Tabla 1. Comparación de los tiempos de ejecición para varios algoritmos de ordenamiento para diferentes tamaños de arreglo.

		Tiempo de ejecución (ms)				
	Tamaño del arreglo	**100**	**1000**	**10000**	**100000**	**1000000**
Algoritmo de ordenamiento	**Bubble**	3	37	702	82561	8305478
	Merge	1	4	20	173	805
	Quick	0	2	16	68	524
	Java	1	2	12	79	484

Parte 3

1) $f(n) = 4f(n-2) + 3^n \qquad F(0) = 1 \qquad F(1) = 2$

$f(n) - 4f(n-2) = 3^n$

- $h(n) - 4h(n-2) = 0$

$\lambda^2 - 4\lambda^{2-2} = 0$

$\lambda^2 - 4 = 0$

$\lambda = 2$ ó $\lambda = -2$

$h(n) = c_1 2^n + C_2(-2^n)$

- $p(n) = c_3 \cdot 3^n$

$c_3 3^n - 4c_3 3^{n-2} = 3^n$

$c_3 - \frac{4c_3}{9} = 1$

$9c_3 - 4c_3 = 9$

$c_3 = \frac{9}{5} \rightarrow p(n) = \frac{9}{5} \cdot 3^n$

- $F(n) = C_1 2^n + C_2(-2^n) + \frac{9}{5} 3^n$

$F(0) = C_1 2^0 + C_2(-2^0) + \frac{9}{5} 3^0 = 1$

$C_1 + C_2 = 1 - \frac{9}{5} = -\frac{4}{5} \rightarrow C_2 = \frac{-4}{5} - C_1$

$F(1) = C_1 2^1 + C_2(-2^1) + \frac{9}{5} 3 = 2$

$2C_1 - 2C_2 = \frac{10}{5} - \frac{27}{5} = -\frac{17}{5}$

$2C_1 - 2\left(\frac{-4}{5} - C_1\right) = \frac{-17}{5}$

$2C_1 + \frac{8}{5} + 2C_1 = \frac{-17}{5}$

$4C_1 = -5$

$C_1 = -\frac{5}{4} \qquad C_2 = \frac{-16}{20} + \frac{25}{20} = \frac{9}{20}$

$$\boxed{F(n) = \frac{-5}{4} \cdot 2^n + \frac{9}{20}(-2^n) + \frac{9}{5} \cdot 3^n}$$

2). $f(n) = 4 \cdot f(n-2) + 2^n \qquad f(0) = 1 \qquad f(1) = 3$

- $h(n) - 4h(n-2) = 0$

$\lambda^2 - 4 = 0$

$\lambda = 2$ ó $\lambda = -2$

$h(n) = C_1 \cdot 2^n + C_2(-2^n)$

- $p(n) = C_3 \cdot 2^n \cdot n$

$C_3 \cancel{2^n} n - \cancel{4} C_3 (\cancel{2^{n-2}})(n-2) = \cancel{2^n}$

$\cancel{C_3 \cdot n} - \cancel{C_3 n} + 2C_3 = 1$

$C_3 = \frac{1}{2}$

$p(n) = \frac{1}{2} \cdot n \cdot 2^n$

- $f(n) = C_1 \cdot 2^n + C_2 \cdot (-2^n) + \frac{1}{2} n \cdot 2^n$

$f(0) = C_1 + C_2 = 1$

$f(1) = 2C_1 - 2C_2 + 1 = 3$

$C_1 - C_2 = 1$

$C_1 - (1 - C_1) = 1$

$2C_1 = 2$

$C_1 = 1 \qquad C_2 = 0$

$$\boxed{f(n) = 2^n + \frac{1}{2} n 2^n = 2^n \left(\frac{1}{2} + n\right)}$$

3). $f(n) = 5f(n-1) - 6f(n-2) + n$ $\qquad f(0)=0$

$f(1)=2$

- $h(n) - 5h(n-1) + 6h(n-2) = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$(\lambda-2)(\lambda-3) = 0$

