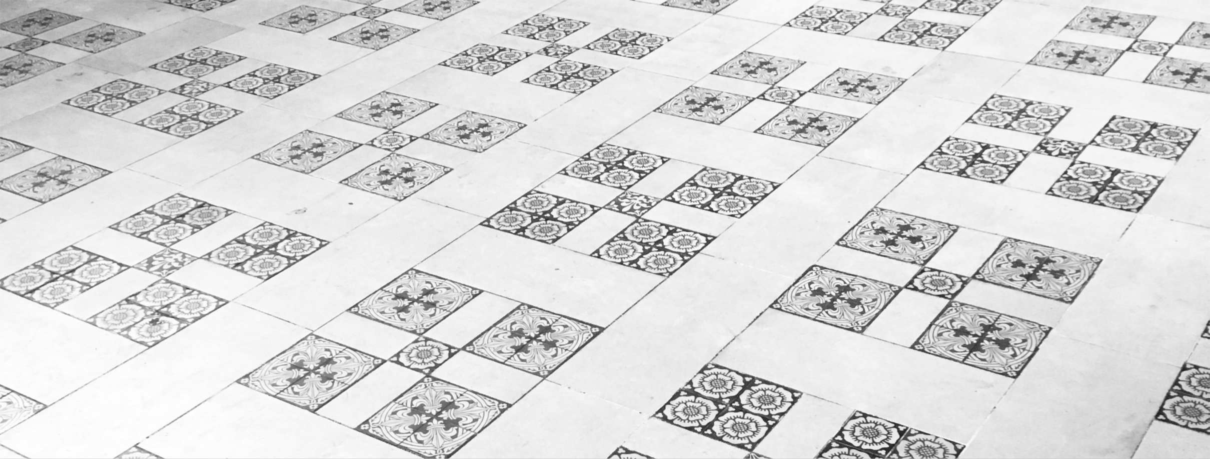
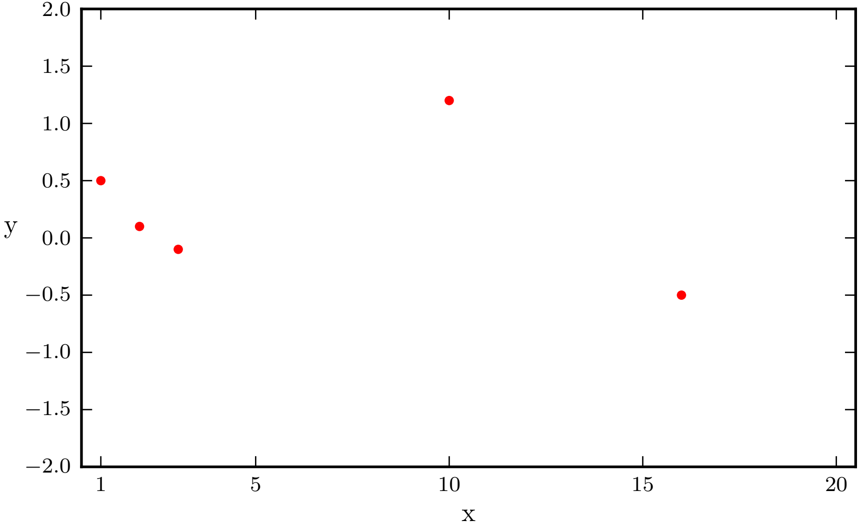


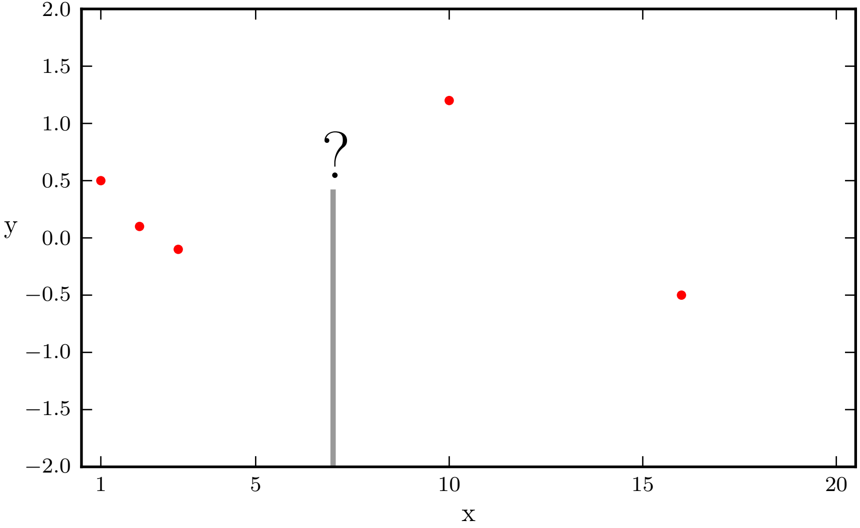
An Introduction to Gaussian Processes

Richard E. Turner
University of Cambridge

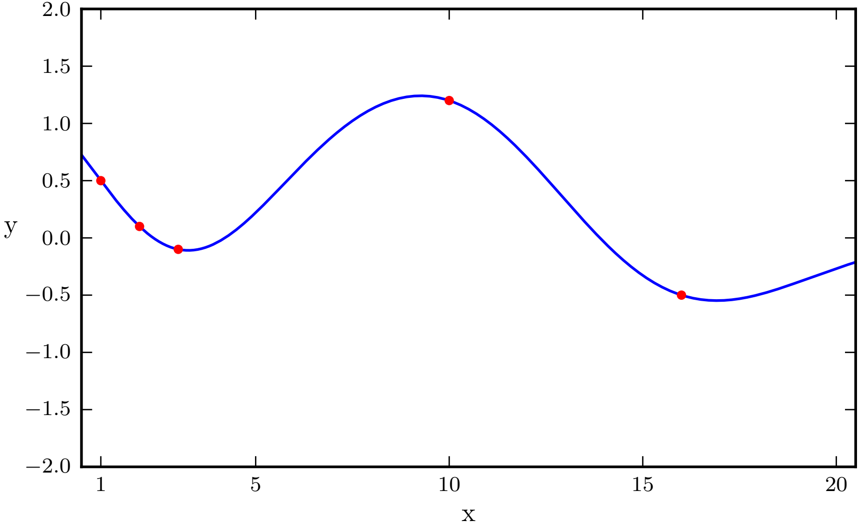
Motivation: non-linear regression



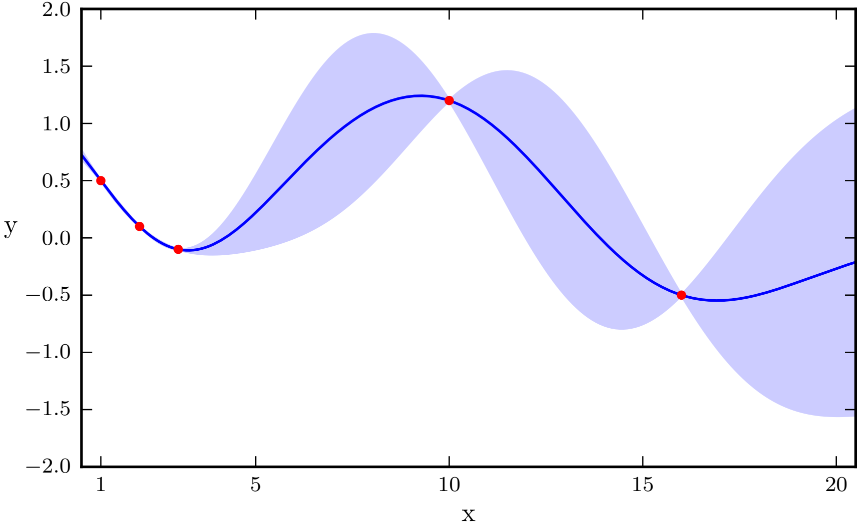
Motivation: non-linear regression



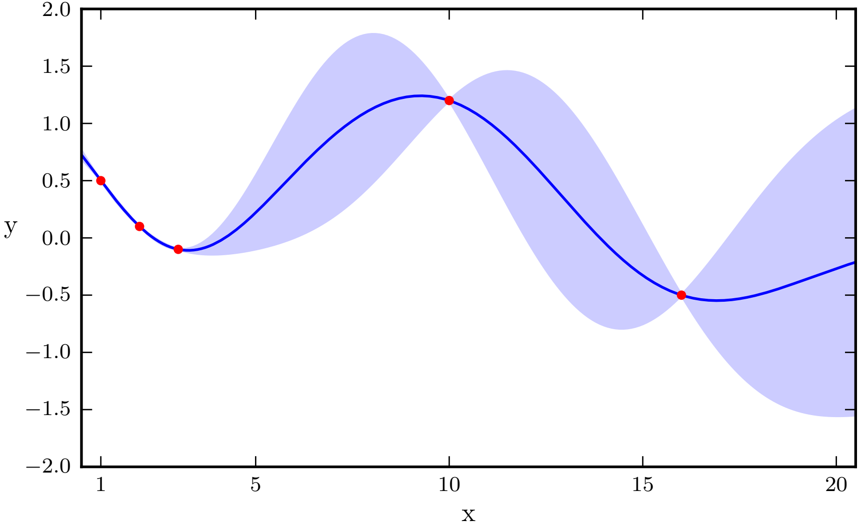
Motivation: non-linear regression



Motivation: non-linear regression

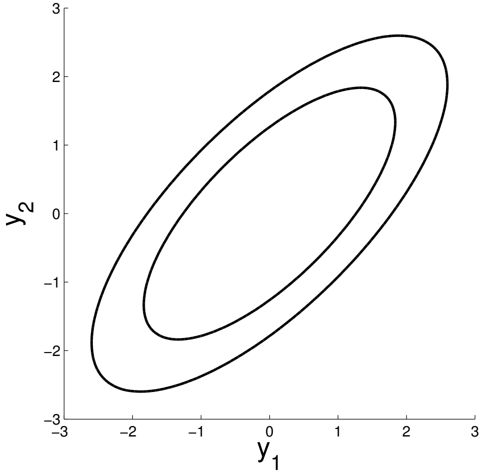


Motivation: non-linear regression. Can we do this with a plain old Gaussian?



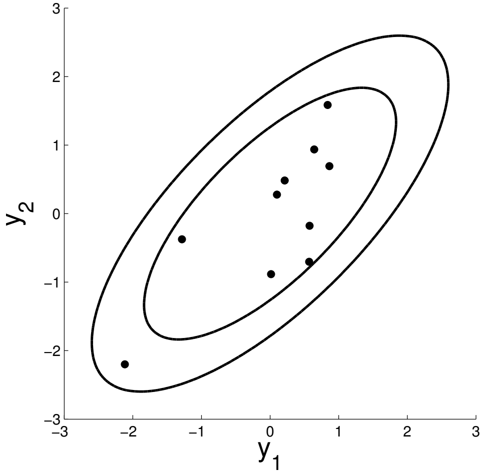
Gaussian distribution

$$p(\mathbf{y}|\Sigma) \propto \exp\left(-\frac{1}{2}\mathbf{y}^\top \Sigma^{-1} \mathbf{y}\right) \qquad \Sigma = \begin{bmatrix} 1 & .7 \\ .7 & 1 \end{bmatrix}$$



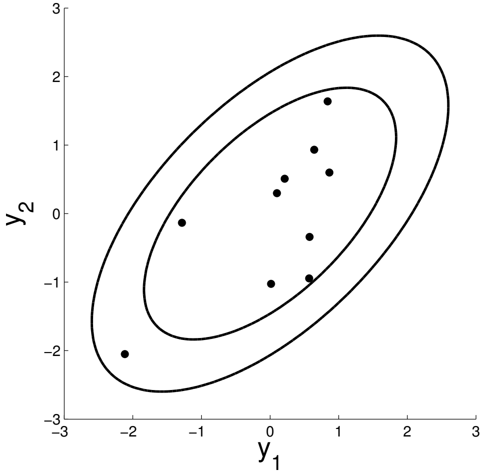
Gaussian distribution

$$p(\mathbf{y}|\Sigma) \propto \exp\left(-\frac{1}{2}\mathbf{y}^\mathsf{T}\Sigma^{-1}\mathbf{y}\right) \qquad \Sigma = \begin{bmatrix} 1 & .7 \\ .7 & 1 \end{bmatrix}$$



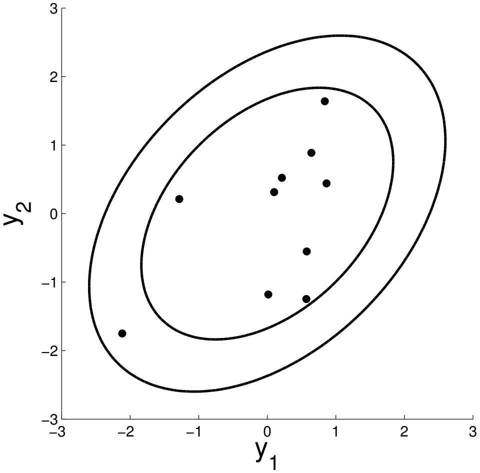
Gaussian distribution

$$p(\mathbf{y}|\Sigma) \propto \exp\left(-\frac{1}{2}\mathbf{y}^\top \Sigma^{-1} \mathbf{y}\right) \qquad \Sigma = \begin{bmatrix} 1 & .6 \\ .6 & 1 \end{bmatrix}$$



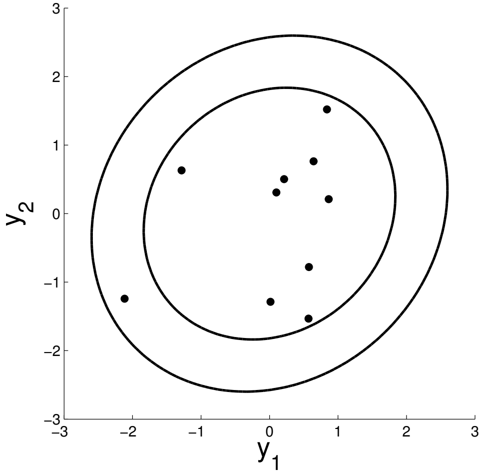
Gaussian distribution

$$p(\mathbf{y}|\Sigma) \propto \exp\left(-\frac{1}{2}\mathbf{y}^{\mathsf{T}}\Sigma^{-1}\mathbf{y}\right) \qquad \Sigma = \begin{bmatrix} 1 & .4 \\ .4 & 1 \end{bmatrix}$$



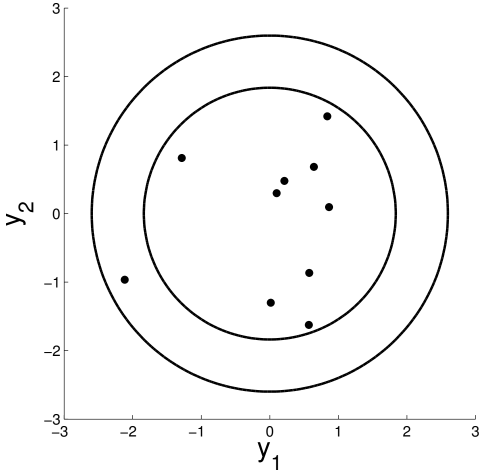
Gaussian distribution

$$p(\mathbf{y}|\Sigma) \propto \exp\left(-\frac{1}{2}\mathbf{y}^\mathsf{T}\Sigma^{-1}\mathbf{y}\right) \qquad \Sigma = \begin{bmatrix} 1 & .1 \\ .1 & 1 \end{bmatrix}$$



Gaussian distribution

$$p(\mathbf{y}|\Sigma) \propto \exp\left(-\frac{1}{2}\mathbf{y}^\mathsf{T}\Sigma^{-1}\mathbf{y}\right) \qquad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



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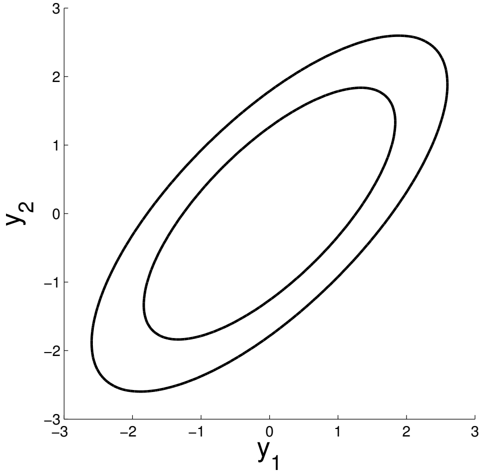

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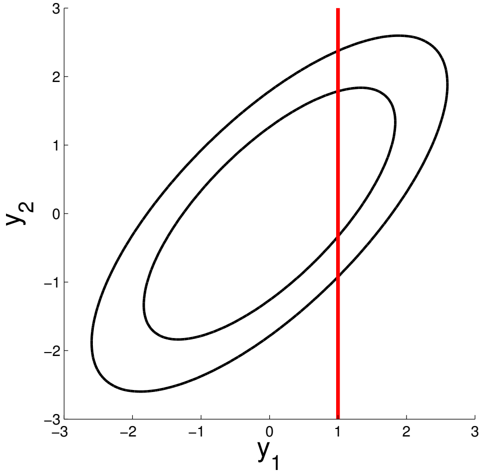
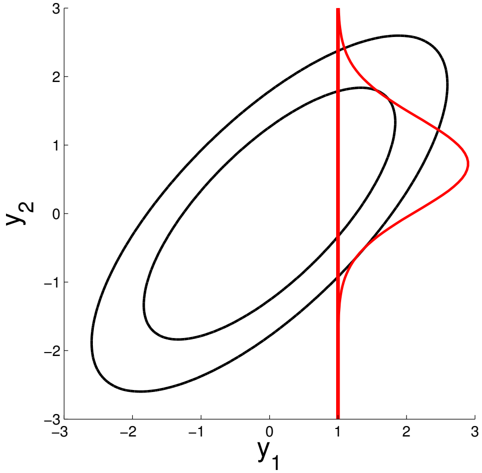


Gaussian distribution

$$p(\mathrm{y}_2|\mathrm{y}_1, \Sigma) \propto \exp\left(-\frac{1}{2}(\mathrm{y}_2 - \mu_*)\Sigma_*^{-1}(\mathrm{y}_2 - \mu_*)\right)$$



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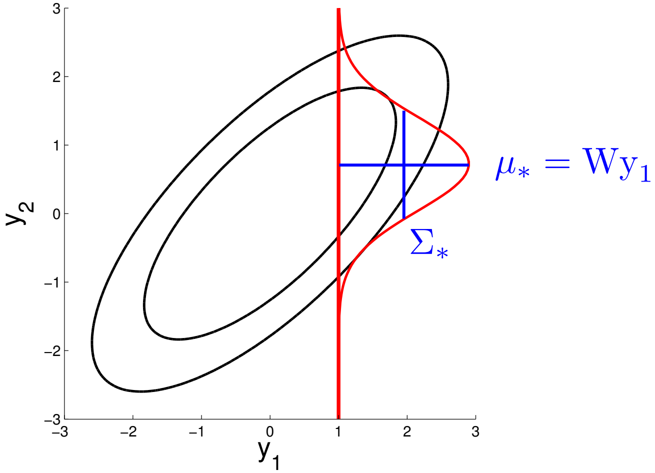

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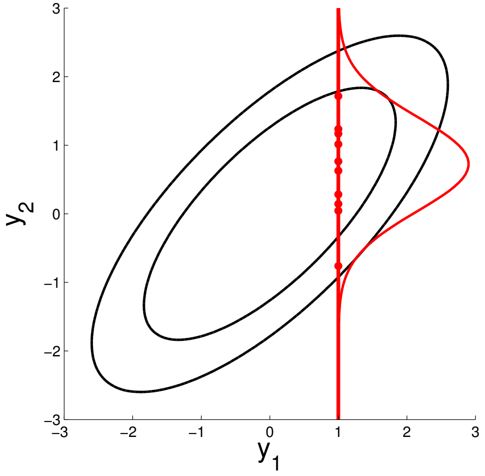

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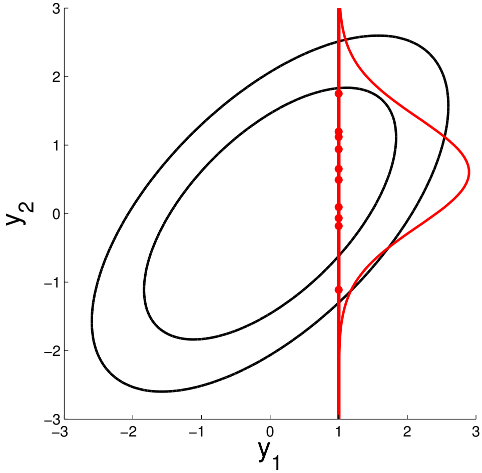

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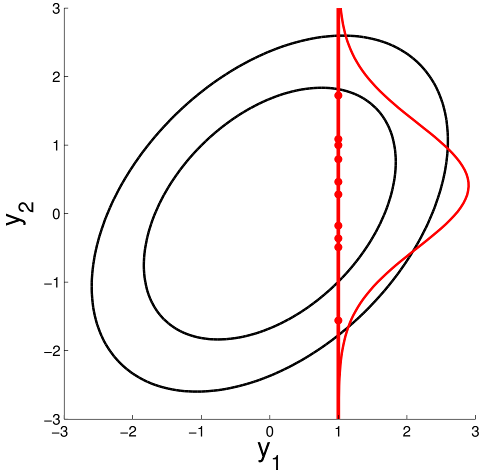

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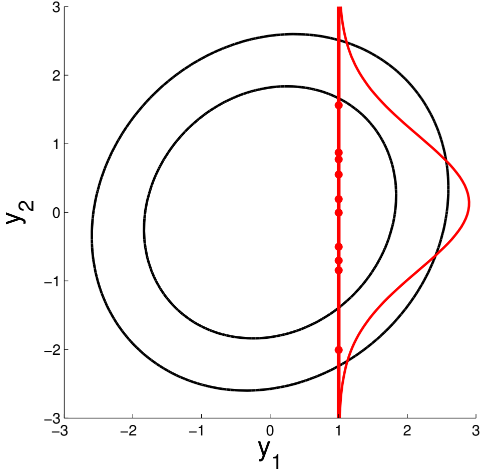

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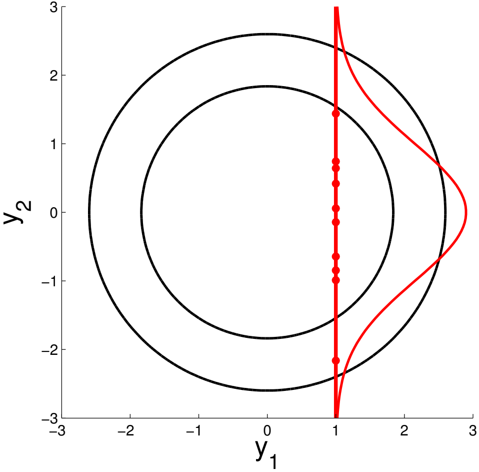
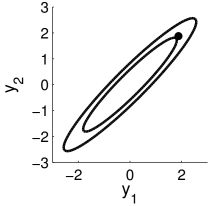
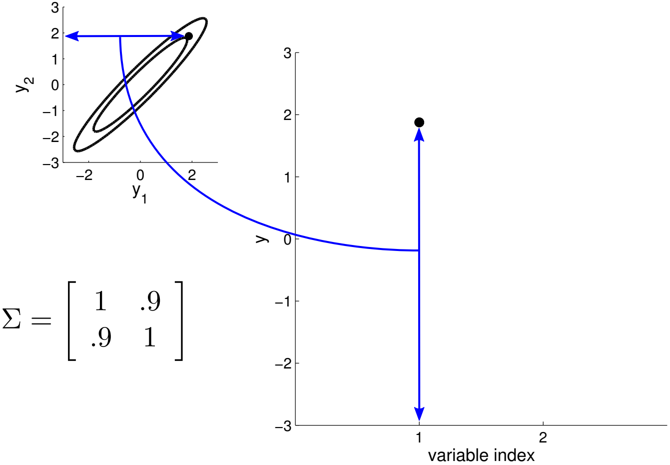


New visualisation



$$\Sigma = \begin{bmatrix} 1 & .9 \\ .9 & 1 \end{bmatrix}$$

New visualisation



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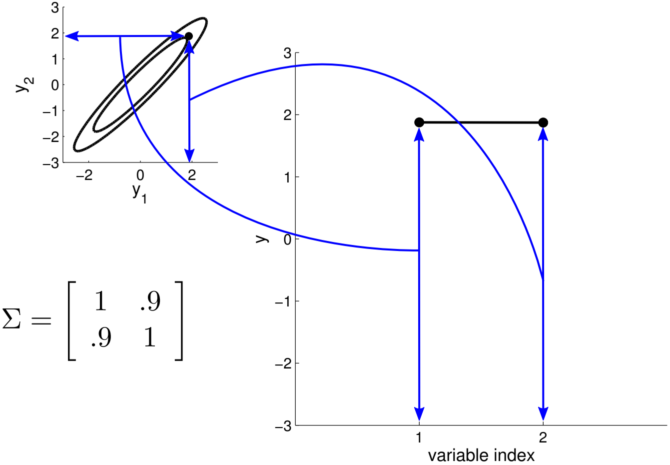

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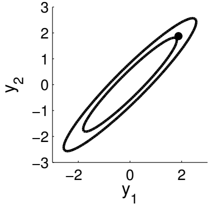

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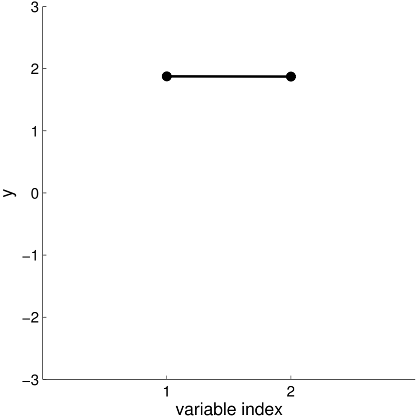

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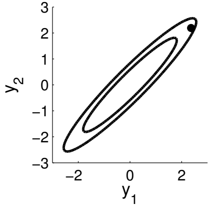

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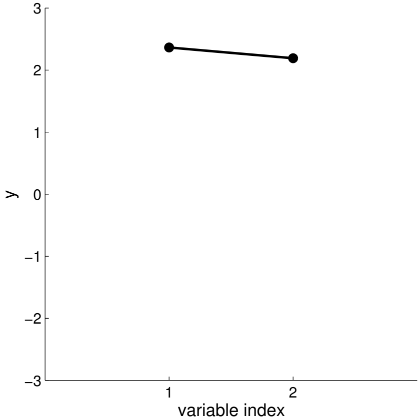

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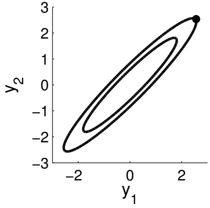

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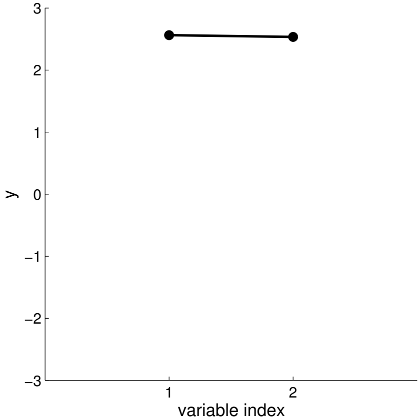

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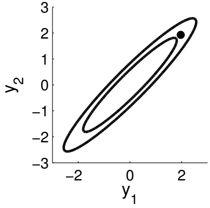

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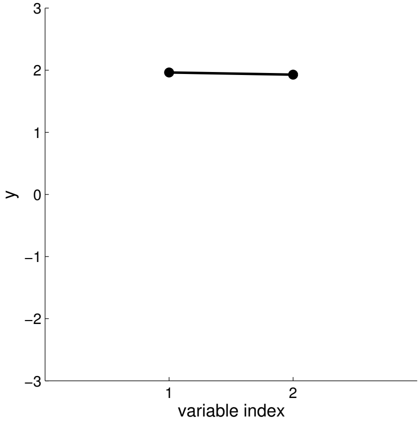

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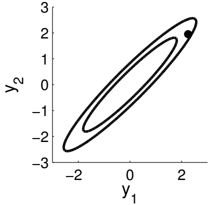

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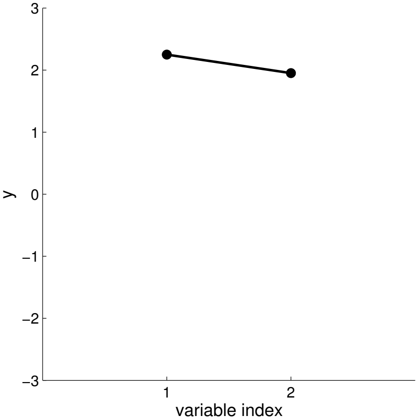

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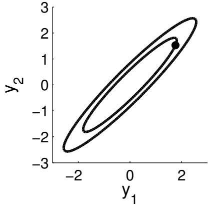

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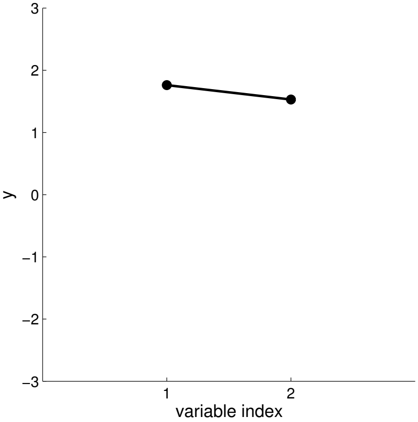

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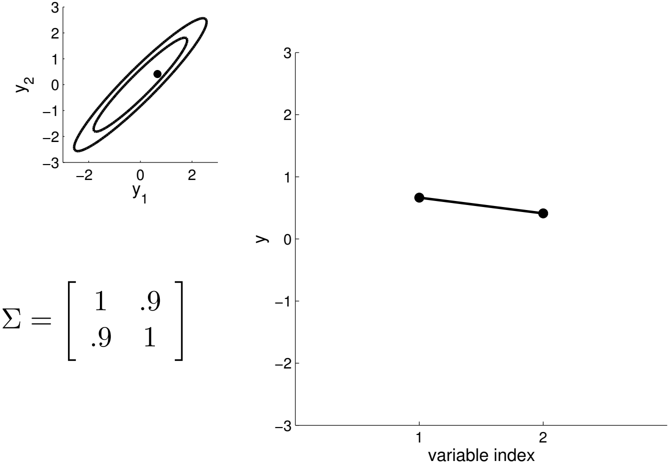

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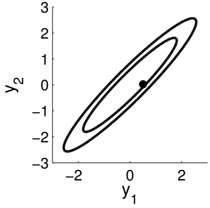

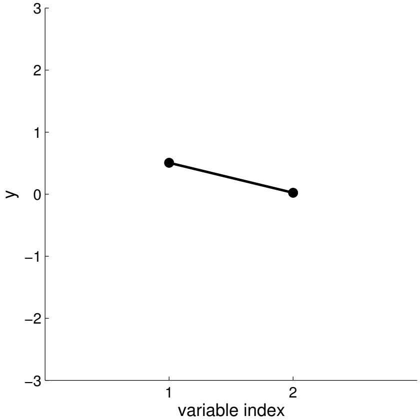

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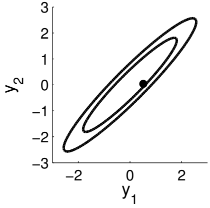

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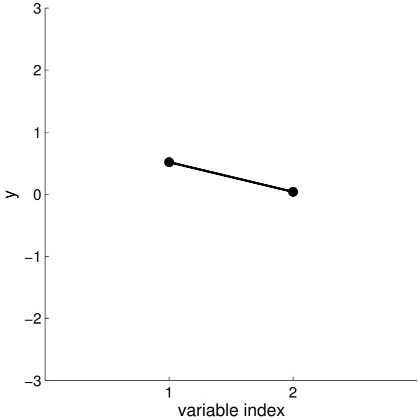

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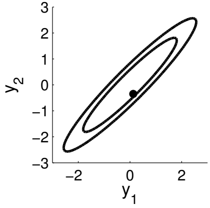

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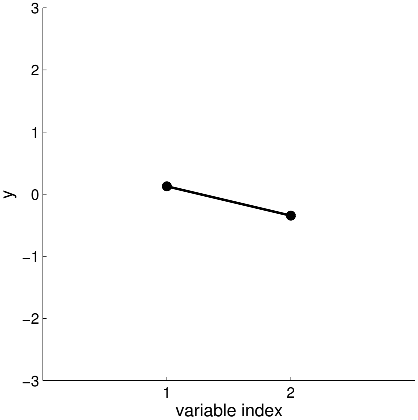

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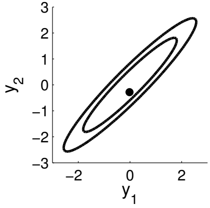

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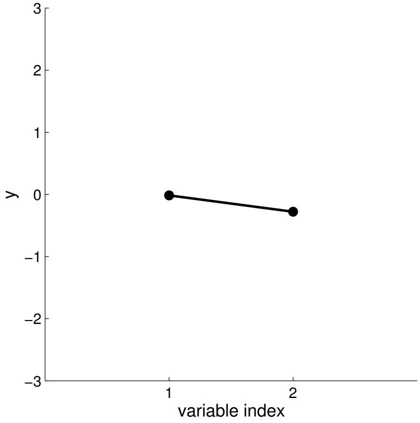

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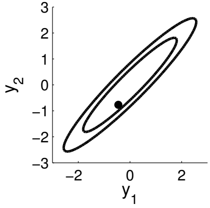

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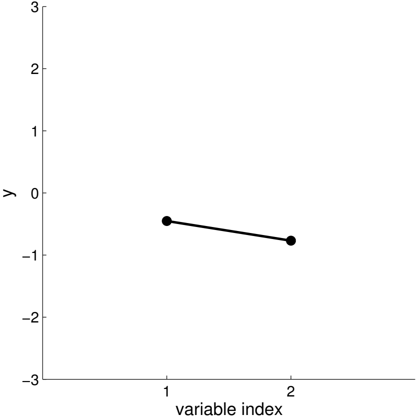

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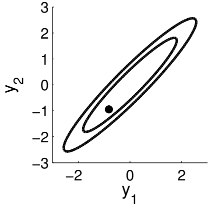

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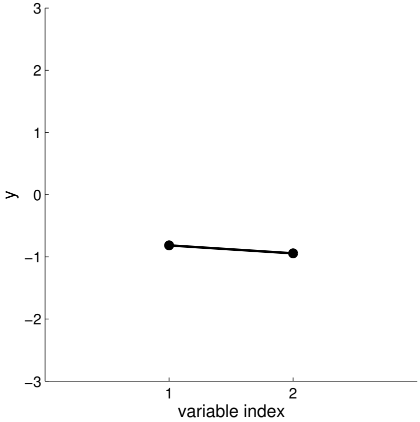

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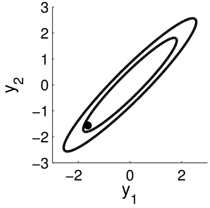

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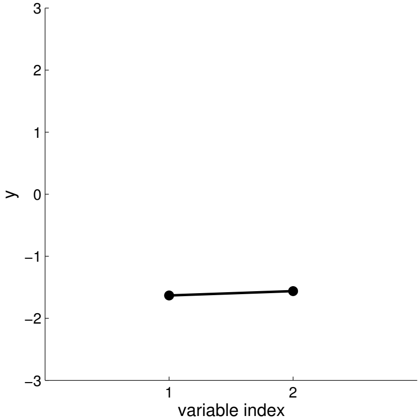

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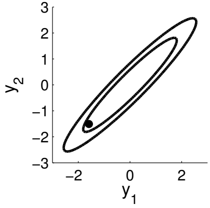

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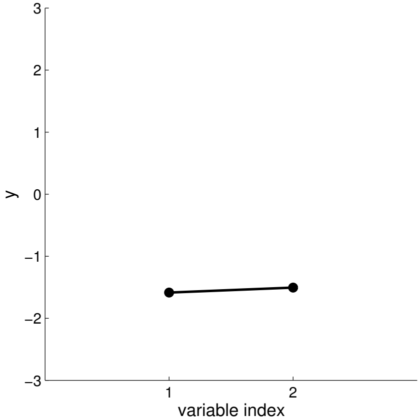

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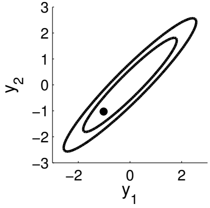

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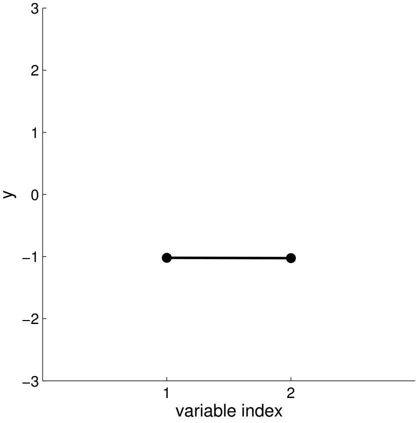

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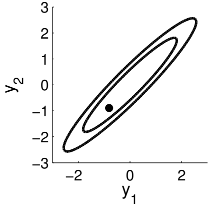

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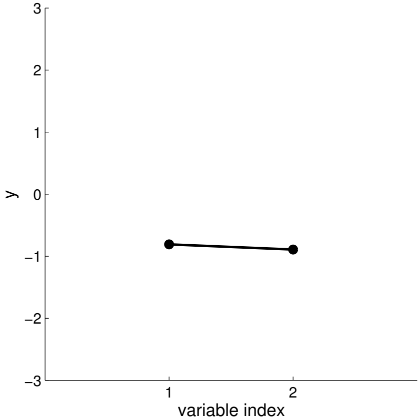
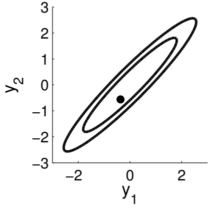


New visualisation



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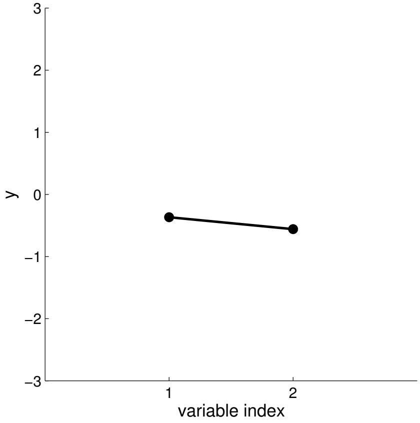
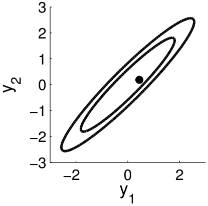
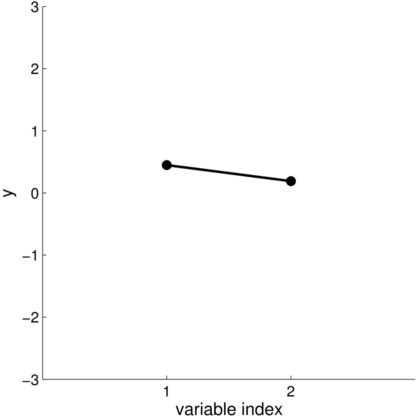


New visualisation





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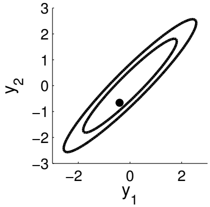
New visualisation



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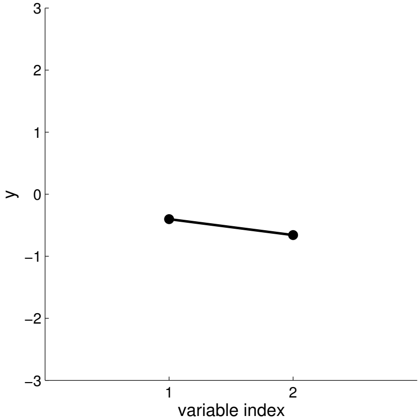

