unit in turn. This is continuity enough to reckon with the like succession of time, where every present moment leads directly to the next, leaving no gap in the event.

But what about the matter of recurrence? For this does seem to distinguish a temporal succession, where every moment leads directly to the next, from any numerical succession where we reach some number in our count only to start the count over at 1; and for reasons, so it seemed, having nothing to do with time, and everything to do with space. We count to 365, or to 30, or to 24, so I said, in following the circuits of celestial bodies. But in fact the most important examples of a recurring count are not found in our calendars and clocks, but rather in a waltz, or a line of iambic pentameter. And in those cases, the sense of the count's recurrence seems to lie within rather than without, as we can hear the recurrence in sound without needing to see anything in space make a circuit, or even have what we hear in sound be made explicitly numerical. If I simply go: da-dah, da-dah, da-dah; or, dah-da-da, dah-da-da, dah-da-da—the recurrence of a count is plain. And this recurrence is made articulate in the arts

of poetry and music, which we credit for having somehow shaped time rather than space into a whole. So even if the recurring counts in our calendars and clocks typically follow some completion in space, these same recurring counts ought to make a temporal sense in themselves, as if they produced a completion in time.

But what kind of sense could this be? Indeed, how could there be a whole in time, if the succession of moments that constitutes time can only compose this whole part by part, but leaves every such part at that moment in time, preventing the parts from ever coming together as a whole? I think there is an answer to this question; and it returns me, at last, to the strange implication about the present moment that I mentioned before. For let us suppose there can be more than one present moment at once. As an example, take again the time on my phone, which is now 8:41. And I will again interpret this time as involving two counts of two present moments, the hour that belongs to 8, and the minute that belongs to 41. And again, we might reduce the two present moments to one, if we could say that the hour at 8 contained the minute at 41. But we can't say this, I continue to think, without wrongly dividing the hour we tell time by into parts. But what I think we can say is almost the opposite: it is not that the hour contains the minute, but rather that the minute contains the hour, in the following sense. This minute, or present moment, belongs to a count that will go to 60, and then start over at 1. But why? Why start the count over? We start this count over, not to follow the circuit of any celestial body, but rather to follow the count of the hour at 8, as it proceeds to 9. And by going from 60 back to 1 as 8 proceeds to 9, we take what was encompassed in the count to 60, and relate it at once to what was reached in the count to 8, in a still-larger encompassment. Or in more temporal terms: we contain the present hour in a succession of minutes. And since this hour is whole, the succession of minutes becomes whole. If this analysis is right, then it suggests that there can indeed be temporal wholes, but only if there can be more than one present moment at once.

There is, however, a question that remains if everything I have said so far is correct. For again, I have all but identified time with number, by reducing them both to elements that lack any position or parts. It is difficult, then, to tell any difference between the present moment and the unit; and difficult in turn to tell any difference between time and number. But is there any difference? Aristotle, for one, thought that time is a number, and I hope my lecture has given a sense of how definitive a thought this might be. For the answer to the question could well be simply no. And in that case, there is no difference between time and number precisely because time is a number—and perhaps every number in turn. If this is right, then our telling of time tells the truth, and perhaps the whole truth. And the next time someone asks you "but what, then, is time?" you might look at your phone, and say: it's 8:45.

## Afterthoughts

I tried to do without two things in this lecture that one finds explicit in Aristotle's account of time: the first is motion, insofar as time, for Aristotle, is a number of motion; and the second is the soul, insofar as this number of motion requires an enumerator. I was hoping, by contrast, to make sense of time's relation to number as if it were an identity, which means as if the enumeration could happen on its own, by the very nature of a unit without position or parts; and also as if the enumeration made sense without any additional reference to a motion (whether this be the motion of a planet, or a pendulum, or an atom).

Or to put what I was hoping to do without another way, I wanted to make sense of time as if the clocks we tell it by were made by time rather than by

The Question Period (for which I thank all the participants) showed me that it may be impossible to make complete sense of time on this level of abstraction, even if I still think it was worth the try.