$\lambda = 2$ ó $\lambda = 3$

$h(n) = C_1 2^n + C_2 3^n$

- $p(n) = C_3 n + C_4$

$C_3 n + C_4 - 5(C_3 n - C_3 + C_4) + 6(C_3 n - 2C_3 + C_4) = n$

$C_3 n + C_4 - 5C_3 n + 5C_3 - 5C_4 + 6C_3 n - 12C_3 + 6C_4 = n$

$2C_3 n + 2C_4 - 7C_3 = n$

$2C_3 n - n = 7C_3 - 2C_4$

$n(2C_3 - 1) = 7C_3 - 2C_4$

$2C_3 - 1 = 0 \qquad 7C_3 - 2C_4 = 0$

$C_3 = \frac{1}{2} \qquad C_4 = \frac{7}{4}$

$p(n) = \frac{1}{2}n + \frac{7}{4}$

- $f(n) = C_1 2^n + C_2 3^n + \frac{1}{2}n + \frac{7}{4}$

$f(0) = C_1 + C_2 + \frac{7}{4} = 0$

$C_1 + C_2 = -\frac{7}{4}$

$f(1) = 2 \cdot C_1 + 3 \cdot C_2 + \frac{1}{2} + \frac{7}{4} = 2$

$2C_1 + 3C_2 + \frac{9}{4} = \frac{8}{4}$

$2C_1 + 3C_2 = \frac{-1}{4}$

$2C_1 + 3\left(\frac{-7}{4} - C_1\right) = \frac{-1}{4}$

$2C_1 - \frac{21}{4} - 3C_1 = -\frac{1}{4}$

$-1C_1 = \frac{-1}{4} + \frac{21}{4}$

$-C_1 = 5$

$C_1 = -5 \qquad C_2 = -\frac{7}{4} + \frac{20}{4} = +\frac{13}{4}$

$$\boxed{f(n) = -5 \cdot 2^n + \frac{13}{4} \cdot 3^n + \frac{1}{2}n + \frac{7}{4}}$$

4) $f(n) = 4f(n-1) - 4f(n-2) + 5$ $\quad F(0)=0$, $f(1)=1$

$f(n) - 4f(n-1) + 4f(n-2) = 5$

· $h(n) - 4h(n-1) + 4h(n-2) = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$(\lambda-2)^2 = 0$

$\lambda = 2$ multiplicidad 2

$h(n) = c_1 \cdot 2^n + C_2 n 2^n$

· $p(n) = C_3$

$C_3 - \cancel{4C_3} + \cancel{4C_3} = 5$

$C_3 = 5$

· $f(n) = C_1 2^n + C_2 \cdot n \cdot 2^n + 5$

$f(0) = C_1 + 5 = 0$

$C_1 = -5$

$f(1) = C_1 2^1 + C_2 \cdot 2^1 + 5 = 1$

$C_2 = \frac{-4+10}{2} = 3$

$f(n) = -5 \cdot 2^n + 3n \cdot 2^n + 5$

$$\boxed{f(n) = 2^n \cdot (-5 + 3n) + 5}$$

5). $F(n) = 4F(n-1) - 4F(n-2) + 2^n$ $\quad f(0)=1$, $f(1)=4$

- $h(n) - 4h(n-1) + 4h(n-2) = 0$

  $\lambda^2 - 4\lambda + 4 = 0$

  $\lambda = 2$ multiplicidad 2

  $h(n) = c_1 2^n + c_2 n 2^n$

- $p(n) = c_3 n^2 2^n$ $\quad$ (2^n y $n2^n$ en $h(n)$)

  $c_3 n^2 2^n - 4c_3 (n-1)^2 \cdot 2^{n-1} + 4c_3 (n-2)^2 2^{n-2} = 2^n$

  $c_3 n^2 2^n - \frac{\cancel{4}^{2} c_3 2^n}{\cancel{2}}(n^2-2n+1) + \frac{4c_3 2^n}{4}(n^2-4n+4)$

  $\cancel{c_3 n^2 2^n} - \cancel{2c_3 n^2 2^n} + \cancel{2c_3 \cdot 2n \cdot 2^n} - 2c_3 2^n + \cancel{c_3 n^2 2^n} - \cancel{4c_3 n 2^n} + 4c_3 2^n$

  $2c_3 2^n = 2^n$

  $c_3 = \frac{1}{2}$

- $f(n) = c_1 2^n + c_2 n 2^n + \frac{1}{2} n^2 2^n$

  $f(0) = c_1 + 0 = 1$

  $c_1 = 1$

  $f(1) = 2^1 + 2c_2 + \frac{1}{\cancel{2}}\cancel{2} = 4$

  $c_2 = \frac{1}{2}$

  $f(n) = 2^n + \frac{1}{2} n 2^n + \frac{1}{2} n^2 2^n$

  $$\boxed{f(n) = 2^n\left(\frac{1}{2}n^2 + \frac{1}{2}n + 1\right)}$$

6). $f(n) = 2f\left(\frac{n}{4}\right) + 10 \quad f(1) = 1$

$f(4^m) = 2f\left(\frac{4^m}{4}\right) + 10 \qquad n = 4^m$

$f(4^m) - 2f(4^{m-1}) = 10$

$f(g(m)) - 2f(g(m-1)) = 10$

$\phi(m) - 2\phi(m-1) = 10$

- $h(m) - 2h(m-1) = 0$

$\lambda - 2 = 0$

$\lambda = 2$

$h(m) = c_1 2^m$

- $p(m) = 10c_2$

$10c_2 - 20c_2 = 10$

$c_2 = -1$

$\phi(m) = c_1 2^m - 10 \qquad m = \log_4 n$

$f(n) = c_1\sqrt{n} - 10 \qquad 2^{\lg_4 n} = \sqrt{n}$

$f(1) = c_1\sqrt{1} - 10 = 1$

$c_1 = 11$

$$\boxed{f(n) = 11\sqrt{n} - 10}$$