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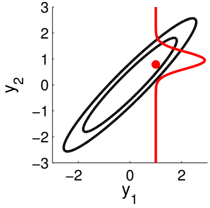

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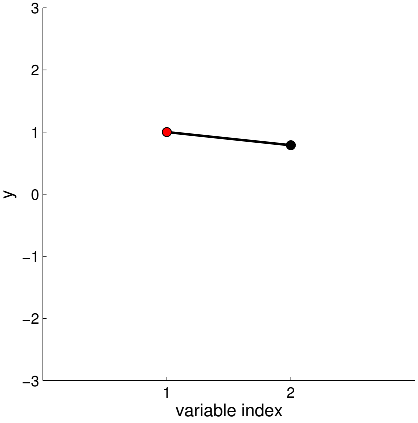

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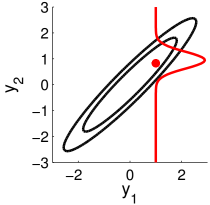

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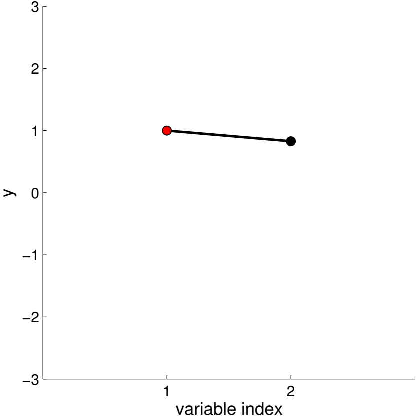

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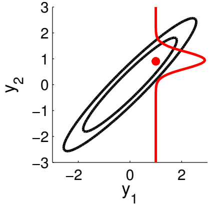

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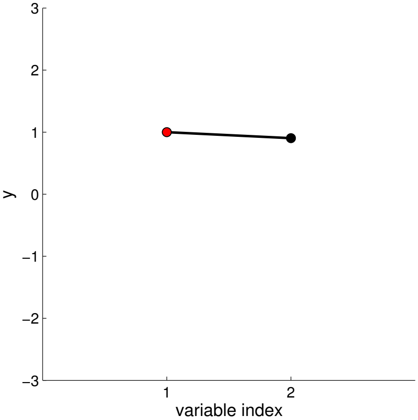

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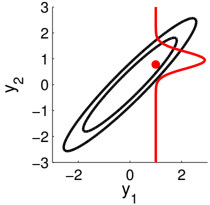

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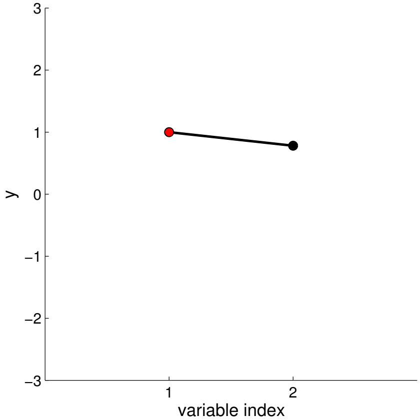

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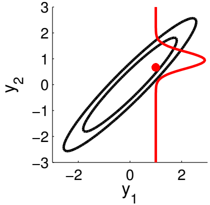

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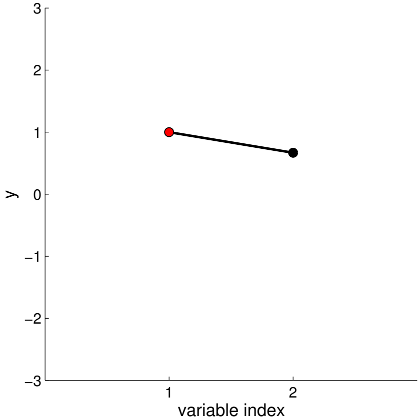

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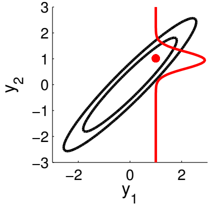

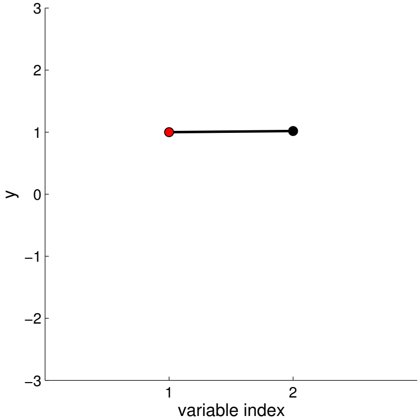

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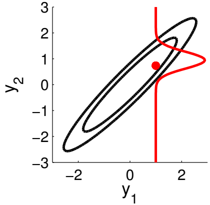

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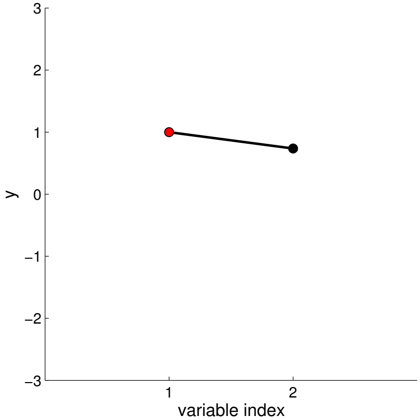

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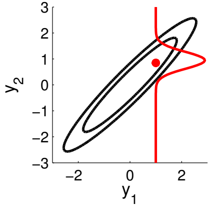

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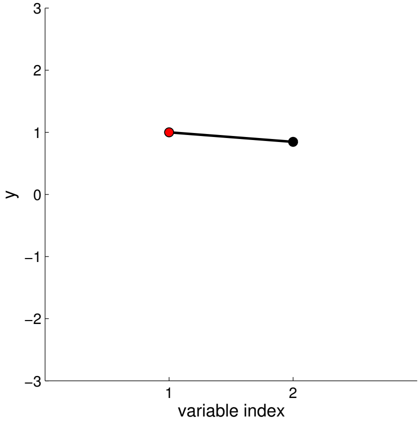

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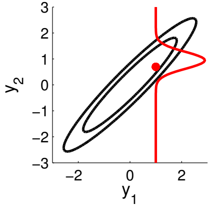

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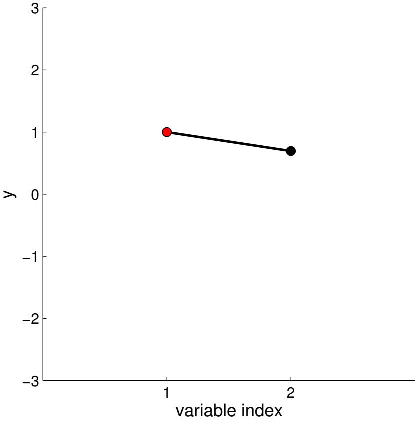

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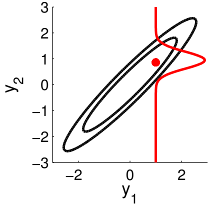

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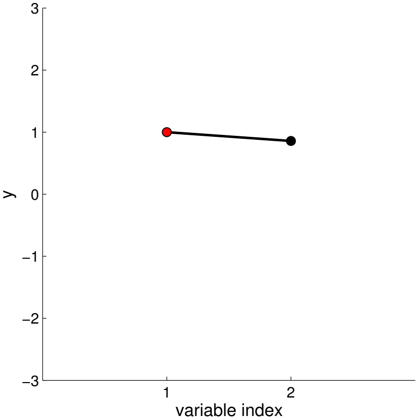

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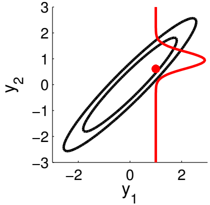

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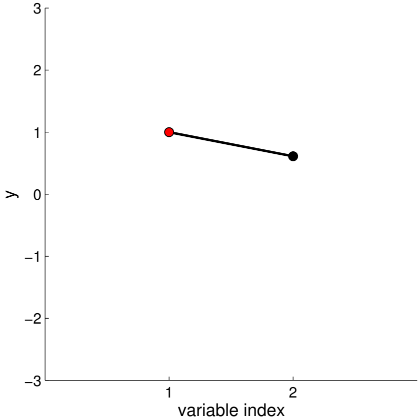

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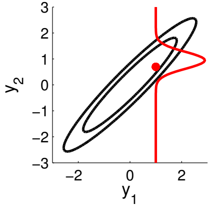

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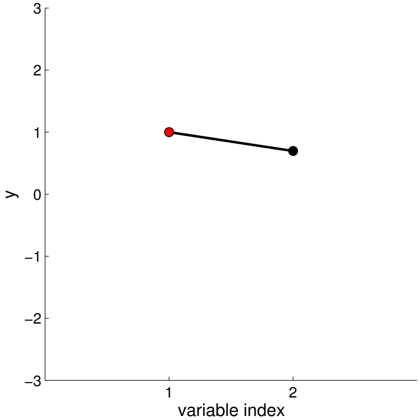

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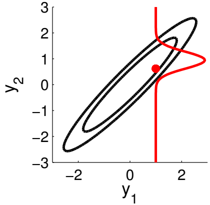

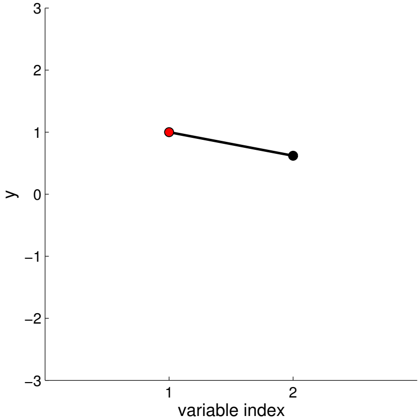

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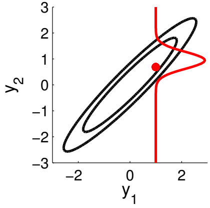

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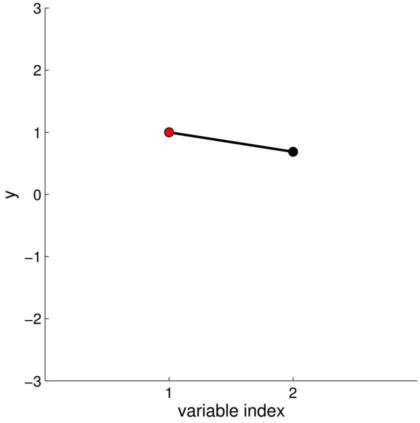

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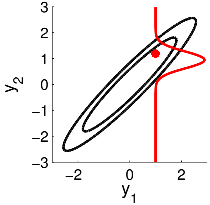

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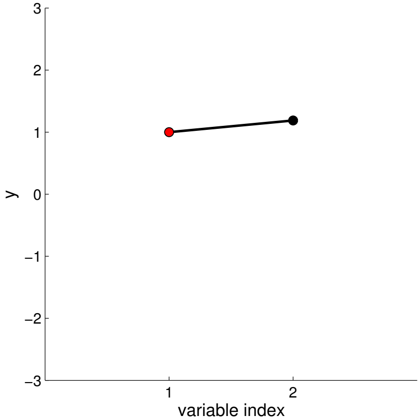

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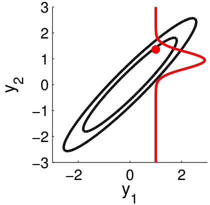

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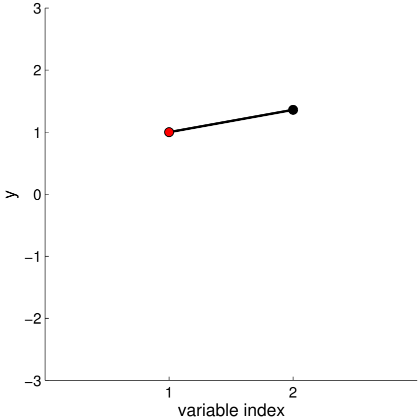

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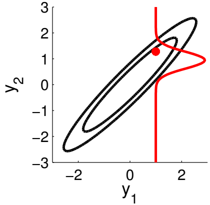

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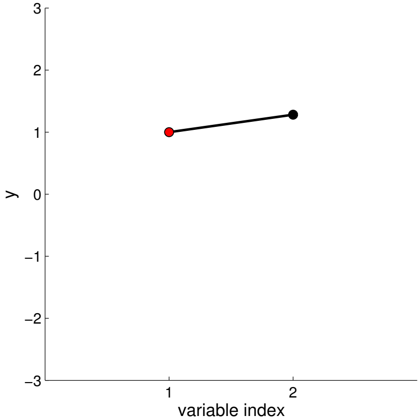

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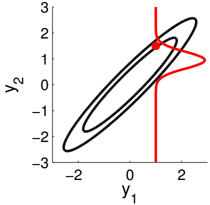

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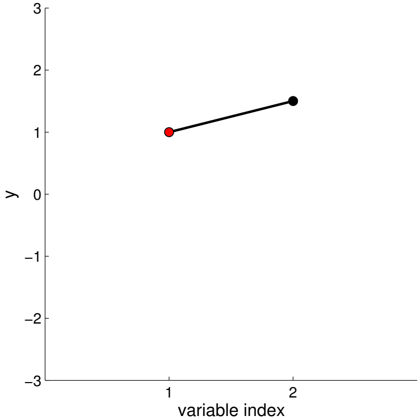

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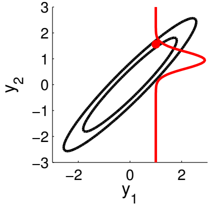

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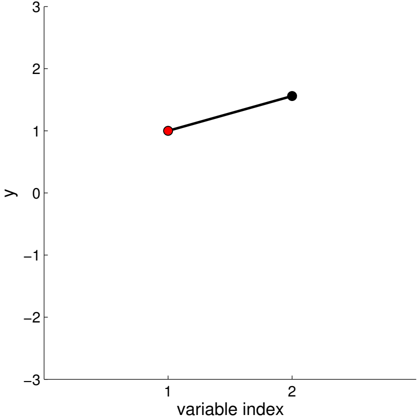

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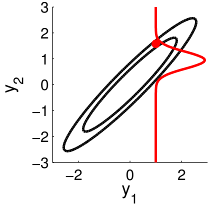

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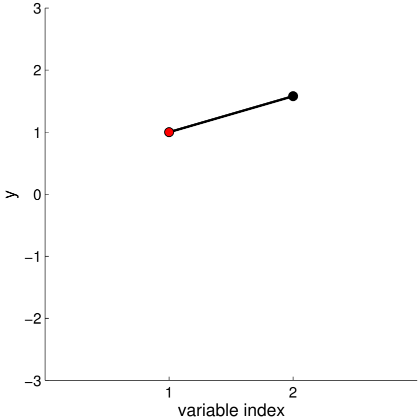

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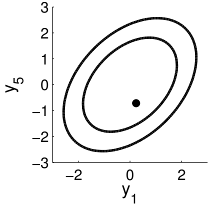

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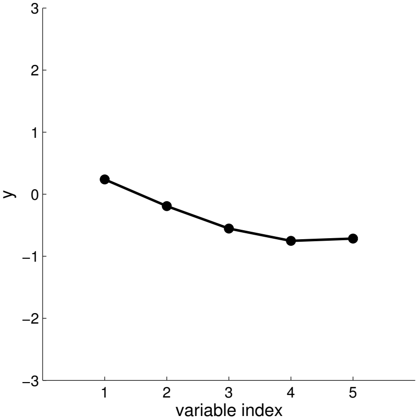

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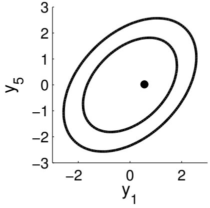

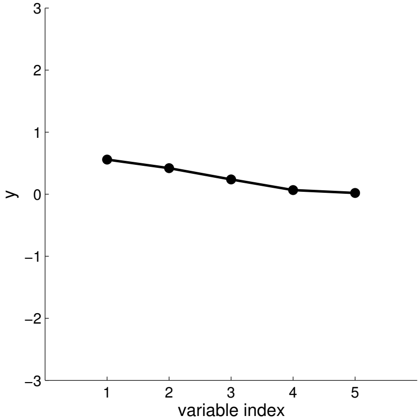

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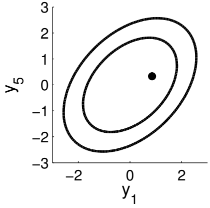
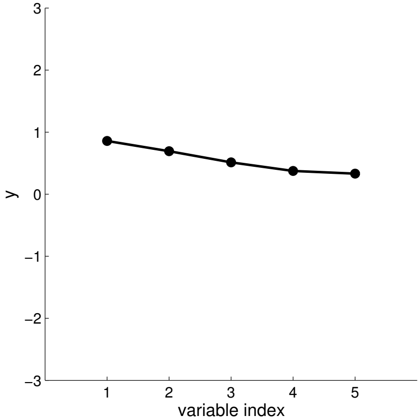
New visualisation





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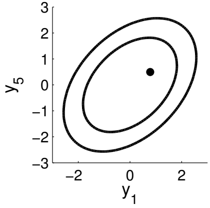
New visualisation



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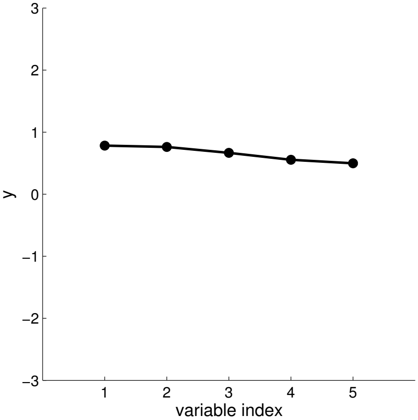

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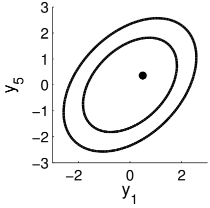

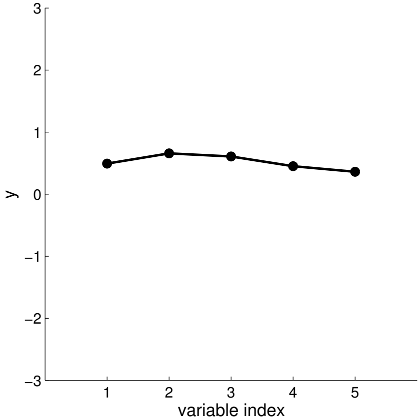

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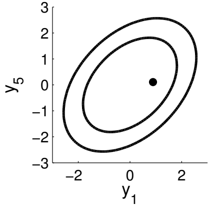
New visualisation



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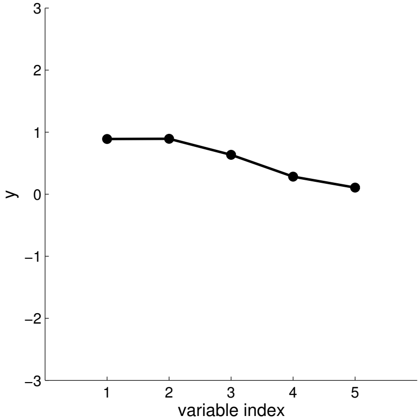

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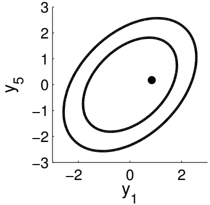

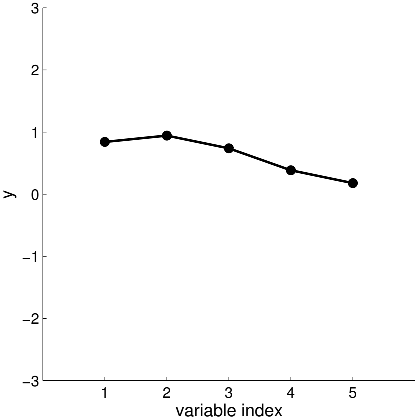

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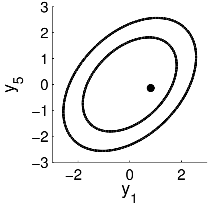
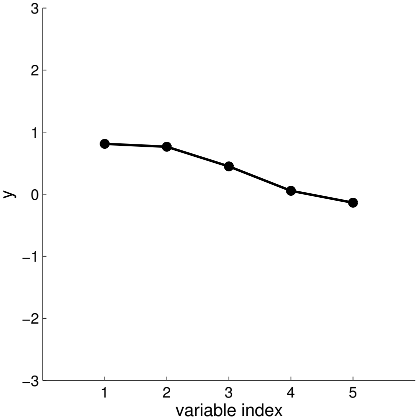
New visualisation





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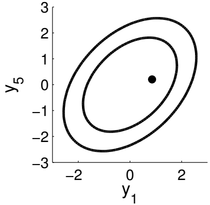
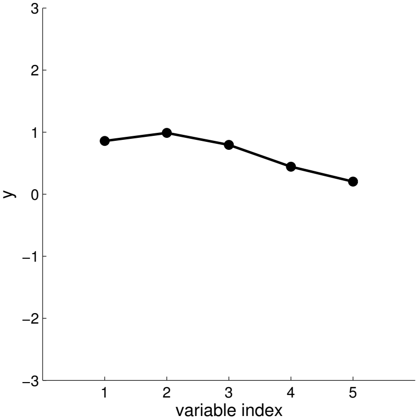
New visualisation





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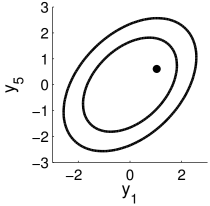
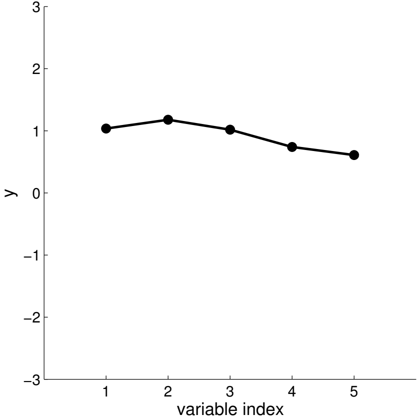
New visualisation





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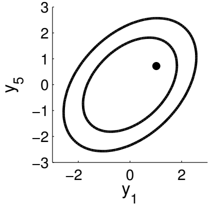
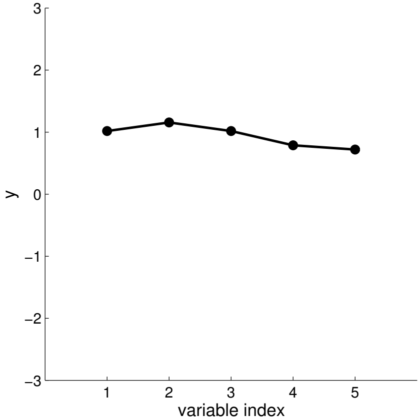
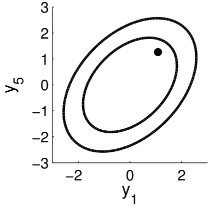
New visualisation

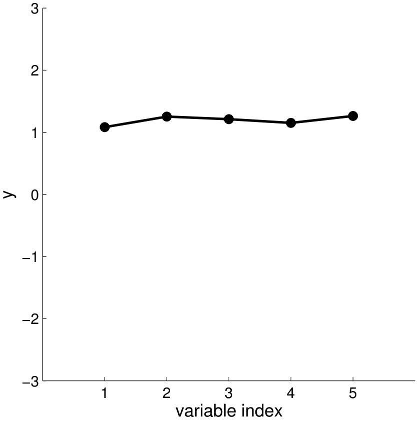




$$\Sigma = \begin{bmatrix} 1 & .9 & .8 & .6 & .4 \\ .9 & 1 & .9 & .8 & .6 \\ .8 & .9 & 1 & .9 & .8 \\ .6 & .8 & .9 & 1 & .9 \\ .4 & .6 & .8 & .9 & 1 \end{bmatrix}$$

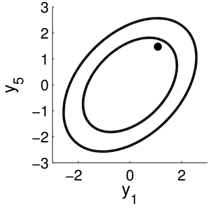
New visualisation

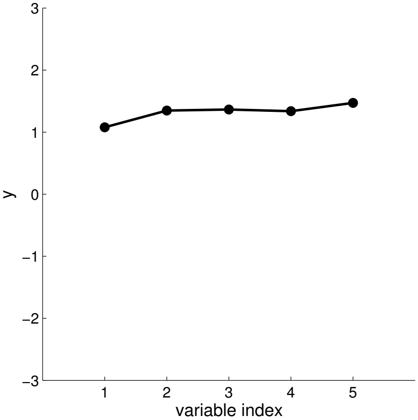




$$\Sigma = \begin{bmatrix} 1 & .9 & .8 & .6 & .4 \\ .9 & 1 & .9 & .8 & .6 \\ .8 & .9 & 1 & .9 & .8 \\ .6 & .8 & .9 & 1 & .9 \\ .4 & .6 & .8 & .9 & 1 \end{bmatrix}$$

New visualisation





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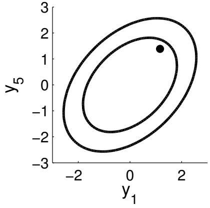
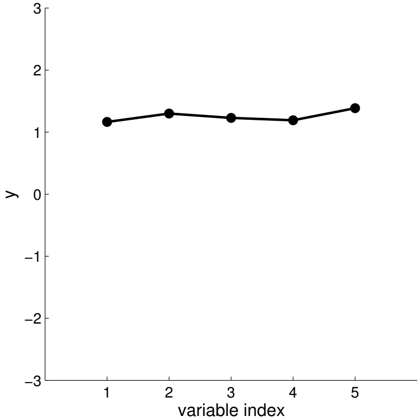
New visualisation





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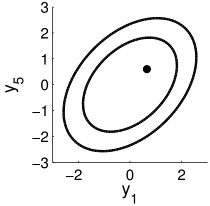
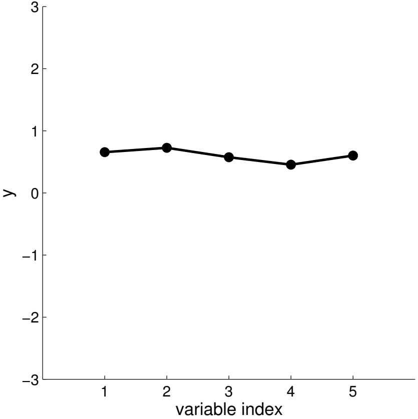
New visualisation





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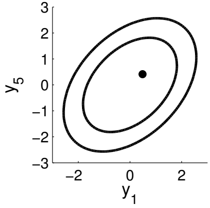
New visualisation



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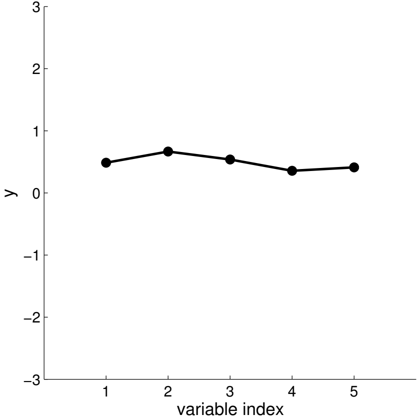

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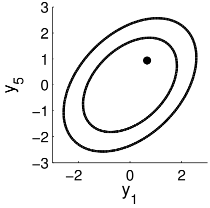

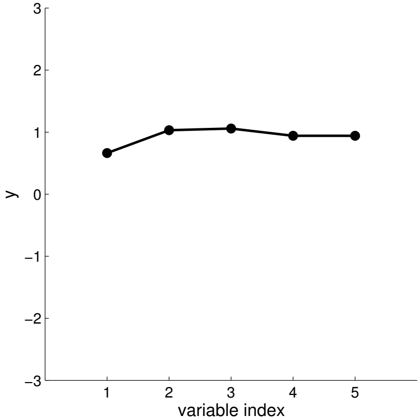

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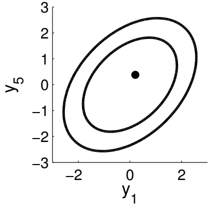
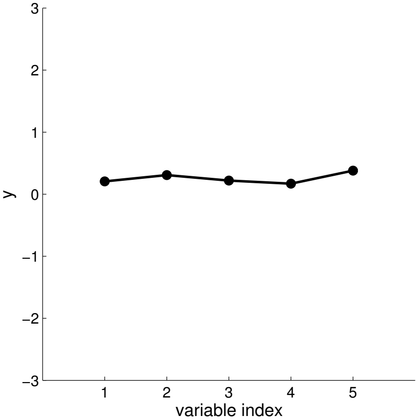
New visualisation





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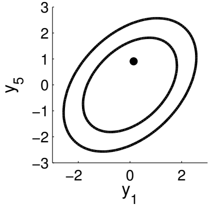
New visualisation



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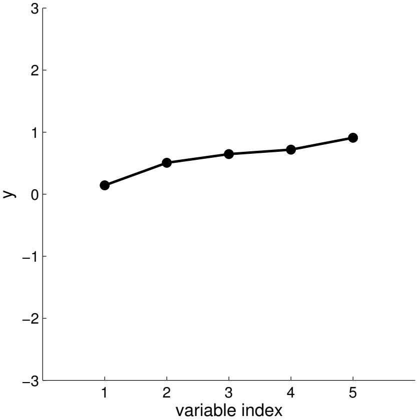
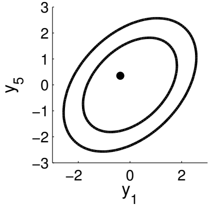
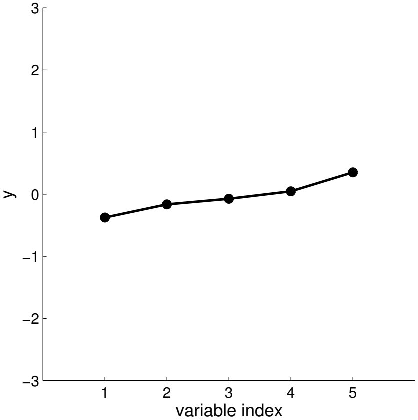


New visualisation





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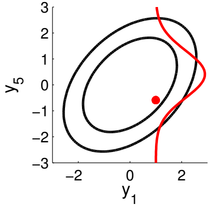
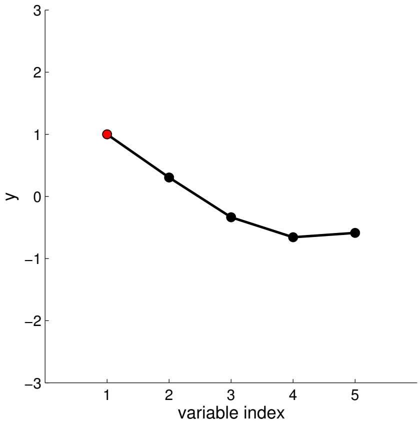
New visualisation





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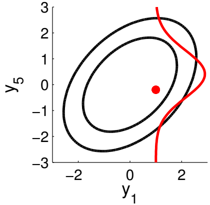
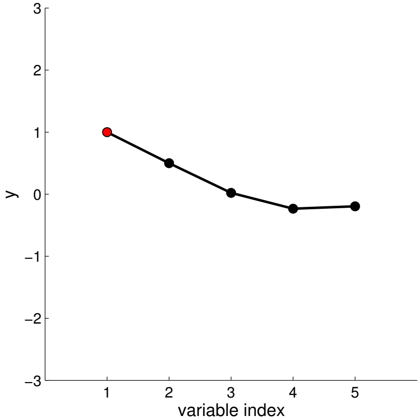
New visualisation





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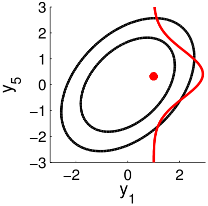
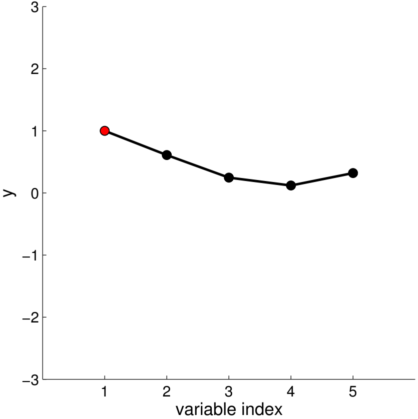
New visualisation





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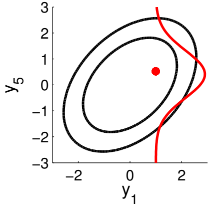
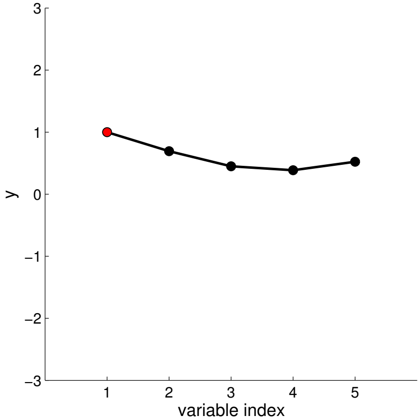
New visualisation





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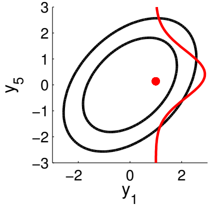
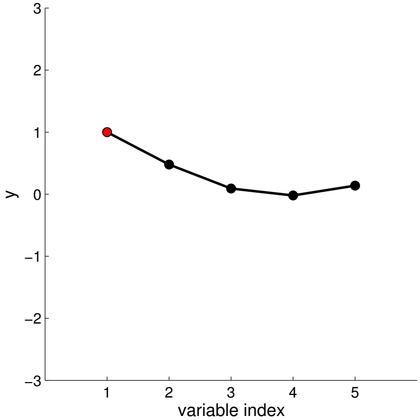
New visualisation





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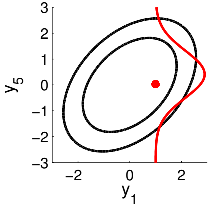
New visualisation



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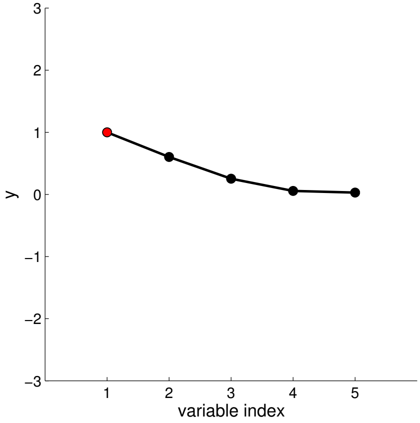
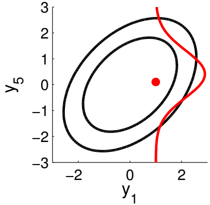


New visualisation



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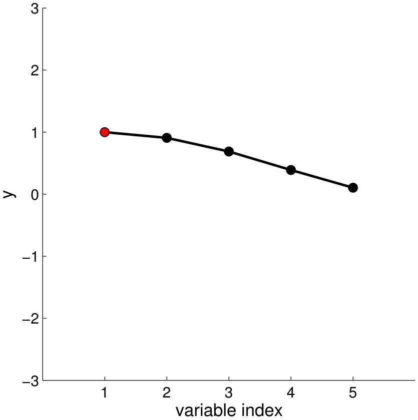
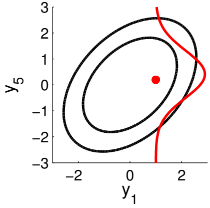
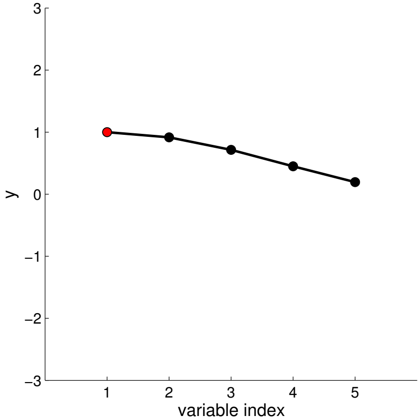


New visualisation





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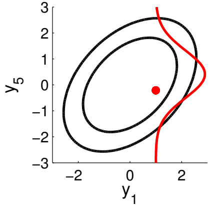
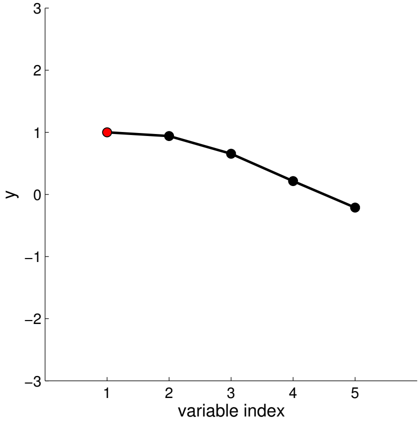
New visualisation





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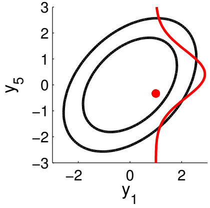
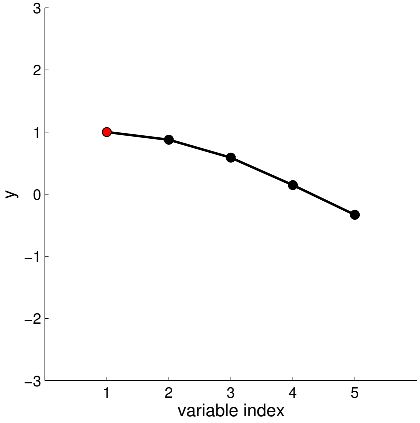
New visualisation





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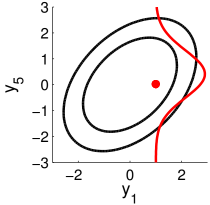
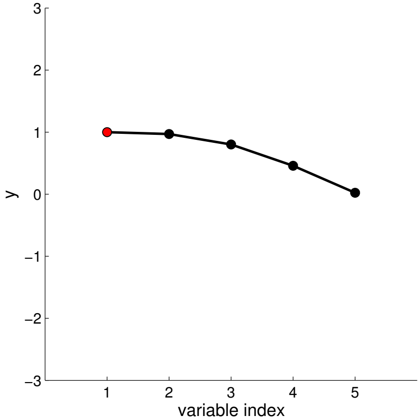
New visualisation





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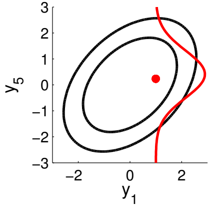
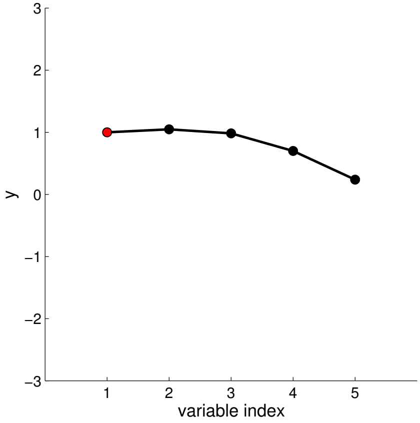
New visualisation





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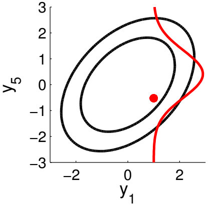
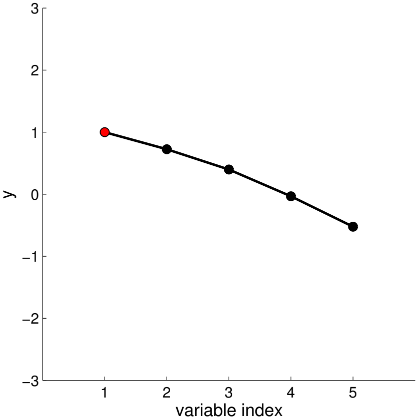
New visualisation





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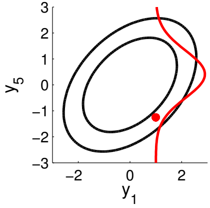
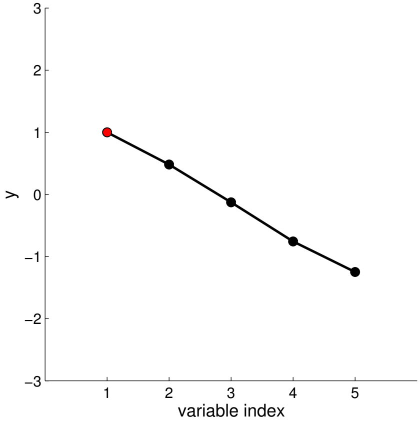
New visualisation





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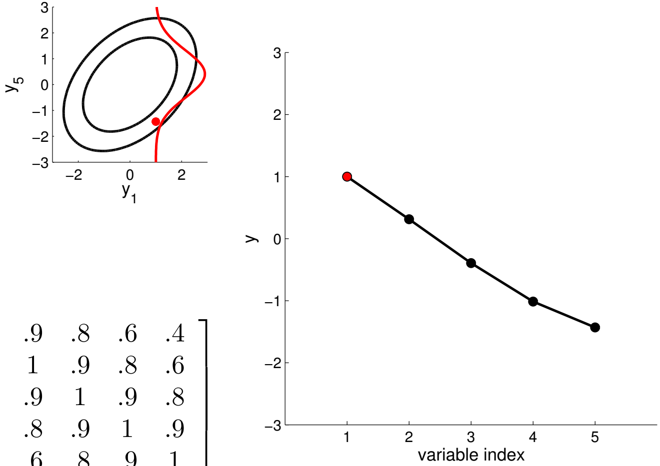
New visualisation



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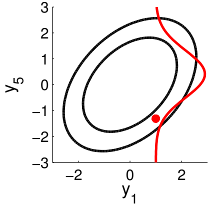
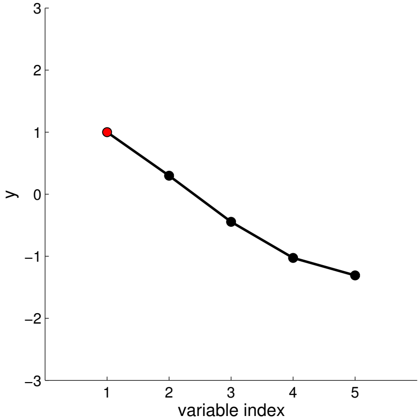
New visualisation





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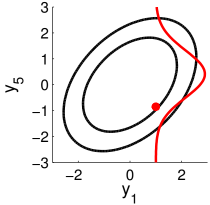
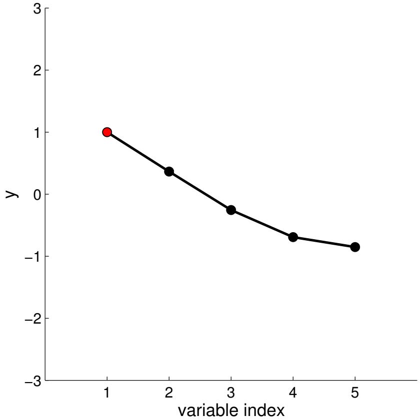
New visualisation





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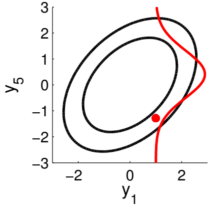
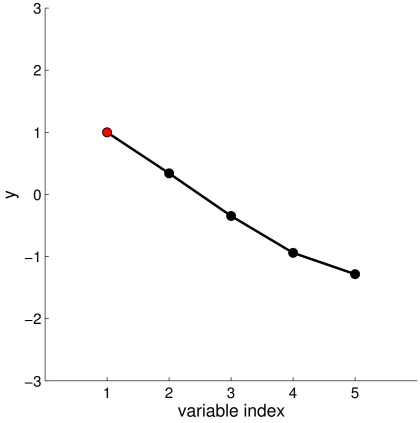
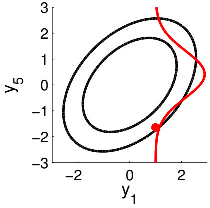
New visualisation



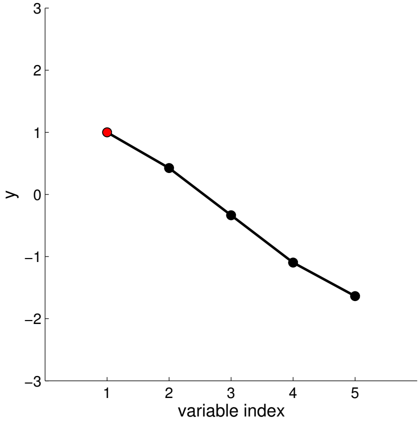


$$\Sigma = \begin{bmatrix} 1 & .9 & .8 & .6 & .4 \\ .9 & 1 & .9 & .8 & .6 \\ .8 & .9 & 1 & .9 & .8 \\ .6 & .8 & .9 & 1 & .9 \\ .4 & .6 & .8 & .9 & 1 \end{bmatrix}$$

New visualisation



$$\Sigma = \begin{bmatrix} 1 & .9 & .8 & .6 & .4 \\ .9 & 1 & .9 & .8 & .6 \\ .8 & .9 & 1 & .9 & .8 \\ .6 & .8 & .9 & 1 & .9 \\ .4 & .6 & .8 & .9 & 1 \end{bmatrix}$$



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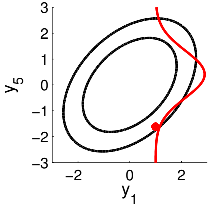
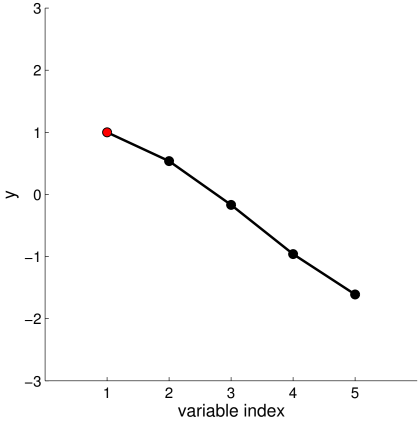


$$\Sigma = \begin{bmatrix} 1 & .9 & .8 & .6 & .4 \\ .9 & 1 & .9 & .8 & .6 \\ .8 & .9 & 1 & .9 & .8 \\ .6 & .8 & .9 & 1 & .9 \\ .4 & .6 & .8 & .9 & 1 \end{bmatrix}$$



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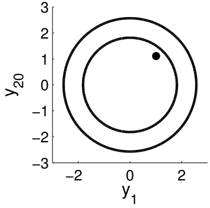

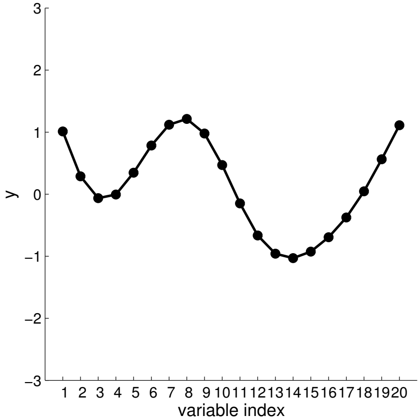

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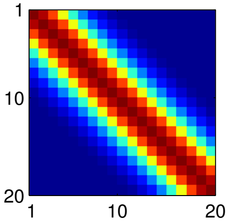

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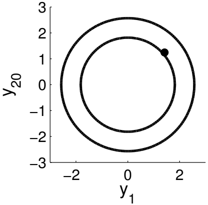

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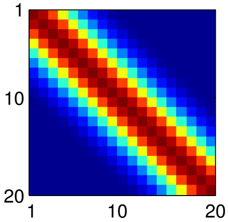

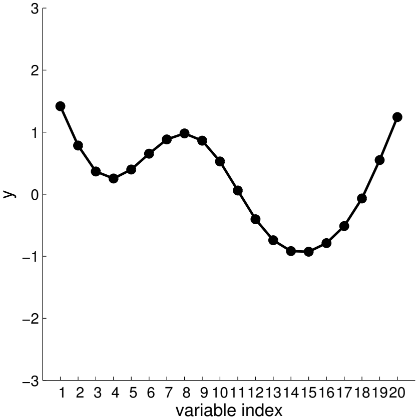

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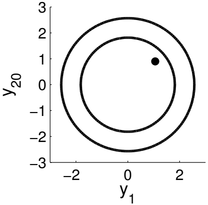

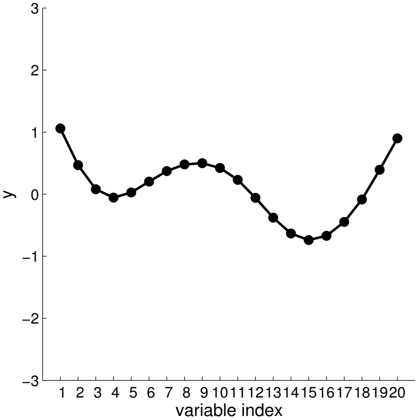

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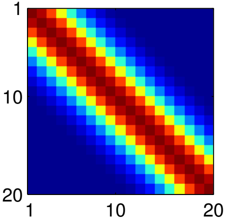

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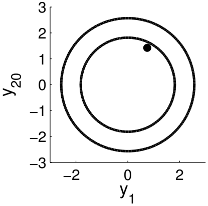

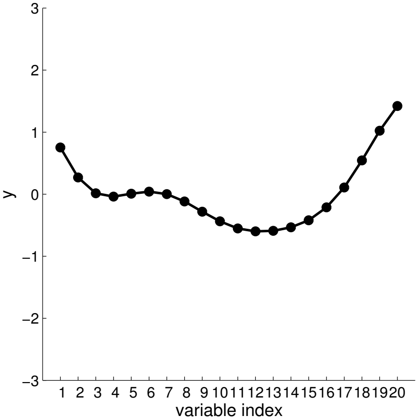

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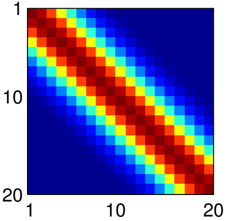

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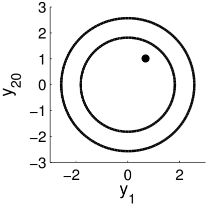

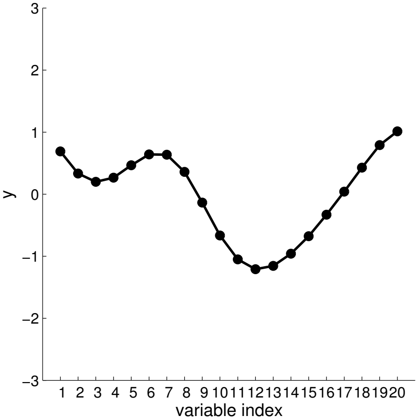

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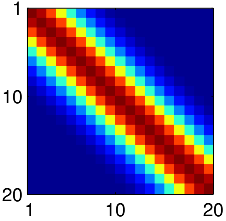

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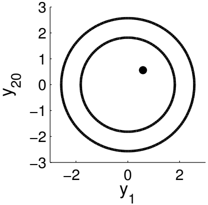

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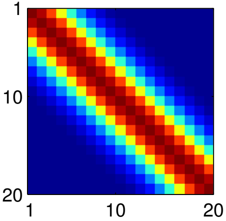

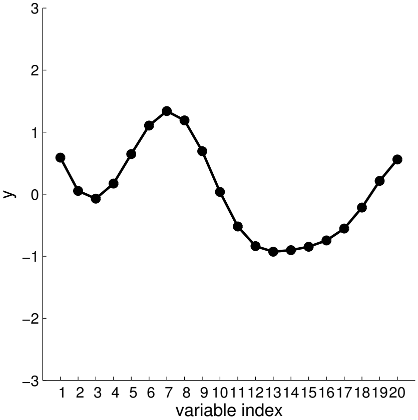

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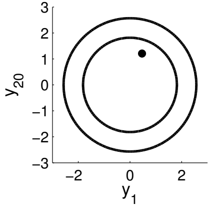

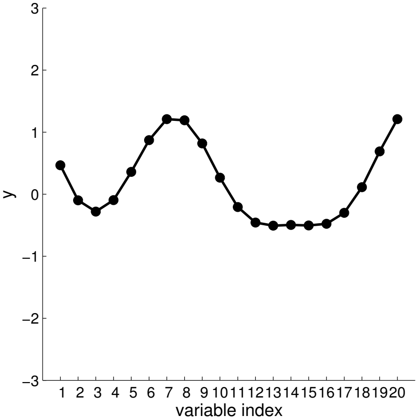

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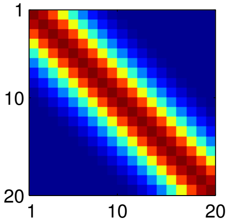

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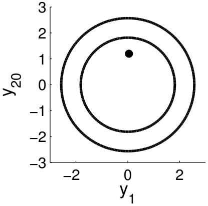

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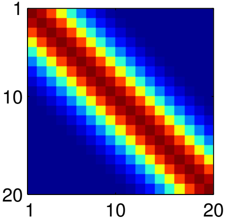

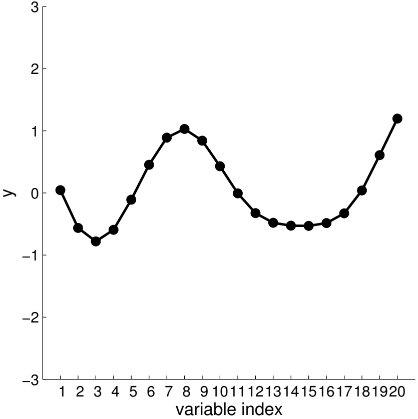

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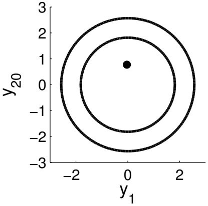

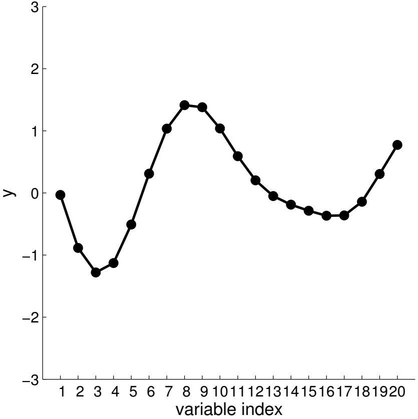

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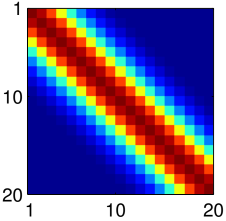

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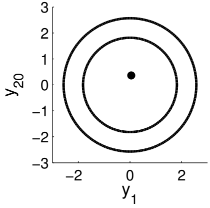

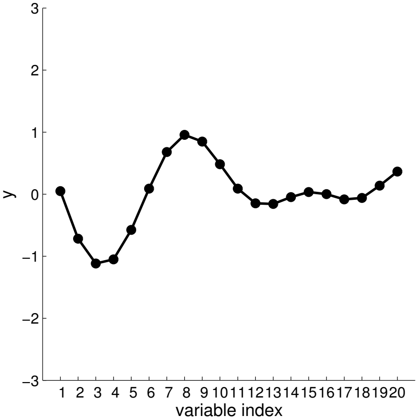

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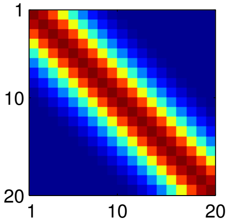

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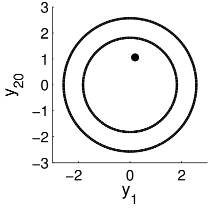

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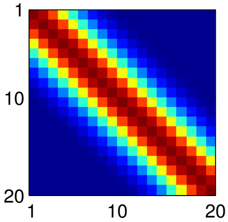

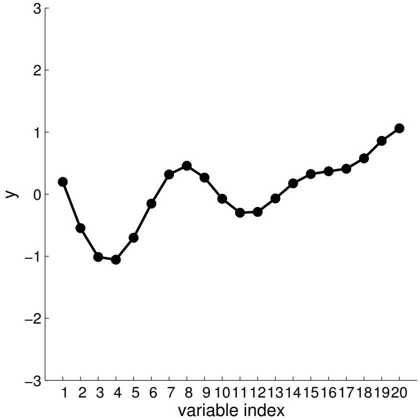

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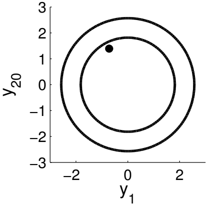

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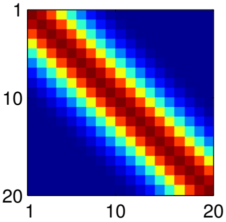

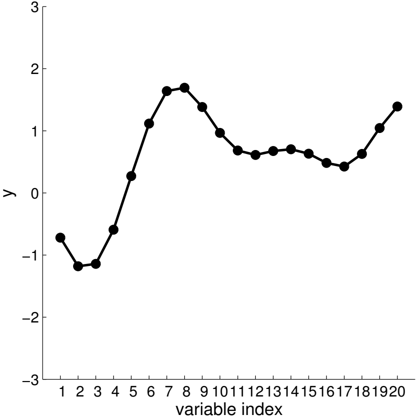

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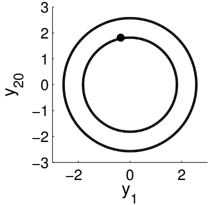

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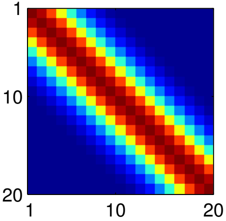

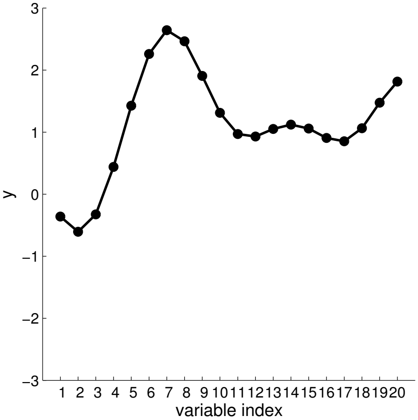

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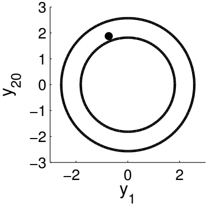

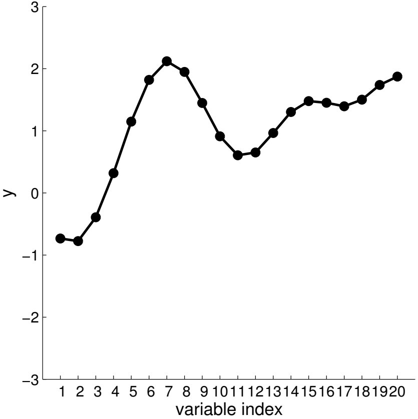

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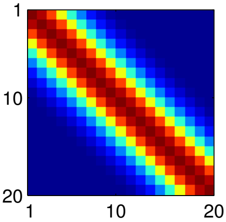

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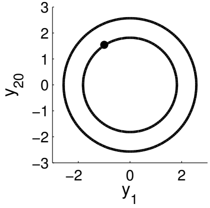

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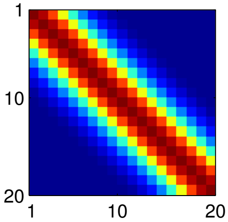

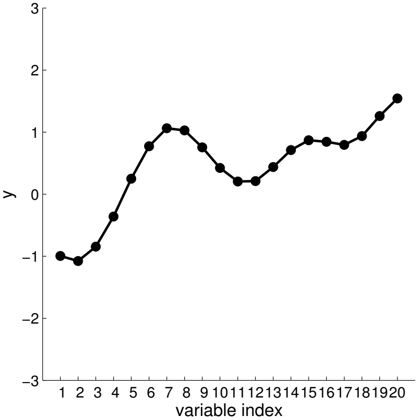

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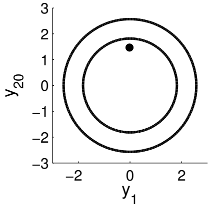

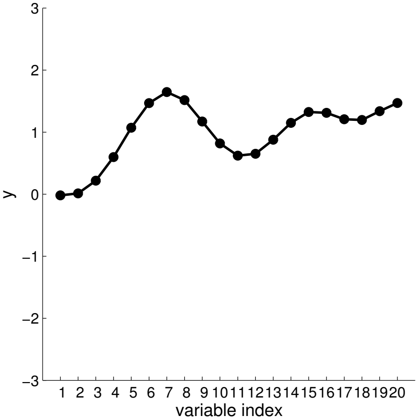

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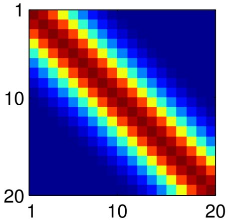

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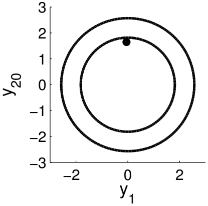

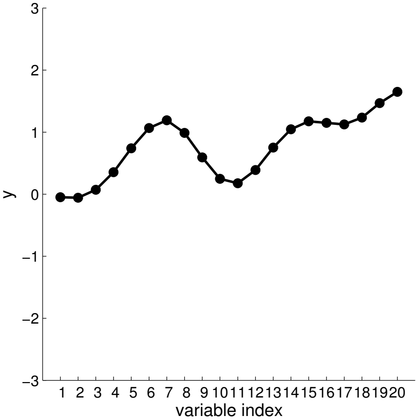

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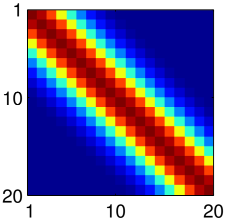

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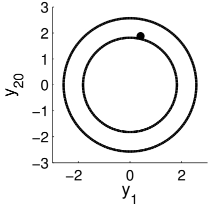

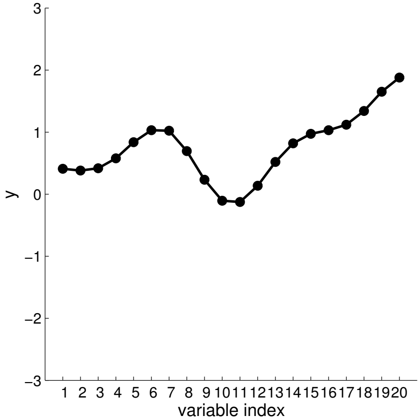

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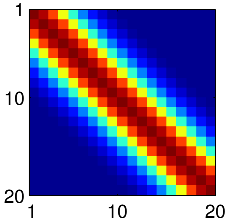

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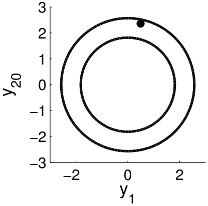

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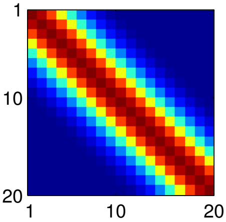

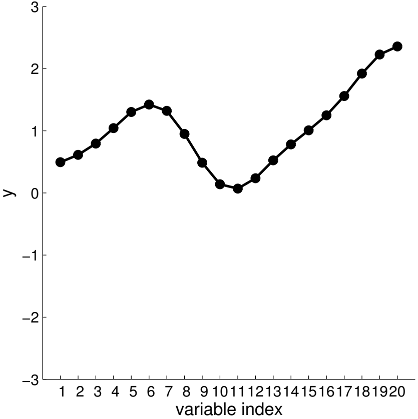

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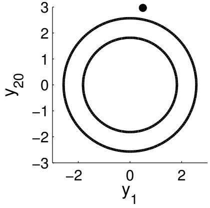

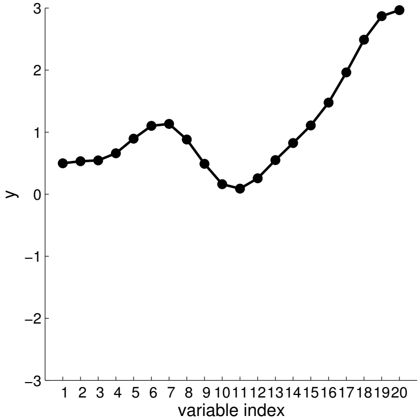

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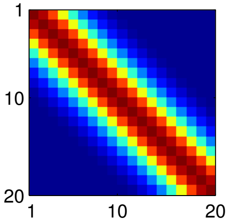

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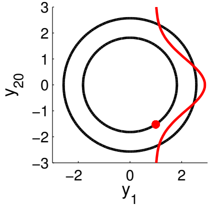

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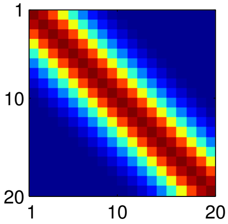

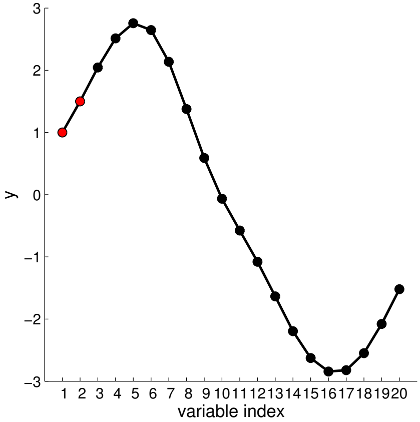

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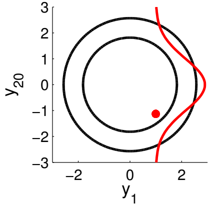

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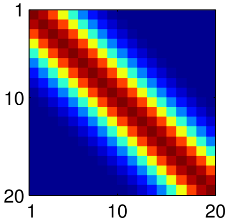

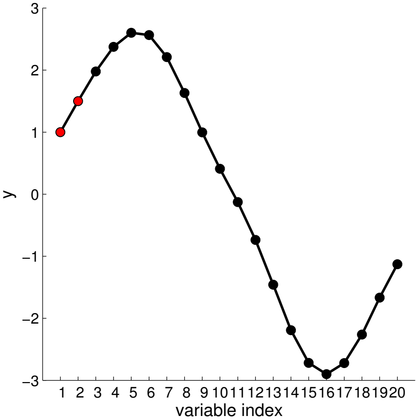

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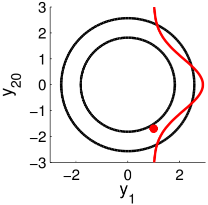

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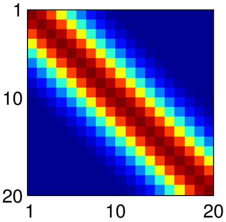

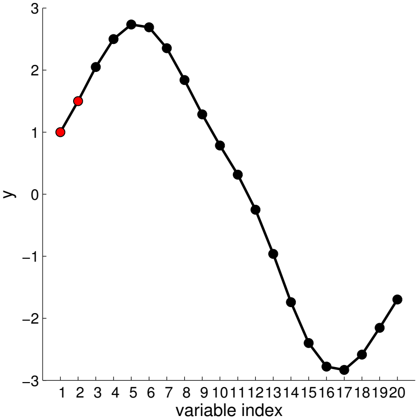

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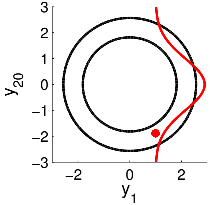

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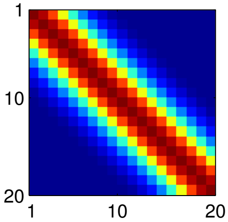

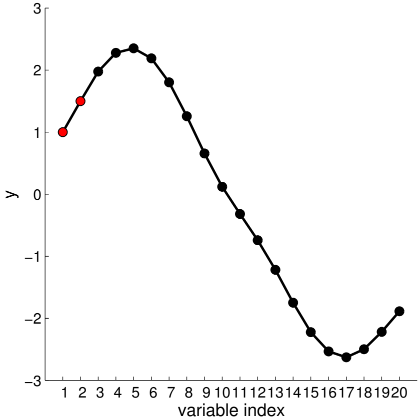

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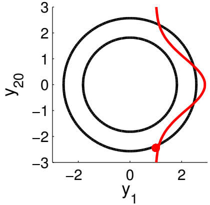

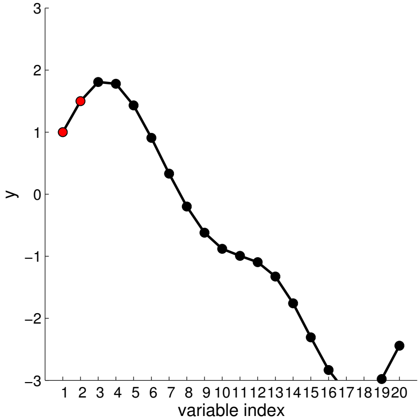

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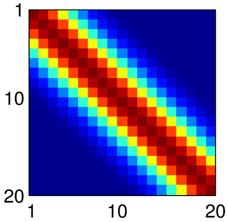

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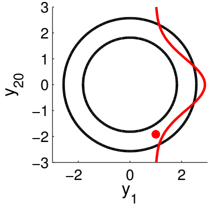

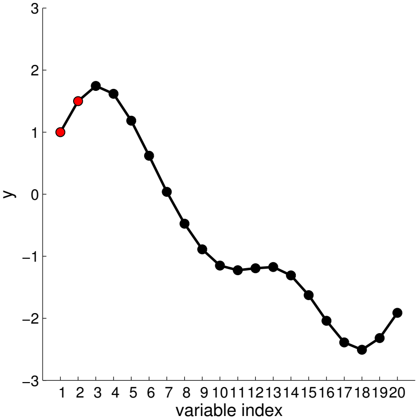

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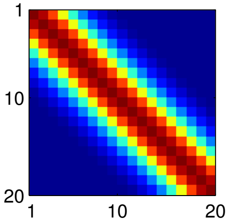

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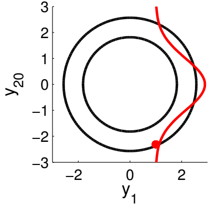

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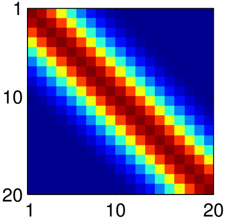

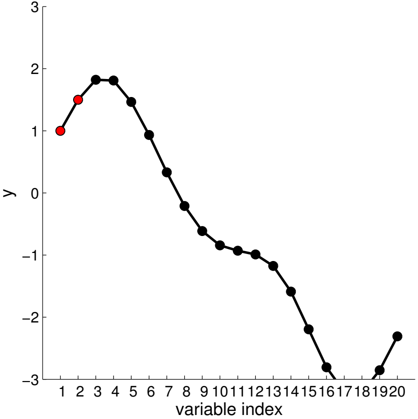

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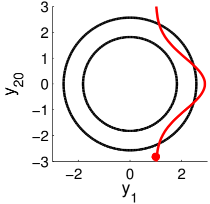

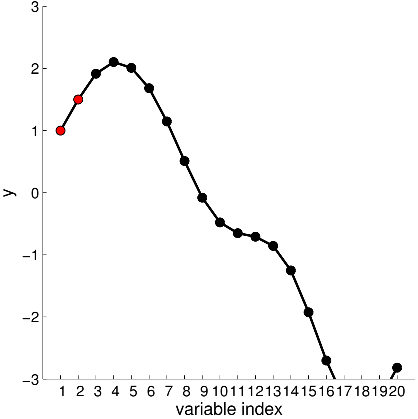

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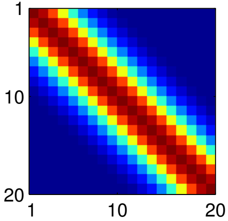

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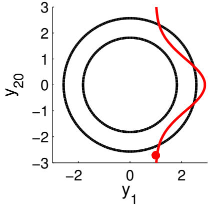

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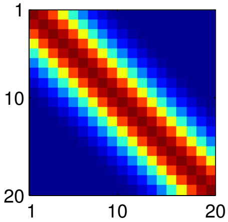

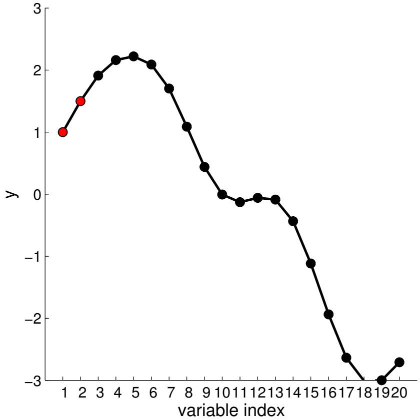

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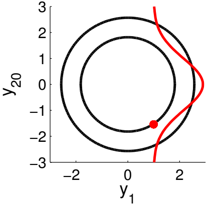

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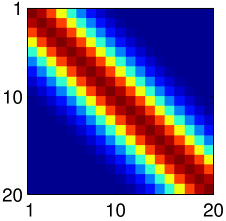

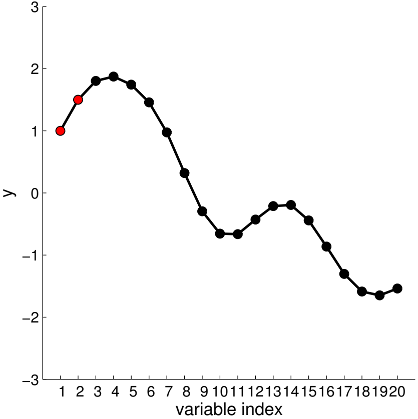

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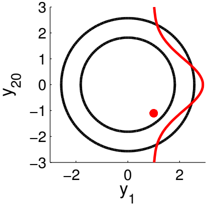

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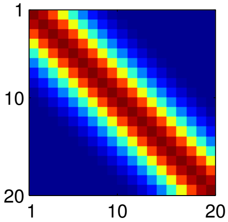

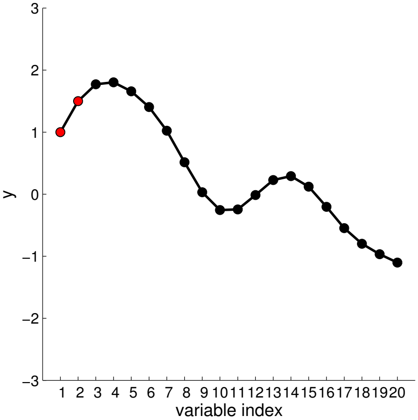

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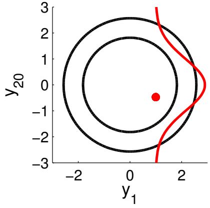

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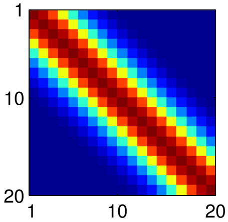

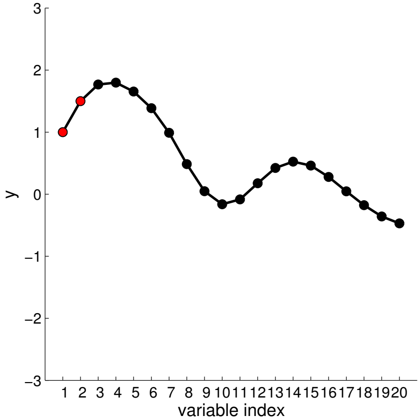

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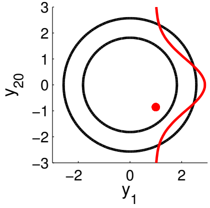

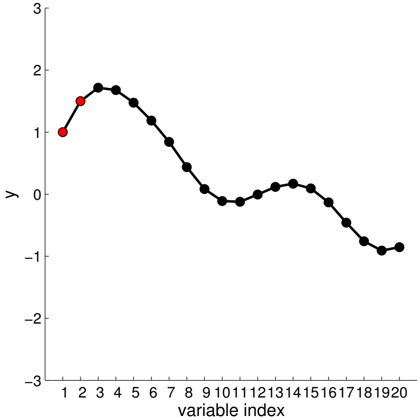

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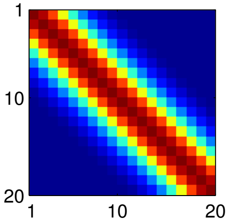

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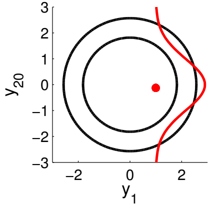
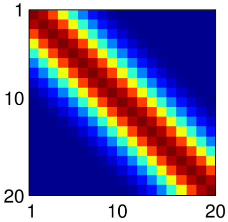
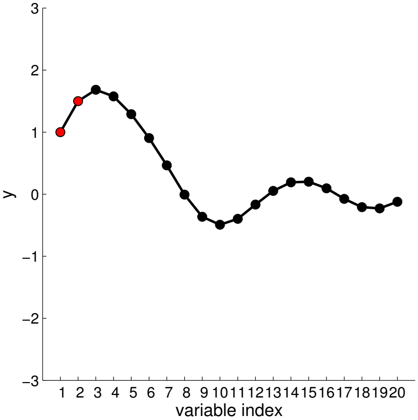


$\Sigma =$ 



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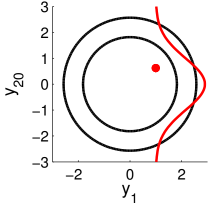

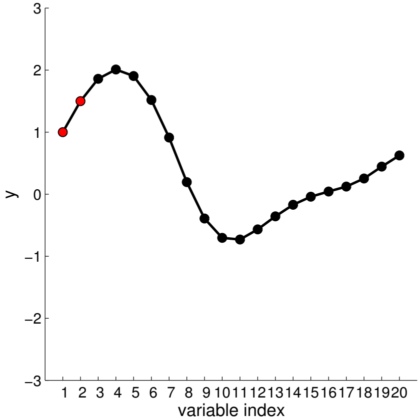

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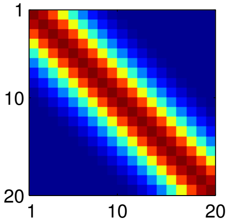

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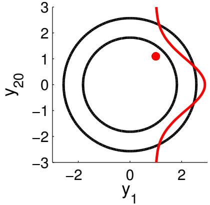

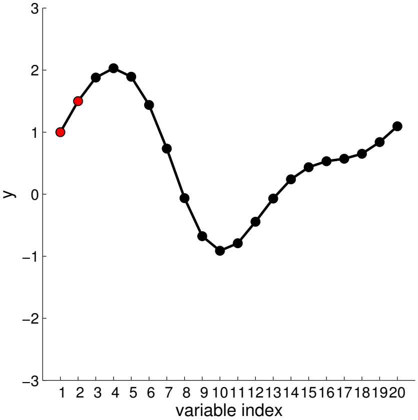

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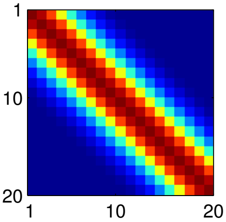

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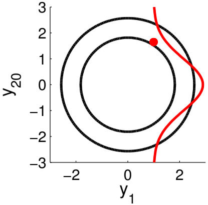

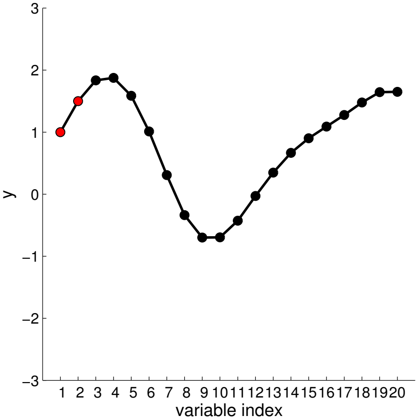

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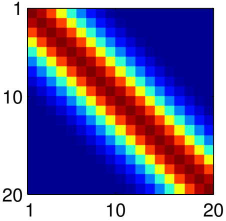

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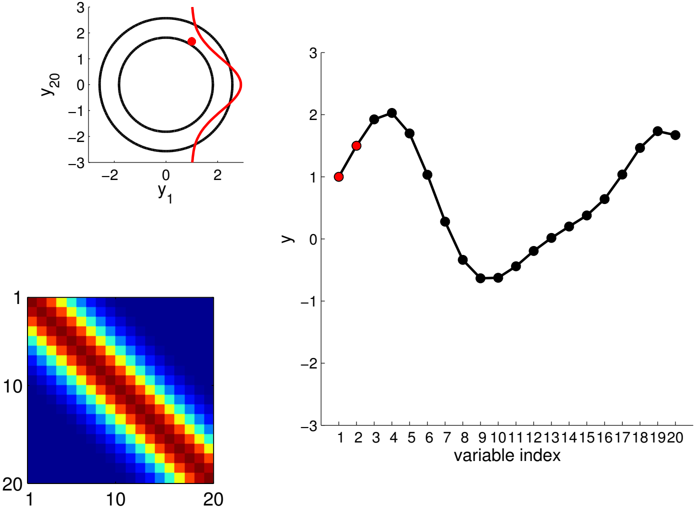

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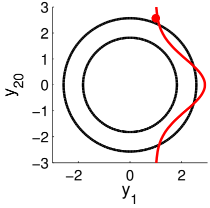
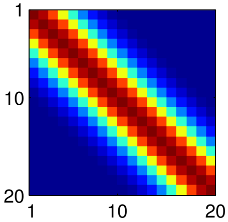
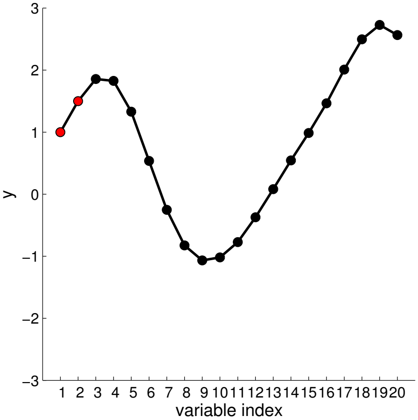


$\Sigma =$ 



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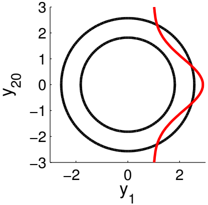

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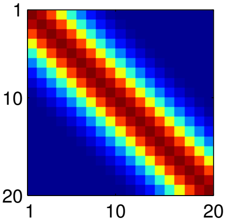
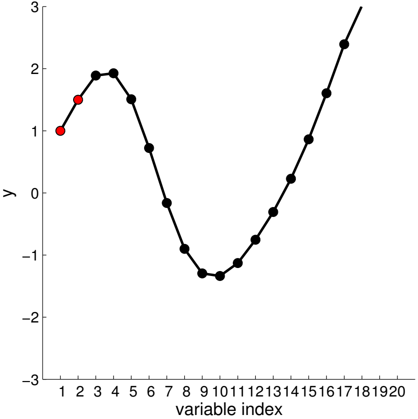
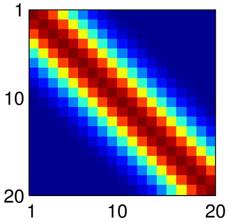
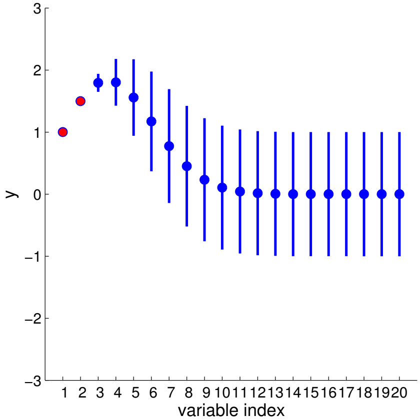




Regression using Gaussians

$\Sigma =$ 



Regression using Gaussians

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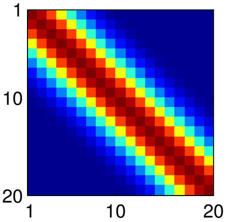
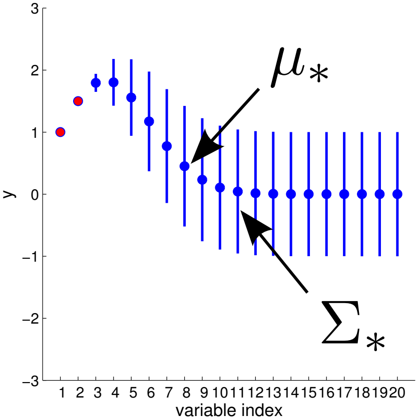
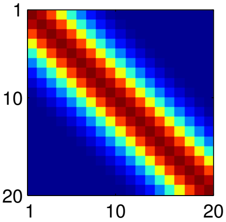
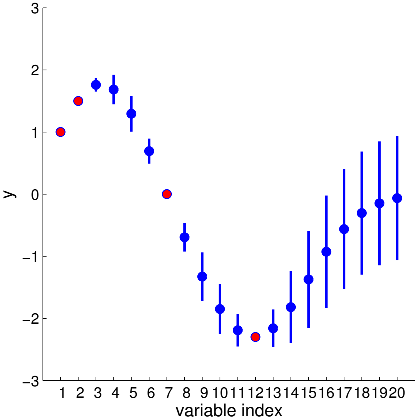




Regression using Gaussians

$\Sigma =$ 

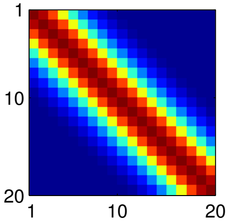


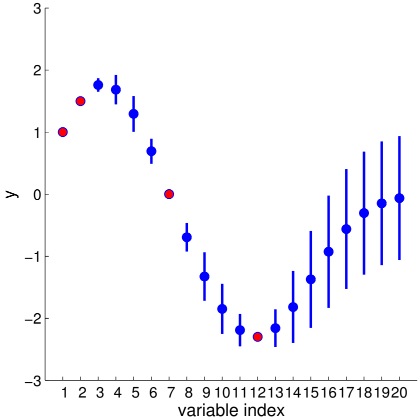
Regression using Gaussians

$$\Sigma(x_1, x_2) = K(x_1, x_2) + I\sigma_y^2$$

$$K(x_1, x_2) = \sigma^2 e^{-\frac{1}{2l^2}(x_1 - x_2)^2}$$

$\Sigma =$



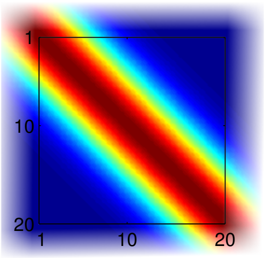


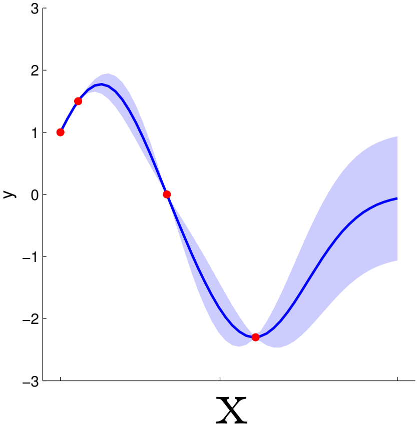
Regression: probabilistic inference in function space

$$\Sigma(x_1, x_2) = K(x_1, x_2) + I\sigma_y^2$$

$$K(x_1, x_2) = \sigma^2 e^{-\frac{1}{2l^2}(x_1 - x_2)^2}$$

$\Sigma =$





Regression: probabilistic inference in function space

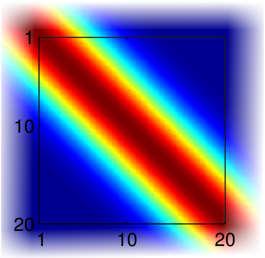
Non-parametric (∞-parametric)

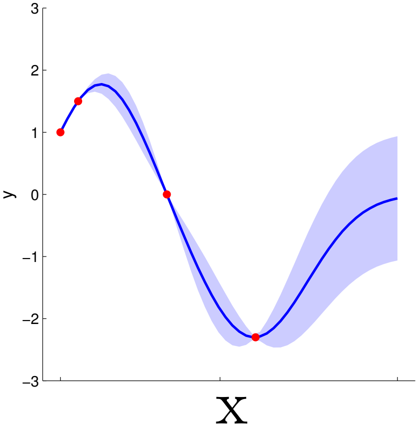
$$p(\mathsf{y}|\theta) = \mathcal{N}(y; 0, \Sigma)$$

$$\Sigma(\mathsf{x}_1, \mathsf{x}_2) = \mathsf{K}(\mathsf{x}_1, \mathsf{x}_2) + \mathsf{I}\sigma_{\mathsf{y}}^2$$

$$\mathsf{K}(\mathsf{x}_1, \mathsf{x}_2) = \sigma^2 \mathrm{e}^{-\frac{1}{2\mathsf{l}^2}(\mathsf{x}_1 - \mathsf{x}_2)^2}$$

$\Sigma =$





Parametric model

$$\mathsf{y}(\mathsf{x}) = f(\mathsf{x}; \theta) + \sigma_{\mathsf{y}}\epsilon$$

$$\epsilon \sim \mathcal{N}(0, 1)$$

Regression: probabilistic inference in function space

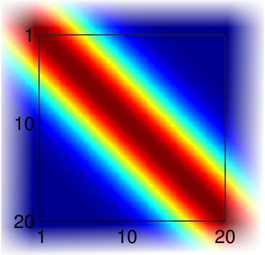
Non-parametric (∞-parametric)

$$p(\mathrm{y}|\theta) = \mathcal{N}(y; 0, \Sigma)$$

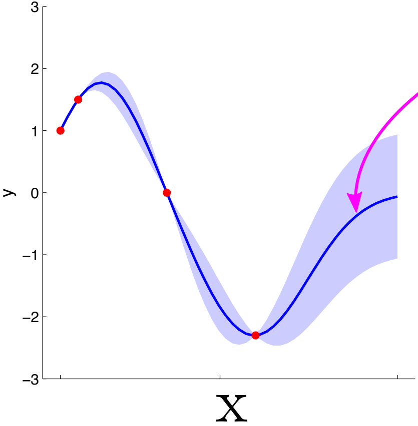
$$\Sigma(\mathrm{x}_1, \mathrm{x}_2) = \mathrm{K}(\mathrm{x}_1, \mathrm{x}_2) + \mathrm{I}\sigma_\mathrm{y}^2$$

$$\mathrm{K}(\mathrm{x}_1, \mathrm{x}_2) = \sigma^2 e^{-\frac{1}{2l^2}(\mathrm{x}_1 - \mathrm{x}_2)^2}$$

$\Sigma =$



function estimate
with uncertainty



Parametric model

$$\mathrm{y}(\mathrm{x}) = f(\mathrm{x}; \theta) + \sigma_\mathrm{y}\epsilon$$

$$\epsilon \sim \mathcal{N}(0, 1)$$

Regression: probabilistic inference in function space

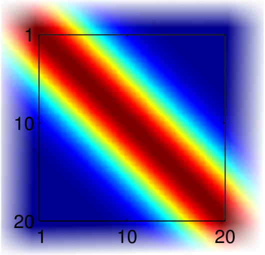
Non-parametric (∞-parametric)

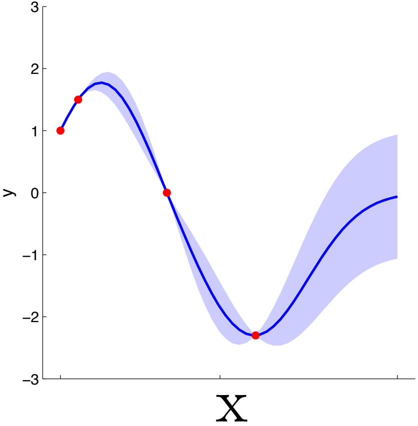
$$p(\mathrm{y}|\theta) = \mathcal{N}(y; 0, \Sigma)$$

observation noise

$$\Sigma(\mathrm{x}_1, \mathrm{x}_2) = \mathrm{K}(\mathrm{x}_1, \mathrm{x}_2) + \mathrm{I}\sigma_{\mathrm{y}}^2$$

$$\mathrm{K}(\mathrm{x}_1, \mathrm{x}_2) = \sigma^2 e^{-\frac{1}{2l^2}(\mathrm{x}_1 - \mathrm{x}_2)^2}$$

$\Sigma =$ 



Parametric model

$$\mathrm{y}(\mathrm{x}) = f(\mathrm{x}; \theta) + \sigma_{\mathrm{y}}\epsilon$$

$$\epsilon \sim \mathcal{N}(0, 1)$$

Regression: probabilistic inference in function space

Non-parametric (∞-parametric)

$$p(\mathrm{y}|\theta) = \mathcal{N}(y; 0, \Sigma)$$

$$\Sigma(\mathrm{x}_1, \mathrm{x}_2) = \mathrm{K}(\mathrm{x}_1, \mathrm{x}_2) + \mathrm{I}\sigma_\mathrm{y}^2$$

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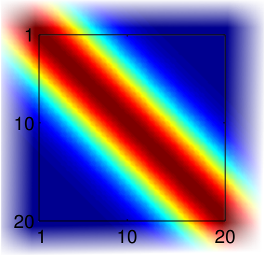
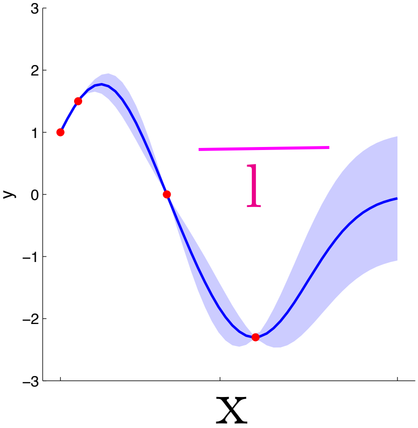
horizontal-scale

$\Sigma =$





Parametric model

$$\mathrm{y}(\mathrm{x}) = f(\mathrm{x}; \theta) + \sigma_\mathrm{y}\epsilon$$

$$\epsilon \sim \mathcal{N}(0, 1)$$

Regression: probabilistic inference in function space

Non-parametric (∞-parametric)

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vertical-scale   horizontal-scale

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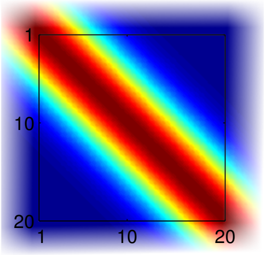

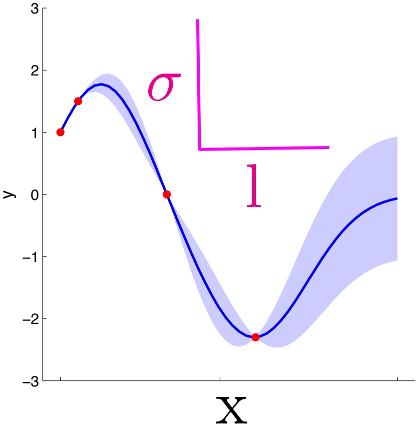

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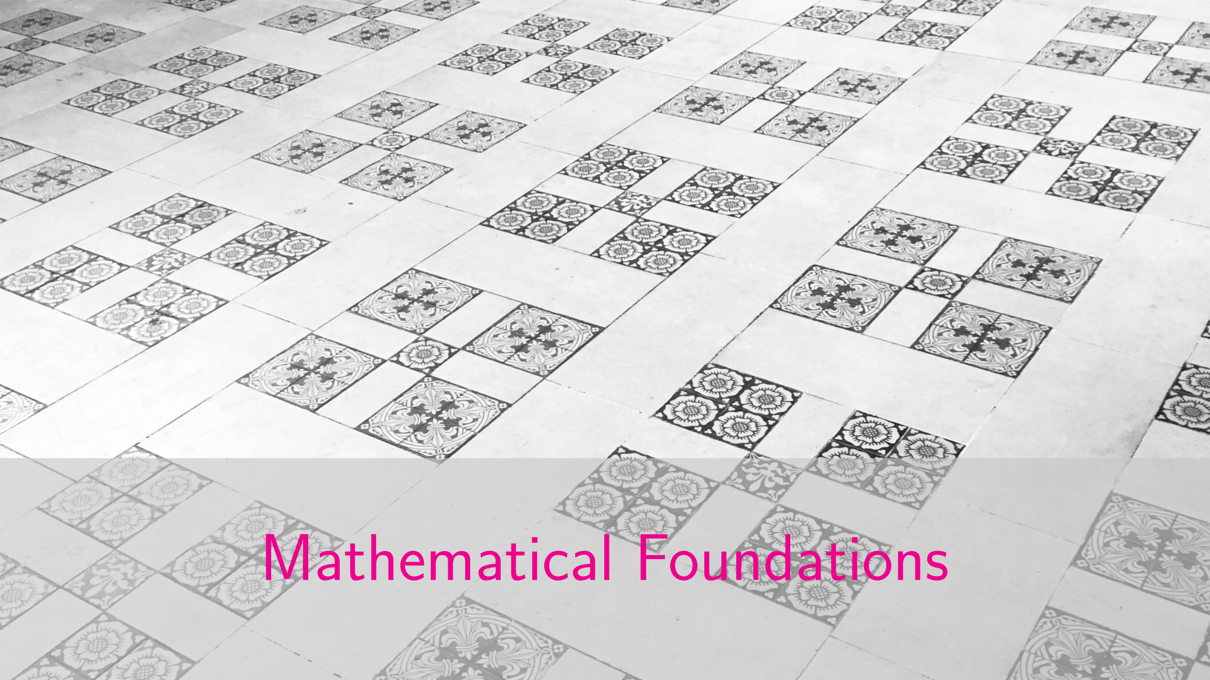
Outline of the tutorial

- **An Introduction to GPs**
  - Mathematical foundations
  - Hyper-parameter learning
  - Covariance functions
  - Multi-dimensional inputs
- **Using GPs: Models, Applications and Connections**
  - Models and more on covariance functions
  - Applications
  - Connections
- **GPs for large data and non-linear models**
  - Scaling through pseudo-data
  - Variational Inference
  - General Approximate inference



Mathematical Foundations

Mathematical Foundations: Definition

Gaussian process = generalisation of multivariate Gaussian distribution to infinitely many variables.

Definition: a Gaussian process is a collection of random variables, any finite number of which have (consistent) Gaussian distributions.

A Gaussian distribution is fully specified by a mean vector, $\boldsymbol{\mu}$, and covariance matrix Σ:

$$\mathbf{f} = (\mathsf{f}_1, \ldots, \mathsf{f}_n) \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma), \quad \text{indices} \quad i = 1, \ldots, n$$

A Gaussian process is fully specified by a mean function $m(\mathbf{x})$ and covariance function $\mathsf{K}(\mathbf{x}, \mathbf{x}')$:

$$f(\mathbf{x}) \sim \mathcal{GP}\left(m(\mathbf{x}), \mathsf{K}(\mathbf{x}, \mathbf{x}')\right), \quad \text{indices} \quad \mathbf{x}$$

Mathematical Foundations: Regression

Q1. What's the formal justification for how we were using GPs for regression?

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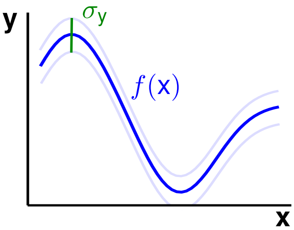
generative model (like non-linear regression)

$$y(x) = f(x) + \epsilon \sigma_y$$

$$p(\epsilon) = \mathcal{N}(\epsilon; 0, 1)$$



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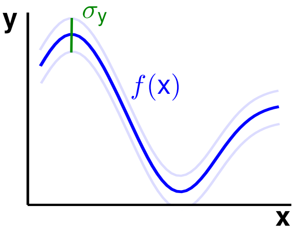

place GP prior over the non-linear function

$$p(f(\mathrm{x})|\theta) = \mathcal{GP}(f(\mathrm{x}); 0, \mathrm{K}_{\theta}(\mathrm{x}, \mathrm{x}'))$$

$$\mathrm{K}(\mathrm{x}, \mathrm{x}') = \sigma^2 \exp\left(-\frac{1}{2l^2}(\mathrm{x} - \mathrm{x}')^2\right)$$ (smoothly wiggling functions expected)

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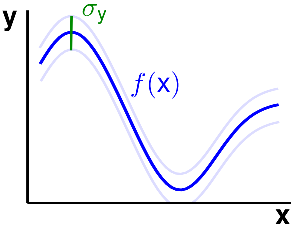

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sum of Gaussian variables = Gaussian: induces a GP over $y(x)$

$$p(y(x)|\theta) = \mathcal{GP}(y(x); 0, K_\theta(x, x') + I\sigma_y^2)$$

Mathematical Foundations: Marginalisation

Q2. A GP is "like" a Gaussian distribution with an infinitely long mean vector and an "infinite by infinite" covariance matrix, so how do we represent it on a computer?

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$\implies$ Entries in a precision matrix depend on what other data we are considering

Mathematical Foundations: Prediction

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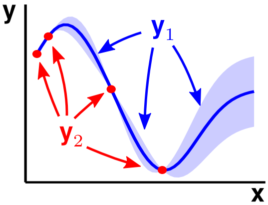

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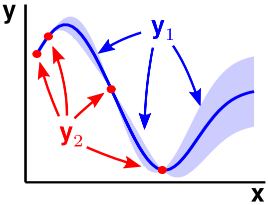

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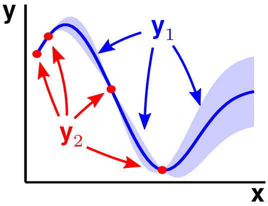

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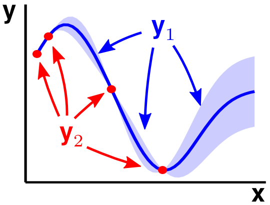

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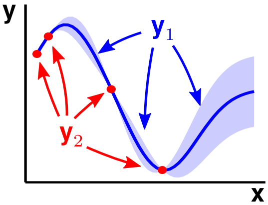

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predictive mean

$$\mu_{\mathbf{y}_1|\mathbf{y}_2} = \mathbf{a} + \mathsf{B}\mathsf{C}^{-1}(\mathbf{y}_2 - \mathbf{b})$$

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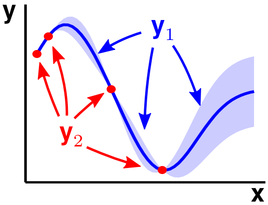

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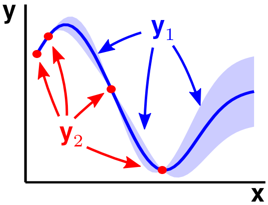

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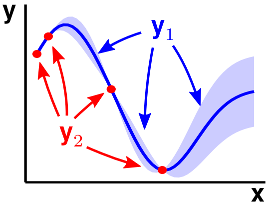

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linear in the data

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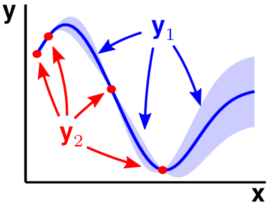

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$$= \mathrm{B}\mathrm{C}^{-1}\mathbf{y}_2$$

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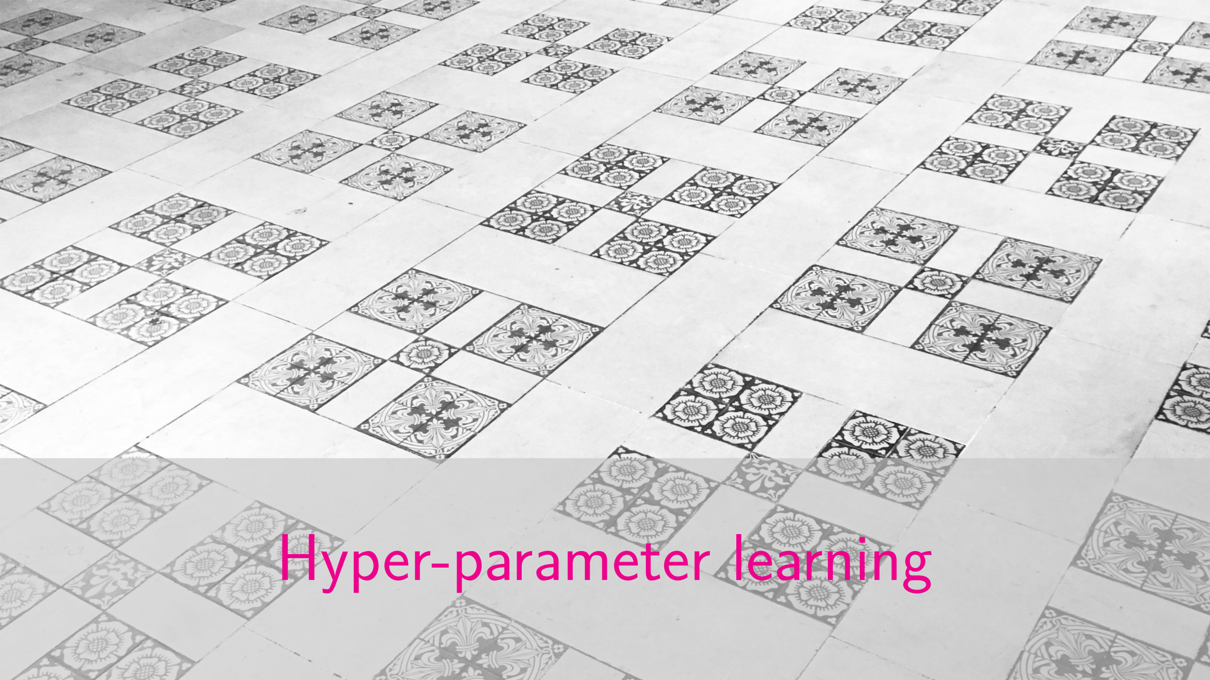
linear in the data

predictive covariance

$$\Sigma_{\mathbf{y}_1|\mathbf{y}_2} = \mathrm{A} - \mathrm{B}\mathrm{C}^{-1}\mathrm{B}^\top$$

predictive uncertainty = prior uncertainty − reduction in uncertainty

predictions more confident than prior



Hyper-parameter learning

What effect do the hyper-parameters have?

Non-parametric (∞-parametric)

$$p(\mathrm{y}|\theta) = \mathcal{N}(y; 0, \Sigma)$$

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vertical-scale horizontal-scale

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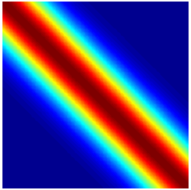

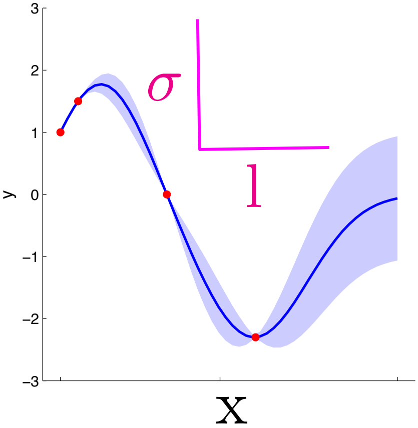

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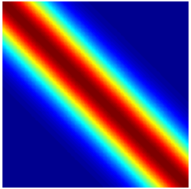
$\Sigma =$ 

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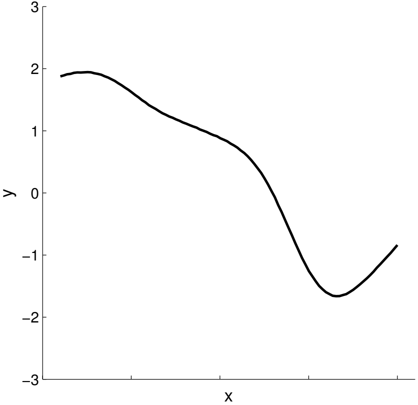

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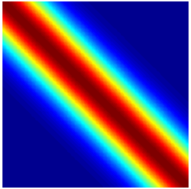
$\Sigma =$ 

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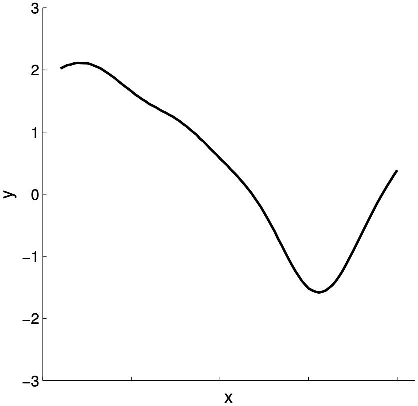

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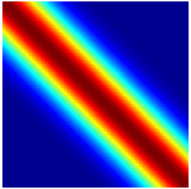
$\Sigma =$ 

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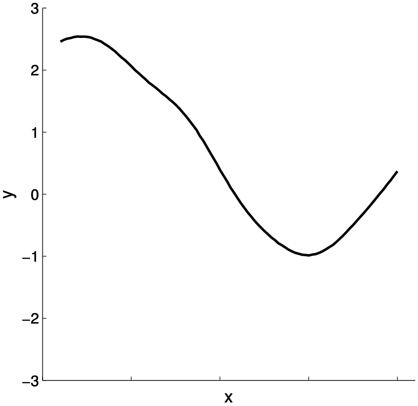

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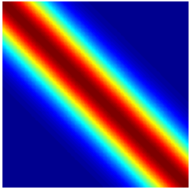
$\Sigma =$ 

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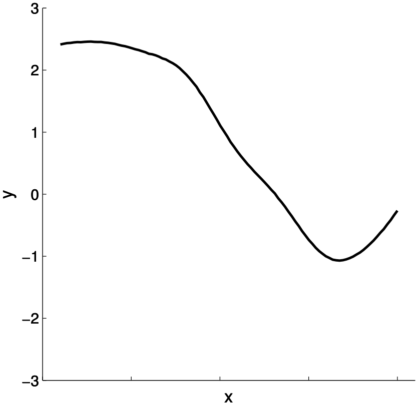

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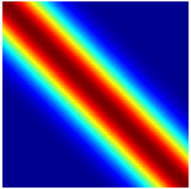
$\Sigma =$ 

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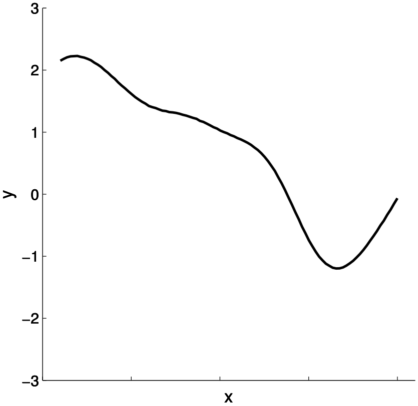

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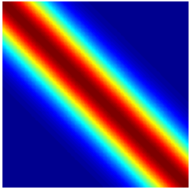
$\Sigma =$ 

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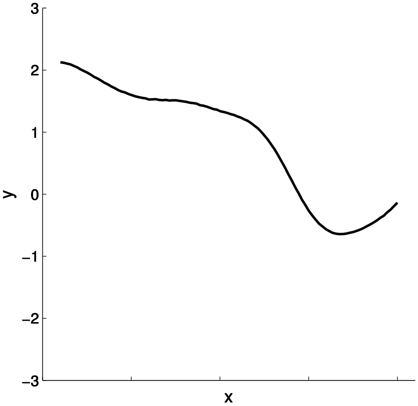

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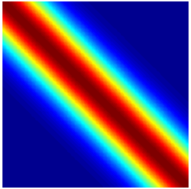
$\Sigma =$ 

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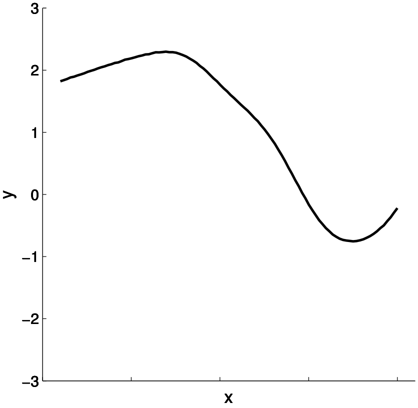

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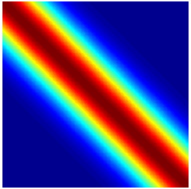
$\Sigma =$ 

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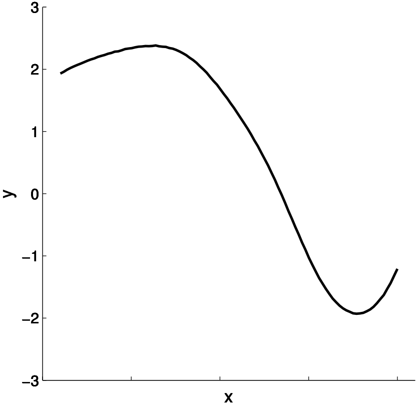

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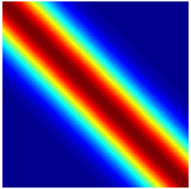
$\Sigma =$ 

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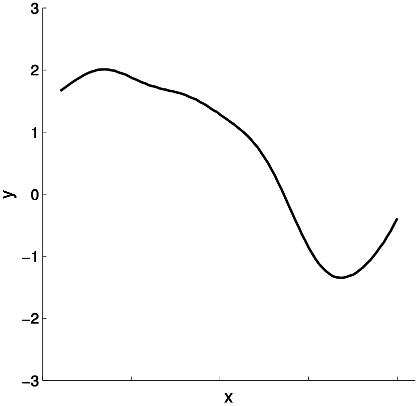

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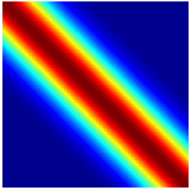
$\Sigma =$ 

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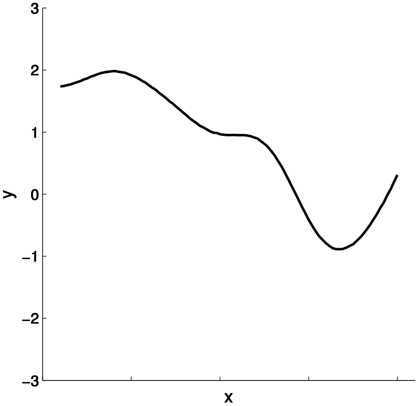

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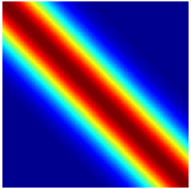
$\Sigma =$ 

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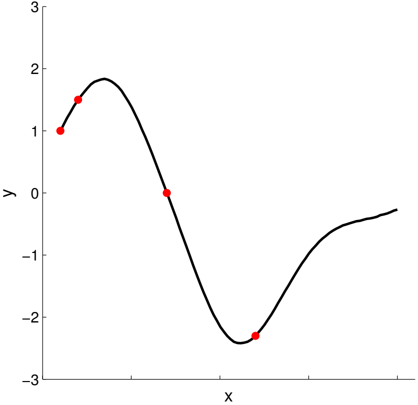

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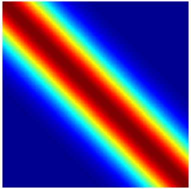
$\Sigma =$ 

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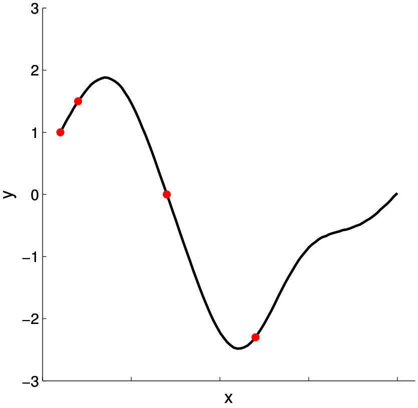

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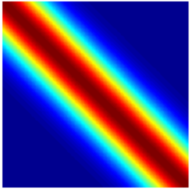
$\Sigma =$ 

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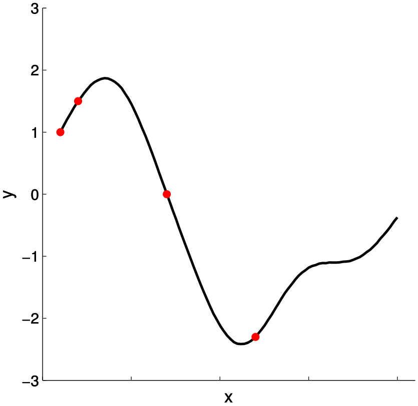

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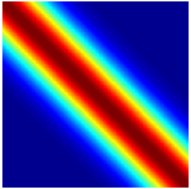
$\Sigma =$ 

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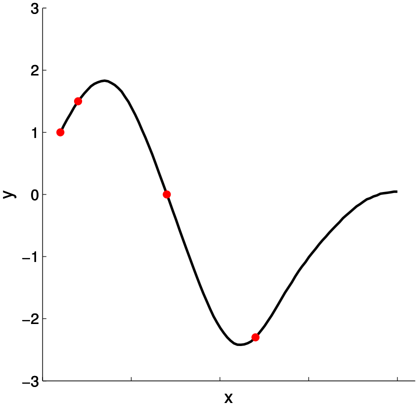

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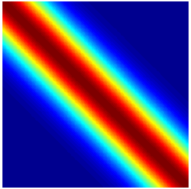
$\Sigma =$ 

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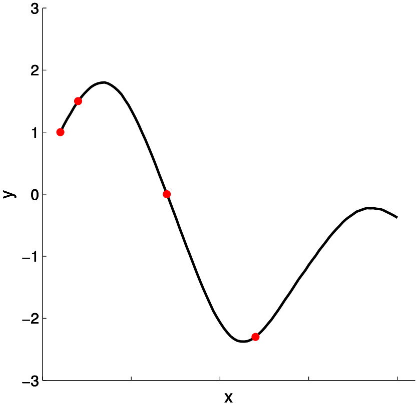

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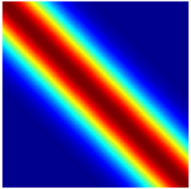

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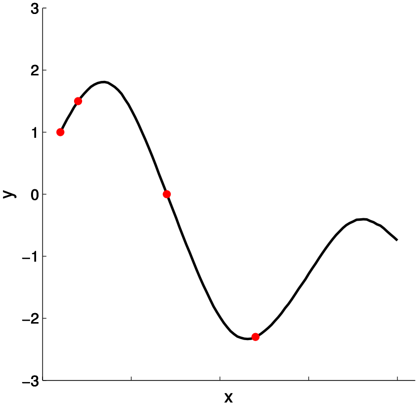

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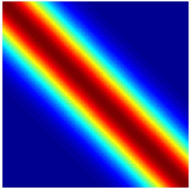

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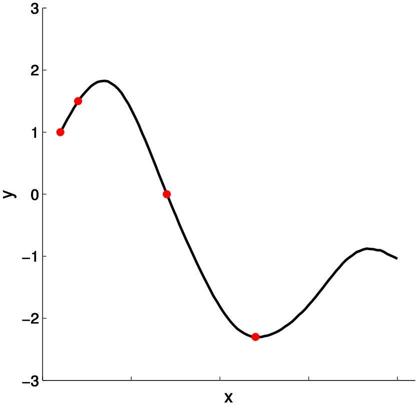

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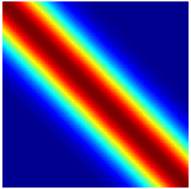
$\Sigma =$ 

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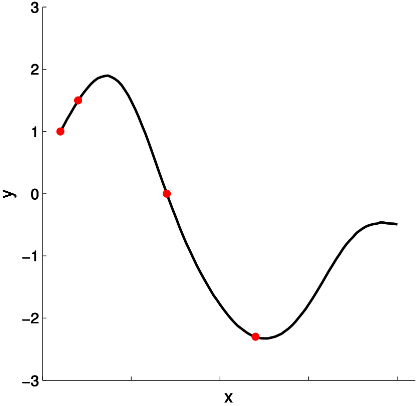

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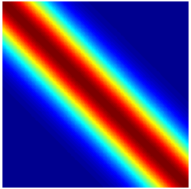
$\Sigma =$ 

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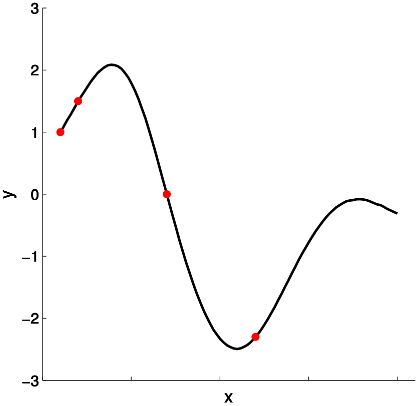

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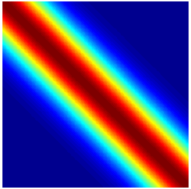
$\Sigma =$ 

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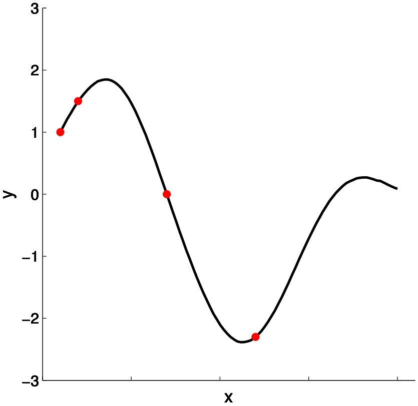

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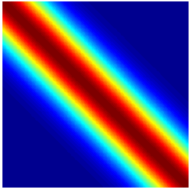
$\Sigma =$ 

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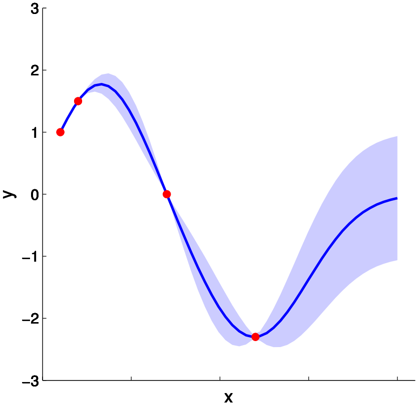

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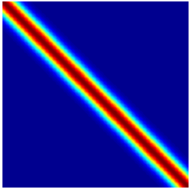
$\Sigma =$ 

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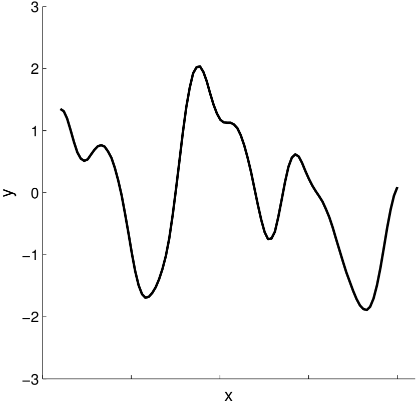

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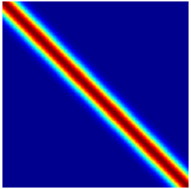
$\Sigma =$ 

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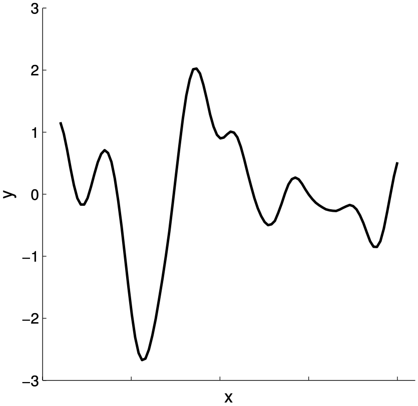

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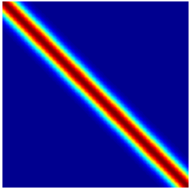
$\Sigma =$ 

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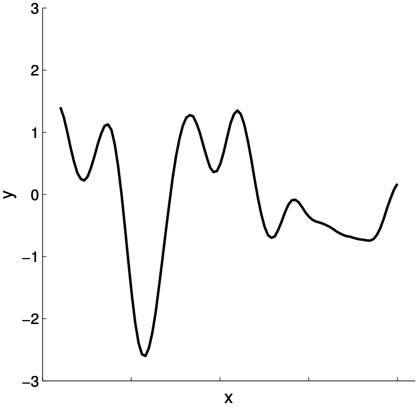

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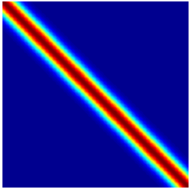
$\Sigma =$ 

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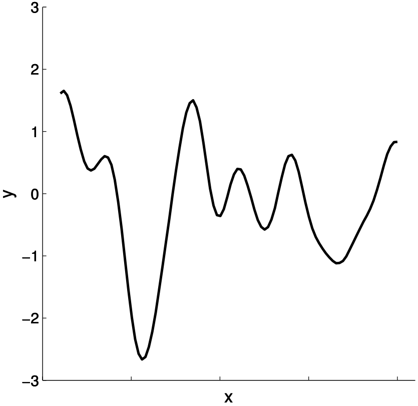

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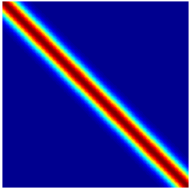

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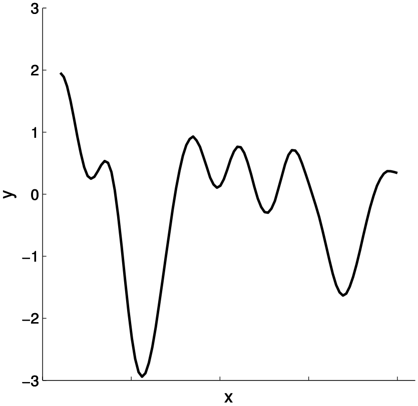

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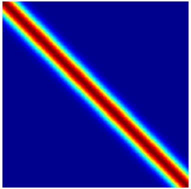
$\Sigma =$ 

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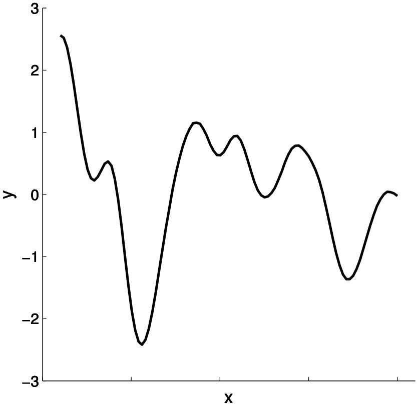

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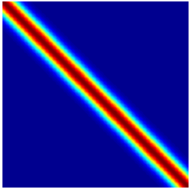

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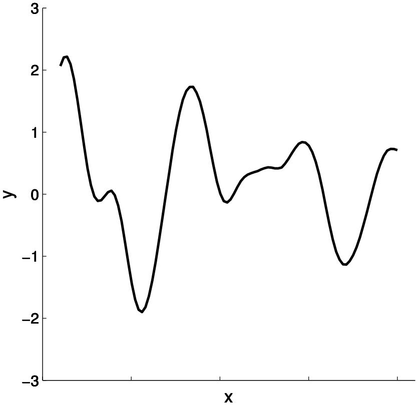

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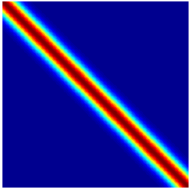
$\Sigma =$ 

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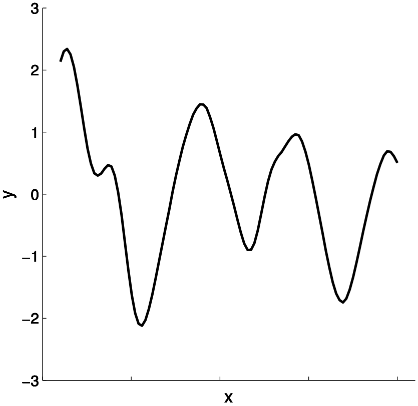

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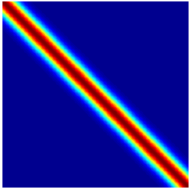

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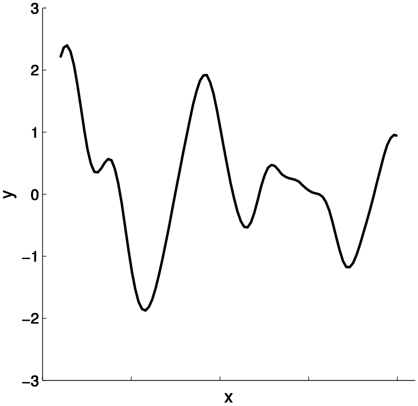

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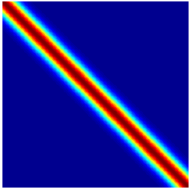

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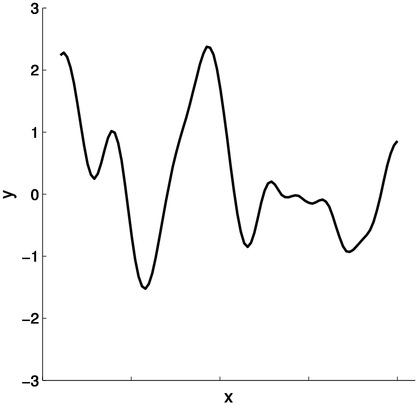

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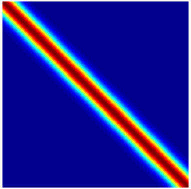
$\Sigma =$ 

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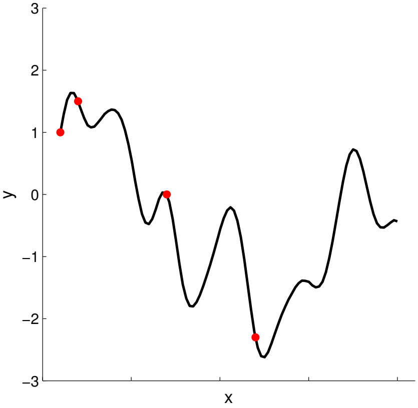

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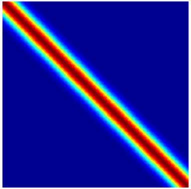
$\Sigma =$ 

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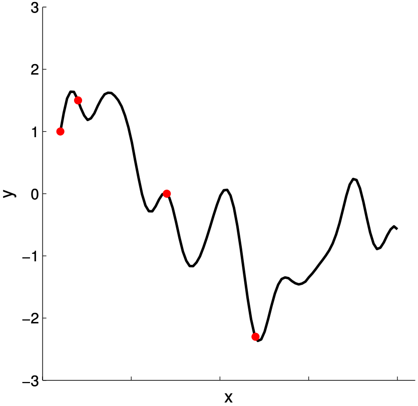

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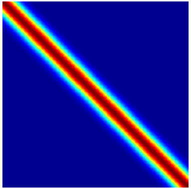
$\Sigma =$ 

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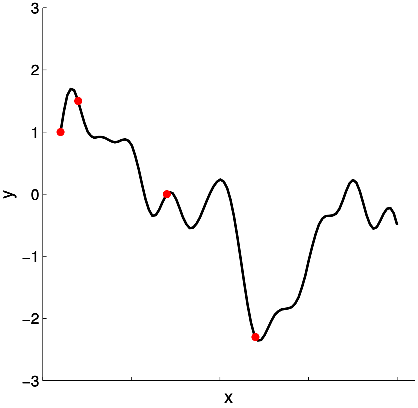

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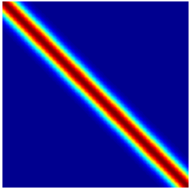
$\Sigma =$ 

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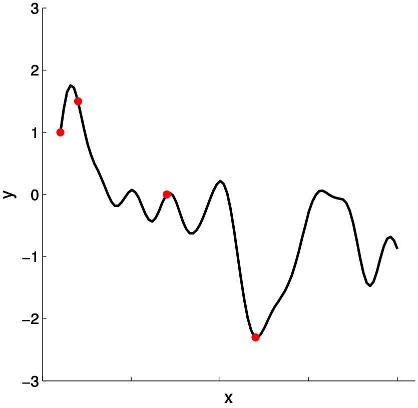

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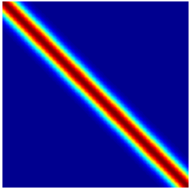
$\Sigma =$ 

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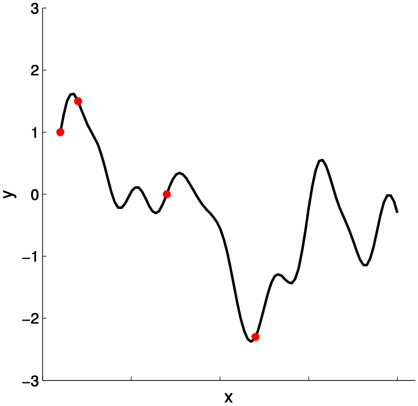

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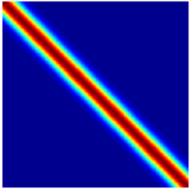
$\Sigma =$ 

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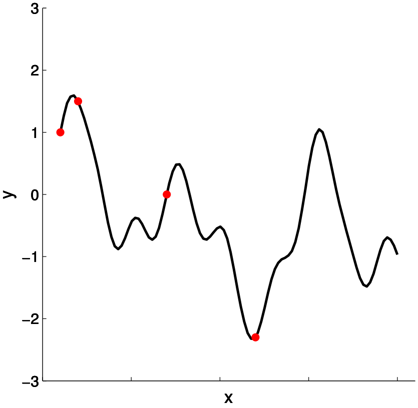

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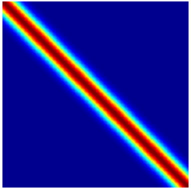
$\Sigma =$ 

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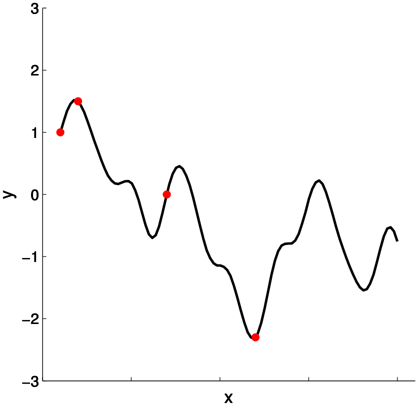

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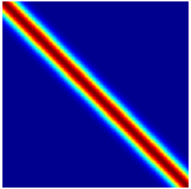
$\Sigma =$ 

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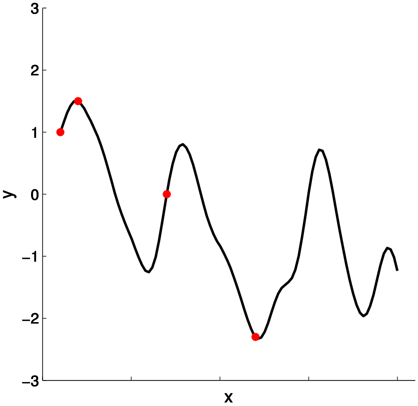

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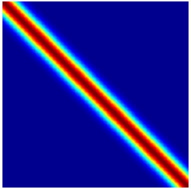
$\Sigma =$ 

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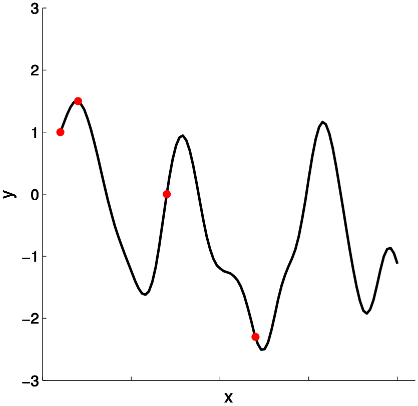

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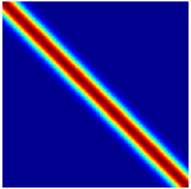

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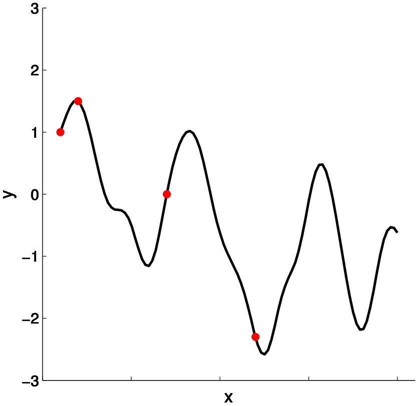

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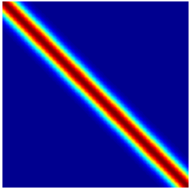
$\Sigma =$ 

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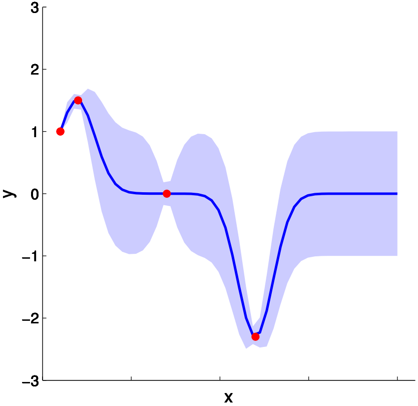

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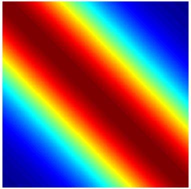

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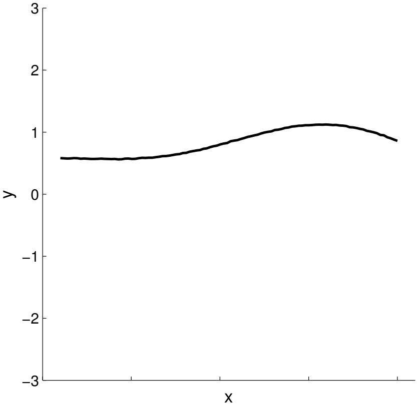

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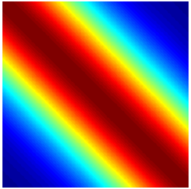
$\Sigma =$ 

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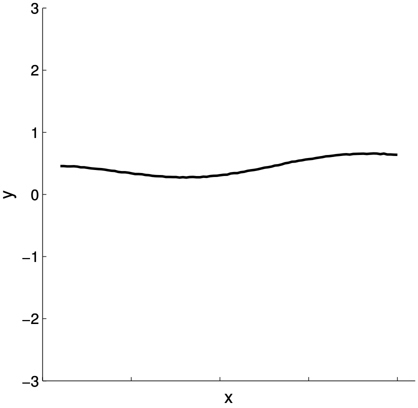

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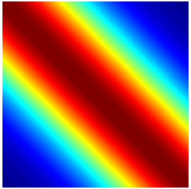
$\Sigma =$ 

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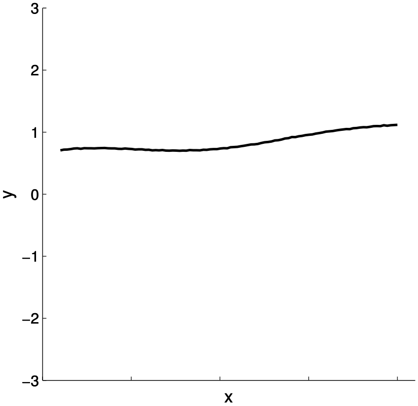

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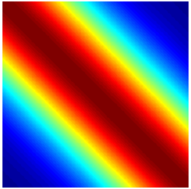

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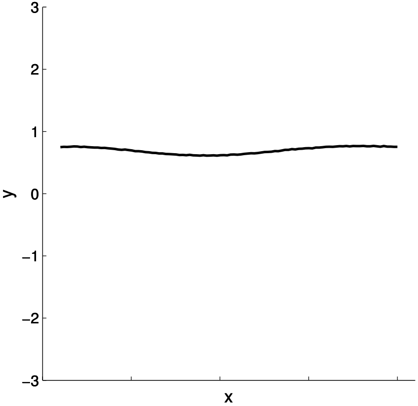

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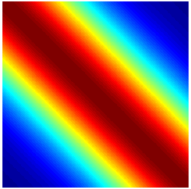
$\Sigma =$ 

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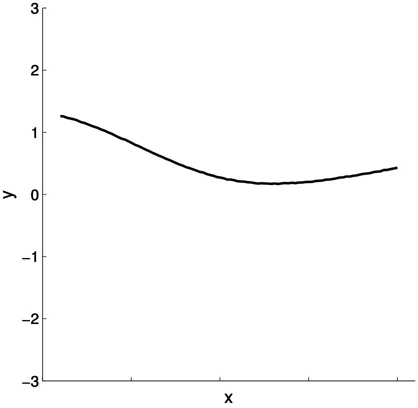

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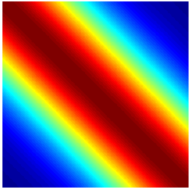
$\Sigma =$ 

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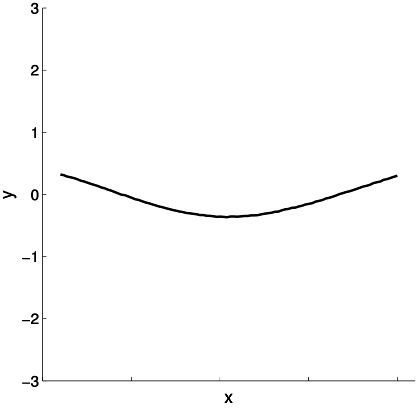

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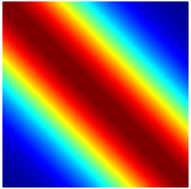
$\Sigma =$ 

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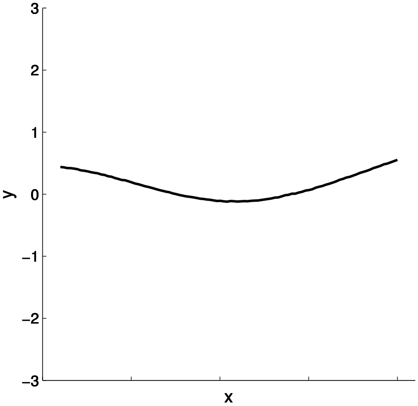

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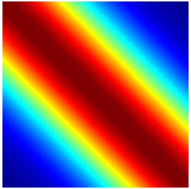
$\Sigma =$ 

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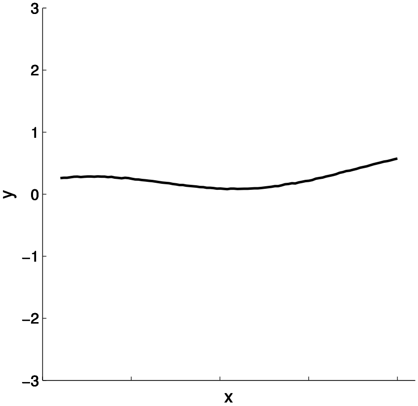

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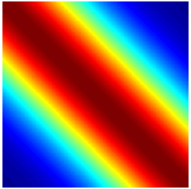
$\Sigma =$ 

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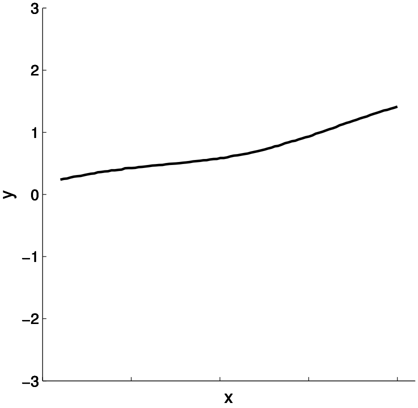

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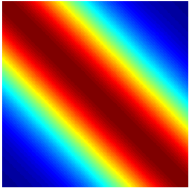
$\Sigma =$ 

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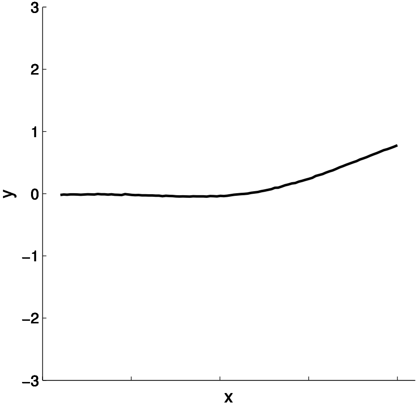

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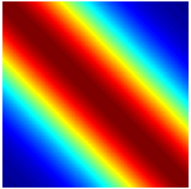
$\Sigma =$ 

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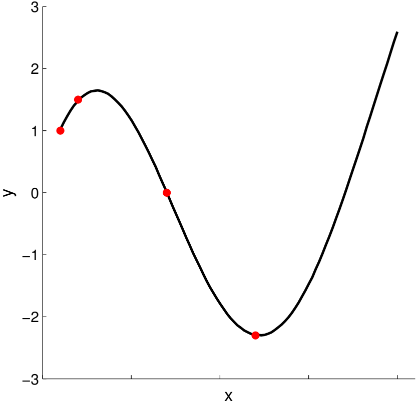

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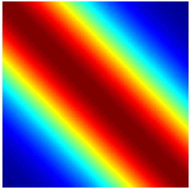
$\Sigma =$ 

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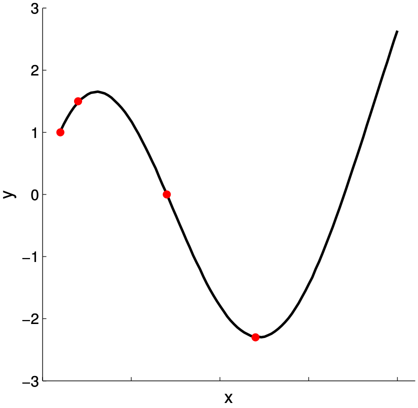

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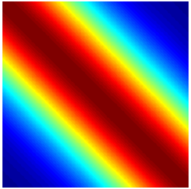
$\Sigma =$ 

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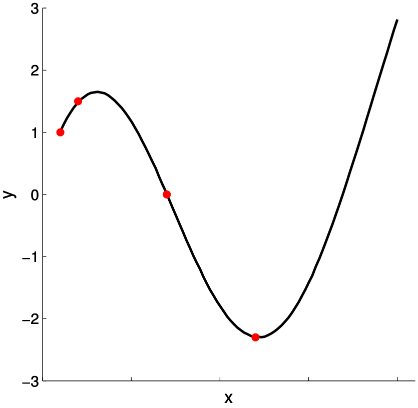

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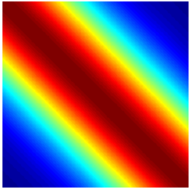
$\Sigma =$ 

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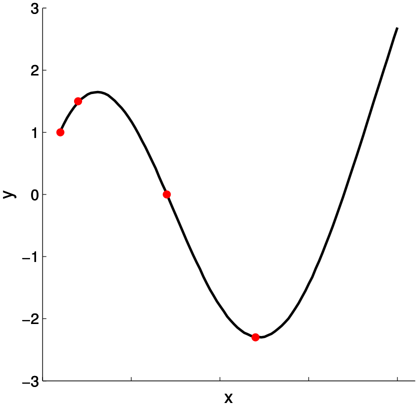

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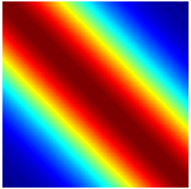
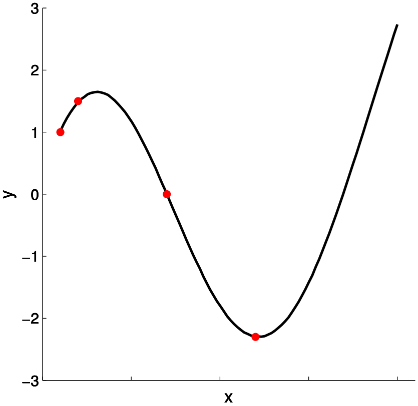
$\Sigma =$ 

long horizontal length-scale



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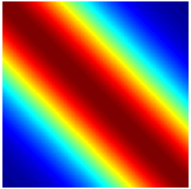
$\Sigma =$ 

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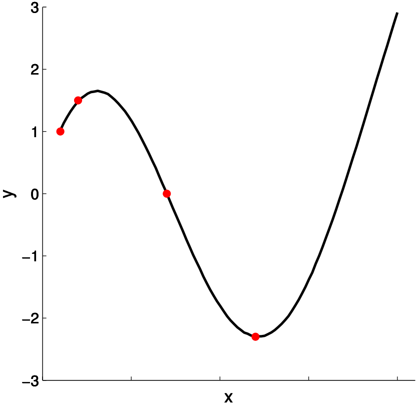

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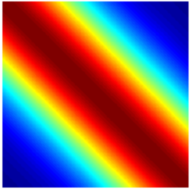
$\Sigma =$ 

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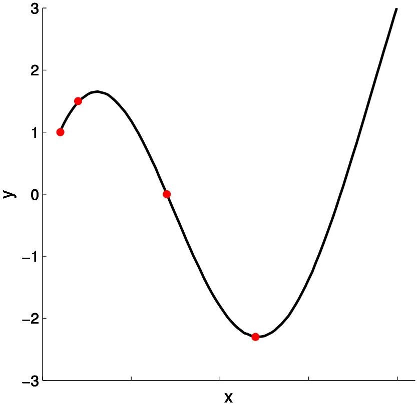

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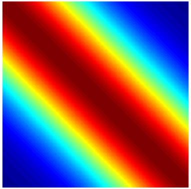
$\Sigma =$ 

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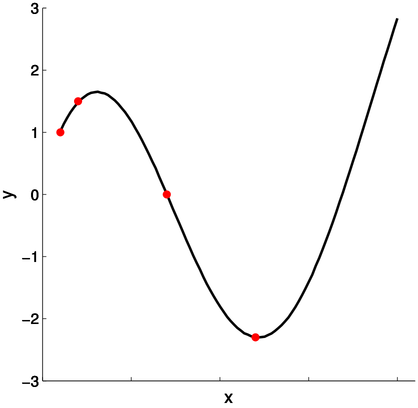

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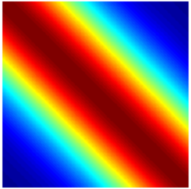
$\Sigma =$ 

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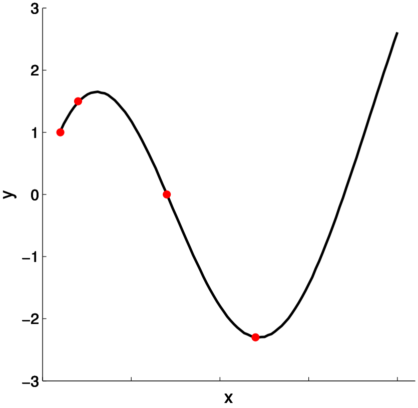

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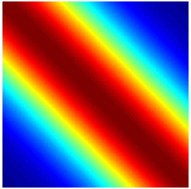
$\Sigma =$ 

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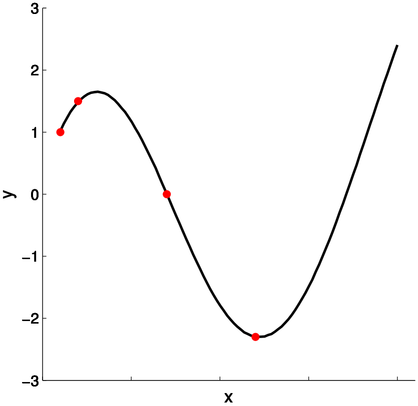

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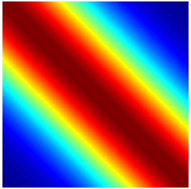
$\Sigma =$ 

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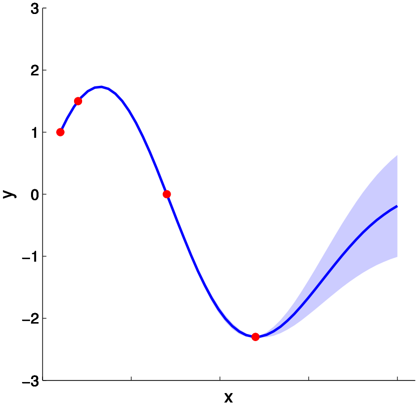

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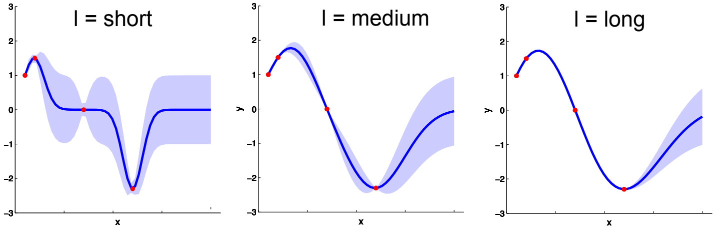
What effect do the hyper-parameters have?

$$K(x_1, x_2) = \sigma^2 \exp\left(-\frac{1}{2l^2}(x_1 - x_2)^2\right)$$

- Hyper-parameters have a strong effect
  - l controls the horizontal length-scale
  - σ^2 controls the vertical scale of the data
- $\Longrightarrow$ need automatic learning of hyper-parameters from data



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$p(\mathbf{y}_{1:N}|\theta)$ = likelihood of the parameters

= how well did θ predict the data we observed

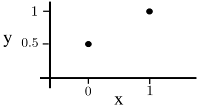
$$p(\mathbf{y}_{1:N}|\theta) = \frac{1}{\det(2\pi\Sigma(\theta))^{-1/2}} \exp\left(-\frac{1}{2}\mathbf{y}_{1:N}^{\top}\Sigma^{-1}(\theta)\mathbf{y}_{1:N}\right)$$

How do we choose the hyper-parameters?

data

$$\mathrm{y}_{1:N} = [1, 0.5]$$

two datapoints



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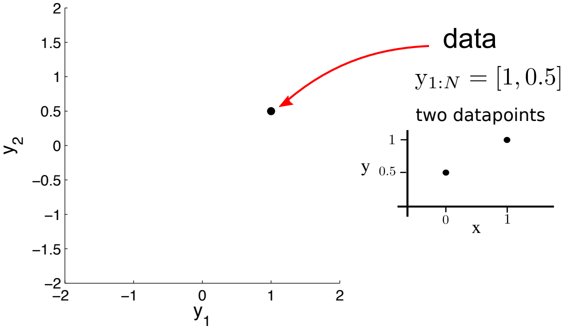

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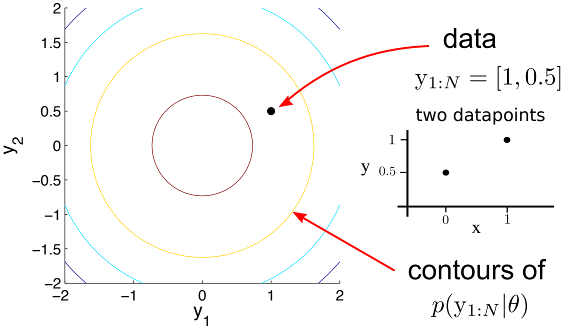

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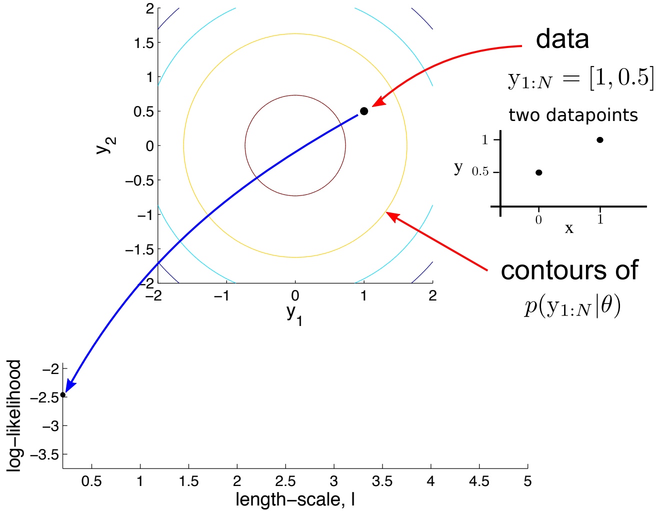

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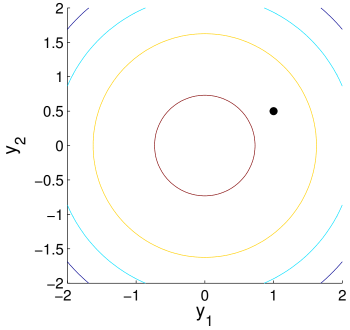

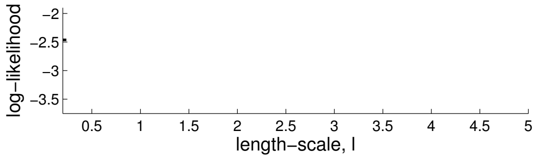

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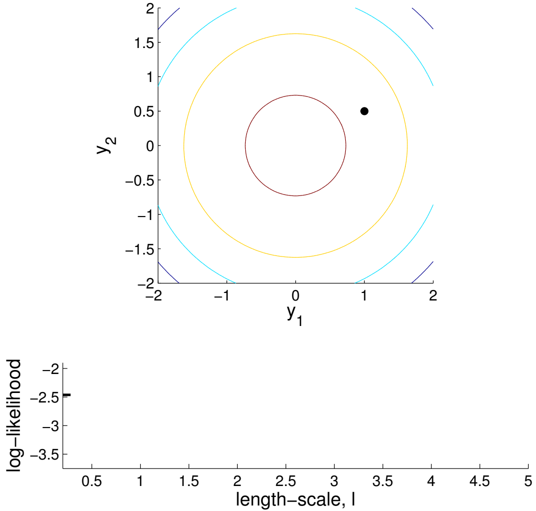

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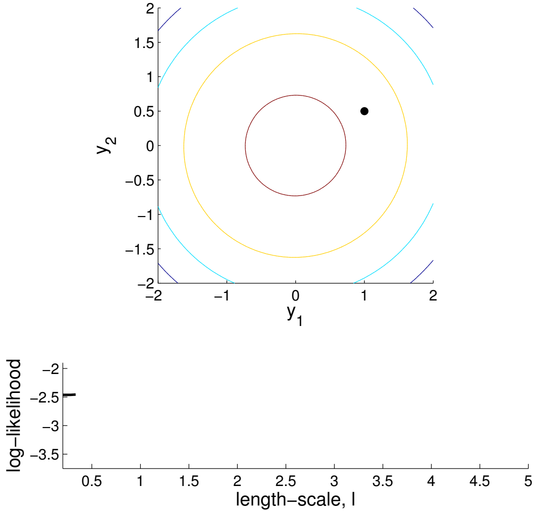

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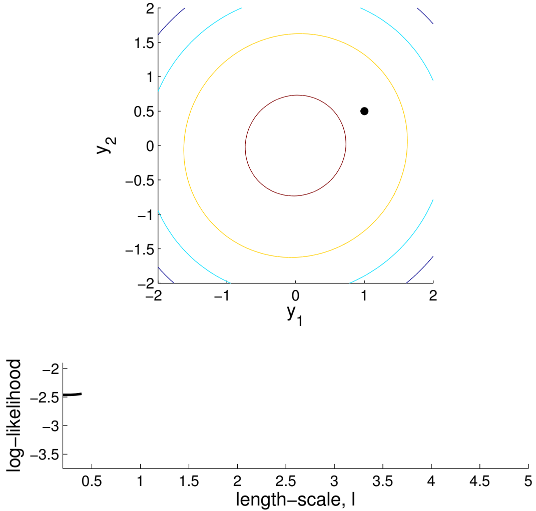

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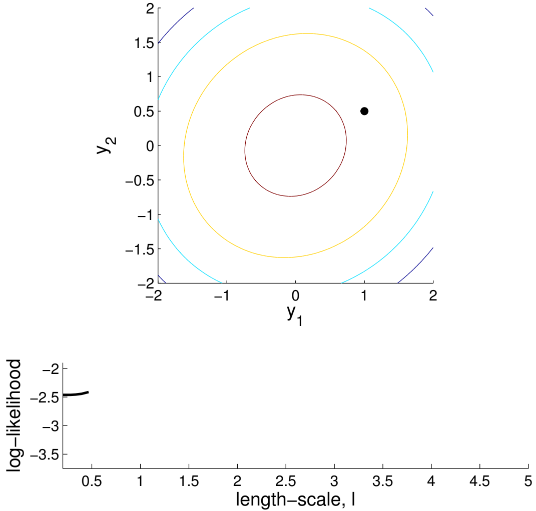

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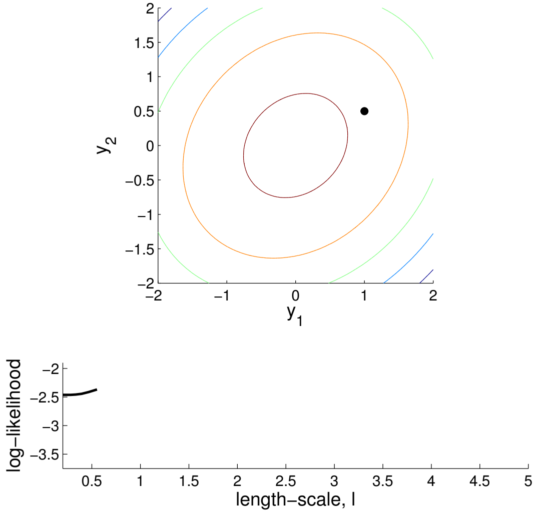

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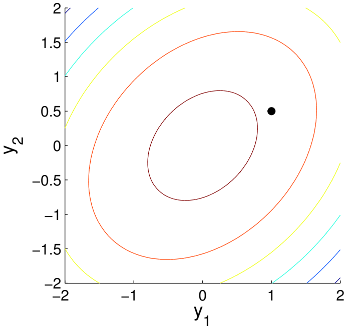

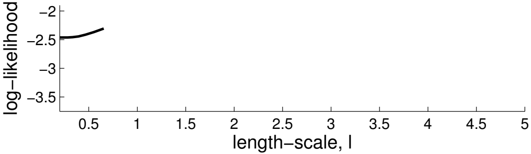

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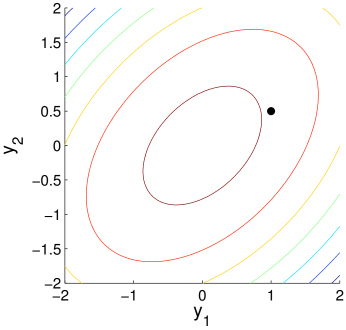

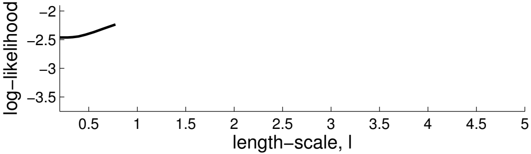

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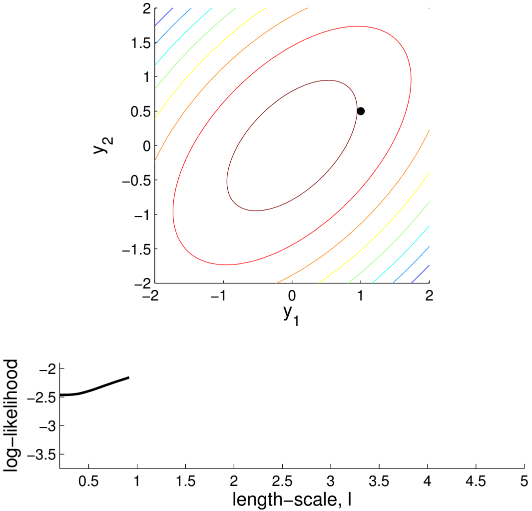

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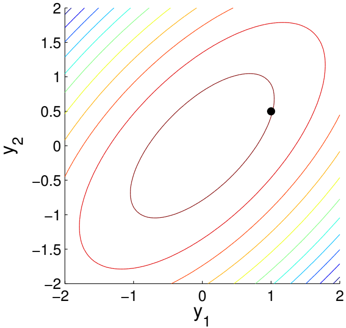

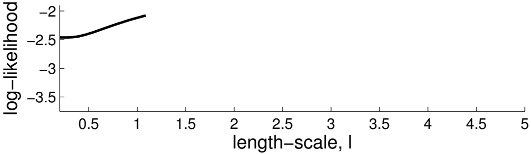

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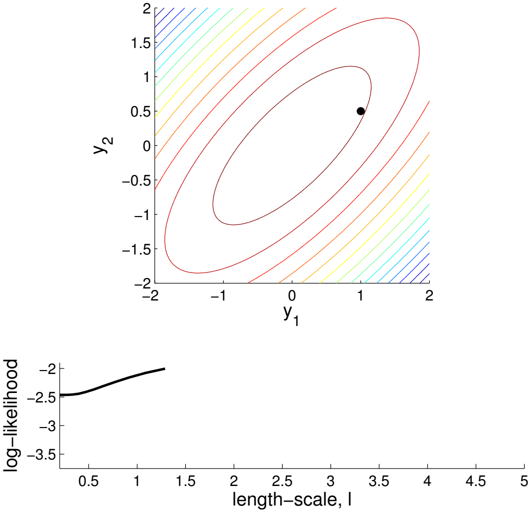

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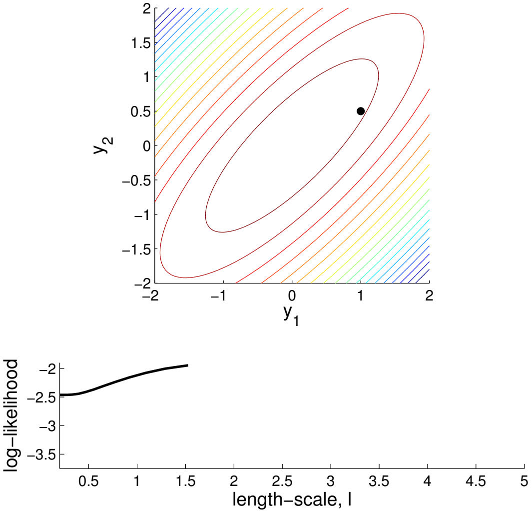

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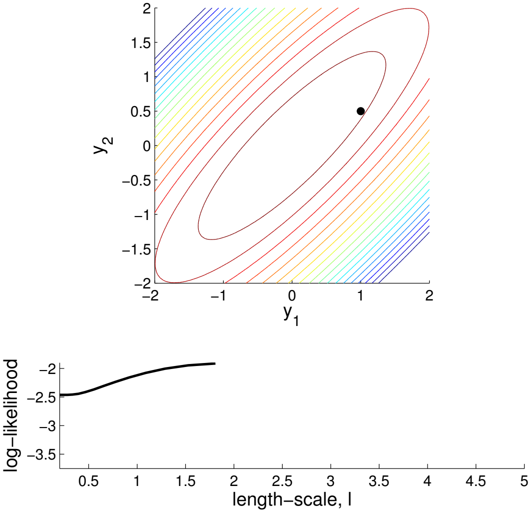

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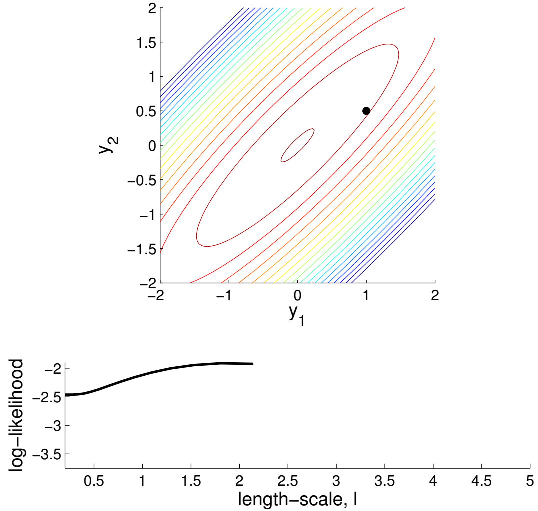

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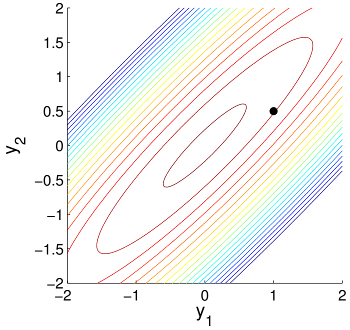

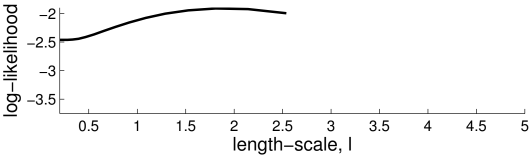

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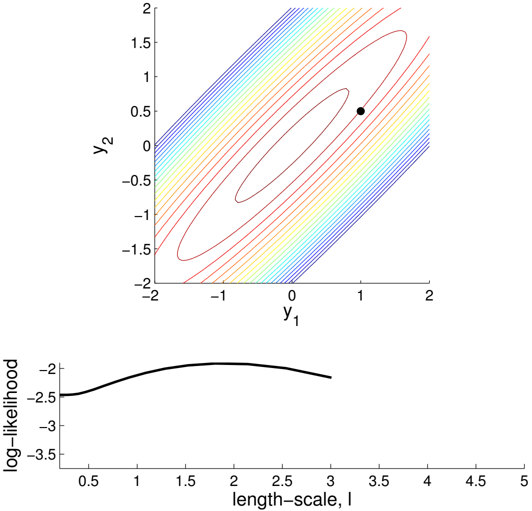

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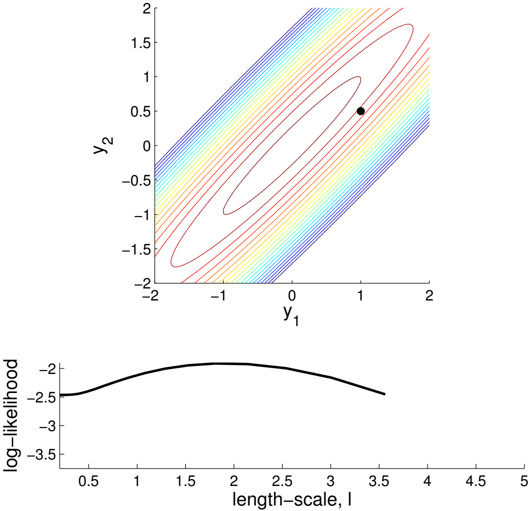

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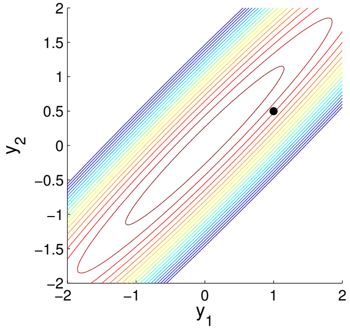

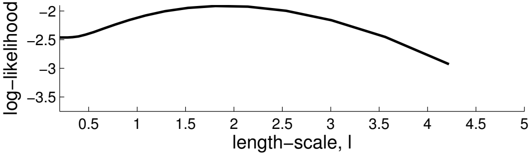

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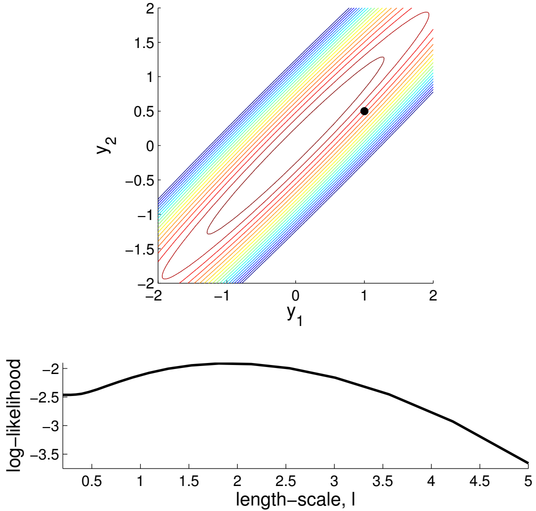

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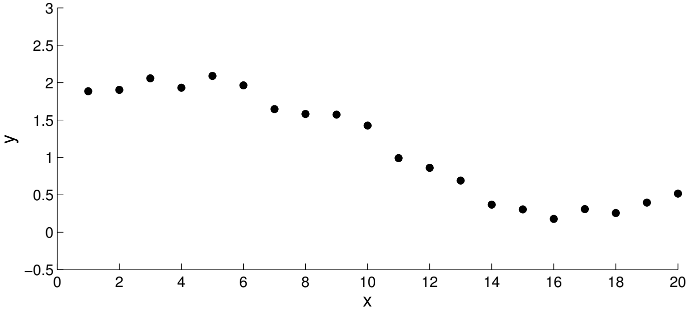

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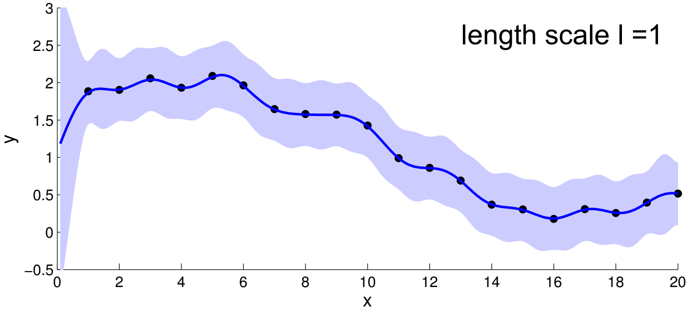

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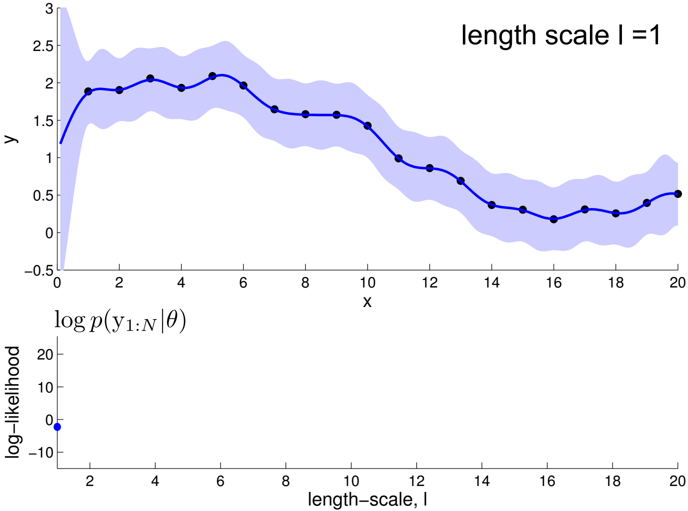

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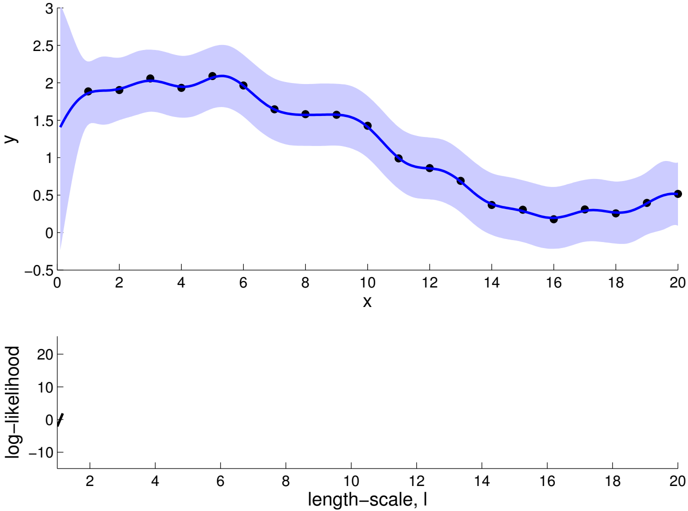

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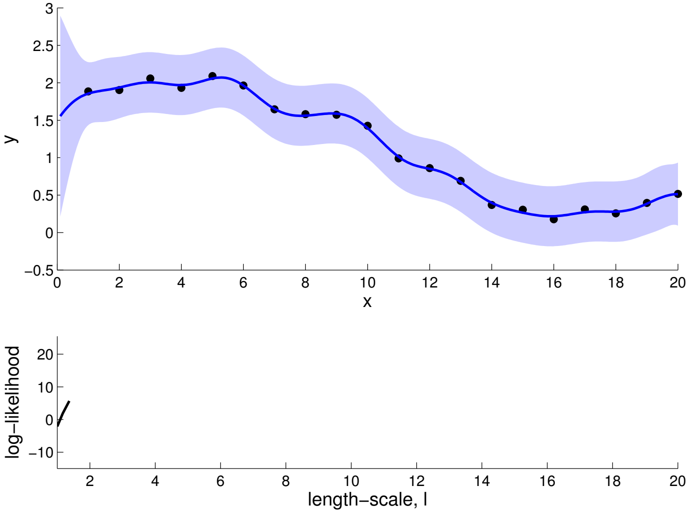

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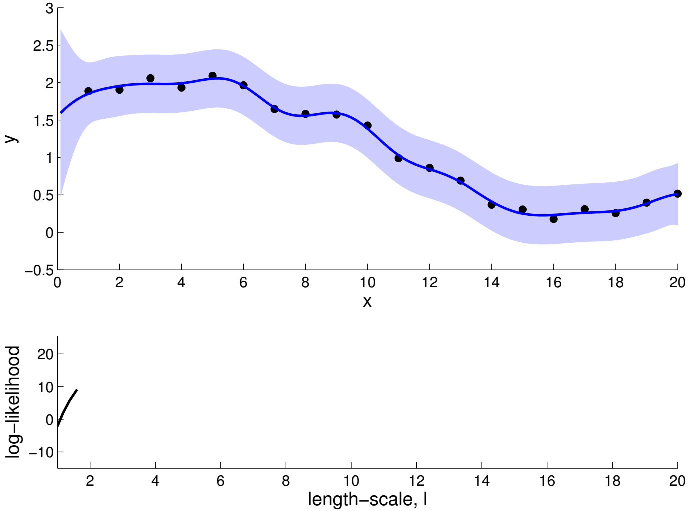

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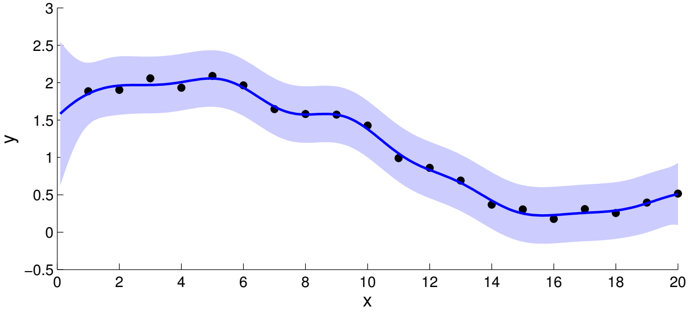

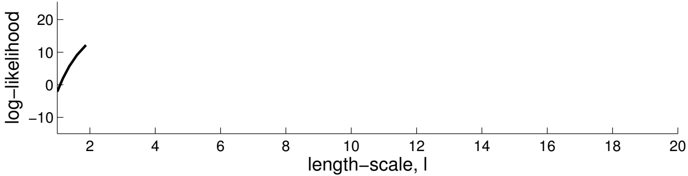

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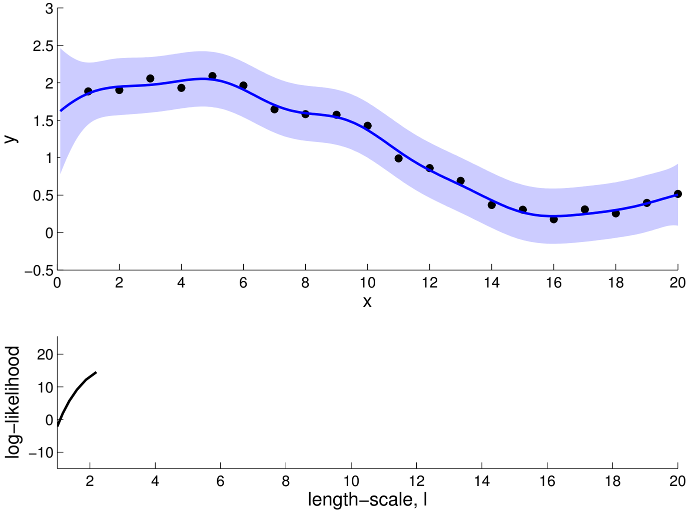

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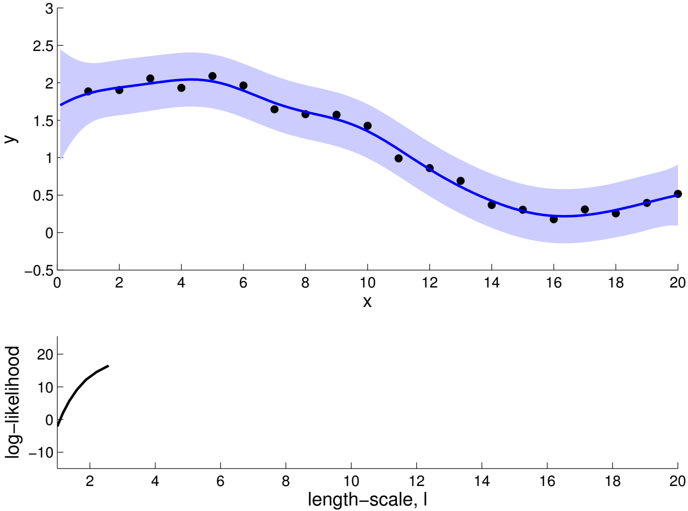

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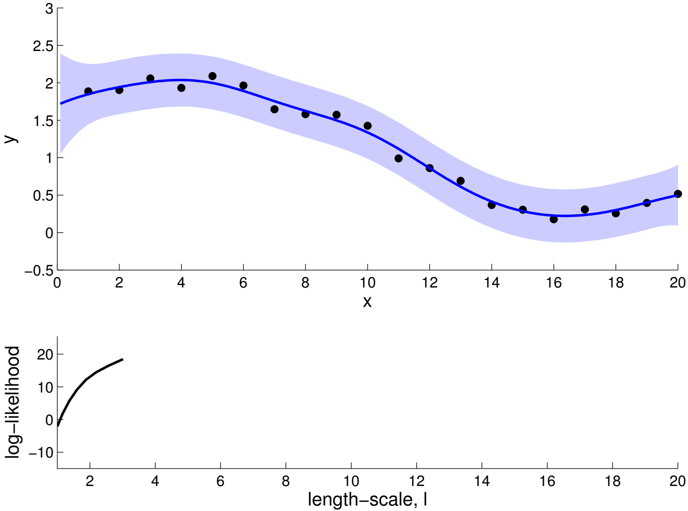

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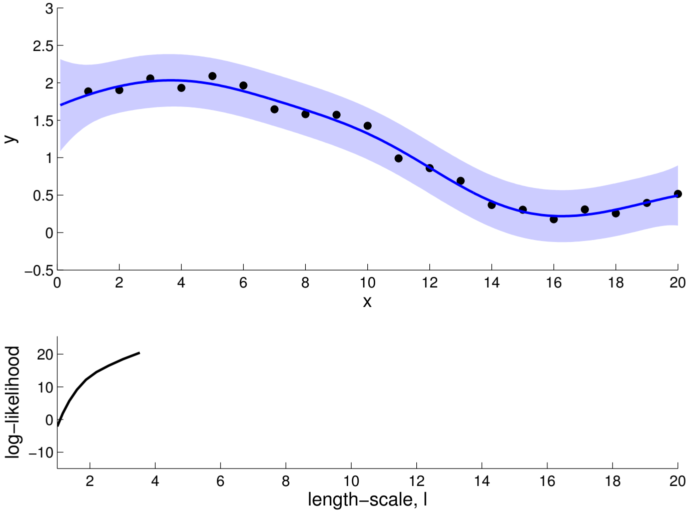

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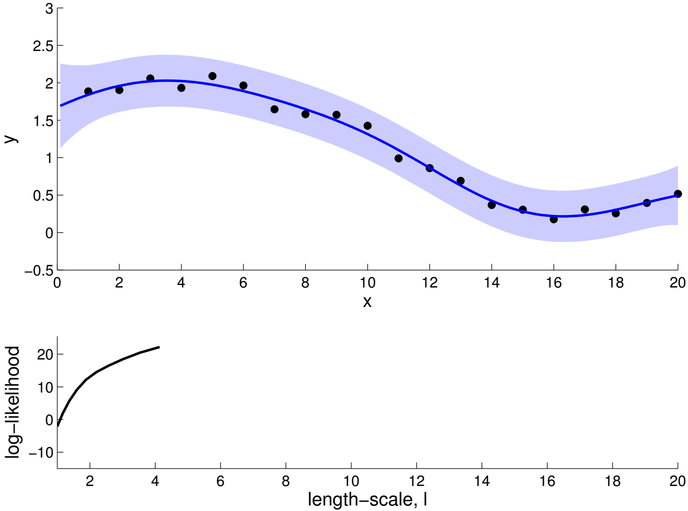

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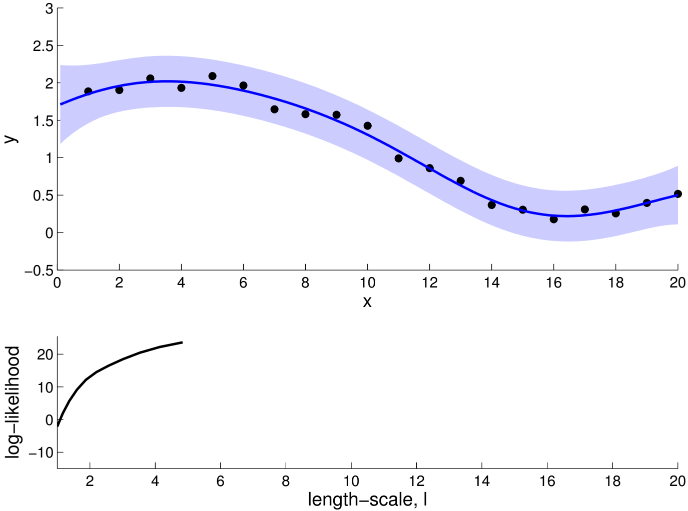

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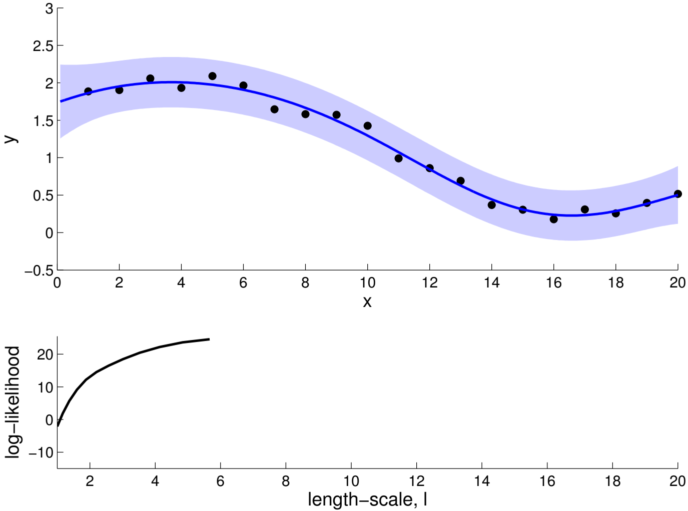

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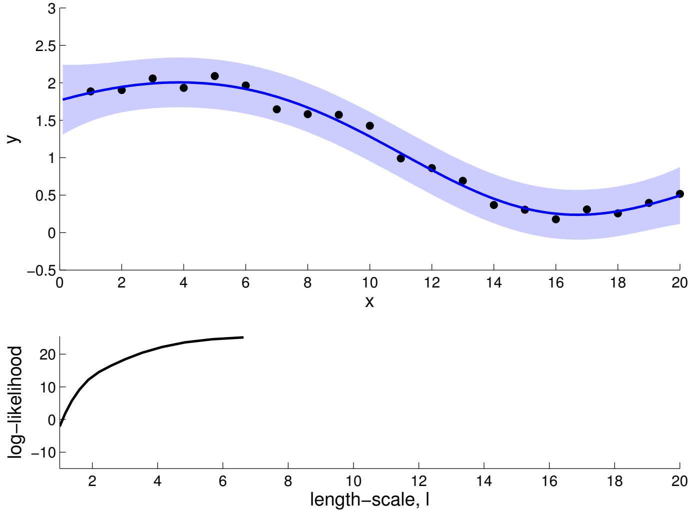

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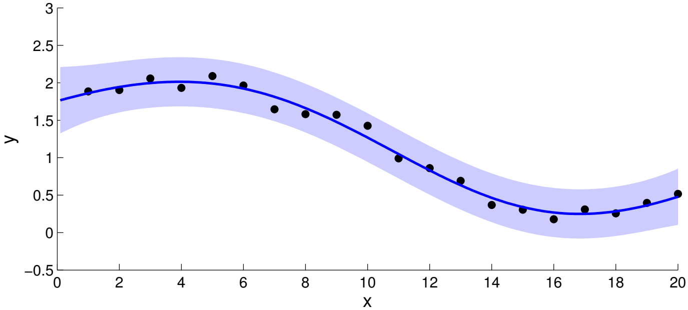

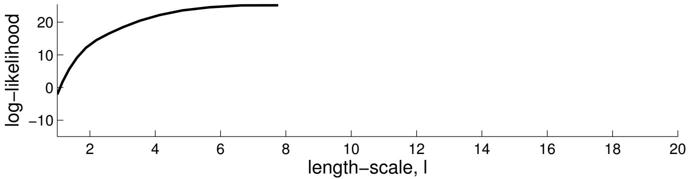

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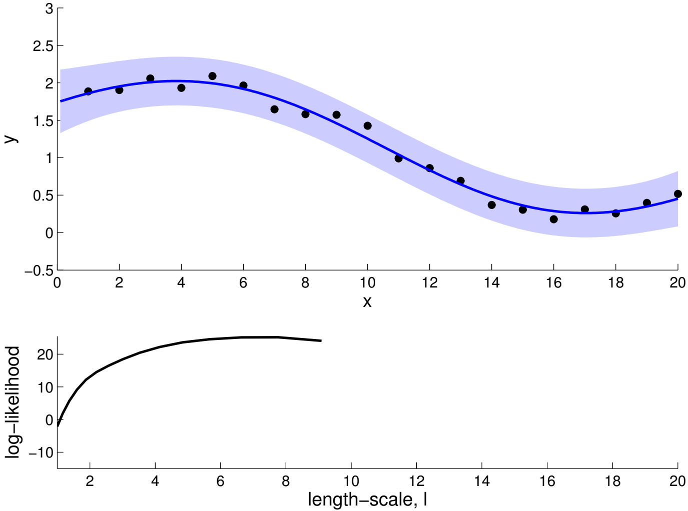

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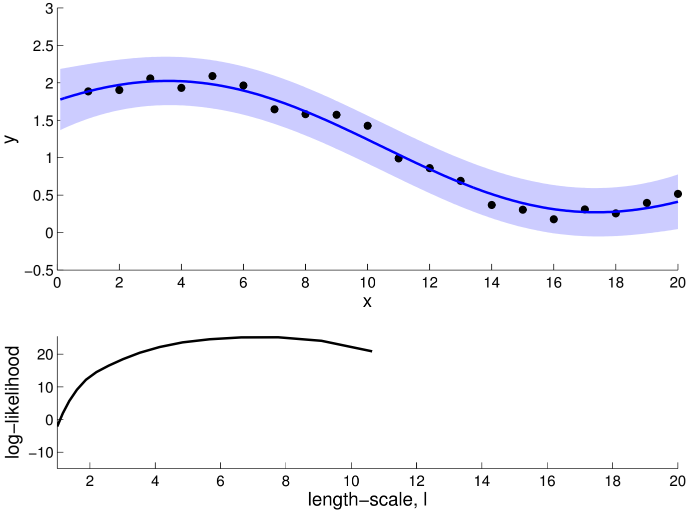

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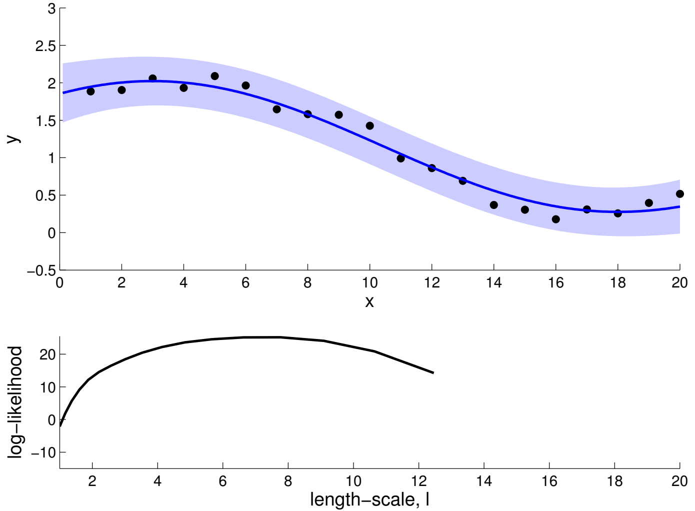

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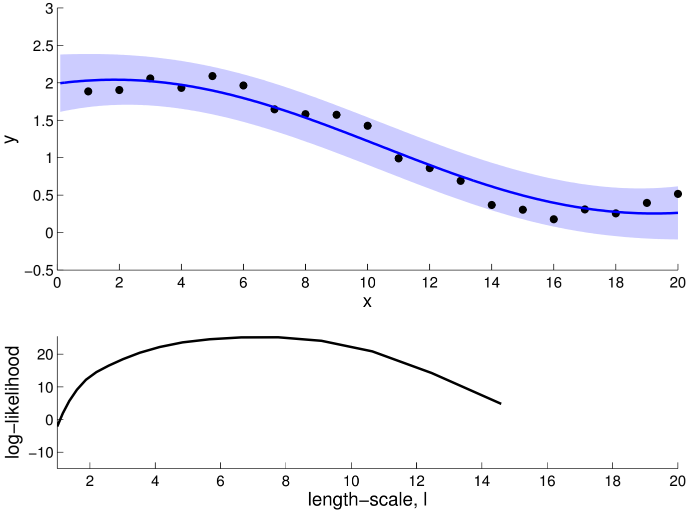

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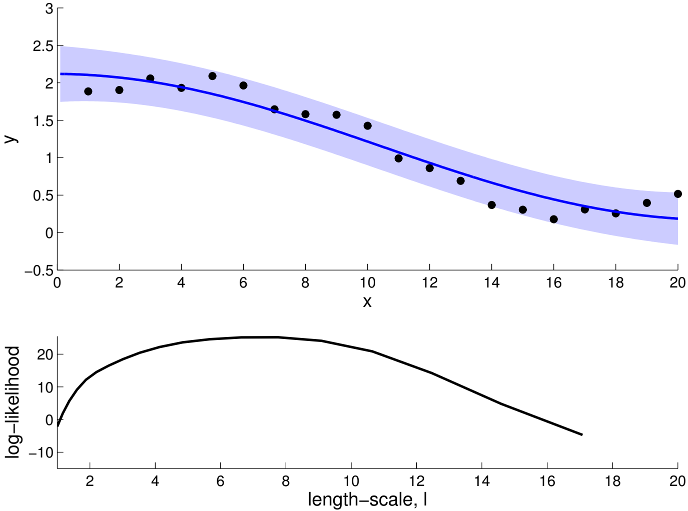

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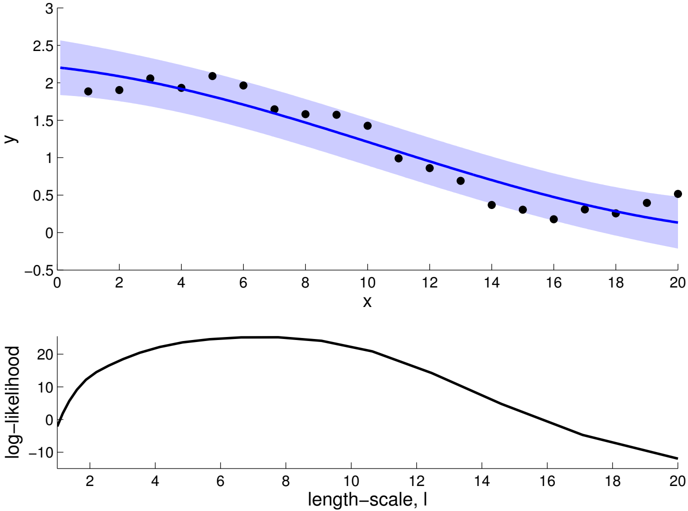

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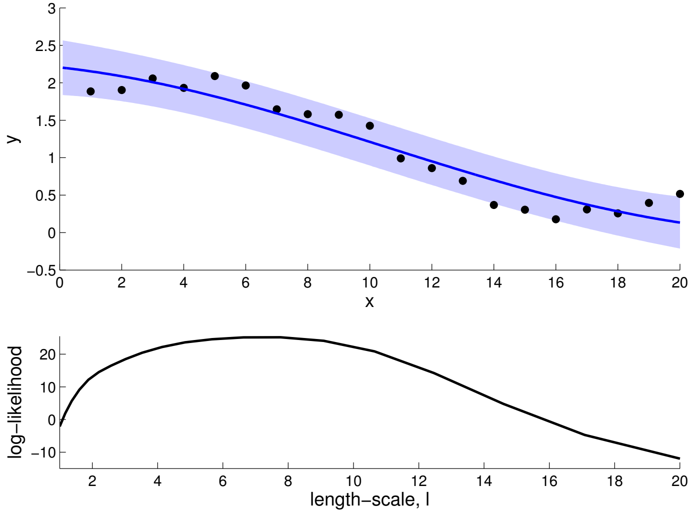

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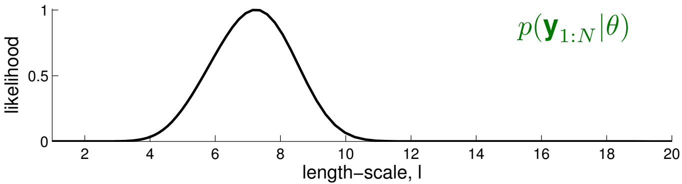

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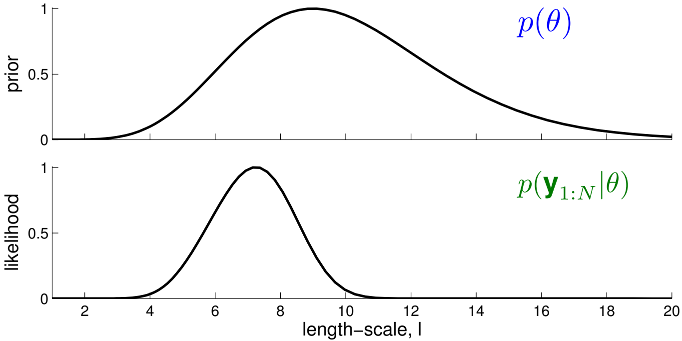

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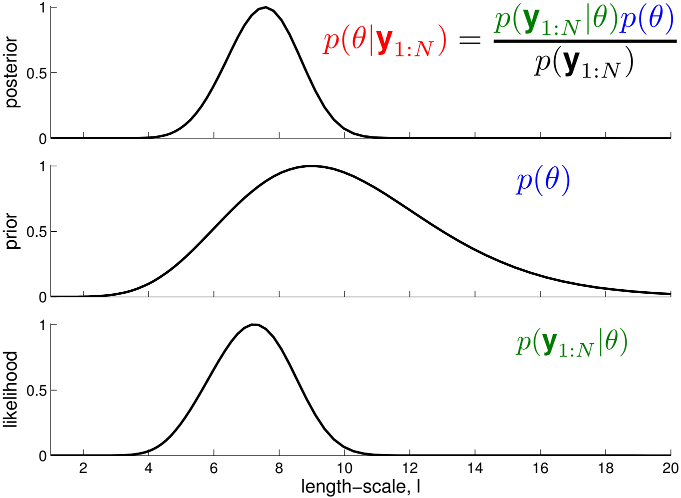
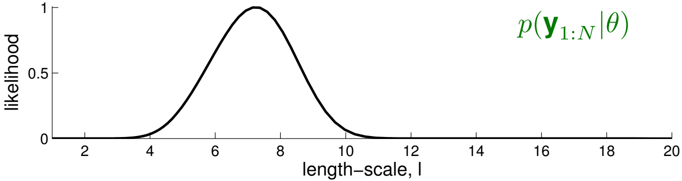


Why does Bayesian inference work?



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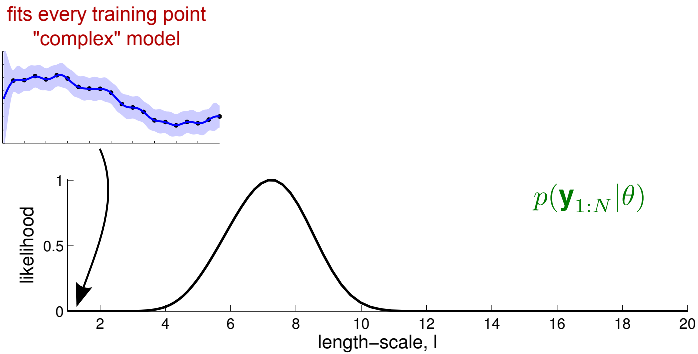

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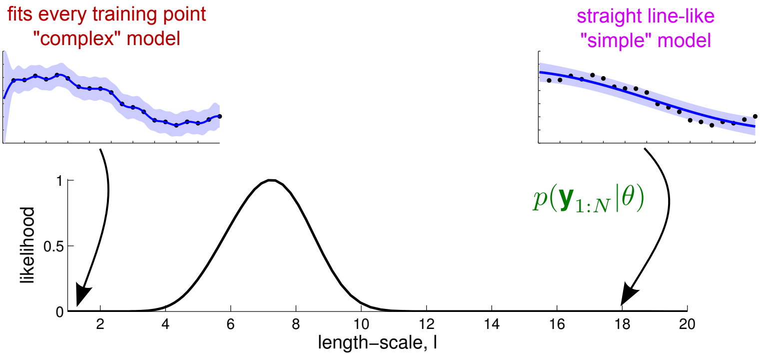

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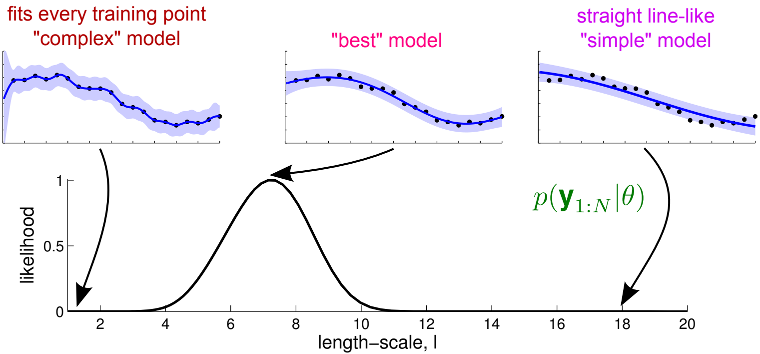

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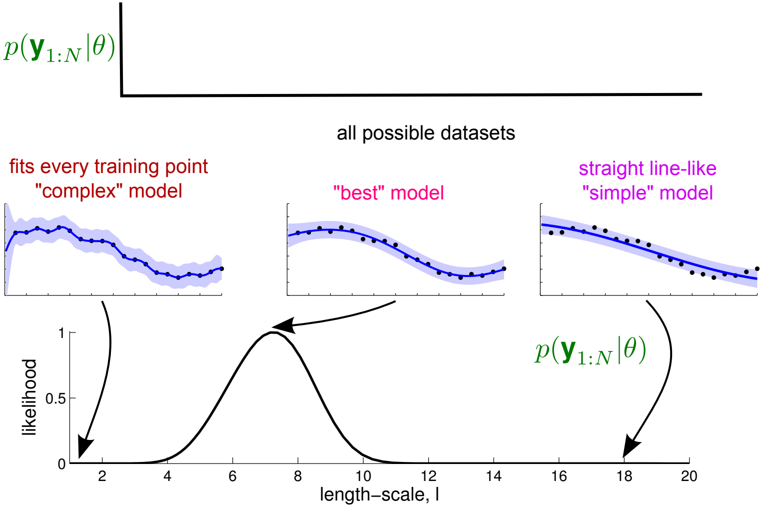

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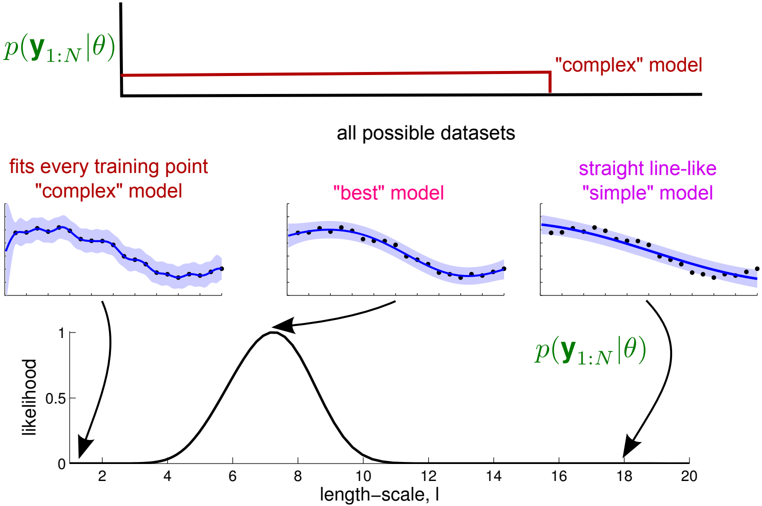

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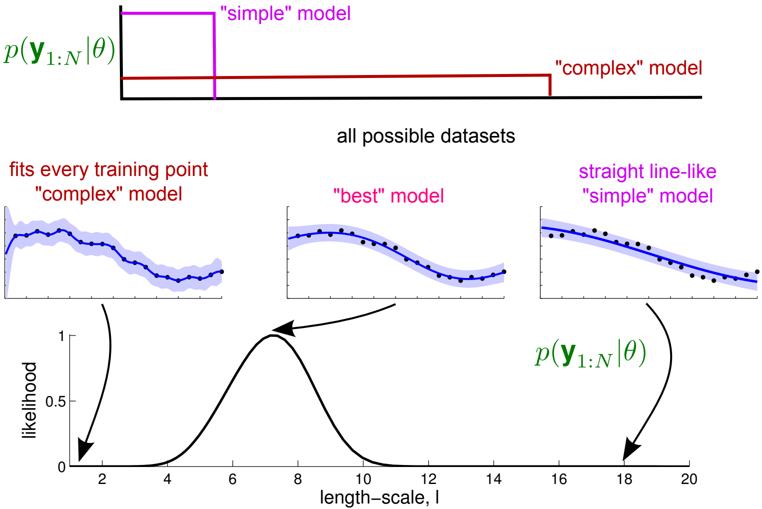

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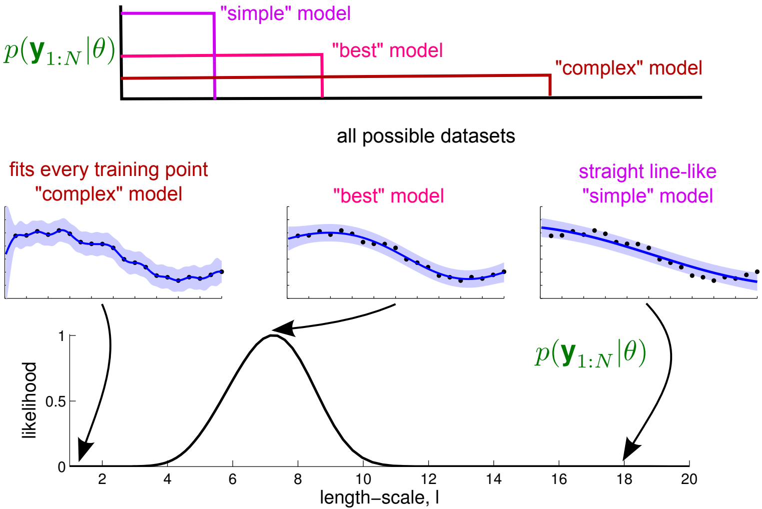
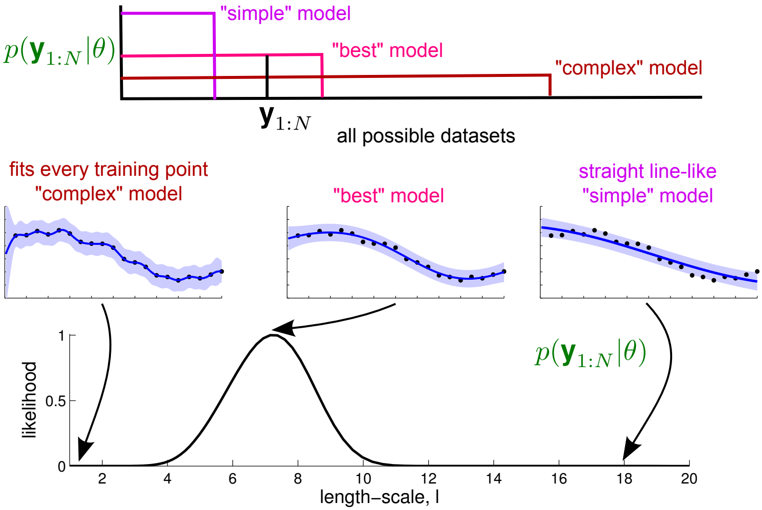
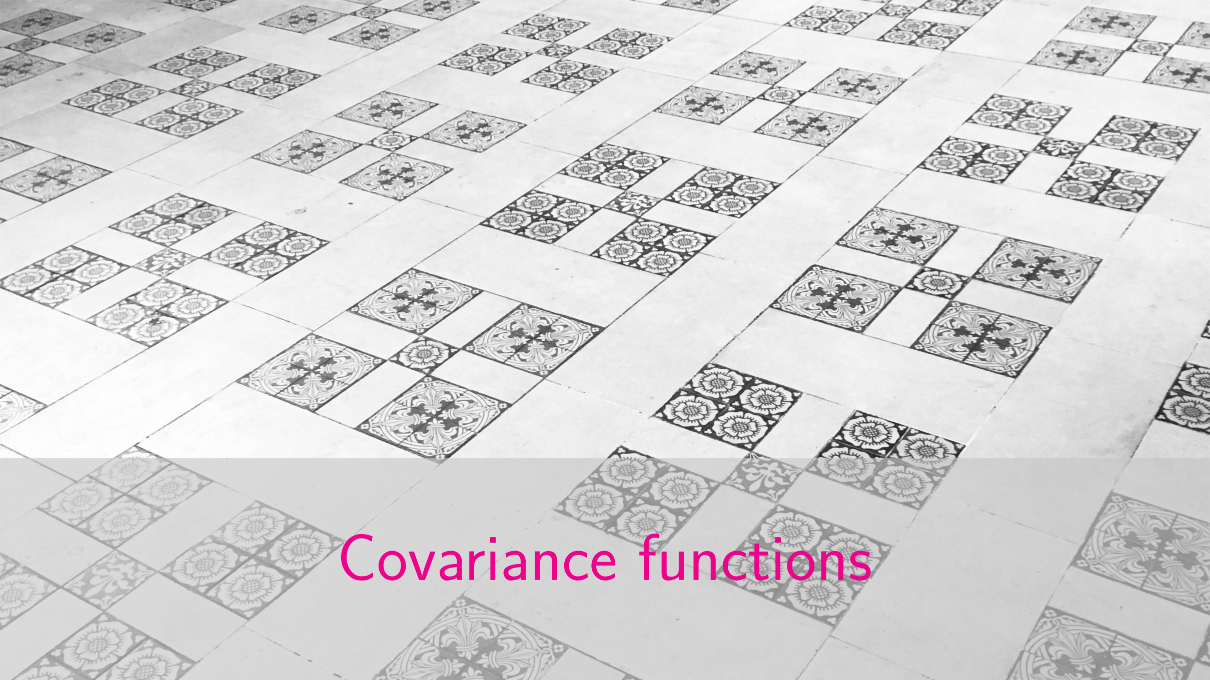


Why does Bayesian inference work? Occam's Razor.





Covariance functions

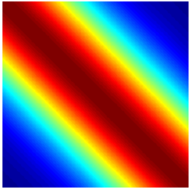
What effect does the form of the covariance function have?

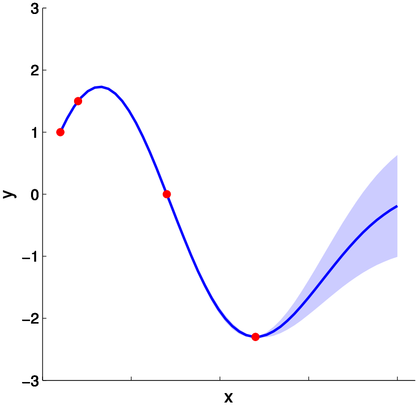
$$K(x_1, x_2) = \sigma^2 \exp\left(-\frac{1}{2l^2}(x_1 - x_2)^2\right)$$

Squared exponential

Exponentiated Quadratic

RBF covariance function

$\Sigma =$ 



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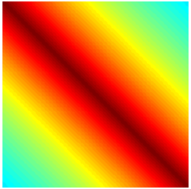
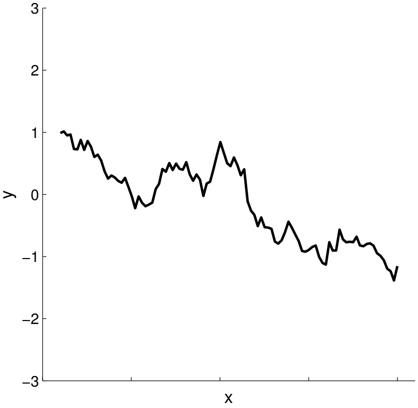
Laplacian covariance function

Browninan motion

Ornstein-Uhlenbeck

$\Sigma =$ 



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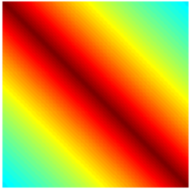
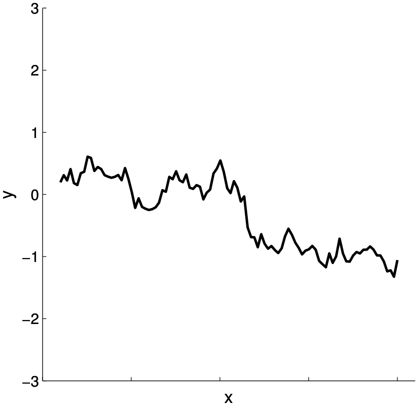
$\Sigma =$ 



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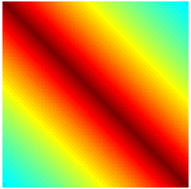
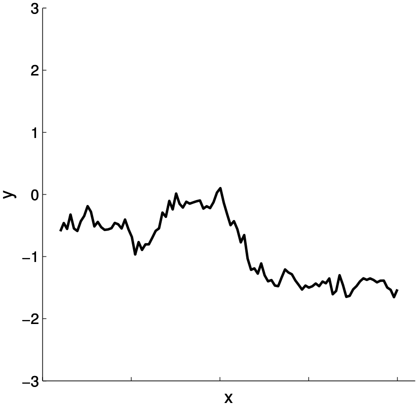
$\Sigma =$ 



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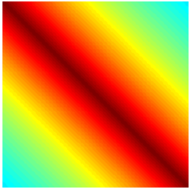

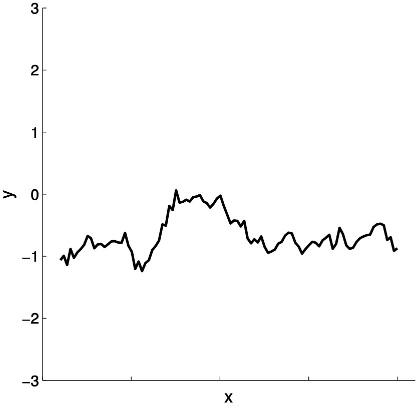

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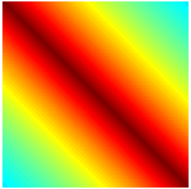

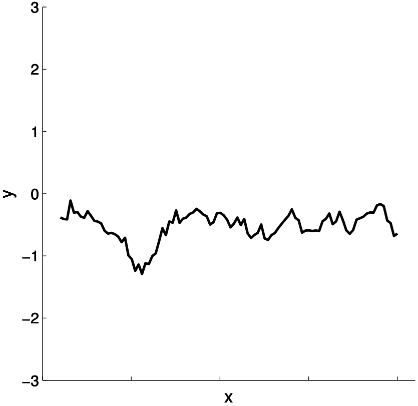

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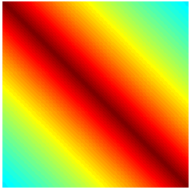

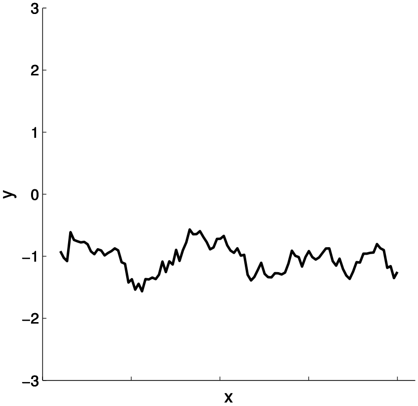

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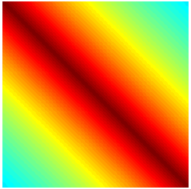
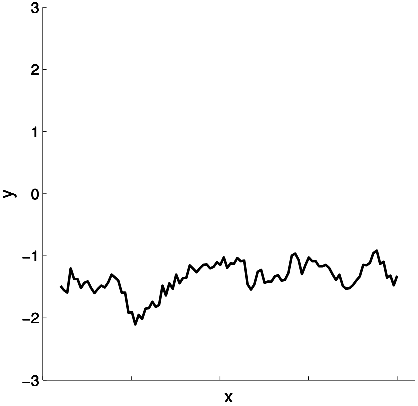
$\Sigma =$ 



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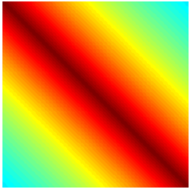

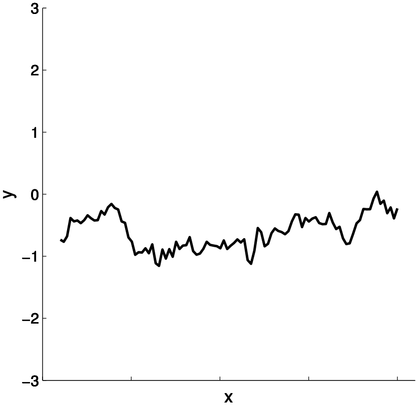

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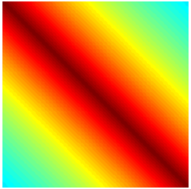

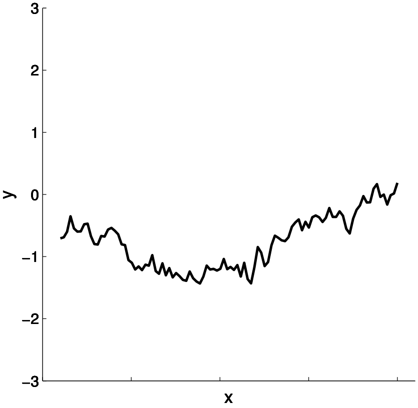

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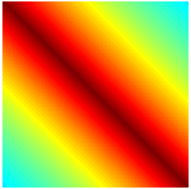
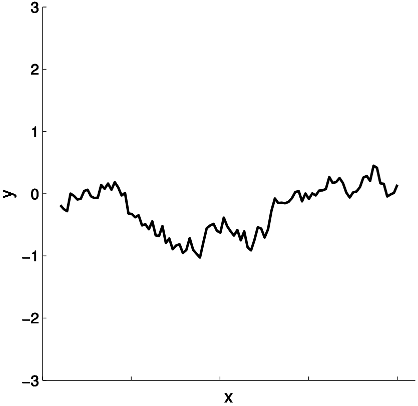
$\Sigma =$ 



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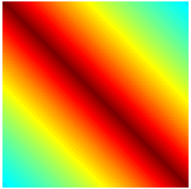
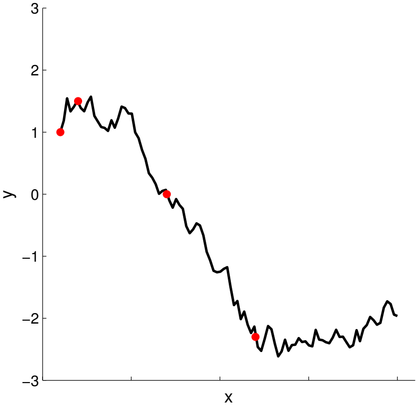
$\Sigma =$ 



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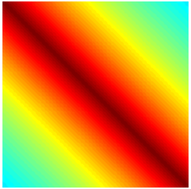

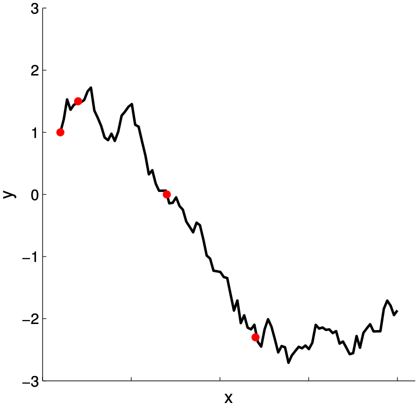

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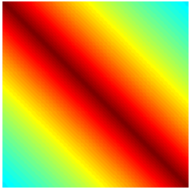

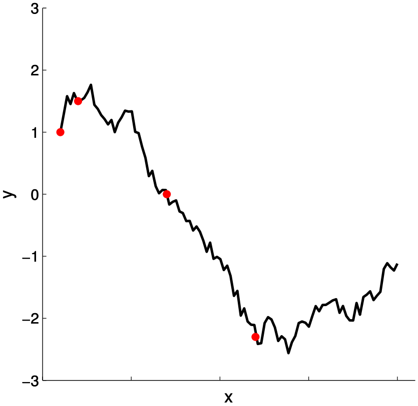

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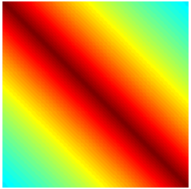
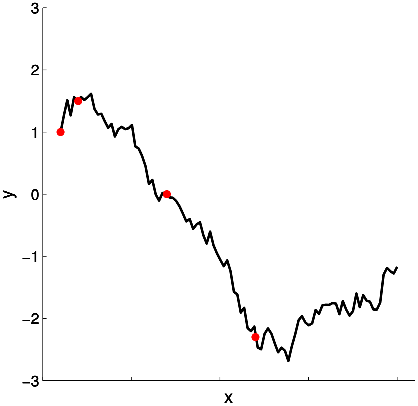
$\Sigma =$ 



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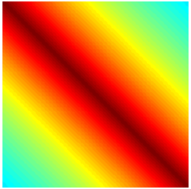
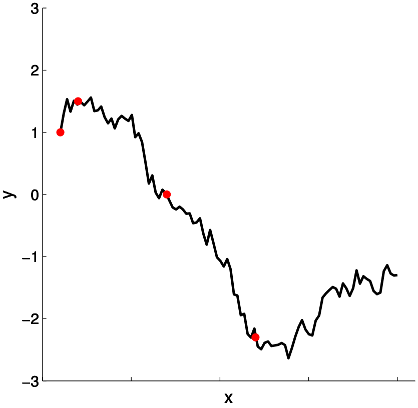
$\Sigma =$ 



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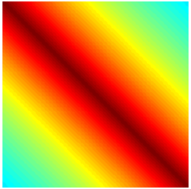
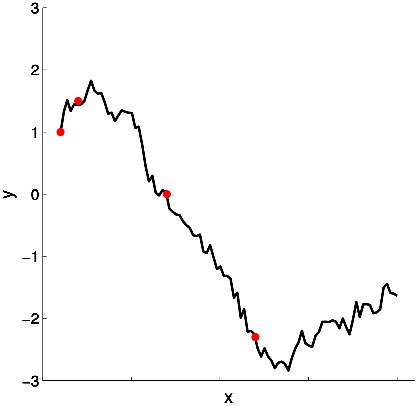
$\Sigma =$ 



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Laplacian covariance function

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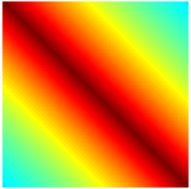
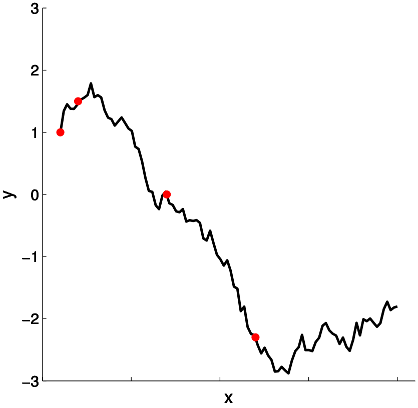
$\Sigma =$ 



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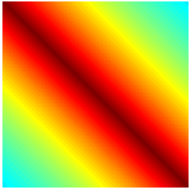
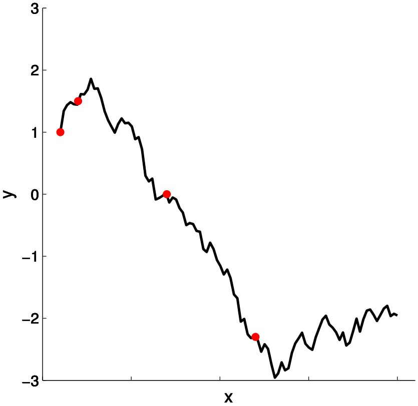
$\Sigma =$ 



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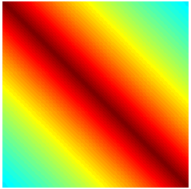
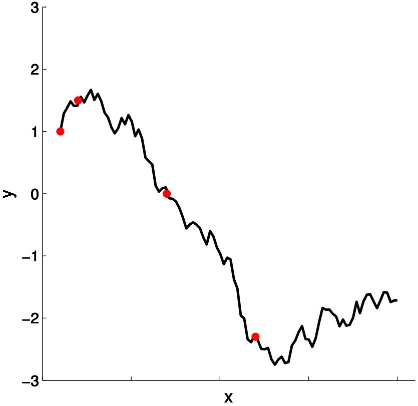
$\Sigma =$ 



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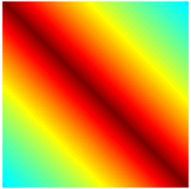
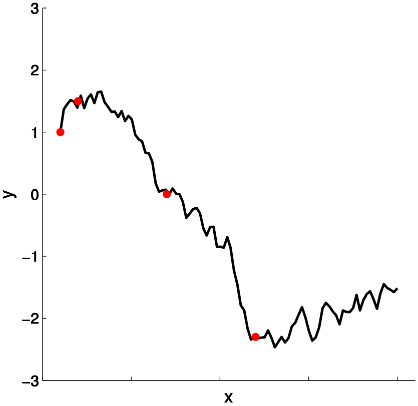
$\Sigma =$ 



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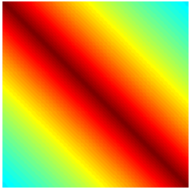
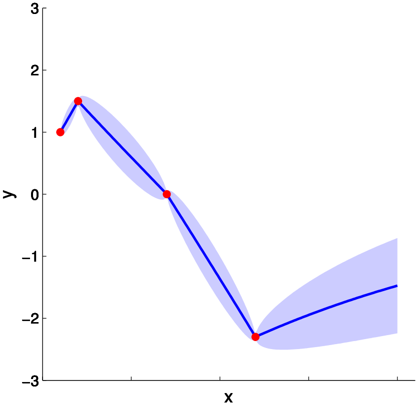
$\Sigma =$ 



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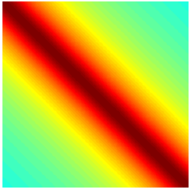
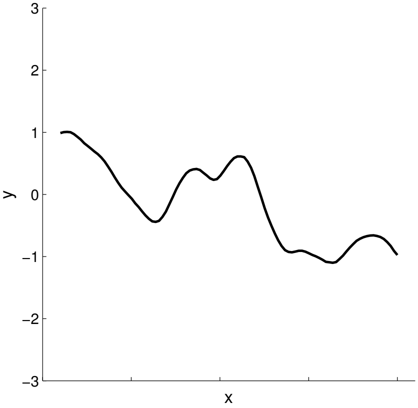
$\Sigma =$ 



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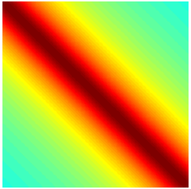
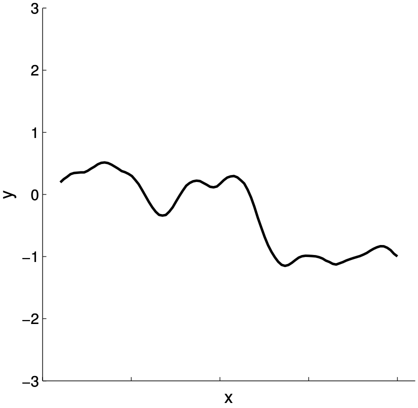
$\Sigma =$ 



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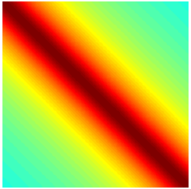
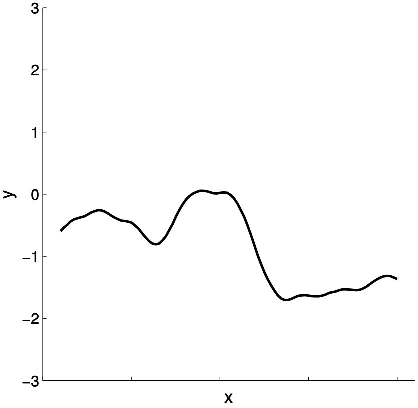
$\Sigma =$ 



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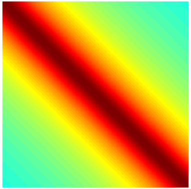

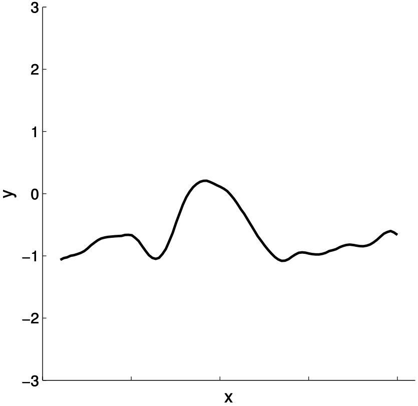

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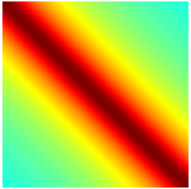

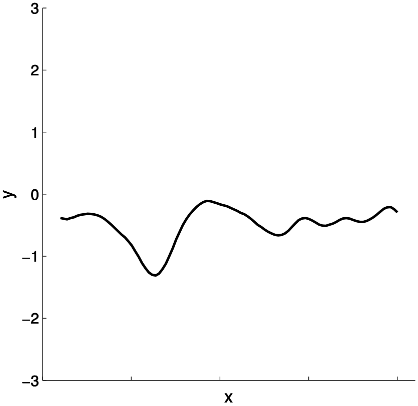

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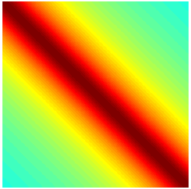
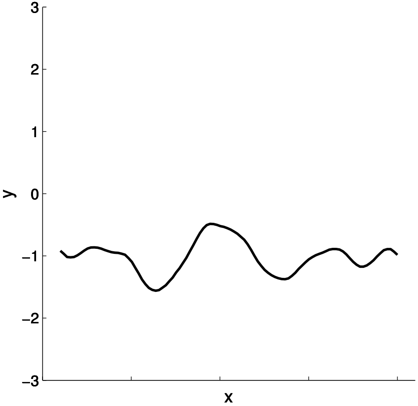
$\Sigma =$ 



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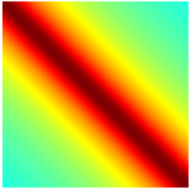

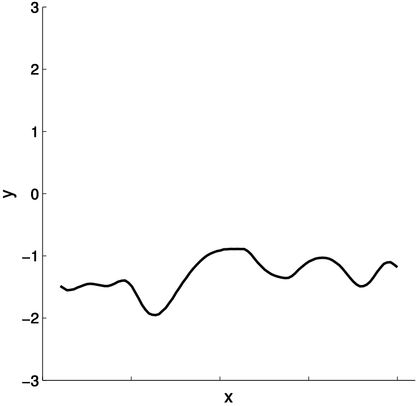

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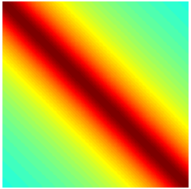

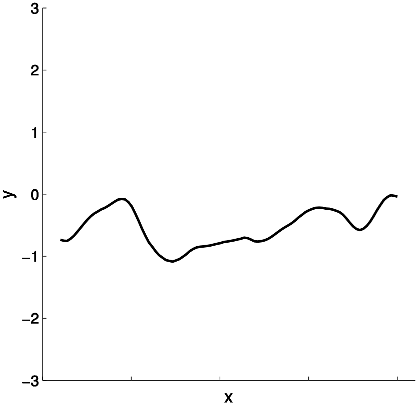

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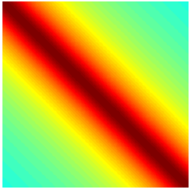
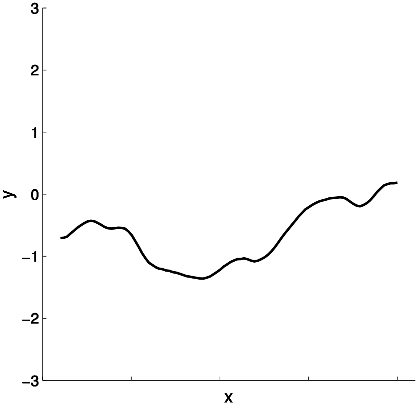
$\Sigma =$ 



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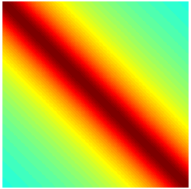

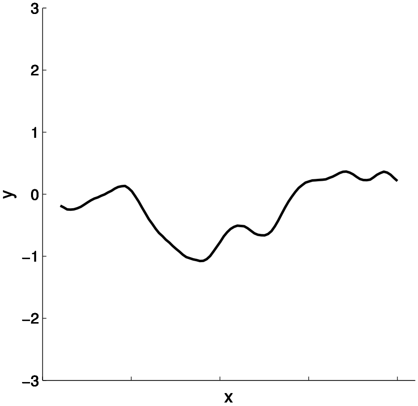

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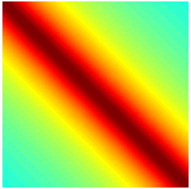
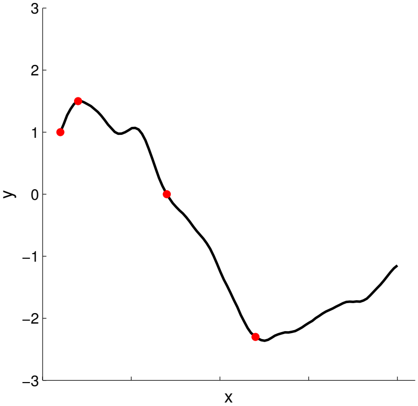
$\Sigma =$ 



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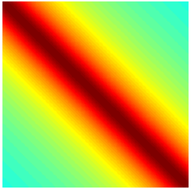

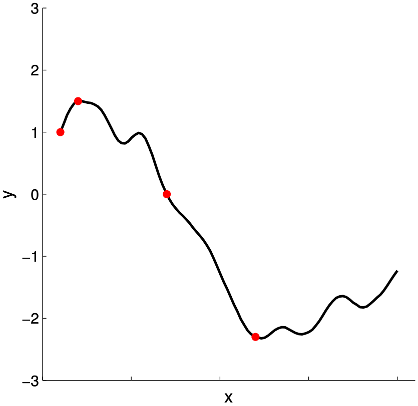

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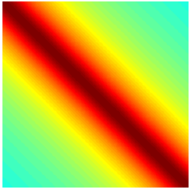

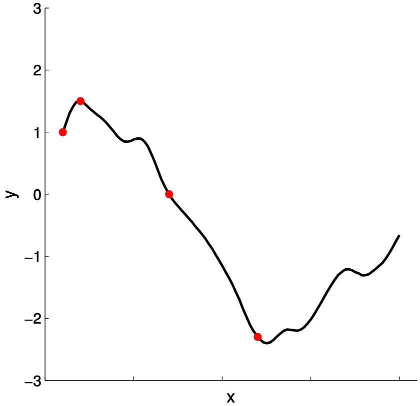

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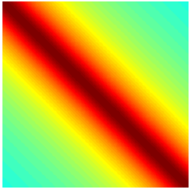
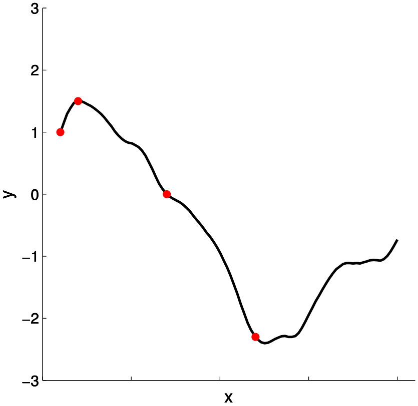
$\Sigma =$ 



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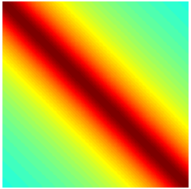
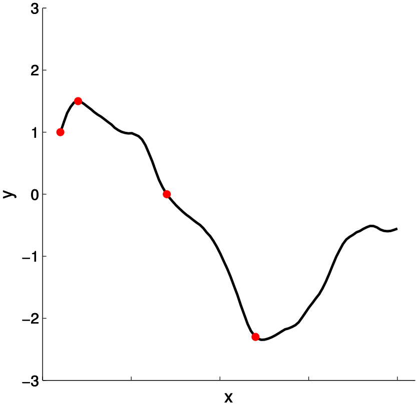
$\Sigma =$ 



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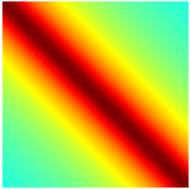
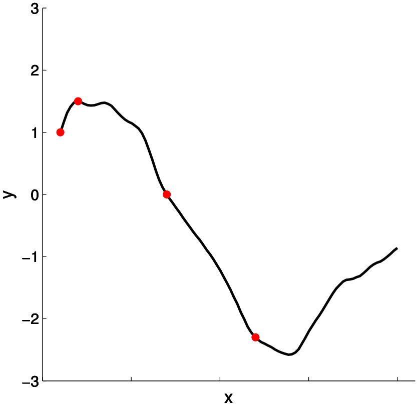
$\Sigma =$ 



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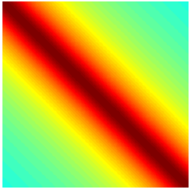
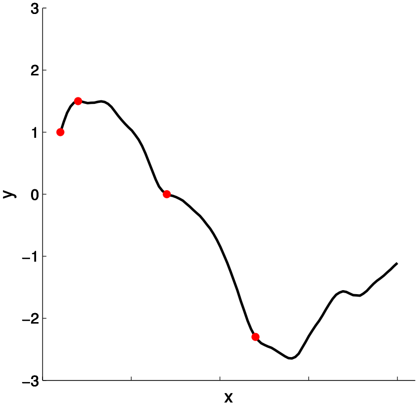
$\Sigma =$ 



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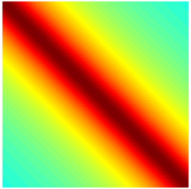
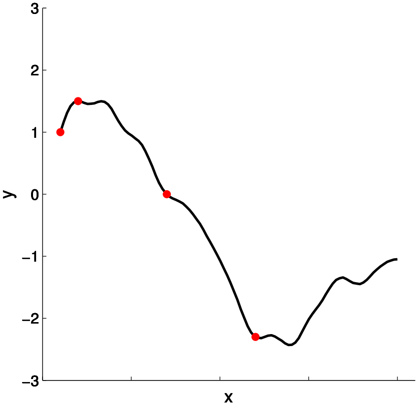
$\Sigma =$ 



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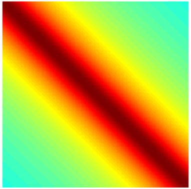
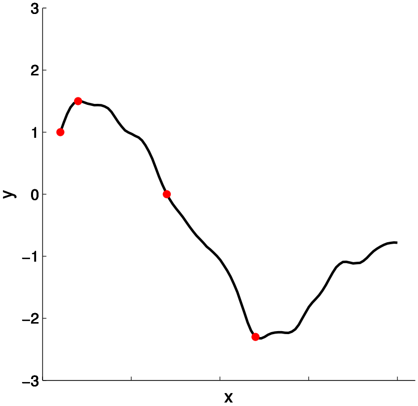
$\Sigma =$ 



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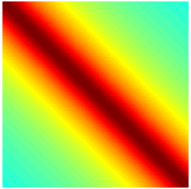

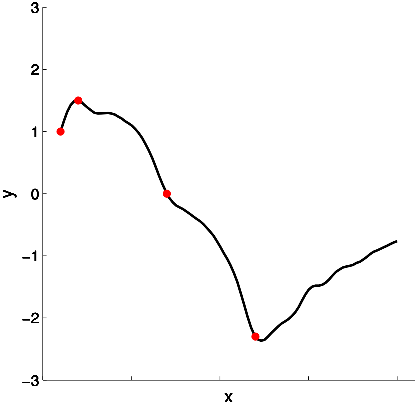

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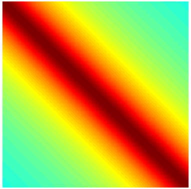
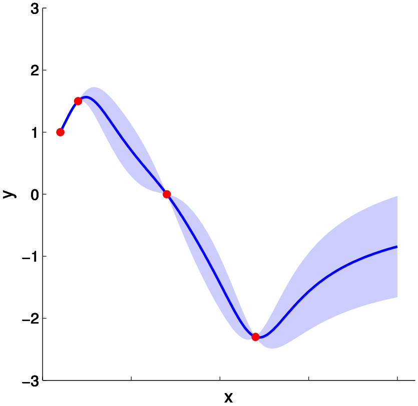
$\Sigma =$ 



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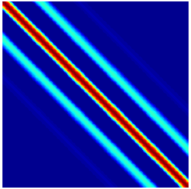
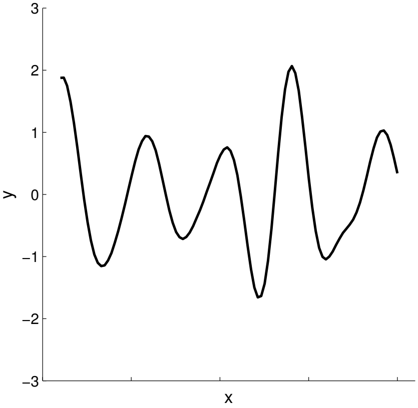
Periodic

sinusoid $\times$ squared exponential

$\Sigma =$ 



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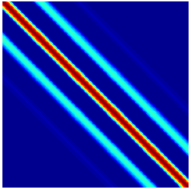

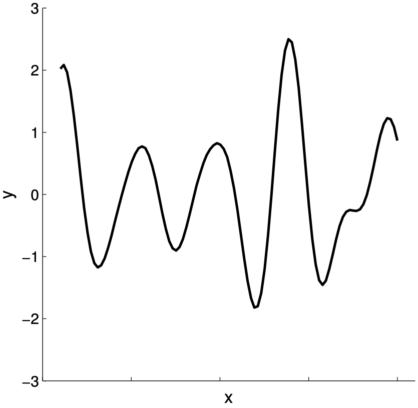

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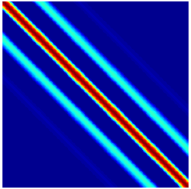
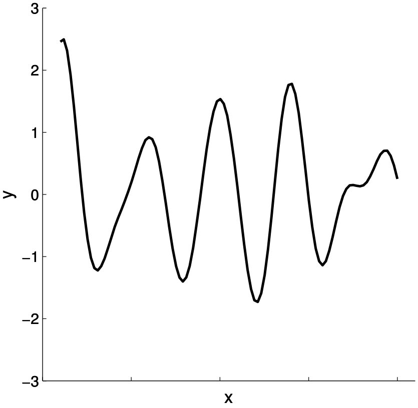
$\Sigma =$ 



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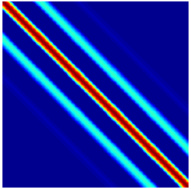
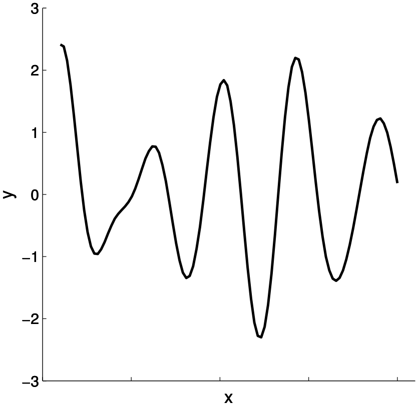
$\Sigma =$ 



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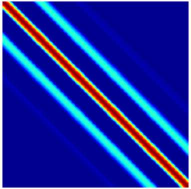

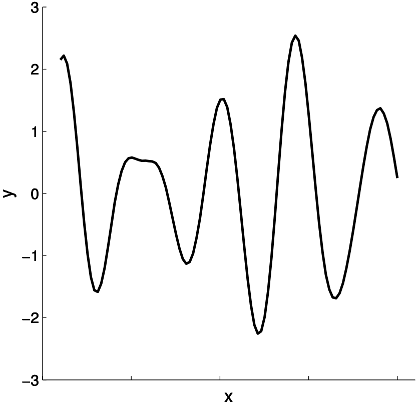

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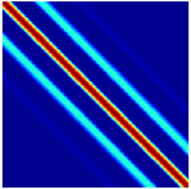
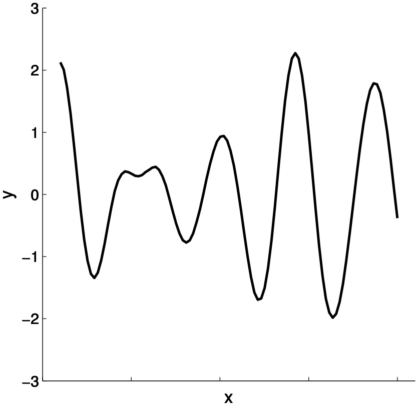
$\Sigma =$ 



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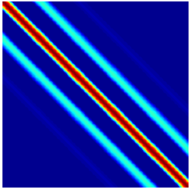
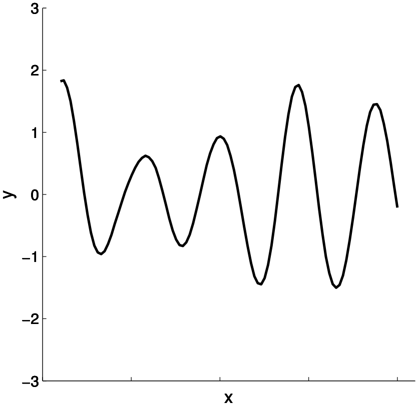
$\Sigma =$ 



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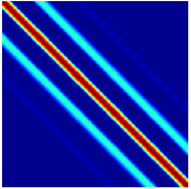
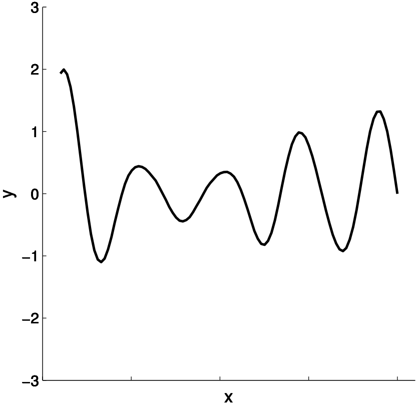
$\Sigma =$ 



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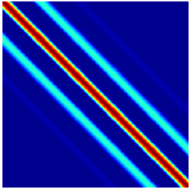
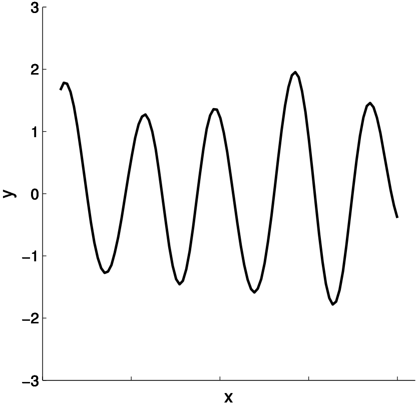
$\Sigma =$ 



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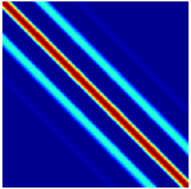
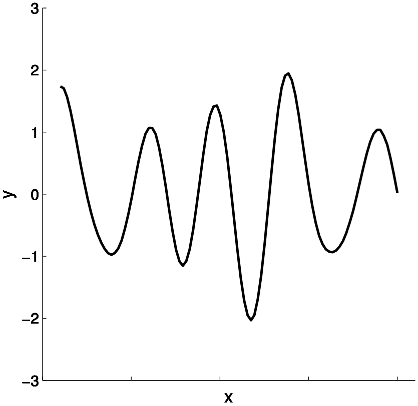
$\Sigma =$ 



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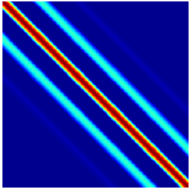
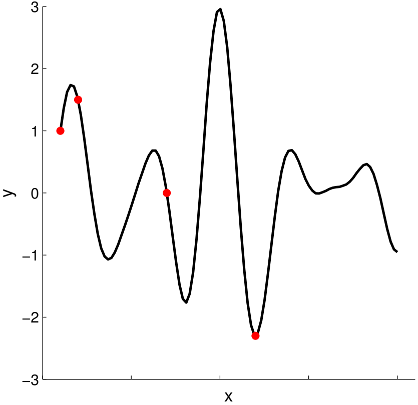
$\Sigma =$ 



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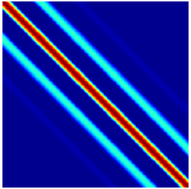

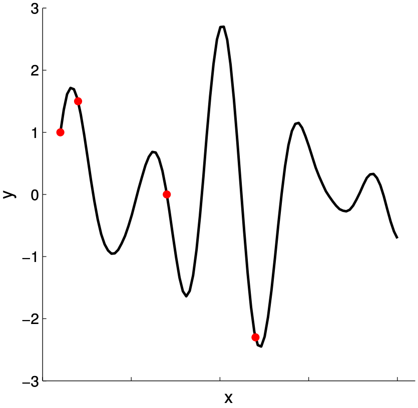

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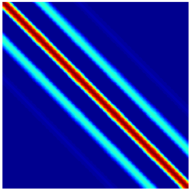
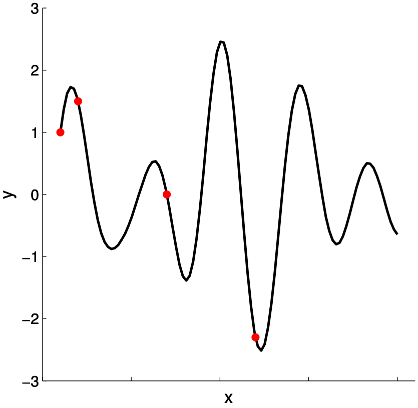
$\Sigma =$ 



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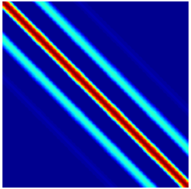
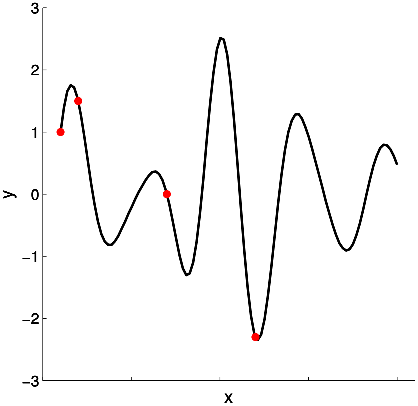
$\Sigma =$ 



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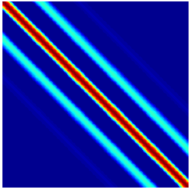
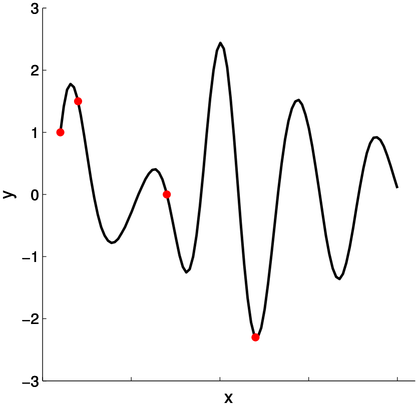
$\Sigma =$ 



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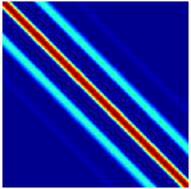
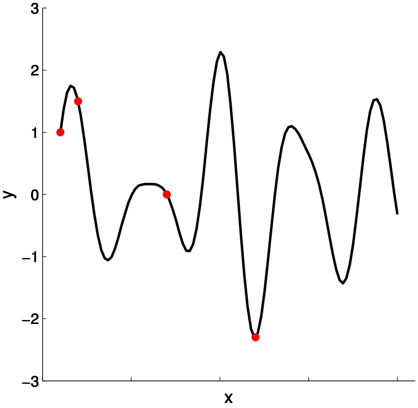
$\Sigma =$ 



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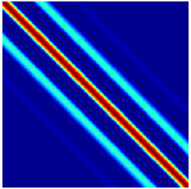
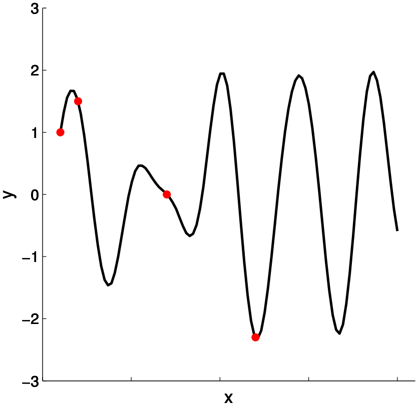
$\Sigma =$ 



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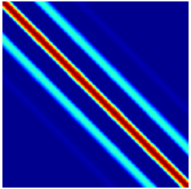

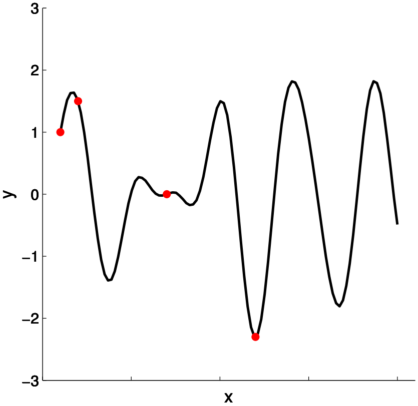

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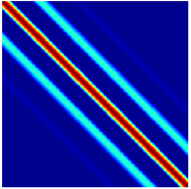
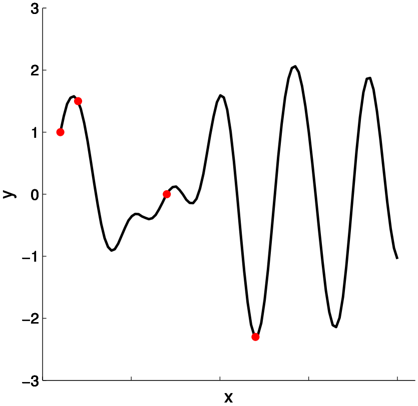
$\Sigma =$ 



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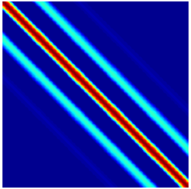
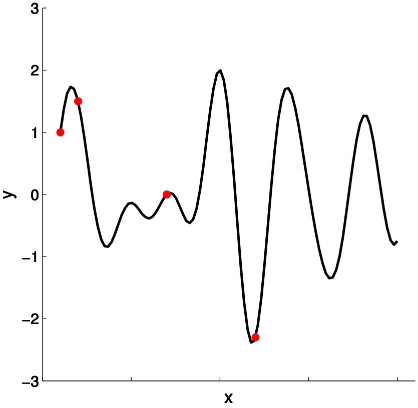
$\Sigma =$ 



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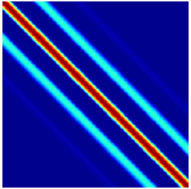
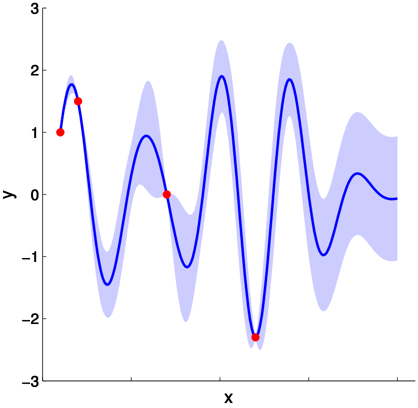
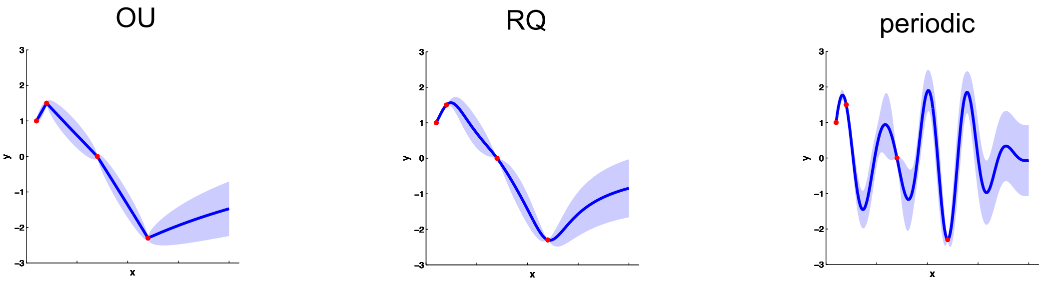
$\Sigma =$ 



The covariance function has a large effect



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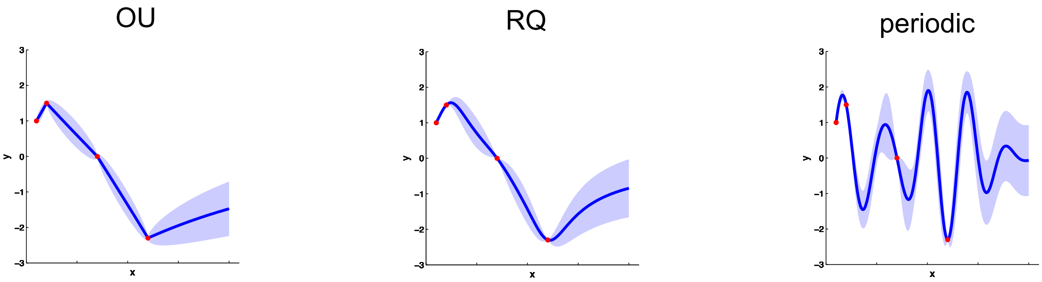

Bayesian model comparison:

$$p(M|\mathbf{y}_{1:N}) = \frac{p(\mathbf{y}_{1:N}|M)p(M)}{\sum_{M'} p(\mathbf{y}_{1:N}|M')p(M')}$$

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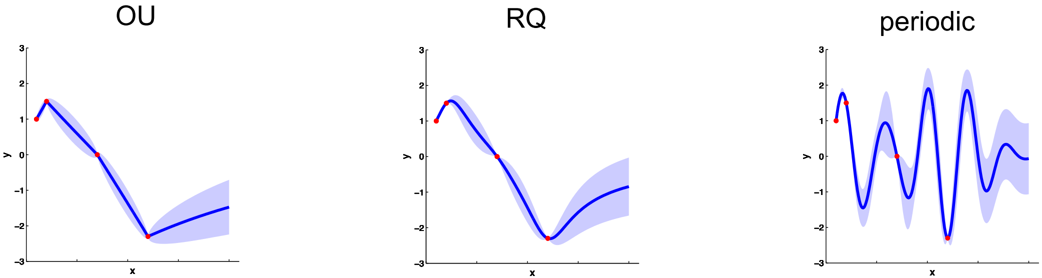

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prior over models

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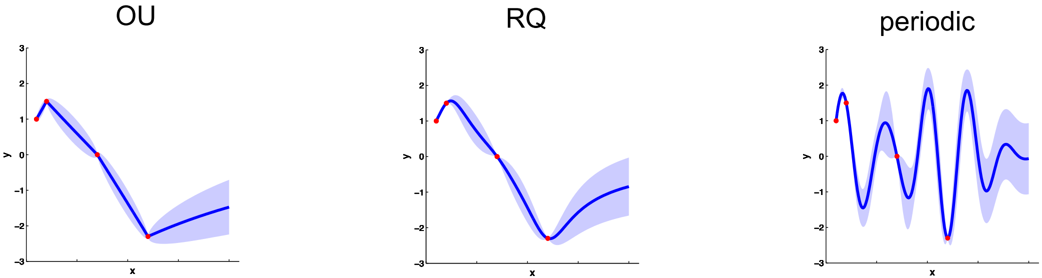

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prior over models

marginal likelihood

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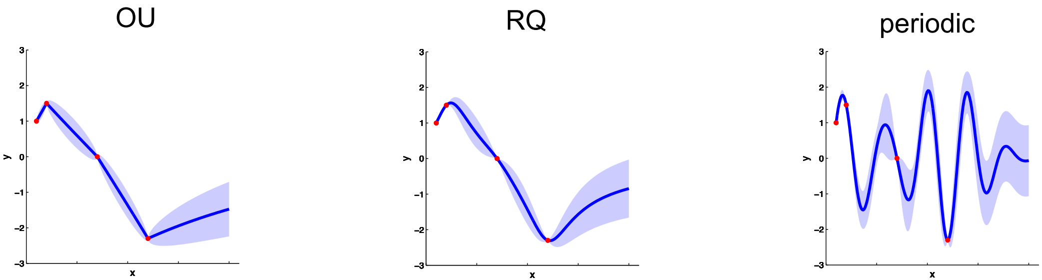

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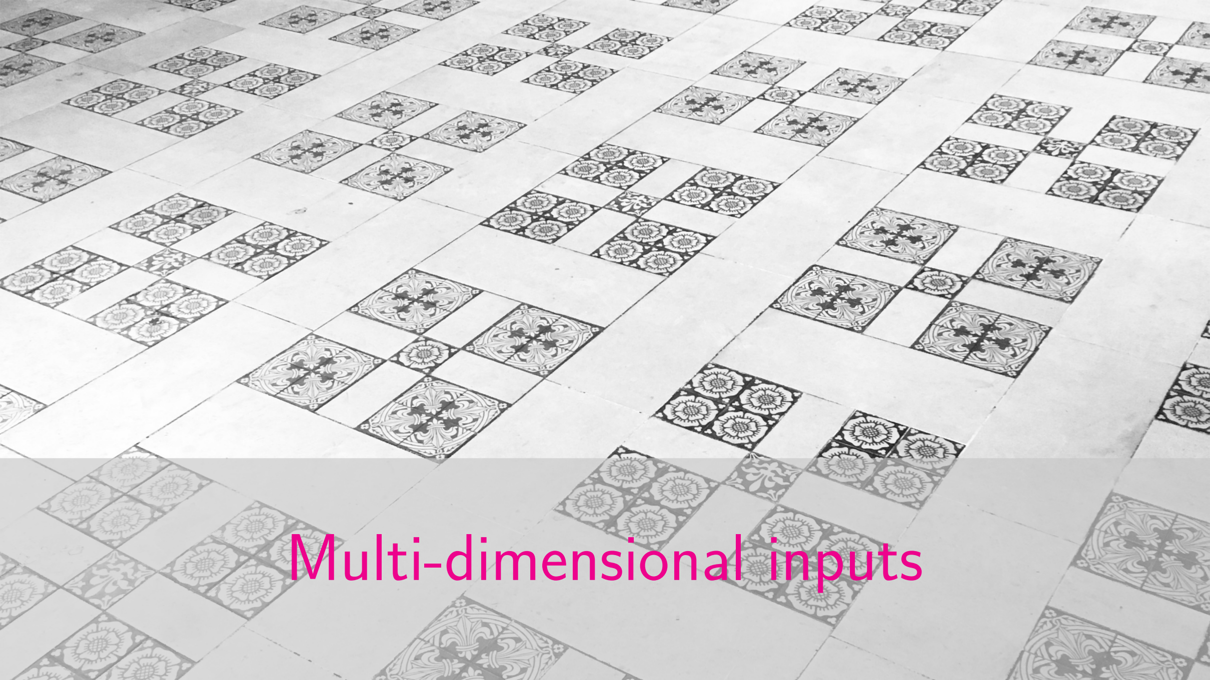
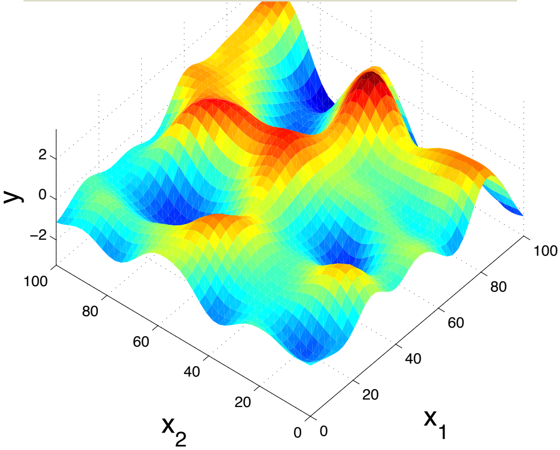
Health warnings:
Hard to compute (need approx.)
Results very sensitive to priors



Multi-dimensional inputs

Higher dimensional input spaces

$$K(\mathbf{x}, \mathbf{x}') = \sigma^2 \exp\left(-\frac{1}{2l_1^2}(\mathsf{x}_1 - \mathsf{x}_1')^2 - \frac{1}{2l_2^2}(\mathsf{x}_2 - \mathsf{x}_2')^2\right)$$



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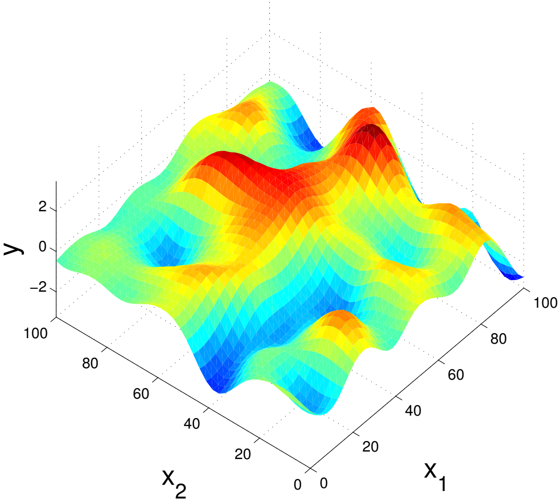

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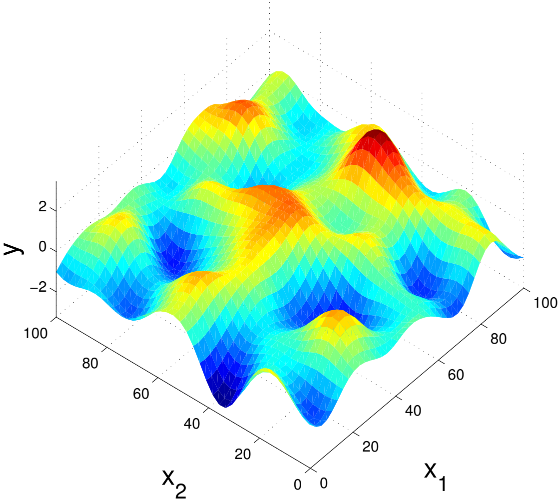

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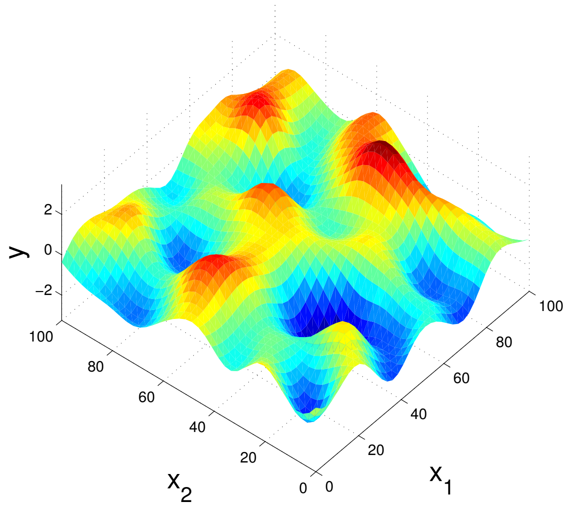

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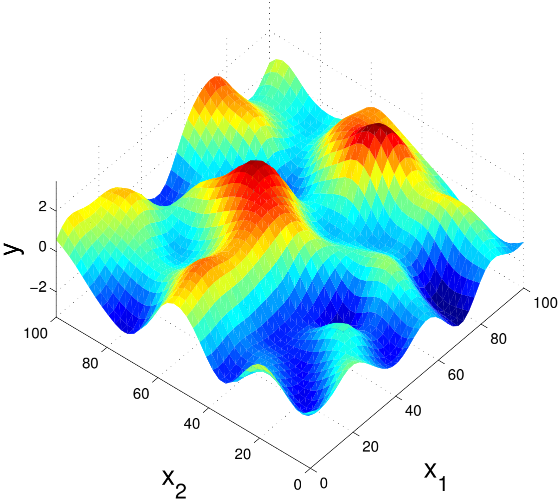

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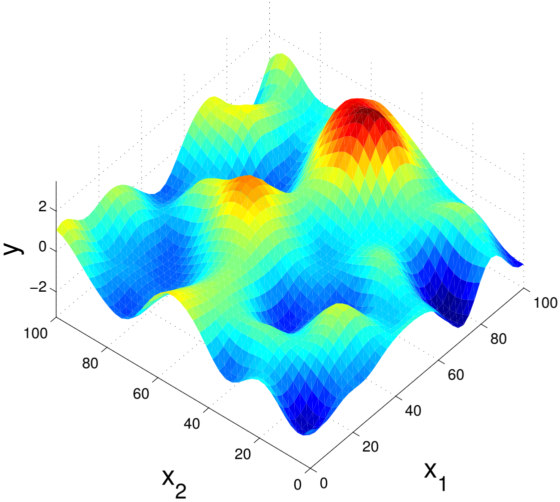

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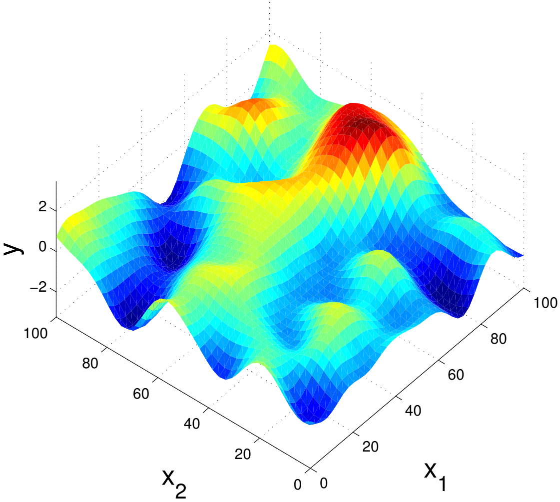

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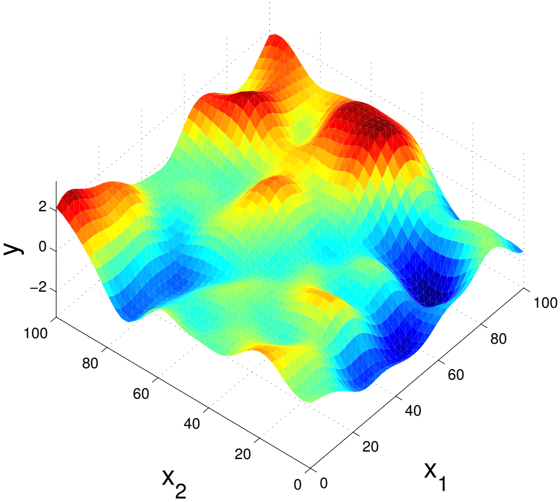

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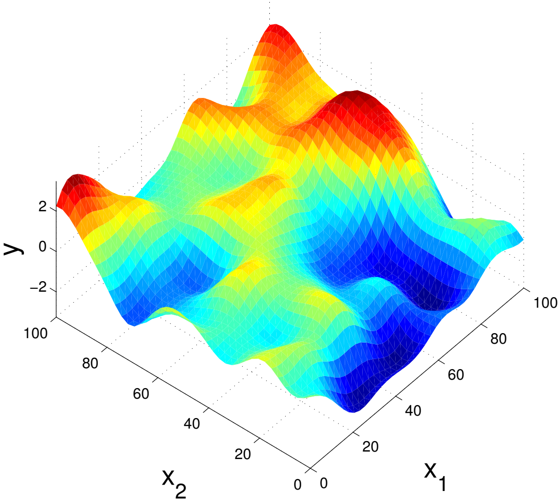

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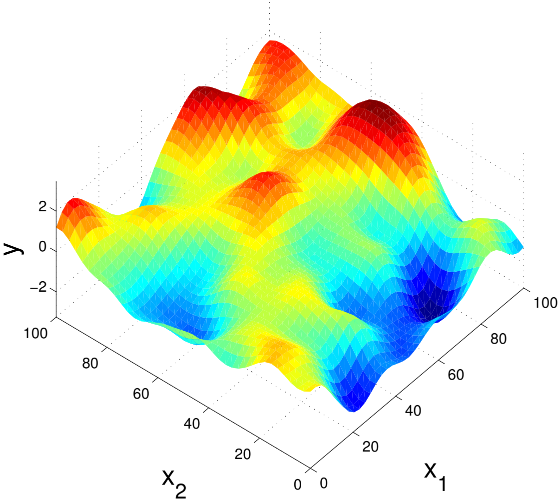

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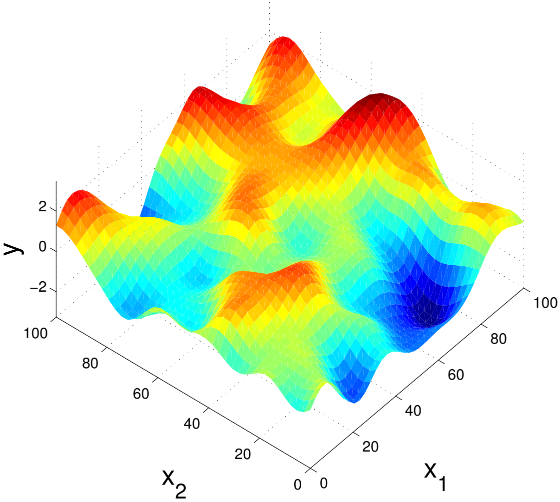

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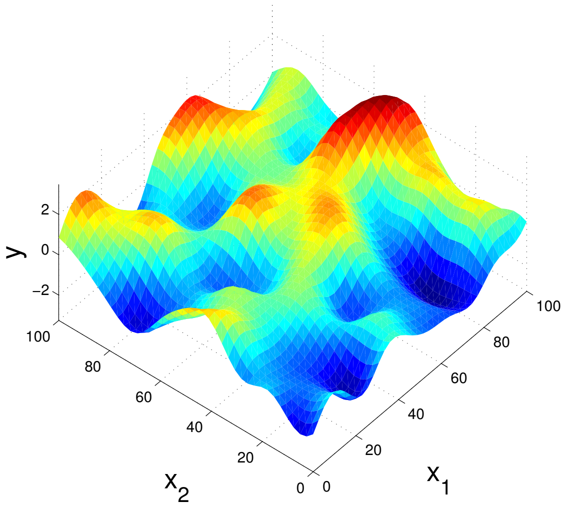

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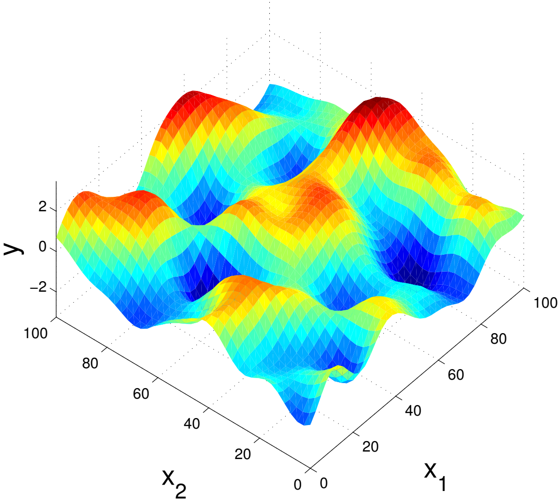
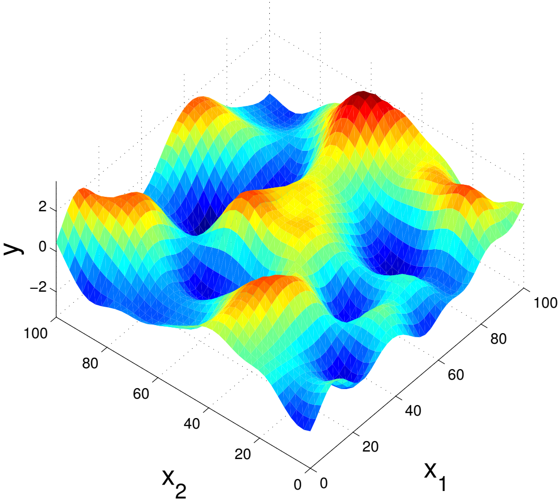
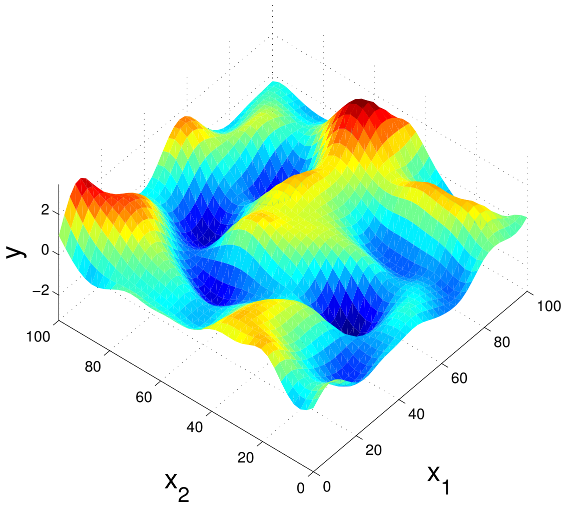


Higher dimensional input spaces



Higher dimensional input spaces



References (hyperlinked)

Great textbook available online:

- Gaussian Processes for Machine Learning, Rasmussen and Williams, 2006

Great Summer and Winter School:

- Gaussian Process Summer School, Neil Lawrence and colleagues

Software:

- GPy: Gaussian Processes in Python
- GPflow: Gaussian Processes and tensorflow
- GPML: Gaussian Processes in Matlab
- GP Stan: Gaussian Processes in probabilistic programming